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By

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TEST PAGE

THESIS ABSTRACT

TODO SOMETHING ABOUT THE ABSTRACT THAT IS ABOUT THIS LONG OR SO

by LOGAN REED

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The topic that I chose to explore for this thesis is a study of the eigenvalues of the Dirichlet Laplacian on a two dimensional domain and ...

Acknowledgment

I would be remiss if I did not take a moment to express appreciation for all of the people who have helped me through this process.

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Introduction

This is the intro.

Chapter 1

Background

1.1 Physical Motivation

Consider a homogeneous elastic drumhead, or membrane, stretched over a rigid frame. We will represent the frame as a domain $\Omega \subset \mathbb{R}^2$. Take the function $u(x, y, t)$ to be the vertical displacement of the membrane from its resting position. Then for any disk $D \subset \Omega$, Newton's second law of motion states that

$$\int_{\partial D} T \frac{\partial u}{\partial \mathbf{n}} \, dS = \int_D \rho u_{tt} \, dA$$

where T is the constant tension, ρ is the density constant, and $\mathbf{n}$ is the outward normal of the boundary. By the divergence theorem, we have

$$\int_D T \Delta u \, dA = \int_D \rho u_{tt} \, dA$$

where Δ is the Laplace operator. From this we can get the wave equation on Ω

$$u_{tt} = c^2 \Delta u$$

where we define u to be 0 on the boundary and where $c = \sqrt{T/\rho}$. We can solve this wave equation using $u(x, y, t) = T(t)V(x, y)$ which gives us

$$\frac{T''}{c^2 T} = \frac{\Delta V}{V} = -\lambda$$

and finally we have reduced our problem to the Dirichlet Laplacian

$$\Delta V = -\lambda V$$

where V on the boundary is zero.

NOTE: The best reference I could find is Logan's Applied Partial Differential Equations. I could also use Ryans paper

In the next section, we will start from the Dirichlet Laplacian and introduce

the conjectures in a formal setting.

1.2 Polya-Szego's Conjecture

1. Introduce Rigorous Definitions from 1.1.2 Henrot

2. Dirichlet Laplacian eigenvalues prereqs

3. Faber Krahn

4. Polya-Szego Conjecture [1]

1.3 Known Results

1. All Explicit Cases

2. Tools for n=3 and n=4

1.4 Numerical Analysis Tools

Chapter 2

Hearing the shape of a triangle

2.1 Section One

Bibliography

[1] G. Polya and G. Szego. *Isoperimetric inequalities in mathematical physics*. Princeton University Press, 1951.