Contents

Chapter 1

General Background

1.1 Notation

1. Notation
2. That isn't obvious
3. Goes here

Definition 1.1.1.

Theorem 1.1.2 (First derivative of a Dirichlet Eigenvalue)**.** *Let Ω be a bounded open set. We assume that $\lambda_k'(\Omega)$ is simple. Then, the functions $t \to \lambda_k(t), t \to u_t \in L^2(\mathbb{R}^N)$ are differentiable at $t = 0$ with*

$$\lambda_k'(0) := -\int_\Omega \operatorname{div}(|\nabla u|^2 V)\,\mathrm{d}x.$$

If, moreover, Ω is of class C^2 or if Ω is convex, then

$$\lambda_k'(0) := -\int_\Omega \left(\frac{\partial u}{\partial n}\right)^2 V.n\,\mathrm{d}\sigma$$

and the derivative u' of u_t is the solution of

$$\begin{cases} -\Delta u' = \lambda_k u' + \lambda_k' u & \text{in}\Omega \\ u' = -\frac{\partial u}{\partial n} V.n & \text{on}\partial\Omega \\ \int_\Omega u u'\,\mathrm{d}\sigma = 0. \end{cases}$$