

# 1 Definitions

Let  $m$  = base of the counter

$MSR$  = most significant digit region

$c_0$  = starting value of counter

$d = \lceil \log_m(c_0) \rceil$ , number of digits per row

$\mathcal{DR} = \left\lceil \frac{d}{3} \right\rceil$ , digit regions per row

$c_f = m^d - 1$ , final value of the counter

$c_\Delta = c_f - c_0$ , number of rows

$l = \lceil \log m \rceil + 2$ , bits needed to represent each digit plus 2 for MSR and MSD

$\mathcal{DR}_{height} = 3 \cdot (l + 30)$ , height of a row

$h = c_\Delta \cdot \mathcal{DR}_{height}$ , height of constuction without roof tiles

In order to determine the number of digits per row, given some value  $k$ , let  $R = k \bmod 6$ . There are 3 distinct cases to consider.

- if  $0 \leq R \leq 1$ , then  $\{ d \in \mathbb{N} \mid 3 \text{ divides } d \}$   
MSR  $\leftarrow$  3 digits
- if  $2 \leq R \leq 3$ , then  $\{ d \in \mathbb{N} \mid 3 \text{ divides } d - 1 \}$   
MSR  $\leftarrow$  1 digit
- if  $4 \leq R \leq 5$ , then  $\{ d \in \mathbb{N} \mid 3 \text{ divides } d - 2 \}$   
MSR  $\leftarrow$  2 digits

In order to account for odd values of  $k$ , when  $k$  is odd, one vertical filler tile is added to increase the width of the rectangle. O(1)

$$\mathcal{DR}_{1st} = 3l + 90 + 1$$

$$\mathcal{DR}_{general} = 3l + 90$$

$$\mathcal{DR}_{last} = 3l + 75$$

Let  $n$  be the number of counter unit rows, then

$$\begin{aligned} h &= (n - 1) \cdot \mathcal{DR}_{general} + \mathcal{DR}_{1st} + \mathcal{DR}_{last} \\ &= (m^d - 1)(3l + 90) + 6l + 166 \end{aligned}$$

$$m = \left\lceil \left( \frac{N}{93} \right)^{\frac{1}{d}} \right\rceil$$

$$\begin{aligned} N &= 93 \left( \frac{N}{93} \right) = 93 \left( \left( \frac{N}{93} \right)^{\frac{1}{d}} \right)^d \leq 93 \left\lceil \left( \frac{N}{93} \right)^{\frac{1}{d}} \right\rceil^d \\ &= 93(m^d - 1) \leq 3lm^d + 90m^d \leq 3lm^d + 90m^d + 3l + 91 \\ &= (m^d - 1)(3l + 90) + 6l + 166 \end{aligned}$$