3R Robot

Other name - Three rotation joint Robot

DOF - 3

Denavit hartenberg (DH) Parameter

T_i = Minimum number of Parameters to describe forward kinematics

$= Rot_{z,\theta_i}, Trans_{z,d_i}, Trans_{x,a_i}$
Rot_{x,α_i}

$$= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where,

a_i - link length
α_i - link twist
d_i - link offset
θ_i - Joint angle

Final DH matrix

$$= \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics

Joint 1

$\theta = 90^\circ$ $a_i = 0.3m$ $\alpha_i = 0$ $d_i = 0$

$$T_1 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here,

$\cos 90 = 0$ $\cos 0 = 1$
$\sin 90 = 1$ $\sin 0 = 0$

Joint 2

$\theta = 90^\circ \quad a_i = 0.25m \quad \alpha_i = 0 \quad d_i = 0$

$$T_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0.25 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint 3

$\theta = 90^\circ \quad a_i = 0.1m \quad \alpha_i = 0 \quad d_i = 0$

$$T_3 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = T_1 \, T_2 \, T_3$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0.25 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 & -0.25 \\ 0 & -1 & 0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

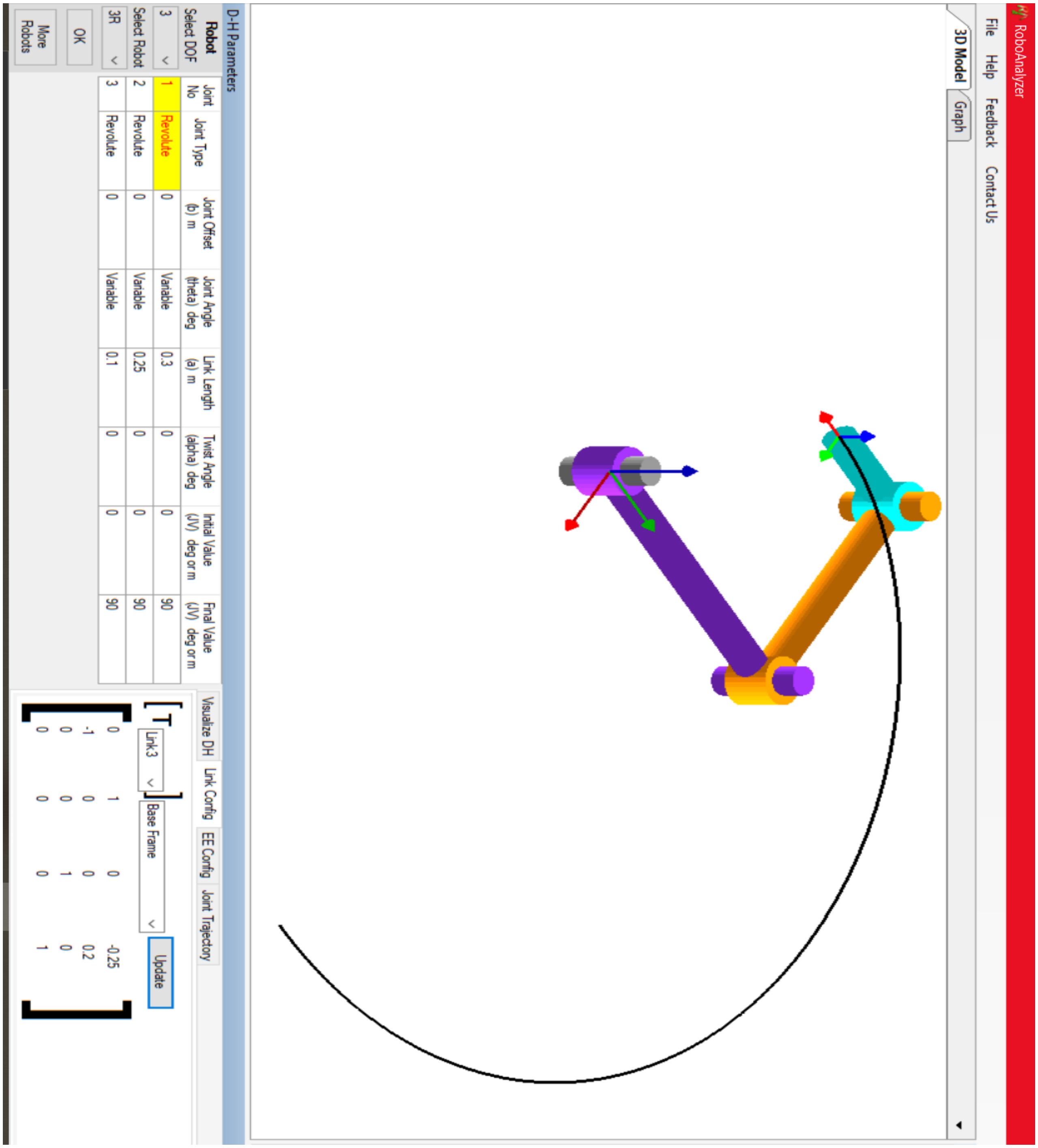
$$= \begin{bmatrix} 0 & 1 & 0 & -0.25 \\ -1 & 0 & 0 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics of 3R robot after set on desired position from origin point

$$= \begin{bmatrix} 0 & 1 & 0 & -0.25 \\ -1 & 0 & 0 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - (2)$$

∴ Thus the above matrix is verified based on "RoboAnalyser" software.

VERIFICATION



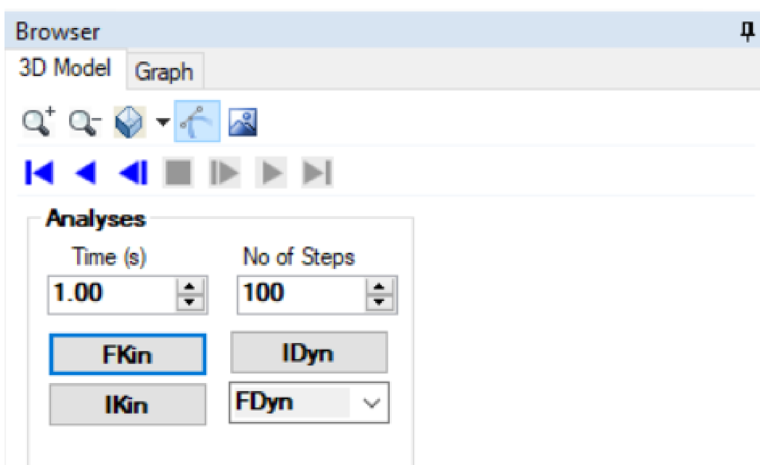
PROCEDURE VIDEO FOR ROBOANALYZER [DEMO VIDEO]

PROCEDURE

1. Open the RoboAnalyzer software.

2. Select **3** on the **select DOF** dropdown list and also select **3P (3 PRISMATIC JOINT ROBOT)** on the **select robot** dropdown list.

3. Press **FKin** button on the 3D model tab to simulate forward kinematics matrices for the final (or) desired position from origin position (or) point.



4. Then click Link Config tab to select **link 3** and change previous frame to **base frame**. Eventually press the **Update** button to get the **DH matrix** for 3P Robot.

5. Finally, I compared the software DH matrix with an observed DH matrix that I solved.

Inverse kinematics

Joint 1

$\theta = \theta_1$ (variable) $\quad a_i = 0.3\,m \quad \alpha_i = 0 \quad d_i = 0$

$$A_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0.3\,C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & 0.3\,S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint 2

$\theta = \theta_2$ (variable) $\quad a_i = 0.25\,m \quad \alpha_i = 0 \quad d_i = 0$

$$A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0.25\,C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & 0.25\,S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint 3

$\theta = \theta_3$ (variable) $\quad a_i = 0.1\,m \quad \alpha_i = 0 \quad d_i = 0$

$$A_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0.1\,C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & 0.1\,S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_1 A_2 A_3$$

$$= \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0.3C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & 0.3S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0.25C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & 0.25C\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0.1C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & 0.1S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad — (1)$$

Sub $\theta = 180^\circ$ in eqn (1)

$\cos 180^\circ = -1$

$\sin 180^\circ = 0$.

$$= \begin{bmatrix} -1 & 0 & 0 & -0.3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & -0.25 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & -0.1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & -0.1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} -1 & 0 & 0 & -0.15 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

An above matrix does not match with equation 2.

So,

Sub $\theta = 270°$ in eqn (1)

$\cos 270° = 0$

$\sin 270° = -1$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -0.25 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 & -0.25 \\ 0 & -1 & 0 & -0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & -0.25 \\ 1 & 0 & 0 & -0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

An above matrix partially match with eauation 2.

so,

Sub, $\theta = 90^\circ$ in eqn (1)

Then you get a matrix

$$= \begin{bmatrix} 0 & +1 & 0 & -0.25 \\ -1 & 0 & 0 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ "$\theta = 90^\circ$" is match with eqn (2).