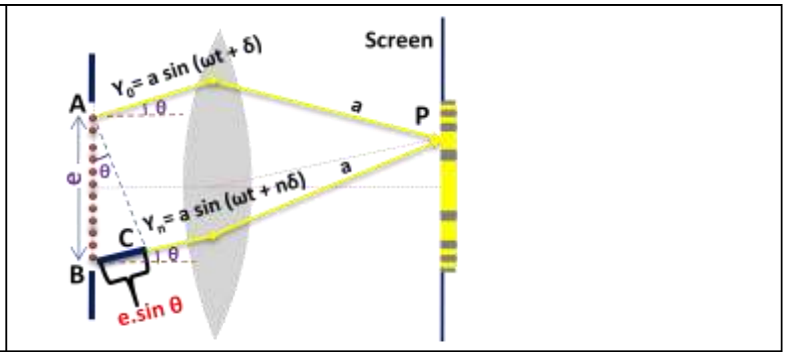
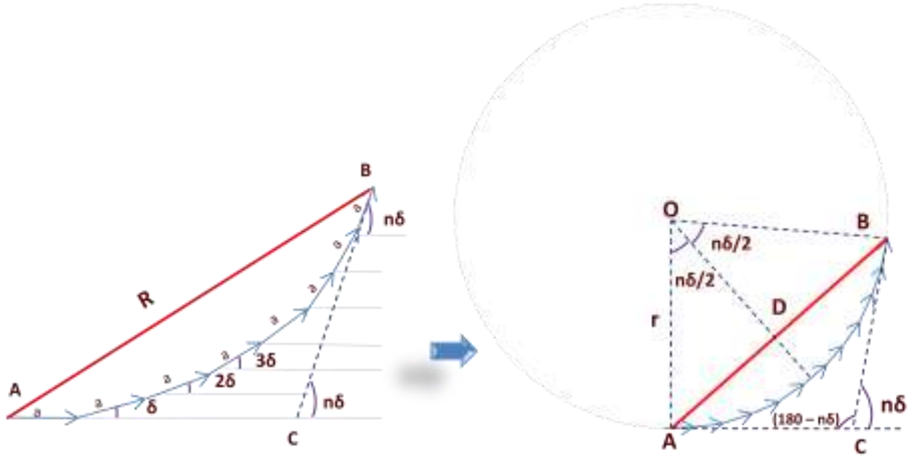
Article:

To ***derive the resultant amplitude (R)*** due to n secondary waves of amplitude (a) that reach at point P on the screen.

If the resultant amplitude (R) is known then we can find the intensity at point P **[Intensity α (resultant amplitude)^2].**





To find the **resultant amplitude (R)** due to all the amplitudes (a) reaching at point P on the screen, we construct a polygon of each arm **a** (amplitude). AB is the resultant amplitude (R).

From fig. AB =2 x AD and AD = r Sin nδ/2

Therefore, AB = 2r Sin nδ/2 ...... [1]

Again from fig. nδ = na/r (since angle=arc/radius and total arc length = na)

Or r = na/ nδ ...... [2]

[2] into [1] –

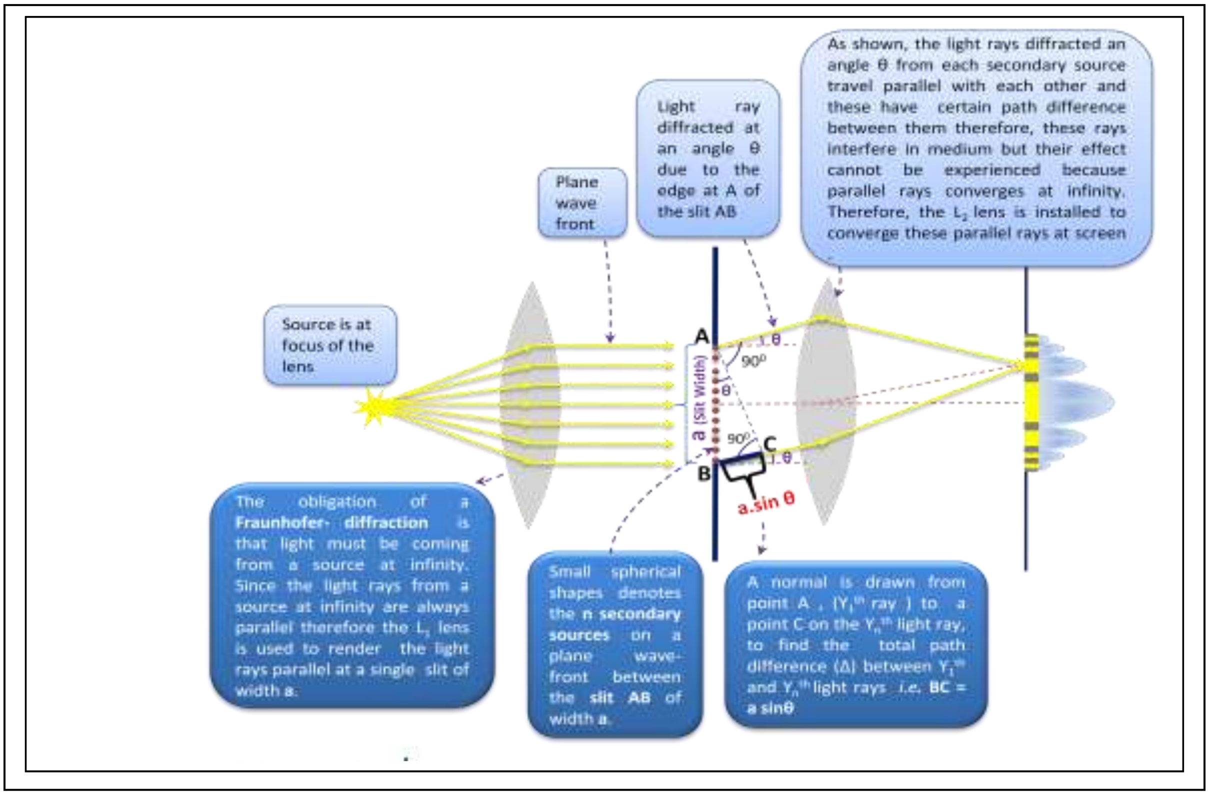
$$AB = \frac{2\,na\;\;\sin\frac{\mathrm{n\delta}}{2}}{\mathrm{n\delta}}$$

$$\text{Or } \mathbf{AB} = \frac{\mathbf{na\;\;sin}\frac{\mathbf{n\delta}}{\mathbf{2}}}{\frac{\mathbf{n\delta}}{\mathbf{2}}}$$

$$\text{let } \mathbf{na} = \mathbf{A}\ \ \&\ \ \boldsymbol{\alpha} = \frac{\mathbf{n\delta}}{\mathbf{2}}$$

$$\boxed{\boldsymbol{R} = \frac{\boldsymbol{A}\ \mathbf{sin}\,\boldsymbol{\alpha}}{\boldsymbol{\alpha}}}$$

Q.1 Derive the intensity and position of central maxima, minima, and secondary maximum in <u>FRAUNHOFER DIFFRACTION at Single Slit:</u>



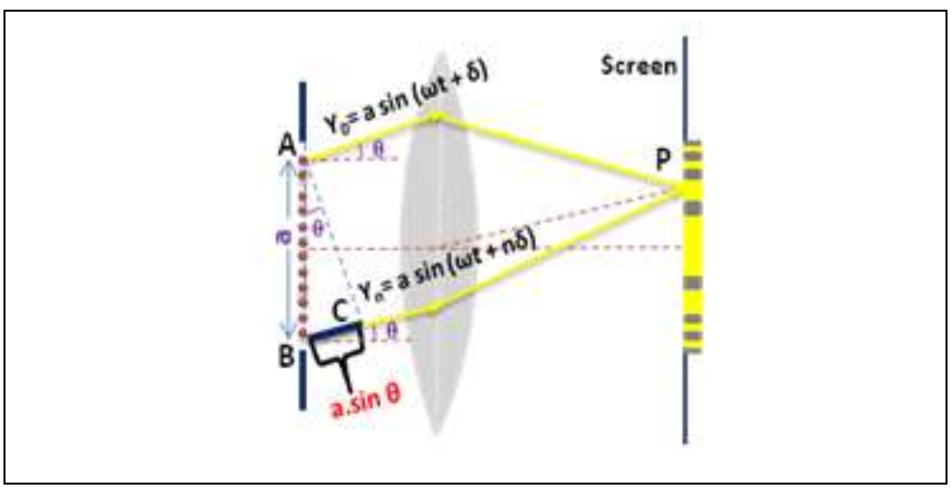
From Fig. nδ is the total phase difference between Y_0 and Y_n wave and the corresponding path difference Δ =BC= **e** sin θ. Then from relation ϕ = 2π/λ. Δ

$$\boxed{n\delta = \frac{2\pi}{\lambda} e \sin\theta}$$

$$\frac{n\delta}{2} = \frac{\pi}{\lambda} e \sin\theta$$

$$\text{let } \alpha = \frac{n\delta}{2}$$

$$\boxed{\alpha = \frac{\pi}{\lambda} e \sin\theta}$$



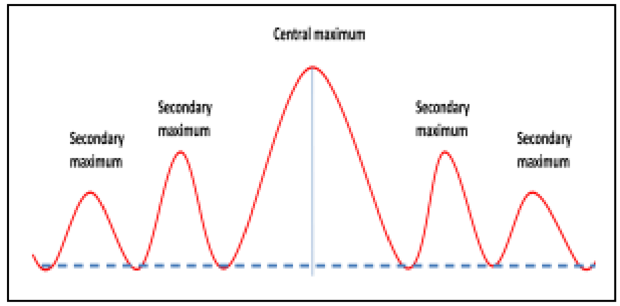
As we know that resultant amplitude (R) due to all the amplitudes (a) reaching at point P on the screen is given by:

$$\left(R = \frac{na \ \sin\frac{n\delta}{2}}{\frac{n\delta}{2}} \quad \text{Let na } = \text{A (a significant no. which represents an amplitude)and } \alpha = \frac{n\delta}{2} \quad , Already\ derived \right)$$

$$\boxed{\boldsymbol{R = \frac{A\ \sin\alpha}{\alpha}}}$$

Therefore, the intensity at point P is proportional to square of amplitude (R) i.e.,

$$\mathbf{I = R^2 = \frac{A^2\ (\sin\alpha)^2}{\alpha^2}}$$



Now the explanation of intensity curve:

Central Maxima:

When α tends to zero then R becomes maximum.

$$R_{max} = A \lim_{\alpha \to \infty} \left[\frac{\sin\alpha}{\alpha}\right] \qquad \left[since \lim_{\alpha \to \infty} \left[\frac{\sin\alpha}{\alpha}\right] = 1\right]$$

$$R_{max} = A$$

Or Intensity at central maximum **I =** $\boldsymbol{R_{max}^2 = A^2}$

Minima:

R becomes minimum when sin α tends to zero but α should not tend to zero.

$or\ R_{min} \rightarrow \sin\alpha = 0$ or $\alpha = \pm n\pi$

$$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \dots \dots \pm n\pi$$

Since $\alpha = \frac{\pi}{\lambda}$ e sinθ

Therefore $\frac{\cancel{\pi}}{\lambda}$ e sinθ $= \pm n\cancel{\pi}$

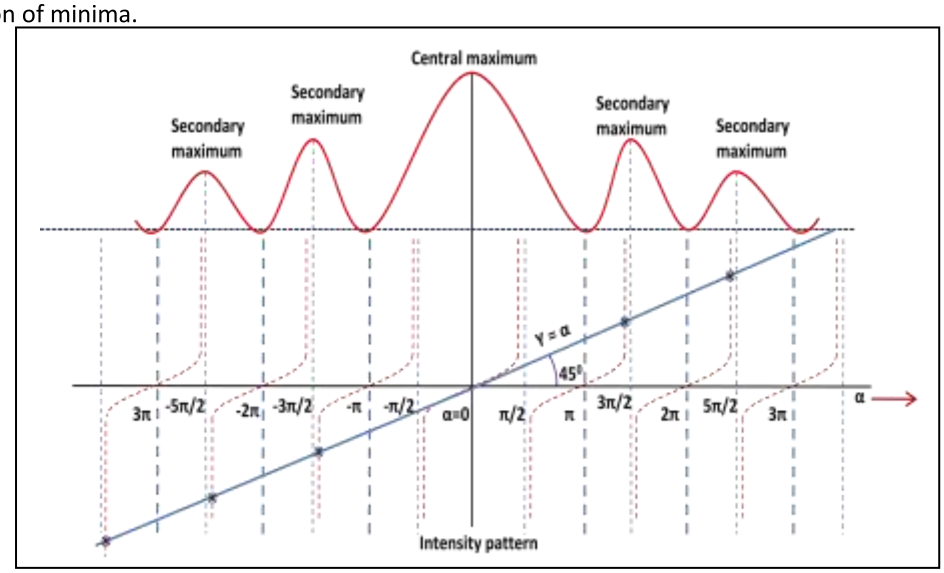
$$\boxed{\mathbf{e\ sin\theta} = \pm \boldsymbol{n\lambda}}$$

This is the expression to find the position of minima.

Secondary Maxima:

Since $\mathrm{I = R^2 = A^2}\left[\frac{\sin\alpha}{\alpha}\right]^2$

Therefore $\frac{dI}{d\alpha} = 0$ will define each point on this curve.



$$\frac{dI}{d\alpha} \rightarrow \frac{d(R^2)}{d\alpha} = A^2 \frac{d}{d\alpha}\left[\frac{\sin\alpha}{\alpha}\right]^2 = 0$$

$$A^2\ 2\frac{\sin\alpha}{\alpha}\frac{d}{d\alpha}\left[\frac{\sin\alpha}{\alpha}\right] = 0$$

$$A^2\ 2\frac{\sin\alpha}{\alpha}\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

Here $A^2 \neq 0$

$\alpha = 0$ is already used to defined the central maxima

$\sin\alpha = 0$ is already used to defined the minima.

Therfore $\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$ will define the positon of secondary maxima.

Thus $\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$

$\alpha\cos\alpha - \sin\alpha = 0$

$$\boxed{\boldsymbol{\alpha = \tan\alpha}}$$

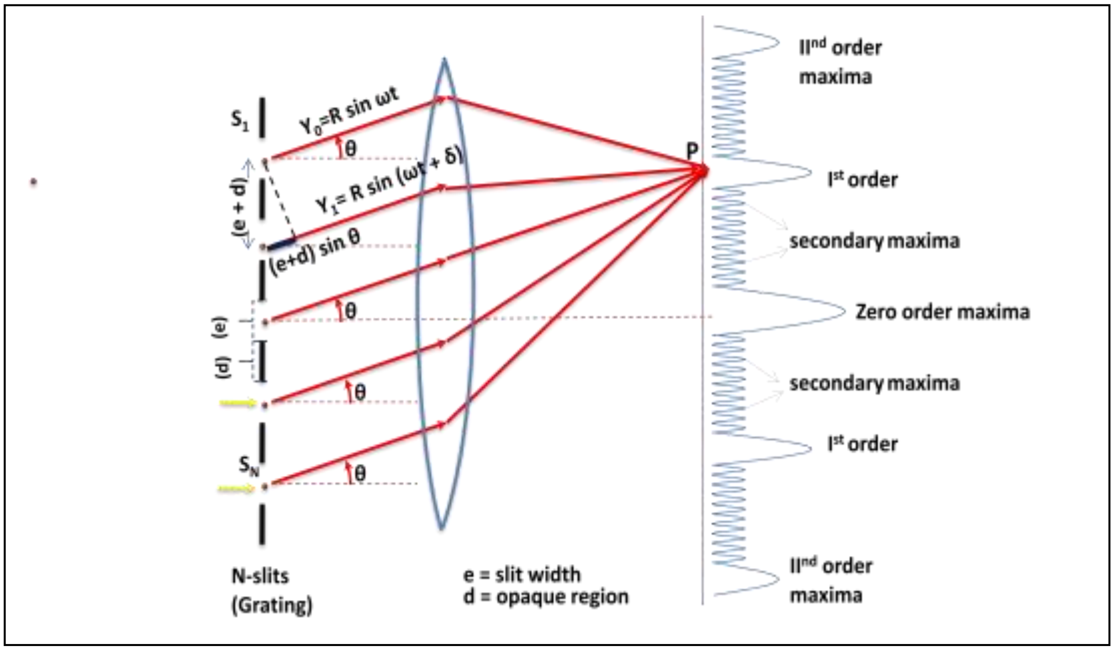
Here the value of α which satisfies above relation, gives the values of position of secondary maxima. To find the position of secondary maxima we use a graphical method of simultaneous equations. For this we draw y = α and y= tan α simultaneously and find their intersecting points except intersecting point at α = 0 (because α = 0 corresponds to the position of central maximum) other intersecting points gives the position of secondary maxima.

As we find that other intersecting points of y = α and y= tan α are at

α = ±3π/2, ±5π/2, ±7π/2 ….. ± (2n+1) π/2 except α = 0.

(Position of secondary maxima is shown in above Figure)

Q.2 Derive the intensity and position of central maxima, minima and secondary maxima.



N-slits (Transmission Grating): From Figure. δ is the total phase difference between Y_0 and Y_1 wave and the corresponding path difference Δ =BC= **(e+d) sin θ**. Then from relation ϕ = (2π/λ) Δ. Therefore,

$$\boldsymbol{\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta}$$

Or $$\boxed{\boldsymbol{\frac{\delta}{2} = \frac{\pi}{\lambda}(e+d)\sin\theta}}$$

Let **δ/2= β**

$$\boxed{\boldsymbol{\beta = \frac{\pi}{\lambda}(e+d)\sin\theta} \qquad \textbf{...... [1]}}$$

As we know the resultant amplitude reaching at P is given as

$$\boxed{R = \frac{na\ \sin\frac{n\delta}{2}}{\frac{n\delta}{2}}}$$

Comparing it with above N-slits figure, we find that

Amplitude (**a**) = **R**, **n**(hypothetical secondary sources) =**N** (N physical sources) and **δ/2= β.** Therefore, the resultant amplitude (R') due to N slits at Point P is

$$\boxed{R' = \frac{R \ \sin N\beta}{\sin \beta}}$$

Principle maximum: For R' to be maximum, sinβ→0. When β=0 then we get an indeterminate form. Therefore, we apply **L-Hospital rule**

$$R'_{max} = R \lim_{\beta \to n\pi} \left[\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin \beta)} \right]$$

$$R'_{max} = R \lim_{\beta \to n\pi} \left[\frac{N\cos N\beta}{\cos \beta} \right]$$

$R'_{max} = RN$ Or $I(\text{principle maximum}) = (R'_{max})^2 = R^2N^2$

Since $\beta = \pm n\pi$ and $\boldsymbol{\beta} = \frac{\pi}{\lambda}\,(\mathbf{e}+\mathbf{d})\,\mathbf{sin\theta}$

$\frac{\cancel{\pi}}{\lambda}\,(e+d)\sin\theta = \pm n\cancel{\pi}$

$\mathbf{(e+d)sin\theta = \pm n\lambda}$

This is expression to find the principal maxima.

Minima: For R' to be minimum, sinNβ→0

or $\sin N\beta = 0$

or $N\beta = \pm m\pi$

$N\frac{\cancel{\pi}}{\lambda}\,(e+d)\sin\theta = \pm m\cancel{\pi}$

$\mathbf{N(e+d)sin\theta = \pm m\lambda}$

Here, if we put m=nN or m=0, N, 2N, 3N.......nN then above expression reduces to the expression of maxima. Therefore, to define minima **m ≠ nN** then it can have values between two consecutive principal maxima. Say zero order and first principal maxima (0-N) then are 1,2,3,.......N-1minima.

Secondary Maxima:

since $I = R'^2 = R^2 \left[\frac{\sin N\beta}{\sin\beta}\right]^2$

Therefore $\frac{dI}{d\beta} = 0$ will define each point on this curve.

$$\frac{dI}{d\beta} \to \frac{d(R'^2)}{d\beta} = R^2 \frac{d}{d\beta}\left[\frac{\sin N\beta}{\sin\beta}\right]^2 = 0$$

$$R^2\ 2\frac{\sin N\beta}{\sin\beta}\frac{d}{d\beta}\left[\frac{\sin N\beta}{\sin\beta}\right] = 0$$

$$R^2\ 2\frac{\sin N\beta}{\sin\beta}\left[\frac{N\cos N\beta\sin\beta - \cos\beta\sin N\beta}{\sin\beta^2}\right] = 0$$

Here $R^2 \neq 0$

$\sin\beta = 0$ is already used to defined the principal maxima

$\sin N\beta = 0$ is already used to defined the minima.

$$\left[\frac{N\cos N\beta - \cos\beta \sin N\beta}{\sin\beta^2}\right] = 0$$

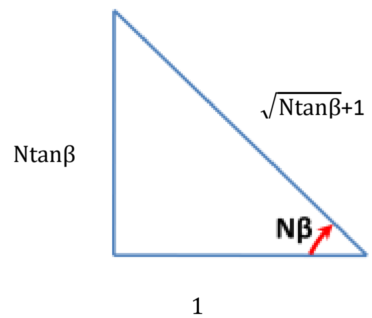
$$N\cos N\beta \sin\beta - \cos\beta \sin N\beta = 0$$

$$N\cos N\beta \sin\beta = \cos\beta \sin N\beta$$

$$\boxed{N\tan\beta = \tan N\beta}$$

Above expression provides the condition for secondary maxima

Therefore, for the requirement of $\sin^2 N\beta$ in expression of Intensity of secondary maxima the $\sin^2 N\beta$ is obtained from expression $N\tan\beta = \tan N\beta$ .....by drawing a right triangle of angle $N\beta$.



Therefore, from Figure.... $\sin^2 N\beta = \frac{(N\tan\beta)^2}{\left(\sqrt{N\tan\beta+1}\right)^2} \rightarrow \frac{N^2\tan\beta^2}{N^2\tan\beta^2+1}$

or the term $\dfrac{\sin^2 N\beta}{\sin^2\beta} = \dfrac{N^2\tan\beta^2}{\left(N^2\tan\beta^2+1\right)\sin^2\beta}$

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\cancel{\tan\beta^2}}{\cancel{\tan\beta^2}\left(N^2 + \dfrac{1}{\tan\beta^2}\right)\sin^2\beta}$$

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{\left(N^2\sin^2\beta + \dfrac{\sin^2\beta}{\tan\beta^2}\right)} \rightarrow \frac{N^2}{\left(N^2\sin^2\beta + \cos^2\beta\right)} \rightarrow \frac{N^2}{\left[N^2\sin^2\beta + 1 - \sin^2\beta\right]} \rightarrow \frac{N^2}{\left[1 + \sin^2\beta\left(N^2 - 1\right)\right]}$$

$$I_{\text{Secondary maximum}} = R'^2 = \frac{R^2 \sin^2 N\beta}{\sin^2\beta} \rightarrow \frac{R^2 N^2}{\left[1 + \sin^2\beta\left(N^2-1\right)\right]} \rightarrow \frac{\mathbf{I_{0(Principal\ maximum}}}{\left[1 + \sin^2\beta\left(N^2-1\right)\right]}$$

$$\boxed{I_{\text{Secondary maximum}} = \frac{\mathbf{I_{0(Principal\ maximum)}}}{[1 + \sin^2\beta\,(N^2 - 1)]}}$$

It means, if N (no. of lines on a grating) increases the intensity of secondary maxima decreases. Practically, say a grating of 20,000 *lpi* has less blur region between two successive maxima (between two successive colored spectral lines) than a 15,000 *lpi* grating.

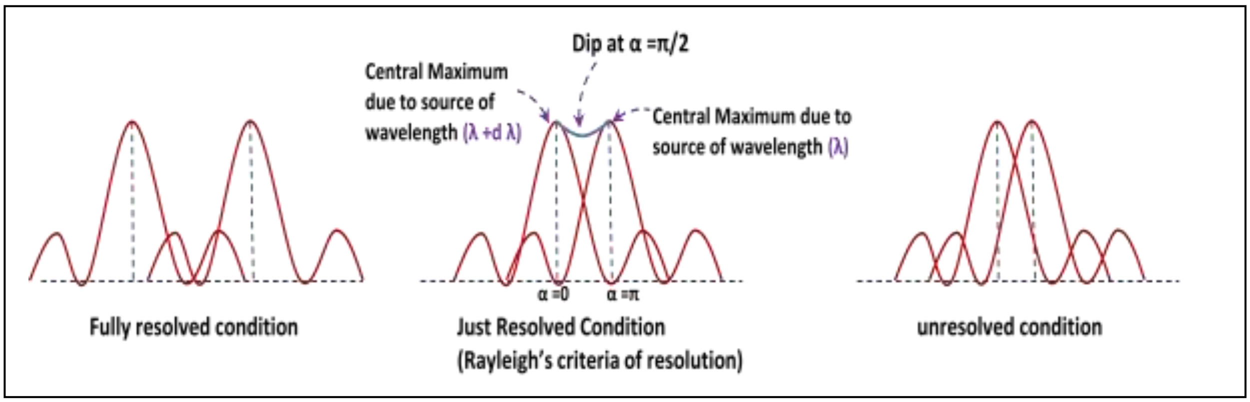
Q. Define the Rayleigh criterion of Resolution and its Application.

Or

Find intensity at mid in Rayleigh criterion of Resolution (i.e., for just resolved condition).

Rayleigh criterion of Resolution:

This criterion is applicable for the evaluation of resolving powers of **telescope, microscope, grating** and **prism** etc.



According to Raleigh, the two point sources or equally intense spectral lines [say sodium light D_1 (λ)=5890 A^0 and $D_2(\lambda + d\lambda)$ =5896 A^0] are just resolved by an optical instrument, i.e. when the central maximum of diffraction pattern due to one source (λ), falls exactly on the first minimum of diffraction pattern of the other (λ+d λ) and vice versa.

The Intensity at the Dip (mid) (for just resolved condition)

$\mathbf{I_{mid} = 2I_{\alpha=\pi/2}}$ (where $I_{\alpha=\pi/2}$ is the intensity due to one spectral line at α=π/2)

(Here I_{mid} is twice of $I_{\alpha=\pi/2}$ because the two intensity are reaching at position π/2, i.e., one is due to source λ and another due to λ+dλ.)

$$I_{mid} = 2.\frac{A^2\left(\sin\frac{\pi}{2}\right)^2}{\left(\frac{\pi}{2}\right)^2} \qquad \text{(since intensity due to sigle slit is given by } I = \frac{A^2\,(\sin\alpha)^2}{\alpha^2}\text{)}$$

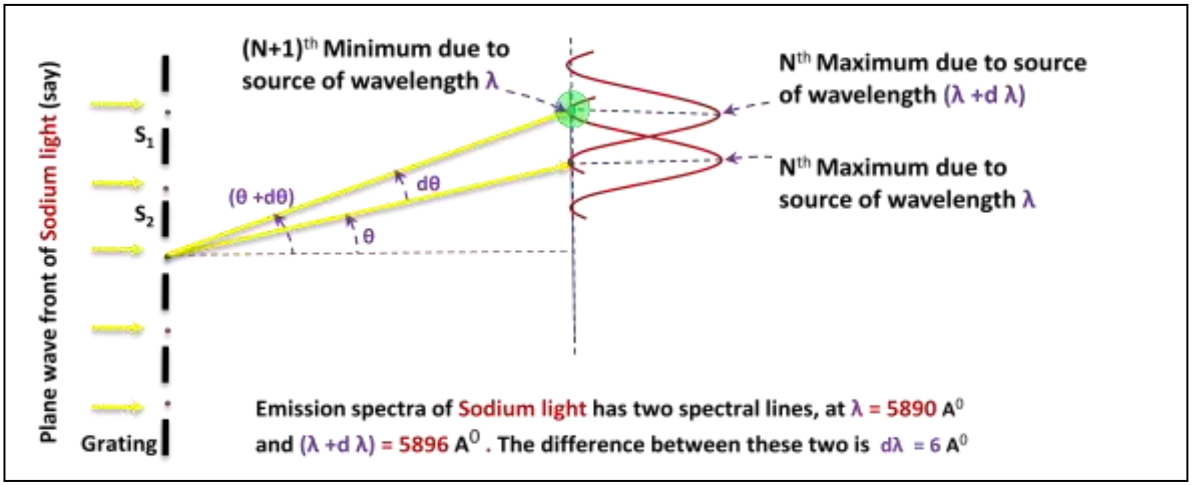
$$I_{mid} = 8\frac{A^2}{\pi^2} \rightarrow \frac{8I_0}{\pi^2}$$

or

$$\boxed{\frac{I_{mid}}{I_0} = \frac{8}{\pi^2}}$$

Q. E n of resolving power of a Grating.

Resolving Power of a Grating:



According to Raleigh, the two equally intense spectral lines can be resolved if central maximum of diffraction pattern due to one spectral line fall on minimum of diffraction pattern due to another (spectral line). *(Concentrate at encircled point in above figure only, there you find).*

Maximum due to spectral line (λ+dλ):

$(e + d)\sin(\theta + d\theta) = \pm n(\lambda + d\lambda)$ .... [1]

And minimum due to spectral line (λ)

$N(e + d)\sin(\theta + d\theta) = \pm(nN + 1)\lambda$

$N(e + d)\sin(\theta + d\theta) = nN\lambda + \lambda$ .... [2]

Multiplying eq. [1] by N

$N(e + d)\sin(\theta + d\theta) = \pm nN(\lambda + d\lambda)$ (Since expression for minimum is $N(e + d)\sin\theta = \pm m\lambda$)

or $N(e + d)\sin(\theta + d\theta) = nN\lambda + nNd\lambda$ .... [3]

Eq. [2] = [3]

$\cancel{nN\lambda} + \lambda = \cancel{nN\lambda} + nNd\lambda$

$$\boxed{\frac{\lambda}{d\lambda} = nN}$$

λ/dλ is defined as **resolving power of a Grating.** It depends upon the order (n) and no of lines on a grating (N).

Q. Discuss the Dispersive power of a Grating.

<u>Dispersive power of Grating:</u>

Since maximum in a grating is given by

$$(e + d)\sin\theta = \pm n\lambda$$

Or $(e + d)\cos\theta \frac{d\theta}{d\lambda} = n$

$$\boldsymbol{\frac{d\theta}{d\lambda} = \frac{n}{(e + d)\cos\theta}}$$

Here dθ/dλ is defined as Dispersive power of a grating. This depends upon (e+d) grating element. And (e+d) = 2.54/N cm or we can say if N (lines per inch in a grating) increases then (e+d) decreases. Thus, dispersive power (dθ/dλ) of a grating increase.