MATHS

IInd Sem

Topic
Expt No. Date Page No.

UNIT - 1 Ordinary Differential Equation

An equation which involves differential coefficient is called a differential equation.

eg:- $\frac{dy}{dx} = \frac{1-x^2}{1+y^2}$, $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

Order and degree of differential equation

The order of a differential eq^n is the order of the highest differential coefficient present in the eq^n

The degree of a differential eq^n is the degree of the highest derivative after removing the radical sign & fraction.

- $a_0\left(\frac{d^2y}{dx^2}\right)^1 + a_1\left(\frac{dy}{dx}\right)^3 + a_2y = F(x)$

here a_0, a_1, a_2 are Constant.

order = 2, degree = 1

- $\left(\frac{d^3y}{dx^3}\right)^1 + 4\left(\frac{d^2y}{dx^2}\right)^2 + 5y = 0$

order = 3, degree = 1

Teacher's Signature

Q Find degree & order of following differential eq^n

- $$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

order = 2
degree = 2

- $$\left(1+\frac{dy}{dx}\right)^{1/3} = \left(\frac{d^2y}{dx^2}\right)^2$$

order = 2
degree = 6

Q ~~Find of~~

- $$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$$

Sq both side

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

order = 2, degree = 2

Differential Eq^n of First Order & First degree

Order = 1, degree = 1

Here, the standard ~~methods~~ types of ~~solving~~ the differential eq^n of order = 1, degree = 1 are as follows

1. eq^n solvable by separation of variables
2. Homogeneous eq^n
3. Linear eq^n of 1st order
4. Exact differential eq^n ← Syllabus

Topic

Expt No. Date Page No.

① Variable Separable

if a differential eqⁿ can be written in the form

$$f(y)\,dy = \phi(x)\,dx$$

we say that variables are separable

Q $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Sol

$$\frac{1}{\sqrt{1-y^2}}\cdot dy = \frac{1}{\sqrt{1-x^2}}\cdot dx$$

integrating both side

$$\int \frac{1}{\sqrt{1-y^2}}\cdot dy = \int \frac{1}{\sqrt{1-x^2}}\cdot dx$$

$$\sin^{-1}y \cdot \cancel{dy} = \sin^{-1}x \cdot \cancel{dx} + C$$

Q $(x+1)\frac{dy}{dx} = x(y^2+1)$

$$\frac{1}{1+y^2}\cdot dy = \frac{x}{1+x}\cdot dx$$

$$\int \frac{1}{1+y^2}\cdot dy = \int \frac{x}{1+x}\cdot dx$$

$$\tan^{-1}y \cdot dy = \cancel{x}\, x - \log(1+x) + C$$

$\frac{1}{1+x}$ $(+1) - 1$

$\frac{x}{x+1}$

$\frac{x+1}{x+1} - \frac{1}{x+1}$

$1+x = t$

$dx = dt$

$\frac{t-1}{t}$

$\frac{1+x}{x}$

$t - \log t$

$(1+x) - \log(1+x)$

Teacher's Signature

$\frac{x+1-1}{x+1}$

Q $\sec^2 x \tan y \, dx + \sec^2 y \tan x \cdot dy = 0$

$\sec^2 x \tan y \, dx$

$\sec^2 y \tan x \, dy = -\sec^2 x \tan y \cdot dx$

$\frac{1}{\cos^2 y} \times \frac{\cos y}{\sin y}$

$\frac{\sec^2 y}{\tan y} \cdot dy = -\frac{\sec^2 x}{\tan x} \cdot dx$

$\cos y \sin y$

$\boxed{\begin{array}{l}\tan x = t \\ \sec^2 x \cdot dx = dt\end{array}}$

$\int \frac{1}{t} dt = -\int \frac{1}{z} \cdot dz$

$\log \tan y = -\log \tan x + \log c$

$\log \tan y + \log \tan x = \log c$

$\log \tan x \tan y = \log c$

$$\boxed{\tan x \tan y = c}$$

② Linear Differential eq^n

$\boxed{\frac{dy}{dx} + Py = Q}$ is linear Diff. eq^n if P & Q are func. of x, The solution of Diff eq^n is

$$\boxed{y(I.f) = \int Q(I.f) \cdot dx + C}$$

Topic
Expt No. Date Page No.

☆ $\boxed{I.f. = e^{\int P \cdot dx}}$

Q $\frac{dy}{dx} + \frac{1}{x} y = x^3 - 3$ — (A)

$P = \frac{1}{x}$

$Q = x^3 - 3$

Sol

$I.f = e^{\int \frac{1}{x} \cdot dx}$

$= e^{\log_e x} = x$

sol. of differentinal eqn (A) is

$y \cdot x = \int (x^3 - 3) x \cdot dx + c$

$= \int x^4 - 3 \int x$

$xy = \frac{x^5}{5} - \frac{3x^2}{2} + c$

Q Solve, $(2y - 3x) dx + x\, dy = 0$

W.R.t Dividing by x

$\frac{2y - 3x}{x} dx + dy = 0$

$\frac{dy}{dx} + \frac{2}{x} y - 3 = 0$

$\boxed{I.f = e^{\int \frac{2}{x} dx} = x^2}$

$\frac{dy}{dx} + \frac{2}{x} y = 3$

sol. of Diff eqn is

$y \cdot x^2 = \int 3 \cdot x^2 \cdot dx$ ~~+c~~

$y \cdot x^2 = x^3 + C$

$y = x + \frac{C}{x^2}$

Teacher's Signature

Syllabus Start

(4) # $\boxed{\text{Exact diff eq}^n}$ (E.D)

A differential eq^n of the form :-

$\boxed{Mdx + Ndy = 0}$; (A) where $M = f(x, y)$
$N = g(x, y)$

we say that W.S.T Differential eq^n (A) is E.D eq^n if,

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

and Solution of Diff. eq. (A) can be written as

$$\int M\,dx + \int N\,dy = C$$

$\int M\,dx$ ↓ Treating y as Constant

$\int N\,dy$ ↓ Do not consider those terms which contains x treat them as 0.

Q find value of λ for :- $\underbrace{(xy^2 + \lambda x^3 y^2)}_{M} dx + \underbrace{(x^3y + yx)x}_{N} dy = 0$ is exact.

M dx + N dy will be exact if

Sol

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$\frac{\partial}{\partial y}(xy^2 + \lambda x^3 y^2) = \frac{\partial}{\partial x}(x^3y + yx)\cdot x$$

$$2xy + 2\lambda x^3 y = x\cdot(3x^2y + y) + (x^3y + yx)$$

Topic

Expt No. Date Page No.

$2x + 2\lambda x^3 = 3x^3 + x + x^3 + x$

$2\lambda x^3 + 2x = 4x^3 + 2x$

$\lambda x^2 + 1 = 2x^2 + 1$

$\lambda x^2 = 2x^2$ if $x \neq 0$ & x is constant

$\boxed{\lambda = 2}$

Q Solve the following Differential eqⁿ

$$\underbrace{(x+y-10)}_{M}dx + \underbrace{(x-y-2)}_{N}dy = 0 \quad \text{--- (1)}$$

Check eqⁿ (A) is given in the form of

$$Mdx + Ndy = 0$$

$M = x+y-10$

$N = x-y-2$

$\frac{\partial M}{\partial y} = 1, \quad \frac{\partial N}{\partial y} = 1 \;;$ i.e. $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}}$

which means Diff eqⁿ (1) is exact Diff. eqⁿ

Now, solution of Diff. eqⁿ (1) is

$$\int M dx + \int N dy = C$$

$$\int (x+y-10)\,dx + \int (x-y-2)\,dy = C$$

$$\frac{x^2}{2} + yx - 10x - \frac{y^2}{2} - 2y = C$$

Teacher's Signature

Rule 1: if,

$$\boxed{\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x) \text{ only}, \text{ then } \underbrace{I.f = e^{\int f(x)dx}}}$$

Q $(2x \log x - xy)dy + 2y dx = 0$ ——— (1)

First check if it is exact or not.

T.G.D.E.I.T.F.O.

$Mdx + Ndy = 0$

Here $N = 2x\log x - xy$

$M = 2y$

Now, $\frac{\partial M}{\partial y} = 2$, $\frac{\partial M}{\partial x} = 2\left[\frac{x}{x} + \log x\right] - y$

i.e.: $\frac{\partial M}{\partial y} \neq \frac{\partial M}{\partial x}$, means eq^n is not exact

Now,

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2 - 2(1+\log x) + y}{2x\log x - xy}$$

$$= \frac{-2\log x + y}{2 \cdot x\log x - xy}$$

$$= -\frac{[2\log x - y]}{x[2\log x - y]}$$

$$= \frac{-1}{x} = f(x)$$

Topic ______ Date ______ Page No. ______
Expt No. ______

$$\int (y^2 - x^2)\,dx + \int 2xy\,dy = C$$

$$y^2x - \frac{x^3}{3} + 0 = C \text{ Ans}$$

Q $\underbrace{(y\sec^2 x + \sec x \tan x)}_{M}dx + \underbrace{(\tan x + 2y)}_{N}dy = 0$

$$\frac{\partial M}{\partial y} = \sec^2 x$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial N}{\partial x} = \sec^2 x$$

Now,

$$\underset{y \text{ constant}}{\int (y\sec^2 x + \sec x \tan x)\,dx} + \underset{x \to 0}{\int (\tan x + 2y)\,dy} = C$$

$$y\tan x + \sec x + y^2 = C \text{ Ans}$$

Conversion to exact/Reducible

Sometimes a differential eqn which is not exact may become exact on multiplication by a suitable funˢ. known as integrating factor

Teacher's Signature ______

Q Solve $\underbrace{(e^y+1)\cos x}_{m}\,dx + \underbrace{e^y \sin x}_{N}\,dy = 0$ — ①

Diffⁿ eqⁿ is in the form of

$$\frac{\partial m}{\partial x} = \frac{\partial N}{\partial y}$$

$$\frac{\partial m}{\partial y} = -(e^y+1)\sin x \quad e^y \cos x$$

$$\frac{\partial m}{\partial x} = e^y \sin x \quad e^y \cos x$$

i.e. $\boxed{\frac{\partial m}{\partial y} = \frac{\partial N}{\partial x}}$

$$\int m\,dx + \int N\,dy = c$$

$$\int (e^y+1)\cos x\,dx + \int e^y \sin x\,dy = c$$

zero

$e^y \sin x +$

$(e^y+1)\sin x = c$

Q $(y^2-x^2)\,dx + 2xy\,dy = 0$

$m \cdot dx + N\,dy$

$$\left.\begin{aligned}\frac{\partial m}{\partial y} &= 2y \\ \frac{\partial N}{\partial x} &= 2y\end{aligned}\right\}$$ equal means exact ✓

Topic
Expt No. Date Page No.

$M = \frac{y}{x^2} - 2x \qquad \frac{\partial M}{\partial y} = \frac{1}{x^2}$

$N = y - \frac{1}{x} \qquad \frac{\partial N}{\partial x} = +\frac{1}{x^2}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ meas eq (11) is exact

Solution is

$\int \left(\frac{y}{x^2} - 2x\right) dx + \int \left(y - \frac{1}{x}\right) dy = c$

$\Rightarrow -\frac{y}{x} - x^2 + \frac{y^2}{2} = c$ Ans

Q $(x \sec^2 y - x^2 \cos y) dy = (\tan y - 3x^4) dx$

$\underbrace{(\tan y - 3x^4)}_{M} dx + \underbrace{(x^2 \cos y - x \sec^2 y)}_{N} dy = 0$ —(1)

$\frac{\partial M}{\partial y} = \sec^2 y \quad , \frac{\partial N}{\partial x} = 2x \cos y - \sec^2 y$

$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$

Now Applying Rule 1

Teacher's Signature

$m\,dx + N\,dy = 0$

Q $\underbrace{(y-2x^3)}_{M}dx - \underbrace{x(1-xy)}_{N}dy = 0$ —— ①

$-x + x^2y$

$\frac{\partial M}{\partial y} = 1 \qquad \frac{\partial N}{\partial x} = -1 + 2xy$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ i.e ① is not exact

Now, Applying Rule ①

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{1+1-2xy}{-x+x^2y}$$

$$= \frac{2-2xy}{x^2y-x} \Rightarrow \frac{2(1-xy)}{-x(1-xy)}$$

$$\cancel{I.f} = \frac{2}{-x} = f(x)$$

multiplying I.f with eq ① we get.

$\cancel{-\frac{2}{x}(y-2x^3)dx - \left[-\frac{2}{x}\{x-x^2y\}\right.}$

$$I.f = e^{\int -\frac{2}{x}dx} = e^{-2\log x} = \frac{1}{x^2}$$

multiplying I.f with eq① we get

$\left(\frac{y}{x^2} - 2x\right)dx + \left(-\frac{1}{x} + \frac{y}{\cancel{x}}\right)dy$ —— ⑪

Topic ______
Expt No. ______ Date ______ Page No. ______

i.e. $I.F = e^{\int -\frac{1}{x} dx} = e^{-\log x}$

$= \boxed{\frac{1}{x}}$

Now multiply $1/x$ with eqⁿ ① & ~~we~~ it will become exact.

multiplying Diff eqⁿ ① by I.F., we get

$\frac{1}{x}\left\{[2x\log x - xy]dy + 2y\,dx\right\} = 0$

$(2\log x - y)dy + 2\frac{y}{x}.dx = 0$ —— ⑪

Here.

$M = 2\frac{y}{x}$; $N = 2\log x - y$

Now, $\frac{\partial M}{\partial y} = \frac{2}{x}$ & $\frac{\partial N}{\partial x} = \frac{2}{x}$

i.e. $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ means Diff eqⁿ ⑪ is exact

and sol, is

$\int 2\log x - y \cdot dy + \int 2\frac{y}{x} \cdot dx = c$

$x \to 0$ y ~~constant~~

$2yx^{-1}$

$-2y$

$-\frac{y^2}{2} \cancel{+0} + 2y\log x = c$

$2y\log x - \frac{y^2}{2} = c$ Ans

Teacher's Signature ______

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{\sec^2 y - 2x\cos y + \sec^2 y}{x^2\cos y - x\sec^2 y} = 0$$

$$= \frac{2\sec^2 y - 2x\cos y}{x^2\cos y - x\sec^2 y} = 0$$

$$\Rightarrow \frac{2(\sec^2 y - x\cos y)}{-x(\sec^2 y - x\cos y)} = 0$$

$$\Rightarrow -\frac{2}{x} = f(x)$$

$$I.f = e^{\int -\frac{2}{x}} \Rightarrow \frac{1}{x^2}$$

multiply eq ①

$$\left(\frac{\sec^2 y}{x} - \cancel{x\cos y}\right)d$$

$$\underbrace{\left(\frac{\tan y}{x^2} - 3x^2\right)}_{M} dx + \underbrace{\left(\cos y - \frac{\sec^2 y}{x}\right)}_{N} dy = 0 \quad \text{(11)}$$

$$\frac{\partial M}{\partial y} = \frac{\sec^2 y}{x^2}, \quad \frac{\partial N}{\partial x} = \frac{\sec^2 y}{x^2}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$ means eq (11) is exact

Topic
Expt No. Date Page No.

solution is

$$\int\left(\frac{\tan y}{x^2} - 3x^2\right)\cdot dx + \int\left(\cos y - \frac{\sec^2 y}{x}\right)\cdot dy = c$$

$$-\frac{\tan y}{x} - x^3 + \sin y = c$$

$$\boxed{\begin{array}{l}\text{Rule 2: if,}\\ \dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = f(y)\\ \text{then } I.f = e^{\int f(y)\,dy}\end{array}}$$

Q solve $(y^4+2y)dx + (xy^3+2y^4-4x)dy = 0$ — (1)

Sol $Mdx + Ndy = 0$

$\Rightarrow M = y^4 + 2y$
$N = xy^3 + 2y^4 - 4x$

$\frac{\partial M}{\partial y} = 4y^3 + 2$; $\frac{\partial N}{\partial x} = y^3 - 4$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ i.e. Hence not exact

∴ applying Rule 2:

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{y^3-4-4y^3-2}{y^4+2y} \Rightarrow \frac{-3y^3-6}{y^4+2y}$$

Teacher's Signature

$$= -\frac{3}{y} = f(y)$$

$I.f = \cancel{f(\cdot)}^3 \quad e^{\int -3/y} \quad e^{\int -3\log y} \Rightarrow \frac{1}{y^3}$

$I.f \times eq ①$

$$\frac{1}{y^3}(y^4+2y)dx + \frac{1}{y^3}(xy^3+2y^4-4x)\,dy$$

$$\underbrace{\left(y+\frac{2}{y^2}\right)}_{m}dx + \underbrace{\left(x+2y-\frac{4x}{y^3}\right)}_{N}dy$$

$$\left.\begin{aligned}\frac{\partial m}{\partial y} &= 1-\frac{4}{y^3}\\ \frac{\partial N}{\partial x} &= 1-\frac{4}{y^3}\end{aligned}\right\}\text{exact}$$

Solution is

$$\int\left(y+\frac{2}{y^2}\right)dx + \int x+2y-\frac{4x}{y^3}\,dy$$

$$yx+\frac{2x}{y^2}+y^2=c$$

Q Solve:- $y(x^2y+e^x)dx - e^x dy = 0$ —— ①

$m = x^2y^2+y\cdot e^x \Rightarrow \frac{\partial m}{\partial y} = 2x^2y+e^x$

$N = -e^x$

$\frac{\partial N}{\partial x} = -e^x \qquad \frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$

Rule 2 not satisfied

$$Q = \frac{\cancel{2x^2y+e^x}}{y(x^2y+e^x)} \neq f(y)$$

i.e ~~we cannot~~ obtain the solution by Rule 2

x^{-2+1}

$\frac{x^{-1}}{-1}$

Topic

Expt No. Date Page No.

$= -\frac{1}{2xy} + \frac{1}{2}\log x + -\frac{1}{2}\log y = c$

$= \frac{1}{2}\log x - \frac{1}{2}\log y - \frac{1}{2xy} = c$

Q solve $(y-xy^2)\,dx - (x+x^2y)dy = 0$ — ①

$\underbrace{y(1-xy)dx}_{M} - \underbrace{x(xy+1)}_{N} dy = 0$

$M = y(1-xy) \qquad \frac{\partial M}{\partial y} = 1-2xy$

not exact

$N = -x(1+xy)$

$\frac{\partial N}{\partial x} = -1-2xy$

$\frac{1}{Mx - Ny} = \frac{1}{xy - x^2y^2 + xy + x^2y^2}$

$I.f = \frac{1}{2xy}$

$I.f \times eq^n$ ①

$\frac{1}{2xy}(y-xy^2)dx - \frac{1}{2xy}(x+x^2y)dy = 0$

$\left[\frac{1}{2x} - \frac{y}{2}\right]dx - \left[\frac{1}{2y} + \frac{x}{2}\right]dy = 0$

This is exact

Teacher's Signature

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Rule III :- If M is of the form $M = y f_1(xy)$

and N is of the form $N = x f_2(xy)$

$$I.f. = \frac{1}{Mx - Ny}$$

Q $y(1+xy)dx + x(1-xy)dy = 0$ —— (A)

Comparing with $Mdx + Ndy = 0$

$$M = y f_1(xy) = y(1+xy)$$

$$N = x f_2(xy) = x(1-xy)$$

Rule 3 :-

$$I.f = \frac{1}{Mx - Ny} = \frac{1}{xy(1+xy) - xy(1-xy)}$$

$$= \frac{1}{xy + x^2y^2 - xy + x^2y^2} = \frac{1}{2x^2y^2}$$

multiply diff eq^n (A) by I.f. we get :-

$$\frac{1}{2x^2y^2}(y+xy^2)\,dx + \frac{1}{2x^2y^2}(x - x^2y)\,dy = 0$$

$$\left[\frac{1}{2x^2y} + \frac{1}{2x}\right]dx + \left[\frac{1}{2xy^2} - \frac{1}{2y}\right]dy = 0$$

It is exact $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Solution

$$\int \frac{1}{2x^2y}\,dx + \int \frac{1}{2x}\,dx + \int \frac{1}{2xy^2}\,dy - \int \frac{1}{2y}\,dy = c$$

Topic
Expt No. Date Page No.

Now, $\dfrac{\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}}{M} = \dfrac{-e^x - 2yx^2 - e^x}{y(x^2y+e^x)}$

$= \dfrac{-2yx^2 - 2e^x}{y(x^2y+e^x)}$

$= \dfrac{-2(x^2y+e^x)}{y(x^2y+e^x)} = \dfrac{-2}{y} = f(y)$

$I.f = e^{\int -2/y} = \dfrac{1}{y^2}$

$I.f \times eq(1)$

$\dfrac{1}{y}(x^2y+e^x)\,dx - \dfrac{e^x}{y^2}\,dy = 0$

$\dfrac{\partial M}{\partial y} = \cancel{x^2} - \dfrac{e^x}{y^2}$ exact.

$\dfrac{\partial N}{\partial x} = -\dfrac{e^x}{y^2}$

$\int x^2 + \dfrac{e^x}{y} - \cancel{\int \dfrac{e^x}{y^2}\,0}$

$\dfrac{x^3}{3} + \dfrac{e^x}{y} = c$ Ans

Teacher's Signature

$$\left[\frac{y}{2}\tan xy + \frac{1}{2x}\right]dx + \left[\frac{x}{2}\tan xy - \frac{1}{2y}\right]dy$$

$$\frac{\partial M}{\partial y} = \frac{1}{2}\tan xy + \frac{y}{2}\cdot\sec^2 xy\cdot x$$

$$= \text{exact}$$

$$\frac{\partial N}{\partial x} = \frac{1}{2}\tan xy + \frac{x}{2}\sec^2 xy\cdot y$$

$$\int \frac{y}{2}\tan xy + \frac{1}{2x}\,dx + \int \cancel{\frac{x}{2}\tan xy} - \frac{1}{2y}\,dy = c$$

$$\cancel{\frac{y}{2}\log\sec xy\tan xy} + \frac{1}{2}\log x - \frac{1}{2}\log y = c$$

$$\frac{y}{2}\log\sec xy$$

☆ RULE IV :

If a Diff^n eq^n is given in the form of $Mdx + Ndy = 0$ & eq^n is Homogeneous & $Mx + Ny \neq 0$

Then $I.f. = \dfrac{1}{Mx+Ny}$

Solve : $\dfrac{dy}{dx} = \dfrac{x^3+y^3}{xy^2}$ ——

$$xy^2\,dy = (x^3+y^3)\,dx$$

$$\underbrace{(x^3+y^3)}_{M}dx - \underbrace{xy^2}_{N}dy = 0 \quad \text{—— (1)}$$

given diff eq^n is Homogeneous

Topic
Expt No. Date Page No.

$$\frac{1}{y^2}(3x^2y^4+2xy)\,dx+\frac{1}{y^2}(2x^3y^3-x^2)\,dy=0$$

$$\left(3x^2y^2+\frac{2x}{y}\right)dx+\left(2x^3y-\frac{x^2}{y^2}\right)dy=0$$

$$\boxed{\frac{\partial M}{\partial y}=6x^2y-2x/y^2} \quad \text{exact} = \quad \boxed{\frac{\partial N}{\partial x}=6x^2y-\frac{2x}{y^2}}$$

solution

$$\int\left(3x^2y^2+\frac{2x}{y}\right)dx+\int\left(2x^3y-\frac{x^2}{y^2}\right)dy=C$$

$$x^3y^2+\frac{x^2}{y}=C \quad \text{Ans}$$

Q $\underbrace{(xy\sin xy+\cos xy)\,y}_{M}\,dx+\underbrace{(xy\sin xy-\cos xy)\,x}_{N}\,dy=0$ — ①

$$M=y\,f_1(xy),\quad N=x\,f_2(xy)$$

$$\frac{1}{Mx-Ny}=\frac{1}{x^2y^2\sin xy+xy\cos xy-x^2y^2\sin xy+xy\cos xy}$$

$$\text{I.f}=\frac{1}{2xy\cos xy}$$

I.f × eq ①

$$\frac{1}{2xy\cos xy}\,y(xy\sin xy+\cos xy)\,dx+\frac{1}{2xy\cos xy}\,x(xy\sin xy-\cos xy)\,dy$$

Teacher's Signature

$$\int\left[\frac{1}{2x}-\frac{y}{2}\right]dx+\int\left[\frac{-1}{2y}-\frac{x}{2}\right]dy=c$$

$$\frac{1}{2}\log x-\frac{xy}{2}+-\frac{1}{2}\log y=c$$

Random Ques :-

$$\underbrace{(3x^2y^4+2xy)}_{M}\,dx+\underbrace{(2x^3y^3-x^2)}_{N}\,dy=0 \quad \text{(1)}$$

$$\frac{\partial M}{\partial y}=12x^2y^3+2x$$

$$\frac{\partial N}{\partial x}=6x^2y^3-2x$$

Applied : Rule 2

$$\frac{6x^2y^3-2x-12x^2y^3-2x}{3x^2y^4+2xy}$$

$$=-\frac{6x^2y^3-4x}{3x^2y^4+2xy}\Rightarrow\frac{-2\cancel{(3x^2y^3+2x)}}{y\cancel{(3x^2y^3+2x)}}$$

$$=-\frac{2}{y}$$

$$\text{I.f } e^{\int -2/y}=\frac{1}{y^2}$$

I.f × (1)

Topic
Expt No. Date Page No.

M & N are homogeneous

$\frac{\partial M}{\partial y} = 3y^2 \quad \frac{\partial N}{\partial x} = -y^2$

given $\frac{1}{Mx+Ny} =$ I.f.

$Mx+Ny = x^4 + xy^3 - xy^3 = x^4 \neq 0$

$I.f = \frac{1}{x^4}$ → it will make non exact to exact

I.f × eq^n ①

$\frac{1}{x^4}(x^3+y^3)\,dx - \frac{xy^2}{x^4}dy = 0$

$\left(\frac{1}{x} + \frac{y^3}{x^4}\right)dx - \frac{y^2}{x^3}dy$ → This is exact

$\int \frac{1}{x} + \frac{y^3}{x^4} - \int \frac{y^2}{x^3} = C$

$\log x - \frac{y^3}{3x^3} = C$ Ans

Q $x^2y\,dx - (x^3+y^3)dy = 0$ — ①

② $Mx+Ny = x^3y - x^3y - y^4 = -y^4 \neq 0$

Teacher's Signature

$\frac{\partial m}{\partial y} = x^2, \frac{\partial N}{\partial x} = -3x^2$ i.e. Not exact

since given ~~Ho~~ Diff eqn is Homo &

$$I.f. = \frac{1}{xM+Ny} = -\frac{1}{y^4}$$

I.f. x eq (1)

$$-\frac{x^2y}{y^4}dx + \frac{1}{y^4}(x^3+y^3)dy = 0$$

$$\left[-\frac{x^2}{y^3}\right]dx + \left[\frac{x^3}{y^4} + \frac{1}{y}\right]dy = 0$$

$$\frac{\partial m}{\partial y} = \frac{3x^2}{y^4} ; \frac{\partial N}{\partial y} = \frac{3x^2}{y^4}$$

exact.

$$\int -\frac{x^2}{y^3} + \int \frac{x^3}{y^4} + \frac{1}{y} = c$$

$$-\frac{x^3}{3y^3} + \log y = c$$

- LC

First order first degree - Complete Now

~~Second~~ Higher order first degree - start

Topic
Expt No.
Date
Page No.

Linear Differential equation of n^{th} Order with Constant Coefficient

* Linear Diff eqn (LDE) :- If degree of depended variable and all derivative is 1 such diff eqn is called LDE.

$y = f(x)$ y depends on x

eg :- $\left(\frac{d^2y}{dx^2}\right)^1 + 4\frac{dy}{dx} + y^1 = 0$ LDE.

$\left(\frac{d^4y}{dx^4}\right)^1 + (y)^1 = \sin x$

degree & dependent variable ki degree should be same.

$\frac{d^2y}{dx^2} + y\frac{dy}{dx} + 4y = 0$ NLDE (non)

$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ NLDE

* Non Linear Diff eq (NLDE) :- If degree of dependent variable, the degree of its derivate is greater than 1 then diff eqn is called NLDE.

* Note :- A diff eqn is also said to be non-linear if derivative of dependent variable is multiplied with dependent variable.

$\frac{d^2y}{dx^2} + y^2 = 0$ NLDE

$\frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0$

Teacher's Signature

* Roots are complex

① Auxilary eqn of degree 2, then

suppose $m_1 = \alpha + i\beta$
$m_2 = \alpha - i\beta$

not any case of repeated root

$$C.F = e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x)$$

② Auxilary eqn of degree 3, then

$m_1 = \alpha$ (real)
$m_2 = \alpha_1 + i\beta_1$
~~$m_3 = \alpha_1 - i\beta_1$~~ $m_3 = \alpha_1 - i\beta_1$

α_1 is common is m_2 & m_3

$$C.F = C_1 e^{\alpha x} + e^{\alpha_1 x}(C_2 \cos\beta_1 x + C_2 \sin\beta_1 x)$$

③ Auxilary eqn of degree 4

Case I :- $m_1 \neq m_2 \neq m_3 \neq m_4$

Like $(x^2+1)(x^2+4)=0$

Let $m_1 = \alpha_1 + i\beta_1$
$m_2 = \alpha_1 - i\beta_1$
$m_3 = \alpha_2 + i\beta_2$
$m_4 = \alpha_2 \mp i\beta_2$

$$C.F = e^{\alpha_1 x}(C_1 \cos\beta_1 x + C_2 \sin\beta_1 x) + e^{\alpha_2 x}(C_3 \cos\beta_2 x + C_4 \sin\beta_2 x)$$

Case II :-

$m_1 = m_3 \neq m_2 = m_4$

eg :- $(x^2+1)^2 = 0$

$m_1 = \alpha_1 + i\beta_1$, $m_3 = \alpha_1 + i\beta_1$
$m_2 = \alpha_1 - i\beta_1$, $m_4 = \alpha_1 - i\beta_1$

Topic
Expt No. Date Page No.

Rules to find Complimentary function

* Roots are real

① Auxilary eqⁿ is of degree 2

m_1, m_2 are roots

C_1, C_2, C_3 are arbitary constant.

Case I $m_1 \neq m_2$

$$C.F = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

Case II $m_1 = m_2$

$$C.F = (C_1 + C_2 x) e^{m_1 x}$$

⑪ Auxilary eqⁿ of degree 3

Case I: $m_1 \neq m_2 \neq m_3$ then

$$C.f. = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x}$$

Case II: $m_1 = m_2 \neq m_3$ then

$$C.f = (C_1 + C_2 x) e^{m_1 x} + C_3 e^{m_3 x}$$

Case III: $m_1 = m_2 = m_3$ then

$$C.f = (C_1 + C_2 x + C_3 x^2) e^{m_1 x}$$

Teacher's Signature

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

P & Q = Constant

R = function of x

- Linear differential equation of 2nd order with Constant Coefficient.

The general form of LDE of 2nd order is

$$\boxed{\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R}$$, where P & Q = Constant and R = function of x or Constant

This differential equation can be written as:

$$(D^2 + PD + Q)y = R \quad ; \text{where } D = \frac{d}{dx}$$

$$D^2 = \frac{d^2}{dx^2}$$

Complete solution is equal to

= Complementary func + Particular integral

i.e.

$$C.S = C.F + P.I$$

$$y = C.F. + P.I.$$

Topic. Date Page No.
Expt No.

Q $(D^3+2D^2-D-2)y = e^x$

Sol Auxilary eqⁿ of given diffⁿ eqⁿ is

$m^3+2m^2-m-2=0$

~~$m^2(m+2)$~~

$m^2(m+2)-1(m+2)=0$

$\Rightarrow (m^2-1)(m+2)=0$

i.e. $m=-1, 1, -2$

$C.F = C_1 e^{-x} + C_2 e^{x} + C_3 e^{-2x}$

P.I. $\dfrac{1}{D^3+2D^2-D-2} \cdot e^x$, $f(a)=0$ $\boxed{a=1}$

~~means~~

$= x \cdot \dfrac{1}{3D^2+4D-1} \cdot e^x$ $(f'(D))$

$= \dfrac{x \cdot e^x}{3+4-1}$ $f'(a)$ $= \dfrac{xe^x}{6}$

$C.S = C.F + P.I$

$y = C_1 e^{-x} + C_2 e^{x} + C_3 e^{-2x} + \dfrac{xe^x}{6}$

Teacher's Signature

Rules to find P.I.

(1) $\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$; $f(a) \neq 0$

(2) If $f(a)=0$; then $\frac{1}{f(D)}e^{ax} = x\frac{1}{f'(a)}e^{ax}$; $f'(a)\neq 0$

If $f'(a)=0$; then $\frac{1}{f(D)}e^{ax} = x^2\frac{1}{f''(a)}e^{ax}$; $f''(a)\neq 0$

Q Solve $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 5e^{3x}$

Sol $(D^2+6D+9)y = 5e^{3x}$; $D=\frac{d}{dx}$

Auxilary eqn of Diff eqn is

$$m^2+6m+9=0 \Rightarrow m=-3,-3$$

$$C.F = (C_1+C_2x)e^{-3x}$$

$$P.I = \frac{1}{f(D)}e^{ax} = \frac{1}{D^2+6D+9}5e^{3x}$$

replace D by a

$$P.I = \frac{1}{3^2+6\cdot3+9}\times 5e^{3x} = \frac{5e^{3x}}{36}$$

$C.S = C.F + P.I.$

i.e. $y = (C_1+C_2x)e^{-3x} + \frac{5}{36}e^{3x}$

Topic ______ Date ______ Page No. ______
Expt No. ______

$$C.f = e^{\alpha_1 x}\left((C_1 + C_2 x)\cos\beta_1 x + (C_3 + C_4 x)\sin\beta_1 x\right)$$

Q Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 30y = 0$

Sol $D^2y + Dy - 30y = 0$ — (1)

where $D = \frac{d}{dx}$

Auxilary eqⁿ of diff eqⁿ (1) is

$D = m,\ y = 1$

$$m^2 + m - 30$$

on solving we get

$$(m+6)(m-5) = 0$$

$$m = 5, -6.$$

$$C.F = C_1 e^{5x} + C_2 e^{-6x}$$

Q Solve $\frac{d^2y}{dx^2} + \mu^2 y = 0$

$D = \frac{d}{dx}$

$$(D^2 + \mu^2)y = 0$$

Auxilary eqn is $\Rightarrow D = m, y = 1$

$$m^2 + \mu^2 = 0$$

$$m = \pm \mu i$$

Real part $= 0$, img part $= \pm \mu i$

$$m = 0 \pm \mu i$$

$$C.f = e^{0x}(C_1 \cos\mu(x) + C_2 \sin\mu(x)).$$

Teacher's Signature ______

$$= C_1 \cos\mu x + C_2 \sin\mu x$$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$; $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

$\cosh\theta = \dfrac{e^{\theta} + e^{-\theta}}{2}$; $\sinh\theta = \dfrac{e^{\theta} - e^{-\theta}}{2}$

hyperbolic

hyperbolic → means no boundary so remove i now it tends to ∞.

Q $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = e^{3x}$

sol $(D^2 - 6D + 9)y = e^{3x}$; $D = \dfrac{d}{dx}$

Aux eqn $m^2 - 6m + 9 = 0$

$\Rightarrow m = 3, 3$

$C.f = (C_1 + C_2 x)e^{3x}$

Now,

$P.I = \dfrac{1}{D^2 - 6D + 9 \Rightarrow f(D)} \cdot e^{3x}$ $a = 3$

replace D by a $f(a) = 0$ means use Rule 2

$= x \cdot \dfrac{1}{2D - 6} \cdot e^{3x}$

$a = 3$ $f'(a) = 0$

again $x^2 \cdot \dfrac{1}{2} e^{3x} \Rightarrow P.I = \dfrac{x^2}{2} e^{3x}$

Topic ______ Date ______ Page No. ______
Expt No. ______

$C.S = C.F + P.I$

$y = (C_1 + C_2 x)e^{3x} + \frac{x^2 e^{3x}}{2}$

* Rule to find P.I of $\sin ax / \cos ax$

$$\boxed{\begin{array}{l} \frac{1}{f(D^2)} \sin ax/\cos ax = \frac{1}{f(-a^2)} \sin ax/\cos ax \; ; f(-a^2) \neq 0 \\ \text{if } f(-a)^2 = 0 \text{ then} \\ \frac{1}{f(D^2)} \sin ax/\cos ax = x . \frac{1}{f'(-a^2)} \sin ax/\cos x \end{array}}$$

$-a^2$

$\boxed{-2^2 = -4}$

Q solve $(D^2+4)y = \sin 3x$

Sol- Auxilary eqn is $D = m$, $y = 1$

$m^2 + 4 = 0$

$m = \pm 2i$

$C.F = C_1 \cos 2x + C_2 \sin 2x$

$P.I = \frac{1}{D^2+4} \sin 3x$

Replace D^2 with $-a^2$ $\Rightarrow \boxed{a=3}$

$= \frac{1}{-3^2+4} \sin 3x = -\frac{1 \sin 3x}{5}$

$C.S = C.F + P.I$.

$y = C_1 \cos 2x + C_2 \sin 2x - \frac{\sin 3x}{5}$

Teacher's Signature ______

Q solve $(D^2+4)y = e^x + \sin 2x$

Aux eqn

$$m^2+4=0$$
$$m = \pm 2i$$

$$C.F = C_1 \cos 2x + C_2 \sin 2x$$

$$P.I = \frac{1}{D^2+4}(e^x + \sin 2x)$$

$$= \frac{e^x}{D^2+4} + \frac{1}{D^2+4}\sin 2x$$

$$= \frac{1}{1^2+4}e^x + x \cdot \frac{1}{2D}\sin 2x$$

$D = \frac{d}{dx}$

$$= \frac{e^x}{5} + x \cdot \frac{D}{2D^2}\sin 2x$$

$D^2 = -a^2$

$$= \frac{e^x}{5} - \frac{x}{4}\cos 2x$$

Q $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^{2x} + \cos x$

Sol $(D^2 - 4D + 3)y = e^{2x} + \cos x$

Aux eqn

$$m^2 - 4m + 3 = 0$$
$$\Rightarrow m = 1, 3$$

$$C.F = C_1 e^x + C_2 e^{3x}$$

Topic
Expt No.
Date
Page No.

* Rule to find P.I. for Polynomials. (x^n)

$$\frac{1}{f(D)} x^n = [f(D)]^{-1} x^n$$

$$(1-x)^{-1} = 1+x+x^2+x^3+x^4+ - - - -$$

$$(1+x)^{-1} = 1-x+x^2-x^3+x^4-x^5+ - - - - -$$

Q Solve $(D^2+5D+4)y = 3-2x$

Sol

Auxilary eq^n of Diff^n eq^n ① can be written as

$$m^2+5m+4=0$$

$$\Rightarrow m = -4, -1$$

$$C.F = C_1 e^{-x} + C_2 e^{-4x}$$

$\frac{1}{D} \rightarrow \int$ integration

$$P.I = \frac{1}{D^2+5D+4}(3-2x) =$$

$D \rightarrow$ Derivative

$$= \frac{1}{4\left[1+\left[\frac{D^2+5D}{4}\right]\right]} \cdot (3-2x)$$

$$= \frac{1}{4}\left[1+\left(\frac{D^2+5D}{4}\right)\right]^{-1}(3-2x)$$

$\therefore$ Let $\frac{D^2+5D}{4} = t$

Teacher's Signature

$\frac{d}{dx}y$ $\frac{d}{dt}x$ $x \to t$

Q $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = \sin t$

$(D^2 + 2D + 3)x = \sin t$ $\quad D = \frac{d}{dt}$

Aux eqⁿ

$m^2 + 2m + 3 = 0$

$m = \frac{-2 \pm \sqrt{4 - 4\times1\times3}}{2\times1}$ $\quad \sqrt{-8} = i\sqrt{8}$ $\quad i2\sqrt{2}$

$= \frac{-2 \pm \sqrt{-8}}{2} \Rightarrow -1 \pm i\sqrt{2}$

$C.F = e^{-t}(C_1 \cos\sqrt{2}t + C_2 \sin\sqrt{2}t)$

$P.I = \frac{1}{D^2 + 2D + 3}\sin t$ $\quad f(D^2)$ $\downarrow$ $-a^2$ $\quad a = 1$

$\frac{1 \times \sin t}{-1 + 2D + 3} = \frac{\sin t}{2 + 2D}$

$\frac{(2 - 2D)\sin t}{(2+2D)(2-2D)}$

$= \frac{(2 - 2D)\sin t}{4 - 4D^2}$ $\quad D^2 = -a^2 = -1^2 = -1$

$= \frac{2\sin t - 2\cos t}{4 - 4(-1)} = \frac{1}{4}(\sin t - \cos t)$

$C.S = C.F + P.I$

$= e^{-t}(C_1 \cos\sqrt{2}t + C_2 \sin\sqrt{2}t) + \frac{1}{4}(\sin t - \cos t)$

Topic ______ Date ______ Page No. ______
Expt No. ______

P.I. $\dfrac{1}{D^2-4D+3}(e^{2x}+\cos x)$

$= \dfrac{e^{2x}}{D^2-4D+3} + \dfrac{\cos x}{D^2-4D+3}$ $a=1$

$a=2$ Replace D by a   Replace D^2 by $-a$

$4-8+3$

$= -e^{2x} + \dfrac{\cos x}{-1^2-4D+3}$

$= -e^{2x} + \dfrac{1}{2-4D}\cos x$

$= -e^{2x} + \dfrac{2+4D}{(2-4D)(2+4D)}\cos x$

$= -e^{2x} + \dfrac{2+4D}{4-16D^2}\cdot\cos x$

$= -e^{2x} + \dfrac{(2+4D)\cos x}{4-16(-1^2)}$

$= -e^{2x} + \dfrac{1}{20}(2+4D)\cos x$

$= -e^{2x} + \dfrac{1}{10}(\cos x - 2\sin x)$

~~e^{2x}~~

C.S = C.F + P.I.

i.e. $y = c_1e^{x} + c_2e^{3x} - e^{2x} + \dfrac{1}{10}(\cos x - 2\sin x)$

Teacher's Signature ______

Q $(D^2-4D+3)y = x^3$

m^2-4m+3

$m^2-3m-m+3$

$m(m-3)-1(m-3)$

$m=1, m=3$

$C.F = C_1 \cdot e^{x} + C_2 e^{3x}$

$P.I = \frac{1}{(D^2-4D+3)} \cdot x^3$

$= \frac{1}{3\left(1+\left(\frac{D^2-4D}{3}\right)\right)} x^3$

x^3
$D = 3x^2$
$D^2 = 6x$
$D^3 = 6$
$D^4 = 0$

$= \frac{1}{3}\left(1+\left(\frac{D^2-4D}{3}\right)\right)^{-1} \cdot x^3$

$= \cancel{\frac{1}{3}\left(1-\frac{1}{3}\left(\frac{D^2-4D}{3}\right)^{-1}\right)}$ $\frac{1}{3}(1+t)^{-1} \cdot x^3$

$= \frac{1}{3}(1-t+t^2+ - - -)x^3$

$= \frac{1}{3}\left[\cancel{x^3 - \frac{1}{3}(D^2-4D)x^3 + \frac{1}{3}(D^2+4D)^2 x^3 + 0}\right]$

$D^4+16D^2+8D^3$

$= \frac{1}{3}[x^3 - \frac{1}{3}(D^2 \cdot x^3 - 4D \cdot x^3) + \frac{1}{3}(D^4 x^3 + 16D^2 x^3 + 8D^3 \cdot x^3)]$

$= \frac{1}{3}[x^3 - \frac{1}{3}\{6x - 12x^2\} + \frac{1}{3}(0 + 16 \times (6x) + 8 \cdot 6)$

$\cancel{6}\{x - 2x^2\}$

3

min order derivative $= 3$

higher order $= 0$ Ans

Topic ______ Date ______ Page No. ______
Expt No. ______

Note: $\frac{(D^2+5D)^2}{4^2} \cdot (3-2x)$

$= \frac{1}{16}[D^4+25D^2+10D^3](3-2x)$ (minimum)
differentiate it two time we get Zero

Q. $(D^2+2D+1)y = x.$

Sol. Aux eq^n
$m^2+2m+1=0$
$m=-1,-1$
$C.f = (C_1+C_2x)e^{-x}$

$P.I = \frac{1}{D^2+2D+1}x = \frac{1}{1+(2D+D^2)}x$

$= [1+(2D+D^2)]^{-1}.x$

$= [1-(2D+D^2)+(2D+D^2)^2-(2D+D^2)^3+---]x$

$= [x-(2D+D^2)x+0]$

$= \{x-[2Dx+D^2.x]\}$

$= x-[2\times1+0]$
$= x-2$

$C.S = C.f + P.I.$

$y = (C_1+C_2x)e^{-x}+(x-2).$

Teacher's Signature ______

$D = \frac{d}{dx}$

$= \frac{1}{4}[1 - t + \underbrace{t^2 + - -}_{\text{It gives zero two times}}](3-2x)$

It gives zero two times

diff. (3-2x) = -2 = 0 vanish.

$= \frac{1}{4}\left[3-2x - \frac{1}{4}[D^2+5D](3-2x) + 0\right]$

$= \frac{1}{4}\left[(3-2x) - \frac{1}{4}[0 + 5D(3-2x)]\right]$

$D(3-2x)$

$= \frac{1}{4}\left[3-2x - \frac{1}{4} 5[D3 - D(2x)]\right]$

$-D(3-2x)$

$= \frac{1}{4}\left[3-2x - \frac{5}{4}[0-2]\right]$

$= \frac{1}{4}\left[3-2x - \frac{5}{4}[-2]\right]$

$= \frac{1}{4}\left[3-2x + \frac{5}{2}\right]$

$= \frac{1}{4}\left[\frac{6-4x+5}{2}\right]$

$P.I = \frac{1}{8}[11-4x]$

CONote.

$a^3 - b^3 - 3ab(a-b)$

$a^3 - b^3 - 3a^2b + 3ab^2$

64×6

128

Topic
Expt No.
Date
Page No.

$\frac{64 \times \cancel{6}^{2}}{27 \times 8}$

$\cancel{\frac{1}{3}}\left[x^3 - \cancel{\frac{6x}{3}}^{2}[1 - 2x]\right.$

$\frac{1}{3}\left[x^3 - x^3\left[\frac{D^2 - 4D}{3}\right] + \left[\frac{D^2 - 4D}{3}\right]^2 - \left(\frac{D^2 - 4D}{3}\right)^3 + \cdots\right]x^3$

$\frac{1}{3}\left[1 - \left[\frac{D^2 - 4D}{3}\right] + \frac{1}{9}\left[D^4 + 16D^2 - 8D^3\right) - \left(\frac{-64 D^3}{27}\right) + \cdots\right]x^3$

$\frac{1}{3}\left[x^3 - \frac{1}{3}\left[D^2 \cdot x^3 - 4D \cdot x^3\right] + \frac{1}{9}\left[D^4 \cdot x^3 + 16D^2 \cdot x^3 - 8D^3 \cdot x^3\right] + \frac{64}{27}D^3 \cdot x^3\right.$

$\frac{1}{3}\left[x^3 - \frac{[6x - 12x^2]}{3} + \frac{1}{9}\left[\overset{0}{\cancel{6x}} + 96x - 48\right) + \frac{64}{27} \times 6\right]$

$\frac{1}{3}\left[x^3 - 2[2x - 2x^2] + \frac{[96x - 48]}{9} + \frac{64 \times 6}{27}\right.$

$\frac{x^3}{3} - \frac{2x}{3}[1 - x] + \frac{96x - 48}{27} + \frac{\cancel{128}\,4}{\cancel{27}}$

$27 \times 4 = 128$

$\frac{x^3}{3} - \frac{2x}{3}[1 - x] + \frac{32x - 16}{9} + 4$

OR

$\frac{1}{27}(9x^3 + 36x^2 + 78x + 80$

Teacher's Signature

Rule to find P.I. of $e^{ax}V$; where V is a func. of x.

$$\frac{1}{f(D)} e^{ax} \cdot V = e^{ax} \frac{1}{f(D+a)} V$$

Q $(D^2-4D+4)y = x^3 \cdot e^{2x}$

Auxilary eqn. of the given differential eqn

$m^2 - 4m + 4 = 0$

$\Rightarrow m = 2, 2$

$C.F = (C_1 + C_2 x)e^{2x}$

$P.I = \frac{1}{D^2-4D+4} \cdot e^{2x} \cdot x^3$ $\quad \boxed{a=2}$

$= e^{2x} \cdot \frac{1}{(D+2)^2 - 4(D+2)+4} \cdot x^3$

$= e^{2x} \cdot \frac{1}{D^2+4+4D-4D-8+4} \cdot x^3$

$= e^{2x} \cdot \frac{1}{D^2} \cdot x^3$

$= e^{2x} \iint x^3 = \frac{e^{2x} \cdot x^5}{20}$

$C.S = C.f + P.I$

$= (C_1 + C_2 x)e^{2x} + \frac{e^{2x} \cdot x^5}{20}$

Topic
Expt No.
Date
Page No.

Q Obtain the general solution of differential equation

$$(D^2-2D+2)y = x + e^x \cos x$$

Aux eq^n

$$m^2 - 2m + 2 = 0$$

$$\frac{-2 \pm \sqrt{2 - 4\times1\times2}}{2} \Rightarrow 1 \pm i$$

$$C.F = e^x(C_1 \cos x + C_2 \sin x)$$

$$P.I = \frac{1}{(D^2-2D+2)}(x + e^x \cos x)$$

$$= \frac{x}{D^2-2D+2} + \frac{e^x \cos x}{D^2-2D+2}$$

$$= \frac{1}{2}\frac{x}{\left(\frac{D^2}{2} - D + 1\right)} \quad \frac{1}{2}\frac{x}{\left[1 + \frac{D^2-2D}{2}\right]} + \frac{e^x \cos x}{D^2-2D+2}$$

$$= \frac{1}{2}\left[1 + \frac{D^2-2D}{2}\right]^{-1} x + e^x \frac{1}{(D+1)^2 - 2(D+1) + 2}\cos x$$

$$= \frac{1}{2}\left[1 - \frac{D^2-2D}{2} + 0\right] x + e^x \frac{1}{D^2+1+2D-2D-2+2}\cos x$$

$$= \frac{1}{2}\left[x + (0 - 1)\right] + e^x \frac{1}{D^2+1}\cos x$$

$$\boxed{\frac{1}{D}\cos x = \sin x}$$

Teacher's Signature

Rule

$$+ e^x . x . \frac{1}{2D}\cos x$$

$$= \frac{1}{2}(x+1) + \frac{1}{2} x e^x \sin x$$

$$= e^{-3x} \frac{1}{D^2 - 6D + 9 + 6D - 18 + 9} x^{-3}$$

$$e^{-3x} \frac{x^{-3}}{D^2}$$

$$\iint x^{-3}$$

$$= \int -\frac{x^{-2}}{2}$$

$$-\left[\frac{x^{-1}}{2 \times -1}\right]$$

$$\frac{1}{2x}$$

$$P.I = \frac{1}{2x} e^{-3x}$$

$$C.S = C.F + P.I.$$

$$(C_1 + C_2 x) e^{-3x} + \frac{1}{2x} e^{-3x} \text{ Ans}$$

Rule:- If $x^n \sin ax$ or $x^n \cos ax$

Polynomial x Trignometry

$$e^{iax} = \underbrace{\cos ax}_{R.P.} + i \underbrace{\sin ax}_{I.P}$$

$x^n \sin ax$

$$\frac{1}{f(D)} \cdot x^n \cdot \sin ax = I.P \text{ of } e^{iax} \frac{1}{f(D+ia)} x^n$$

$\cos ax = R.P.$ of e^{iax}

$\sin ax = I.P.$ of e^{iax}

$$\frac{1}{f(D)} x^n \cdot \cos ax = R.P. \text{ of } e^{iax} \frac{1}{f(D+ia)} x^n$$

Topic ______ Date ______ Page No. ______
Expt No. ______

$$= -e^{x} \frac{(3D-2)}{(3D+2)(3D-2)} \cos 2x$$

$$= -\frac{e^{x}(3D-2)\cos 2x}{9D^2-4} \qquad D^2=-4$$

$9\times -4 = -36$

$$\frac{+e^{x}(3D\cos 2x - 2\cos 2x)}{-40}$$

$$\frac{e^{x}(-3\times 2\sin 2x - 2\cos 2x)}{40}$$

$$P.I = -\frac{x e^{x}(3\sin 2x + \cos 2x)}{\cancel{40}\ 20}$$

$C.S = C.F + P.I$

$$= C_1 e^{2x} + C_2 e^{3x} - \frac{e^{x}}{20}(3\sin 2x + \cos 2x) \text{ Ans}$$

Q $(D^2+6D+9)y = \dfrac{e^{-3x}}{x^3}$

Aux eq^n

m^2+6m+9

$m = -3, -3$

$C.F = (C_1+C_2x)e^{-3x}$

$$P.I = \frac{1}{D^2+6D+9} e^{-3x} \times x^{-3}$$

$$= e^{-3x} \frac{1}{(D-3)^2+6(D-3)+9} x^{-3}$$

Teacher's Signature ______

$C.S = C.F + P.I$

$= e^x(C_1\cos x + C_2\sin x) + \frac{1}{2}(x+1) + \frac{1}{2}xe^x\sin x$ Ans

Q $(D^2 - 5D + 6)y = e^x\cos 2x$

Aux eq^n

$m^2 - 5m + 6 = 0$

$\Rightarrow m = 2, 3$

$C.F = C_1 e^{2x} + C_2 e^{3x}$

$P.I = e^x \frac{1}{D^2 - 5D + 6}\cos 2x$

$= e^x \frac{1}{(D+1)^2 - 5(D+1) + 6}\cos 2x$ ← Here $a = 1$

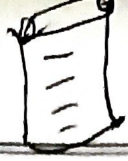
$= e^x \frac{1}{D^2 + 1 + 2D - 5D - 5 + 6}\cos 2x$

$= e^x \frac{1}{D^2 - 3D + 2}\cos 2x$ ← Here $a = 2$ -4

$= e^x \frac{1}{-4 - 3D + 2}\cos 2x$ $D^2 = -a^2$

$= e^x \frac{1}{-3D - 2}\cos 2x$

$= -e^x \frac{1}{(3D+2)}\cos 2x$



Topic
Date
Page No.
Expt No.

$$R\cdot P \text{ of } (\overbrace{\cos x}^{Real} + \overbrace{i\sin x}^{Img})\left[\overbrace{\frac{1}{2}}^{Real} + \overbrace{i\frac{(1-x)}{2}}^{Img}\right]$$

$$= \frac{\cos x}{2} - \frac{(1-x)\sin x}{2} \text{ Ans}$$

Q $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$$

$$(D^2 - 2D + 1)y = xe^x \sin x$$

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2 = 0$$

$$m = 1, 1$$

$$C\cdot F = (C_1 + C_2 x)e^x$$

$$P\cdot I = \frac{1}{D^2 - 2D + 1} e^x \cdot x\sin x$$

$$= I\cdot P \text{ of } \frac{1}{D^2-2D+1} x\cdot e^x(\cos x + i\sin x)$$

$$= \cancel{e^x}\cdot\ I\cdot P \text{ of } e^{ix}\frac{1}{D^2-2D+1} x\cdot e^x$$

$$= I\cdot P \text{ of } e^{ix}e^x \frac{1}{D^2-2D+1} x\cdot \cancel{e^x}$$

lengthy

$$= R.P. \text{ of } e^{ix} \frac{1}{2i\left[1+\left(\frac{D^2+2D(i+1)}{2i}\right)\right]} . x$$

$$= R.P. \text{ of } e^{ix} \frac{1}{2i}\left[1+\left(\frac{D^2+2D(i+1)}{2i}\right)\right]^{-1} x$$

$$= R.P \text{ of } e^{ix} \frac{1}{2i}\left[1-\left[\frac{D^2+2D(i+1)}{2i}\right]+(\;)^2+\cdots\right]x$$

$$\frac{D^2}{2i}x = 0$$

$$= R.P \text{ of } e^{ix} \frac{1}{2i}\left[x - \frac{2(i+1)}{2i} + 0\right]$$

$$2D(x) = 2$$

$$= R.P. \text{ of } e^{ix} \frac{1}{2i}\left[x - \frac{(i+1)}{i} \times \frac{i}{i}\right]$$

$$= R.P. \text{ of } e^{ix} \frac{1}{2i}\left[x + (i-1)\right]$$

$$= R.P. \text{ of } e^{ix} \frac{i}{2i^2}\left[x + (i-1)\right]$$

$$= R.P \text{ of } e^{ix}\left(-\frac{1}{2}\right)\left[xi + \left[i^2 - i\right]\right]$$

$$= R.P \text{ of } e^{ix}\left(-\frac{1}{2}\right)\left[-1 + i(x-1)\right]$$

$$= R.P \text{ of } e^{ix}\left[\frac{1}{2} + \frac{(1-x)i}{2}\right]$$

Topic
Expt No.
Date
Page No.

Q $(D^2+2D+1)y = x\cos x$

Aux eqⁿ

$m^2+2m+1=0$

$m=-1,-1$

$C.F = (C_1+C_2x)e^{-x}$

$P.I = \frac{1}{(D^2+2D+1)} x\cos x$ $\quad a=1, n=1$

~~$\cos x = R.P$ of e^{ix}~~

~~$\cos x = R.P$ of e^{ix}~~

$= R.P.$ of $\frac{1}{D^2+2D+1}(x[\cos x + i\sin x])$

$= R.P.$ of $\frac{1}{D^2+2D+1}(x.e^{ix})$

$= R.P.$ of $e^{ix}\frac{1}{(D+i)^2+2(D+i)+1} x$

$= R.P.$ of $e^{ix}\frac{1}{D^2+i^2+2Di+2D+2i+1}$

$= R.P$ of $e^{ix} \times \frac{1}{D^2-\cancel{1}+2D(i+1)+2i+\cancel{1}}$

$=$ ~~R.P of e^{ix}~~ $\cdot$ $\frac{1}{\cancel{D^2+2D+x}}$

Teacher's Signature

$$= \text{I.P. of } e^{ix} e^{x} \frac{1}{[(D+i+1)^2 - 2(D+i+D+1)]} x$$

$$= \text{I.P. of } e^{x(i+1)} [1 + \underbrace{(D+i+1)^2 - 2(D+i+1)}_{x}]^{-1} \cdot x$$

$$= \text{I.P of } e^{x(i+1)} [1 - (D+i+1)^2 + 2(D+i+1)$$

wrong

↳ non-termaniting

, (D) is present

$$P.I = \frac{1}{D^2 - 2D + 1} x e^{x} \sin x$$

$$= \frac{1}{(D-1)^2} x e^{x} \sin x$$

$$= e^{x} \frac{1}{(D-1+1)^2} x \sin x$$

$$= e^{x} \frac{1}{D^2} \overset{I}{x} \overset{II}{\sin x}$$

$$x \int \sin x - \int \frac{dx}{dx} \int \sin$$

$$- x \cos x + \int \cos x$$

$$= e^{x} \iint x \sin x$$

$$\int - x \cos x + \sin x + c$$

Topic ______ Date ______ Page No. ______

Expt No. ______

Q $(D^2+4)y = 3x\sin x$

$m^2+4=0$

$\Rightarrow m = \pm 2i$

$C.F = C_1\cos 2x + C_2\sin 2x$

$P.I = \frac{1}{D^2+4} 3x\sin x$

$= \text{I.P. of } 3\frac{1}{D^2+4}[x(\cos x + i\sin x)]$

$= 3\text{ I.P. of } e^{ix}\frac{1}{(D+i)^2+4}x$

$= 3\text{ I.P. of } e^{ix}\frac{1}{D^2+i^2+2Di+4}x$

$= 3\text{ I.P. of } e^{ix}\frac{1}{D^2+2Di+3}x$

$= \cancel{3}\times\text{I.P. of } e^{ix}\frac{1}{\cancel{3}\left[1+\frac{D^2+2iD}{3}\right]}x$

$= \text{I.P. of } e^{ix}\left[1-\frac{D^2+2iD}{3}+0\right]x$

$\text{I.P. of } e^{ix}\left[x-\left\{0-\frac{2i}{3}\right\}\right]$

$= \text{I.P. of } e^{ix}\left[x-\frac{2}{3}i\right]$

$\left(\frac{D^2+2iD}{3}\right)^2 x = 0$

$(D^2+2iD)^2$

$D^4+4i^2D^2-D^3$

Teacher's Signature ______

$C.S = C.F + \text{I.P}$

$= C_1\cos 2x + C_2\sin 2x$

$= I.P.\ of \left[(\cos x + i\sin x)\left(x - \frac{2i}{3}\right)\right]$

To make imaginary

imaginary x real

$= I.P. of [(\cos$

$= -\frac{2}{3}\cos x + x\sin x$

i.e. $PI = x\sin x - \frac{2}{3}\cos x$

$C.S = C.f + PI$

$= C_1\cos 2x + C_2\sin 2x + x\sin x - \frac{2}{3}\cos x$

Cauchy - Euler

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1}\frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2}\frac{d^{n-2} y}{dx^{n-2}} + \text{---} + a_n y$$

$x\frac{dy}{dx} =$

Let $x = e^z \Rightarrow z = \log x$; $\frac{dz}{dx} = \frac{1}{x}$

$$\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx} \Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{x}\frac{dy}{dz}}$$

$$x\frac{dy}{dx} = \frac{dy}{dz}$$

Topic
Expt No.
Date
Page No.

$$x^3 \frac{d^3y}{dx^3} = x^3 D^3 = D_1(D_1-1)(D_1-2)y$$

eq^n (A) will become

$$(D_1(D_1-1)(D_1-2) + D_1(D_1-1) - 2)y = e^z - e^{-3z}$$

$$\Rightarrow (D_1(D_1^2-3D_1+2) + D_1^2 - D_1 - 2)y = e^z - e^{-3z}$$

$$\Rightarrow (D_1^3 - 2D_1^2 + D_1 - 2)y = e^z - e^{-3z}$$

Now it is L.D.E of Constant coefficient.

AUX eq^n is

$$m^3 - 2m^2 + m - 2 = 0 \quad \text{— (II)}$$

$-1-2-1-2$

$1-2+1-2$

$8-8+2-2$

$m = 2$ satisfies eq (II)

$$m-2 \overline{\big)\, m^3 - 2m^2 + m - 2}\ \ m^2+1$$

$m^3 - 2m^2$

$\times \quad m-2$

$m-2$

$\times$

$m^2+1=0$

$m = \pm i$

$$(m^2+1)(m-2) = 0$$

$$m = \pm i, 2$$

$$C_1 e^{2z} + (C_2 \cos z + C_3 \sin z)$$

Teacher's Signature

Aux eqⁿ of Diff eqⁿ (x) is

$m^2 - 3m - 4 = 0$

$\Rightarrow m = 4, -1$

$C.F. = C_1 e^{4z} + C_2 e^{-z}$

$$P.I. = \frac{1}{D^2 - 3D - 4} e^{4z}$$

$$= \frac{1}{(4)^2 - 3(4) - 4} \cdot e^{4z} = \frac{1}{0}, \quad f(a) = 0$$

Since denominator is zero

then, $z \frac{1}{2D - 3} e^{4z}$

$$= z \cdot \frac{1}{2(4) - 3} e^{4z} = z \cdot \frac{e^{4z}}{5}$$

$C.S. = C.F + P.I.$

i.e. $y = C_1 e^{4z} + C_2 e^{-z} + \frac{z}{5} e^{4z}$

$y = C_1 x^4 + \frac{C_2}{x} + \frac{\log x}{5} x^4$

since $x = e^z$

$x^{-1} = e^{-z}$

Q $(x^3 D^3 + x^2 D^2 - 2) y = x - \frac{1}{x^3}$ —— (A)

$\boxed{\text{Let } x = e^z \Rightarrow z = \log x}$

$x \frac{dy}{dx} = xD = D_1 y$ where $D_1 = \frac{d}{dz}$

$x^2 \frac{d^2 y}{dx^2} = x^2 D^2 = D_1(D_1 - 1) y$

$$x\frac{dy}{dx} = Dy \qquad \boxed{D=\frac{d}{dz}}$$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y$$

$$x^3\frac{d^2y}{dx^3} = D(D-1)(\overset{D-2}{\cancel{D-3}})y$$

$$x^4\frac{d^2y}{dx^4} = D(D-1)(D-2)(D-3)y$$

Q $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$

Sol Let $x = e^z$ i.e. $z = \log x$

mean, $\boxed{x^4 = e^{4z}}$

&

$$x\frac{dy}{dx} = Dy$$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y \quad \therefore D=\frac{d}{dz}$$

Diff eqⁿ ① will become

$$D(D-1)y - 2Dy - 4y = e^{4z}$$

i.e.

$$(D^2 - D - 2D - 4)\,y = e^{4z}$$

$$(D^2 - 3D - 4)y = e^{4z} \quad \text{——} \; (\times)$$

$a^3+b^3+3a^2b+3ab^2$ $+3ab(a\mp b)$

$\left(\frac{d^2y}{dx^2}\right)^{1}$ degree

$= \frac{e^{3z}}{27}\left(1+\frac{D_1}{3}\right)^{-3}\cdot z$ $\qquad (1+x)^{-1} = 1-x+x^2+$

$= \frac{e^{3z}}{27}\left[1-\frac{D_1}{3}+\cancel{\frac{D_1^2}{9}+(\ )^3+\cdots}\right]z$

$\cancel{\frac{e^{3z}}{27}\cdot\left[z-\frac{1}{3}\right]}$

using Binomial theorem

$= \frac{e^{3z}}{27}\left[1-3\cdot\frac{D_1}{3}+\cdots\right]z$ $\qquad D_1 = \frac{d}{dz}z = 1$

$= \frac{e^{3z}}{27}\left[1-D_1\right]z$ $\qquad D_1 z = 1$

$= \frac{e^{3z}}{27}\left[z-1\right]$

$P\cdot I = \frac{x^3}{27}(\log x - 1)$

$C\cdot S = C\cdot F + P I$

$= C_1 + C_2\log x + C_3(\log x)^2 + \frac{x^3}{27}(\log x - 1)$

Topic ______ Date ______ Page No. ______

Expt No. ______

Q $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = x^3 \log x$ — (A)

Let $x = e^z \Rightarrow x = \log z$

$x\frac{dy}{dx} = xD = D_1 y \equiv D_1 = \frac{d}{dxz}$

$x^2\frac{d^2y}{dx^2} = x^2D^2 = D_1(D_1 - 1)y$

$x^3\frac{d^3y}{dx^3} = x^3D^3 = D_1(D_1-1)(D_1-2)y$

$D_1(D_1-1)(D_1-2) + 3\cancel{x} D_1(D_1-1) + D_1 = e^{3z}\cdot z$

$\cancel{D_1^3 - 2D_1^2 + D_1 - 2 + 3D_1^2 - 3D_1 + D_1}$

$D_1^3 - 3\cancel{D_1^2} + 2\cancel{D_1} + 3\cancel{D_1^2} - 3\cancel{D_1} + \cancel{D_1}$

Reduce L.D.E of constant coefficient $m^3 = 0$

$D_1^3 = e^{3z}\cdot z$ — (B)

Aux. eqn

$m^3 = 0$

$C.f = \cancel{(C_1 + C_2 z + C_3 z^2)}\, e^{0z}$ $(C_1 + C_2 z + C_3 z^2) e^{0z}$

$C.f = C_1 + C_2 \log x + C_3 (\log x)^2$

$P.I = \frac{1}{D_1^3} e^{3z}\cdot z$

$e^{3z} \frac{1}{(D_1+3)^3}\cdot z$

Teacher's Signature ______

$= e^{3z} \frac{1}{27} \frac{1}{\left(1+\frac{D_1}{3}\right)^3}\cdot z$

$$P.I = \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax} \quad f(a) \neq 0$$

$$= \frac{1}{D_1^3 - 2D_1^2 + D_1 - 2} \cdot e^{z} - \frac{1}{D_1^3 - 2D_1^2 + D_1 - 2} e^{-3z}$$

$a = 1$ $\qquad$ $a = -3$

$$= \frac{1}{1 - 2 + 1 - 2} e^{z} - \frac{1}{(-3)^3 - 2(-3)^2 - 3 - 2} e^{-3z}$$

$2 - 4$

$-27 - 18 - 5$

$$= -\frac{e^{z}}{2} + \frac{1}{50} e^{-3z}$$

$\begin{array}{r} 23 \\ 27 \\ \hline 50 \end{array}$

$$P.I = -\frac{x}{2} + \frac{1}{950x^3}$$

$$C.S = C.f + P.I$$

$$= C_1 e^{2z} + C_2 \cos 2 + C_3 \sin z - \frac{e^{z}}{2} + \frac{e^{-3z}}{50}$$

$$y = C_1 x^2 + C_2 \cos(\log x) + C_3 \sin(\log x) - \frac{x}{2} + \frac{1}{50x^3}$$

Ans

Topic
Expt No.
Date
Page No.

Variation Of Parameter

order − 2

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = X$$
where a, b, c are constant
and $\boxed{X = f(x)}$

(A)

Solution of diff. eqn (A) by variation of parameter method

eg: $c_1 \cos x + c_2 \sin x$ (y_1, y_2)

$C.F = Ay_1 + By_2$ — y_1 & y_2 are func. of x

$P.I = uy_1 + vy_2$ — we get y_1 & y_2 from C.F.

$$\boxed{\begin{aligned} u &= -\int \frac{y_2 X}{W}\,dx \\ v &= \int \frac{y_1 X}{W}\,dx \end{aligned}}$$

where :
W = Wronskian
$$= \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

Q $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

$(D^2+1)y = \operatorname{cosec} x$

A.E $m^2+1=0$
$m = \pm i$

Teacher's Signature

$C.F = C_1 \cos x + C_2 \sin x$
$\quad\;\; A\, y_1 \;+\; B\, y_2$

Let $y_1 = \cos x$, $y_2 = \sin x$ $\quad nB = x_n$ $\quad \int \tan x = \sec x$

$P.I = u y_1 + v y_2$

where

$$U = -\int \frac{y_2 X}{W} dx \quad \& \quad V = \int \frac{y_1 X}{W} dx$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

$$U = -\int \sin x \cdot \operatorname{cosec} x \, dx$$

$$= -\int 1 \, dx$$

$$U = -x$$

$$V = \int \cos x \cdot \operatorname{cosec} x \, dx$$

$$V = \int \cot x \cdot dx$$

$$\cancel{-\operatorname{cosec}^2 x}$$

$$V = \log \sin x$$

$$P.I = -x\cos x + \log(\sin x) \cdot \sin x$$

Now $C.S = CF + PI$

i.e. $y = C_1 \cos x + C_2 \sin x - x\cos x + \sin x (\log \sin x)$

Q $\dfrac{d^2y}{dx^2} - y = \dfrac{2}{1+e^x}$

$$D^2 - 1 = \frac{2}{1+e^x} \quad ; D = \frac{d}{dx}$$

AUX eq^n : $m^2 - 1 = 0$

$m = \pm 1$

$C.F = C_1 e^x + C_2 e^{-x}$

$P.I = u y_1 + v y_2$

$y_1 = e^x \quad , \quad y_2 = e^{-x}$

Topic ______ Date ______ Page No. ______
Expt No. ______

$$W = \begin{vmatrix} e^{x} & e^{-x} \\ e^{x} & -e^{-x} \end{vmatrix}$$

$$\frac{e^{-x}(1-e^{x})}{(1+e^{x})(1-e^{x})}$$

$$\frac{e^{-x}-1}{1-e^{2x}}$$

$$W = -1 - 1 = -2$$

$$U = -\int \frac{y_2 \cancel{e} X}{W} \cdot dx$$

$$V = \int \frac{y_1 X}{W} dx$$

$$U = -\int \frac{e^{-x}}{\cancel{2}} \times \frac{\cancel{2}}{1+e^{x}}$$

$$V = \int \frac{-e^{x}}{\cancel{2}} \times \frac{\cancel{2}}{1+e^{x}}$$

$$U = +\int \frac{e^{-x}}{1+e^{x}} dx$$

$$V = -\int \frac{e^{x}}{1+e^{x}} \cdot dx$$

$$U = \int \frac{1}{e^{x}(1+e^{x})} dx$$

$$V = -\log(1+e^{x})$$

$$\boxed{1+e^{x} = t}$$

$$e^{x} \cdot dx = dt$$

$$\frac{1}{e^{x}(1+e^{x})} = \frac{A}{e^{x}} + \frac{B}{1+e^{x}}$$

$$\frac{1}{e^{x}} - \frac{1}{1+e^{x}}$$

$$1 = A(1+e^{x}) + Be^{x}$$

$$A = 1, B = -1$$

i.e. $\displaystyle\int \frac{1}{e^{x}(1+e^{x})dx} = \int \frac{1}{e^{x}} dx - \int \frac{1}{1+e^{x}} dx$

$$= \int e^{-x} dx - \int \frac{e^{-x}}{(1+e^{x})e^{-x}} dx$$

$$= -e^{-x} - \int \frac{e^{-x}}{e^{-x}+1} dx \quad \text{—— (1)}$$

Teacher's Signature ______

$$= -e^{-x} + \int \frac{-e^{-x}}{e^{-x}+1} \cdot dx$$

Let $e^{-x}+1 = t$

$\Rightarrow -e^{-x} \cdot dx = dt$

eqn ① will become

$$-e^{-x} + \int \frac{dt}{t}$$

$$= -e^{-x} + \log t$$

$$v = -e^{-x} + \log(e^{x}+1)$$

$$P.I = \{-e^{-x} + \log(e^{x}+1)\}e^{x} + \{-\log(1+e^{x})\}e^{-x}$$

$$= -1 + e^{x}\log(e^{x}+1) - e^{-x}\log(1+e^{x})$$

$$P.I = \log(1+e^{x})\{e^{x} - e^{-x}\} - 1$$

$$C.S = C_1 e^{x} + C_2 e^{-x} + \log(1+e^{x})\{e^{x} - e^{-x}\} - 1$$

Q $(D^2-D)y = x e^{x} \sin x$

Sol

solve $\left\{ e^{x}\ I.P.\ \text{of}\ \frac{1}{D(D+2)}\, x \sin x \right.$

$$\frac{1}{2}\left[\frac{1}{D} - \frac{1}{D+2}\right] x \sin x \qquad \frac{D+2-D}{D(D+2)}$$

$$P.I = e^{x}\left[I.P.\ \text{of}\ e^{ix}\frac{1}{(2i-1)}\left[1 + \frac{D^2 + 2D(i+1)}{(2i-1)}\right]^{-1} x\right]$$

$$PI = e^{x}\left[I.P\ \text{of}\ e^{ix}\frac{1}{(2i-1)}\left[x - \frac{2(i+1)}{2i-1}\right]\right]$$

$$= e^{x}\left\{I.P.\ \text{of}\ e^{ix}\frac{1}{(2i-1)^2}\left[(2i-1)x - 2(i+1)\right]\right.$$

D^2-1

$D(D-1)$

Topic

Expt No. Date Page No.

$$= e^x \left[I.P. \text{ of } e^{ix} \frac{1}{-4+1-4i} \left[-(x+2) + i(2x-2)\right] \right]$$

$$= e^x \left[I.P. \text{ of } e^{ix} \frac{1}{-(3+4i)} \frac{(3-4i)}{(3-4i)} \left[-(x+2) + i(2x-2)\right] \right]$$

$$= e^x \left[I.P. \text{ of } e^{ix} \frac{(3-4i)}{-25} \left[-(x+2) + i(2x-2)\right] \right]$$

$$= -\frac{e^x}{25} \left[I.P. \text{ of } e^{ix} (3-4i) \left[-(x+2) + i(2x-2)\right] \right]$$

$$= -\frac{e^x}{25} \left[I.P. \text{ of } e^{ix} \left[-3(x+2) + 8(x-1) + i\,4(x+2) + 6(x-1)\right] \right]$$

$$= -\frac{e^x}{25} \left[I.P. \text{ of } (\cos x + i\sin x)(-3(x+2) + 8(x-1) + i\left[4(x+2) + 6(x-1)\right] \right]$$

$$= -\frac{e^x}{25} \sin x\left[-3(x+2) + 8(x-1) + \cos x\left[4(x+2) + 6(x-1)\right]\right]$$

Q Apply method of variation of parameters to solve:

(i) $(D^2+1)y = \tan x$

(ii) $(D^2+1)y = \sec x$

(iii) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$

Teacher's Signature