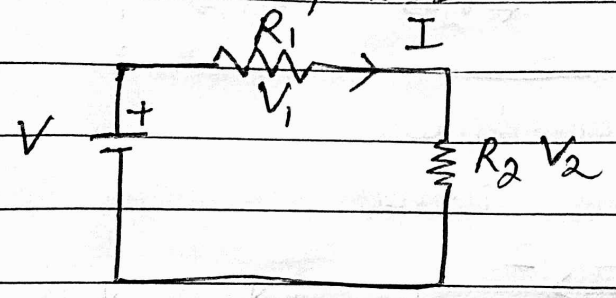
DC Circuit Analysis

Types of Electrical Circuit

- AC Ckts
  - based on
    - Types of sources
    - Types of components
- DC Ckts
  - based on
    - Types of Sources
    - Types of component used



DC Ckt

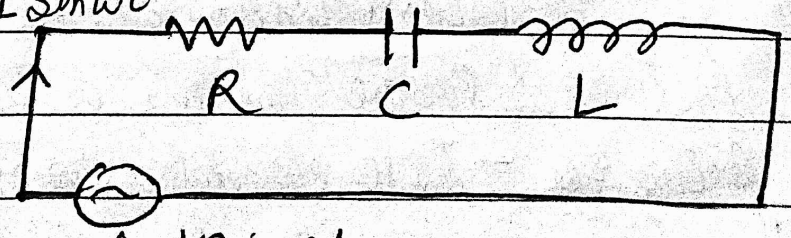
V_1 & V_2 are responses of particular ckts.

Circuits :- combination of supply & source parts

AC ckts → R, L, C diodes

→ AC sign/symbol (sinusoidal wave)

$i = I \sin \omega t$



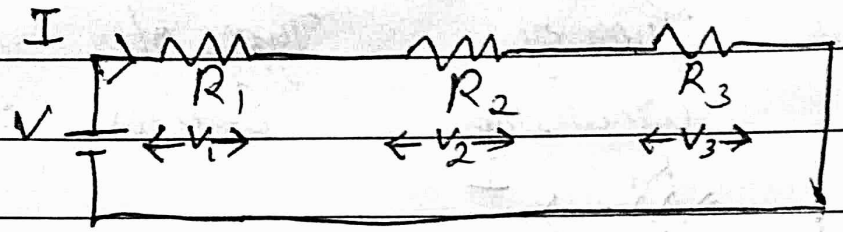
$V = V \sqrt{2} \sin \omega t$

Capacitor – stores energy in form of charge
Inductor – " " " " " magnetic field.

$$V_L = L\frac{di}{dt}$$

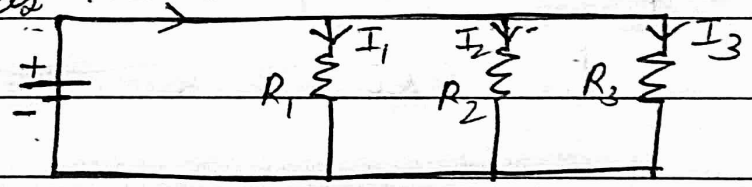
$$V_C = \frac{1}{C}\int i\,dt$$

→ For series connection :-

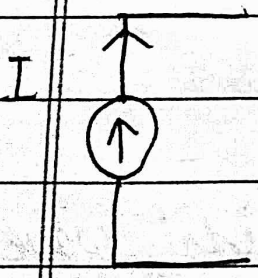


* Current should be same
* Voltage " " divided

→ For // ckts :-



* I should be divided.
* Voltage should be same accross each resistor (Potential diff)



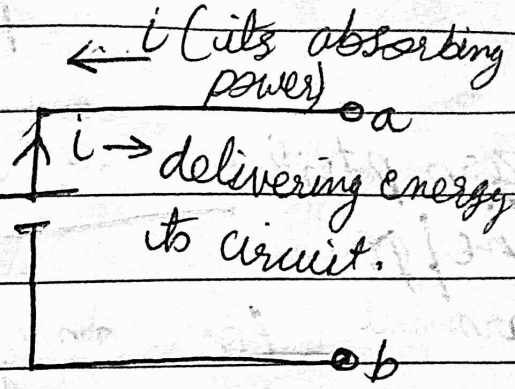
⇒ Current source is represented by this symbol

Classification of Elements

1) Active & Passive Elements :- source (Active) & components (Passive)

Active Ele. – These ele. deliver power to other elements which are present in an electric circuit. Sometimes

they may absorb power like passive ele's that means active ele's have capability of both delivering & absorbing power. Ex- voltage & i source.



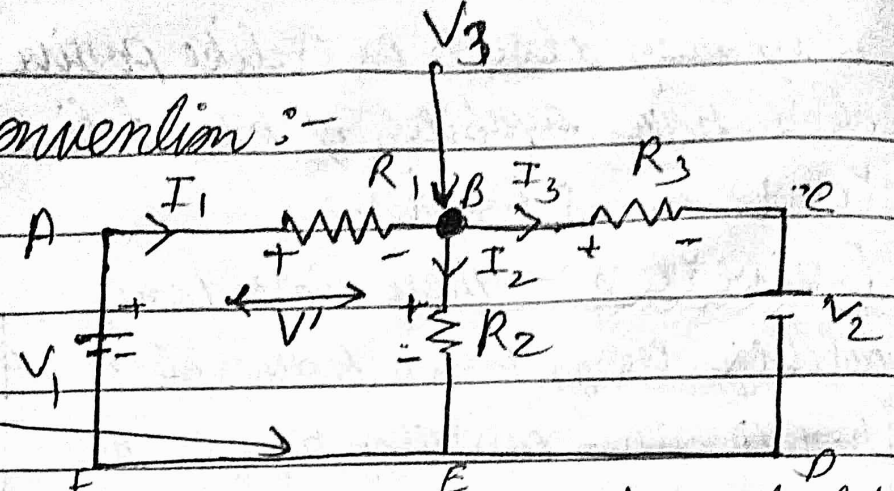
Passive Ele's :- These ele's cant deliver energy/power to other ele's however, they can absorb power that means these ele's either dissipate power in form of heat or store energy in form of either magnetic field or electric field. Ex- R, L, C.

2) Linear & Non linear Ele's :- linear ele's are ele's that show linear relationship b/w voltage & current. Ex- R, L, & C. Similarly non linear ele - these ele's are those that don't show a linear relation b/w voltage & current. ex - diode.

3) Bilateral & Unilateral Ele — Bilateral Ele are the ele that ~~show linear relationship b/w~~ allow current in both directions & offer same impedance in either direction of current flow. Ex - R, L & C.
A unilateral element has diff relation b/w voltage & current for 2 possible direction of current. Ex- diode.

16/4/2022

1 Sign Convention :-

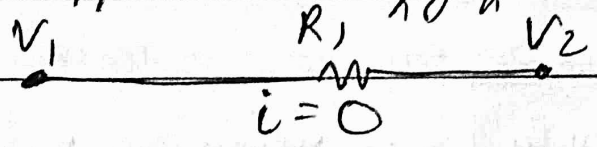


zero potential line / grounded terminal

For applying KVL & KCL we have to take closed circuit.

Only

- For KVL net potential diff will be 0.
- For KCL applicable only on junction or node.



if $V_1 = V_2 \Rightarrow V_{12} = 0$

$V_1 = 5V$, $V_2 = 0V$

$V_{12} = 5 - 0 = 5V$

or

$0 - 5 = -5V$

For above diagram:-

$\Rightarrow$ Accord to KVL

In ABEF, $V - (I_1 R_1) - (I_2 R_2) = 0$ —(1)

In BCDE, $-I_3 R_3 - V_2 - (-I_2 R_2) = 0$

$-(I_3 R_3) - V_2 + I_2 R_2 = 0$ —(2)

$V_1 - (IR_1) - I_3 R_3 - V_2 = 0$ —(3)

B node can be said as principal node.

$\Rightarrow$ Accord to KCL $\rightarrow$

$$I_1 = I_2 + I_3$$

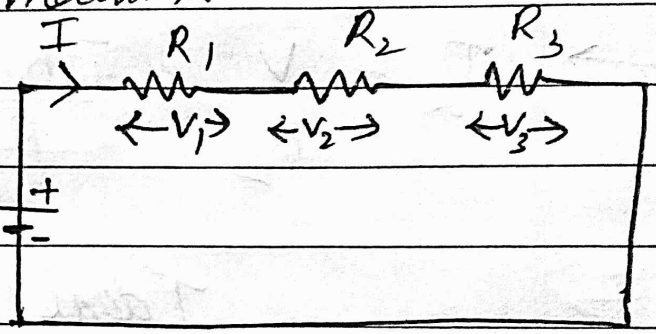
$$\frac{V_1 - V_3}{R_1} = \frac{V_3 - 0}{R_2} + \frac{V_3 - V_2}{R_3}$$

[V_1 is +ive as leaving terminal is +ive]

[V_2 is -ive as its leaving from -ive terminal.]

2. Series & parallel connections:-

1) Series :-



Voltage Division Rule →

→ $V_1 = IR_1$

$$V_1 = \frac{V}{R_{eq}} R_1$$

$$\boxed{V_1 = \frac{VR_1}{R_1 + R_2 + R_3}}$$

Condition :-

* i should be same
* Voltage should be diff across each resistor

→ $V_2 = IR_2$ *

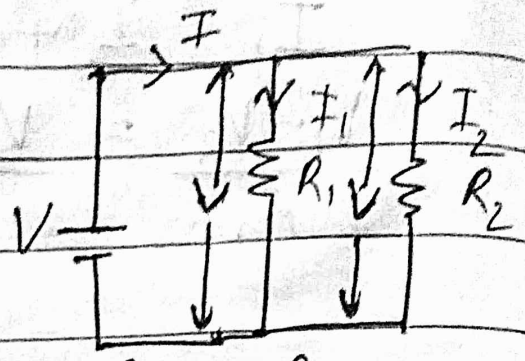
$$V_2 = \frac{V}{R_{eq}} R_2$$

$$\boxed{V_2 = \frac{VR_2}{R_1 + R_2 + R_3}}$$

→ $$\boxed{V_3 = \frac{IR_3}{R_1 + R_2 + R_3}}$$

2) Parallel Connection →

Current division Rule :-



→ $I_1 = \frac{V}{R_1}$

$I_1 = \frac{I R_{eq}}{R_1} = \frac{I \cancel{R_1} R_2}{(R_1+R_2)\cancel{R_1}}$

$I_1 = \frac{I R_2}{R_1+R_2} =$

Cond Req :-

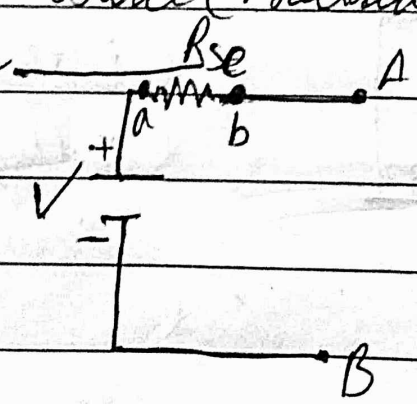
* Voltage remain same across each resistor
* current will divide across each resistor

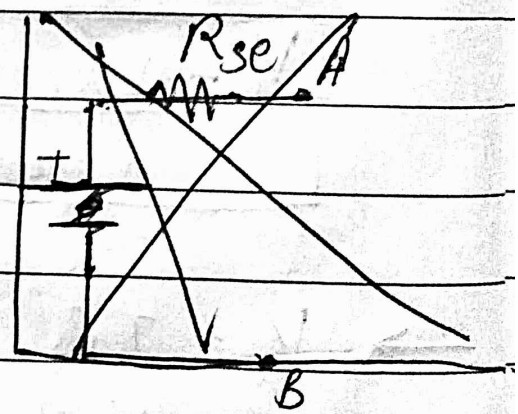
→ $I_2 = \frac{V}{R_2} = \frac{I R_{eq}}{R_2} = \frac{I R_1 \cancel{R_2}}{(R_1+R_2)\cancel{R_2}} = \frac{I R_1}{R_1+R_2}$

Ideal & Practical Sources:

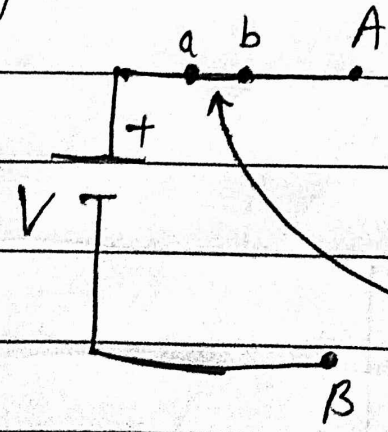
1) Voltage Source (Practical) →

Internal resistance series



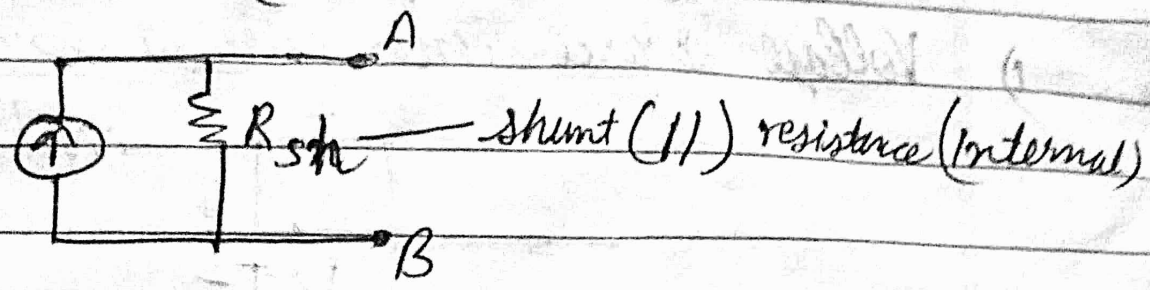


Voltage source (ideal) →

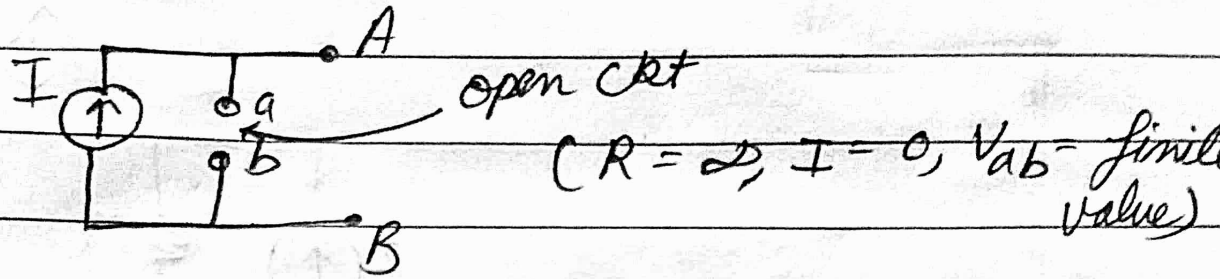


Short ckt (circuit) → ($R_{se}=0$, $I=\infty$, $V_{ab}=0$)

2) Current Source (Practical):-

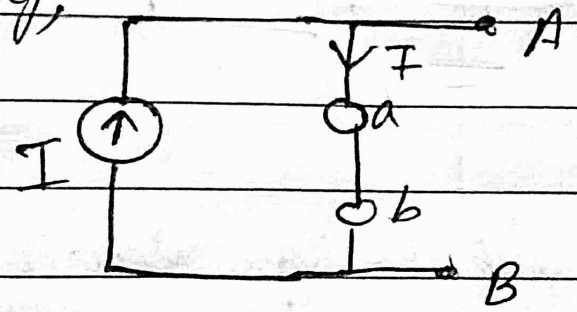


Current Source (Ideal):-



($R = \infty$, $I = 0$, V_{ab} = finite value)

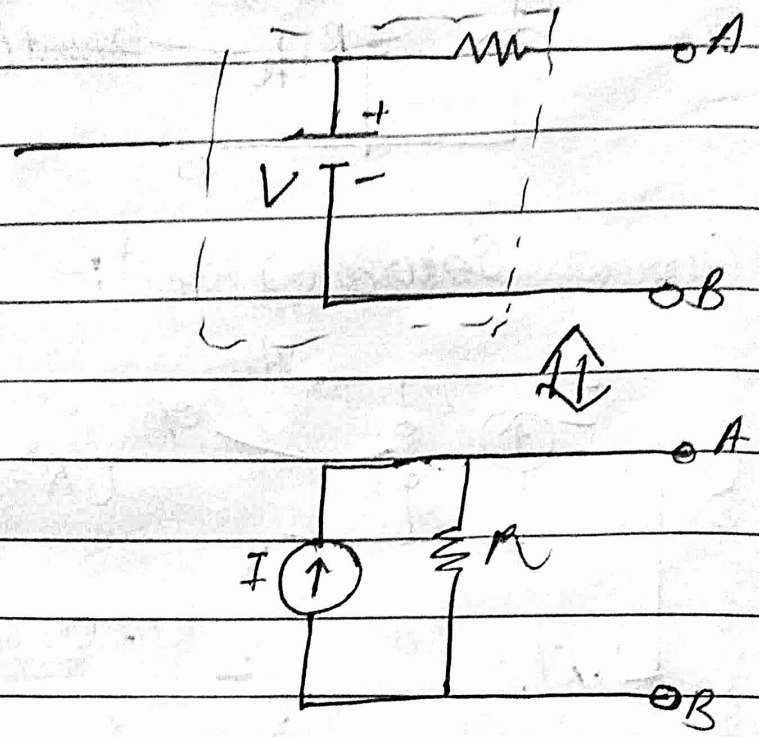
if, a & b is connected through short ckt.



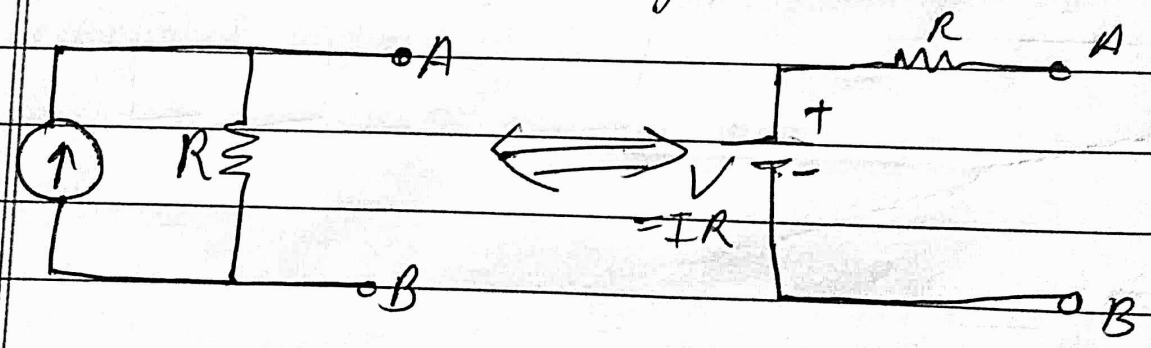
all current will move through a & b not in AB.

Source Transformation.

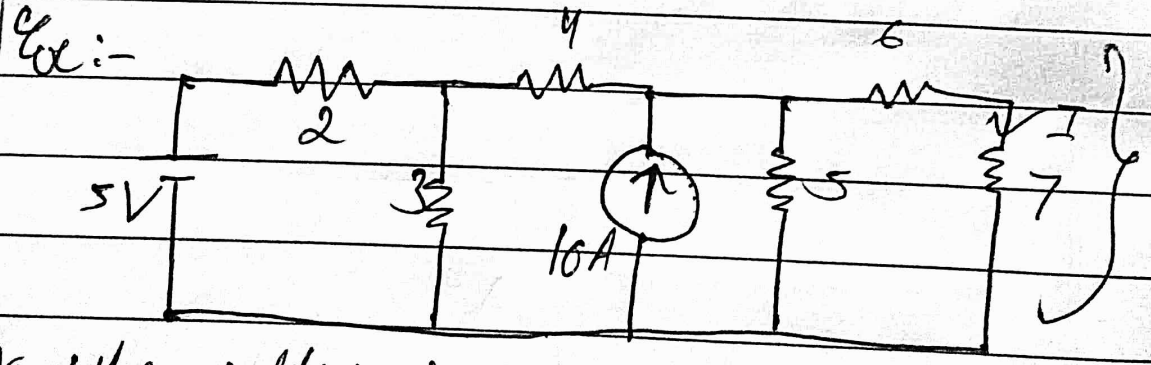
1) Voltage source into current source:-



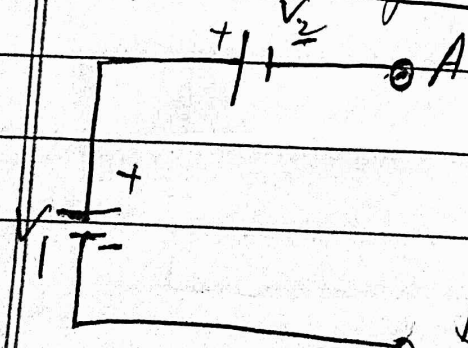
2) Current source into Voltage source:-



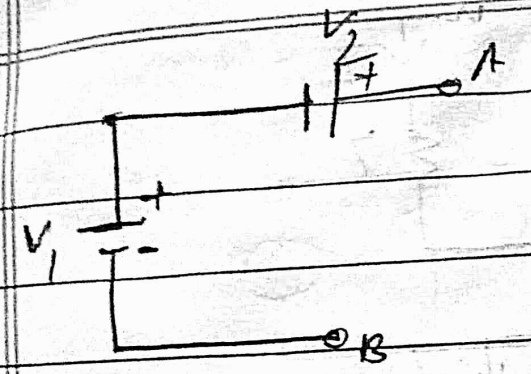
Ex:-



* <u>When voltage sources are connected in series</u> :-

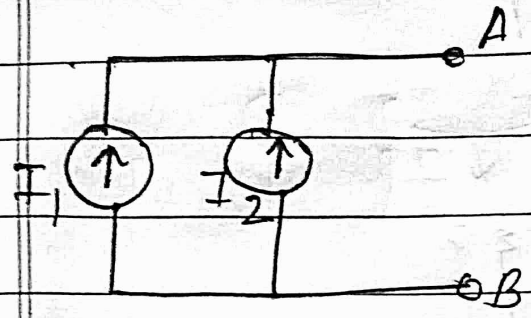


$\Rightarrow V_{AB} = V_1 - V_2$

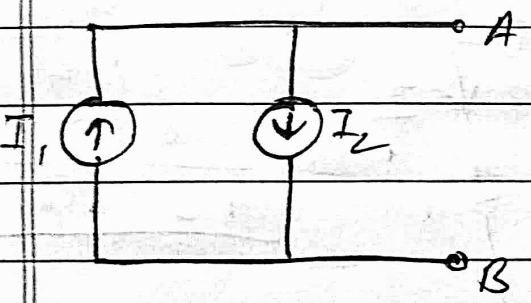


$$V_{AB} = V_1 + V_2$$

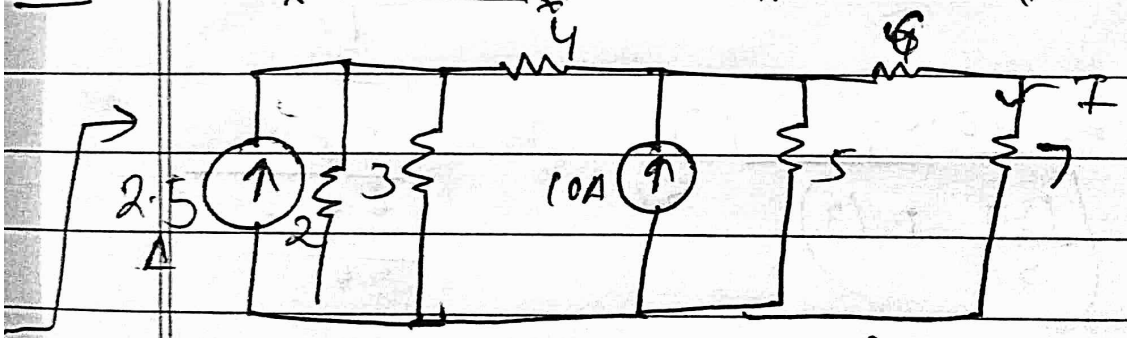
* When current sources are connected in ||. :-



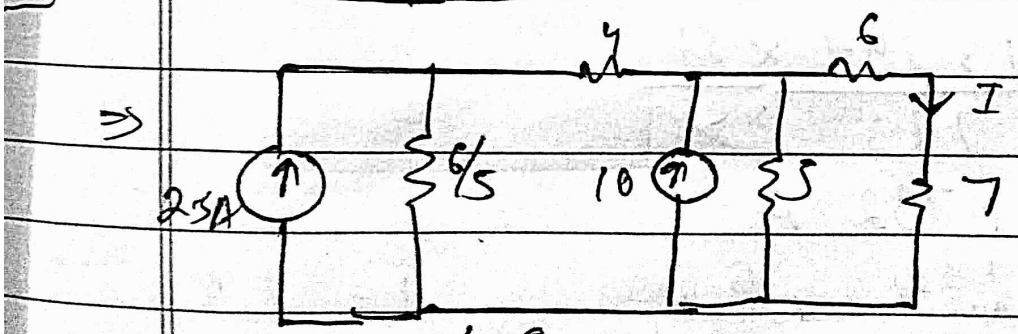
$$I = I_1 + I_2$$



$$I = I_1 - I_2$$

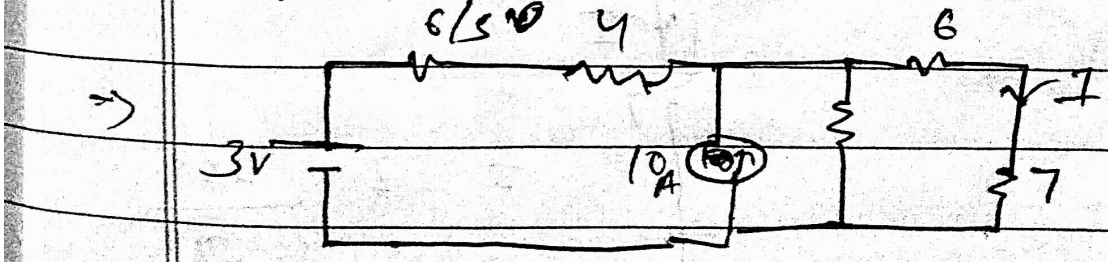


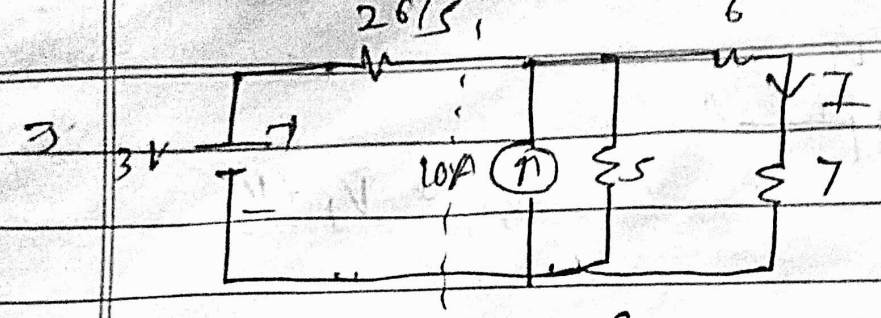
$\frac{1}{2} + \frac{1}{3}$
$= \frac{5}{6}$

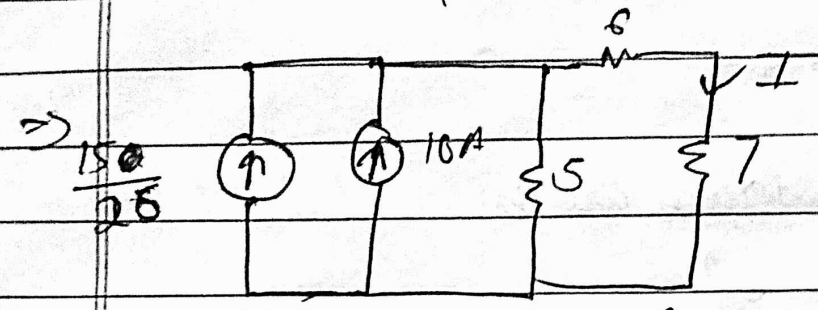


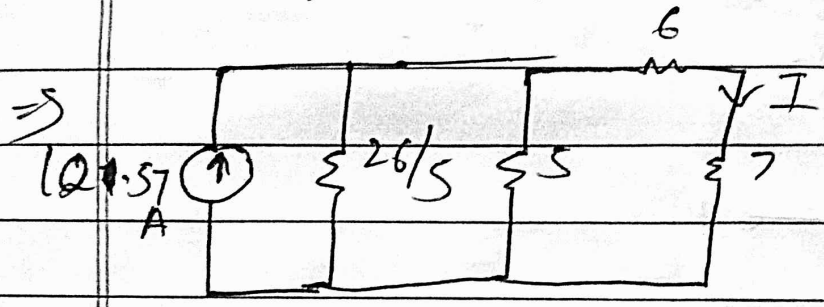
$\frac{6}{5} + 4$

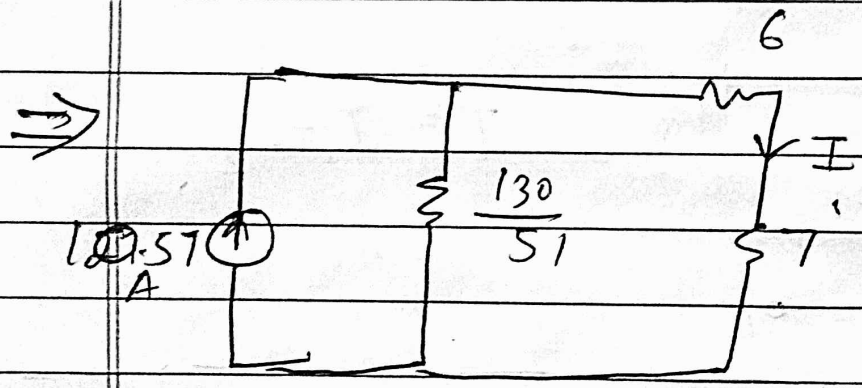
$\frac{26}{5}$











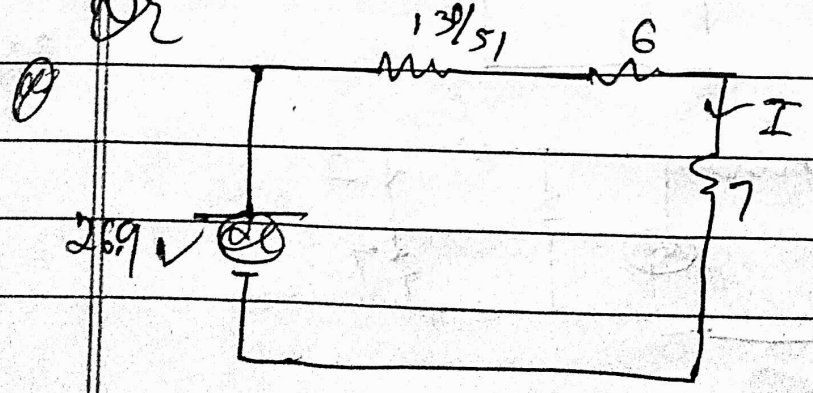
$$\begin{array}{r} 10.57 \\ \times 13 \\ \hline 317 \\ 1057\times \\ \hline 137.41 \end{array}$$

$$I = \frac{10.57 \times 130}{51} \Big/ \left(\frac{130}{51} + 13\right) = \frac{65\times}{663} \quad \begin{array}{r} 13 \\ \times 51 \\ \hline 13 \\ 65\times \\ \hline 663 \\ 130 \\ \hline 793 \end{array}$$

$$= \frac{1374.1 \times \cancel{51}}{793 \times \cancel{51}}$$

$\Rightarrow 1.732\ A$

Or



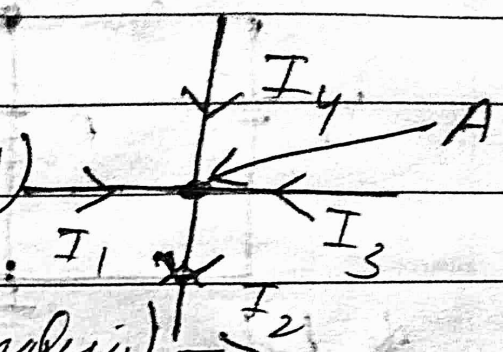
$V = IR$

$= I = \frac{26.9}{15.57}$

$= 1.730\ A$

Kirchoff's Current Law :-

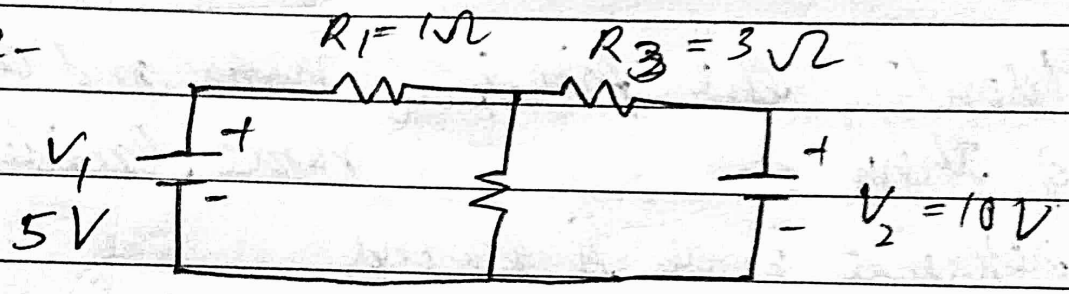
According to KCL the algebric sum of currents meeting at a junction in an electrical circuit is zero.



$$I_1 + I_4 + I_3 - I_2 = 0 \quad -(1)$$

Node voltage Analysis (Nodal Analysis) $\Rightarrow$ Completely relying on KCL.

Ex :-



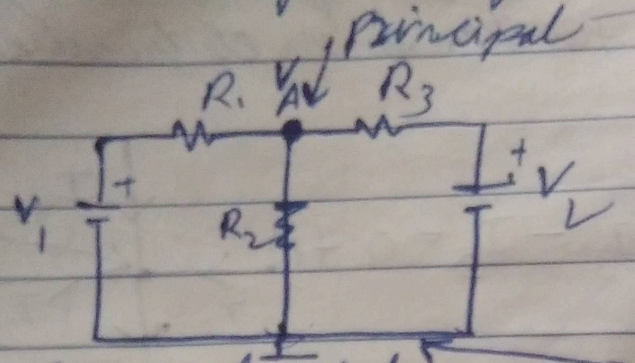
Reference node (ground node) = where potential = 0 (0 resistance)

23/9/2022

Date.
Page No.

Q1

Node Voltage Analysis → (Based on KCL) →



* Principal Node
* Reference Node (ground node)

Step 1:- Identify the principal nodes and choose one of them as reference node. Consider that the reference node is ground.

Step 2:- Label the node voltages with respect to ground from all the principal nodes, except the reference nodes.

Step 3:- Write nodal eqs at all principal nodes except the reference node. Nodal eq is obtained by applying KCL first & then ohms law.

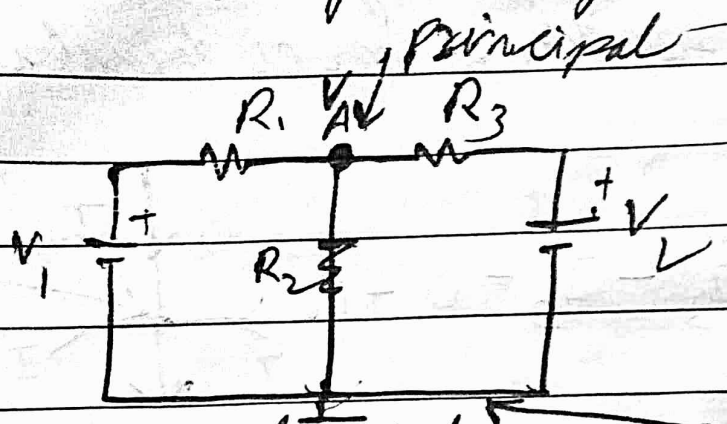
Step 4:- Solve the nodal eqs obtained in step 3 in order to get node voltages.

$$\frac{V_A-(-V_1)-0}{R_1}+\frac{V_A-0}{R_2}+\frac{V_A-(-V_2)-0}{R_3}=0$$

Q2

23/9/2022

Node Voltage Analysis → (Based on KCL) →



* Principal Node
* Reference Node (ground node)

Step 1:- Identify the principal nodes and chose one of them as reference node. Consider that the reference node is ground.

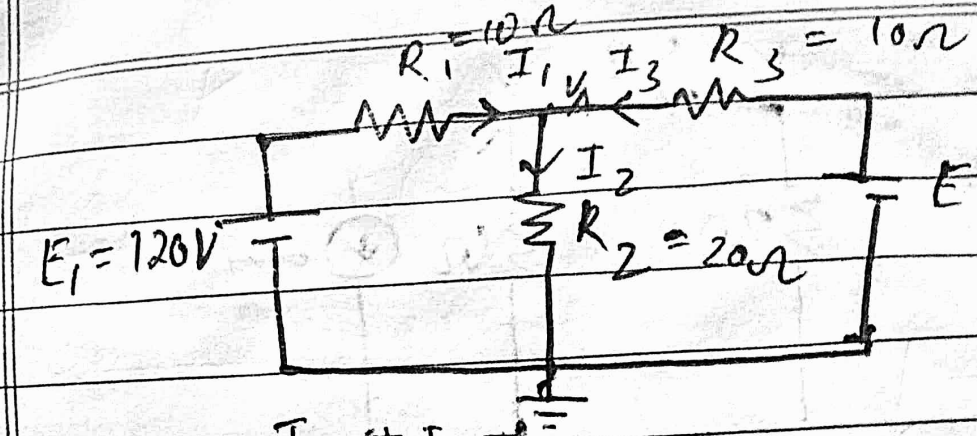
Step 2:- Label the node voltages with respect to ground. for all the principal nodes, except the reference nodes.

Step 3:- Write nodal eqs at all principal nodes except the reference node. Nodal eq is obtained by applying KCL first & then ohms law.

Step 4:- Solve the nodal eqs obtained in step 3 in order to get node voltages.

$$\frac{V_A - (-V_1) - 0}{R_1} + \frac{V_A - 0}{R_2} + \frac{V_A - (-V_2) - 0}{R_3} = 0$$

Q1



$$I_1 + I_3 = I_2$$

$$\frac{120 - V_A}{R_1} + \frac{E_2 - V_A}{R_3} = \frac{V_A - 0}{20}$$

$$\frac{120 - V_A}{10} + \frac{65 - V_A}{10} = \frac{V_A}{20}$$

$$\frac{120 - V_A + 65 - V_A}{1} = \frac{V_A}{2}$$

$$2(185 - 2V_A) = V_A$$

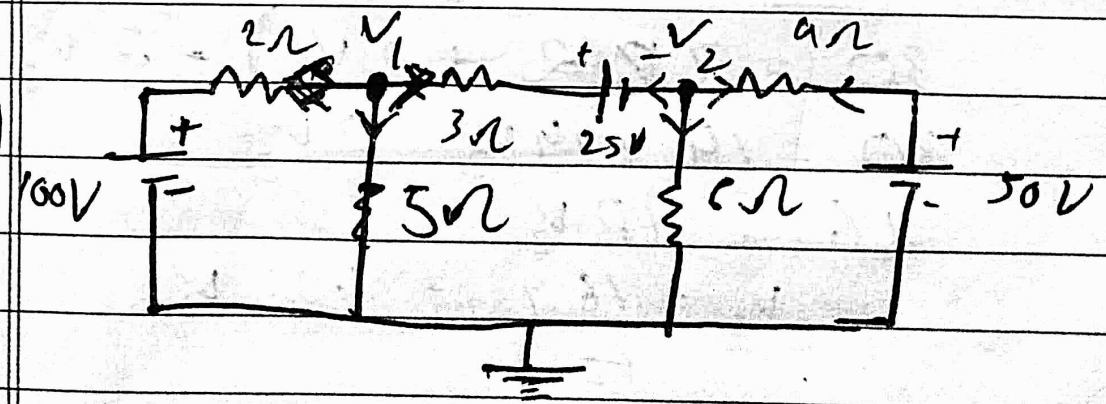
$$185 \times 2 = 5V_A$$

$$37 \times 2 = V_A$$

$$V_A = 74V$$

$$I_2 = \frac{74}{20} = 3.7A$$

Q2)

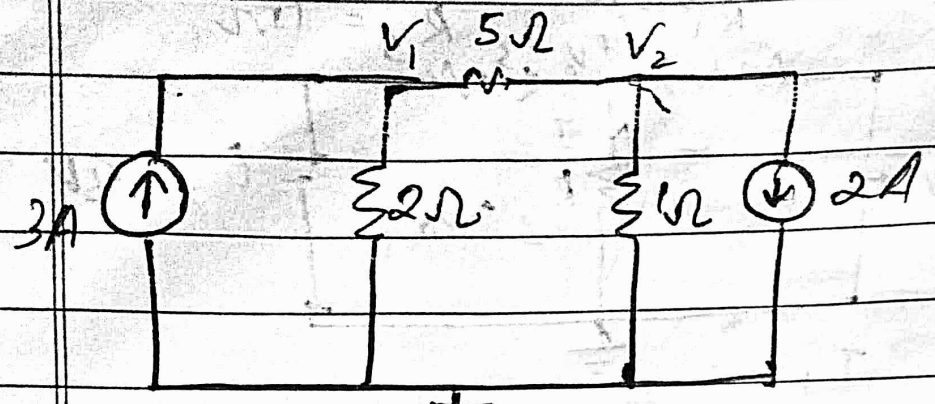


$$\frac{V_1 + (-100)}{2} + \frac{-25 + V_1 - V_2}{3} + \frac{V_1 - 0}{5} = 0 \quad \text{(i)}$$

$$\frac{V_2 + 25 - V_1}{3} + \frac{V_2}{6} + \frac{V_2 - 50}{4} = 0 \quad \text{(ii)}$$

Date.
Page No.

26/9/2022



Find current across each branch by using node voltage analysis?

Soln

$$3 = \left(\frac{V_1 - 0}{2}\right) + \left(\frac{V_1 - V_2}{5}\right) \quad -(1)$$

$$2 + \frac{V_2 - 0}{1} + \frac{V_2 - V_1}{5} = 0 \quad -(2)$$

$$3 = \frac{5V_1 + 2V_1 - 2V_2}{10}$$

$$30 = 7V_1 - 2V_2 \quad -(III)$$

$$2 + 2V_2 - V_1 = 0$$

$$2 + 2V_2 = V_1 \quad -(IV)$$

$$30 = 7(2 + 2V_2) - 2V_2$$

$$30 = 14 + 14V_2 - 2V_2$$

$$16 = 12V_2$$

$$V_2 = \frac{16}{12} = 1.33\ V$$

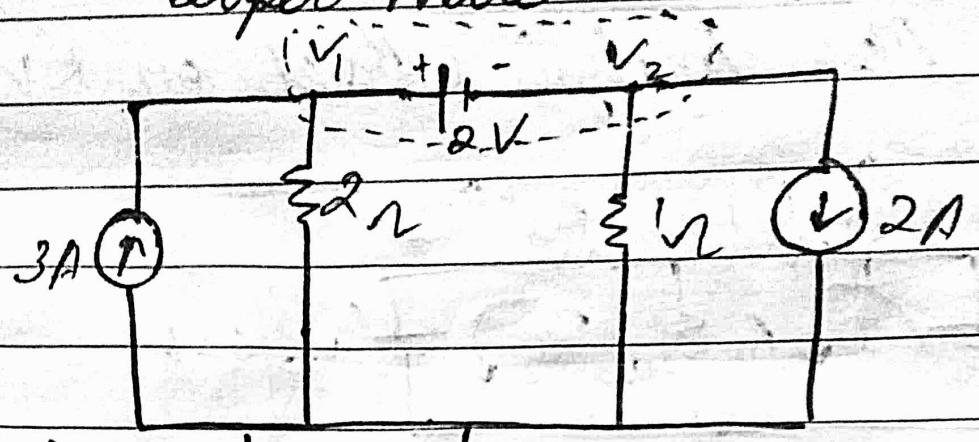
$$V_1 = 2 + 1.66$$

$$V_1 = 3.66\ V$$

$$I_1 = \frac{V_1}{R_1} = \frac{3.66}{2} = 1.83\ A$$

$$I_2 = \frac{V_2}{R_2} = \frac{1.33}{1} = 1.33A$$

Super Node Condition



If there is no resistor b/w 2 nodes, or voltage source.

$$3 = \left(\frac{V_1 - 0}{2}\right) + \left(\frac{V_2 - 0}{1}\right) + 2 \quad \text{—(1)}$$

$$V_1 - 2 - V_2 = 0 \quad \text{—(2)}$$

$$\rightarrow \quad V_1 = 2 + V_2$$

(1) $$6 = V_1 + 2V_2 + 4$$

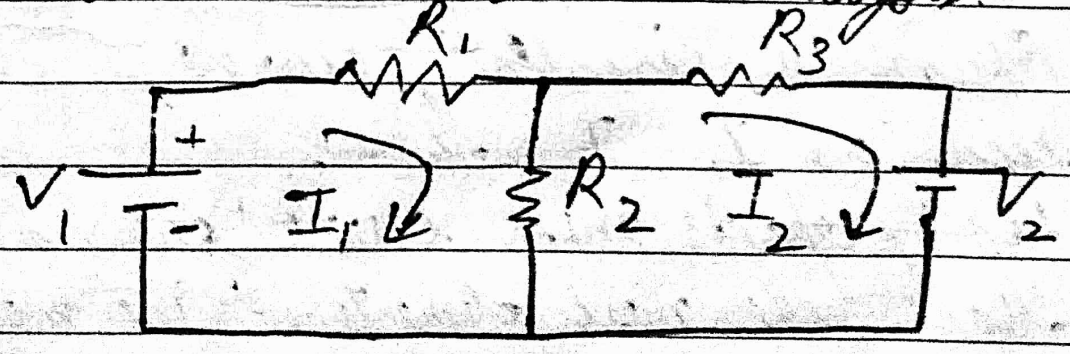
$$2 = V_1 + 2V_2$$

$$\cancel{2} = \cancel{2} + V_2 + 2V_2$$

$$0 = V_2$$

$$V_1 = 2 + 0 = 2V$$

Mesh Analysis (Loop Analysis) or Mesh Current Analysis

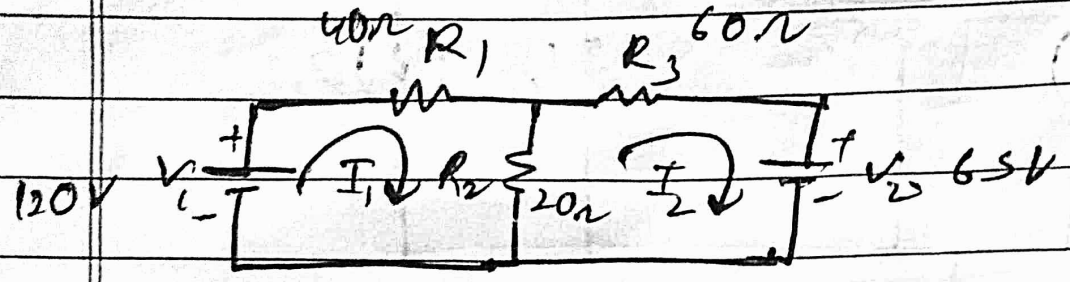


Mesh is an individial path.

$$+V_1 - I_1 R - (I_1 - I_2) \times R_2 = 0 \quad \text{—(1)}$$

$$-V_2 - I_2 R_3 - R_2 (I_2 - I_1) = 0 \quad \text{—(2)}$$

30/9/2022 Mesh Analysis (Based on KVL)



KVL states that the algebric sum of products of current & resistance in each of conductors in any closed path or mesh in a network + algebric sum of EMF's in that path is 0.

$$\sum IR + \sum emf's = 0$$

Steps to solve:-

(1) Identify the no of meshes in given circuit. Mesh is the elementary loop of the network & also known as essential mesh / independent mesh.

(2) Assign a circulating current to each mesh such as I_1, I_2, etc.

(3) ~~Its pref~~ As mesh current enters as well as leave the mesh elements. ∴ The mesh current satisfies KCL. Its preferable to assign the

same direction to mesh currents in each mesh

(4) Write KVL eqns for each mesh.

(5) Solve the mesh " to determine the mesh currents.

← Q Find current across each resistance.

Sol

$$120 - I_1 R_1 - (I_1 - I_2) R_2 = 0$$

$$-60 I_1 + 20 I_2 = -120 \quad -(1)$$

$$-3 I_1 + I_2 = -6 \Rightarrow I_2 = -6 + 3 I_1 = -0.341 A$$

$$-60 \times I_2 - 65 - 20(I_2 - I_1) = 0$$

$$-60 I_2 + 20 I_1 = 65 \quad -(2)$$

$$I_1 = 1.886 A.$$

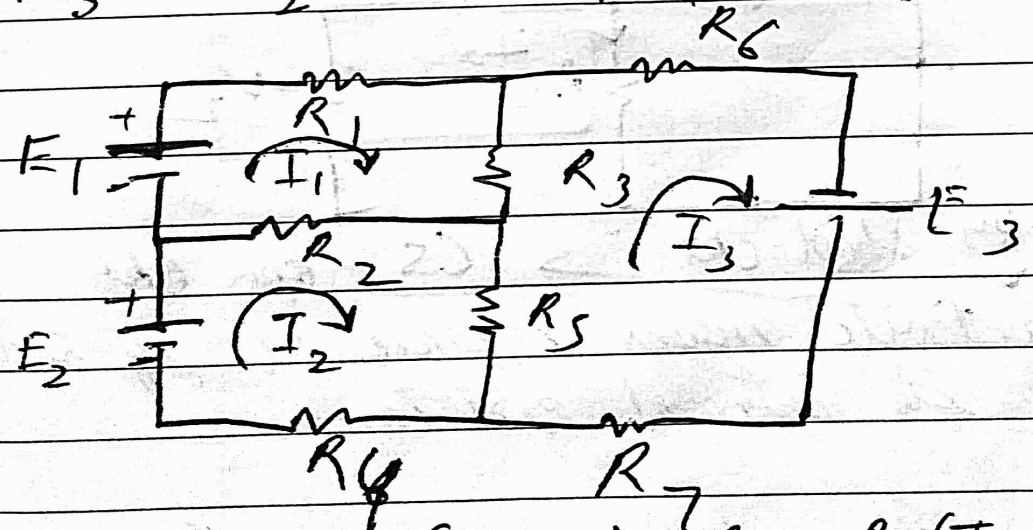
$$\begin{array}{r} 1.887 \\ 0.341 \\ \hline 2.227 \end{array}$$

$R_1 \Rightarrow I_1 = 1.886 A$

$R_2 \Rightarrow 2.227$

$R_3 \Rightarrow I_2 = -0.341 A$

Q



$$E_1 - I_1 R_1 - (I_1 - I_3) R_3 - R_2 (I_1 - I_2) = 0$$

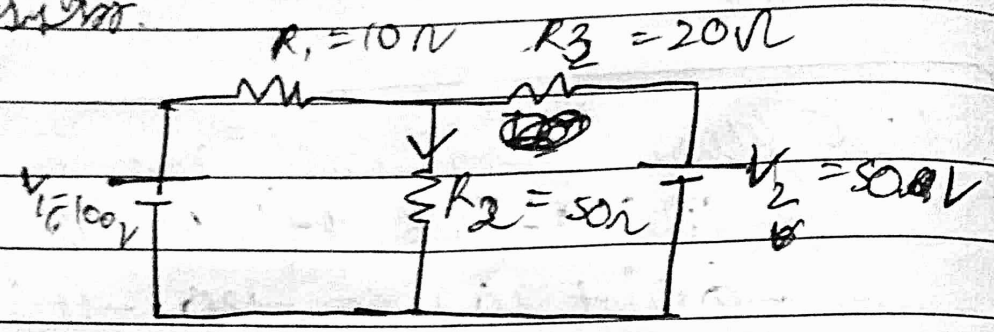
$$E_2 - R_2 (I_2 - I_1) - R_5 (I_2 - I_3) - R_4 I_2 = 0$$

$$E_3 - R_7 \times I_3 - R_5 (I_3 - I_2) - R_3 (I_3 - I_1) = 0$$

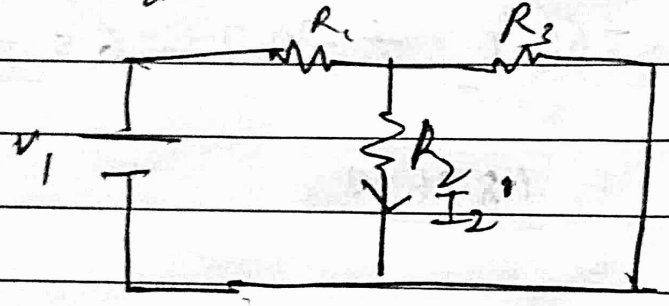
3/10/2022

Superposition Theorem

Only ckt having more than 1 E.S.
To find current through any branch resistor.

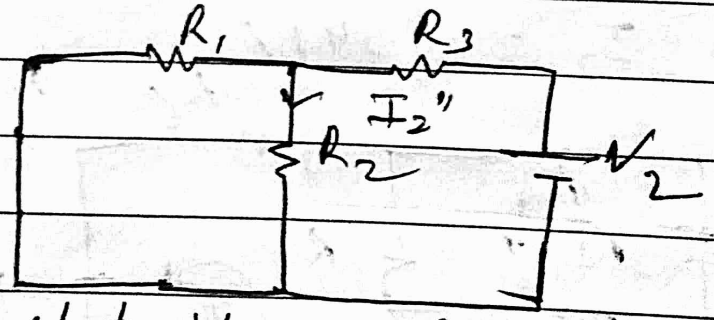


When V_2 is deactivated



$I_2 = I_2' + I_2''$

When V_1 is deactivated



by short ckt → CS - open ckt

Deactivate means replace energy source by its internal resistance.

In a linear bilateral, dc network, containing more than one energy source, the current

through any element is equal to the algebraic sum of current for that element produced by each energy source acting alone in all other sources deactivated.

The voltage source is deactivated by short ckt.

internal resistance = short ckt

Similarly current source is deactivated by open ckt.

internal resistan = infinite.

I_2

Solving the using superposition theorem:-

Step 1:- Activate V_1 & deactive V_2 (by short ckt) Using CDR:-

$$I_2' = \frac{I_{total} \times 20}{20+50}$$ — Resistor of opp brna — Resis of (opp branch + current branch)

$$I_{total} = \frac{V_1}{10 + \frac{50 \times 20}{50+20}} = \frac{10\cancel{0}}{1\cancel{00}} \times 7\cancel{0}$$

$$= \frac{70}{17}$$

$$= 4.11 A.$$

$$I_2 = \frac{4.11 \times 2\cancel{0}}{7\cancel{0}} = 1.76 A.$$

Activate V_2 & deactivate V_1 by (SC)

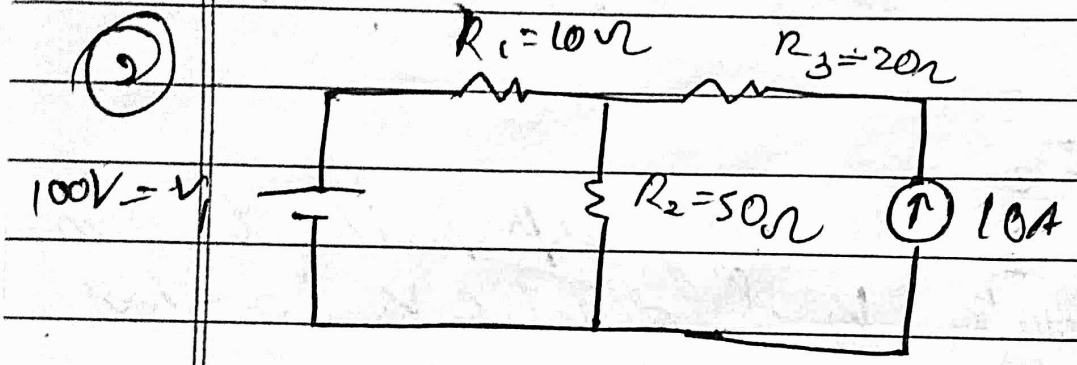
Step 2:- By using CDR:-

$$I_2'' = \frac{I_{total} \times 10}{50+10} = \frac{41.1}{60} = 0.294\,A$$

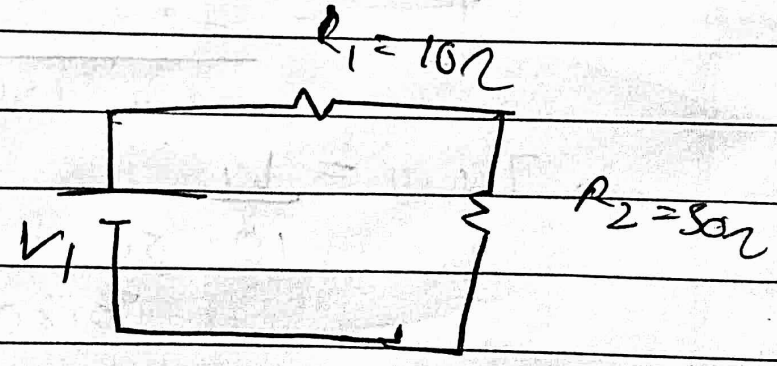
$$I_{total} = \frac{V_2}{\left(\frac{50 \times 10}{50+10}\right) + 20} = 1.76\,A$$

$$I_2 = I_2' + I_2''$$
$$= 1.176 + 0.294$$
$$= 1.470\ A$$

(2)

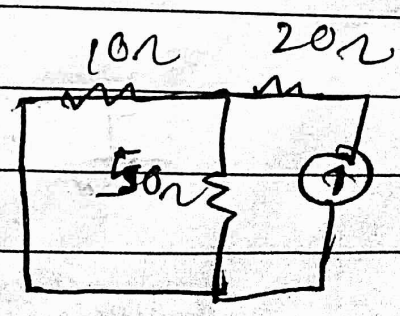


Step 1:- I-2



$$I_2' = \frac{V_1}{60} = \frac{100}{60} = 1.66\,A$$

Step 2

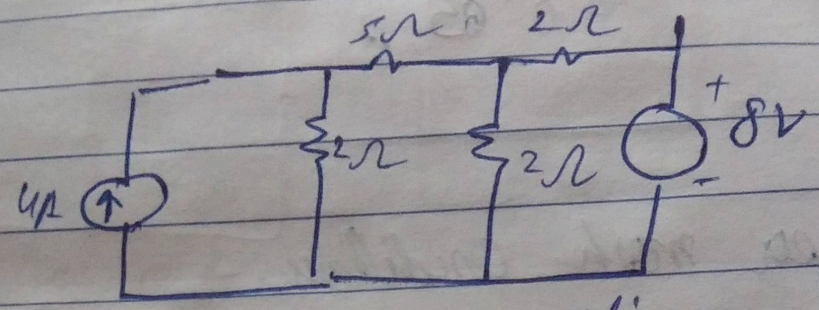


$$I_2'' = \frac{10 \times 10}{50+10}$$
$$= 1.66\,A$$

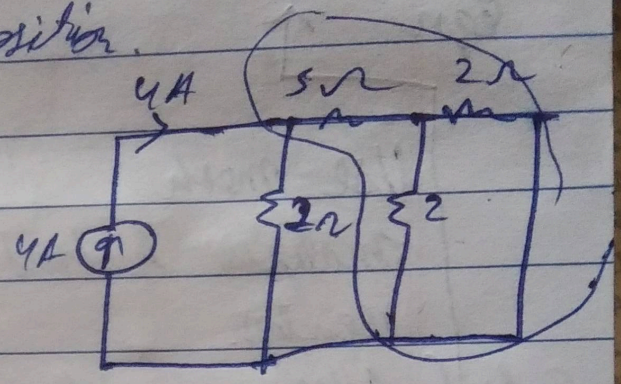
$I_2 = I_2'' + I_2'$

$= 1.66 + 1.66 = 3.32\ A$

(3)



I in 5Ω using superposition.



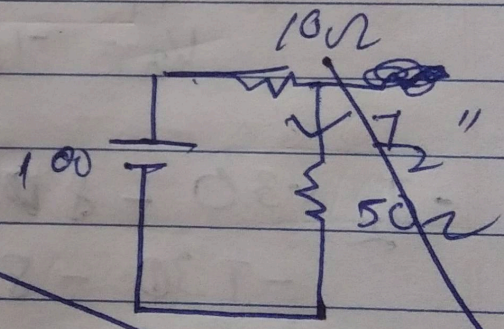
Step1:-

~~Operation~~

$I' = \frac{4 \times 2}{2+6}$ (Resistance of opp)

$= \frac{8}{8} = 1A$

Step2:-



$I_2'' = \frac{100}{60}$

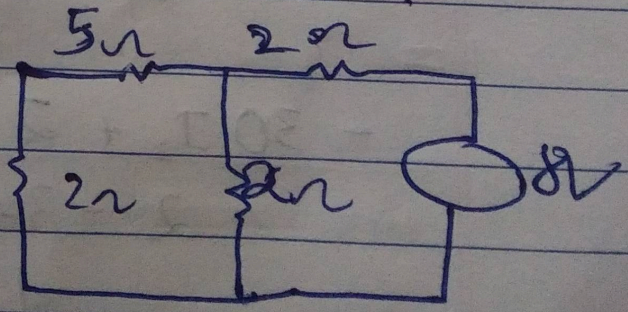
$I_2'' = 1.66\ A$

$I_2 = I_2'' + I_2' = 1 + 1.66$

$= 2.66\ A$

$I'' = \frac{I_{total} \times 2}{2+7}$

$= 0.5\ A$

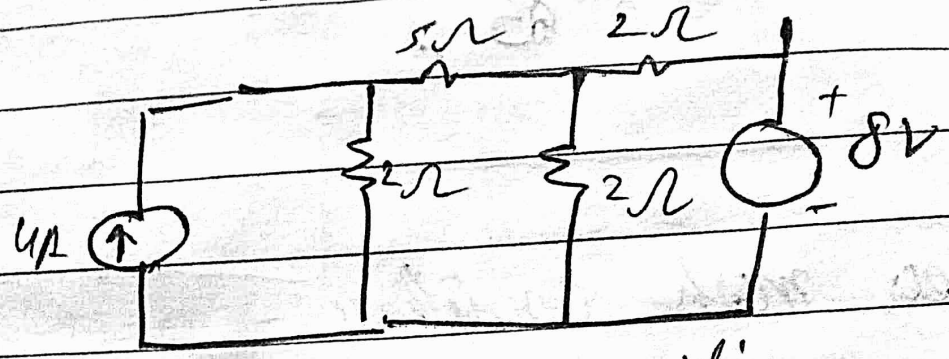


$I_{total} = \frac{8}{2+6}$

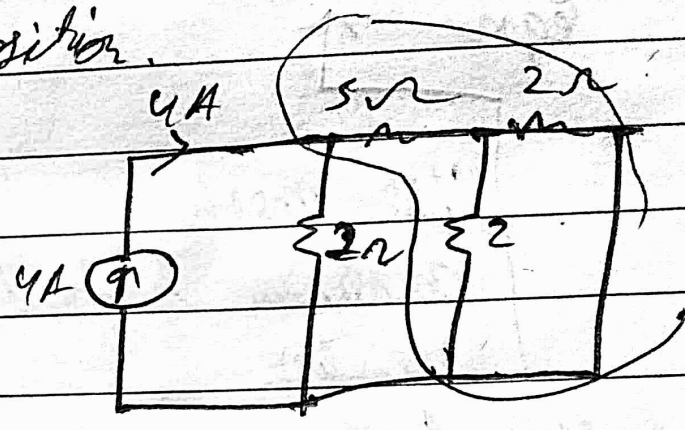
$I_2 = I_2'' + I_2'$

$= 1.66 + 1.66 = 3.32\ A$

(3)



I in 5Ω using superposition.

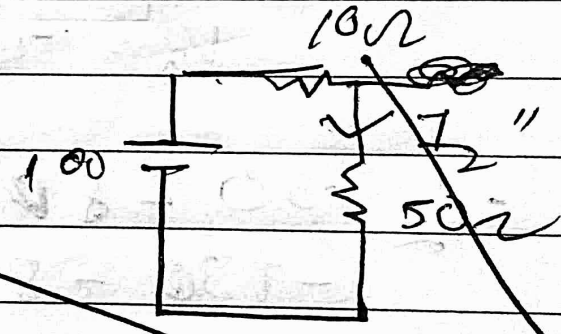


Step1:-

$I' = \frac{4 \times 2}{2+6}$ (Resistance of opp)

$= \frac{8}{8} = 1\ A$

Step2:-

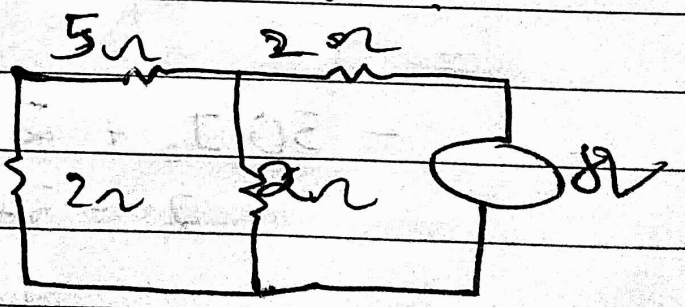


$I_2'' = \frac{100}{60}$

$I_2'' = 1.66\ A$

$I_2 = I_2'' + I_2' = 1 + 1.66$

$= 2.66\ A$



$I'' = \frac{I_{total} \times 2}{2+7}$

$= 0.5\ A$

$I_{total} = \frac{8}{2+6}$

$I = 1 - 0.5$

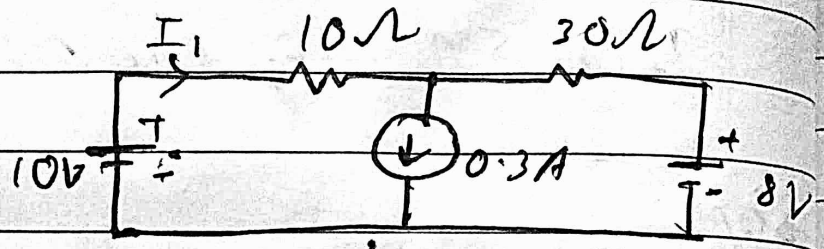
$= 1 - 0.5$

$= 0.5\ A$

7/10/2022

Super mesh conditions :-

Eqn



Use mesh analysis to find current through each resistor.

Solu: Whenever b/w 2 nodes there is a single current source then the card is known as super mesh conditions :-

$10 - I_1 R_1 - V_x = 0$ —(i)

$V_x = -10 + I_1 R_1$

$-I_2 \times 30 - 8 + V_x = 0$ —(ii)

$-I_2 30 - 8 - 10 + I_1 R_1 = 0$

$-30 I_2 - 8 + 10 - 10 I_1 = 0$ —(iii)

$I_1 - I_2 = 0.3A$ —(iv)

$I_1 = 0.3 + I_2$

$-30 I_2 + 2 - 10(0.3 + I_2) = 0$

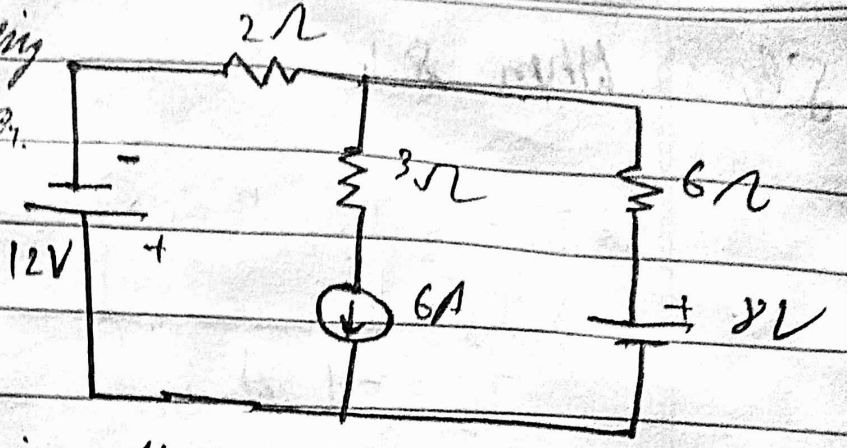
$2 - 3 - I_2 = 30 I_2$

$-1 = 31 I_2$

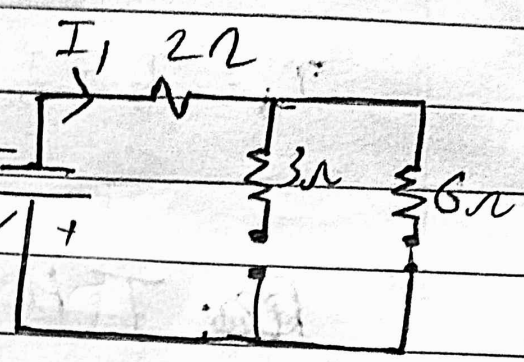
$I_2 = -\frac{1}{31} = -0.028A$

$I_1 = 0.3 - 0.028 = 0.275$

Q2) Find I by using superposition theorem.



Step 1:- Only 12V is active



$\cancel{I_1 = \frac{12}{2+6} = 1.5A}$

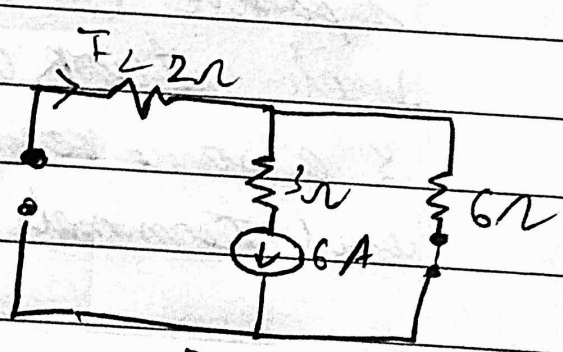
$\frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = 2$

$-12 - 2I_1 - 6I_1 = 0$

$-12 = 8I_1$

$I_1 = -1.5A$

Step 2) When 6A is active.



$I_2 = \frac{6 \times 6}{6+2}$

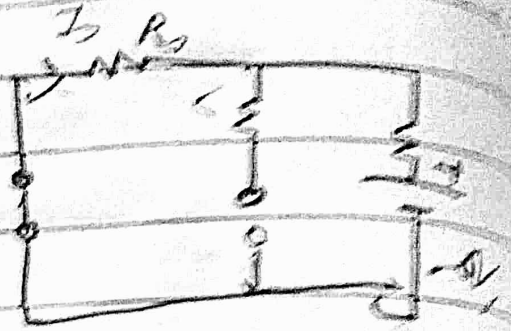
$= \frac{36}{8} = 4.5A$

[C.D.R]

$I_2 = \frac{\text{Total } i \times \text{opposite}}{\text{Sum of opposite}}$

if I_2 direction would be down the the -sign will be ahead of 4.5A.

Step 3 - When 8 V is active →



$-2I_3 - 6I_3 - 8 = 0$

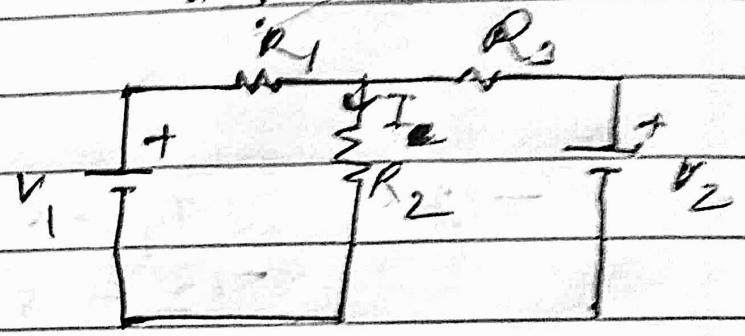
$-8I_3 - 8$

$I_3 = -1\ A.$

$I_0 = I_1 + I_2 + I_3 = -1.5 + 4.5 - 1$

$= 2.0A$

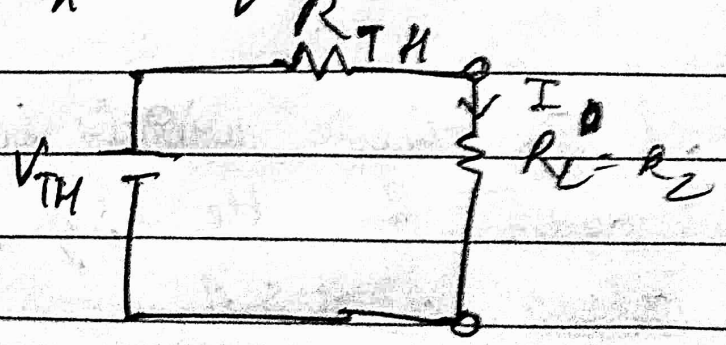
THEVENIN THEOREM



It states that any linear bilateral network containing one or more energy sources & passive elements can be replaced by a simple network containing a single voltage source & a single resistance in series across the load terminals.

⇓

THEVENIN Equivalent Ckt

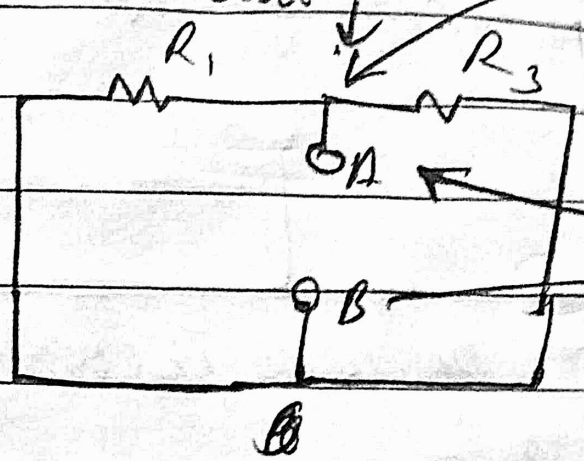


Calculate RTH :-

Step 1 = Remove load resistance & make make it open ckt.

Step 2 = Deactivate / Replace the source by its internal resistance.

RTH ckt dia

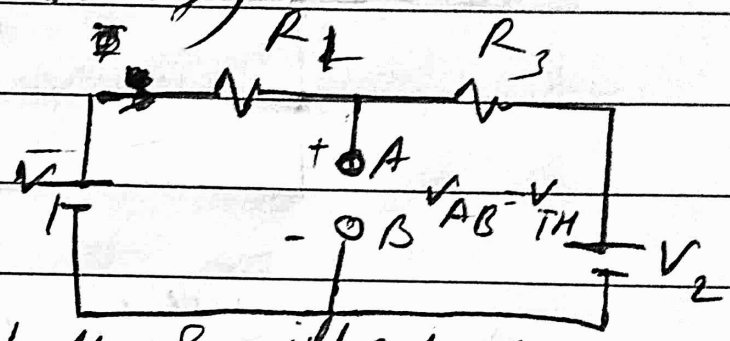


div of current on comp the branch same for opp side.

Same voltage

$$R_{AB} = R_{TH} = R_1 \| R_3$$
$$= \frac{R_1 R_3}{R_1 + R_3}$$

$V_{TH} \rightarrow$ (Open Ckt voltage)

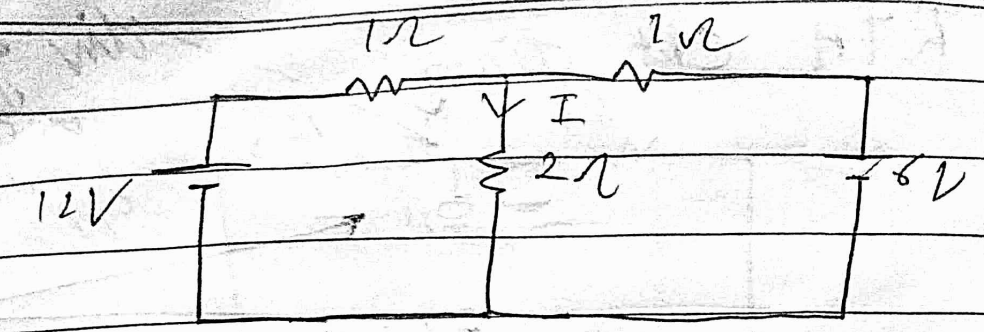


Here we will find Voltage across R_1 & R_2 then subtract both & will get V_{AB}.

As the current is same flowing in R_1 & R_3.

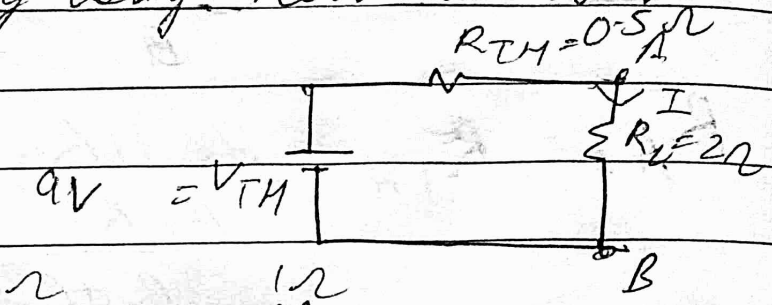
10/10/2021

Q1)

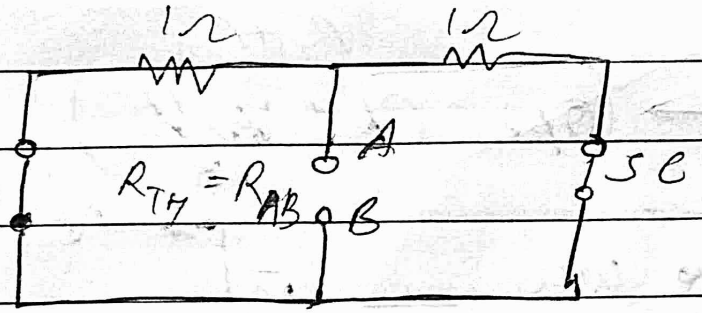


Find I by using Thevenin Theorem

Soln

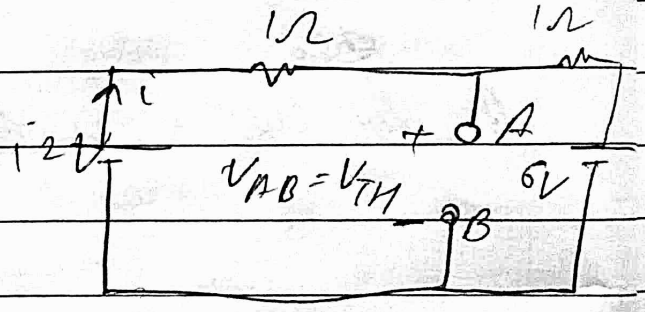


$R_{TH} =$



$R_{TH} = (1 \parallel 1) = 0.5\Omega$

$V_{TH} =$



$12 - i - i - 6 = 0$

$6 = 2i$

$i = 3\ A$

for V_{TH},

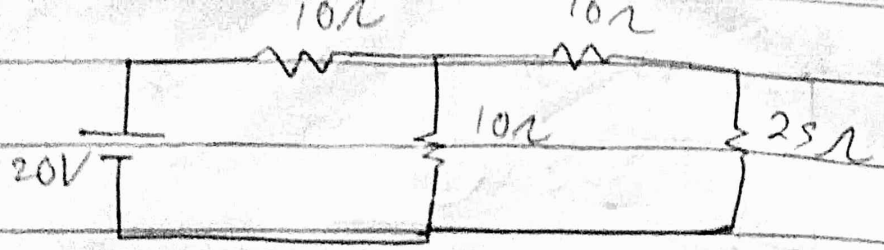
$12 - 3 \times 1 - V_{TH} = 0$

$V_{TH} = 12 - 3$

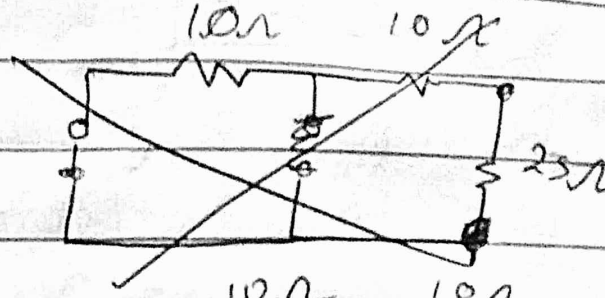
$V_{TH} = 9V$

$I = \frac{V}{R} = \frac{9}{0.5} = 18\ A$

Q) Ex



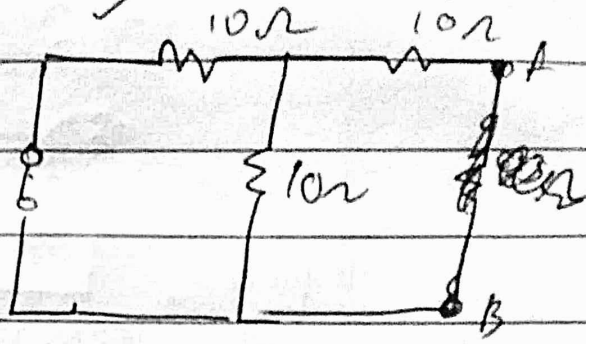
$$R_{TH} = 10 + \left(\frac{1}{10} + \frac{1}{10}\right)$$



$$= 10 + \left(\frac{2}{10}\right)$$

$$\Rightarrow 10 + \left(\frac{10}{2}\right)$$

$$= 15\Omega$$



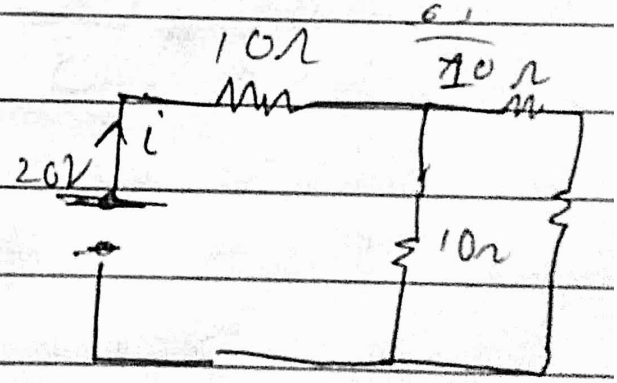
$$= 15\Omega$$

$$\begin{array}{r} 5 \,\big)\, 35-10 \\ 2 \,\big)\, 7-2 \\ 7 \end{array}$$

$$\begin{array}{r} 17 \\ 9 \,\big)\, 160 \\ 70 \\ 61 \end{array}$$

$V_{TH} =$

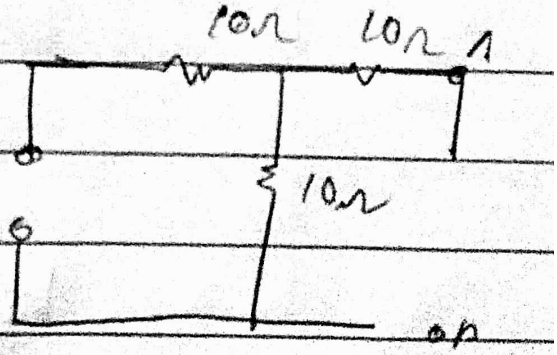
$$i = \frac{20}{10+0} = 1A$$



$$20 - 10 - (10 \times 0) - V_{TH} = 0$$

$$10 = V_{TH}$$

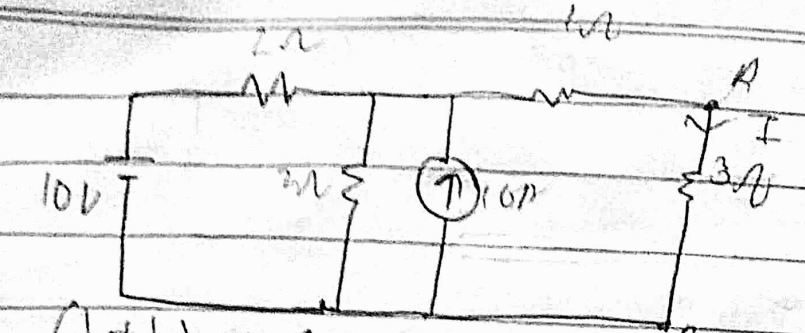
$$V_{TH} = 10V$$



$$R_{eq} = \frac{10}{40\{15+25\}} = 0.25\Omega$$

Date.
Page No.

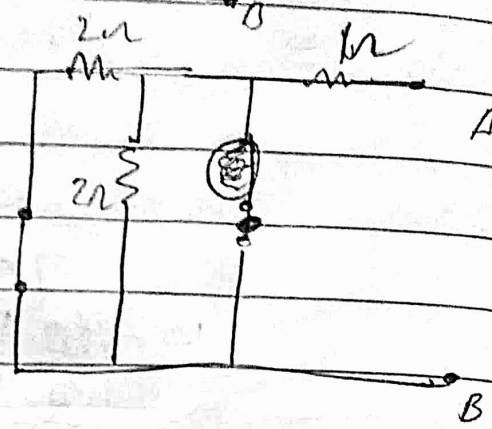
Q3)



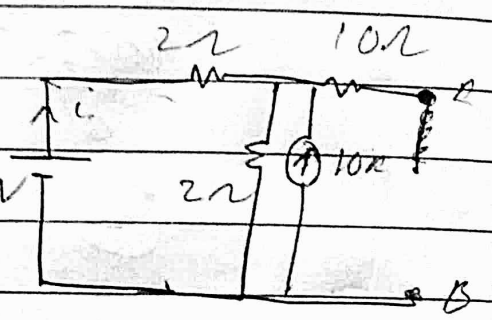
Soln

$\left(\frac{1}{2}+\frac{1}{2}\right) \Rightarrow 1\Omega$

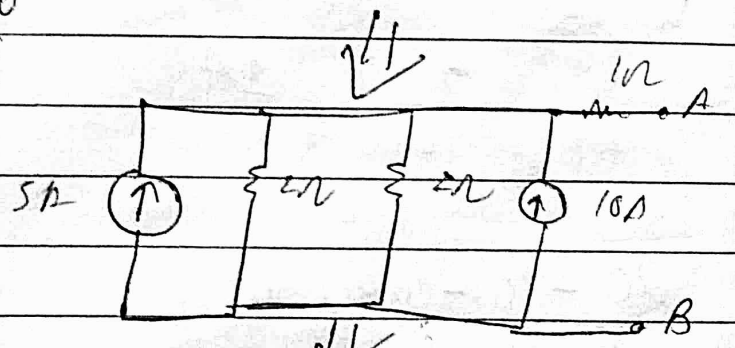
$R_{TH} = 1 + 1$
$= 2\Omega$

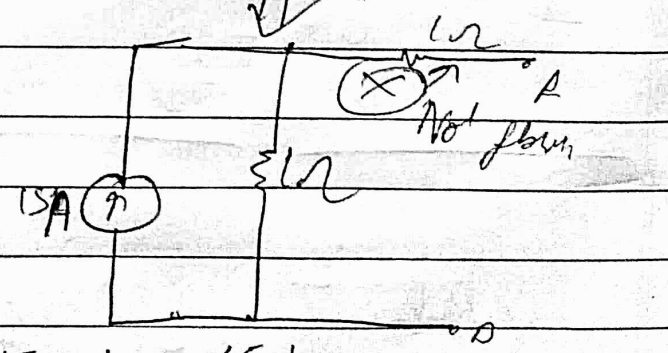


$V_{TH} \Rightarrow$



$10 - 2i - 2i = 0$
$4i = 10$
$i = \frac{10}{3}$





$V_{TH} = 15 \times 1 = 15V$

$I = \frac{V}{R} = \frac{15}{5\{2+3\}} = 3A$

Date
Page No.

14/10/2022

NORTAONS THEOREM :

↓

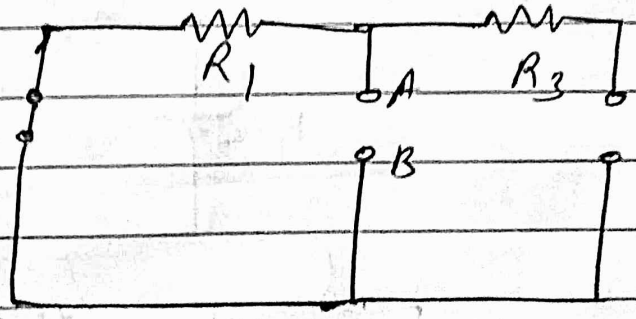
Nortans Equivalent circuit

I_{SC} or I_N

short circuit current.

This theorem states that any linear bilateral network containing one or more energy sources & passive elements can be replaced by a simple network containing a single current source & a single resistance in // across the load terminals.

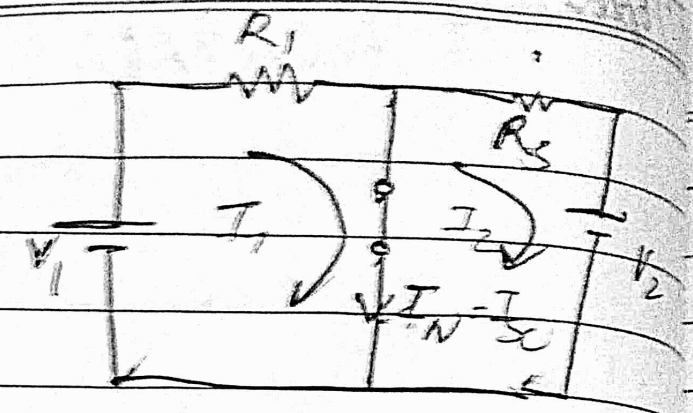
→ Soln :



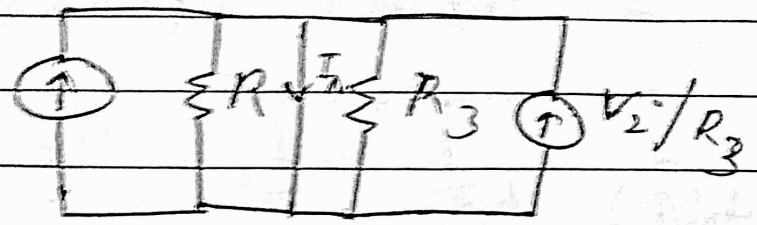
$$R_{AB} = R_N = R_1 \,//\, R_3 = \frac{R_1 R_3}{R_1 + R_3}$$

I_N or I_{SC} ⟹

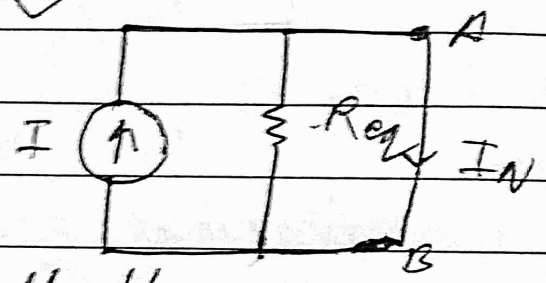
I_N or I_{sc} ⇒



⇓



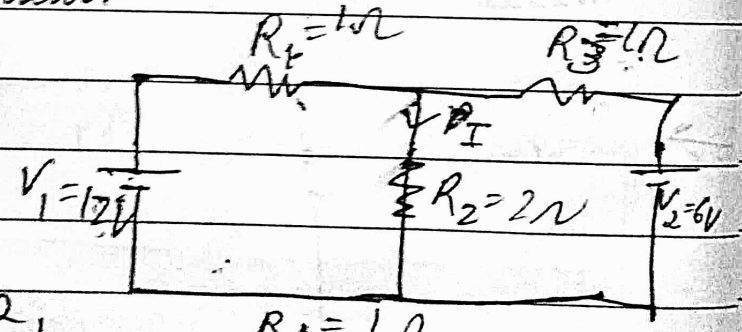
⇓

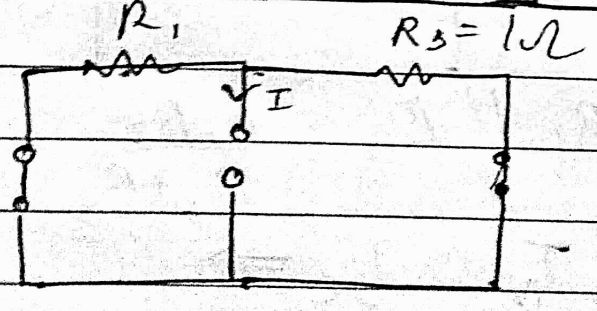


$I_N = I$ (as all the current will flow from ~~A~~ AB as it has 0 resistance)

Numerical.

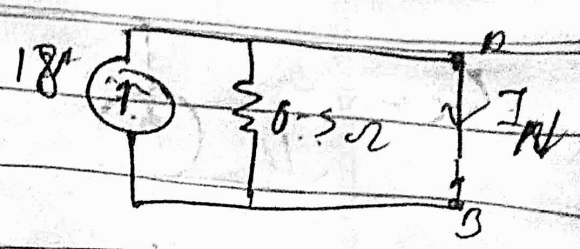
Q1)





$$R_n = \frac{1 \times 1}{1+1} = \frac{1}{2} = 0.5\,\Omega$$

$I_N =$

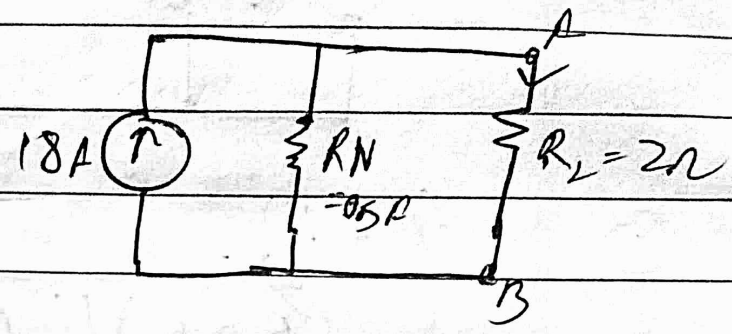


$= 18\,A$

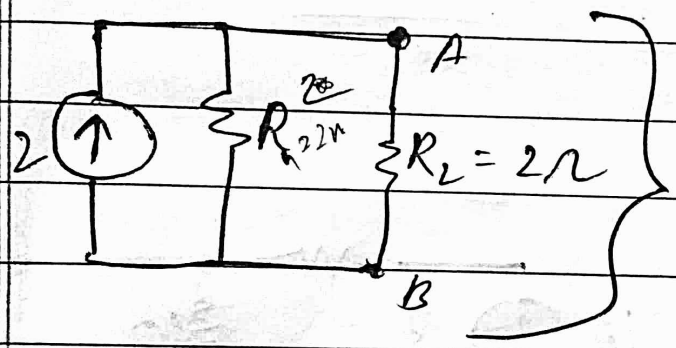
$\cancel{I_N = I_1 R_1}$

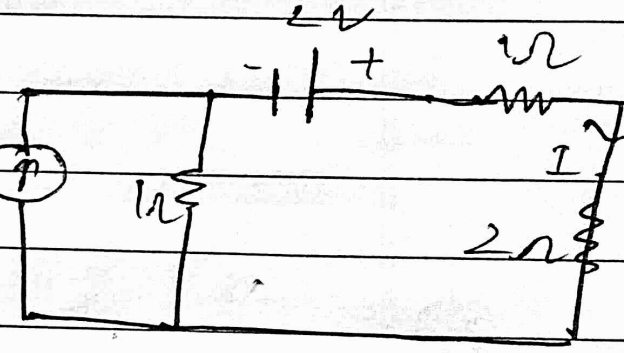
$I = \dfrac{18 \times 0.5}{2 + 5.5}$

$I = 3.6\,A$



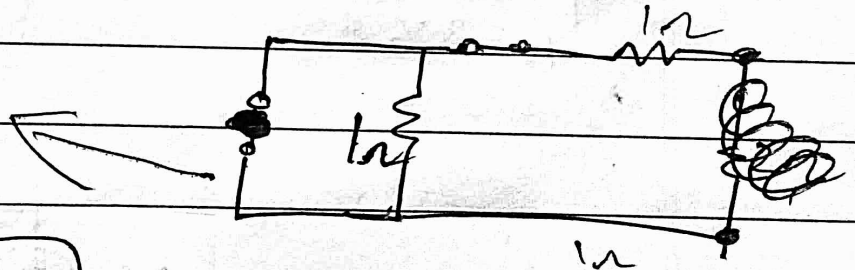
Q2)

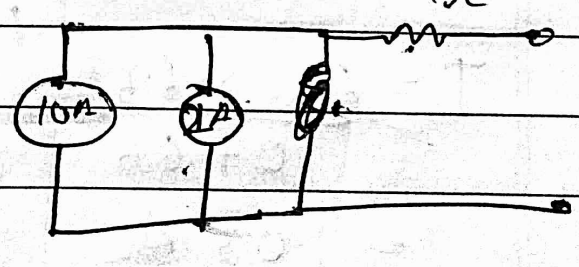




(1)

$R_N = 2\Omega$





(2) $I = \dfrac{12 \times 12}{2 + 12}$

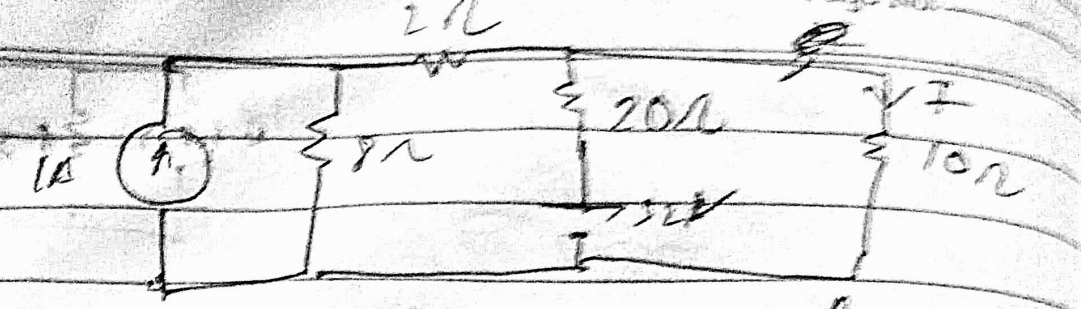
$= \dfrac{144}{14}$

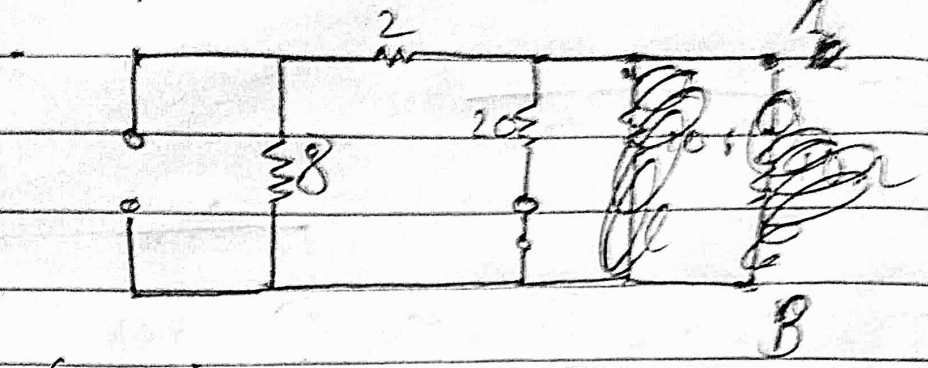
$= 8.8\,A$

(2) $12 = I_n \times 1$

$I_n = 12\,A$

$i_x =$





① $(R_1 + R_2) = 10\,\Omega$

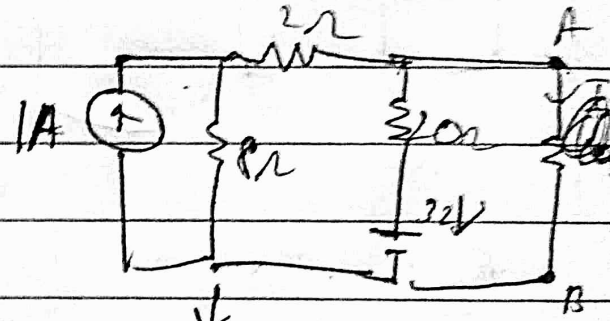
$(R_1 + R_2) \,||\, R_3 = \frac{1}{10} + \frac{1}{20}$

$= \frac{1+2}{20}$

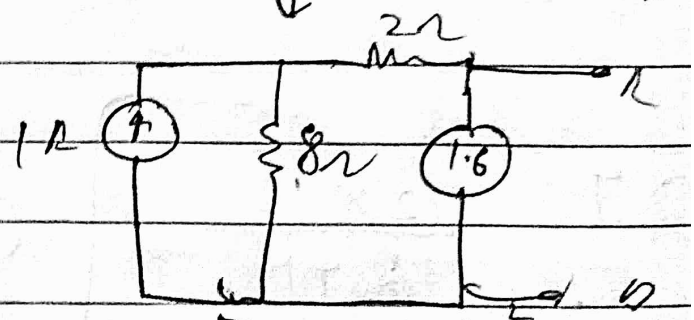
$= \frac{20}{3} = 6.66\,\Omega$

$I = \frac{32}{20} = 1.6$

②

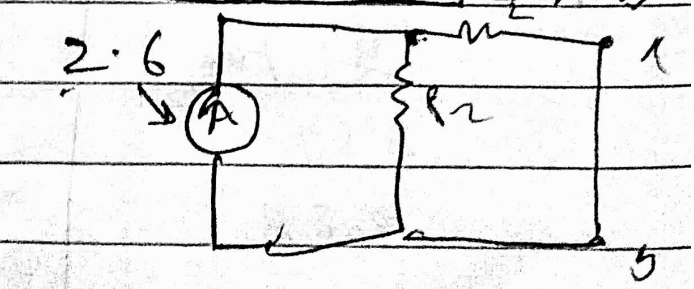


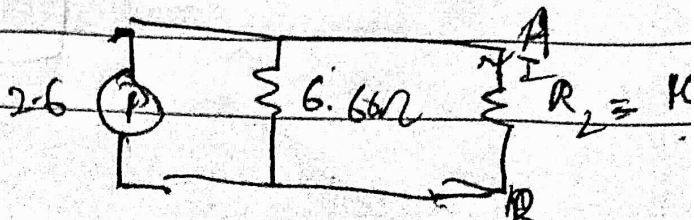
$I_N = 2.6\,A$



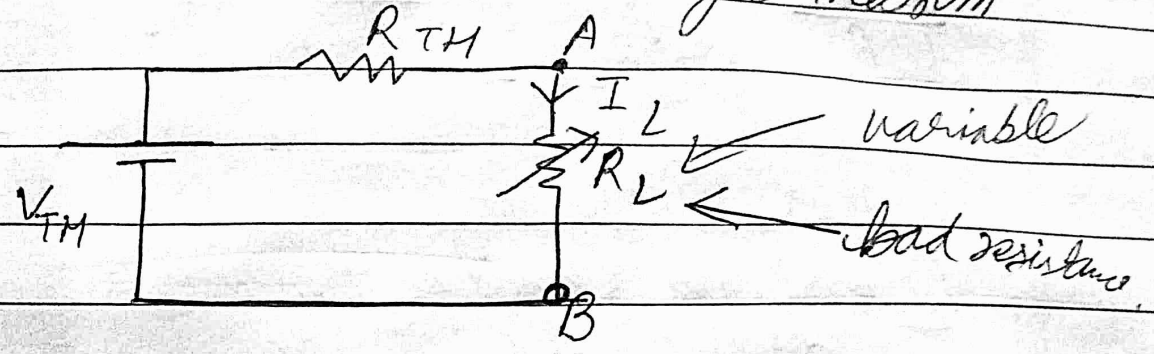
$I = \frac{2.6 \times 6.6}{10 + 6.6}$

$= 1.03\,A$





Maximum Power Transfer Theorem



Power generated by V_{TH} is utilised by R_L or load terminal.

Amt of power generated $= V_{TH} \times I_L$

This theorem states that in dc circuit max power is transfered from source to load when load resistance is equal to ~~the~~ thevenin resistance or internal resistance of the circuit.

Power delivered to the load ~~of~~ or power dissipated by the load:-

$$P_L = I^2 R_L$$

where $I_L = \dfrac{V_{TH}}{R_L + R_{TH}}$

Hence, $P_L = \left(\dfrac{V_{TH}}{R_L + R_{TH}}\right)^2 \cdot R_L$

$$P_L = \frac{V_{TH}^2}{R_L^2 + R_{TH}^2 + 2R_L \cdot R_{TH}}$$

$$P_L = \frac{V_{TH}^2}{R_L + \dfrac{R_{TH}^2}{R_L} + 2R_{TH}}$$

For max power $\Rightarrow$

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[\frac{V_{TH}^2}{R_L + \frac{R_{TH}^2}{R_L} + 2R_{TH}}\right]$$

For max Power:-

Denominator should be minimum =>

$$\frac{d}{dR_L}\left[R_L + \frac{R_{TH}^2}{R_L} + 2R_{TH}\right] = 0$$

$$1 - \frac{R_{TH}^2}{R_L^2} = 0$$

$$\boxed{R_L = R_{TH}}$$

double diff y

$$\frac{d}{dR_L}\left[1 - \frac{R_{TH}^2}{R_L^2}\right]$$

$$0 + \frac{2R_{TH}^2}{R_L^3} = \frac{2R_{TH}^2}{R_L^3}$$

[At $R_L = R_{TH}$ the op is +ive).

$\therefore R_L = R_{TH}$ is cond of max power

$$P_L\,max = \left(\frac{V_{TH}}{R_L + R_{TH}}\right)^2 R_L$$

at $R_L = R_{TH}$

$$P_L\,max = \frac{V_{TH}^2}{4R_{TH}^2} \cancel{\times R_{TH}}$$

$$P_L\,max = \frac{V_{TH}^2}{4R_{TH}}$$

Efficiency of ckt at max Power condition

$$\eta = \frac{\text{Output Power}}{\text{Input "}} \times 100$$

$$\eta_{max} = \frac{\text{Output Power (max)}}{\text{Input " "}} \times 100$$

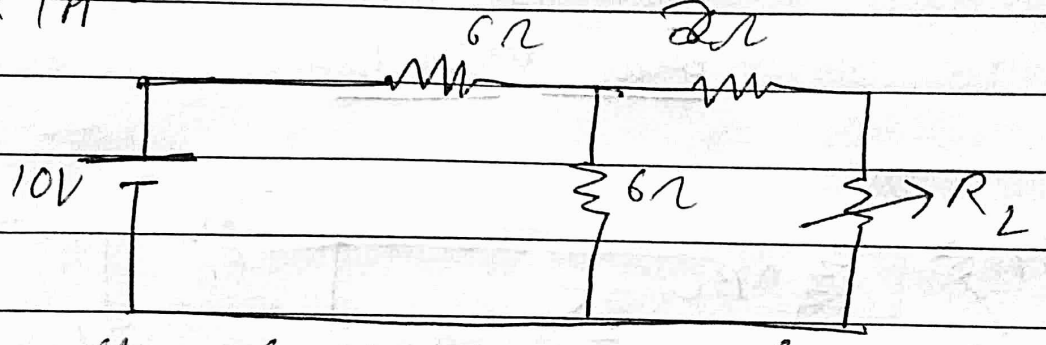
$$P_{L} = V_{TH} \cdot I_L = \frac{V_{TH} \cdot V_{TH}}{R_L + R_{TH}} = \frac{V_{TH}^2}{2R_{TH}}$$

$R_{TH} = R_L$

$$= \frac{V_{TH}^2 / 4R_{TH}}{V_{TH}^2 / 2R_{TH}} \times 100$$

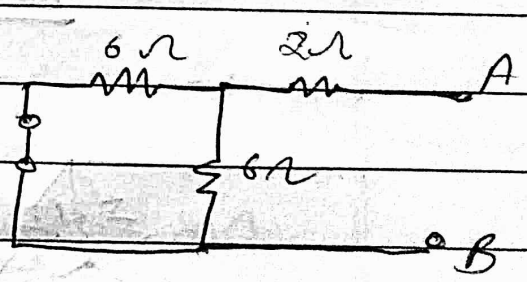
$$= \frac{1}{2} \times 100 = 50\%$$

Q1)



In the network shown in fig determine the value of load resistance to give max power transfer & also find out the power delivered to the load.

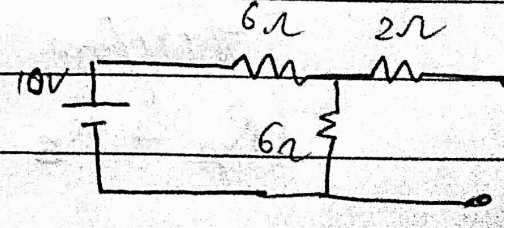
Soln



$$R_{TH} = 6 + \left(\frac{1}{2} + \frac{1}{6}\right)$$

$$= 6 + \left(\frac{3+1}{6}\right)$$

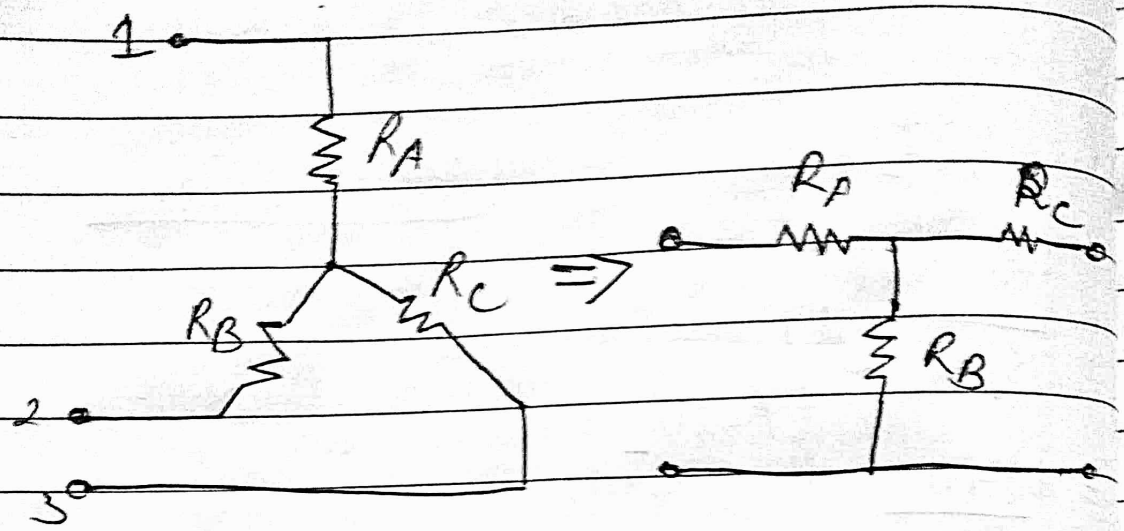
$$= 6 + \frac{6}{4} = \frac{30}{4} = 7.5\ \Omega$$



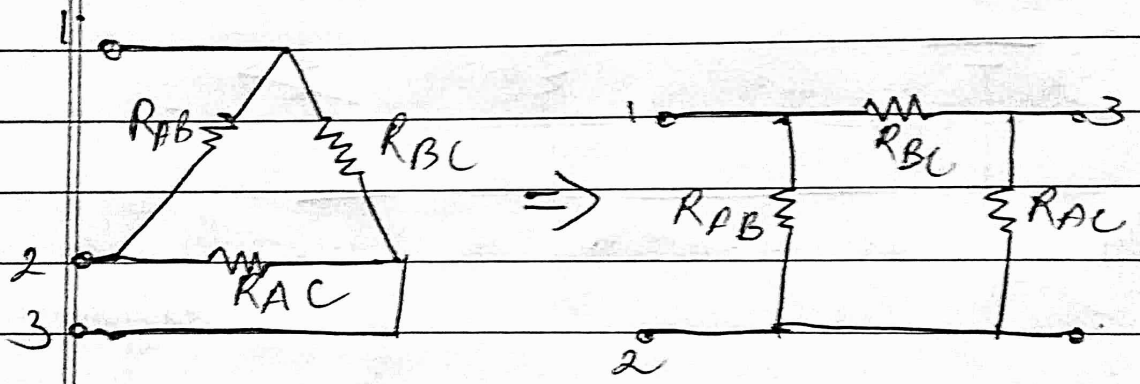
$V_{AB} = 5\ V$

$P_{max} = 1.25\,W$

21/10/2022 Star & Delta Transformation

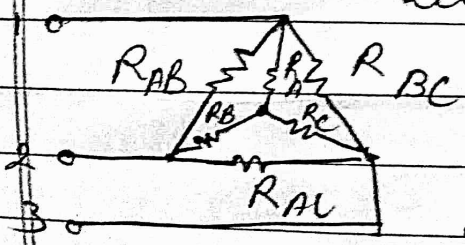


Star Connection



Delta Connection

Delta to Star Transformation



∵ The equivalent resistance b/w any 2 terminal will be same for star & delta connection.

Therefore R_{eq} b/w 1 & 2 :-

$R_A + R_B = R_{AB} \parallel (R_{BC} + R_{AC})$ —(i)

similarly Req b/w terminal 2 & 3 ⇒

$$R_B + R_C = R_{AC} \parallel (R_{AB} + R_{BC}) \quad -(2)$$

∴ Req b/w terminal 1 & 3 ⇒

$$R_A + R_C = R_{BC} \parallel (R_{AB} + R_{AC}) \quad -(3)$$

(1) can be written as.

$$R_A + R_B = \frac{R_{AB} \cdot R_{BC} + R_{AB} \cdot R_{AC}}{R_{AB} + R_{BC} + R_{AC}} \quad -(1)$$

$$R_B + R_C = \frac{R_{AC} \cdot R_{AB} + R_{AC} \cdot R_{BC}}{R_{AB} + R_{BC} + R_{AC}} \quad -(2)$$

$$R_A + R_C = \frac{R_{BC} \cdot R_{AB} + R_{BC} \cdot R_{AC}}{R_{AB} + R_{BC} + R_{AC}} \quad -(3)$$

Adding eq (1), (2), (3).

$$\cancel{2}(R_A + R_B + R_C) = \frac{\cancel{2}(R_{AB} \cdot R_{BC} + R_{BC} \cdot R_{AC} + R_{AC} \cdot R_{AB})}{R_{AB} + R_{BC} + R_{AC}} \quad -(4)$$

~~$R_A + R_B + R_C =$~~

Now (eqn 4 − eqn 1) = R_C

$$R_C = \frac{R_{AC} \cdot R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

Similarly; $R_B = \dfrac{R_{AB} \cdot R_{AC}}{R_{AB} + R_{AC} + R_{BC}}$

$$R_A = \frac{R_{AB} \cdot R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

Star to delta Transformation :-

$$[\text{Eqn}(5) \times \text{Eqn}(6)] + [\text{Eqn}(6) \times \text{Eqn}(7)] + [\text{eqn}(7) \times \text{eqn}(5)]$$

$$\Rightarrow R_C \cdot R_B + R_B \cdot R_A + R_A \cdot R_C = \frac{R_{AC}^2 \cdot R_{AB} \cdot R_{BC} + R_{AB}^2 \cdot R_{AC} \cdot R_{BC} + R_{BC}^2 \cdot R_{AB} \cdot R_{AC}}{(R_{AB} + R_{AC} + R_{BC})^2}$$

$$\Rightarrow \text{RHS}; \quad \frac{R_{AB} \cdot R_{AC} \cdot R_{BC} [R_{AC} + R_{AB} + R_{BC}]}{(R_{AB} + R_{AC} + R_{BC})^2}$$

$$= \frac{R_{AB} \cdot R_{AC} \cdot R_{BC}}{R_{AB} + R_{AC} + R_{BC}} \quad (8)$$

$$\text{LHS}, \quad R_C \cdot R_B + R_B \cdot R_A + R_A R_C$$

Put R_C from eqn (5) into eqn (8) $\Rightarrow$

$$R_{AB} = R_A + R_B + \frac{R_A \cdot R_B}{R_C} \quad \} (9)$$

Put R_B from eqn (6) into eq (8)

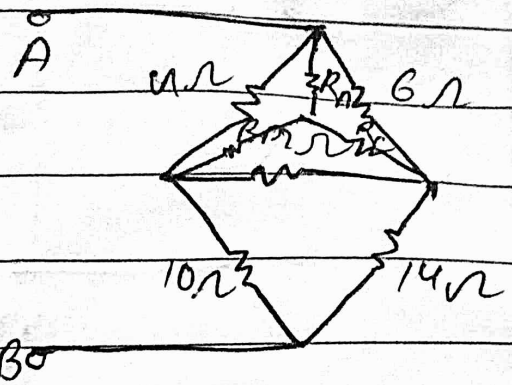
$$R_{BC} = \frac{R_B + R_C + R_A R_C}{R_B} \quad \} (10)$$

Put R_A from eqn (7) into eqn (8)

$$R_{AC} = R_B + R_C + \frac{R_B \cdot R_C}{R_A} \quad \} (11)$$

Ex:-

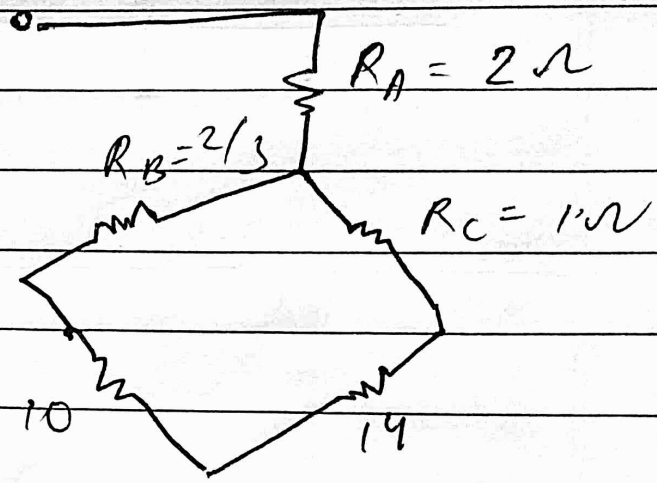
(1) Find $R_{AB} = ?$



$$R_A = \frac{6 \times 4}{6+4+2} = \frac{24}{12} = 2\Omega$$

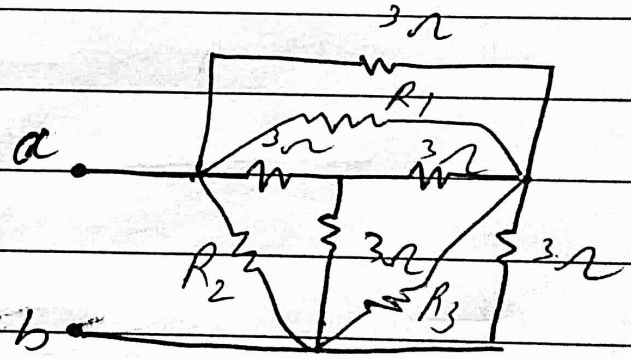
$$R_B = \frac{4 \times 2}{12} = \frac{8}{12} = \frac{2}{3}\Omega$$

$$R_C = \frac{2 \times 6}{12} = 1\Omega$$



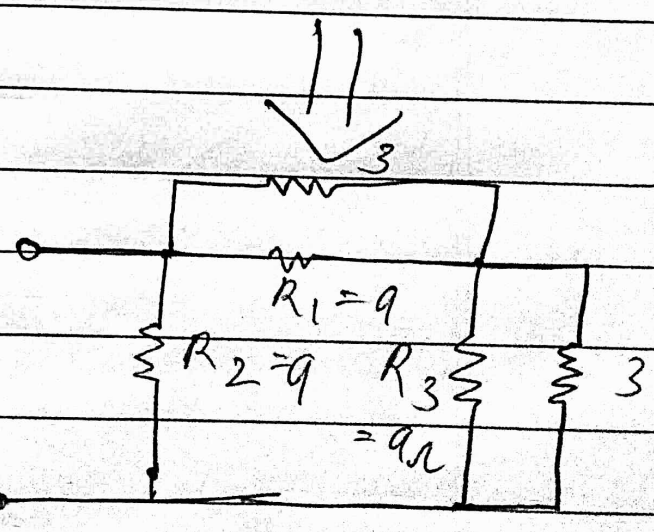
(2) Ex:- $R_{ab} = 8.15\,\Omega$

Find R_{ab}



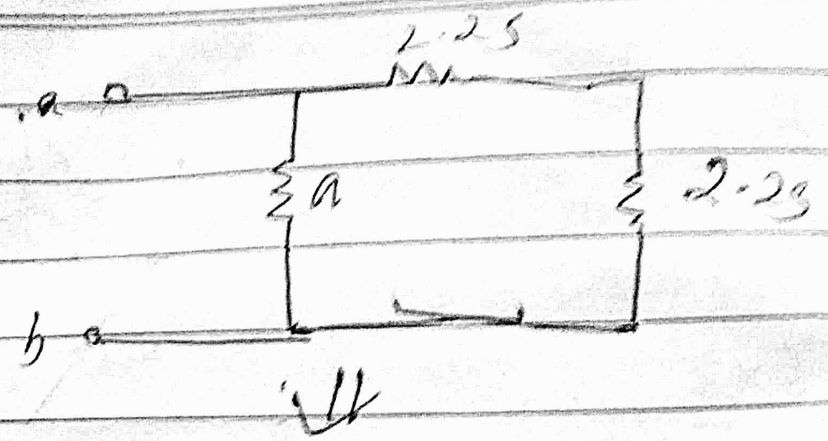
$$R_1 = 3 + 3 + \frac{3 \times 3}{3} = 9\Omega$$

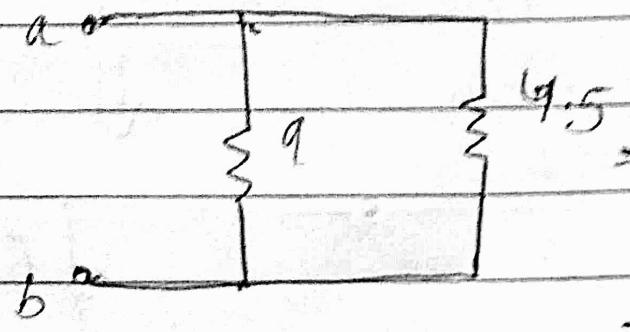
$R_2 = 9\Omega$, $R_3 = 9\Omega$



$$\frac{9 \times 3}{12} = 2.25\,\Omega$$

31/10/2022





$$= \frac{9 \times 4.5}{13.5}$$

$$= 3\Omega$$

11/11/2022

Page No.

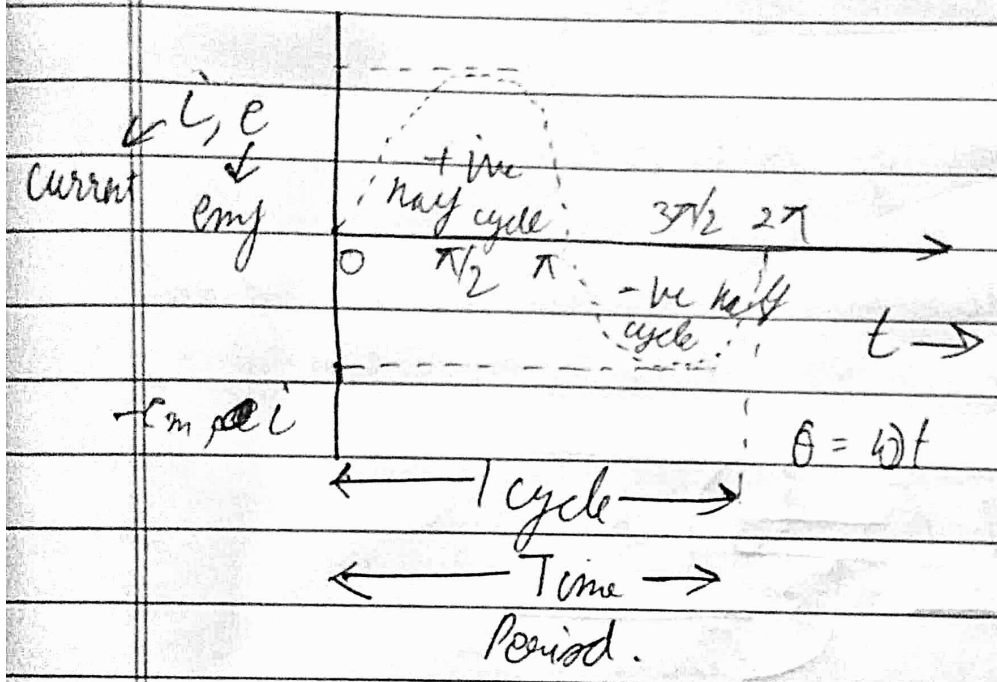
Unit 2 - Introduction to alternating quantity. (AC)

Current

Voltage

1) Phase (ϕ) AC Quantity:-

* What is Alternating Quantity:-



$e = e_m \sin(\theta)$

or

$e = e_m \sin(\omega t)$

or

$e = e_m \sin(2\pi f t)$

or

$e = e_m \sin\left(\frac{2\pi t}{T}\right)$

where θ = angular displacement

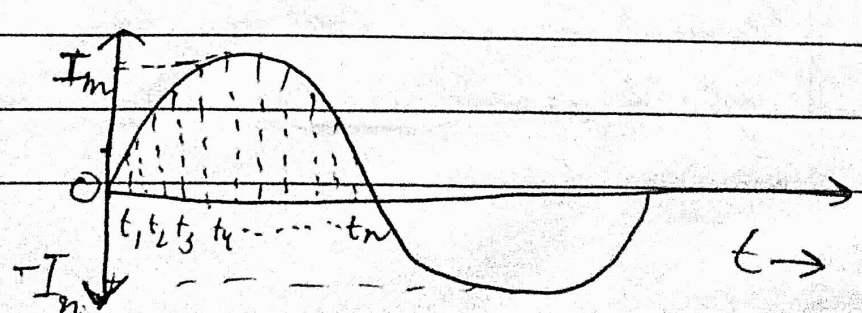
ω = Angular freq (rad/sec)

~~$e = e_m \sin(2\pi f t)$~~ f = freq. as $f = \frac{1}{T}$

Freq is no of cycles completed in one sec.

Instantaneous Value:- Value at a particular instant of time.

$i = I_m \sin(\omega t)$



$$\text{Avg value} = \frac{\text{Area under the curve}}{\text{Base length of the curve}}$$

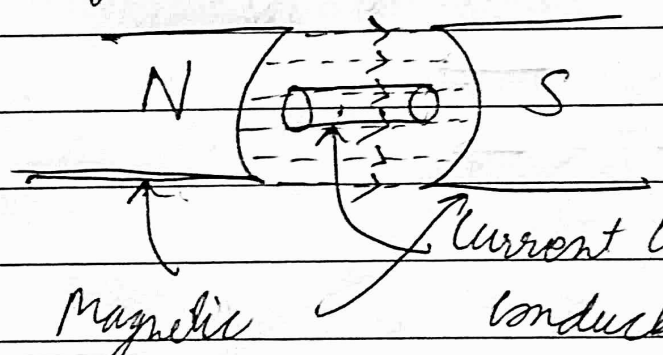
Integration Method

$$\text{Avg value} = \frac{i_1 + i_2 + i_3 + \cdots + i_n}{n}$$

Mid Ordinate method

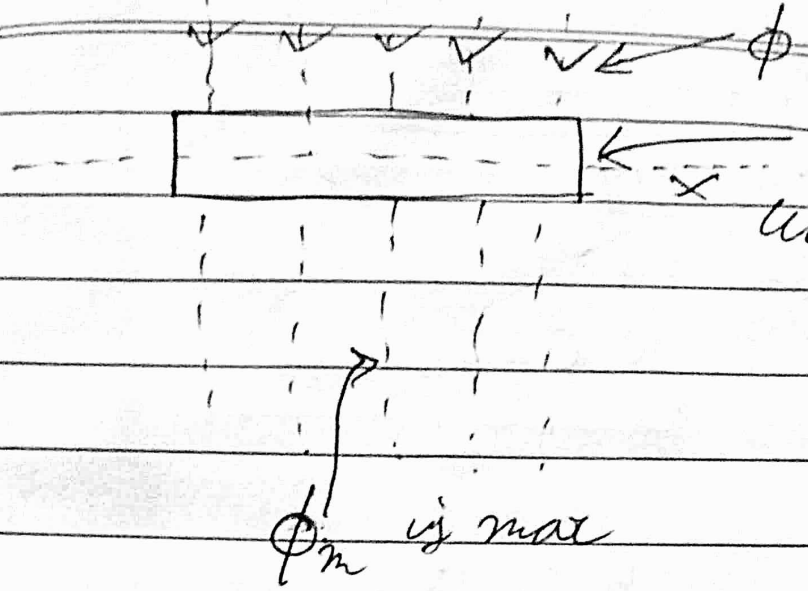
Generation of Alternating ~~current~~ Quantity :-

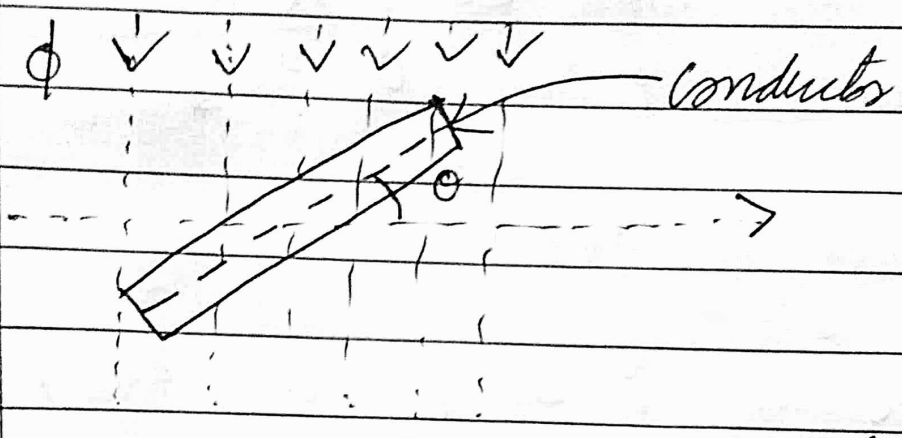
The alternating quantity is based on Faraday Law of Electromagnetic Induction.



The alternating voltage can be generated by two ways:-

1. Magnetic field is stationary & current carrying coil is rotating inside the magnetic field at constant speed.
2. Current carrying coil is stationary and magnetic field around the coil is rotating with a constant speed.





$$e = -N\frac{d\phi}{dt} = -N\frac{d(\phi_m \cos\omega t)}{dt}$$

$$e = -N\phi_m(-\sin\omega t)\cdot\omega$$

$$= N\phi_m\omega\sin(\omega t).$$

$$\boxed{e = e_m \sin\omega t}, \boxed{e_m = N\phi_m\omega}$$

Peak value is the max value.

Q An alternate current i is given by $i = 141.4\sin 314t$

(i) max value

(ii) freq

(iii) time period

(iv) instantaneous value when t is 3 milli sec.

Sol. (i) max value is 141.4

$$2\pi f \cancel{t} = 314 \cancel{t}$$

$$f = \frac{314}{2\times\pi} = 50 \text{ Hz}$$

(iii) $T = \frac{1}{50} = 0.02 \text{ sec}$

(IV) $i = 141.4 \sin \underbrace{2\times\pi\times 50\times 3\times10^{-3}}$

$= 141.4 \sin 0.942$ → radian units degree.

$= 141.4 \times 0.808$

$= 114.387 \text{ A}$.

$141\left(\sin \frac{0.942\times180}{\pi}\right)$

Q An AC is given by $i = 10 \sin 942 t$ Determine the time taken for $t=0$ for current to reach value of $+6$ A for first time.

Sol

$i = 0$

$6 = 10 \sin 942 \times t$

$6 = 10 (-0.459) \times t$

$t = \frac{6}{10(-0.459)} = -1.3 \text{ A}$

$i = 10 \sin 942 t$

$6 = 10 \sin 942 t$

$\sin 942 t = \frac{6}{10}$

$942 t = \sin^{-1}\left(\frac{6}{10}\right)$

$t = \frac{\sin^{-1}(6/10)}{942} = 0.68$ milli sec

Date.
Page No.

Ans should be degree

4/11/2022

~~$E = E_m \sin \theta t$~~

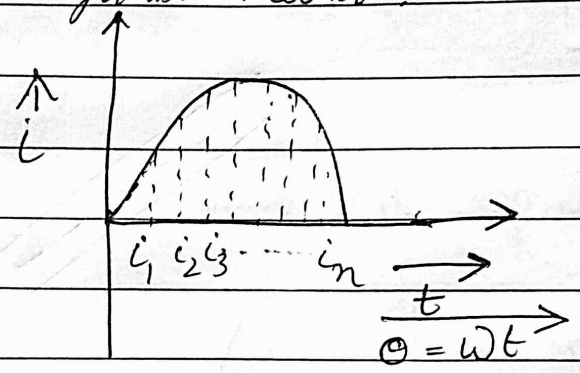
Avg & RMS value for alternating Quantity of sinusoidal wave:-

~~(i) Mid ordinate method~~

(i) Avg value:- It can be calculated by 2 ways.

(i) Mid ordinate method.

(ii) Integration method.



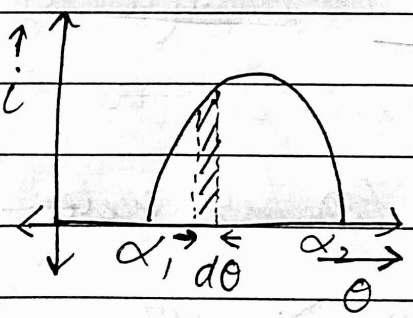
for half cycle.

$$i = I_m \sin\theta$$

$$i_{avg} = \frac{i_1 + i_2 + i_3 \cdots i_n}{n}$$

(ii) Integration method :-

$$i_{avg} = \frac{\text{Area under the curve}}{\text{Base length of the curve.}}$$



$$i_{avg} = \frac{\int_{\alpha_1}^{\alpha_2} i\, d\theta}{(\alpha_2 - \alpha_1)}$$

If we have alternating voltage or current ($V_m \sin\theta$)

then $\rightarrow$ $V_{avg} = \dfrac{\int_0^T V \cdot d\theta}{T} = \dfrac{\int_0^T V_m \sin\theta\, d\theta}{T}$

Date.
Page No.

Calculate avg value of A.C for half cycle & full cycle.

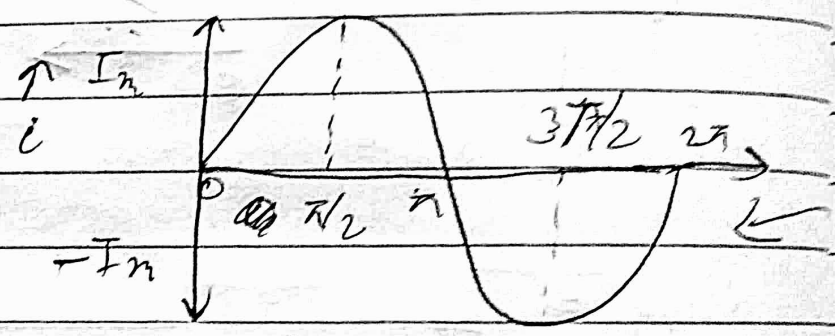
$$i_{avg} = \frac{\int_0^T i \cdot d\theta}{T}$$

$$= \frac{\int_0^T i_m \sin\theta \cdot d\theta}{T}$$

i_{avg} of AC is alltays 0.

$P = i^2 R$

↑ its always rms value.



Full cycle $(0-2\pi)$

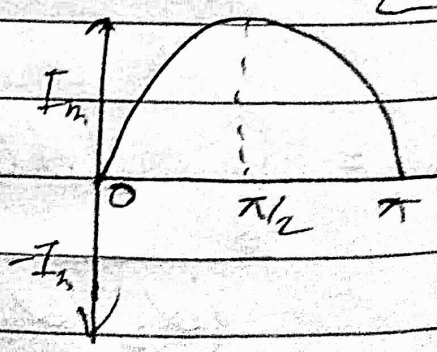
Half cycle $(0-\pi)$:

graph for the.

For half cycle :-

Prove this

$$\cancel{i_{avg} = \frac{\int_0^T}{T}}$$



$$I_{avg} = \frac{2 I_m}{\pi}$$

$$I_{avg} = \frac{1}{\pi}\int_0^{\pi} i\, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \cdot d\theta$$

$$= \frac{I_m}{\pi}\left(-\cos\theta\right)_0^{\pi}$$

$$= \frac{-I_m}{\pi}\left(-\cos\pi - \cos 0\right)$$

$$= -\frac{I_m}{\pi}\left(-1-1\right)$$

$$= \frac{2 I_m}{\pi}$$

For full :-

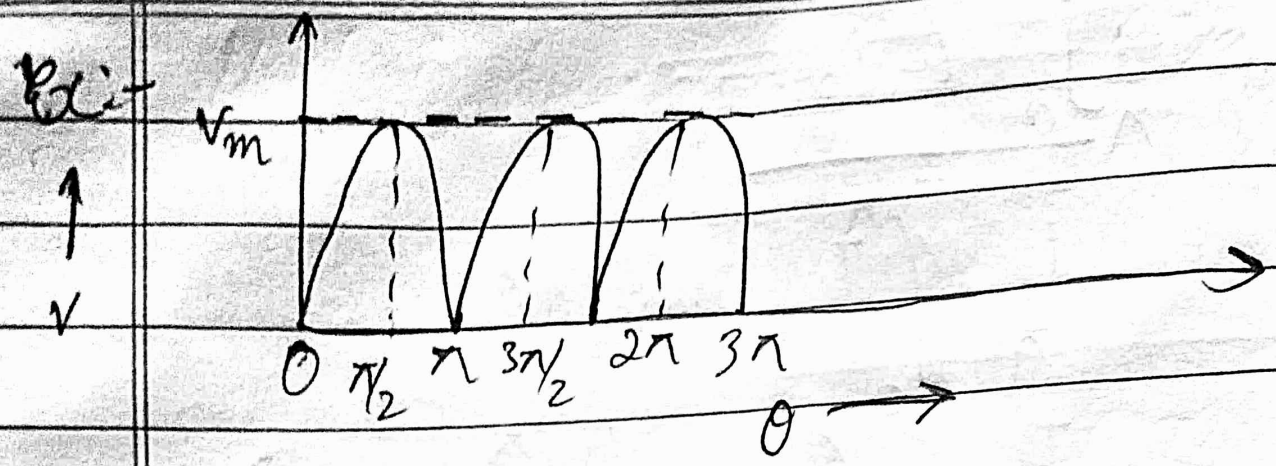
$$i_{avg} = \frac{1}{2\pi}\int_0^{2\pi} i\, d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi} I_m \sin\theta\, d\theta\right]$$

$$= \frac{I_m}{2\pi}\left(-\cos\theta\right)_0^{2\pi}$$

$$= \frac{I_m}{2\pi}\left(-\cos 2\pi - \cos 0\right)$$

$$= \frac{-I_m}{2\pi}\left(1-1\right) = 0$$



Find V_{avg} →

Soln.

$$\int_0^{T} \frac{V_m \sin\theta \, d\theta}{T}$$

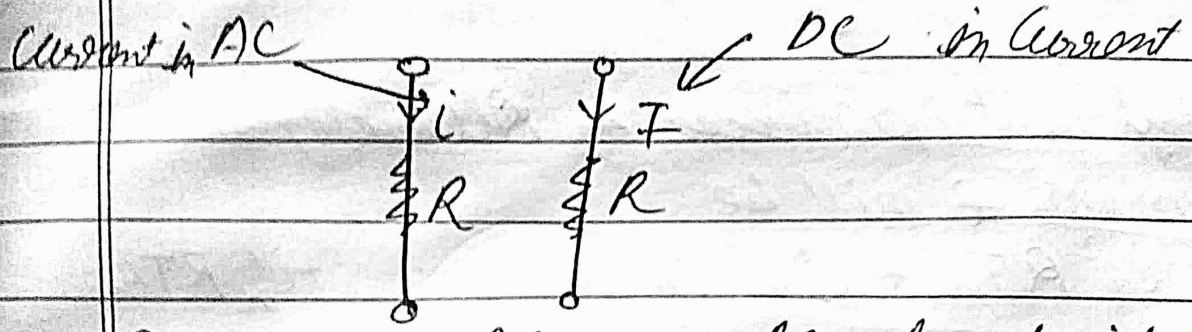
$$\Rightarrow \frac{1}{\pi} \int V \, d\theta$$

18/11/2022

Date.
Page No.

RMS (Root Mean Square Value)

RMS value of an alternating current is known as effective value or its also called as d.c equivalent current.

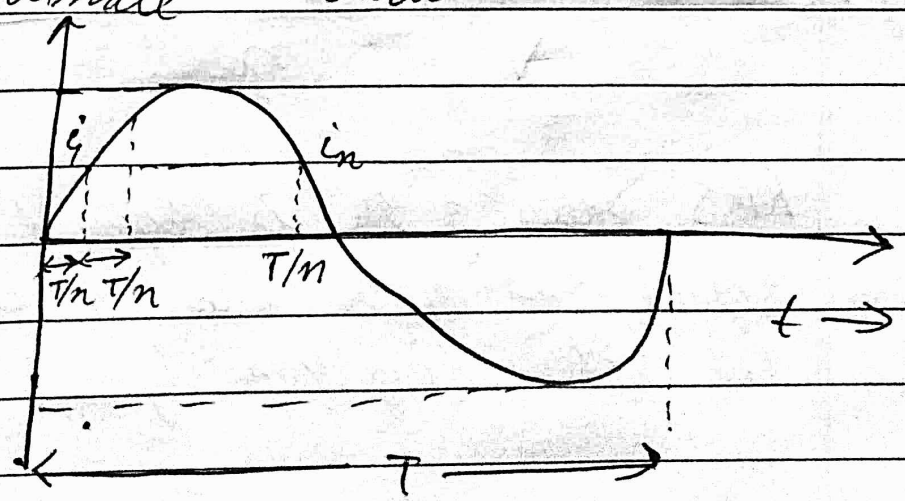


We prefer DC over AC due to its constant magnitude of current so that we can calculate the power loss of the resistor.

RMS value of AC current is defined as DC equivalent current which produces the same amt of heat across the same resistor R in same time interval.

2 ways to calculate the RMS:-

(i) Mid ordinate Method:-



Date.
Page No.

Heat produced by $i_1 = i_1^2 RT/n$
" " " $i_2 = i_2^2 RT/n$
⋮
" " " $i_n = i_n^2 RT/n$

Total heat produced by Alternating current i will be,
$i_1^2 RT/n + i_2^2 RT/n + \cdots + i_n^2 RT/n$

Similarly heat produced by equivalent direct current, $I = I^2 RT$

Now based on the definition, both the expression should be same.

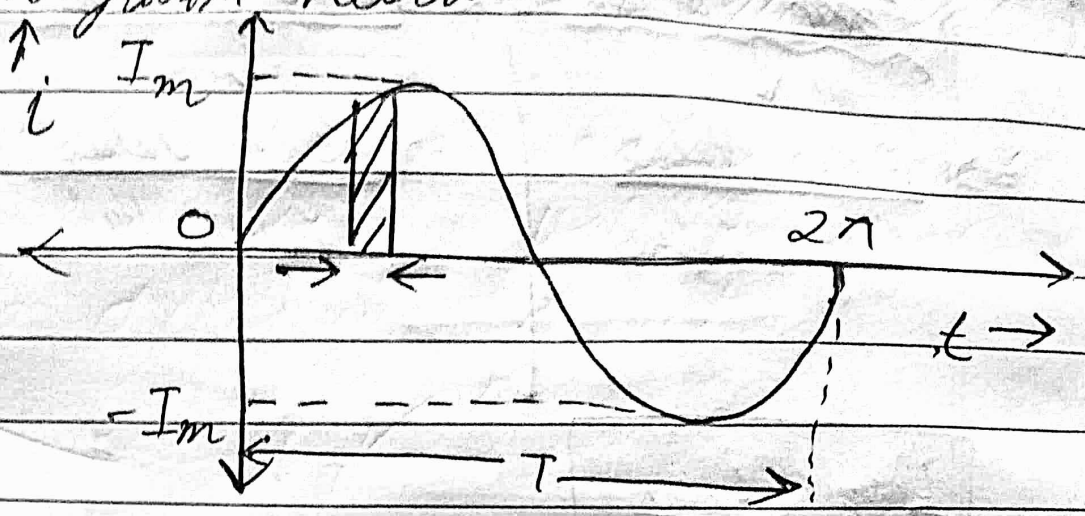
$$i_1^2 RT/n + i_2^2 RT/n \cdots i_n^2 RT/n = I^2 RT$$

$$(i_1^2 + i_2^2 + \cdots + i_n^2) RT/n = I^2 RT$$

RMS of AC $\Rightarrow I = \sqrt{\dfrac{i_1^2 + i_2 + \cdots i_n^2}{n}}$

RMS value of AC is defined as the square root of the mean of the square of the alternating quantity.

(ii) By Integration Method :-



Let an alternating current i is flowing across resistor R, at time period T.
Therefore heat produced by the alternating current i, across R, at time period T will be defined as :-

$$H_{a.c} = \int_0^T i^2 R \, dt$$

Similarly, the heat produced by equivalent direct current I, across R at time period T, will be defined as :-

$$H_{dc} = I^2 R T$$

Now, based on definition both the expression should be same.

$$I^2 \cancel{R} T = \int_0^T i^2 \cancel{R} \, dt$$

$$I^2 = \frac{1}{T} \int_0^T i^2 \, dt$$

$$I = \sqrt{\frac{1}{T} \int_0^T i^2 \, dt}$$

Form Factor & Peak Factor

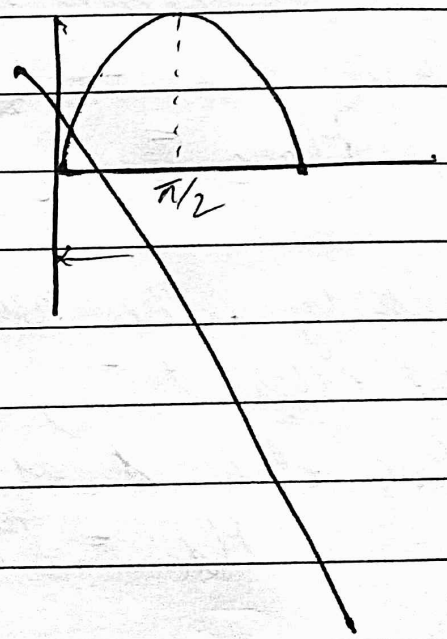
Form Factor	Peak Factor
$\Rightarrow \dfrac{\text{RMS Value}}{\text{Avg Value}}$	$\Rightarrow \dfrac{\text{Peak Value}}{\text{RMS Value}}$

$$\boxed{i_{\cancel{avg}} = \frac{i_{peak}}{\sqrt{2}}}$$

rms

Q Avg value of.
RMS Value of R.

$\Rightarrow$ (i) For Half Cycle.



$$I = \int_0^T i^2 R\,dt$$

$$= \int_0^T$$

$$I_{Rms} = \frac{I_0}{\sqrt{2}}$$

$$I^2 = \frac{1}{T}\int_0^T i^2\,dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2\theta\,d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{2\pi} \frac{(1-\cos\theta)}{2}\,d\theta$$

$$= \frac{I_m^2}{4\pi}\left[(\theta)_0^{2\pi} - \frac{1}{2}(\sin 2\theta)_0^{2\pi}\right]$$

$$I^2 = \frac{I_m^2}{4\pi}\left[2\pi - \frac{1}{2}(\sin 4\pi - \sin 0)\right]$$

$$I^2 = \frac{I_m^2}{4\pi}\,2\pi$$

$$I^2 = \frac{I_m^2}{2}$$

$$I_{rms} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

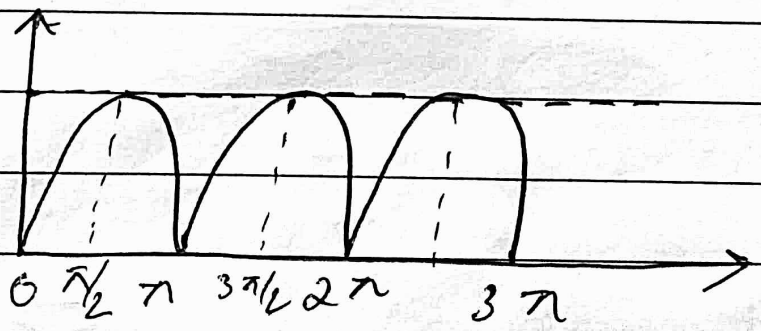
Form Factor for full cycle will be infinite.

as $\frac{\text{RMS value}}{\text{Avg value}} = \frac{\text{Avg value}}{0} = \infty$

Whereas Peak factor $= \frac{\text{Peak Value}}{\text{RMS value}} = \frac{I_m \cdot \sqrt{2}}{I_m} = \sqrt{2}$

$= 1.414.$

Q Ex :-



Find V_{avg}, V_{rms}, form factor & Peak Factor.
$T = \pi$.

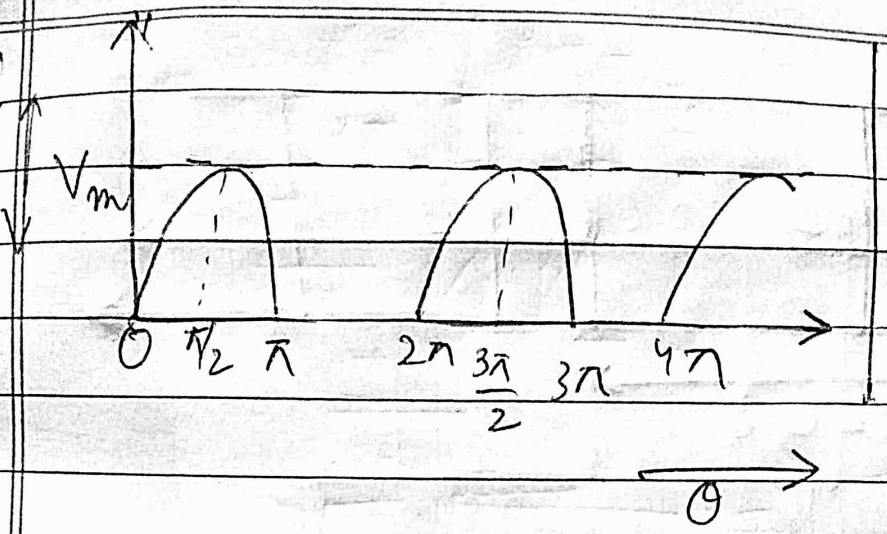
Soln

$$V_{avg} = \int_0^{\pi} \frac{V_m \sin\theta \, d\theta}{\pi}$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{V_m}{\pi} - \left[\cos\theta\right]_0^{\pi}$$

Q



Find (i) avg, (ii) rms, (iii) form factor & (iv) peak factor.

A (i)

$$V_{avg} = \int_0^{2\pi} \frac{V_m \sin\theta \, d\theta}{2\pi}$$

$$'' = \frac{V_m}{2\pi}\left[\int_0^{\pi} \sin\theta \, d\theta + \int_{\pi}^{2\pi} 0 \cdot d\theta\right]$$

$$'' = \frac{V_m}{2\pi} -[\cos\theta]_0^{\pi}$$

$$'' = -\frac{V_m}{2\pi}[-\cos\pi + \cos 0]$$

$$V_{avg} = -\frac{V_m}{2\pi}[-(-1) + 1]$$

$$= \frac{V_m}{\pi}$$

(ii) (V) $V_{rms} = \frac{V_m}{2}$ $\Rightarrow$

$$V_{rms}^2 = V^2 = \frac{1}{T}\int_0^T V^2 d\theta$$

$$V_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta \, d\theta$$

$$= \frac{V_m^2}{2\pi}\left[\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta\right]$$

$$= \frac{V_m^2}{4\pi}\left[(\theta)_0^{2\pi} - \left(\frac{\sin 2\theta}{2}\right)_0^{2\pi}\right]$$

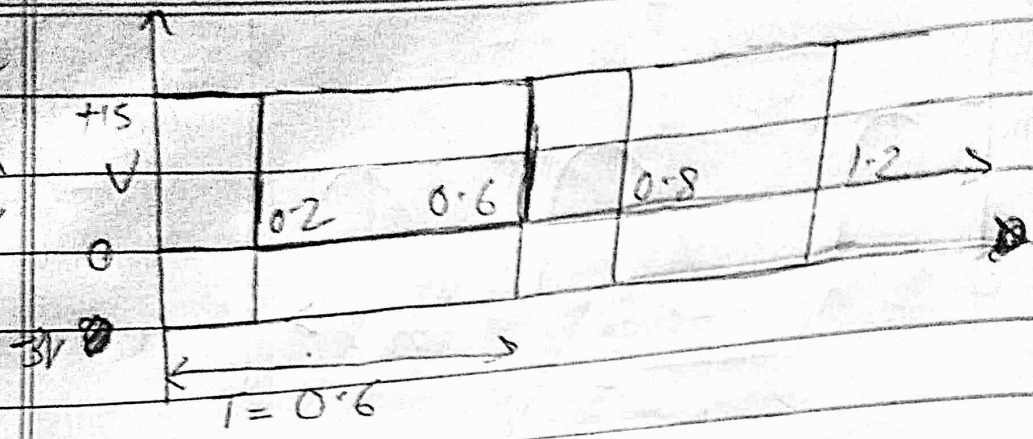
$$= \frac{V_m^2}{4\pi}\pi$$

$$V_{rms} = \sqrt{\frac{V_m^2}{4}} = \frac{V_m}{2}$$

(iii) Form factor $= \frac{\frac{V_m}{2}}{\frac{V_m}{\pi}} = \frac{\pi}{2}$

(iv) Peak factor $= \frac{V_m}{V_m/2} = 2$

Q



Find V_{avg} & V_{rms} ?

Sol

$$V_{avg} = \frac{\int_0^{0.6} v\,d\theta}{0.6} = \frac{\int_0^{0.2} v\,d\theta + \int_{0.2}^{0.6} v\,d\theta}{0.6}$$

$$V_{avg} = \frac{\int_0^{0.2} 15\,d\theta + \int_0^{0.6} (-3)\,d\theta}{0.6}$$

$$= \frac{15(\theta)_0^{0.2} - 3(\theta)_{0.2}^{0.6}}{0.6}$$

$$= \frac{(15 \times 0.2) - 3 \times 0.4}{0.6}$$

$$= \frac{3 - 1.2}{0.6} = \frac{1.8}{0.6}$$

$$= +3\ V$$

$$V_{rms}^2 = \frac{1}{0.6} \int_0^{0.6} v^2\,d\theta$$

$$= \frac{1}{0.6} \left[\int_0^{0.2} (15)^2\,d\theta + \int_{0.2}^{0.6} (-3)^2\,d\theta \right]$$

$$= \frac{1}{0.6} \left[225(\theta)_0^{0.2} + 9(\theta)_{0.2}^{0.6} \right]$$

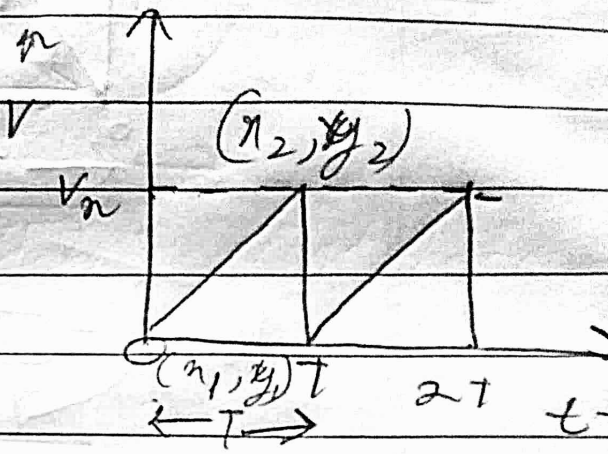
$$= \frac{1}{0.6} \left[45 + 3.2 \right]$$

$$= \frac{1}{0.6}\left[45 + 3.6\right]$$

$$= 81$$

$$V_{rms} = \sqrt{81} = 9V$$

Q Find avg & rms value of Δ waveform?



Sol

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$V - 0 = \frac{V_m - 0}{T - 0}(t - 0)$$

$$V = \frac{V_m}{T} \cdot t$$

$$V_{avg} = \frac{\frac{1}{2} \times T \times V_m}{T} = \frac{T \times V_m}{2T} = \frac{V_m}{T}$$

$$V_{rms} = V^2 = \frac{1}{T}\int v^2 \, d\theta$$

$$= \frac{1}{T}\int \frac{V_m^2}{T^2} t \, d\theta$$

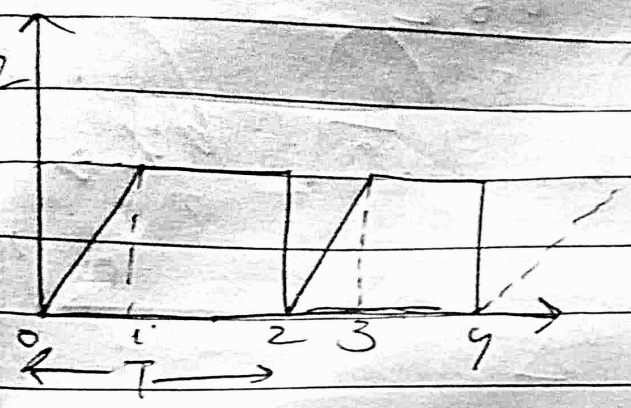
$$= \frac{1}{T^3}\int V_m^2 \, t$$

$$=$$

Date.
Page No.

2/12/2022

Ex:-



$$V_{avg} = \frac{1}{T}\int_0^T v\,dt$$

$$= \frac{1}{2}\int_0^2 v\,dt$$

$$= \frac{1}{2}\left[\int_0^1 t\,dt + \int_1^2 dt\right]$$

$$= \frac{1}{2}\left[\left(\frac{t^2}{2}\right)_0^1 + (t)_1^2\right]$$

$$= \frac{1}{2}\left[\frac{1}{2} + 1\right]$$

$$= \frac{3}{4} = 0.75\ V$$

$$V_{RMS} = \frac{1}{T}\int_0^2 v^2\,dt$$

$$= \frac{1}{T}\left[\int_0^1 t^2\,dt + \int_1^2 dt\right]$$

$$= \frac{1}{2}\left[\frac{1}{3} + (t)_1^2\right]$$

$$= \frac{1}{2}\left[\frac{1}{3} + 1\right] = \frac{4}{6}$$

$$\frac{1}{2} \int_0^3 = \frac{1}{3} \int_0^3$$

$$V_{rms} = \sqrt{\frac{4}{6}} = \sqrt{\frac{2}{3}} = 0.81\ldots$$

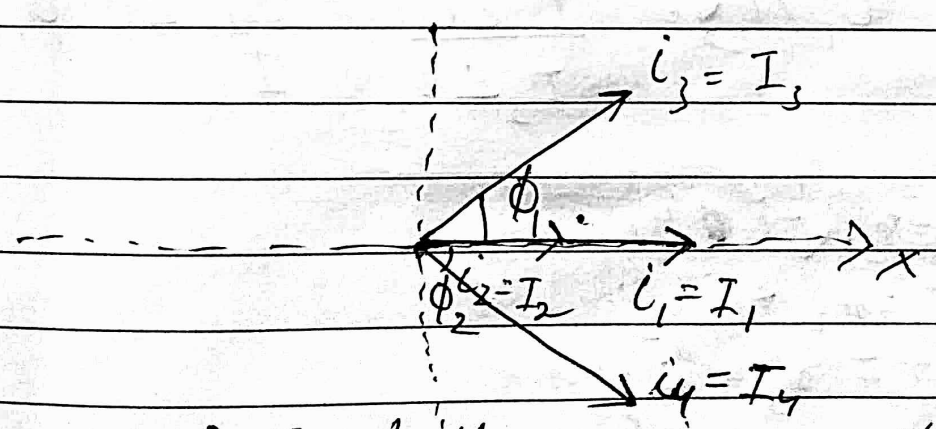
Phaser Representation of Alternating Quantity :-

$$i_1 = I_1 \sin(\omega t)$$
$$i_2 = I_2 \sin(\omega t)$$
$$i_3 = I_3 \sin(\omega t + \phi_1) \leftarrow \text{phase lead of } \phi_1$$
$$i_4 = I_4 \sin(\omega t - \phi_2) \leftarrow \text{phase lag of } \phi_2$$

Polar form :-

$$i_1 = I_1 \angle 0^\circ$$
$$i_2 = I_2 \angle 0^\circ$$
$$i_3 = I_3 \angle \phi_1$$
$$i_4 = I_4 \angle -\phi_2$$

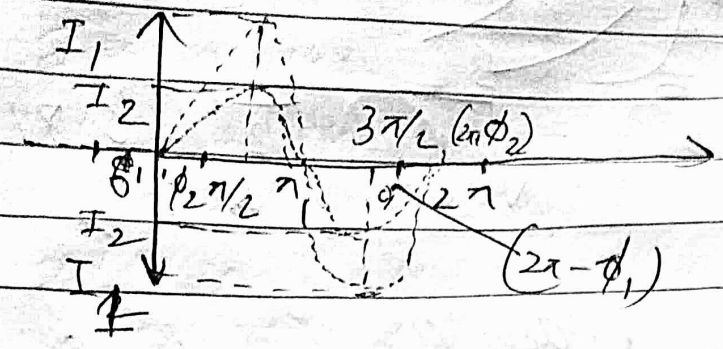
Phaser diagram :- vector representation.



* I_1 & I_2 both are in same phase.
* I_3 is leading from I_1 & I_2 by an angle of ϕ_1
* I_4 is lagging from I_1 & I_2 with an angle of ϕ_2

Date.
Page No.

Waveform :-



Rectangular form :

$x + jy$

x — real part

y — imaginary part

j — always shows a phase shift of $90°$

y & x add phase shift w.r.t to j in x axis.

Multiplication & division

$i_1 \times i_3 = I_1 \angle 0° \times I_3 \angle \phi_1$

$i_1 \times i_3 = I_1 \cdot I_3 \angle 0° + \phi_1$

$$i_1 / i_3 = \frac{I_1 \angle 0°}{I_3 \angle \phi_1} = \frac{I_1}{I_3} \angle 0° - \phi_1$$

$I_1 = x_1 + i y_1$

$I_2 = x_2 + i y_2$

$I_3 = x_3 + i y_3$

$I_4 = x_4 + i y_4$

Polar form ⇒

Polar form :- $r \angle \theta$, where r is the magnitude θ is phase angle. $\Big\}$ $x = r\cos\theta$, $y = r\sin\theta$

Rectangular form :- $x + iy$. $\Big\}$ $r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

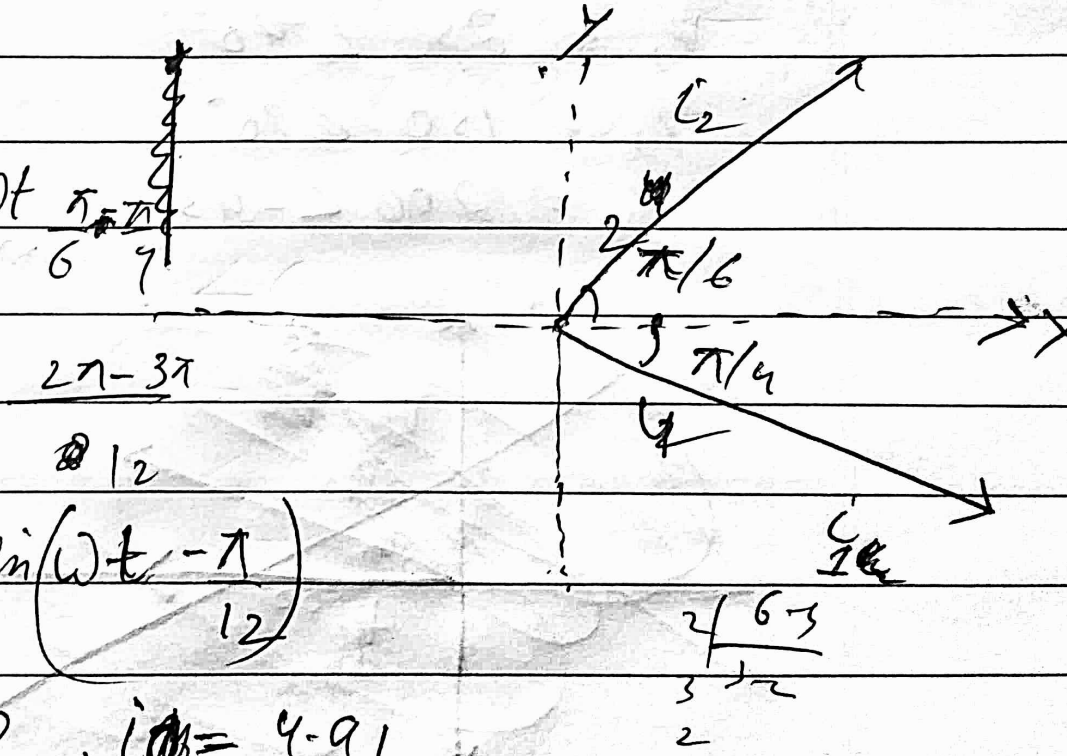
Q) $i_1 = 4\sin(\omega t - \pi/4)$

$i_2 = 2\sin(\omega t + \pi/6)$

Find expression for resultant alternating current.

Soln

$i = (i_1 + i_2)\sin\omega t$



$i = i_1 + i_2 \sin\left(\omega t - \frac{\pi}{12}\right)$

$i = 21.8$, $i = 4.91$

Date.
Page No.

5/12/2022

Q $V_1 = 100 \sin 314t$

$V_2 = 250 \cos 314t$

$V_3 = 150 \sin(314t + \pi/6)$

$V_4 = 200 \sin(314t - \pi/4)$

Find out the resultant voltage!

$V = V_m \sin(\omega t \pm \phi)$.

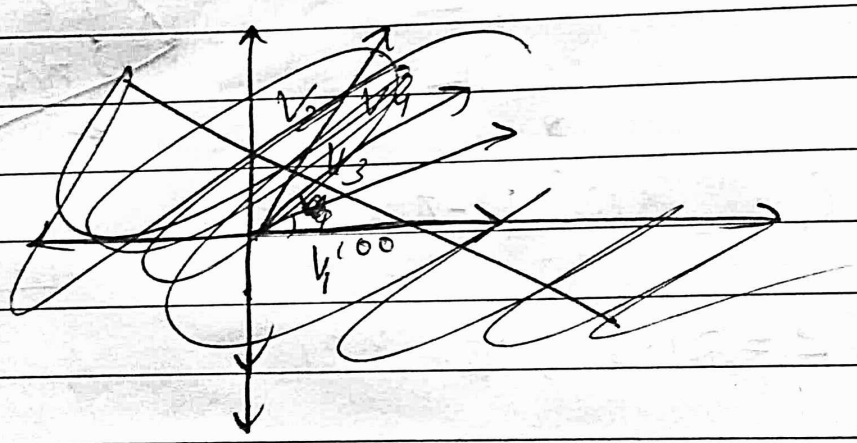
Soln $V_2 = 250 \sin\left(314t + \frac{\pi}{2}\right)$

$V_1 = 100\angle 0^\circ$

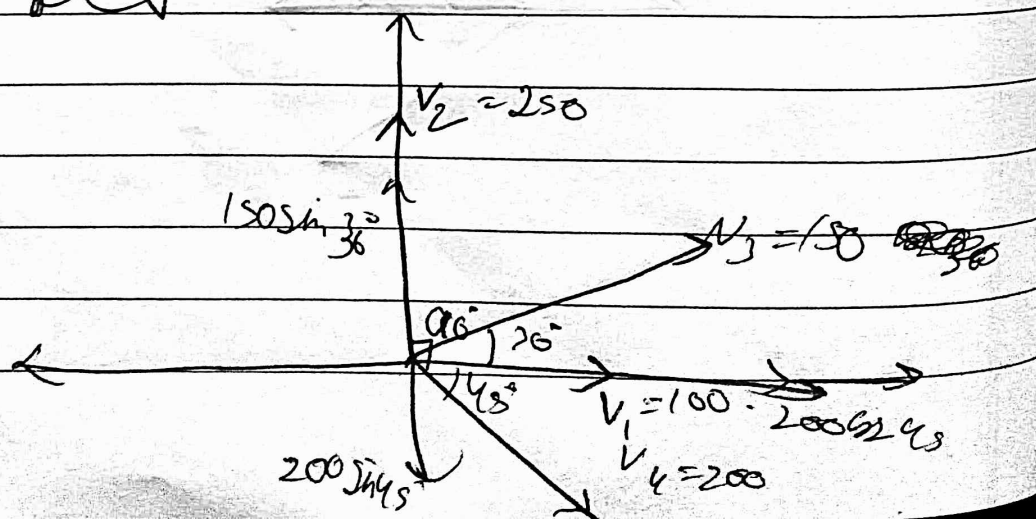
$V_2 = 250\angle 90^\circ$

$V_3 = 150\angle 30^\circ$

$V_4 = 200\angle -45^\circ$



$V = V_m \sin(\omega t \pm \phi)$



Date.
Page No.

$X \rightarrow 100 + 150\cos 30^\circ + 200\cos 45^\circ$

$= 100 + \frac{150 \times \sqrt{3}}{2} + 200 \times \frac{1}{\sqrt{2}}$

$= 371.39.$

$Y = 250 + 150\sin 30^\circ - 200\sin 45^\circ$

$= 250 + \frac{150 \times 1}{2} - \frac{200 \times 1}{\sqrt{2}}$

$= 183.56$

Magnitude $(V_m) = \sqrt{(371.39)^2 + (183.56)^2}$

$= 414.2V$

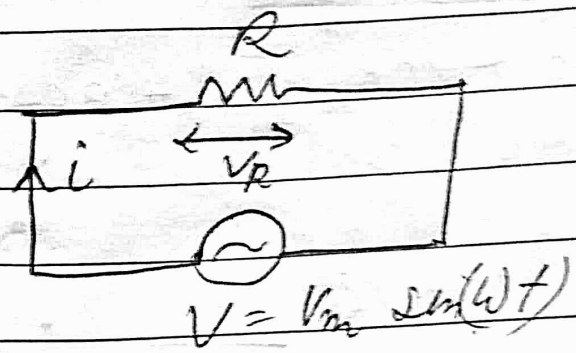
$\phi = \tan^{-1}\left[\frac{183.56}{371.39}\right]$

$= 26.2^\circ$

Date.
Page No.

* 1 - ϕ (phase) AC Ckt Analyses with diff components :-

i) For purely resistive Ckt ⇒



$V = V_m \sin \omega t$ —(1)

Apply KVL

$V - V_R = 0$

$V = V_R$

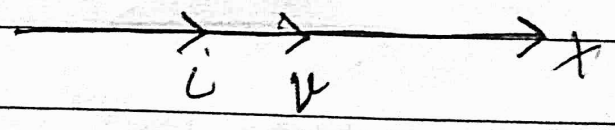
$V_m \sin \omega t = iR$

$i = \frac{V_m}{R} \sin \omega t$

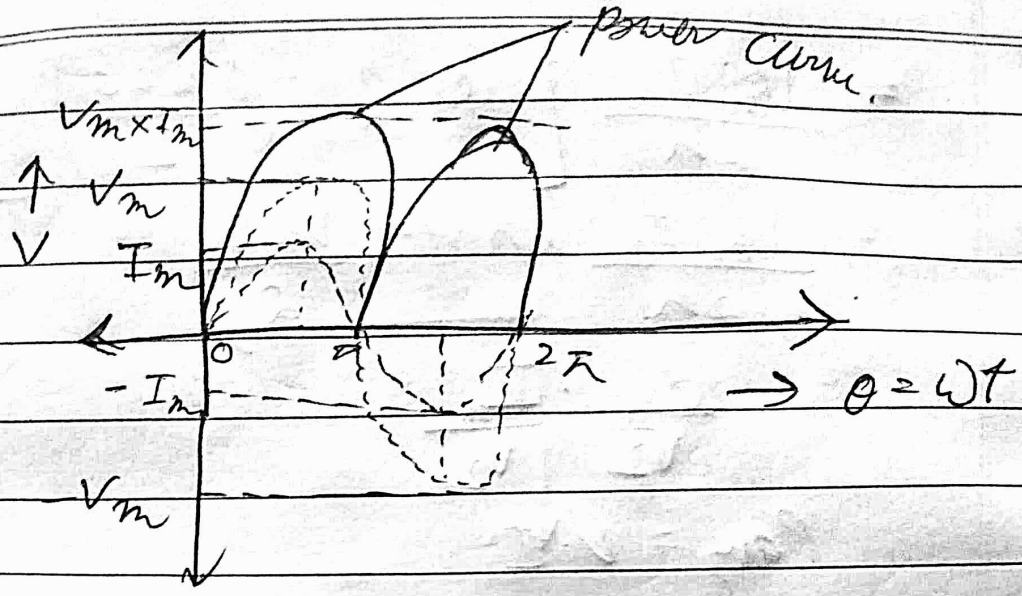
$i = I_m \sin \omega t$

∴ Here Voltage & current both are in same phase.

phasor dia ⇒



waveform ⇒



Time period for $I = 0 - 2\pi$

" " " power $= 0 - \pi$

∵ the instantaneous power will be product of alternating voltage & current.

$$P = V \cdot i = V_m \sin\omega t \cdot I_m \sin\omega t$$
$$= V_m \cdot I_m \sin^2\omega t$$

$$P_{avg} = \frac{1}{T}\int_0^T P\,d\theta$$

$$\cancel{= \frac{1}{T}\int_0^T V_m \cdot I_m \sin^2\omega t\,d\theta}$$

$$\cancel{= \frac{1}{T}\int \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \frac{1}{\pi}\int_0^T P\,d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin^2\theta\,d\theta$$

$$= \frac{V_m I_m}{\pi}\left[\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta\right.$$

Date.
Page No.

$$= \frac{V_m I_m}{2\pi}\left[(\theta)_0^{\pi} - \left(\frac{\sin 2\theta}{2}\right)_0^{\pi}\right]$$

$$= \frac{V_m I_m}{2\pi} \times \pi = \frac{V_m I_m}{2}$$

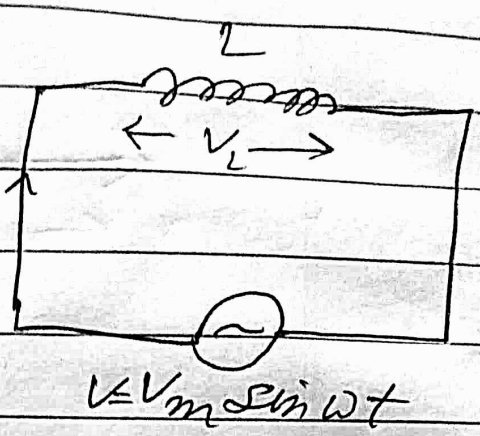
$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$P_{avg} = V \cdot I$$

10/12/2022

Purely Inductive ckt

$V = V_m \sin \omega t$ —(1)



By applying KVL, i

$V - V_L = 0$

$V = V_L$

Applying KVL,

$V - V_L = 0$

$V = V_L$

$$V_m \sin \omega t = L \frac{di}{dt}$$

$$di = \frac{V_m}{L} \sin(\omega t)\, dt$$

$$\int di = \frac{V_m}{L} \int \sin \omega t \, dt$$

$$i = \frac{V_m}{L} \left[\frac{-\cos \omega t}{\omega} \right]$$

$$i = \frac{V_m}{\omega L} \left[\sin(\omega t - \pi/2) \right]$$

$$i = \frac{V_m}{X_L} \left[\sin(\omega t - \pi/2) \right]$$

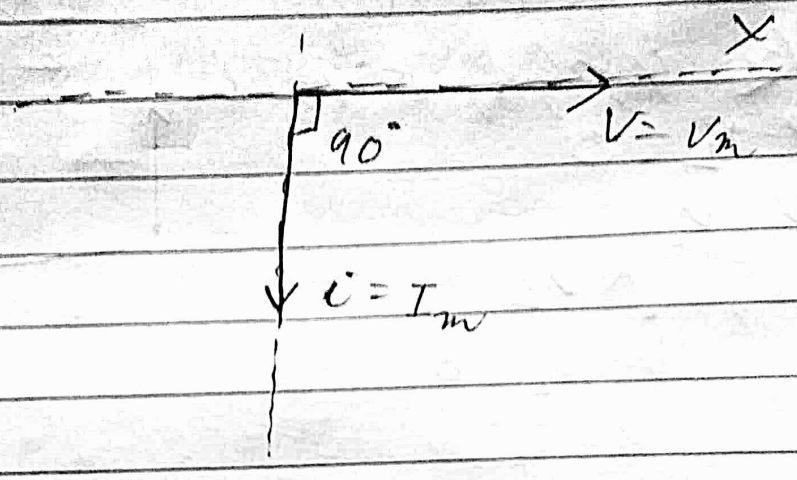
$$i = \frac{I_m}{X_L} \sin(\omega t - \pi/2)$$

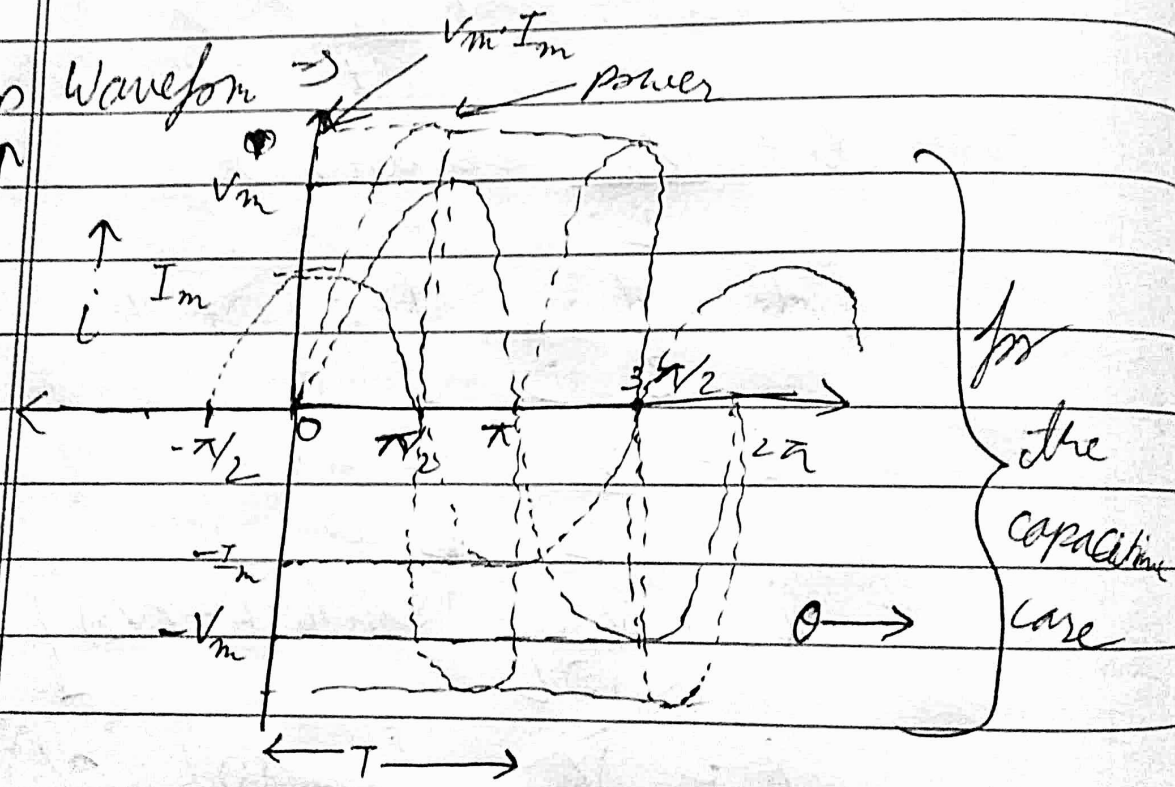
X_L ← reactance (inductive)

From eqn ① & ②

we can say that i is lagging from V by an angle ϕ of $\pi/2$ or $90°$.

Phasor diagram :-





$P_{avg} = v \cdot i$

$P_{avg} = V_m \sin\omega t \cdot I_m \sin(\omega t - \pi/2)$

$P_{avg} = V_m \cdot I_m \cdot \sin\omega t \cdot \cos\omega t$

$P_{avg} = \frac{1}{T}\int_0^T P \cdot dt$

$= \frac{1}{\pi}\left[P \right]_0^{\pi}$

$$\cancel{P_{avg} = \frac{1}{T}\left[V_0 I_m - V_0 I_m\right]}$$

$$\cancel{= \frac{1}{T}}$$

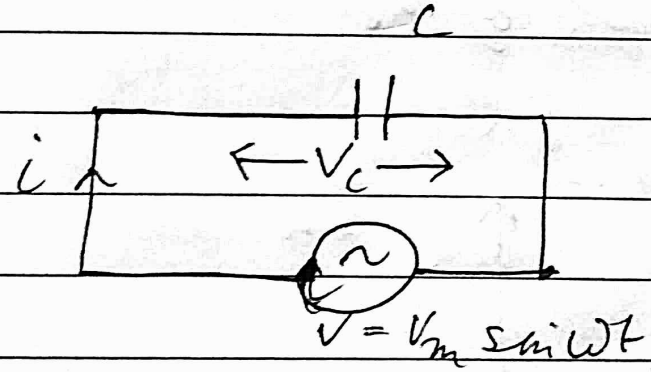
$$P_{avg} = \frac{1}{T}\left[-P\right]_0^{T/2} + \left[P\right]_{T/2}^{T}$$

$$= \frac{1}{T}\left\{\left[V_m I_m\right] + \left[-V_m I_m\right]\right\}$$

$$= \frac{1}{T}\left\{V_m I_m - V_m I_m\right\}$$

$$= \frac{1}{T} \times 0 = 0$$

(iii) Purely Capacitor ckt



$C = \frac{q}{V}$

by applying KVL,

$$V - V_C = 0$$

$$V = V_C$$

$$V_m \sin\omega t = \frac{1}{C}\int i\,dt$$

$$\int i\,dt = V_m \sin(\omega t)$$

$$\frac{d}{dt}\left[\int i\,dt\right] = V_m C \frac{d}{dt}\sin(\omega t)$$

C I V I L

Date.
Page No.

Current will lead the voltage by $\pi/2$ in capacitor ckt.

$$\frac{d}{dt}\left[\int i\,dt\right] = V_m C \cdot \omega \cos(\omega t)$$

$$i = V_m C \omega \sin \omega t$$

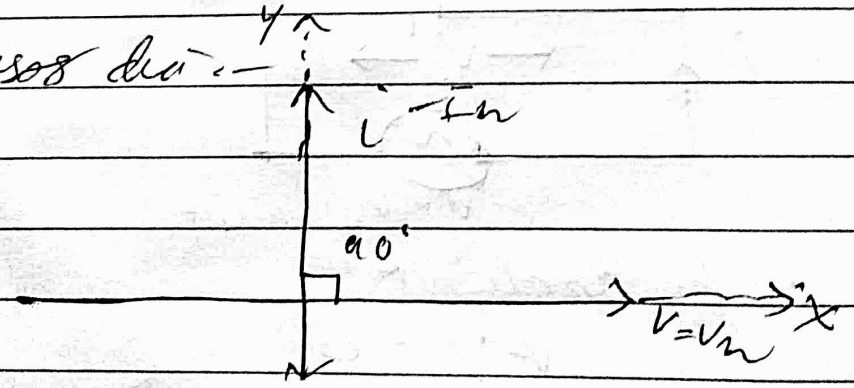
$$i = \left(\frac{V_m}{\frac{1}{C\omega}}\right) \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = \frac{V_m}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

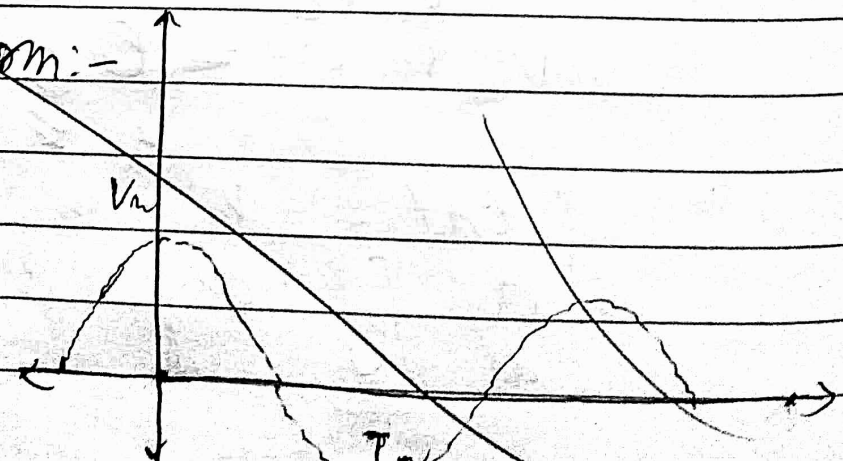
$$i = I_m \sin(\omega t + \pi/2) \quad -(2)$$

From eqn ① & ② we can conclude that i is leading from V by angle of 90°

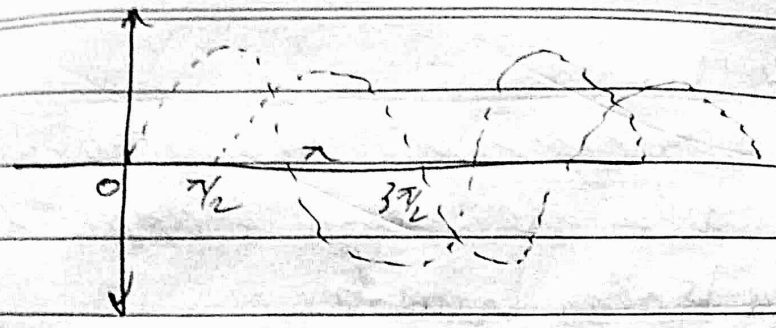
Phasor dia:-



Waveform:-



Date.
Page No.



Q A coil has an inductance of 25 milli Henry. Calculate its reactance.

(a) 50 Hz
(b) 2 KHz
(c) 1.5 MHz

Sol(1) $X_L = \omega L$
$= 2\pi f \times L$
$= 2\pi \times 50 \times 25 \times 10^{-3}$
$= 314 \times 25 \times 10^{-3}$
$= 7.854\ \Omega$

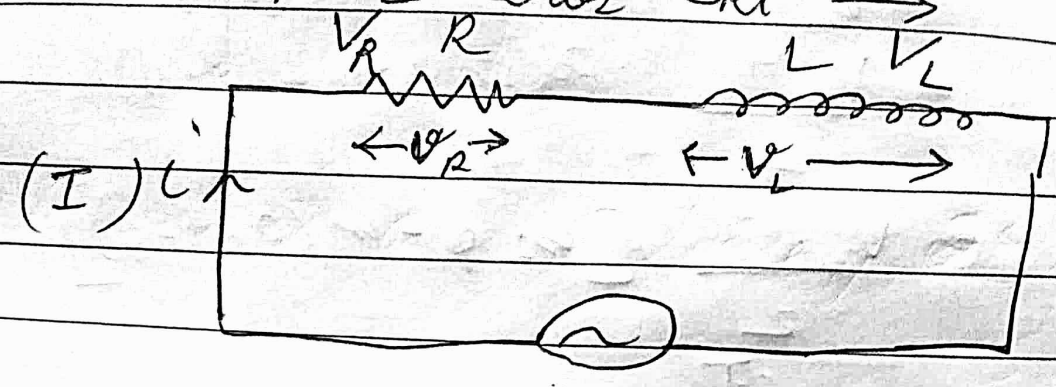
Sol(2) $X_L = \omega L$
$= 2\pi f L$
$= 2 \times 3.14 \times 2 \times 10^{3} \times 25 \times 10^{-3}$
$= 314\ \Omega$

Sol(3) $X_L = \omega L$
$= 2\pi f L$
$= 2 \times 3.14 \times 1.5 \times 10^{6} \times 25 \times 10^{-3}$
$= 235.6 \times 10^{3}\ \Omega$

Q) Write down the freq the r_{ms} & peak value of voltage wave expressed as
$$V = 14.1 \sin 100\pi t.$$

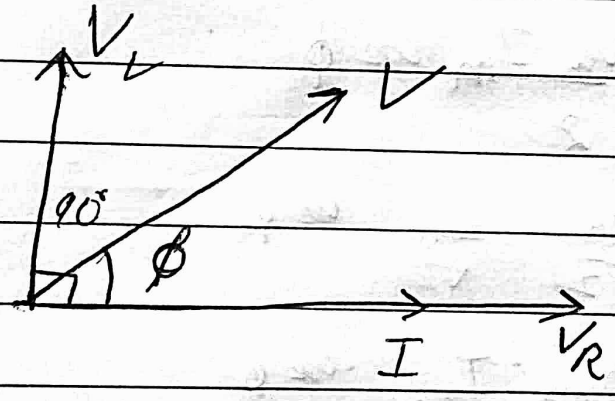
Case 4 :-

R L Series Ckt



where V, I, V_R, V_L are rms value of voltage & current, resultant voltage across resistance & inductor.

Phasor Dig :-

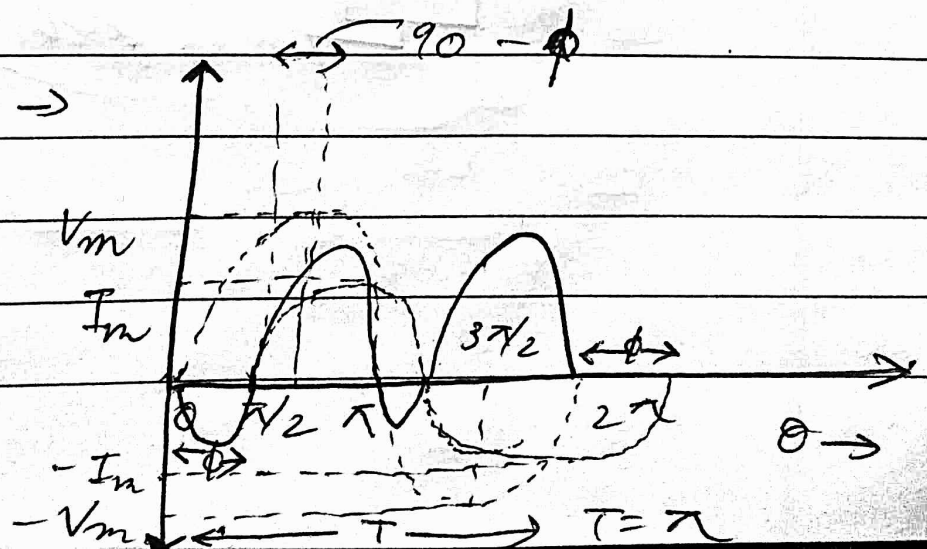


$\phi < 90°$

If $v = V_m \sin(\omega t)$

$i = I_m \sin(\omega t - \phi)$

Waveform →



$$P = V \cdot i$$

$$= V_m \sin \omega t \cdot I_m \sin(\omega t - \phi)$$

$$= V_m \cdot I_m \sin(\omega t) \cdot \sin(\omega t - \phi)$$

$$= \frac{V_m I_m}{2} \cdot 2(\sin \omega t)(\sin(\omega t - \phi))$$

$$= \frac{V_m \cdot I_m}{2} \left[\cos \phi - \cos(2\omega t - \phi)\right]$$

$$P_{avg} = \frac{1}{T} \int_0^T p\,dt$$

$$= \frac{V_m I_m}{2\pi} \int_0^{\pi} \left[\cos \phi - \cos(2\omega t - \phi)\right]$$

$$= \frac{V_m I_m}{2\pi} \int_0^{\pi} -\sin \phi \frac{d\phi}{dt} + \sin \phi$$

Incomplete in b/w

$$\Rightarrow \frac{V_m I_m}{2} \cos \phi$$

$$\Rightarrow \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \phi$$

$$\Rightarrow V_{rms} \cdot I_{rms} \cdot \cos \phi$$

$$\Rightarrow V \cdot I \underbrace{\cos \phi}_{\text{power factor}}$$

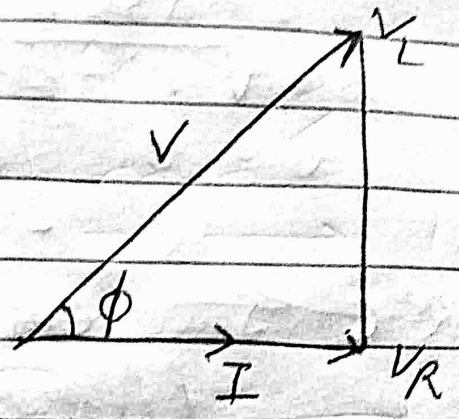
Voltage Triangle →

Here, $V_R = V\cos\theta$

$V_L = \sin\phi$

$V = \sqrt{V_R^2 + V_L^2}$

$\phi = \tan^{-1}\left[\frac{V_L}{V_R}\right]$



Impedance Triangle :-

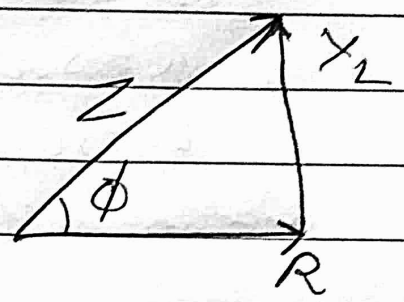
(R, L) or (R, C) or (R, L, C)

Here $R = Z\cos\phi$

$X_L = Z\sin\phi$

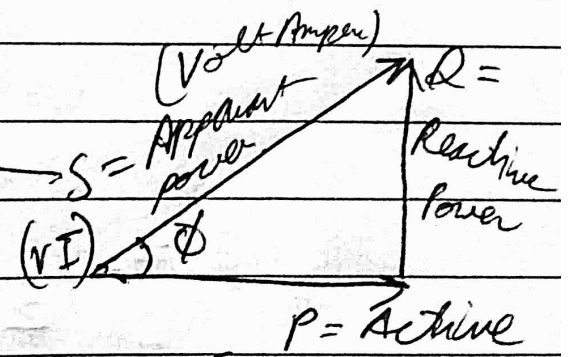
$Z = \sqrt{R^2 + X_L^2}$

$\phi = \tan^{-1}\left[\frac{X_L}{R}\right]$



Power Triangle → [VI$\sin\phi$] Volt ampere Reactive

Power actually generated for the source.



Its the only beneficial or mostly used power & ϕ should be less for Active power.

$$P = S\cos\phi$$

$$Q = S\sin\phi$$

$$S = \sqrt{P^2 + Q^2}$$

$$\phi = \tan^{-1}\left[\frac{Q}{R}\right]$$

~~5) R-C Series Ckt.~~

Numerical :-

Q1) A coil has an inductance of 20 mH, resistance of 5Ω. connected across a supply voltage $v = 50\sin 314t$.

$L = 20\,mH$

$R = 5\Omega$

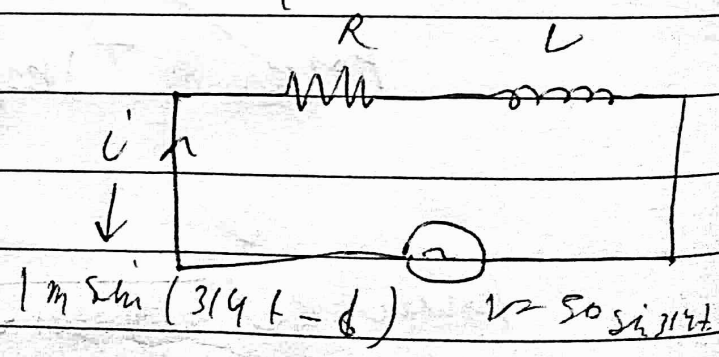
$V = 50\sin 314t$

Obtain a similar exp for supply current.

Soln.

$\omega = 2\pi f$

$314 = 2 \times 3.14 \times \frac{1}{T}$



$$I_m = \frac{V_m}{Z}$$

$$|Z| = \sqrt{R^2 + X_L}$$

$$X_L = \omega L$$

$$= 314 \times 20 \times 10^{-3}$$

$$|Z| = \sqrt{5^2 + 314 \times 20 \times 10^{-3}}$$

$$=$$

$$I_m = \frac{V_m}{Z} = 6.25\,A.$$

Rectangular form → Polar to Rect.

(Real + j (imaginary))

$\downarrow \quad + j \quad \downarrow$

$x \quad y$

$\downarrow \quad \downarrow$

$r\cos\phi \quad r\sin\phi$

$V_1 = 100\cos 0^\circ + j100\sin 0^\circ = 100 + j0$

$V_2 = 150\cos(-30)^\circ + j\,150\sin(-30^\circ) = \frac{150\times\sqrt{3}}{2}$

$- j\,150\times\frac{1}{2}$

$= 129.9 - j\times 75$

Rect to polar form:-

$r = \sqrt{x^2+y^2}$

$= \sqrt{129.9^2+(75)^2}$

$\phi = \tan^{-1}\left[\frac{-75}{129.9}\right]$