Forgetful Phil: A Rebuttal of the Proof of Concept Objection to Epistemic Infinitism

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Motivation

Proof of Concept Objection

Rebutting the Proof of Concept Objection

Forgetful Phil

Motivation

Meta-epistemology versus Epistemology

Foundationalism versus Infinitism versus Mixed Views

Foundationalism

Basic justified belief

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Inferred justified belief

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Inferred justified belief

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Inferred justified belief

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Inferred justified belief

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Infinitism

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Inferred justified belief

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Inferred justified belief

Mixed Views

Infinitism has little appeal for *a posteriori* justification.

Infinitism has some appeal for *a priori* justification.

Proof of Concept Objection

Doxastic Justification versus Propositional Justification

For an epistemic infinitist:

Propositional justification for a belief means having available an infinite and non-repeating sequence of reasons for that belief.

Doxastic justification for a belief means traversing some satisfactory number of those reasons for this belief.

The *satisfactory number* of beliefs in this infinite sequence of reasons is set by the dialectical context.

Proof of Concept Objection

Problem: why believe there are any *infinite* and non-repeating sequences of reasons available to beings like us?

1. For an infinitist to think that a belief is justified [either doxastically or propositionally], they must think that it is supported by an infinite number of reasons. (Premise)
2. There could be no evidence to support the claim that the reasons available to support a belief are infinite as opposed to numerous but finite. (Premise)
3. So an infinitist has no reason to think that any belief that someone holds is justified [doxastically or propositionally].

Finite Minds Objection

An infinitist has another problem pulling in another direction:

Finite Minds Objection: since finite minds cannot actually traverse an infinite and non-repeating sequence of reasons, so if epistemic infinitism is true, then we have no justified beliefs.

Damned no matter what you do

The epistemic infinitist dissolves the finite minds objection by saying that the sequence only has to be available to us.

This avoids the problem that we never actually traverse an infinite sequence: we just need to be able to do so.

This response opens them up to the proof of concept objection. Actually traversing the sequence would show it is infinite.

Rebutting the Proof of Concept Objection

The Reply to the Finite Minds Objection is Right

The infinitist is right to say that the finite minds objection is dissolved by the (appropriately infinitist) distinction between propositional and doxastic justification.

We can 'see' and be justified in believing that certain recursive processes could go on indefinitely.

Some Infinite and Non-Repeating Sequences are Unhelpful

It is easy to conjure up infinite and non-repeating sequences of 'reasons' for beliefs:

$$p_0 \leftarrow \neg\neg p_0 \leftarrow \neg\neg\neg\neg p_0 \leftarrow \neg\neg\neg\neg\neg\neg p_0 \leftarrow \ldots$$

The Goal

We want an infinite and non-repeating sequence of reasons that

1. actually boosts the justification that one has for a belief
2. is available to the subject
3. for which we have evidence that it is infinite

Forgetful Phil

Forgetful Phil

Forgetful Phil is a mathematician who frequently forgets about the logical equivalences of claims.

Phil's long-term memory and his reasoning abilities work fine. Phil also accepts all the axioms of set theory minus Choice.

Phil also often forgets the pieces of logical equivalences that he has proven, but can reprove one half of a logical equivalence with a bit—say, five minutes—of thinking.

The Upshot

Phil will chase logically equivalent claims in just one direction:

$$p \leftarrow q \leftarrow r \leftarrow \ldots$$

Now what *informative* and *plausibly infinite* sequence of logical equivalences could we give Phil to follow?

Equivalences of the Axiom of Choice

The *Axiom of Choice* is the assertion that for every family $\mathcal{F}$ of non-empty sets, there is a choice function f such that for every F in $\mathcal{F}$, we have $f(F) \in F$.

Suppose that Phil has some foundationalist-acceptable grounds for accepting Choice; assume Phil has evidence available to him that any practicing mathematician would.

But Phil, being forgetful, does not remember all the equivalences of Choice.

Boosting Phil's Justification

Happily, Phil begins to recover precisely this evidence with on the spot reflection.

Phil begins a chain that he will never completely traverse by asking himself, reflecting on his pragmatic and other evidence, 'Is Choice really true?'

Phil considers this and then takes as a premise the Well-Ordering Principle (hereafter 'Well-Ordering'), which is logically equivalent to Choice.

Phil validly proves Choice from Well-Ordering in the context of Zermelo-Fraenkel set theory plus the principles of classical logic.

Forgetful Phil Forgets Again

Suppose all this takes exactly five minutes, so that just as Phil signs the proof 'QED,' Phil promptly forgets all about Choice.

Thanks to his forgetfulness, Phil does not bother to prove that the Well-Ordering holds if and only if Choice holds.

This setup ensures Phil's reasoning will be

- non-repeating: Phil will never come back to Choice
- non-trivial: Phil does not believe Choice simply because he believes a biconditional and believes one half of it.

Phil then asks whether Well-Ordering is true. Phil takes as a premise Zorn's Lemma, which equivalent to Well-Ordering.

Similar events transpire again.

And again.

And again.

The Result

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\begin{array}{lllll}
\text{Choice} & \Longleftarrow_{\mathrm{ZF}} \text{Well-Ordering} & & & \\
& \qquad\quad\downarrow & & & \\
& \text{Well-Ordering} & \Longleftarrow_{\mathrm{ZF}} \text{Zorn's Lemma} & & \\
& & \qquad\quad\downarrow & & \\
& & \text{Zorn's Lemma} & \Longleftarrow_{\mathrm{ZF}} \ldots &
\end{array}
$$

Claims

Forget Phil's sequence of beliefs for Choice

1. give Phil further justification for believing Choice
2. is available to Phil
3. is infinite and non-repeating

Takeaway

If these claims about Forgetful Phil are true, then the Proof of Concept objection was rebutted. We have a proof of concept.

Questions?