Image Enhancement in the Spatial Domain

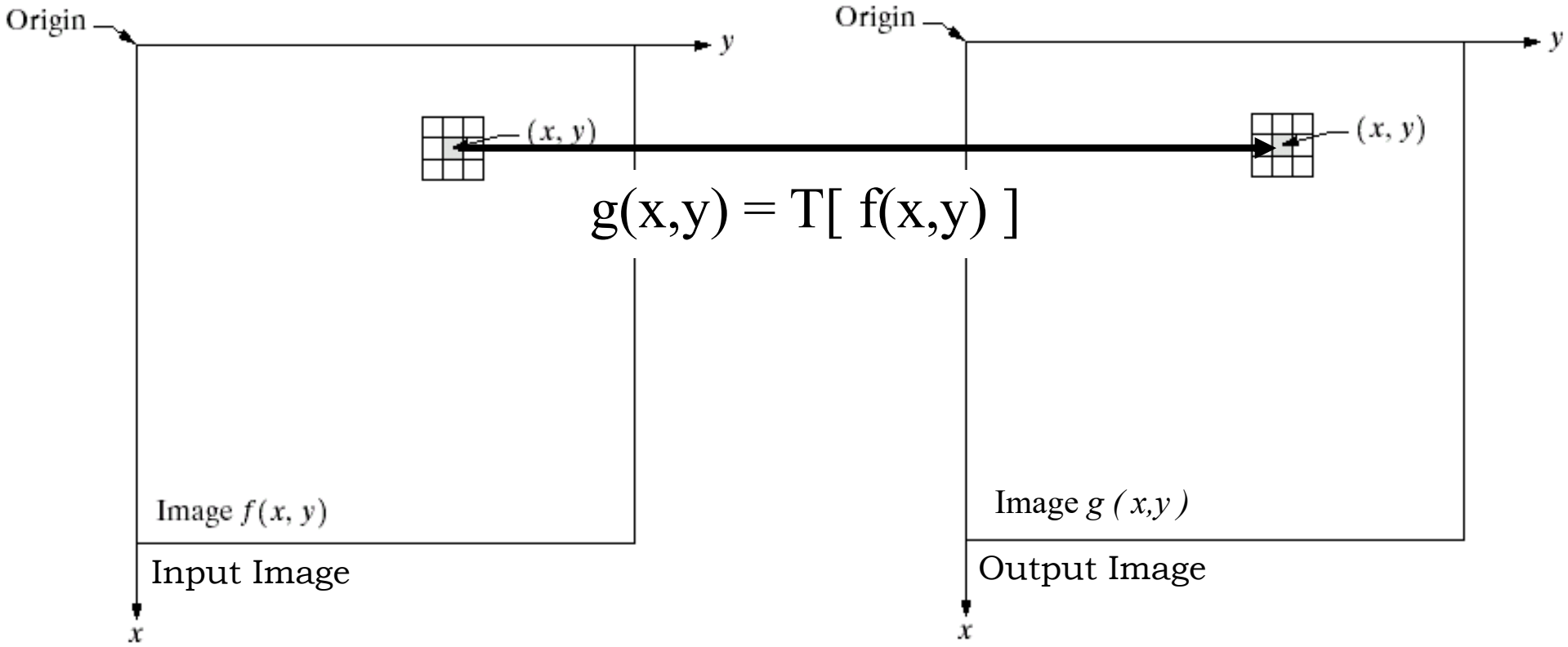
Enhancement

- Objectives
  - It makes an image more suitable than original for specific and problem-oriented applications.
  - It is often for human perception, the viewer is the ultimate judge of how well a particular works.
- Two broad categories
  - Spatial domain
    - Works with pixels
  - Frequency domain
    - Works with frequencies

Spatial Domain

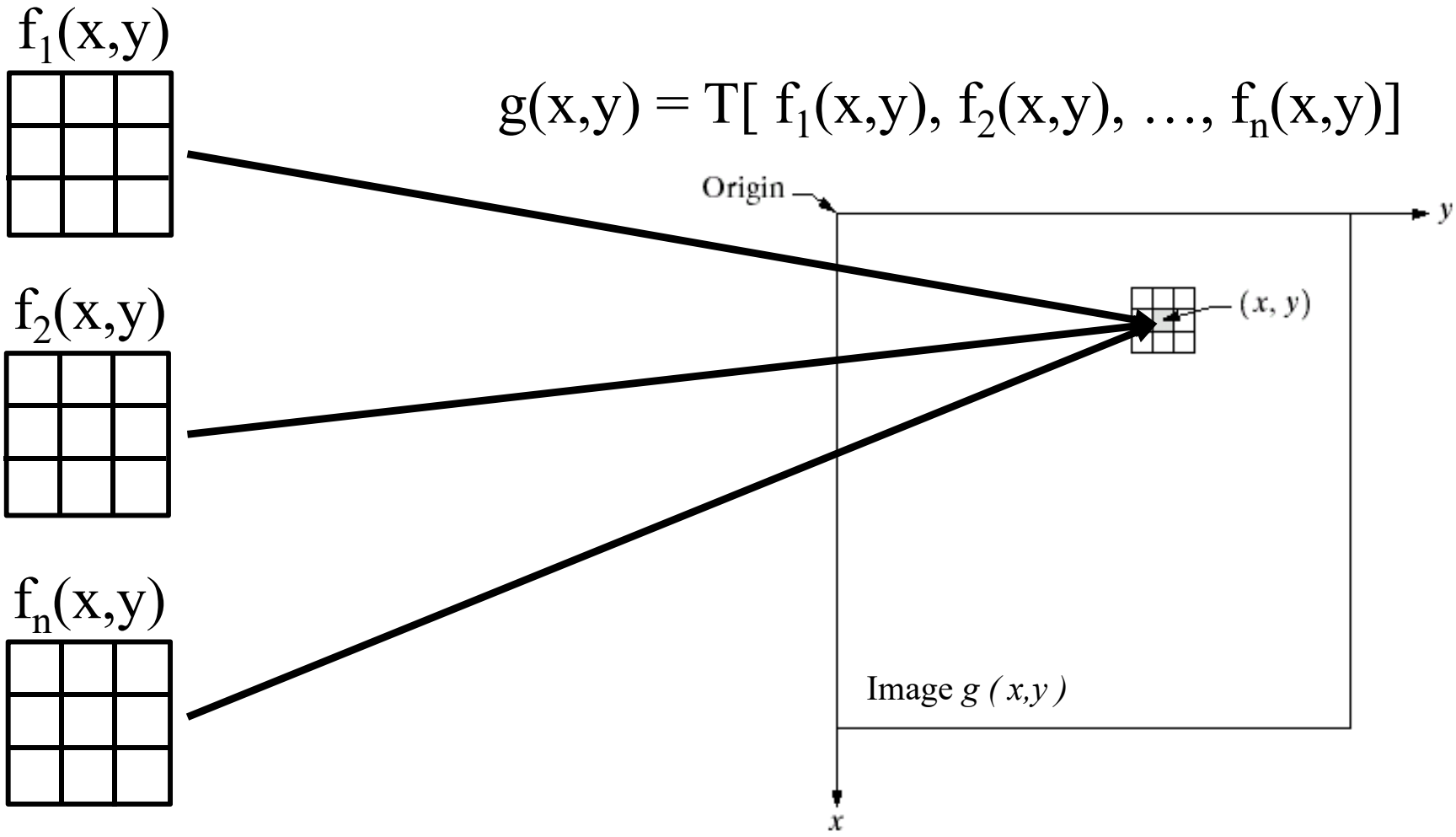
$$g(x,y) = T[\, f(x,y) \,]$$

- f(x,y) input image
- g(x,y) output, processed or transformed image
- T is an operator on f (T = intensity transformation function)
  - T is defined over some neighborhood of (x,y), e.g., square or rectangular sub-image area, or a point of 1x1 size.
  - T can be operated on a single image or a set of images.



T = intensity transformation function

FIGURE 3.1 A 3×3 neighborhood about a point (x, y) in an image.



T = intensity transformation function

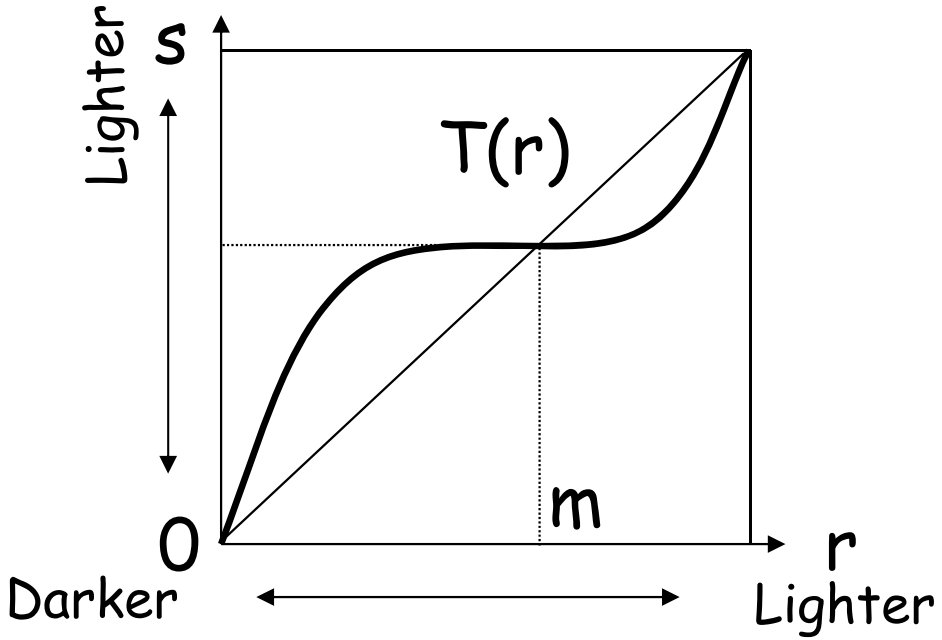
FIGURE 3.1 A 3×3 neighborhood about a point (x, y) in an image.

Gray-level transformation

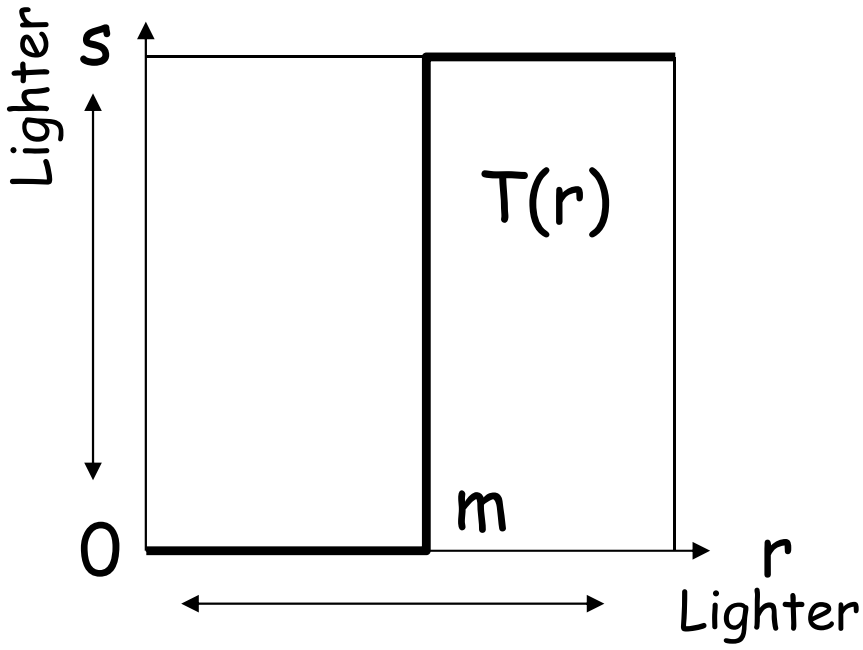
- Simplest form of T
  - neighborhood is 1 x 1 (one pixel)

- Notation: s = T(r)
  - r, s denote gray levels of f(x,y) and g(x,y) for any point (x,y) respectively

Examples of T(r)



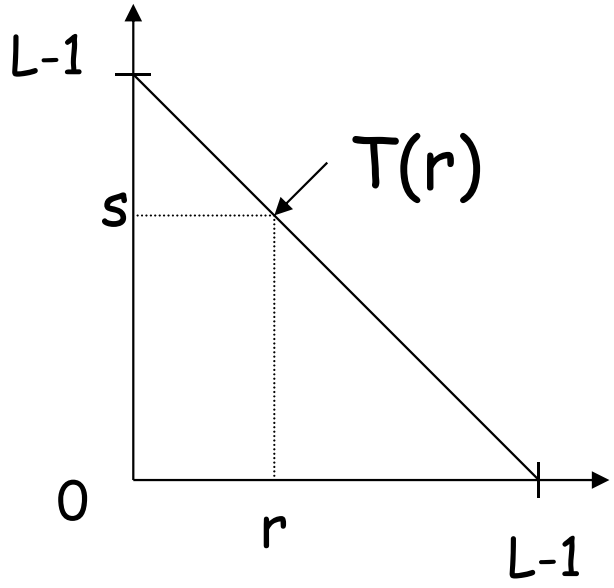
It produces an image of lower contrast than the original by (1) darkening the intensity levels above **m** and (2) brightening the leaves below **m** in the original image.



It produces a binary image with an intensity threshold at **m**.

Low intensity levels below **m** map to zero and high intensity levels above **m** map to **s**.

Basic Gray Level Transforms: Inverse or Image Negatives



$s = T(r) = (L-1)-r$

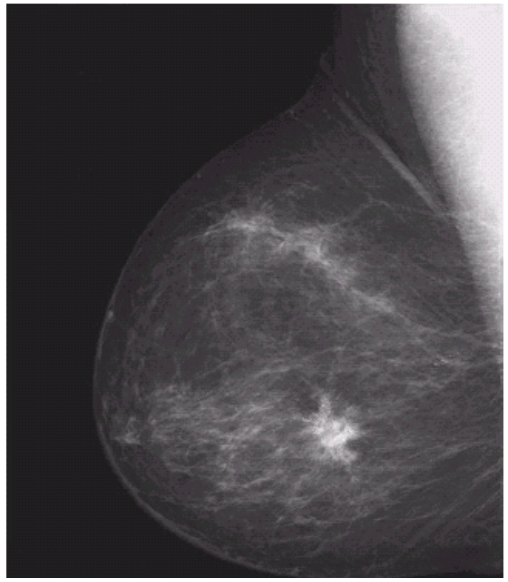
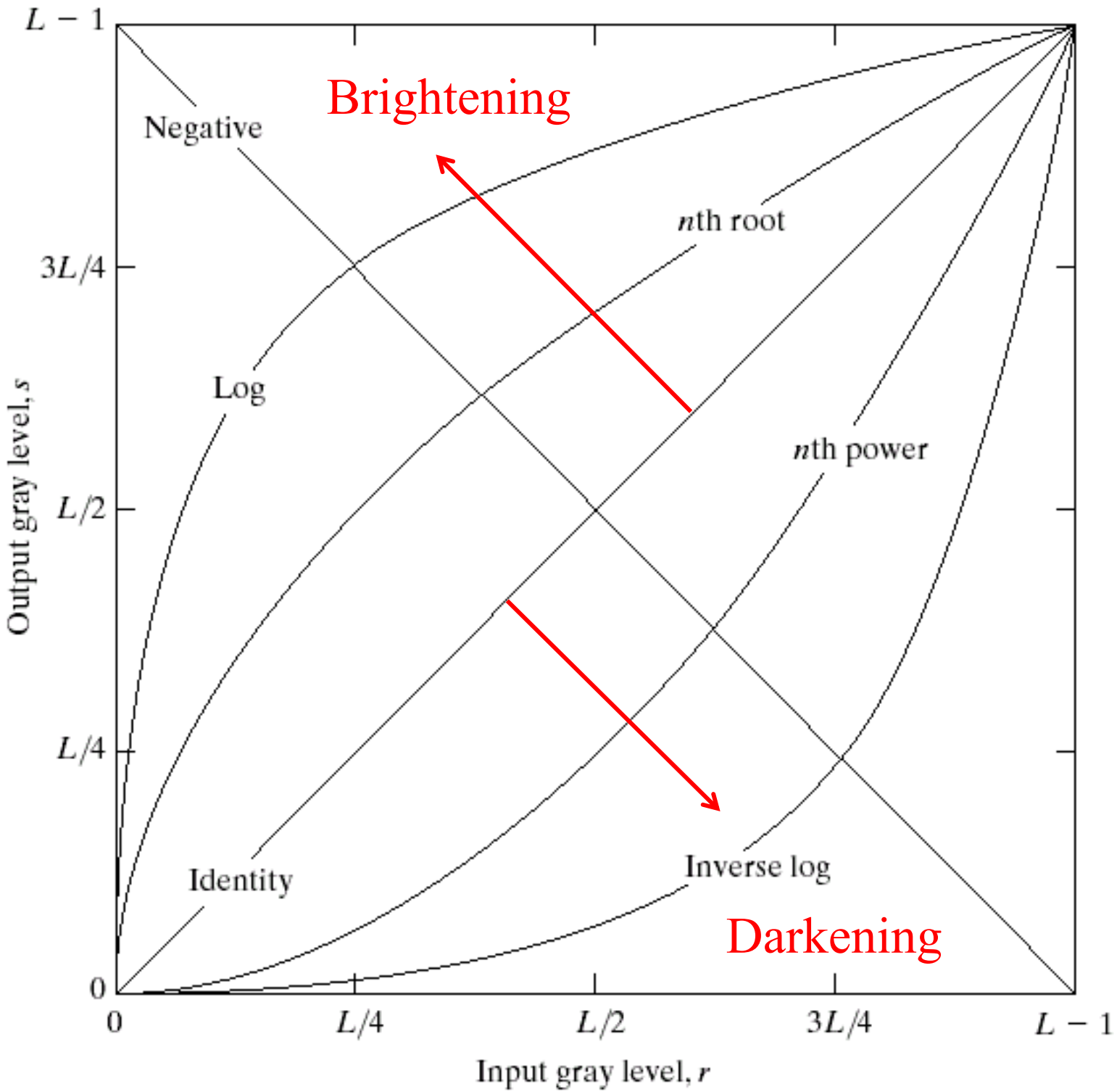
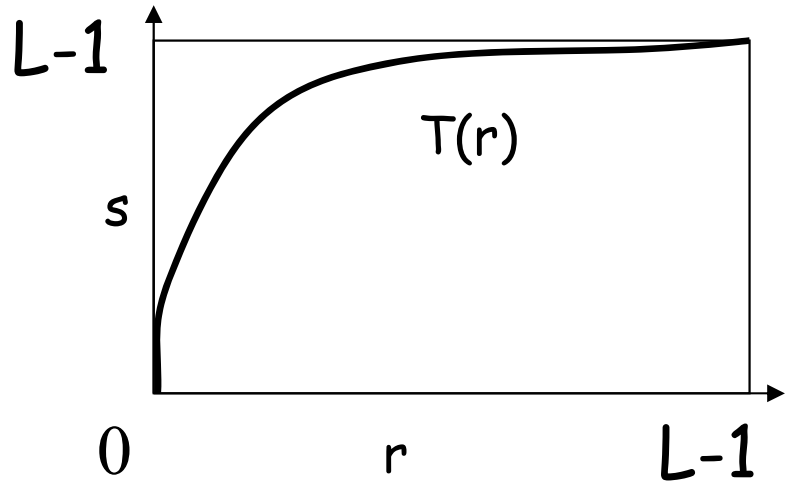


FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.



Log Transformations



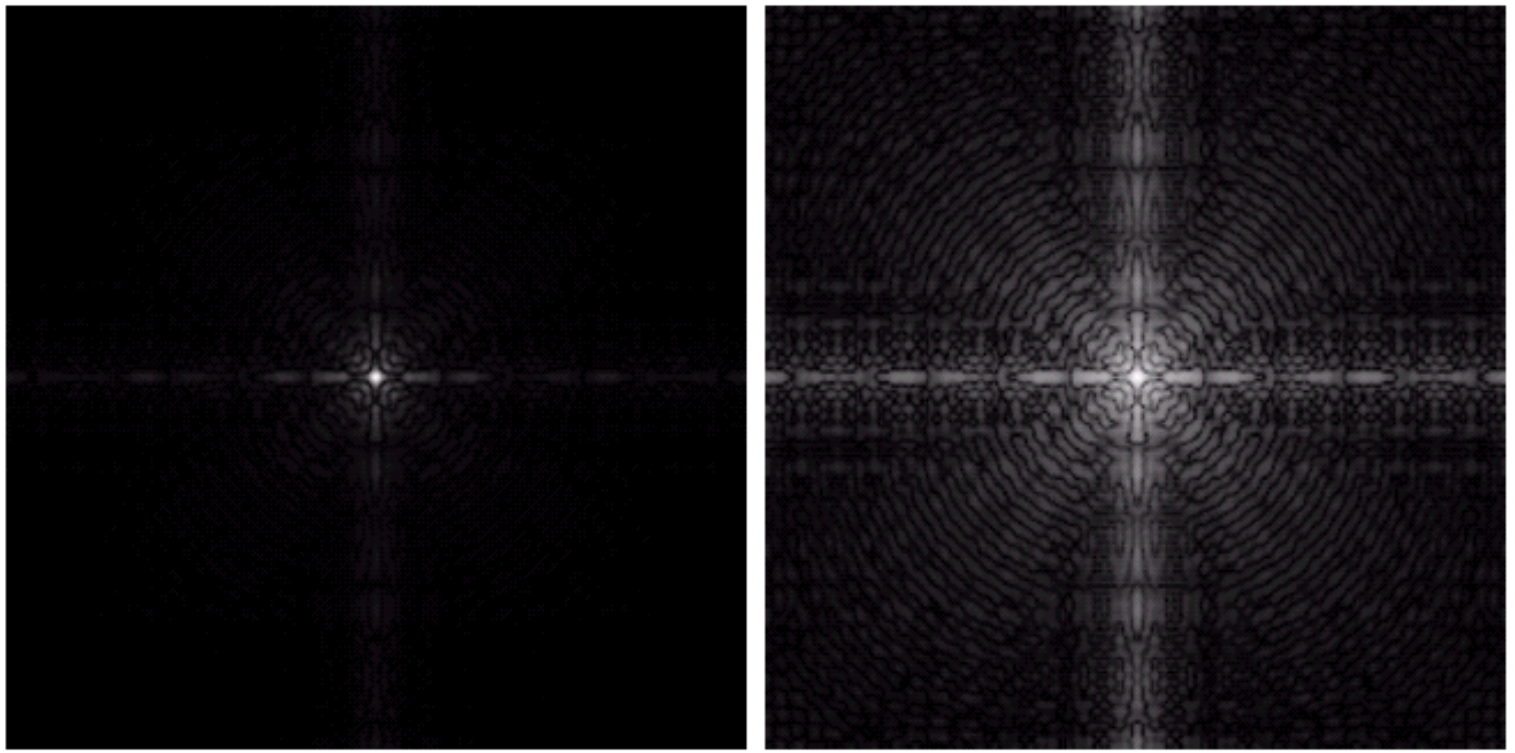
$s = c \log(1 + r)$

(c is some scale-factor)
(intensity rescaling maybe needed for s to fit s into the range [0, L-1])

The image becomes brighter and details are more visible.

a b

FIGURE 3.5
(a) Fourier spectrum.
(b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.



Power-Law Transformations

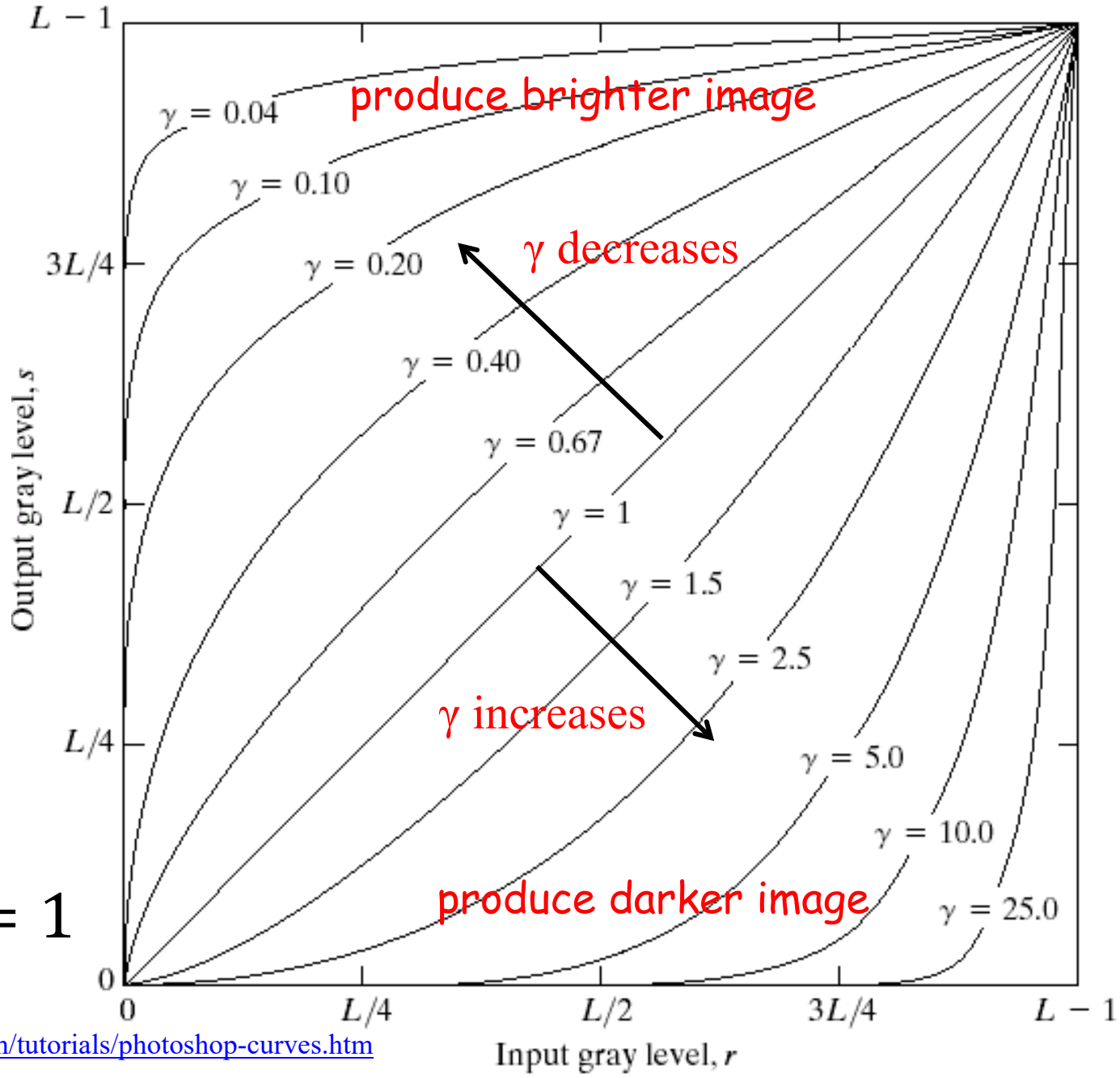
$$s = cr^{\gamma}$$

where c and γ are positive constants.

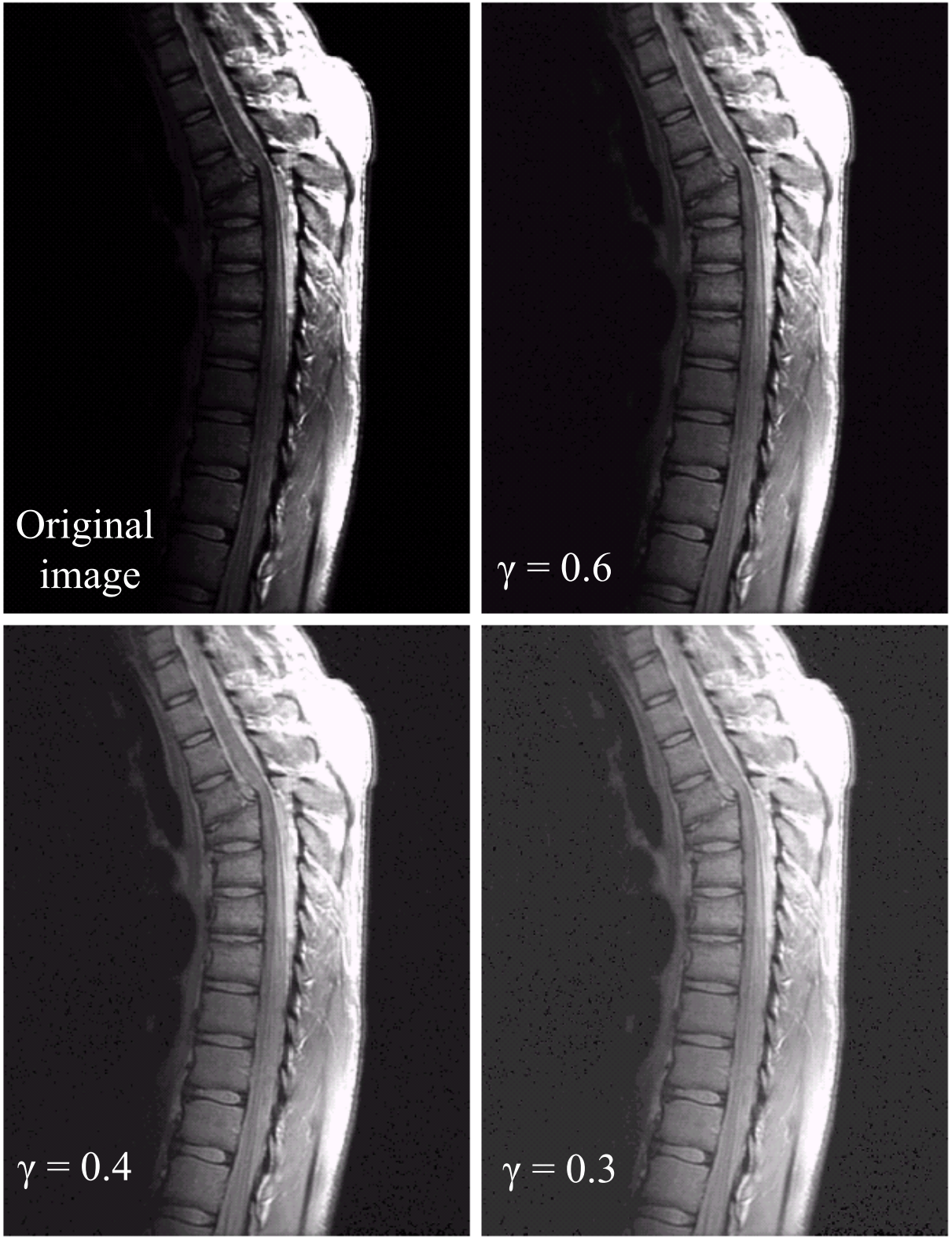
Intensity rescaling may be needed for s to fit s into the range [0, L-1].

Also, known as *gamma* (γ) *correction.*

$C = 1$



Examples: http://www.cambridgeincolour.com/tutorials/photoshop-curves.htm



a b
c d

FIGURE 3.8
(a) Magnetic resonance (MR) image of a fractured human spine.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4$, and 0.3, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

With the help of power-law transformation, fractures near the vertical centre of the spine are now made more visible in the magnetic resonance image.

a b
c d

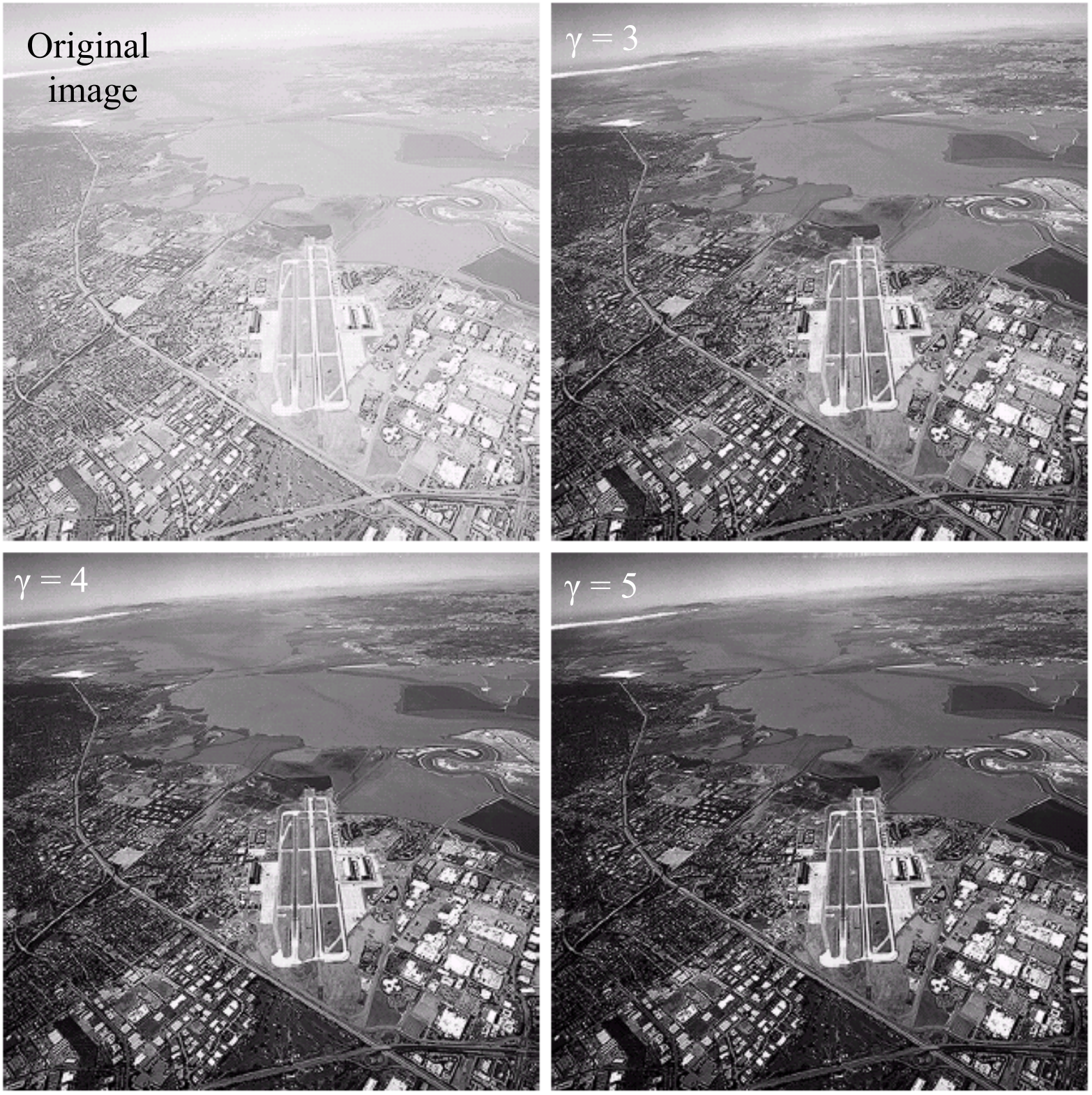
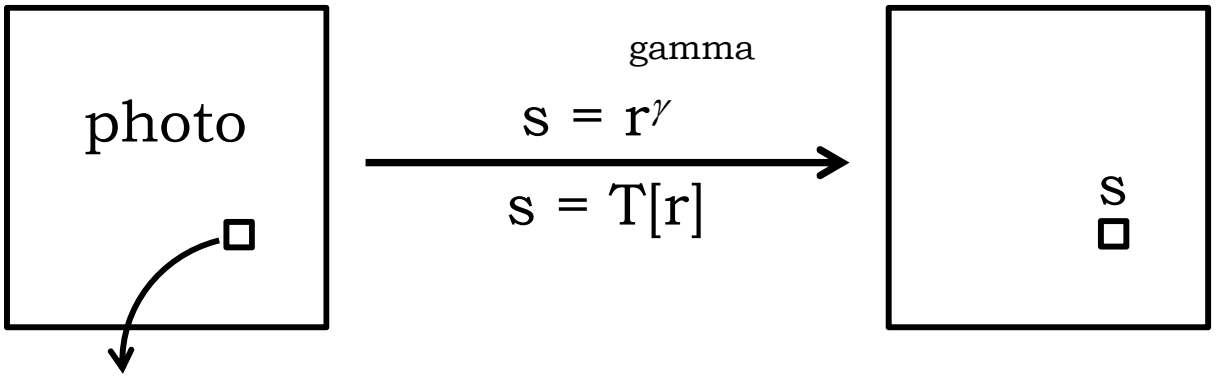


FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

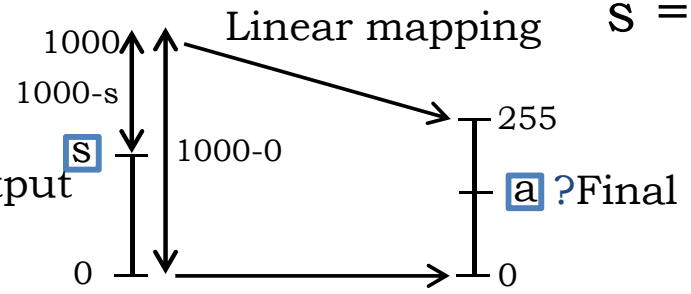
Intensity normalization

r = input intensity

s = output



$r = [0 - 255]$
8 bits



$s = [0 - 1000]$

matrix

max(max(s)) = 1000

min(min(s)) = 0

known

unknown (Final)

Ratio $\left[\frac{1000 - s}{1000 - 0}\right] = \left[\frac{255 - a}{255 - 0}\right]$

(Full range) (Full range)

8 bits

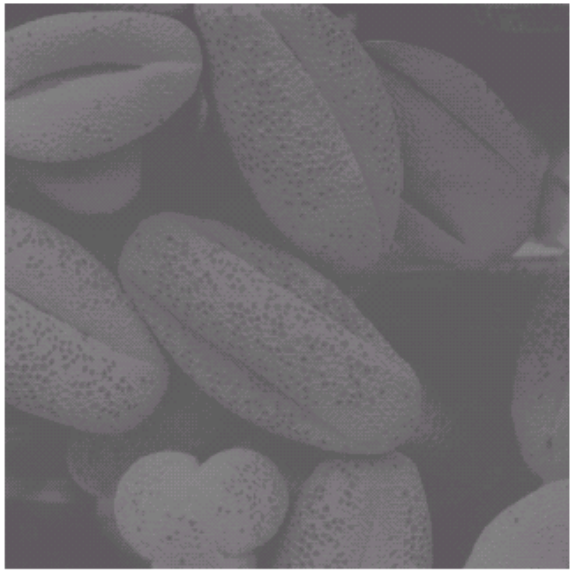
$$a = 255 - \frac{255(1000 - s)}{1000}$$

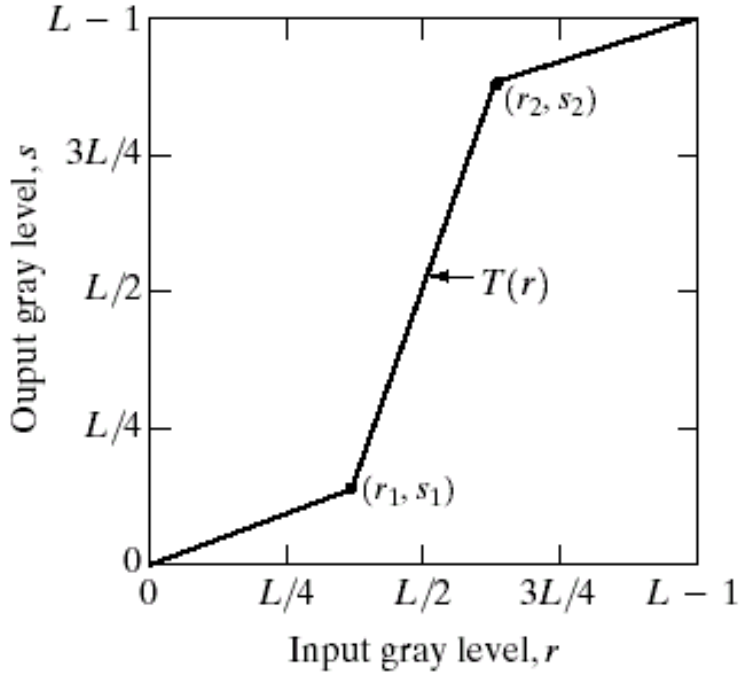
matrix

$a = [0 - 255]$

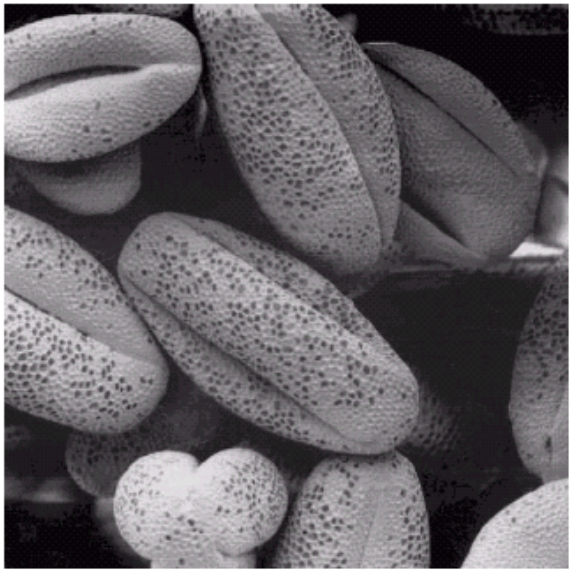
Contrast Stretching

A low-contrast image





Output image



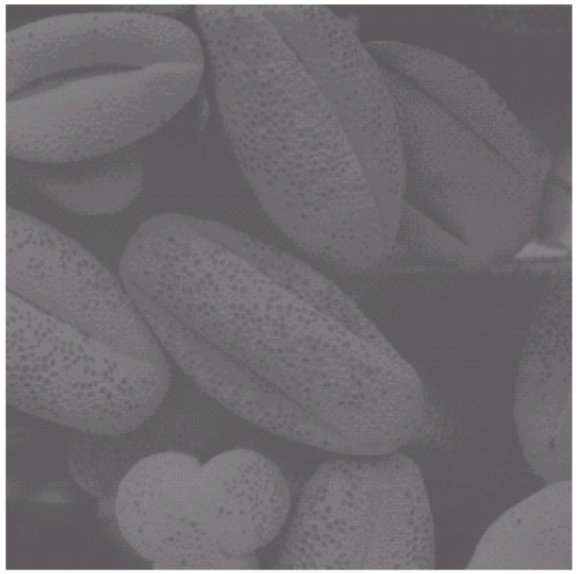
If $r_1 = r_{\min}$ $s_1 = 0$

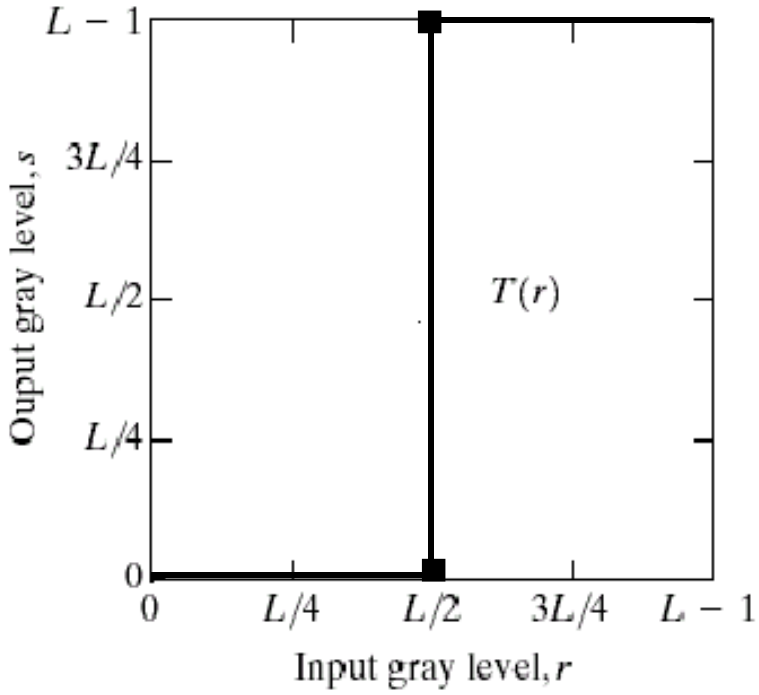
$r_2 = r_{\max}$ $s_2 = L-1$

then intensity levels are stretched to the full intensity range [0, L – 1].

Contrast Stretching

A low-contrast image



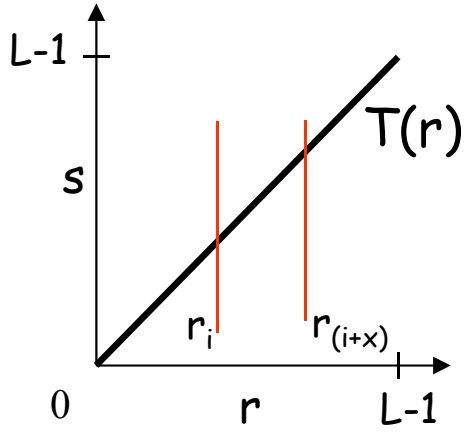


Output binary image

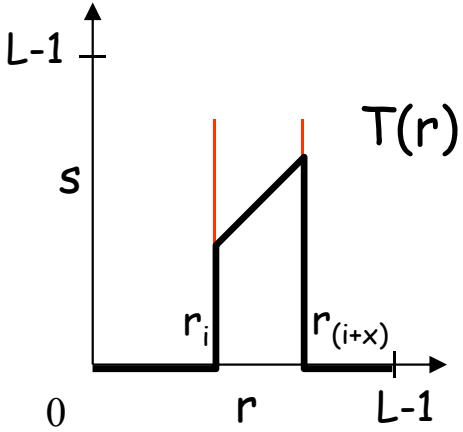
If $r_1 = r_2$ $s_1 = 0$ $s_2 = L-1$

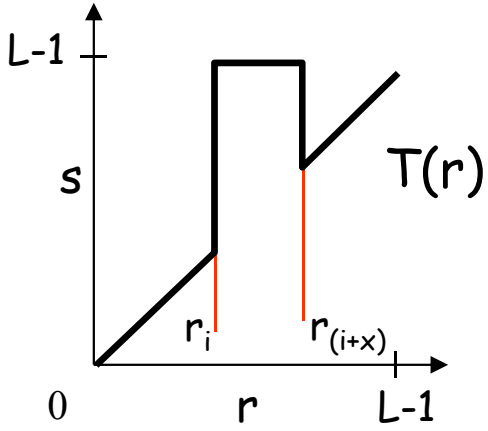
then the image is thresholded and a binary image is produced.

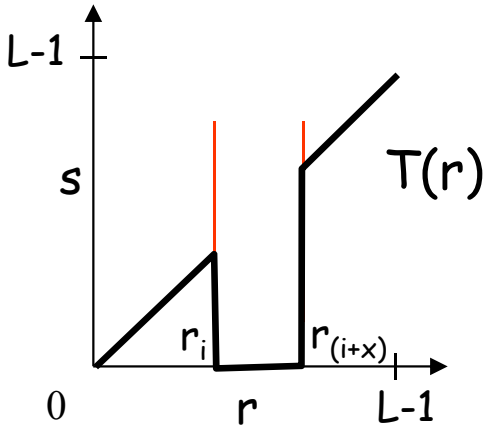
Windowing (or Gray Level Slicing)



Operating with a "window" of gray levels r_i to $r_{(i+x)}$

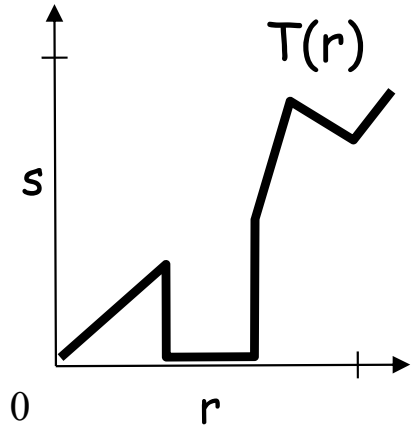






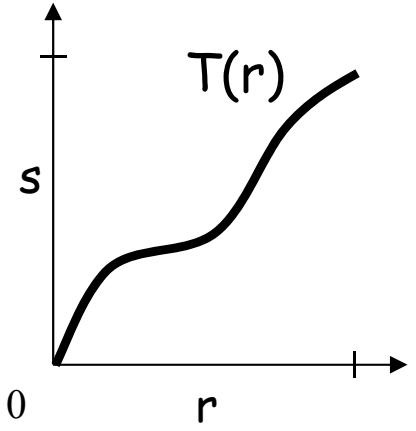
Properties of T(r)

- From r to s,
  one-to-one mapping
- From s to r,
  one-to-many mapping



- No-inverse due to ambiguity
- Does not preserve gray level ordering
- For non-inverse transformation, it is not an one-to-one mapping and is one-to-many mapping.

- One-to-one mapping
- Increasing



- Inverse
- Preserves gray level ordering (increasing)

Image Histogram

- Image Histogram
  - digital image with gray levels [0, L-1]
  - $p(r_k) = n_k/N$, probability of occurrence of gray level r_k
  - r_k is the k^{th} gray level
  - n_k = number of pixels with k^{th} gray level
  - N = total number of pixels
  - k = 0, 1, 2, 3, 4, 5 . . ., L-1

Image Histogram

- $p(r_k)$ is the probability of the occurrence of gray-level r_k

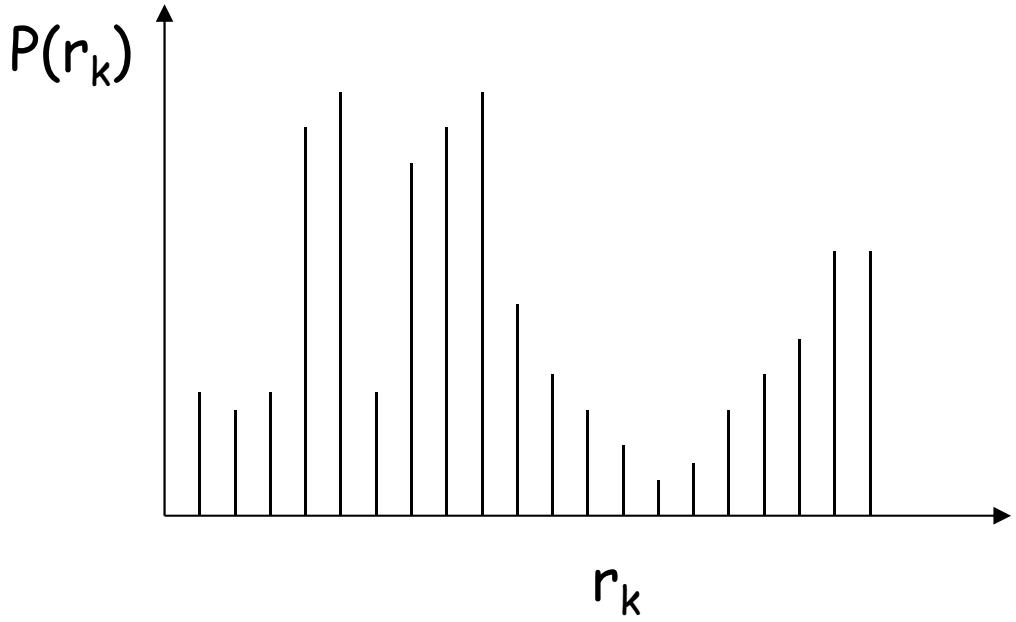


Image Histogram

- Histogram can tell you a lot about an image
- Gray level distribution
- It can be modelled by statistical distribution (red line)

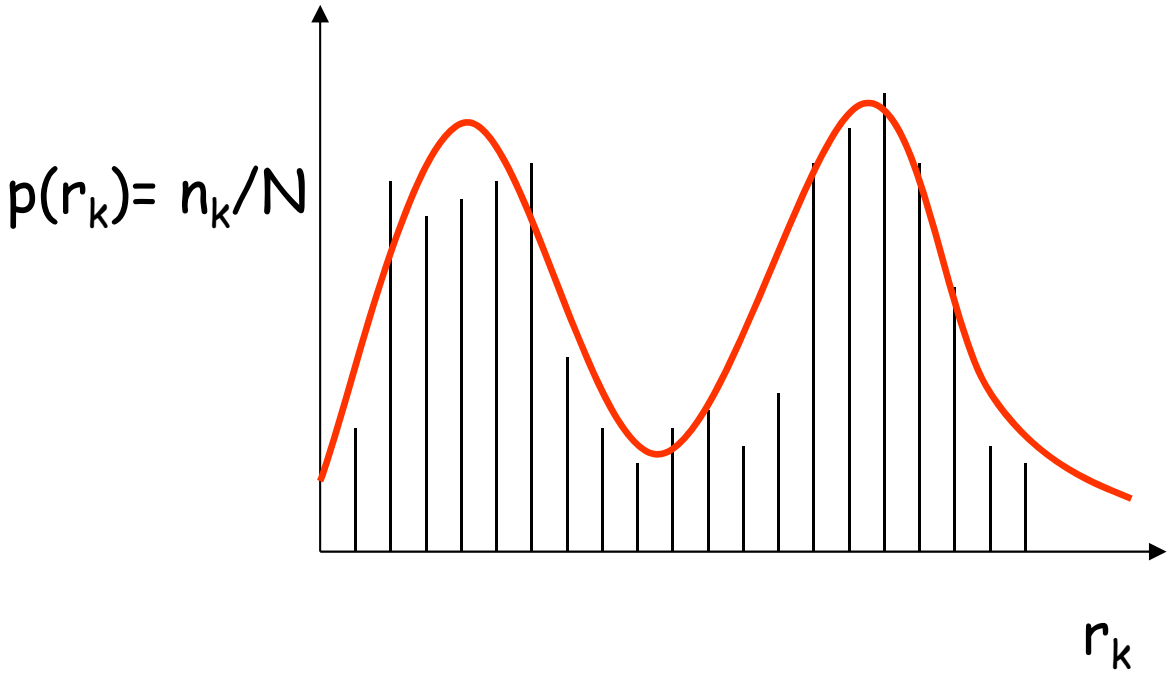


Image Histogram

- Gray level distribution helps define intensity threshold

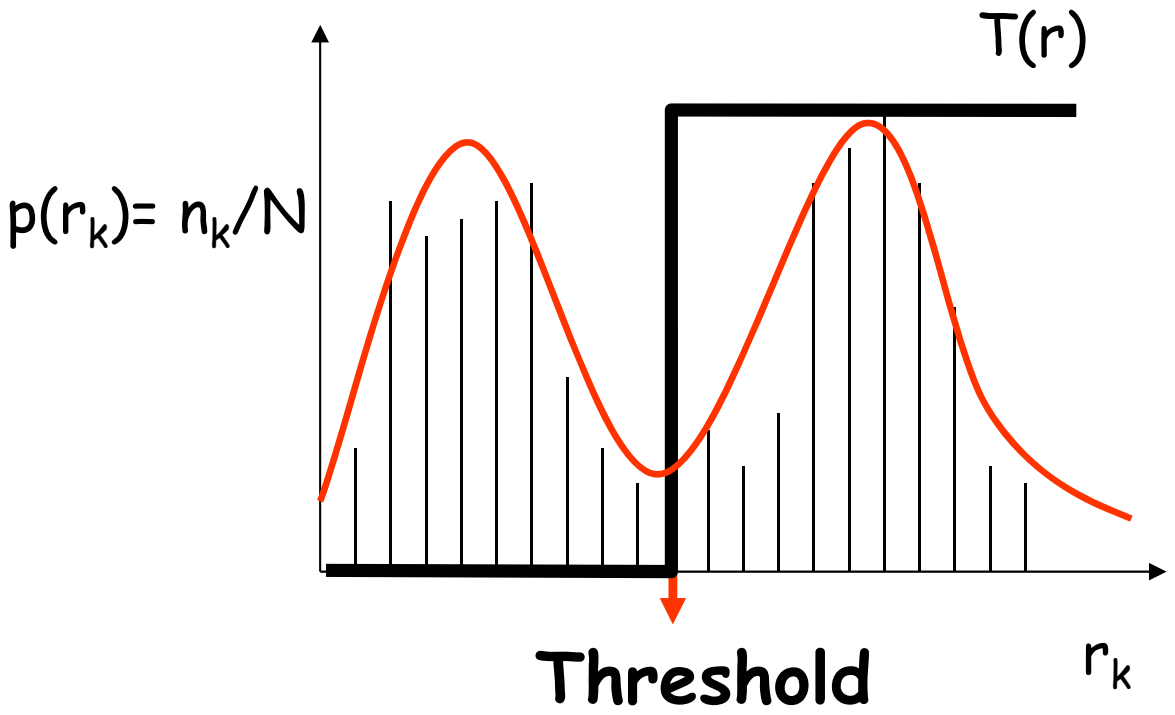
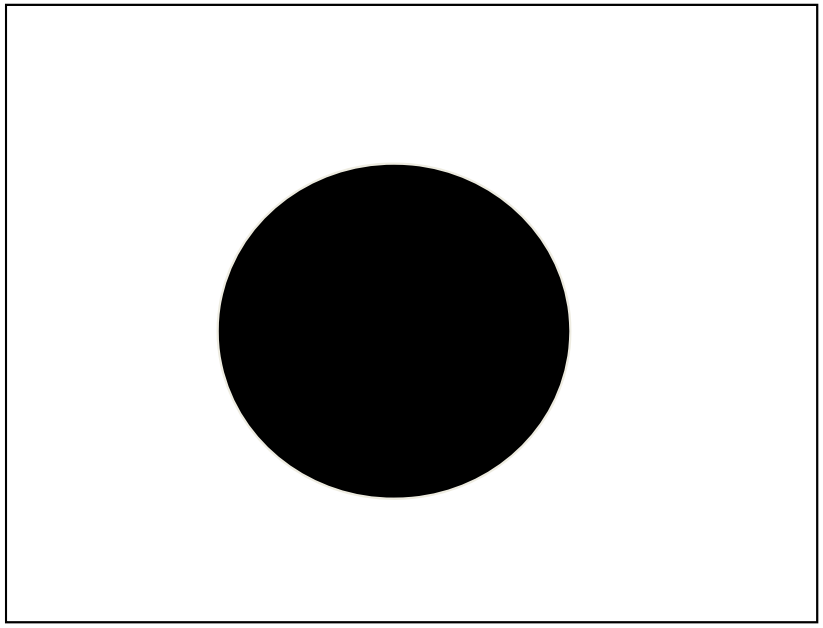


Image Histogram

- Histograms are not unique. Two images below give the same image histogram.
- No spatial information is captured



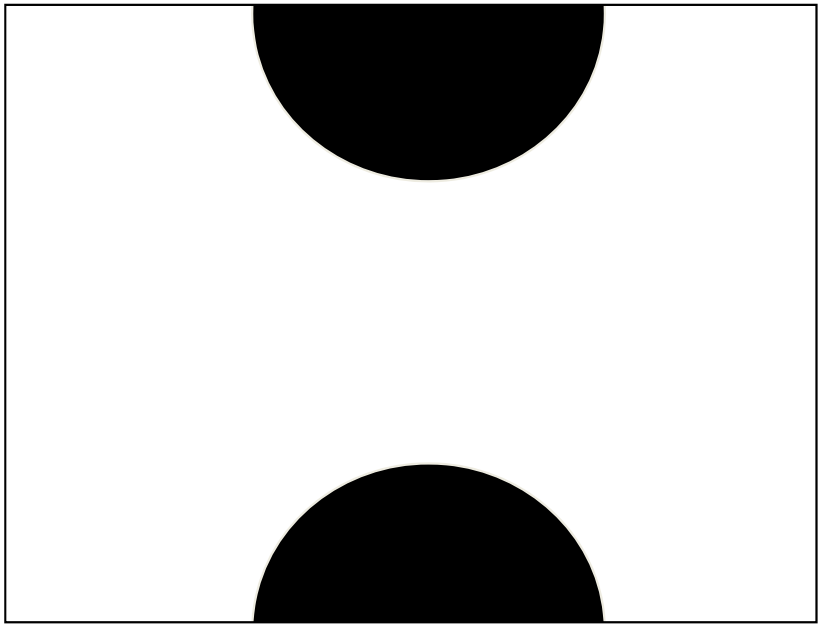


Image Histogram

- Types of Image

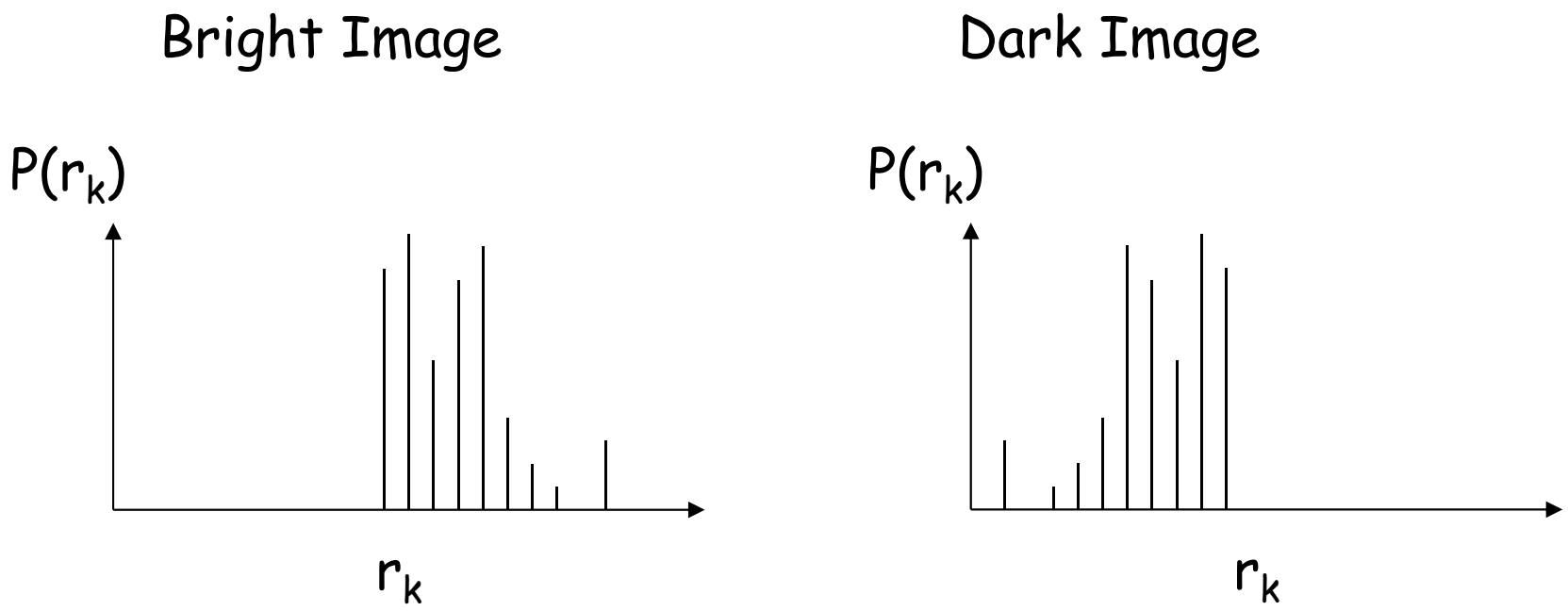
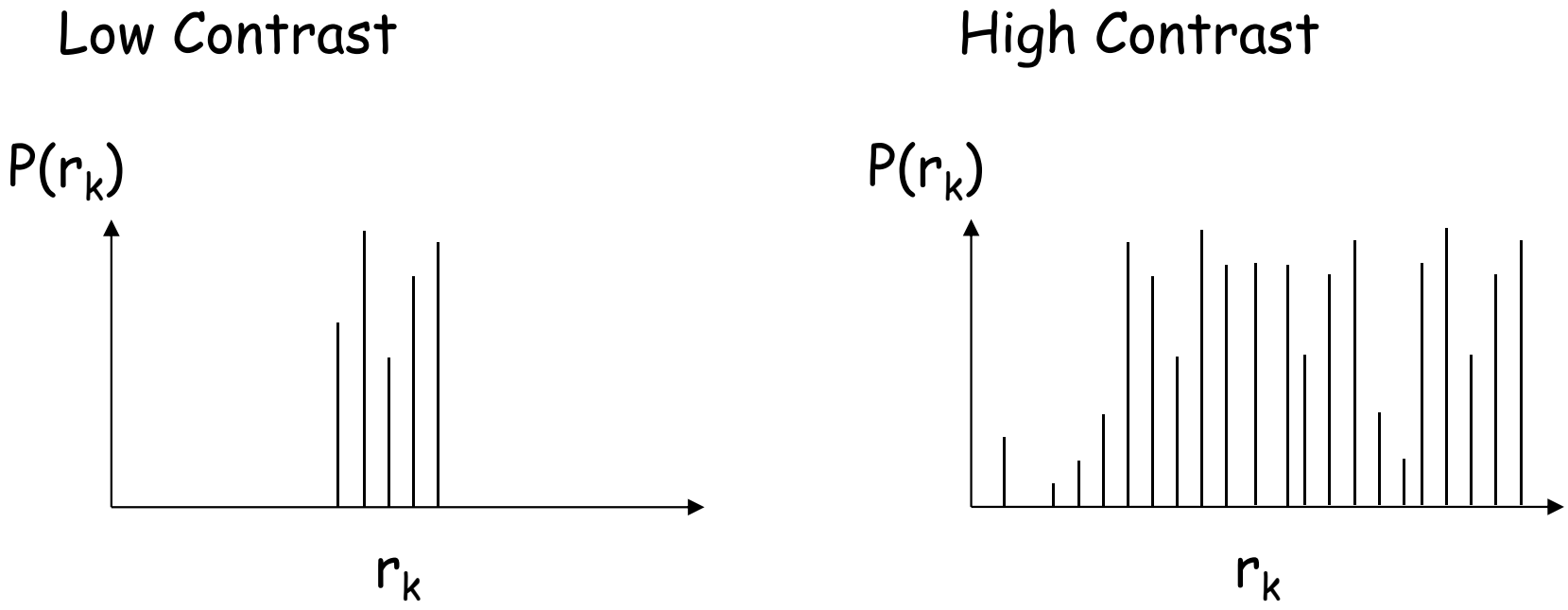
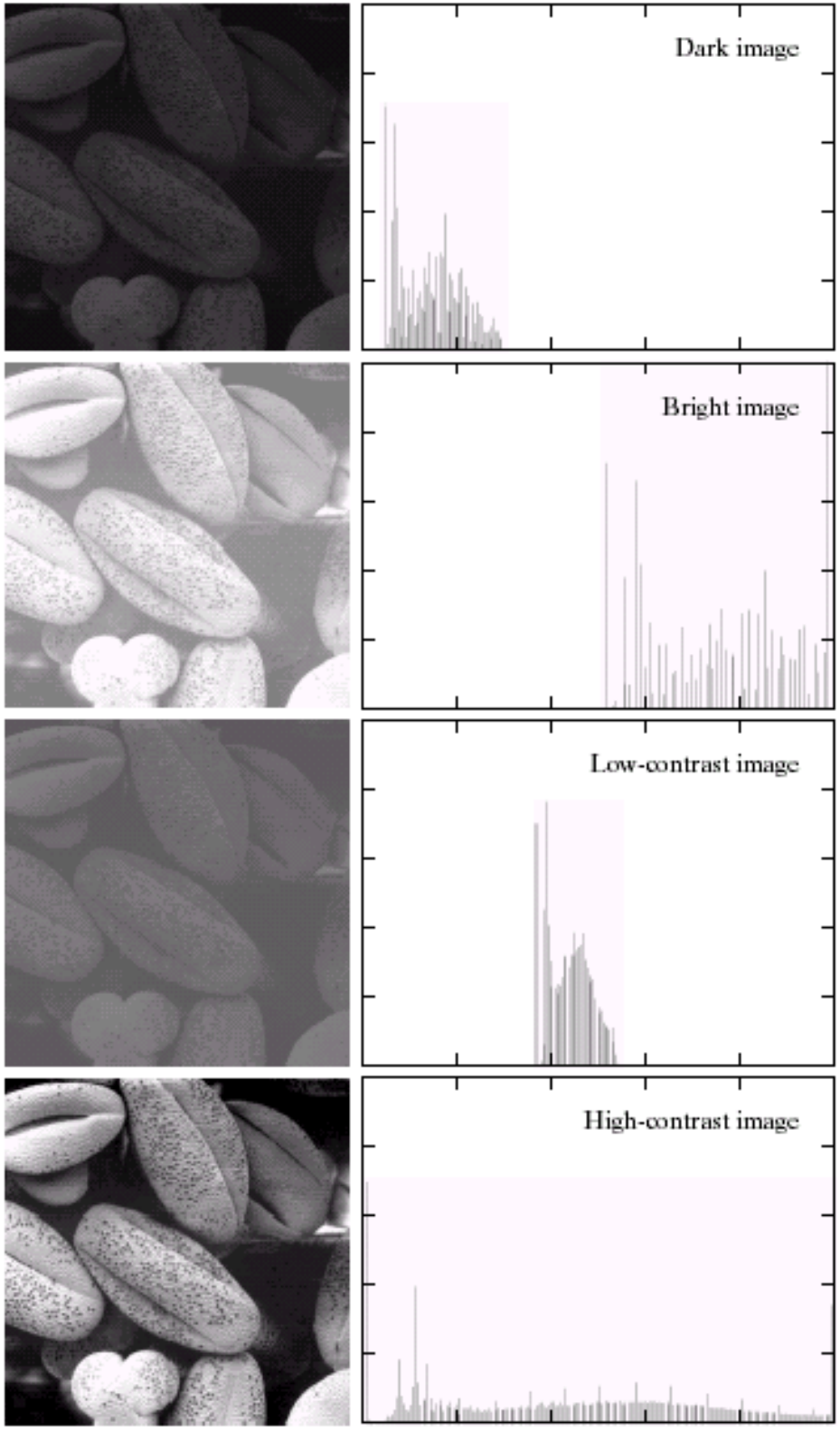


Image Histogram

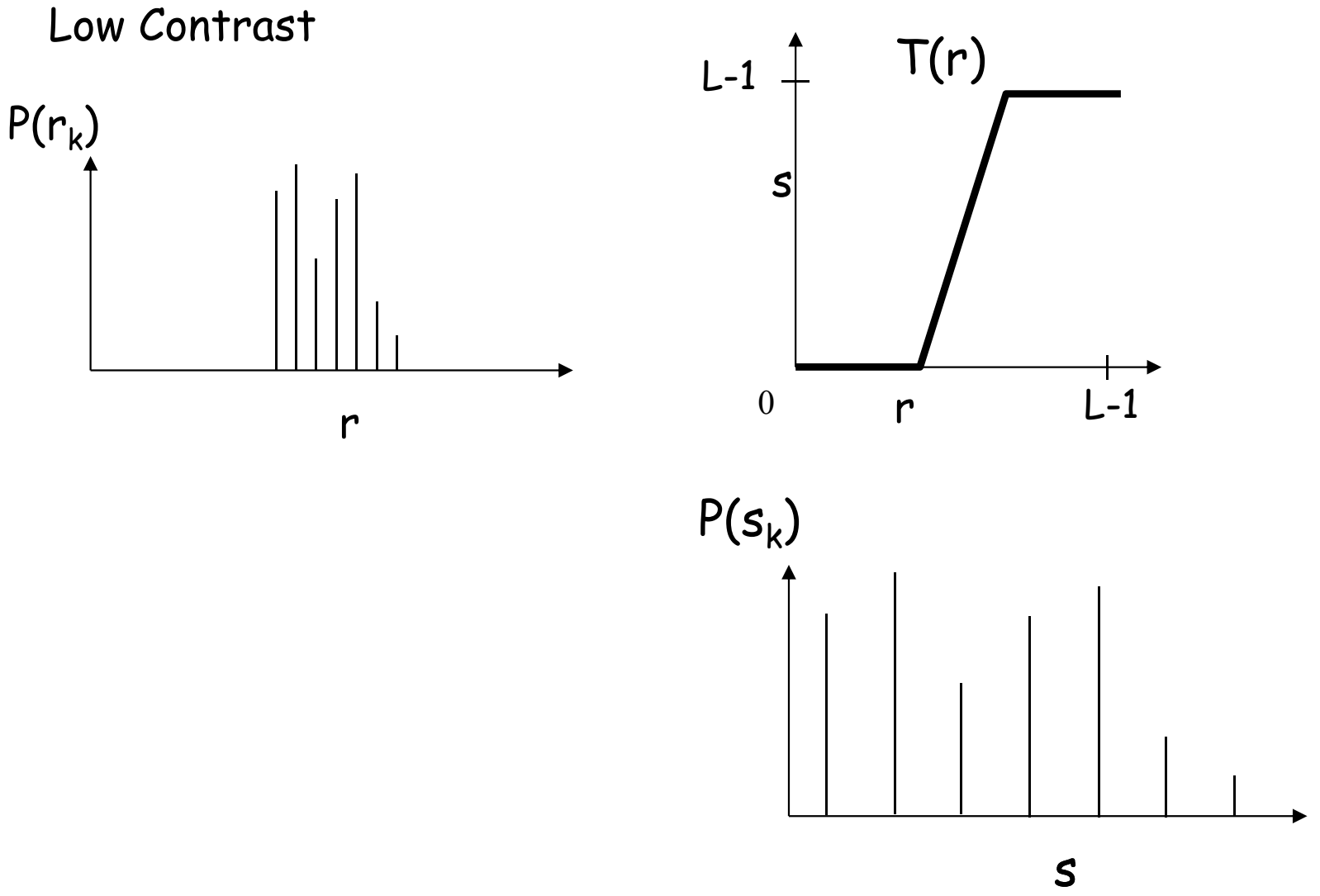
- Types of Image



Some Examples

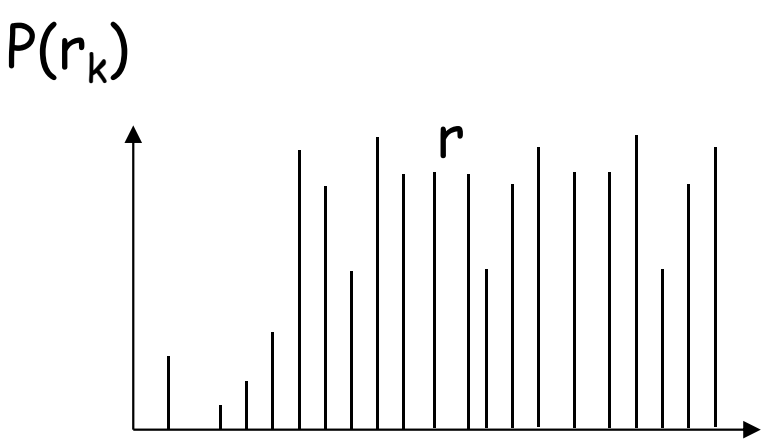


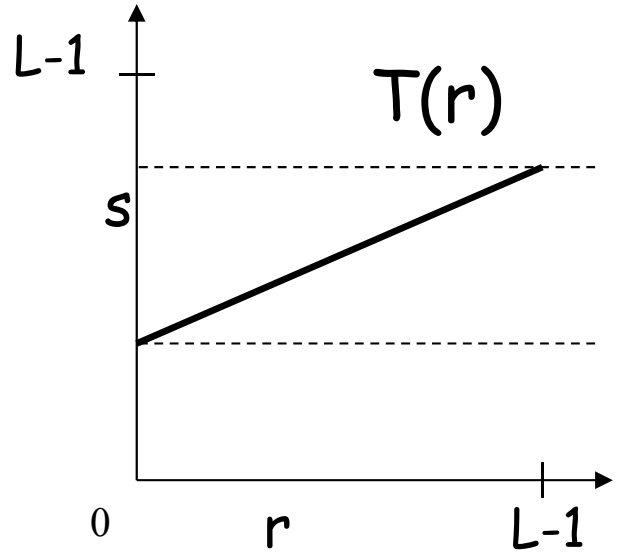
Contrast Stretching

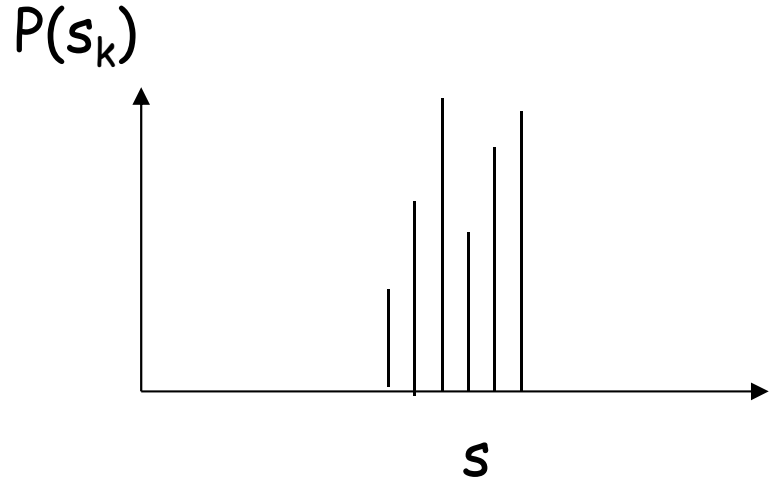


Contrast Compressing

High Contrast







Histogram Equalization

- We want an image with equally many pixels at every gray level, or the output intensity approx. follows *uniform distribution*.

- That is, a flat histogram, where each gray level, r_k, appears (N/r_m) times

  - where "r_m" is the maximum gray level
  - N is number of pixels in the image

- Example
  - http://www.mathworks.com/help/toolbox/images/ref/histeq.html

Histogram Equalization
Image Transformation

- Cumulative distribution function (CDF) of r is represented by (in continuous form)

$$s = T(r) = \int_0^r p_r(w)\,dw$$

T(r) to equalize the histogram

- Assume the input variable “r” has been normalized between [0,1].

- s = T(r), there are two properties
  - (a) T(r) is single-valued in the interval 0 <= r <= 1
  - (b) 0 <= T(r) <= 1 for 0 <= r <= 1

Conditions (a) and (b)

- (a) T(r) is single-valued and non-decreasing in the interval $0 <= r <= 1$
  - Can preserve the order from black to white in the gray scale.
  - Can preserve the basic appearance of an image.

- (b) for $0 <= r <= 1$, $0 <= T(r) <= 1$
  - Can guarantee a mapping that is consistent with the allowed range of pixel values. No intensity rescaling is needed.

CDF is for continuous functions

- Let's look at the discrete approximation

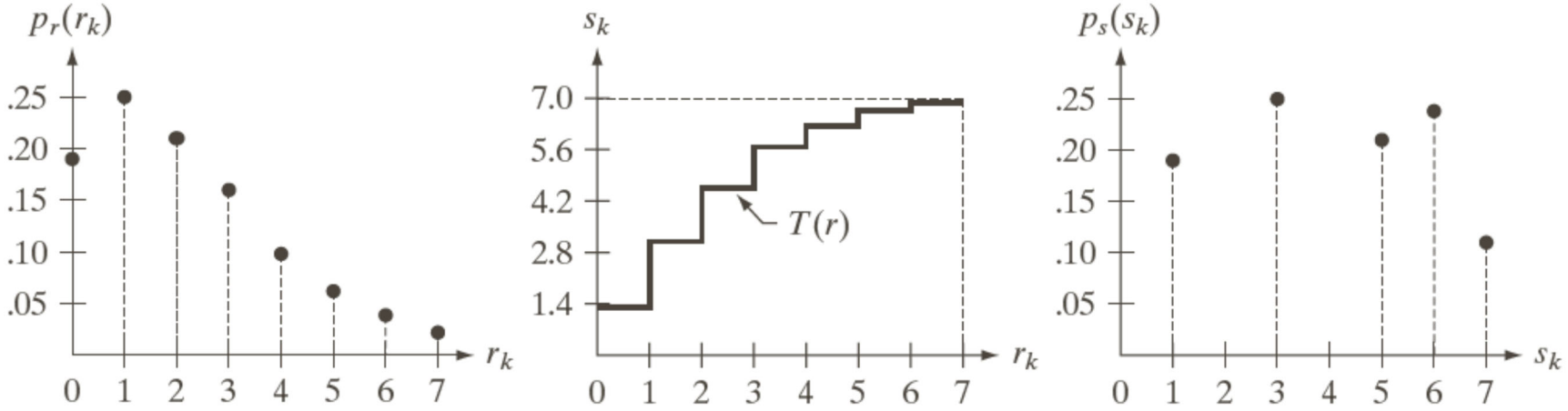
$$s_k = T(r_k) = \sum_{j=0}^{k} n_j / N = \sum_{j=0}^{k} p_r(r_j)$$

s_k is output intensity
r_k is input intensity
n_j is number of pixels with jth gray level
k = 0, 1, 2, 3, . . L-1 (gray levels)

Example

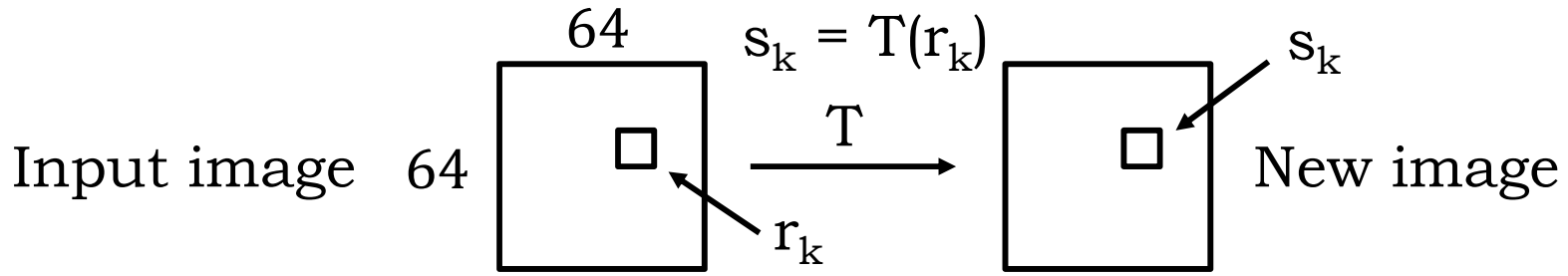
r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.



Total number of pixels = 64*64 = 4096 pixels

$\frac{2^3-1=7}{2^8-1=255}$

k	0	1	2	3	4	5	6	7
r_k	0/7	1/7	2/7	3/7	4/7	5/7	6/7	7/7
$Pr(r_k)$	$\frac{790}{4096}$	$\frac{1023}{4096}$	$\frac{850}{4096}$	$\frac{656}{4096}$	$\frac{329}{4096}$	$\frac{245}{4096}$	$\frac{122}{4096}$	$\frac{81}{4096}$
	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02
s_k	0.19	0.44	0.65	0.81	0.89	0.95	0.98	1
Gray levels	1	3	5	6	6	7	7	7

$\sum Pr(r_k) = 1$

Stretching | Compressing | Compressing

0 1 2 → 1 3 5

3 4 → 6

5 6 7 → 7

The discrete formulation of *histogram equalization* is given by

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j),\ k = 0,1,2,\ldots,L-1$$

where r_k and s_k represent the original and transformed grey levels respectively, L denotes the number of discrete grey levels, T is the transformation function, p_r is the probability density function $p_r(r_j) = n_j/n$, n is the total number of pixels and n_j is the number of pixels having intensity level r_j.

(a) Perform histogram equalization on a 3×3 input image, as shown below. Show all steps.

Input image

0	0.33	0.67
0.33	1	0.33
0.67	0.33	0.67

Answers:

Histogram equalized image

0.11	0.56	0.89
0.56	1	0.56
0.89	0.56	0.89

Round to 2 decimal places

r_k	0	0.33	0.67	1
$P_r(r_k)$	1/9	4/9	3/9	1/9
Transformation function S_k	1/9 = 0.11	5/9 = 0.56	8/9 = 0.89	9/9 = 1

(b) Explain why the transformation function $T(r_j)$ is increasing in the interval $0 \le r_j \le 1$, where $j = 0,1,2,\ldots,L-1$.

Answers:

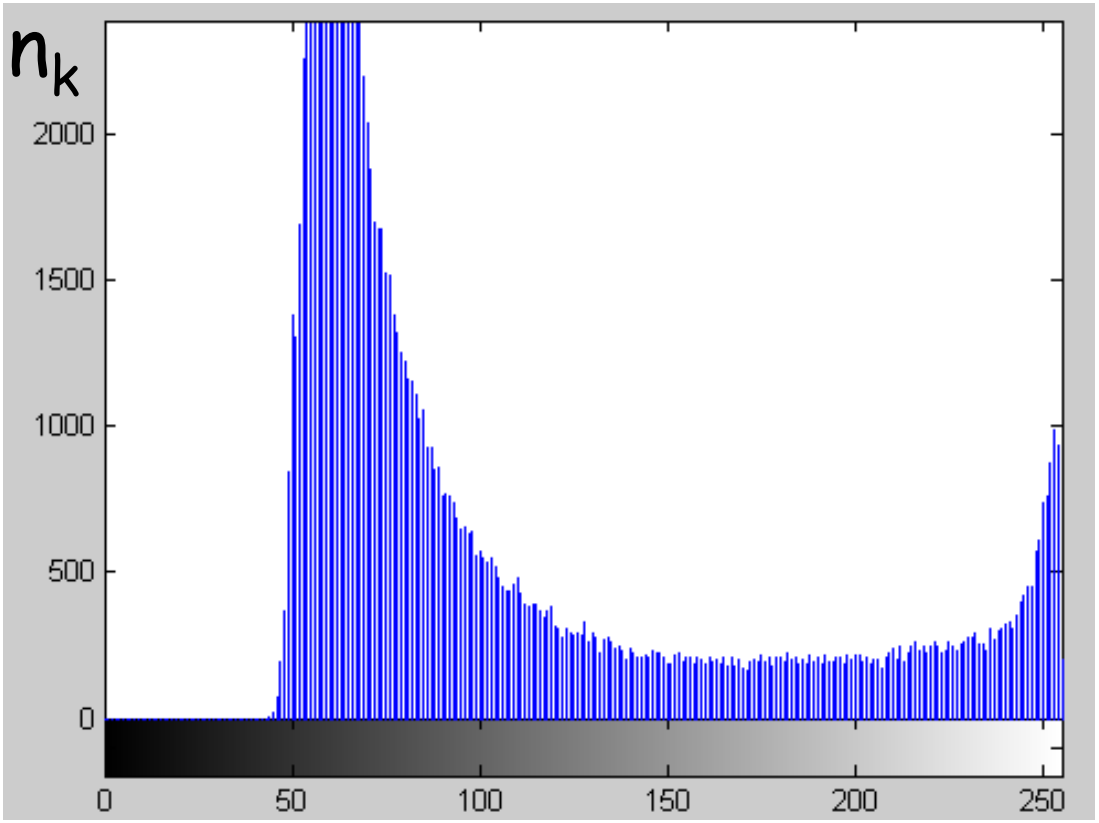
$$S_k = T(r_k) = \sum_{j=0}^{k} P_r(r_j)$$

$P_r(r_j)$ is non-negative

Example

http://www.mathworks.com/help/toolbox/images/ref/histeq.html

Image

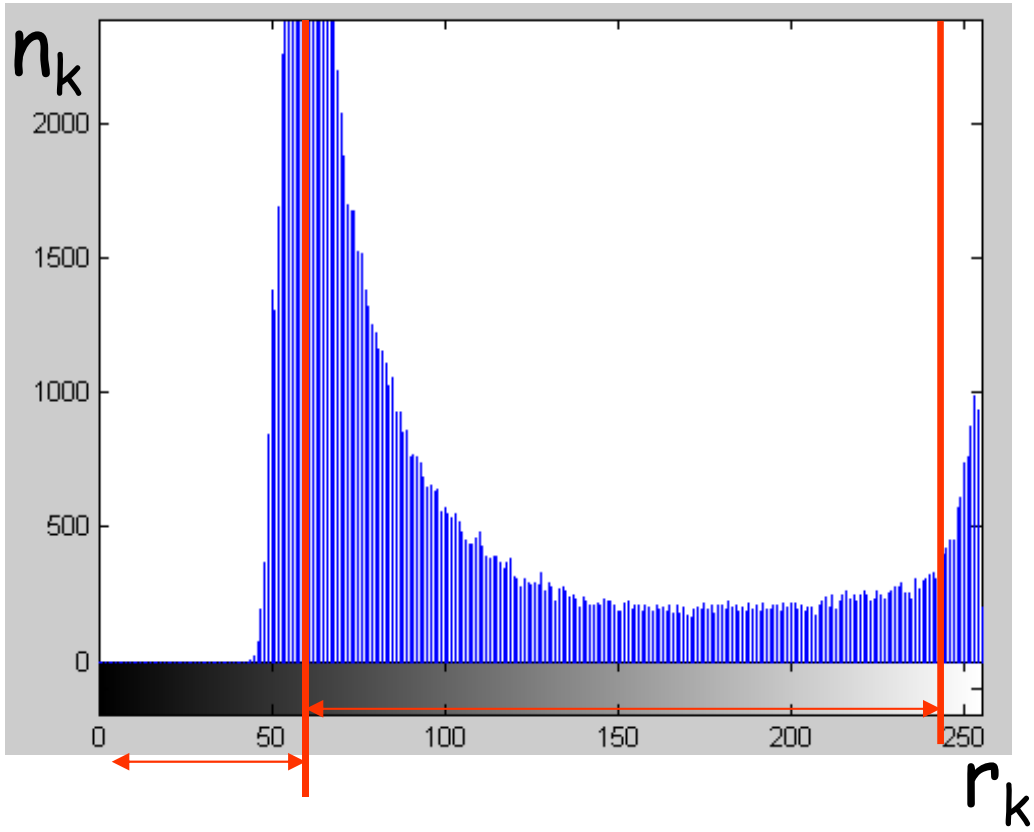


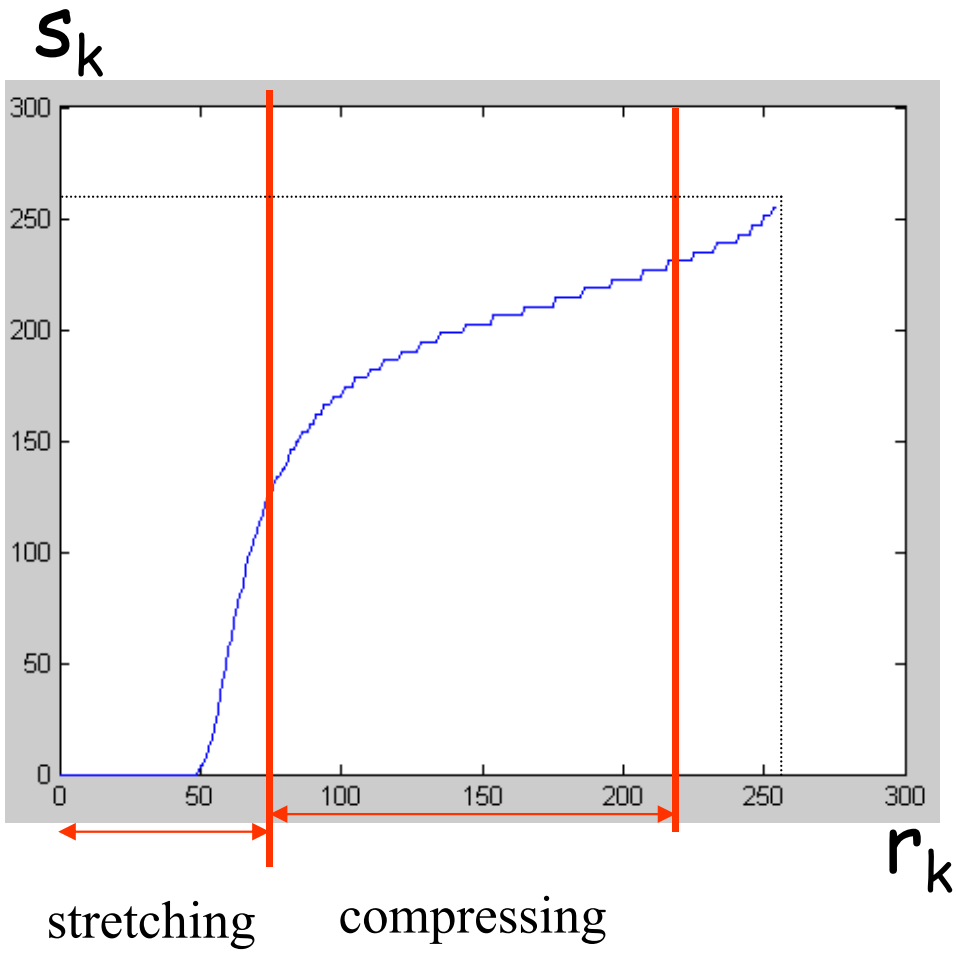
Histogram

Notice, this is not normalized, y axis is n_k.

To normalize, let y axis = $p(r_k) = n_k / N$

Example



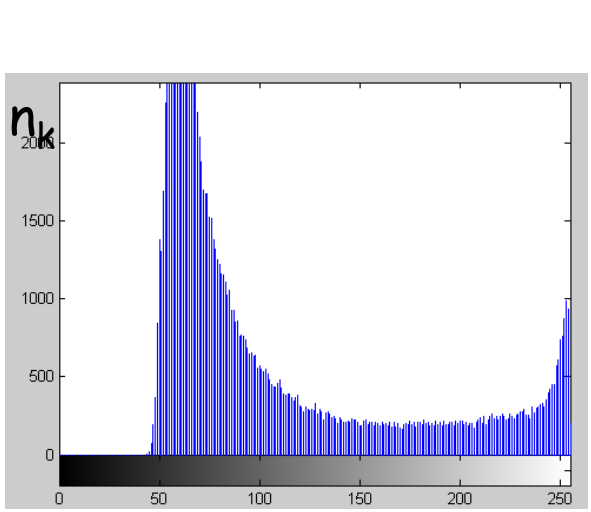


Histogram

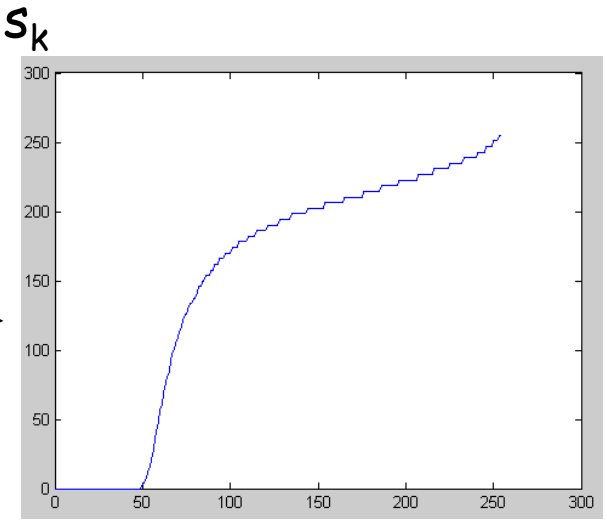
Notice, this is not normalized, y axis is n_k.

To normalize, let y axis = $p(r_k) = n_k / N$

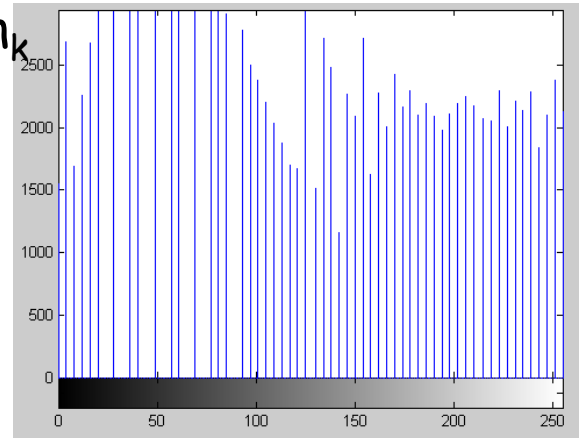
Example



r_k



$T(r)$



s_k

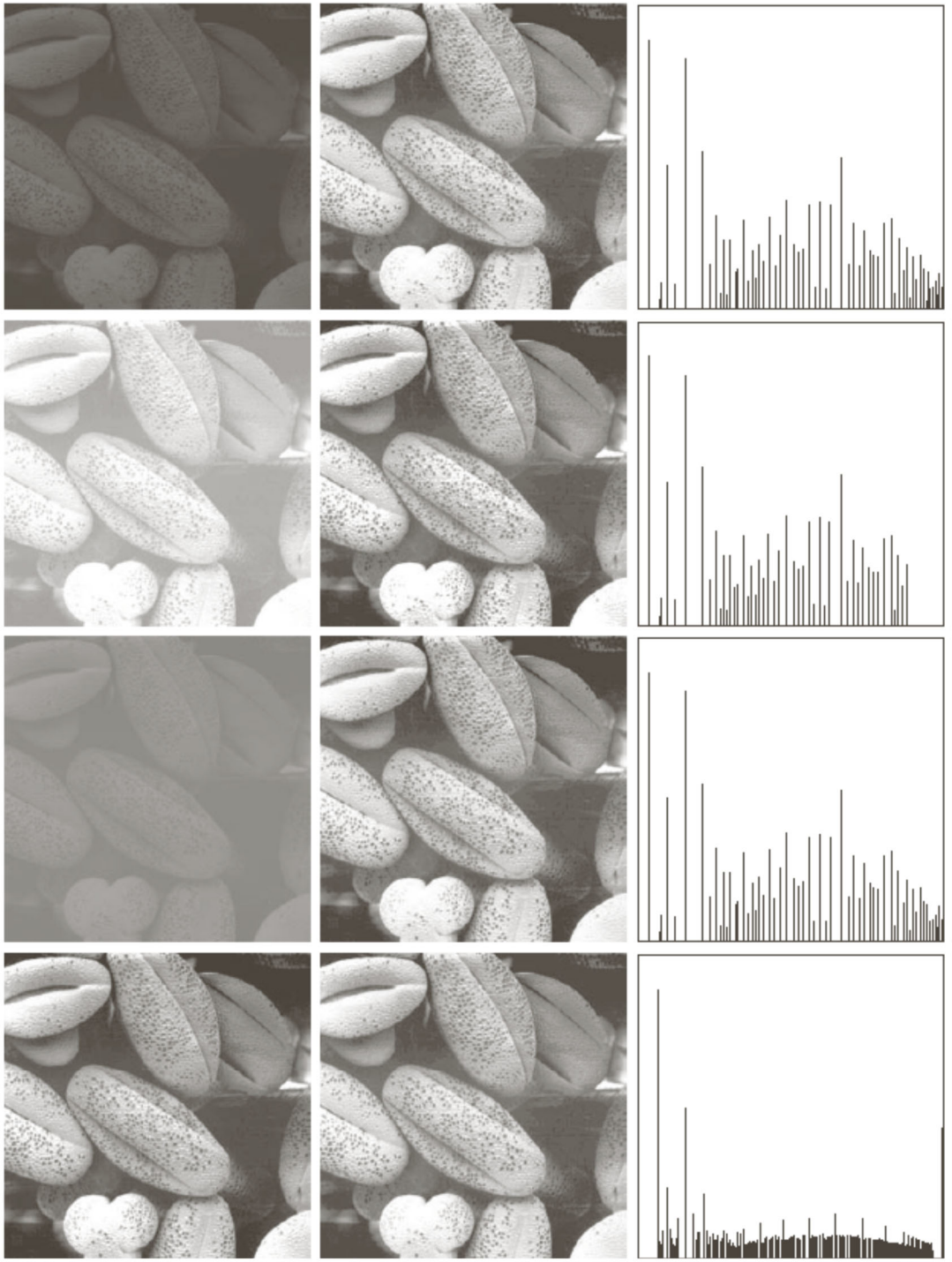


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

Histogram Equalization

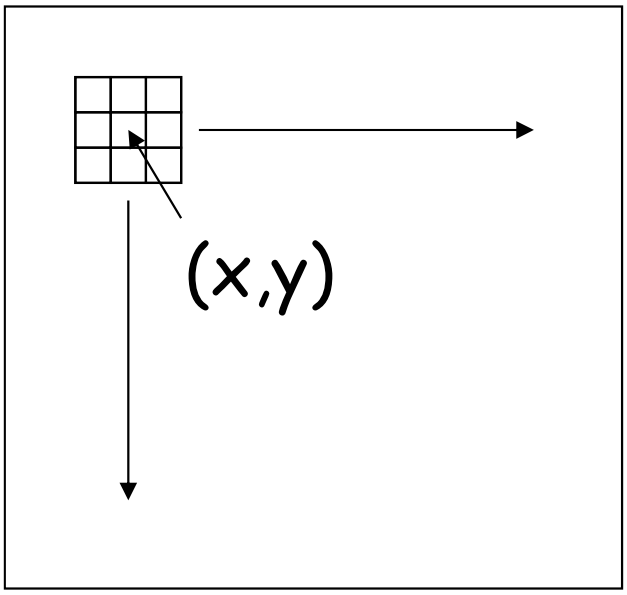
- Can significantly improve image appearance
  - Automatic
  - users doesn’t have to perform manual windowing or manual gray level slicing
- Nice pre-processing step before image comparison
  - Account for different lighting conditions
  - Account for different camera/device properties

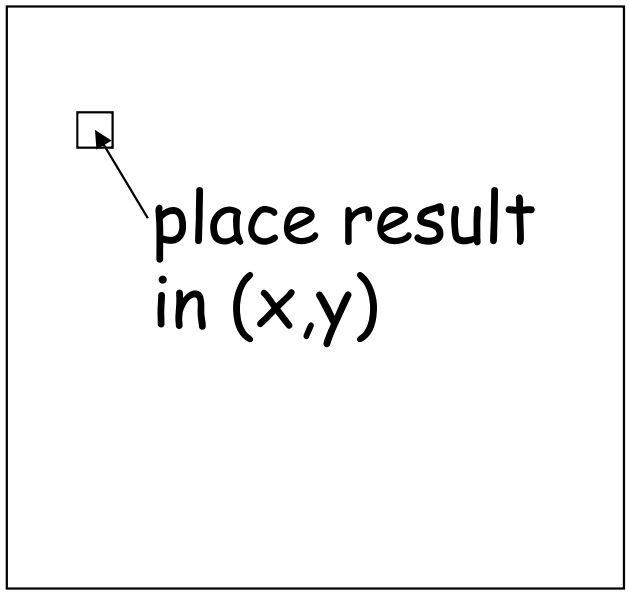
Local Enhancement

- Histogram equalization is a global operation
  - Each pixel is processed based on information of the entire image
  - Often enhances global details

- We would like to enhance details over small areas
  - Each pixel is processed based on information of a small area/sub-image

Local Histogram Equalization





Result

calculate histogram
using neighborhood of mxm
about (x,y)

Local Histogram

- Apply histogram equalization about a neighborhood around (x,y)
- Transform the gray level for pixel (x,y)
- Move the neighborhood over the rest of the image

Local Histogram

Reveals detail in local areas

Original | Global Histogram | Local Histogram

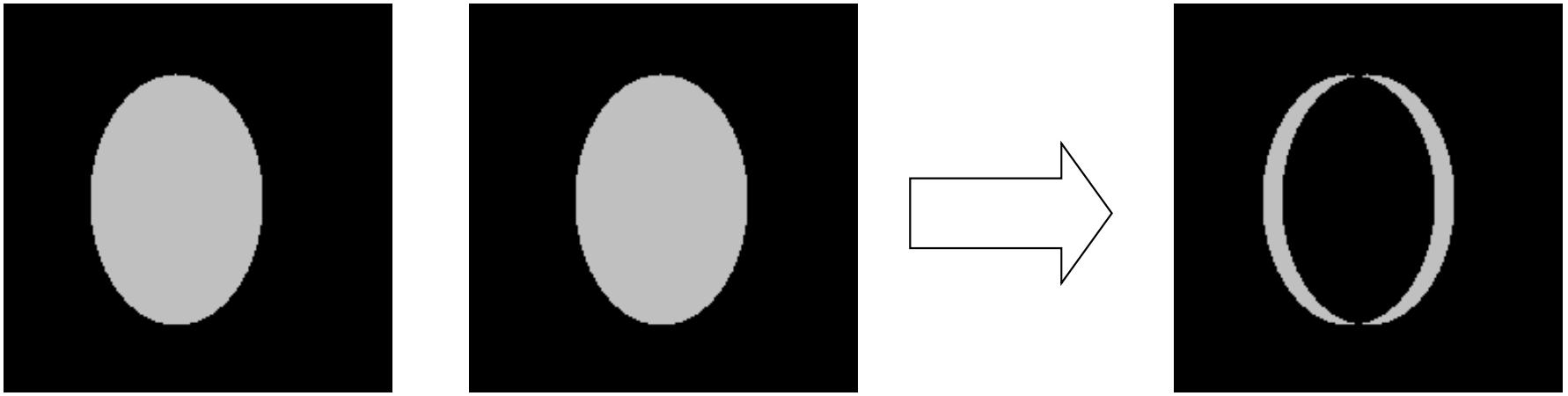
Image Operations

- Arithmetic Operations (p and q are images)
  - p+q
  - p-q
  - p*q (also stated as pq, or p x q)
  - p%q

- Logic Operations (p and q are binary images)
  - p AND q
  - p OR q
  - Not q
  - p XOR q

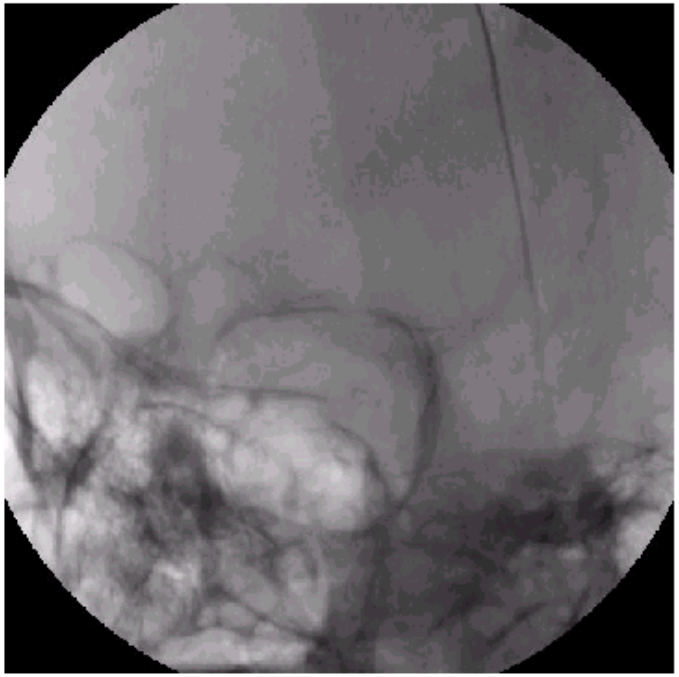
Image Operations

- Subtraction or Absolute Difference
  - Very useful for determining the difference between two images
  - Used for “detection”

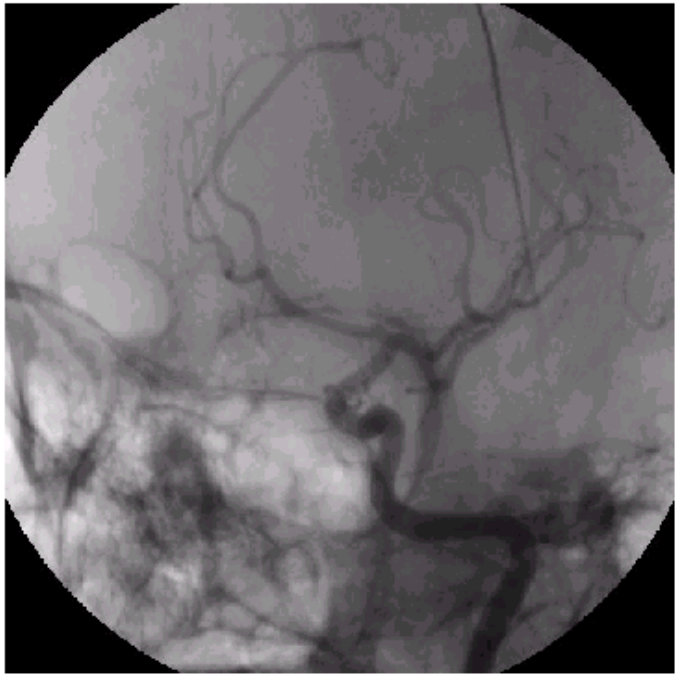


Grey = non-zero, Background = 0.

Example



X-ray brain image without contrast agent injected



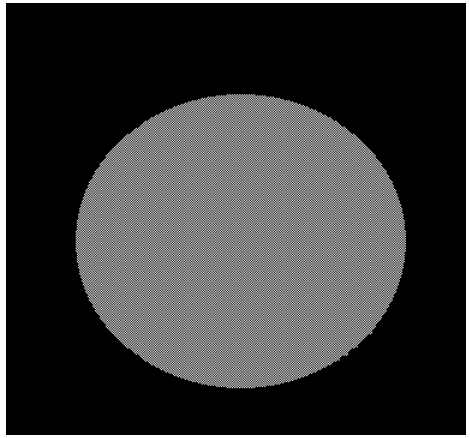
X-ray brain image with contrast agent injected

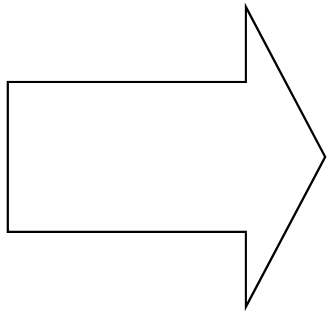
Digital subtraction angiogram (DSA)

http://en.wikipedia.org/wiki/Digital_subtraction_angiography

Image Operation

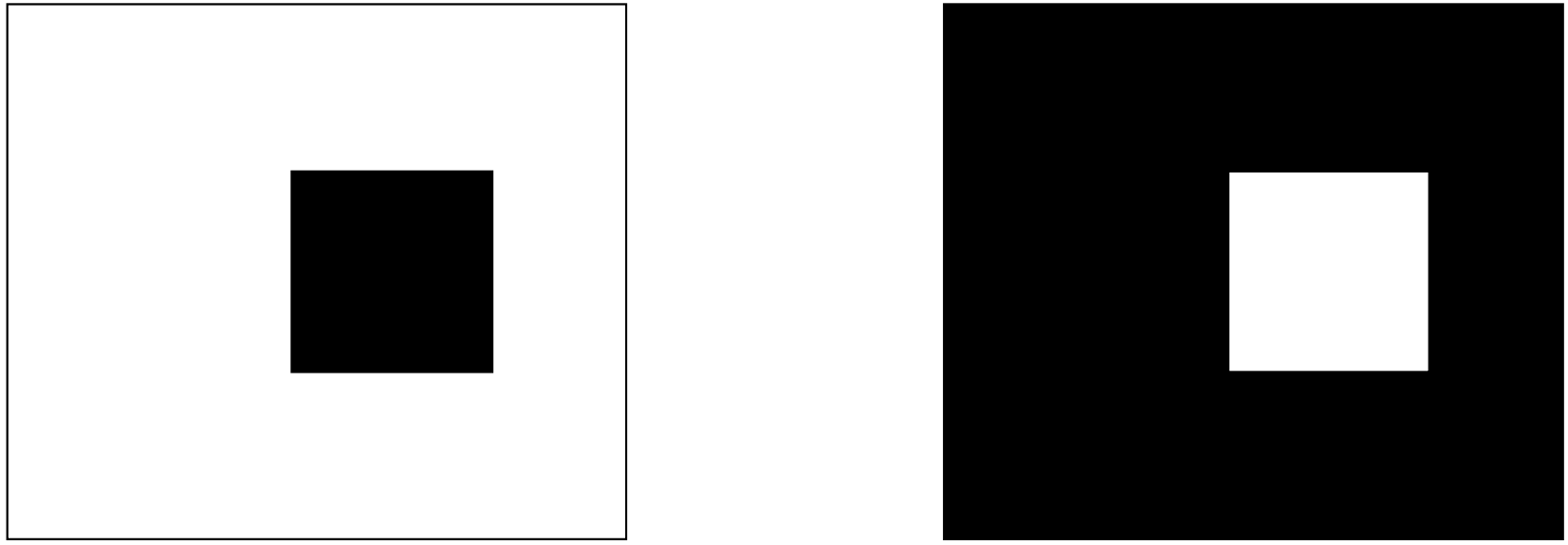
- Addition can blend two images





Logic Operators

- NOT

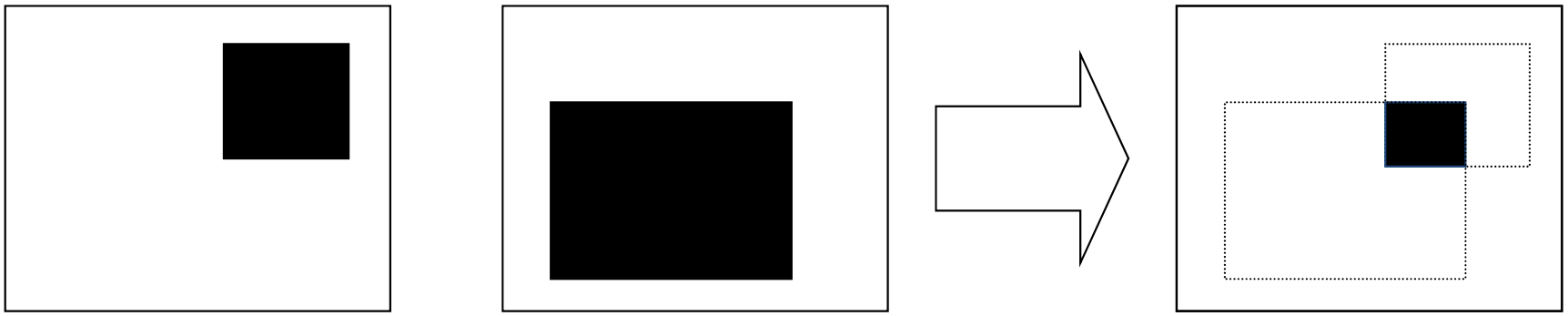


black = 1, white = 0.

Logic Operators

- AND

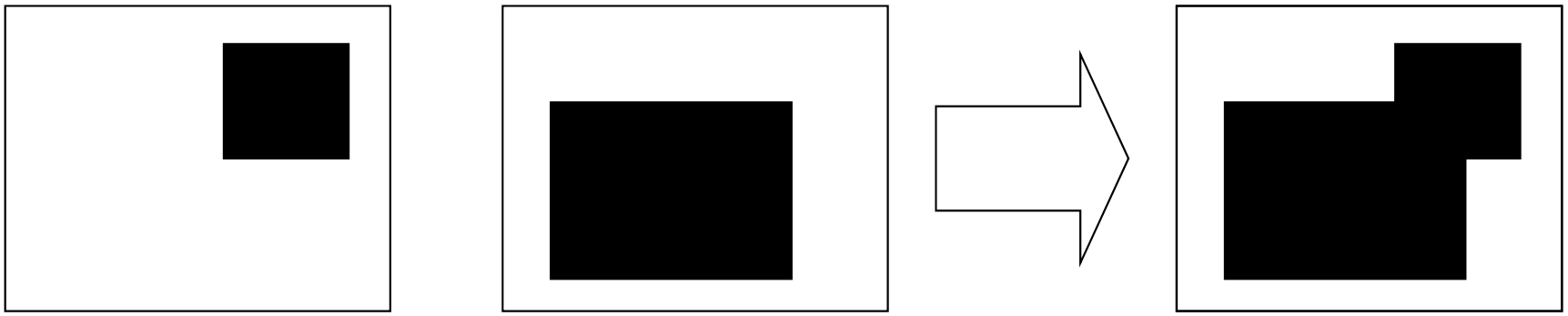
black = 1, white = 0.



Logic Operators

- OR

black = 1, white = 0.



- XOR?

Image Averaging

- If you can capture a scene or acquire an image repeatedly, then averaging all the observed images will give you a clear image with less noise.

- $g_i(x,y)$ = i^{th} observed noisy image, i = 1, ..., K, and K = total number of images.
- f(x,y) = 'true', original image
- $\eta(x,y)$ = (*eta*) is zero mean, random Gaussian noise
- the final average image is given by

$$\overline{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

Image Averaging

- It follows that

Expected value of the average image

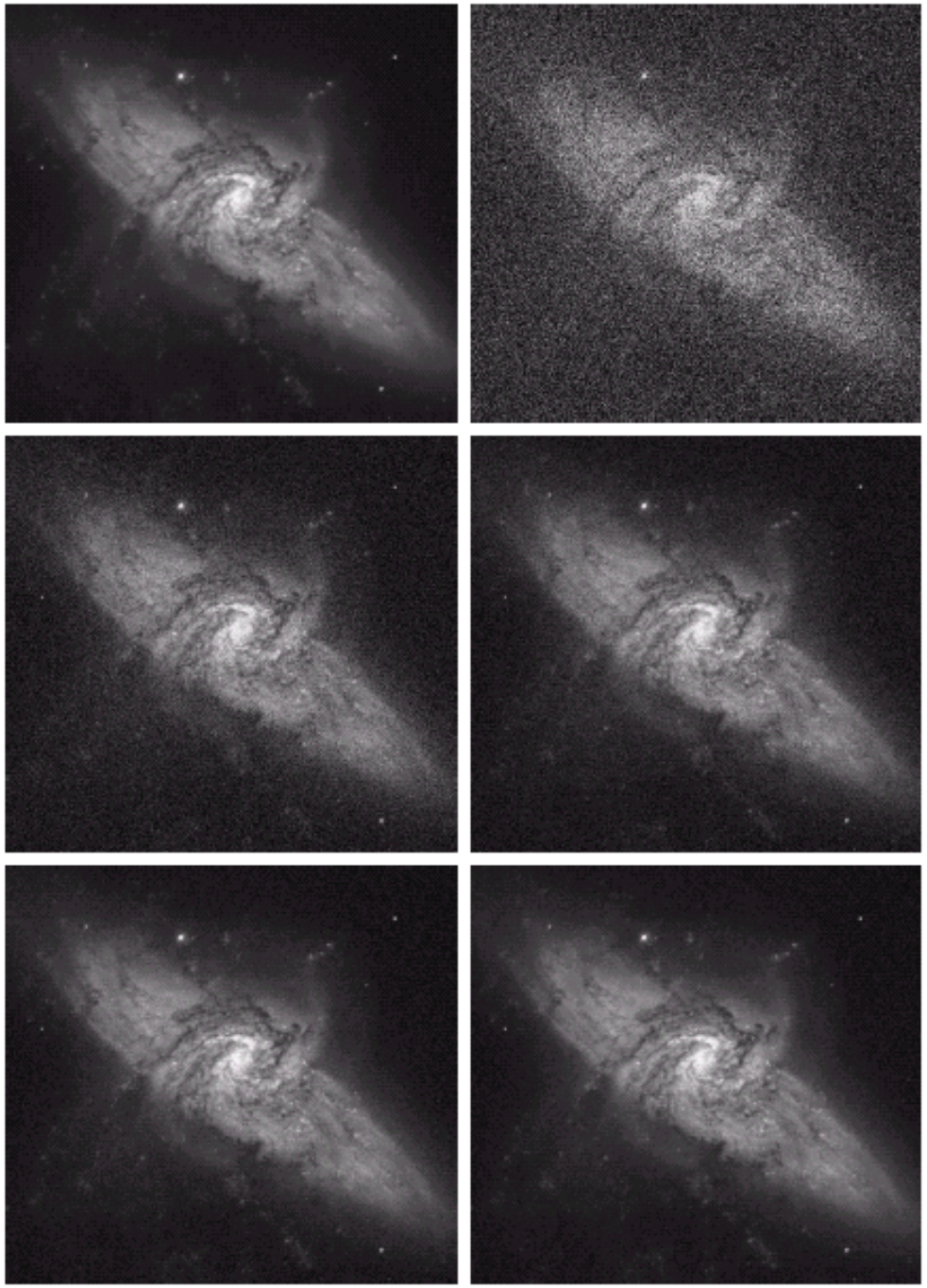
$$E\{\bar{g}(x,y)\} = f(x,y)$$

Variance of the average image

$$\sigma^2_{\bar{g}(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)}$$

- If K is sufficiently large, then the average image will be very close to the true, original image, and the variance (noise) will be small.

Example



a b
c d
e f

FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64,$ and 128 noisy images. (Original image courtesy of NASA.)

More examples

1. Reducing noise by image averaging
http://www.school-of-digital-photography.com/2014/03/reducing-noise-by-image-averaging.html

2. The best noise reduction technique
http://ezbackgrounds.com/blog/noise-reduction-averaging-photoshop.php

3. Noise reduction by image averaging
http://www.cambridgeincolour.com/tutorials/image-averaging-noise.htm

Spatial Filtering using a Mask

- Neighborhood operators

z1	z2	z3
z4	z5	z6
z7	z8	z9

mask/filter

w1	w2	w3
w4	w5	w6
w7	w8	w9

Response of a linear mask/filter, R,

$$R = (w1z1 + w2z2 + w3z3 \ldots + w9z9) = \sum_{i=1}^{9} wizi$$

f(x,y) is centered around "z5"

so g(x,y) = R =filter*f(x,y), *=convolution symbol

Response of a *linear mask*

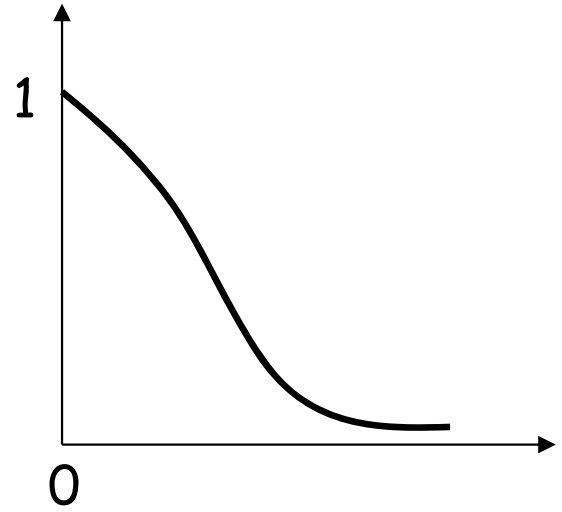
$$R = w1z1 + w2z2 + w3z3 \ldots + w9z9$$

where w_i are mask coefficients (weights) and z_i are pixel intensities.

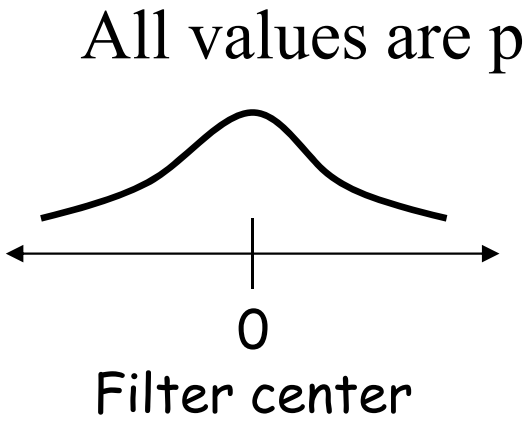
Smoothing Filters

- Smoothing Filters
  - blurring
    - “pre-processing”
      - removal of small details before object extraction
  - noise reduction
    - removal of noise in an image
- Often called “Low-Pass” Filters
  - Filter lets low-frequencies pass
  - Stops high-frequencies

Low-pass spatial filtering



Frequency Domain



Spatial Domain

Low-pass spatial filtering

w1	w2	w3
w4	w5	w6
w7	w8	w9

- Only requirement for a low-pass filter is that w_i be positive
- Note that the result can be larger than the valid output range(L-1)
- Can pre-scale the filter

$$\text{scale\_factor} = \left(\sum w_i\right)^{-1}$$

Low-pass spatial filter

$$\frac{1}{9} * \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Average Filter

Extends to larger filters

1/9 *

1	1	1
1	1	1
1	1	1

1/81 *

1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1

Examples

original

n=5 (nxn mask)

n=15 (nxn mask)

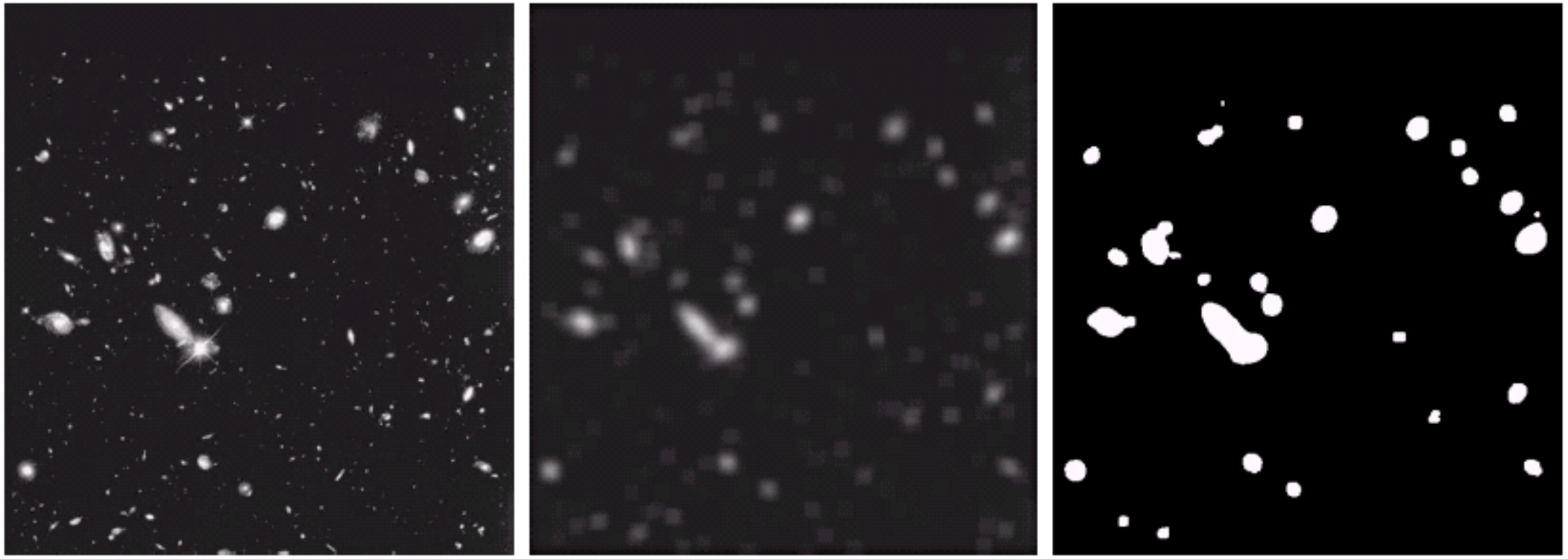
n=25 (nxn mask)

Example

Original image

Smooth image
15x15 averaging filter

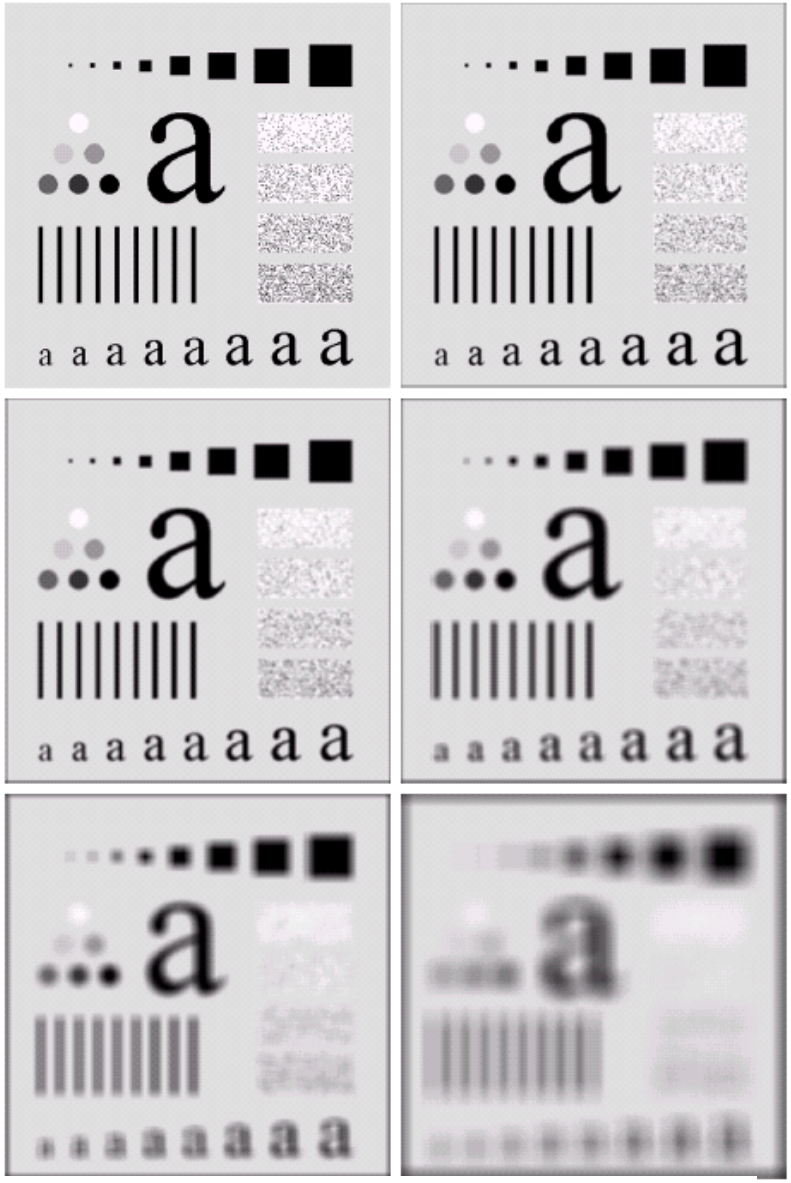
thresholded image



a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Examples



a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Other arrangements

1	1	1
1	2	1
1	1	1

1	1	1	1	1
1	2	3	2	1
1	3	4	3	1
1	2	3	2	1
1	1	1	1	1

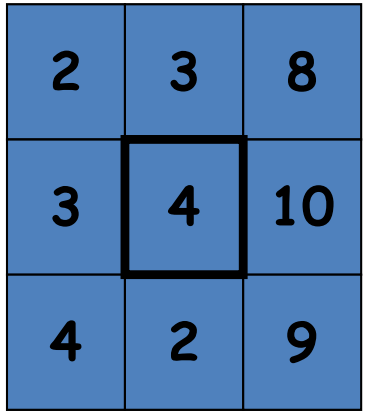
Remember that the resulting gray-levels may be out of the range of the original image.

Linear vs. Non-linear

- Linear Filters
  - Linear operation (output can be expressed as the convolution of the input image with filter)
  - Have corresponding frequency domain filter
- Non-linear Filters
  - Examine neighbors using various orderings
    - Often use Rank or Order Statistics

Median Filter

- Very popular non-linear filter
- Find the median of the window
- Preserves edges
- Removes impulse noise, avoids excessive smoothing



pixel values about (x,y)
window 3x3

neighbor sort = {2,2,3,3,4,4,8,9,10}

f(x,y) = median

Examples

Original

(a)

(b)

Noisy Image

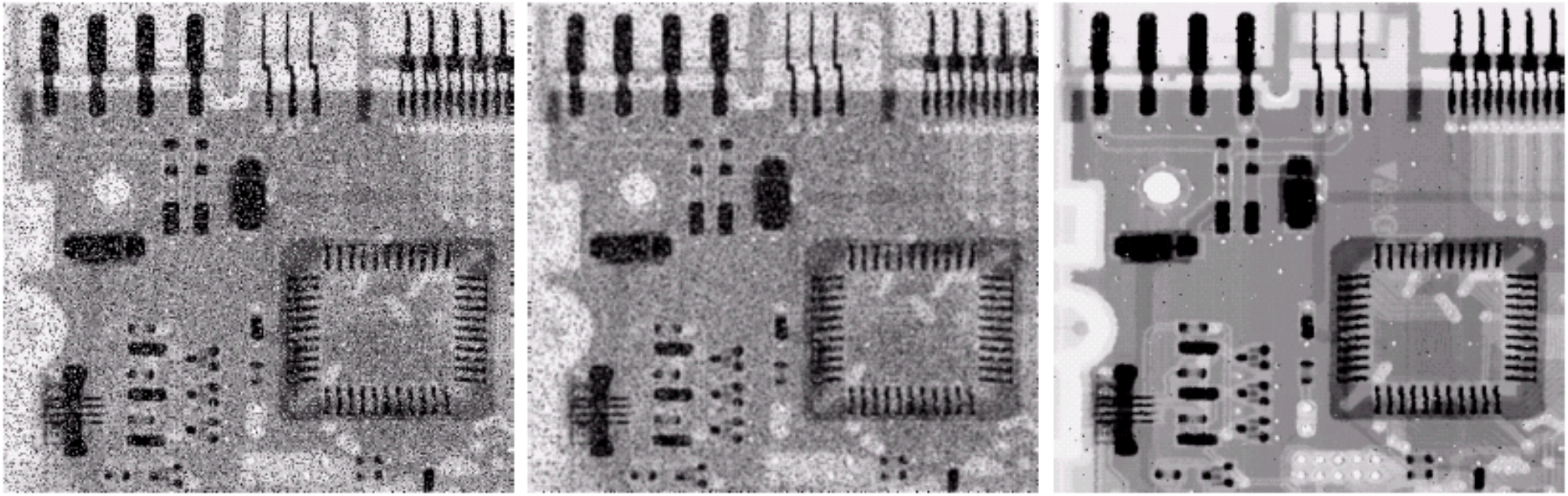
Low-Pass Filter

(c)

(d)

Median Filter

Examples



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Median Filter

- Often referred to as “de-speckle operation”

- It converges
  - That is, if you perform it over and over for many times, eventually the image will not change.

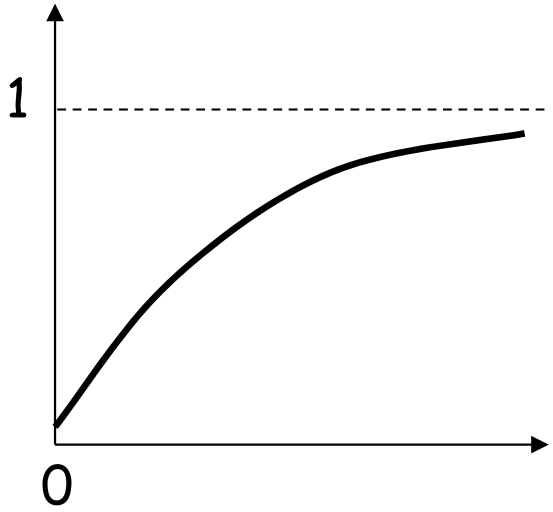
Iterative Smoothing

- Local Averaging
  - Converges to an image with constant intensity
- Median Filter
  - Converges to an image invariant to the filter

Sharpening

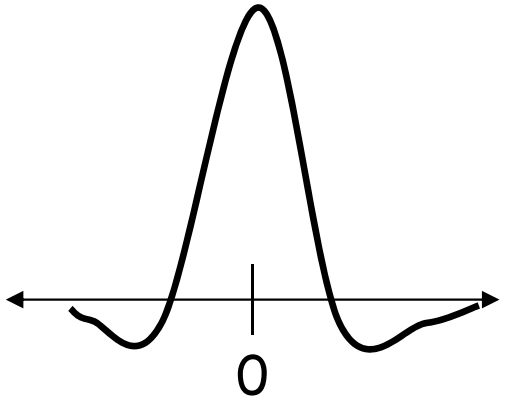
- Objectives of sharpening
  - Highlight fine details in an image
  - Enhance details that have been blurred
  - We can think of this as high-pass filtering
    - Letting high frequencies pass
    - Removing low frequencies

Sharpening



Frequency Domain

Values can be either positive or negative



Filter center

Spatial Domain

Sharpening spatial filter

1/9 *

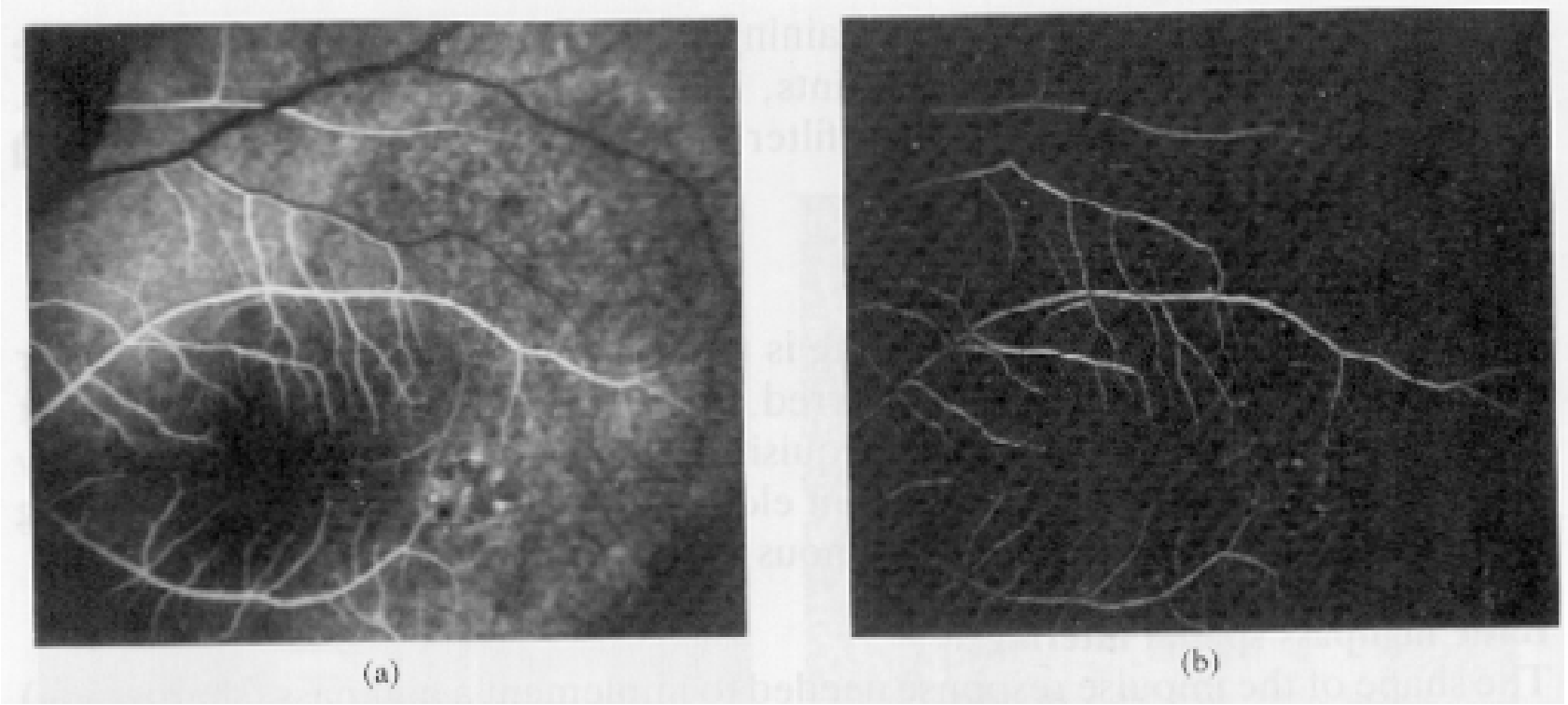
-1	-1	-1
-1	8	-1
-1	-1	-1

- Positive coefficients near its center
- Negative coefficients near the outer periphery
- Note, the sum of the coefficients is zero
- Thus, when the mask is applied over an area of constant intensity, the result is zero
- Output = 0 if the intensity values are constant.

Sharpening spatial filter

- The results may be negative.
- You'll need to scale and/or clip so that the gray levels of the result span [0, L-1].

Example



Original

High-Pass

First Order Derivatives

- Gradient
- Function of 2 variables x, y

$$\nabla f = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

First Order Derivatives

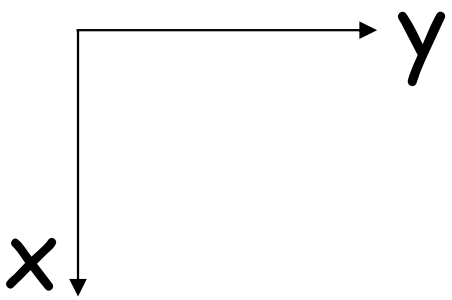
- For each (x,y) you are storing two values:

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

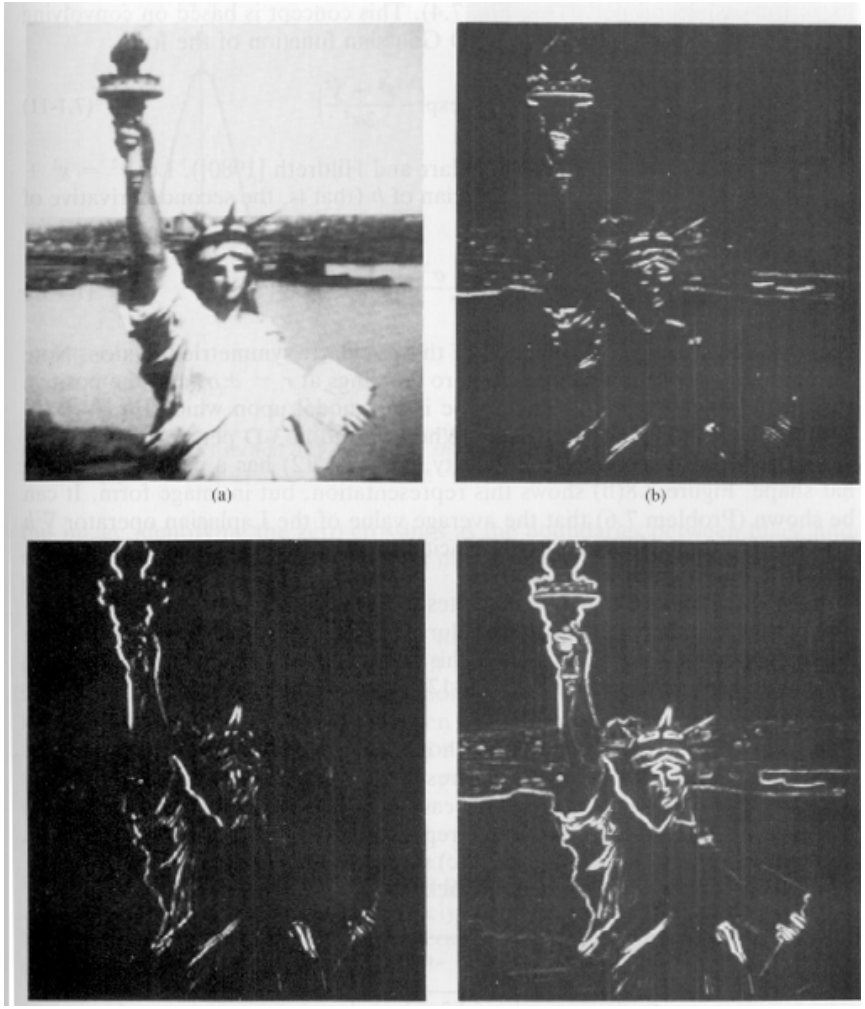
- Often have two images to represent this
- X-Gradient and Y-Gradient
  - Can be computed independently

First Order Derivatives

- X-Gradient and Y-Gradient



Original



$|\partial f/\partial x|$

$|\partial f/\partial y|$

$\text{mag}(\nabla f)$

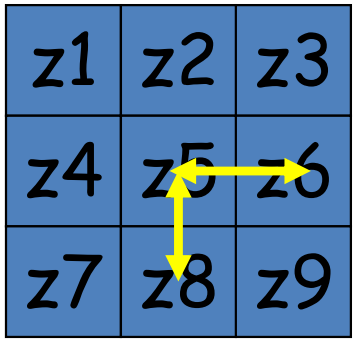
Gradient

- Gradient Magnitude

$$\operatorname{mag}(\nabla \mathrm{f}) = \left[\left(\frac{\partial \mathrm{f}}{\partial \mathrm{x}} \right)^2 + \left(\frac{\partial \mathrm{f}}{\partial \mathrm{y}} \right)^2 \right]^{1/2}$$

Basic Derivative

- Consider the pixels

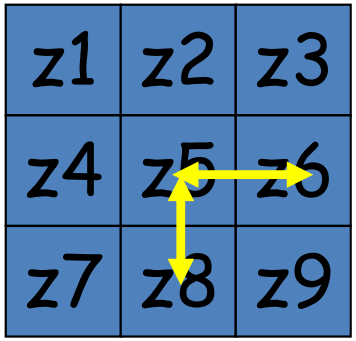


- $\mathrm{mag}(\nabla \mathrm{f})$ at z_5 can be computed

$$\mathrm{mag}(\nabla f) \approx [(z_5 - z_6)^2 + (z_5 - z_8)^2]^{1/2}$$

Basic Derivative

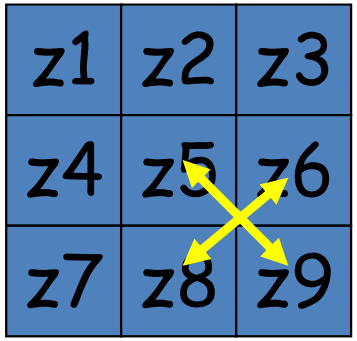
- Consider the pixels



- $\mathrm{mag}(\nabla \mathrm{f})$ at z_5 can be computed quicker

$$\mathrm{mag}(\nabla f) \approx | z_5 - z_6 | + | z_5 - z_8 |$$

Basic Derivative



- $\mathrm{mag}(\nabla \mathrm{f})$ sometimes is computed using the "cross" difference

$$\mathrm{mag}(\nabla f) \approx | z_6 - z_8 | \; + | z_5 - z_9 |$$

Even Sized Masks

$$\text{mag}(\nabla f) \approx |(z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)| +$$
$$|(z_3 + z_6 + z_9) - (z_1 + z_4 + z_7)|$$

- Difference between first and third rows (∂f/∂x)
- Difference between first and third columns (∂f/∂y)

Even sized mask

- Prewitt operator

z1	z2	z3
z4	z5	z6
z7	z8	z9

-1	0	1
-1	0	1
-1	0	1

-1	-1	-1
0	0	0
1	1	1

$$\text{mag}(\nabla f) \approx |(z_7 + z_8 + z_9) - (z_1 + z_2 + z_3)| +$$

$$|(z_3 + z_6 + z_9) - (z_1 + z_4 + z_7)|$$

Even sized mask

- Sobel operator

-1	0	1
-2	0	2
-1	0	1

-1	-2	-1
0	0	0
1	2	1

- Weights closer neighbor a little more