Snakes: Active Contour Models

Figures are provided by the Pearson Education

Snakes: Active Contour Models

1. The position of a 2D contour can be parameterized as

$$\vec{V}(s) = \left(x(s), y(s)\right)^T$$

   where s is a parameter that increases as the contour is traversed, $0 \leq s \leq 1$, and x and y are position variables as well as functions of s. It can be either an open or close contour. Link: http://www.cs.ucla.edu/~dt/papers/ijcv88/ijcv88.pdf

2. Snakes are an attractive approach because they are capable of finding salient image contours – edges, lines and subjective contours – as well as tracking those contours during motion.

   Demo: http://www.markschulze.net/snakes/

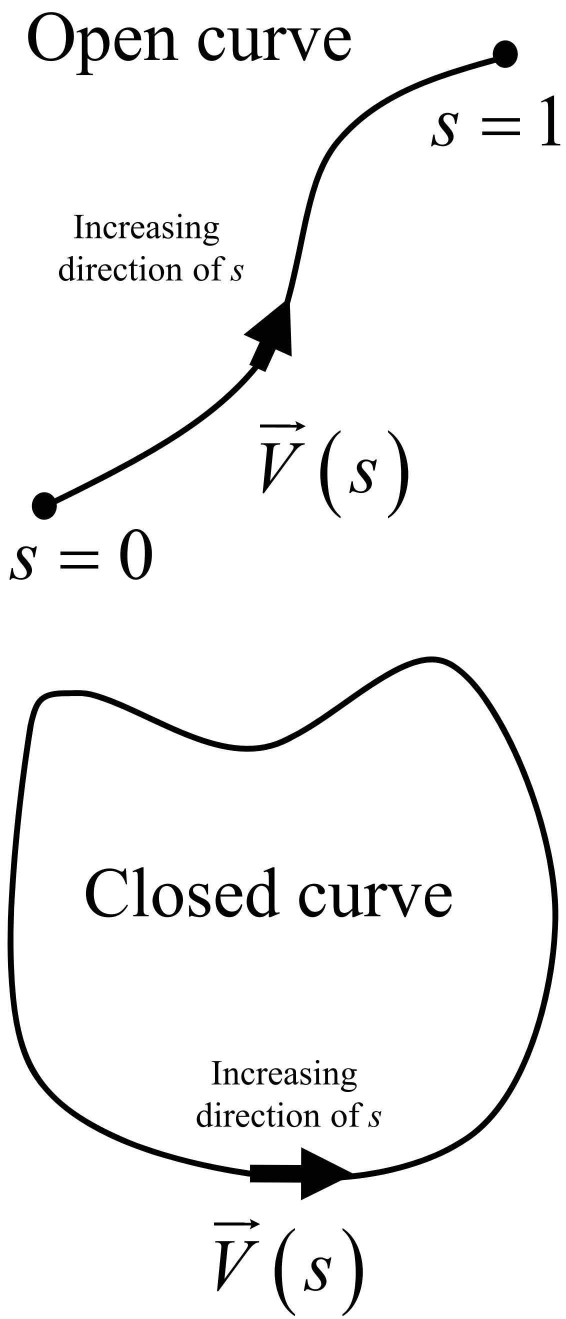
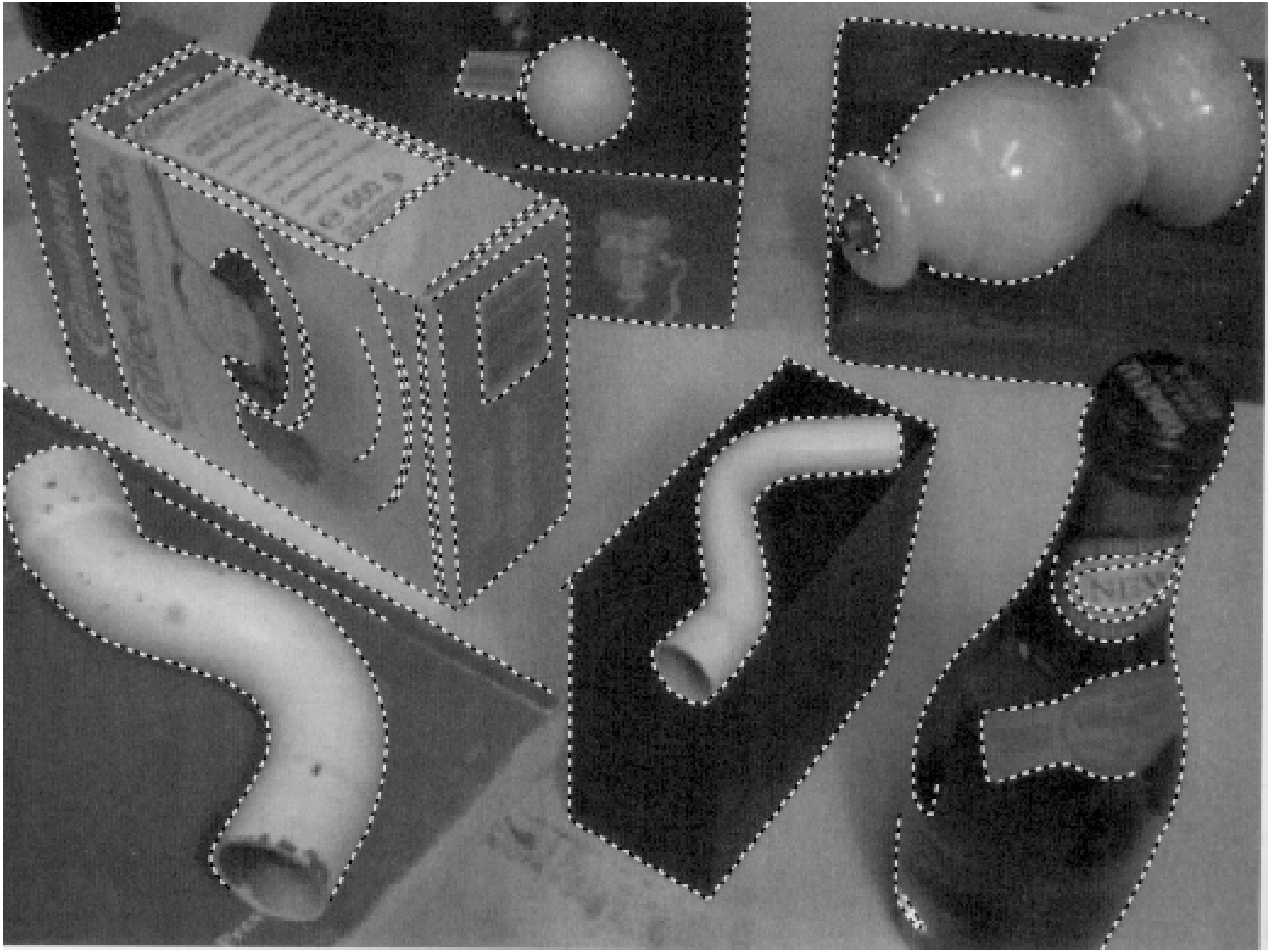


FIGURE 1 Snake (white) attracted to cell membrane in an EM photomicrograph [18].

Handbook of Medical Imaging, p. 129



Examples

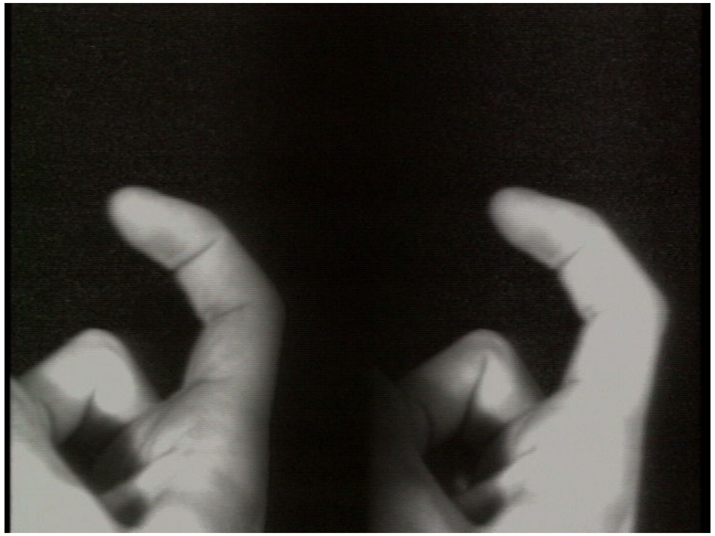
Snakes: Active Contour Model

Tracking

http://www.cs.ucla.edu/~dt/vision.html

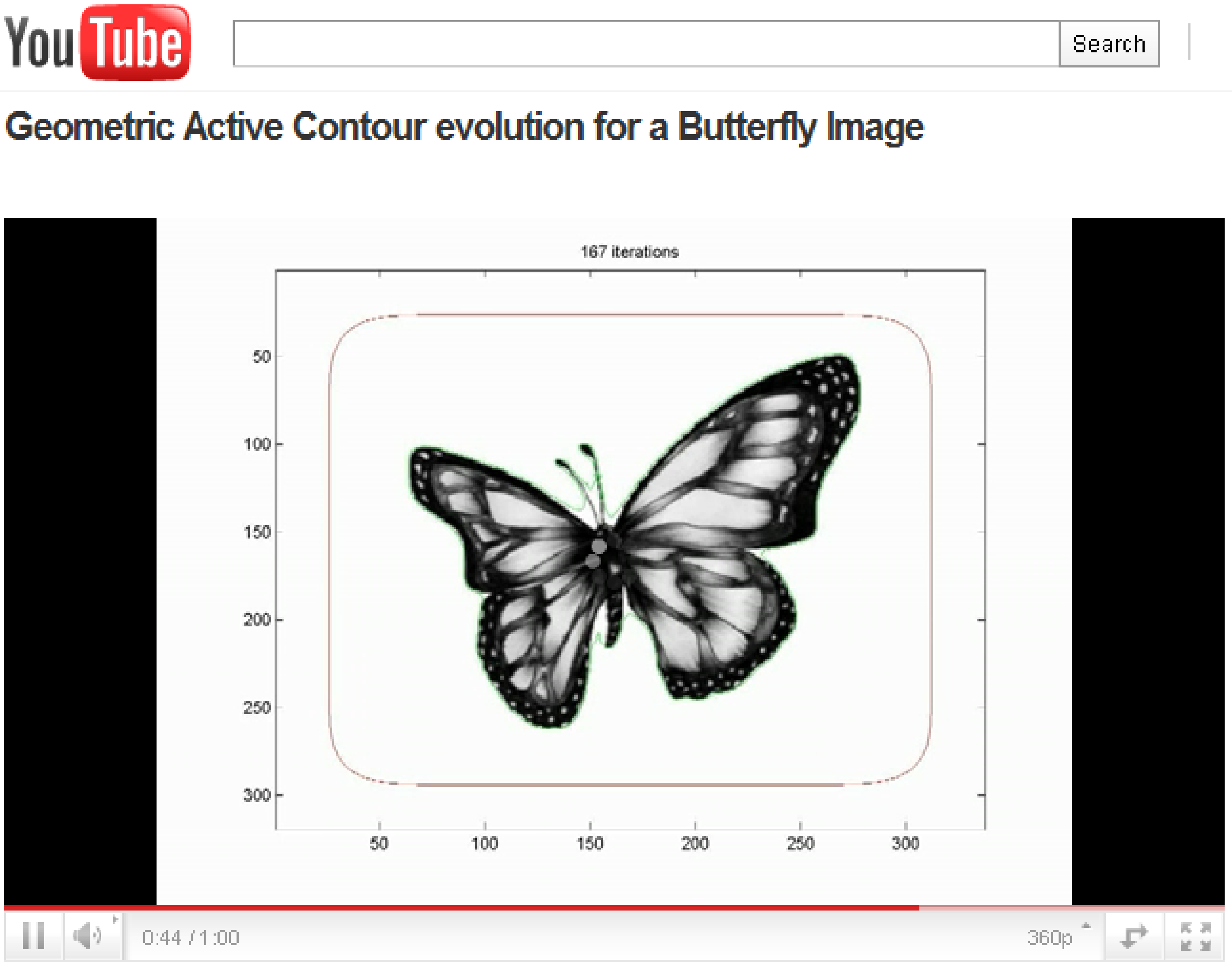
Example

Finger tracking

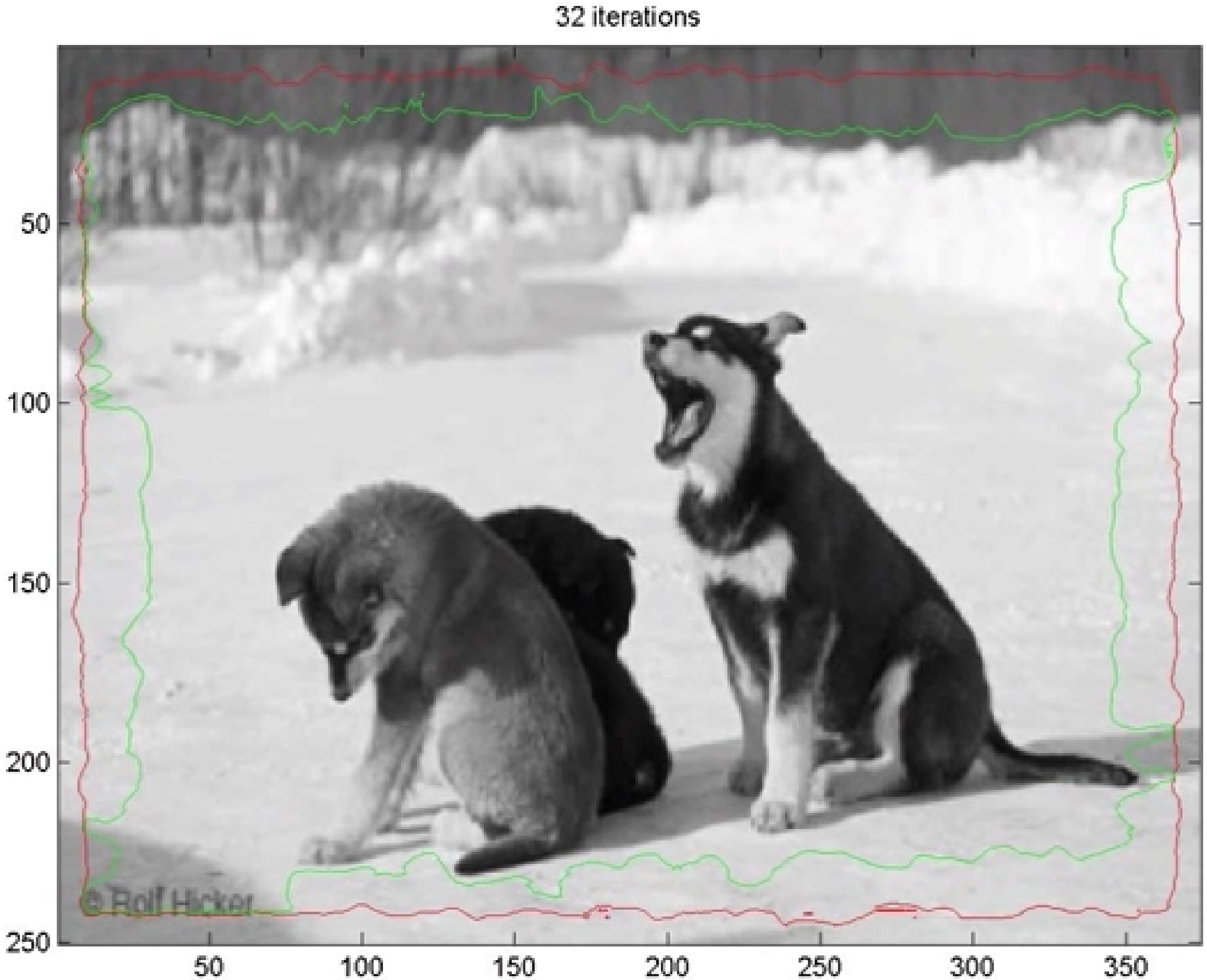


http://www.cs.ucla.edu/~dt/vision.html

http://www.youtube.com/watch?v=5se69vcbqxA&feature=related



https://www.youtube.com/watch?v=qlaJMiARUyg

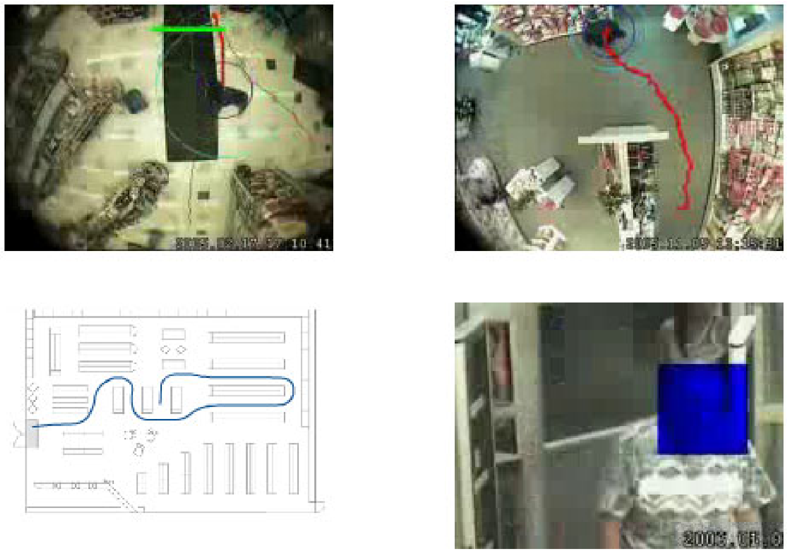


https://www.youtube.com/watch?v=QNk6Zx6Wi_k

VideoMining is changing the way in-store insights are gathered and applied by automating the collection of shopper behavior and segmentation data. VideoMining's patent-protected technologies and processes turn in-store video into actionable intelligence for retailers and consumer product manufacturers.

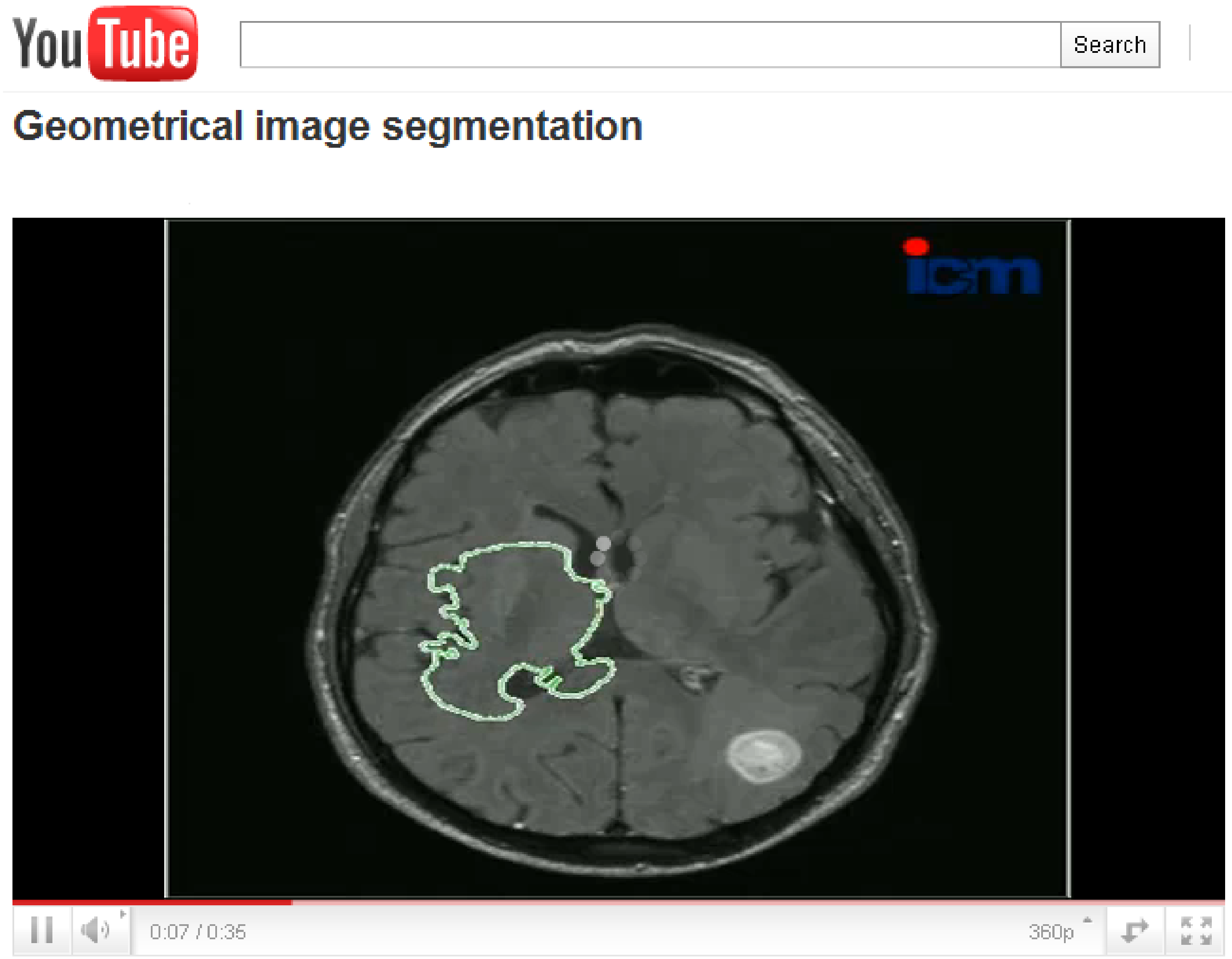
The VideoMining Process

Examples of VideoMining Technologies in Action



People tracking:

http://www.videomining.com/solutions



http://www.youtube.com/watch?v=3imS_9EeNhU&feature=related

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3. The shape of the contour is dictated by the energy functional

$$E\left(\vec{V}\right) = E_{\text{int}}\left(\vec{V}\right) + E_{\text{ext}}\left(\vec{V}\right)$$

4. Internal energy

$$E_{\text{int}}\left(\vec{V}\right) = \frac{1}{2}\int_0^1 \alpha(s)\left|\frac{d\vec{V}}{ds}\right|^2 ds + \frac{1}{2}\int_0^1 \beta(s)\left|\frac{d^2\vec{V}}{ds^2}\right|^2 ds$$

   a. the first term controls the 'tension' of the contour.
   b. the second term controls the 'rigidity' of the contour.

Derivative: https://en.wikipedia.org/wiki/Derivative

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5. External energy

$$E_{\text{ext}}\left(\vec{V}\right)=\int_0^1 E_{image}\left(\vec{V}(s)\right)ds$$

   a. E_{image} represents the scalar potential (gradient) function defined on the image plane, e.g.

$$E_{image}\left(\vec{V}(s)\right)=-c\left|\nabla\left[G_\sigma * I\left(\vec{V}(s)\right)\right]\right|$$

   b. $c>0$ is constant, $G_\sigma * I$ represents an image I convolved with a Gaussian smoothing filter with SD σ .

6. The final shape of the contour $\vec{V}(s)$ corresponds to the minimum of energy E.

Examples

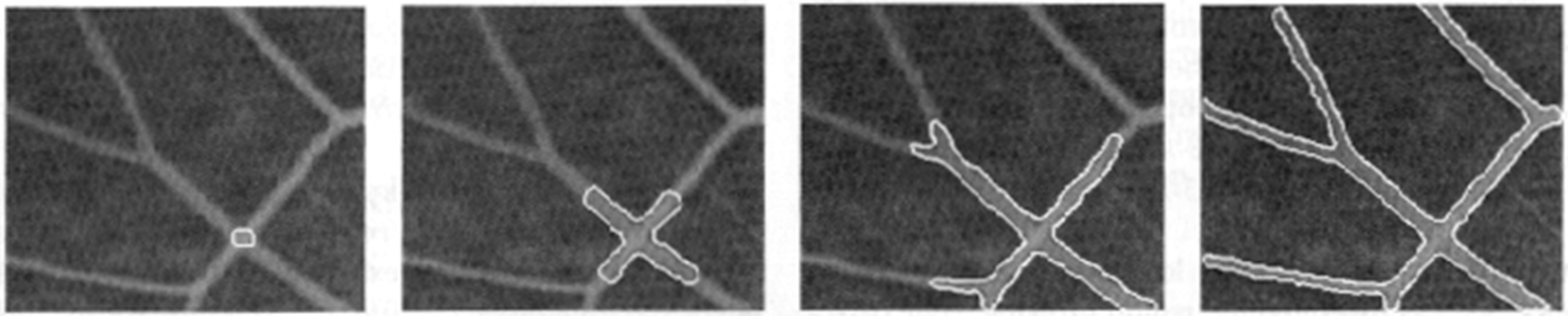


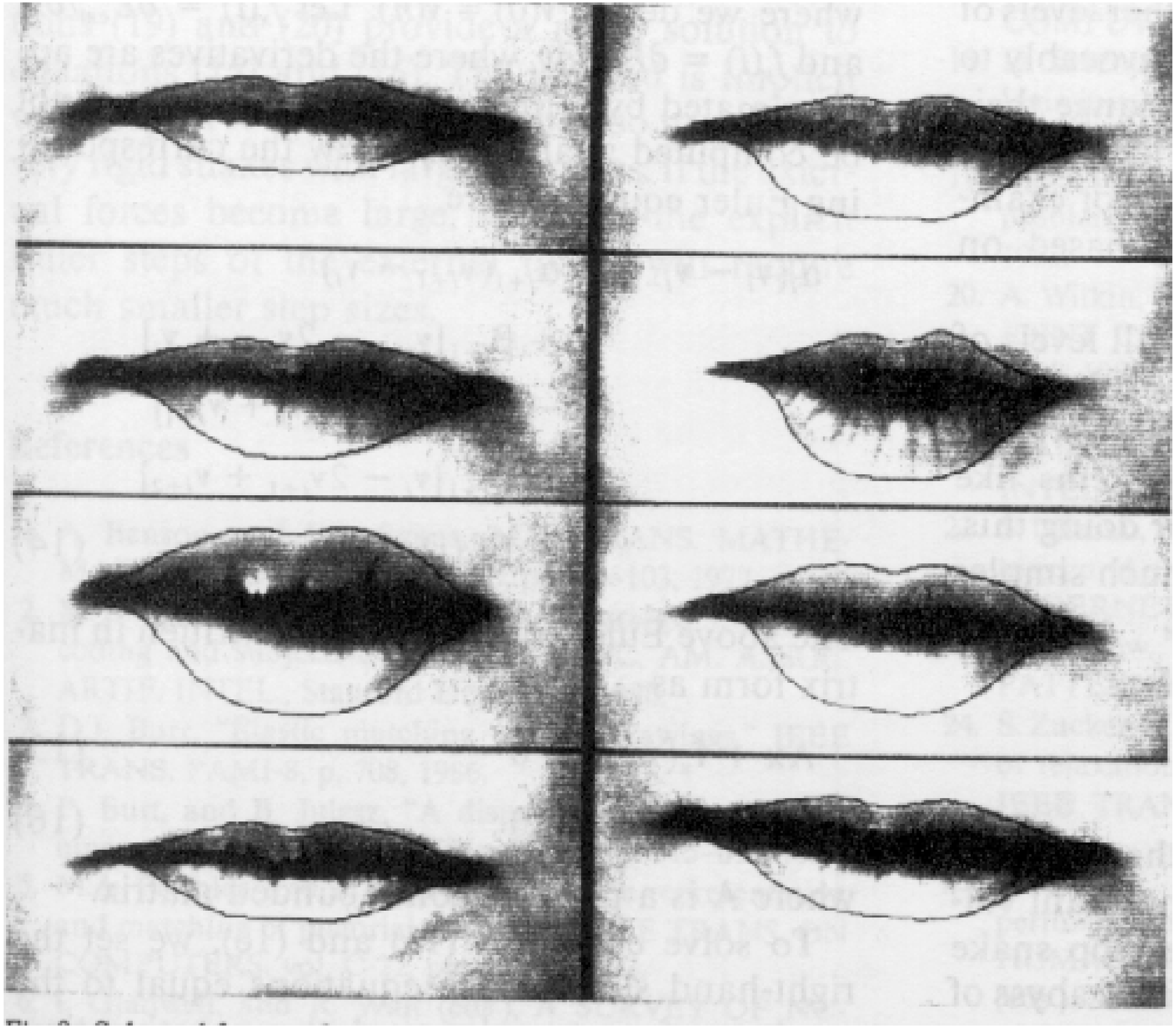
FIGURE 4 Image sequence of clipped angiogram of retina showing an automatically subdividing snake flowing and branching along a vessel [96].

(a) (b) (c) (d)

FIGURE 5 Segmentation of a cross sectional image of a human vertebra phantom with a topologically adaptable snake [96]. The snake begins as a single closed curve and becomes three closed curves.

Handbook of Medical Imaging, p. 132

Example

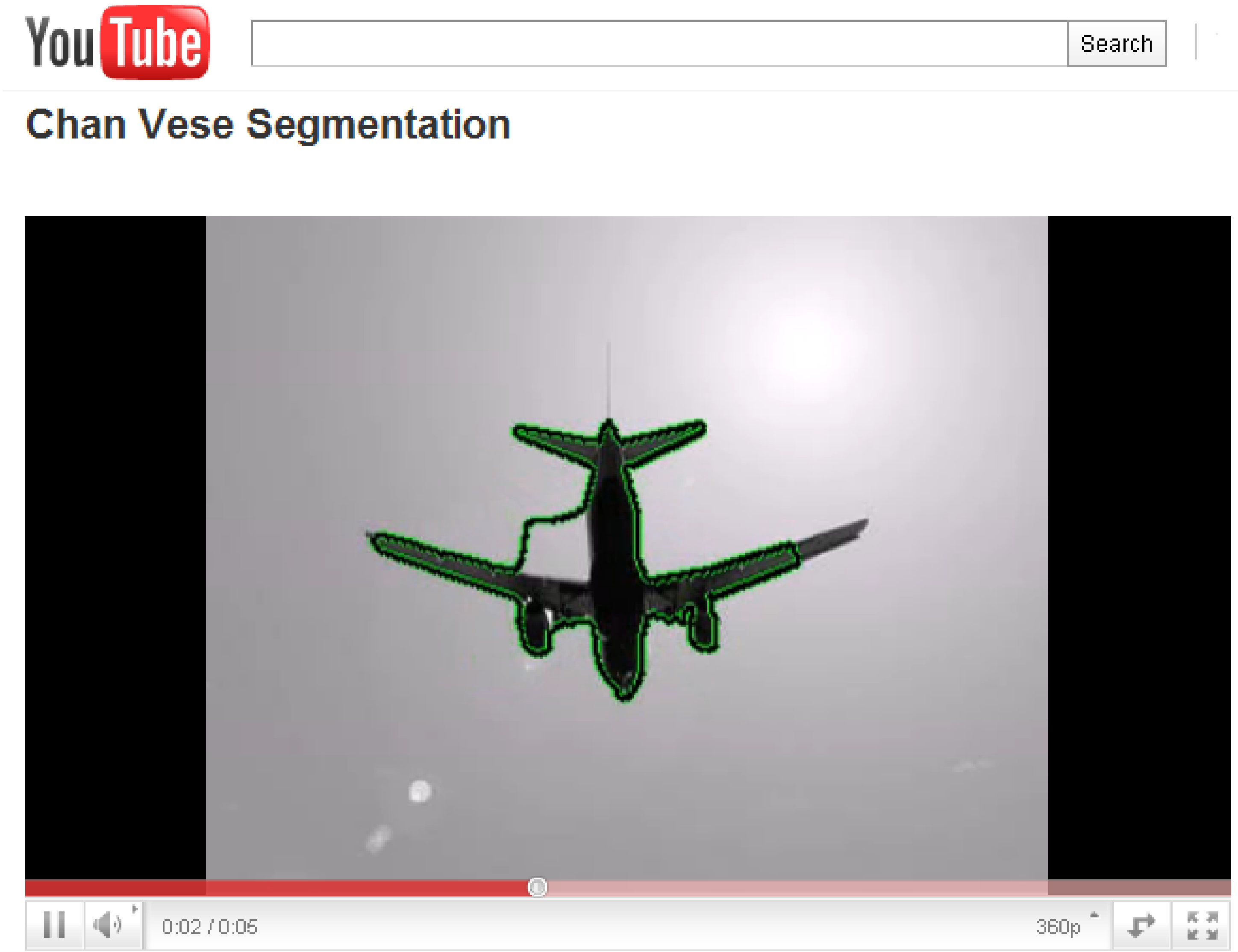


Kass, Witkin and Terzopoulus, 1987

Search

Region Based Segmentation Using Active Contours

https://www.youtube.com/watch?v=celddPk78yA



https://www.youtube.com/watch?v=H3P5N7ZfvEo

Reference: http://cdanup.com/10.1.1.2.1828.pdf

Implementation of Active Contour Models

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7. According to the calculus of variations, the contour $\vec{V}$ which minimizes the energy E must satisfy the Euler-Lagrange equation,

$$-\frac{d}{ds}\left(\alpha(s)\frac{d\vec{V}}{ds}\right)+\frac{d^2}{ds^2}\left(\beta(s)\frac{d^2\vec{V}}{ds^2}\right)+\nabla E_{\text{ext}}\left(\vec{V}\right)=0$$

where $\nabla E_{\text{ext}}(\vec{V}) = \begin{pmatrix} \dfrac{\partial E_{\text{ext}}(\vec{V})}{\partial x} \\ \dfrac{\partial E_{\text{ext}}(\vec{V})}{\partial y} \end{pmatrix}$

https://en.wikipedia.org/wiki/Calculus_of_variations

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8. Discrete formulation based on finite differences

$$-\frac{d}{ds}\left(\alpha(s)\frac{d\vec{V}}{ds}\right)+\frac{d^2}{ds^2}\left(\beta(s)\frac{d^2\vec{V}}{ds^2}\right)+\nabla E_{\text{ext}}\left(\vec{V}\right)=0$$

Using the finite difference scheme in space with a step size of h

$$\frac{1}{h}\left(a_i\left(\vec{V}_i-\vec{V}_{i-1}\right)-a_{i+1}\left(\vec{V}_{i+1}-\vec{V}_i\right)\right)+$$

$$\frac{b_{i-1}}{h^2}\left(\vec{V}_{i-2}-2\vec{V}_{i-1}+\vec{V}_i\right)-2\frac{b_i}{h^2}\left(\vec{V}_{i-1}-2\vec{V}_i+\vec{V}_{i+1}\right)+\frac{b_{i+1}}{h^2}\left(\vec{V}_i-2\vec{V}_{i+1}+\vec{V}_{i+2}\right)-$$

$$\left(f_x(\vec{V}_i),f_y(\vec{V}_i)\right)^{\mathrm{T}}=0$$

where $h = \text{step size}$

$$\vec{V}_i = \left(x(ih), y(ih) \right)^T$$

$$a_i = \frac{\alpha(ih)}{h}$$

$$b_i = \frac{\beta(ih)}{h^2}$$

$$f_x\left(\vec{V}_i\right) = \frac{\partial E_{\text{ext}}}{\partial x} \quad \text{and} \quad f_y\left(\vec{V}_i\right) = \frac{\partial E_{\text{ext}}}{\partial y}$$

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9. Matrix form

$$A\vec{x}+\vec{f}_x=0 \quad \text{and} \quad A\vec{y}+\vec{f}_y=0$$

where **A** is a matrix in terms of a and b.

$$f_x\left(\vec{V}_i\right)=\frac{\partial E_{\text{ext}}}{\partial x} \quad \text{and} \quad f_y\left(\vec{V}_i\right)=\frac{\partial E_{\text{ext}}}{\partial y}$$

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10. Dynamic deformable model. Let $\vec{V} = \vec{V}(s,t)$ be a time-varying contour. The evolution equation is given as

$$\frac{\partial \vec{V}}{\partial t} - \frac{\partial}{\partial s}\left(\alpha \frac{\partial \vec{V}}{\partial s}\right) + \frac{\partial^2}{\partial s^2}\left(\beta \frac{\partial^2 \vec{V}}{\partial s^2}\right) + \nabla E_{\text{ext}}\left(\vec{V}\right) = 0$$

11. Discrete formulation of the dynamic SNAKE (Finite difference) in matrix form

$$\gamma\left(\vec{\mathrm{x}}_{\mathrm{t}} - \vec{\mathrm{x}}_{\mathrm{t-1}}\right) + \mathrm{A}\vec{\mathrm{x}}_{\mathrm{t}} + \vec{\mathrm{f}}_{\mathrm{x}}\left(\mathrm{x}_{\mathrm{t-1}}, \mathrm{y}_{\mathrm{t-1}}\right) = 0$$

$$\gamma\left(\vec{\mathrm{y}}_{\mathrm{t}} - \vec{\mathrm{y}}_{\mathrm{t-1}}\right) + \mathrm{A}\vec{\mathrm{y}}_{\mathrm{t}} + \vec{\mathrm{f}}_{\mathrm{y}}\left(\mathrm{x}_{\mathrm{t-1}}, \mathrm{y}_{\mathrm{t-1}}\right) = 0$$

where γ represents time step size.

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12. Therefore, the dynamic SNAKE can be solved iteratively.

$$\vec{x}_t = (A+\gamma I)^{-1}\left(\gamma\vec{x}_{t-1} - \vec{f}_x\left(x_{t-1}, y_{t-1}\right)\right)$$
$$\vec{y}_t = (A+\gamma I)^{-1}\left(\gamma\vec{y}_{t-1} - \vec{f}_y\left(x_{t-1}, y_{t-1}\right)\right)$$

13. At equilibrium, a stationary contour with minimum internal and external energies is obtained.

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Active Contour by Professor Guillermo Sapiro, Duke University

https://www.youtube.com/watch?v=r610mi5hiHM

Not examined

Quadratic energy functional I

$$E_{\text{ext}} = \frac{-1}{2}(u-v)^2$$

where $u =$ mean in region R_u

$v =$ mean in region R_v

Let $S_u = \int_{R_u} I\,dA$ and $A_u = \int_{R_u} dA$

Then $\nabla S_u = I\,\vec{n}$

$$\nabla A_u = \vec{n}$$

$$\nabla u = \frac{A_u \nabla S_u - S_u \nabla A_u}{A_u^2}$$

Not examined

Yezzi et al.

Quadratic energy functional

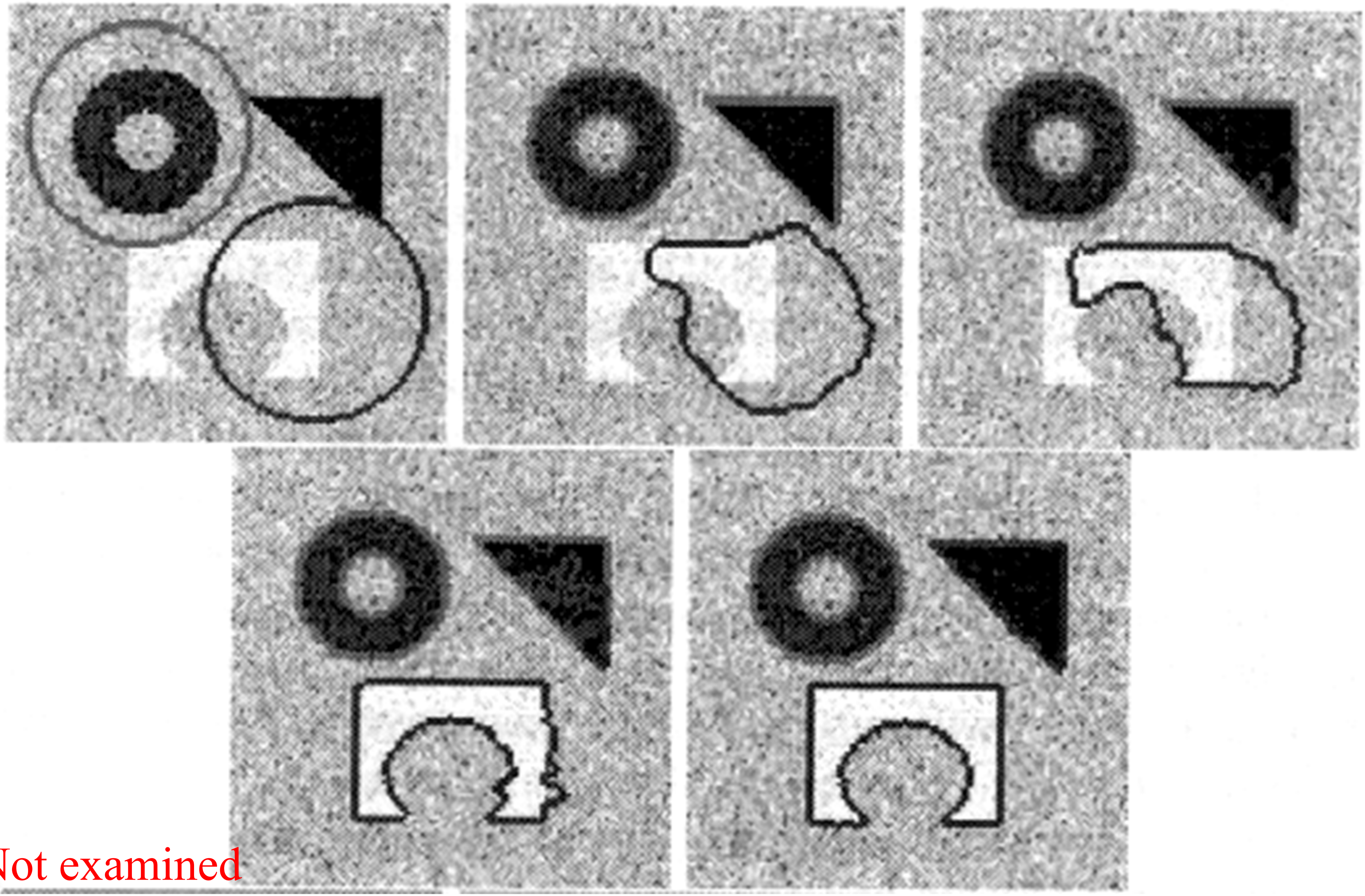
$$\frac{d\vec{V}}{dt} = -\nabla E_{\text{ext}}$$

$$= (u - v)\left(\frac{I - u}{A_u} + \frac{I - v}{A_v}\right)\vec{n}$$

Not examined

Yezzi et al.

Example



Not examined

Quadratic energy functional II

$$E_{\text{ext}} = \int_{\Omega_1} \left| I(x,y) - C_1 \right|^2 dxdy + \int_{\Omega_2} \left| I(x,y) - C_2 \right|^2 dxdy$$

$$\frac{d\vec{V}}{dt} = (|I(x,y) - C_2|^2 - |I(x,y) - C_1|^2)\vec{n}$$

Not examined

Chan and Vese

Example

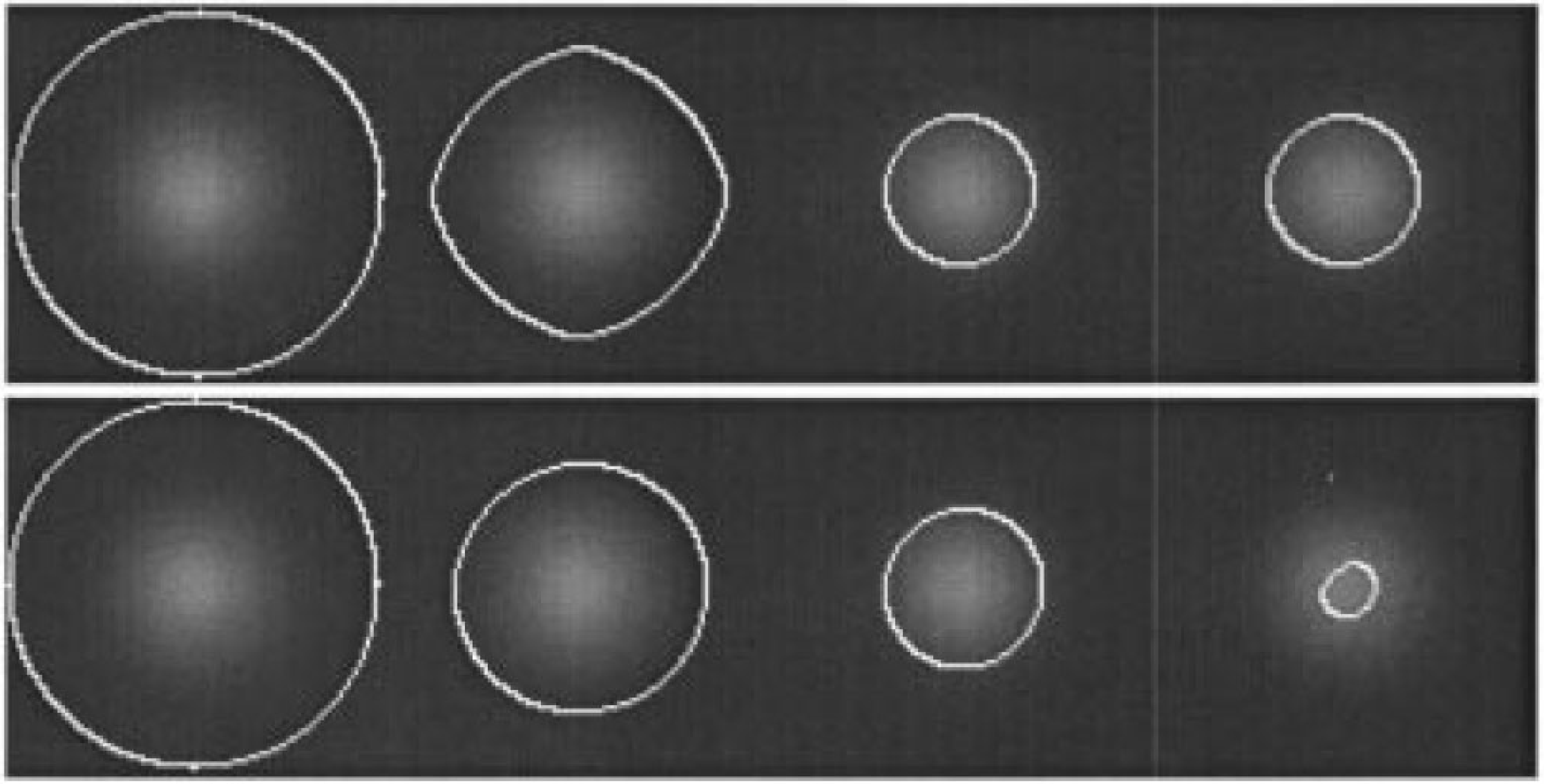
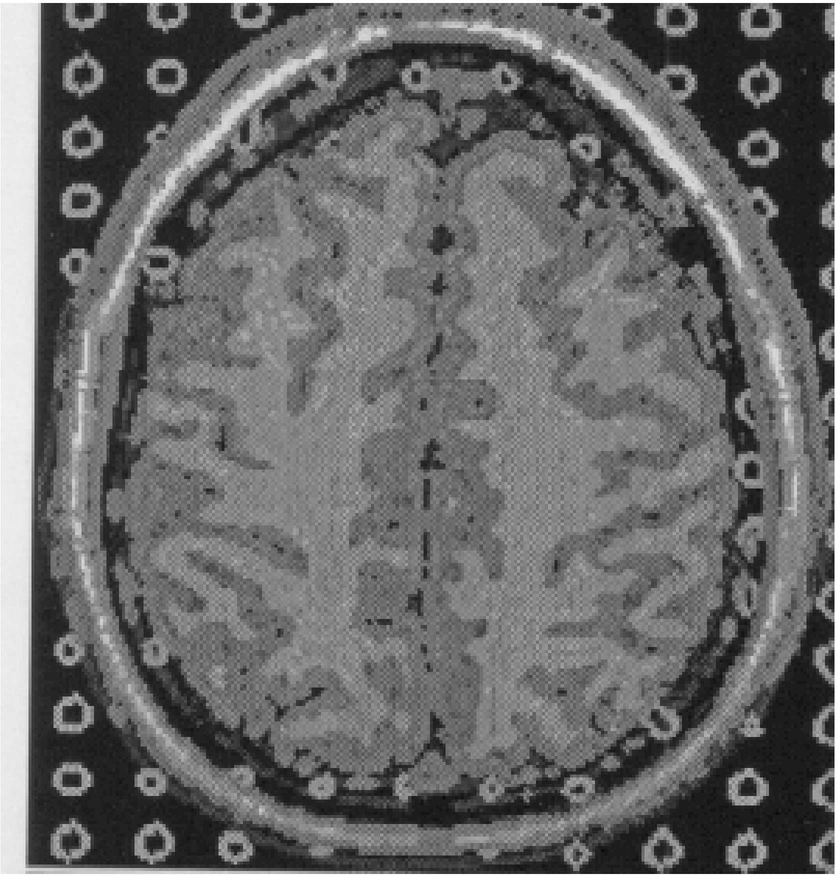


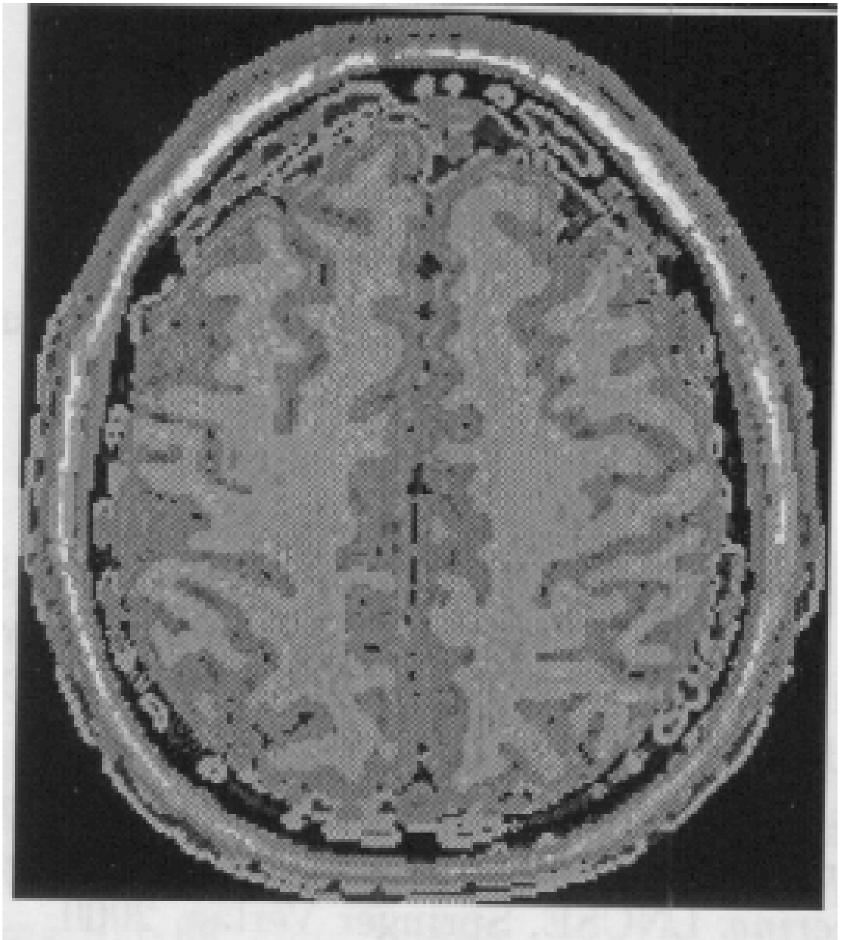
Fig. 9. Object with smooth contour. Top: results using our model without edge-function. Bottom: results using the classical model (2) with edge-function.

Not examined

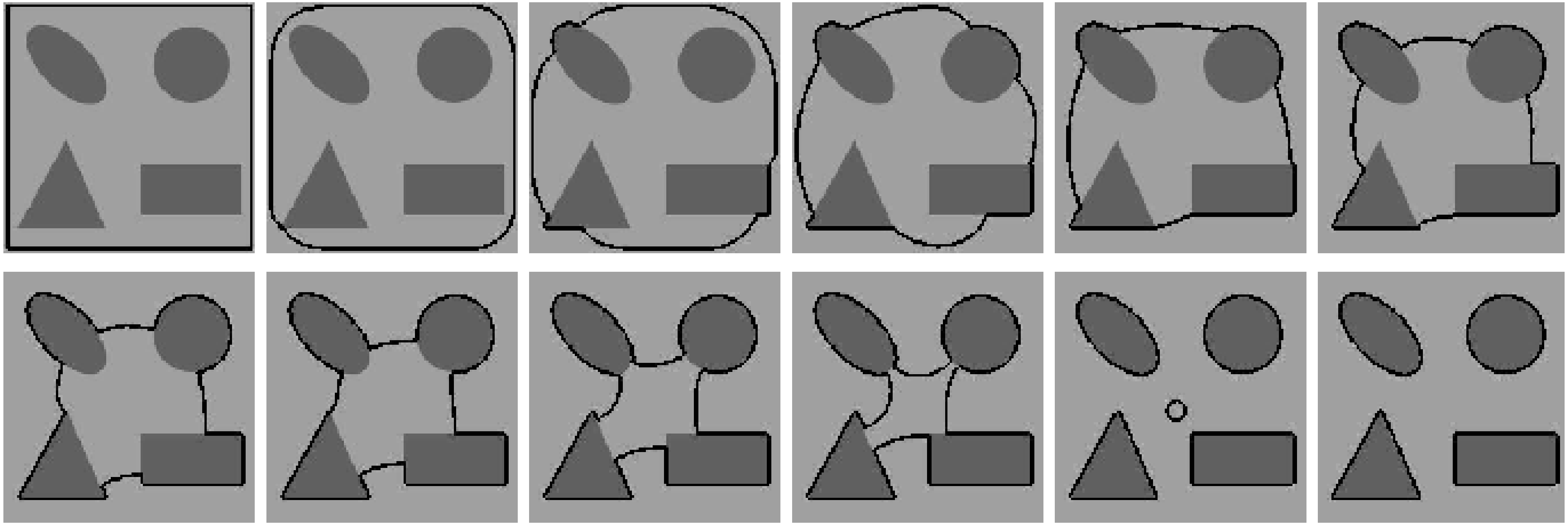
Chan and Vese

Example using level set methods





Not examined



Not examined