Handheld-GPS based Speed-Measurements

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Introduction

With the advance of handheld **GPS** (Global Positioning System) devices speed measurements for example of speedsurfers or speedkiters become affordable for everyone. GPS units receive signals at a frequency of **1575.42Mhz** from typically up to twelve of the 28 satellites orbiting the Earth in about 20.180km height. From the time differences of the satellite signals the distances to the satellites and finally the unit's position (latitude, longitude, and altitude) is calculated.

Position, time, and other measures are stored by the units with a typical rate of **1 per second** in so-called **trackpoints**. Some units store time and positional data only (e.g. Garmin® Geko, Foretrex, Forerunner, Edge), other units like the Locosys® GT-11 save **NMEA**-data (1) in their internal memory or on removable SD-cards. NMEA-data contain much more than positional information like number of tracked satellites, satellite signal strengths, confidence measures like Horizontal Dilution of Precision (HDoP), and Doppler-speed (which is determined directly from the Doppler-shift of the satellite frequencies and thus completely independent from the positional computations), etc. The possible use of instantaneous Doppler-speed data for speed measurements was lately discussed in speedsurfing blogs (2, 3) and will be compared here to the 'traditional' positional data based methods. A nice overview of modern GPS-technology can be found in (4).

Earth model

The Earth's shape can be approximated by a rotational ellipsoid (spheroid) defined by it's two semi axes **a** (equatorial axis) and **b** (pole axis). The flattening parameter **f** is defined by **f=(a-b)/a**. For the worldwide reference ellipsoid **WGS-84** these parameters are: **a=6378137m, b= 6356752m, 1/f=298.257223563**. With **Helmert's formula** ellipsoidal angle co-ordinates that are contained in the GPS-data can be transformed into Cartesian co-ordinates.

Speed Measurements

There are two ways to determine the speed of movement of the GPS-unit:

1. indirectly from positional data differences and the corresponding time differences (the time stamps are very well defined, error << 1µsec)
2. directly from the instantaneous Doppler-speed

The first method can be applied to all GPS-units since position and time is the minimum information contained in the trackpoint-data. The second approach is of course only possible if there is access to the speed data as in the NMEA-data format.

For the positional method there are two flavours to determine the average speed over a given time or distance. One can integrate (accumulate) the 1-second segment distances of the trackpoint trajectory or one can determine the distance from a start position to the actual position and thus have a projected (or linear) speed measurement.

For the Doppler-speed method no positional data are used, this approach is very similar to the accumulative approach of the positional speed computations.

Instantaneous speed **ds** and distance **dp** are linearly related by the corresponding time difference **dt** needed to travel the distance **dp** between the positions $\underline{\mathbf{p}}_i$ and $\underline{\mathbf{p}}_{i+1}$:

$$\mathbf{dp} = |\underline{\mathbf{p}}_{i+1} - \underline{\mathbf{p}}_i| \qquad \mathbf{dt} = \mathbf{t}_{i+1} - \mathbf{t}_i \qquad \mathbf{ds} = \mathbf{dp} / \mathbf{dt} \qquad \mathbf{dp} = \mathbf{ds}\,\mathbf{dt} \qquad (1)$$

The integration over time or distance is a simple summation over consecutive 1-second pieces (**dt** = 1sec) of the whole trajectory giving the total distance **P**, the total time **T**, and the average speed **S**:

$$\mathbf{P = sum\,(dp_i) \quad -> \quad T = sum\,(dt_i) \quad -> \quad S = P\,/\,T} \qquad \mathbf{(2)}$$

Equation 2 applies for both positional and Doppler accumulative methods, the projected positional method uses the distance between a start point (i=s) and an end point (i=e) only, thus the intermediate points (i=s+1…i=e-1) are irrelevant ($\mathbf{p_{ez}} = \mathbf{p_{sz}} = 0$ in a 2-dimnesional representation):

$$\mathbf{P_P = [(p_{ex} - p_{sx})^2 + (p_{ey} - p_{sy})^2 + (p_{ez} - p_{sz})^2]^{0.5}} \qquad \mathbf{(3)}$$

The task of the evaluation program is to determine all possible trajectory-segments that are longer or equal to the given distance (100m, 250, 500m, 1852m=nautical mile) or time (e.g. for the fastest 10sec runs, main ranking criteria for www.gps-speedsurfing.com), and find the fastest non-overlapping segments or the fastest run per leg and finally average the 5 fastest runs for the 5 x 10sec ranking (2).

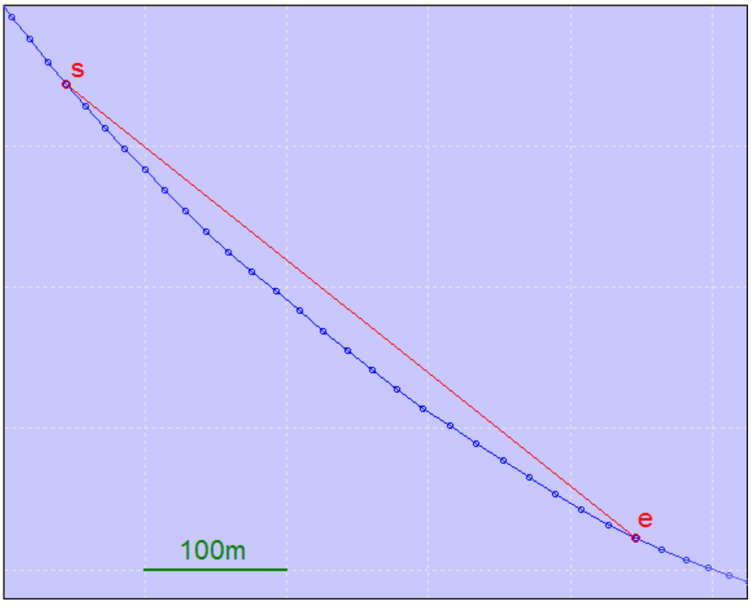

Fig. 1: Example of a trajectory (measured with a GT-11 at Sandy Point, rider: A. Daff, blue line connecting the measured trackpoints shown as blue circles) with accumulated distance and projected distance between points **s** and **e** (red line), the given minimum distance is **500m**: The accumulated distance $\mathbf{P_A}$ is **518.1m**, the projected distance $\mathbf{P_P}$ is **515.1m**, the corresponding average speeds are $\mathbf{S_A}$**=41.961knots** and $\mathbf{S_P}$**=41.718knots**. The average Doppler-speed of the same run is $\mathbf{S_D}$**=41.635knots**. Runtime in all cases **T=24sec**.

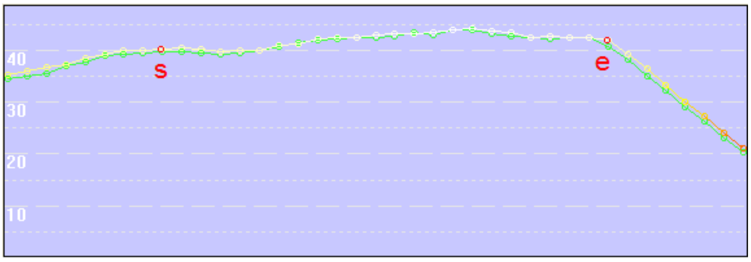

Fig. 2: Speed over time plot, ‘positional’ speed derived from the trackpoint positions shown in white, Doppler-speed in green. Start- and end-trackpoint are shown in red.

Linear Interpolation

From the example above it is obvious that the accumulated distances are always larger than the straight-line projections and thus the projected positional speeds $\mathbf{S_P}$ are always smaller than the accumulated speeds $\mathbf{S_A}$. The larger the curvature of the trajectory, the larger the differences between accumulated and projected speeds. There are special cases where a projection does not reach the given distance, but the accumulated distance does (e.g. a circular trajectory or a trajectory including a jibe).

The accumulated Doppler-speeds $\mathbf{S_D}$ are a completely independent measure and do not need positional information at all, thus a projection in this case is not possible.

However, in general the accumulated distances and also the projected distances are larger than the given run length because of the coarse one-second sampling of the trajectory (for the 10sec ranking this is of course not a problem). A better accuracy can be achieved by linear interpolation of slower start- or end-piece, i.e. only the fraction of the slower sub-segment is used, that is needed to exactly reach the given distance. In the example above the first 1-second sub-section was **20.6m** long with a speed of **40.050knots** (Doppler-speed: 39.64knots), the last sub-section was **21.6m** long with a speed of **41.730knots** (Doppler-speed: 40.61knots). Thus from the first section only 20.6m – (518.1m – 500m) = 2.5m are needed to reach the given 500m run-length. This results in a slightly larger, more realistic speed of **42.033knots** for the accumulated speed (**0.072knots** faster than without linear interpolation, see above).

The same can be applied to the projected and the Doppler-speed results: $\mathbf{S_{PI}=41.796knots}$ (**0.078knots** faster) and $\mathbf{S_{DI}=41.694knots}$ (**0.059knots** faster). For constant speed linear interpolation of course does not make a difference and for runs with faster start and finish sections linear interpolation gives slightly slower average speeds.

Error Bounds

In order to estimate error bounds of the different average speed calculations the error propagation of the individual trackpoints has to be considered. The most simple case is the projected positional speed case where only the first and the last trackpoint need to be considered. If the positional error (standard deviation) of the first point (index **S**) and of the last point (index **E**) is **D** than the total error $\mathbf{D_T}$ in a simplified one-dimensional case is given by $\mathbf{D_T = 2^{0.5} * D= 1.41 * D}$ (Gaussian error propagation, directional influences ignored). If **N** independent values are added like in the accumulation methods the total standard deviation is:

$$\mathbf{D_T = N^{0.5} * D} \qquad \mathbf{(4)}$$

D is the standard deviation of the individual trackpoint and is assumed equal for all trackpoints. If one takes the specifications (5) of the GT-11 (D = 25m for the absolute localization error and D = 0.1m for the Doppler-speed error method (derived from the Doppler-speed error of 0.1m/s = 0.2knots and 1 second sampling rate), one would get the following total errors for the 500m run from above (N = 24: 24 seconds run-time):

$\mathbf{D_{TA} = 24^{0.5} * 25m = 122m}$ $\mathbf{D_{TP} = 2^{0.5} * 25m = 35m}$ $\mathbf{D_{TD} = 24^{0.5} * 0.1m = 0.5m}$

Since the time interval can be considered error-free, these total positional errors would translate into the following speed error margins:

$\mathbf{D_{TA} = 9.9knots}$ $\mathbf{D_{TP} = 2.9knots}$ $\mathbf{D_{TD} = 0.04knots}$

However, from practical experience the errors especially for the positional speeds seem way to large. Otherwise e.g. the trajectory (Fig. 1) with approximately 20m sub-segments and also the rather smooth speed over time curve (Fig. 2) would look much more zig-zagged. The reason is that the relative or short time accuracy is much better than the absolute accuracy and that for the speed calculations only position differences (distances) are used (Eq. 1-3). Thus experiments were performed to get more representative measures.

Experiments

The positional and Doppler-speed standard deviations can be measured by simply analysing data from a stationary unit, since the speed range we are interested in here (~40knots) is rather small compared to the relative speed of a resting unit and the satellites. The satellites finish one full orbit in about 12hours, taking into account the Earth's rotation, the satellites are at the same positions (seen from the Earth) after 24hours. Thus it does not make much difference considering a stationary device or a unit travelling at 40knots, as long as the satellites tracked do not change.

Stationary Measurements

Fig. 3 shows positional and speed data from a stationary GT-11 storing trackpoints for more than one and a half hour. The obvious grid-effect is caused by the limited number of decimals of the latitude and longitude angles in the NMEA-sentences and leads to a spatial resolution of **0.19m** in North-South direction and to 0.19m * cos (53.6°) = **0.11m** in East-West direction (latitude of Hamburg/Germany: 53.6°). The Garmin units suffer from a rather severe grid-effect of **2.39m** in NS- and **1.42m** in EW-direction. With a logging rate of 1 per second this leads a speed resolution of the GT-11 in NS-direction of 0.19m/s = **0.36knots** (Garmin: **4.64knots**) and 0.11m/s = **0.21knots** (Garmin: **2.76knots**) in EW-direction.

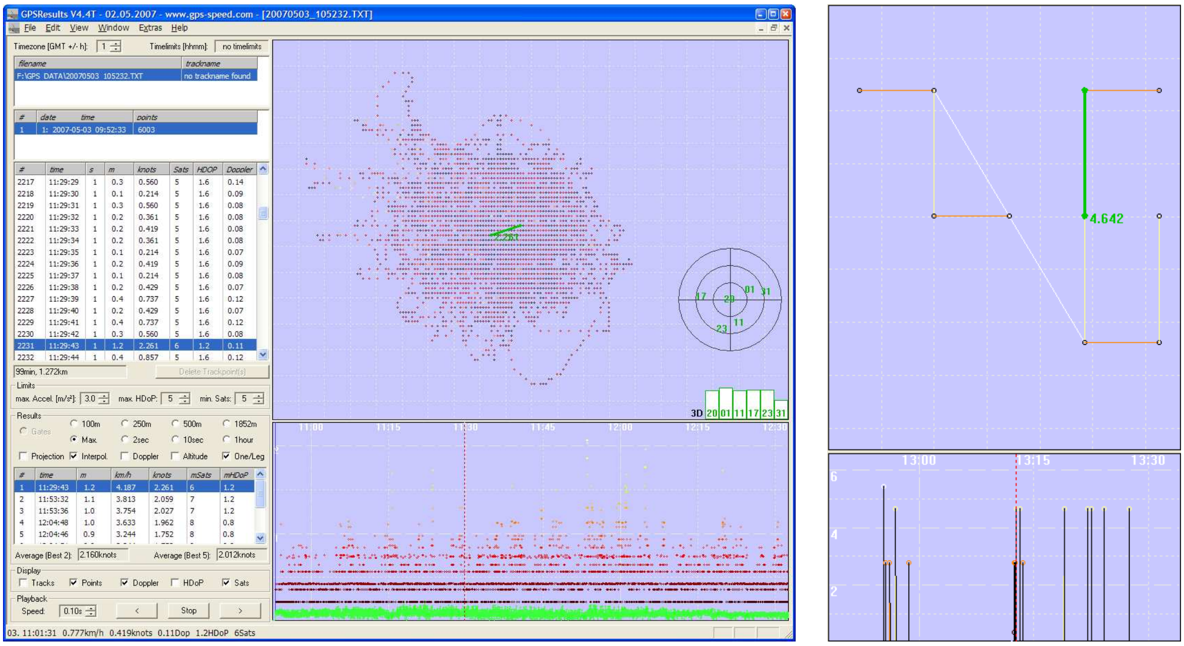

Fig. 3: Stationary GT-11 measurements (left), upper panel: trackpoints and sky-view of the satellites, lower panel: Doppler-speed (green) and grid-effect dominated trackpoint-speeds (black/red/yellow). On the right stationary Foretrex201 data exhibiting a 2.39m grid-effect are shown (dashed grid - size 1m).

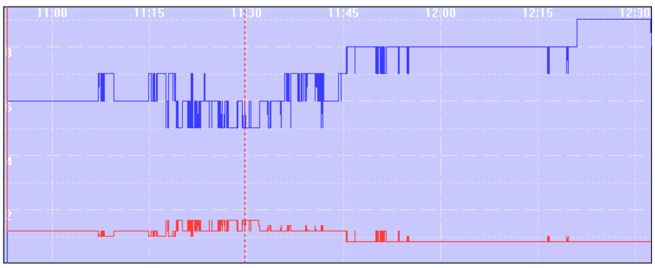

Fig. 4: GT-11: Number of satellites (blue) and Horizontal Dilution of Precision (red).

The effect can clearly be seen in Fig. 3, the intermediate speed values are caused by diagonal positional ‘jumps’ but are all composed of the elementary grid unit: 1*NS+1*EW=0.42knots, 2*EW=0.42knots, 1*NS+2*EW=0.56knots, 3+EW=0.64knots, 2*NS=0.72knots, etc.

The green Doppler-speed curve in Fig. 3 shows how much more stable this more direct speed-measurement is as compared to the derived short time positional speed values. The Doppler-speed resolution is **0.01knots** (from NMEA-sentence GPRMC). Fig. 4 displays the time-courses of the number of satellites and the HDoP-values of the GT-11. For the Foretrex these data are not available. Fig. 5 and 6 show the distributions of the Doppler- and short-time positional speeds respectively. Similar experiments were repeated several times, the results can be found in Tab. 1.

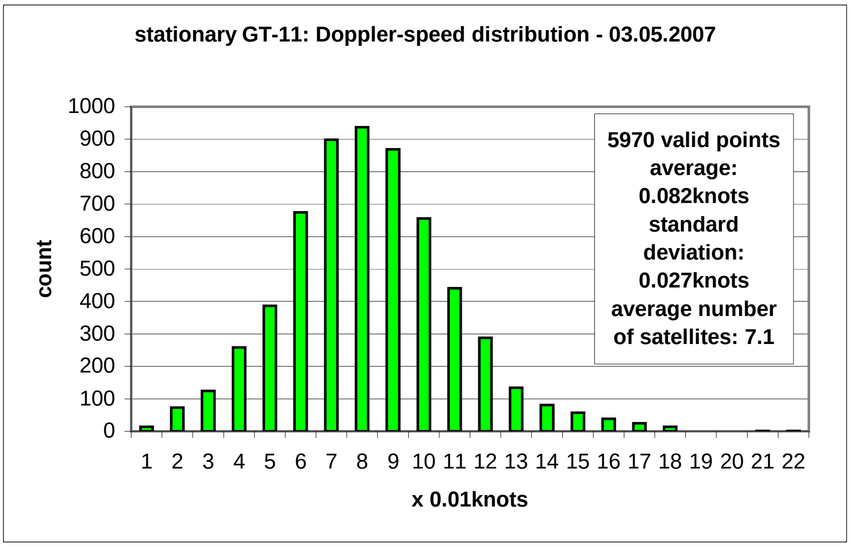

Fig. 5: Distribution of Doppler-speed values of a stationary GT-11 unit. The equivalent average short time positional error is 0.082knots*1sec=**0.042m**.

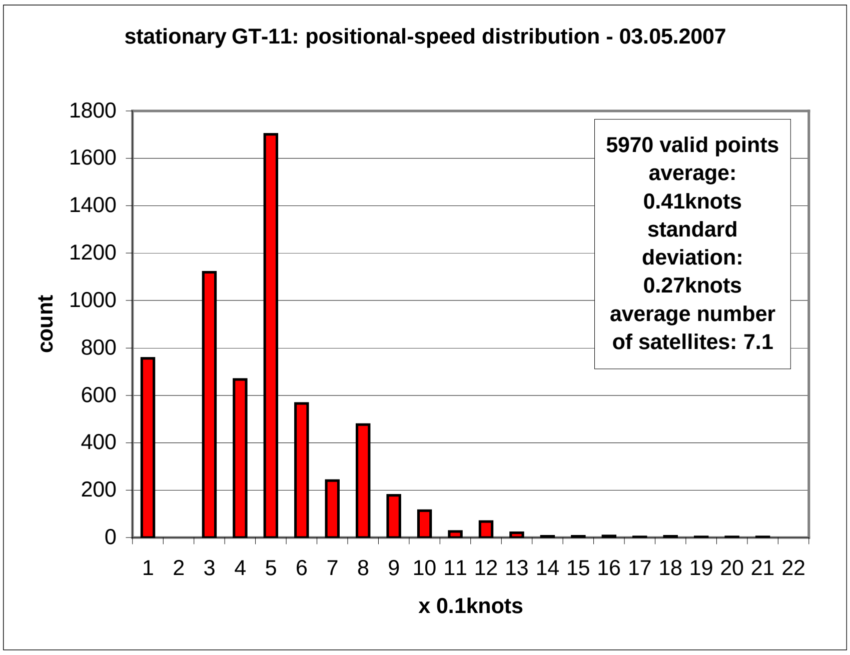

Fig. 6: Distribution of positional, grid-effect dominated speed values of a stationary GT-11 unit (the horizontal scale differs by one order of magnitude from Fig.5). The equivalent average short time (1 second) positional error is 0.41knots*1sec=**0.21m**.

21.04.2007		knots	knots
4104 points	**sats**	**Doppler**	**position**
average	7.508	0.060	0.419
std. dev.	0.505	0.028	0.265
min	7	0	0
max	9	0.25	1.914

03.05.2007		knots	knots
5970 points	**sats**	**Doppler**	**position**
average	7.085	0.082	0.414
std. dev.	1.202	0.027	0.271
min	5	0.01	0
max	9	0.22	2.261

05.05.2007		knots	knots
4065 points	**sats**	**Doppler**	**position**
average	8.014	0.081	0.429
std. dev.	0.743	0.025	0.259
min	6	0.01	0
max	9	0.2	2.097

Table 1: Statistical evaluation of three stationary GT-11 measurement sessions.

Error Bounds Using Short Time Stability

Using the experimentally determined short time (1 second) stability results (0.082knots for Doppler-speed, 0.41knots for positional speed) for the error estimation, the following error bounds for the 500m run discussed above (24 seconds run-time) can be calculated:

Accumulated speed $\mathbf{D_{TA}}$, projected (linear) speed $\mathbf{D_{TP}}$ (for the projected speed the accumulated short time errors have to be used instead of the absolute errors of start- and end-trackpoint only, thus the same result as for the accumulated positional speed is achieved here), and accumulated Doppler-speed $\mathbf{D_{TD}}$:

$D_{TA} = 24^{0.5} * 0.21m = 1.03m$
$D_{TP} = 24^{0.5} * 0.21m = 1.03m$
$D_{TD} = 24^{0.5} * 0.042m = 0.21m$

Since the time interval can be considered error-free, these total positional errors would translate into the following speed error margins:

D_{TA} = 0.09knots D_{TP} = 0.09knots D_{TD} = 0.017knots

These values look rather optimistic and taking the non Gaussian distribution of the positional speed (grid-effect) and the rather larger standard deviation of the average and maximum values (outliers, see Tab. 1) as compared to the corresponding Doppler-speed values into account, a factor of 2 for the positional speed errors seems to be more adequate. If we now use 2 standard deviations (**2σ**), resulting in a confidence limit of **95.4%** (a single standard deviation gives **68.3%** probability margins only), we get:

D_{TA} = 0.34knots D_{TP} = 0.34knots D_{TD} = 0.033knots

The more direct Doppler-speed measurements have about one order of magnitude smaller error bounds as compared to the ‘traditional’ position based speed measurements.

Since the errors depend on the run-time **T**, the general formula for the error bounds at 1 second sampling rate using the experimentally found short time accuracy values is (**2σ**):

$$\mathbf{D_{TA} = D_{TP} = \quad +/\text{-}T^{0.5} * 0.84m \quad => \quad +/\text{-}1.68 / T^{0.5} knots} \qquad \mathbf{(5)}$$
$$\mathbf{D_{TD} = \quad +/\text{-}T^{0.5} * 0.084m \quad => \quad +/\text{-}0.16 / T^{0.5} knots}$$

Thus, e.g. for the **10 second** runs follows:
$\mathbf{D_{TA} = D_{TP} = +/\text{-}0.53}$ **knots**, $\mathbf{D_{TD} = +/\text{-}0.05}$**knots**
and for the average of the fastest 5 runs:
$\mathbf{D_{TA} = D_{TP} = +/\text{-}0.24}$ **knots**, $\mathbf{D_{TD} = +/\text{-}0.02}$**knots**

For the distance based measures (100m, 250m, 500m, 1852m) the errors depend on the speed (-> run-time), but can of course easily be calculated. The faster and the shorter the run, the larger the error, however, there is only a square root dependency (Eq. 5).

Examples

Several data sets from northern Germany (rider: T. Heinig) and one from Sandy Point (rider: A. Daff) were analyzed (Fig. 7) using the three different approaches to determine the fastest (1 second), 2s, 10s, 100m, 250m, and 500m runs using '**GPSResults'** (6):
accumulated $\mathbf{S_A}$, projected (linear) $\mathbf{S_P}$, and accumulated Doppler-speed $\mathbf{S_D}$.

Since the Doppler-speed error margins are by far the smallest (see above), Doppler-speed was used as reference and the differences to accumulated and projected speed were calculated. Tab. 2 shows the results and statistical analyses obtained with the different methods evaluating the 20 fastest runs in each class. The **min**imum and **max**imum speeds are indicated as well as the averages (**av**) and standard deviations (**sd - 1σ**) of the differences **A-D** (accumulated – Doppler) and **P-D** (projected – Doppler).

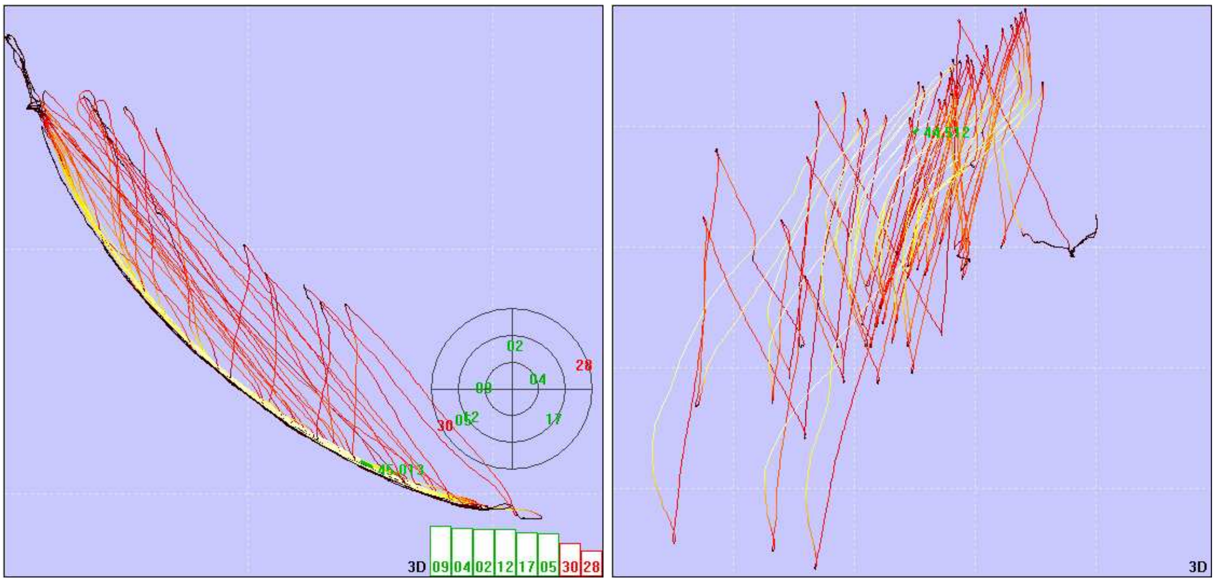

Fig. 7: Analysed runs from Sandy Point (left, A. Daff, speedsurfer) and Westerhever (right, T. Heinig, speedkiter) measured with Locosys GT-11 GPS-devices.

AD/SP	max.	2sec	10sec	100m	250m	500m
min	26.08	25.77	23.99	24.39	22.34	20.71
max	44.92	44.78	44.14	44.42	44.09	42.48
A-D av	0.21	0.20	0.24	0.25	0.23	0.29
A-D sd	0.25	0.23	0.18	0.18	0.19	0.21
P-D av	0.21	0.20	0.19	0.23	0.16	-0.12
P-D sd	0.25	0.23	0.19	0.19	0.20	0.35

TH/WH	max.	2sec	10sec	100m	250m	500m
min	40.50	40.46	39.22	39.90	38.81	37.79
max	44.69	44.14	43.07	43.39	43.09	41.73
A-D av	0.26	0.33	0.30	0.36	0.33	0.26
A-D sd	0.26	0.21	0.18	0.22	0.16	0.12
P-D av	0.26	0.33	0.18	0.32	0.15	-0.07
P-D sd	0.26	0.21	0.19	0.23	0.20	0.21

Table 2: Analyses of the 20 fastest runs in different classes from the two sessions shown in Fig. 7. Accumulated positional speed **A** and projected positional speed **P** are compared which accumulated Doppler-speed **D**. Averages of the differences (**av**) and standard deviations (**sd-1σ**) are shown. All values in **knots**.

Of course projected speeds are always slower than accumulated speeds (Fig. 1), thus the **A-D**-difference values are always slightly larger than the **P-D**-differences. For the **max.** and the **2sec**-speeds there are no differences between **A-D** and **P-D** because in these cases only one or two neighboured trackpoints segments are involved and thus there is no curvature that could cause a difference. The larger the run-length, the larger the differences between accumulated and projected speeds. All differences are close to the expected tolerances from Eq. 5 (which is valid for **2σ** = 95.4% probability, here only **1σ** = 68.3% is used). For the different classes (**1σ**) at 40knots speed (20.58m/s) one gets:

max. (1sec): +/-0.84knots, 2sec: +/-0.59knots, 10sec: +/-0.27knots, 100m: +/-0.38knots, 250m: +/-0.24knots, 500m: +/-0.17knots

Obviously there is a tendency (bias) for larger speeds produced by the accumulated positional speed method compared to the more accurate accumulated Doppler-speed method. This is caused by geometrical (two-dimensional) effects that where not included in the one-dimensional error propagation considerations above. The accumulated travelled distance of the positional method is always slightly larger than the accumulated Doppler method due to the larger error bounds of the first method and the square root of the sum of the squared co-ordinate differences in the distance calculations resulting in higher speeds (longer distance travelled in the same time).

Laser- and Video-Gate Measurements

In order to compare the GPS-based speed measurements against the ‘ground truth’ of laser-gated and video-controlled speed measurement some preliminary experiments over a rather short distance (35m) at rather moderate speeds (walking – running) were performed.

Fig. 8 shows the equipment used, the distance between the gates was measured by a laser range finder (Leica LRF 800) and verified using a measuring tape. The laser-gate time-resolution was 1msec, the cameras produced 25frames per second, thus deviations in the order of +/-40ms (+/-1frame) can be expected due to the unsyncronized cameras (parts of the system were made available by the German Speedsurfing Association VDS (7)).

Results are shown in Table 3. The video-results are in the expected accuracy range with a standard deviation of +/-46ms (about +/-1frame), the GPS-measurements are also very close to the ‘real’ speeds, however, the standard deviation of the Doppler-results is much better than the standard deviation of both positional GPS-results indicating the better reliability of the Doppler-method which is due to the smaller error bounds of this method.

The run-times cover the important 10sec and 250m range, however, more intense comparisons over larger distances (250m, 500m) should be performed in a more realistic speed range (40knots). It is difficult to compare the different GPS-methods against video- or laser-gated measurements because of their relatively small sampling rate (1/sec), so one should move with very constant speed to cover the same (video-controlled) course.

Finally Fig. 9 displays a comparison between projected positional GPS-speeds and hand-stopped, video-controlled speeds during the Sotavento speedweek in Fuerteventura 2006.

Fig. 8: Laser- and video-gated measurements: video-mixer, -timer and -digitizer (left), laser, video-camera and video-transmitter (right).

	Video sec	Video knots	Accumulated knots	Projected knots	Doppler knots
av	0.014	-0.004	0.021	0.017	-0.066
sd	0.046	0.063	0.232	0.234	0.083

Tab. 3: Video-timing, accumulated positional, projected positional, and accumulated Doppler-GPS-speed measurements versus laser-gate-timing. Average (**av**) and standard deviations (**sd**) are shown for 10 runs (speeds 3.5 to 11.5knots, 19 to 5.9sec run-time).

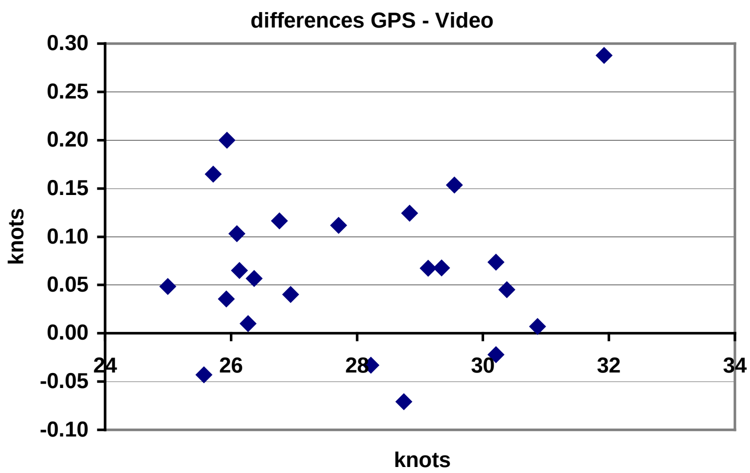

Fig. 9: Comparison between video-controlled speeds on a 500m course and projected positional speeds measured with Garmin Edge205 (1 sec logging rate, no grid-effect) during a speedsurfing competition in Fuerteventura 2006. Average deviation: 0.07knots, standard deviation of the average: +/-0.08knots (1σ).

Conclusion

The error margins of handheld-GPS measured speeds can be drastically improved by the use of Doppler-speed-based NMEA-data as compared to the traditionally used position-based evaluations. About one order of magnitude in accuracy can be gained bringing the Doppler-speeds into the confidence range of video-controlled measurements.

The latter suffer from problems with camera adjustment, exact course-length determination and operator-decisions during evaluation of the runs. The minimum error comes from the limited frame-rate of unsynchronized cameras and is in the order of +/- one frame (+/-40msec). For a 500m course an error of +/-0.066knots at 40knots speed (24.3sec run-time) can be estimated (not taking the other systematic error sources into account).

With accumulated Doppler-speed the errors (2σ=95.4% probability, Eq. 5) are in the order of +/-0.032knots! So even without taking directional effects in the error propagation into account, that would further reduce the error margins, a single NMEA-GPS-unit (e.g. a GT-11) can give better accuracy than the conventionally used video-systems. However, these speeds are integrated and averaged along a possibly curved trajectory and cannot be directly compared to a straight, video-controlled course where only start- and finish-transitions are observed and the really travelled distance is unknown.

GPS-units that store positional data only offer (about one order of magnitude) less accuracy for speed measurements and don't allow prove of the GPS-signal integrity. A severe grid-effect larger than 2m is also not acceptable since it is larger than the short time stability of the devices and dominates the error estimations.

However, positional and other data like number of tracked satellites, satellite-IDs and strength have to be monitored along with the Doppler-speed to increase the confidence in GPS-based measurements by excluding runs with satellite changes or too large HDoP-values. The GPS-evaluations could be constrained by the positional information to a certain competition area, so instead of a fixed 500m course, a slightly larger area could be marked, e.g. 550m * 100m taking the absolute positional GPS-errors into account and only in this area Doppler-speed evaluations are performed by the software.

References

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Appendix (24. May 2007)

Directional Doppler-speed

The averaged accumulated Doppler-speed evaluation described above offers one order of magnitude better accuracy than position (trackpoint) based speed measurements, but can be applied as an accumulative method only, integrating over the 1 second segments of the travelled path. Thus projected, distance based speed measurements (compare Fig. 1) are not possible since there is no positional information and a curved trajectory cannot be distinguished from a straight trajectory.

Traditional, video-controlled speed measurements measure the averaged ‘projected’ speed over a given, distinct distance (e.g. 500m), the really travelled distance (of the possibly curved path) cannot be measured and is of no relevance.

In order to have a comparable measure using the high accuracy Doppler data available from handheld consumer GPS, at least directional information is necessary. This information is contained in NMEA-data (GPRMC-sentence, resolution 0.01degrees) and also in binary data directly obtained from the SiRF chipset of the GT-11 ((8) message ID41: Course over Ground, COG, resolution 0.01degrees).

The scalar summation of the accumulated Doppler-speed is replaced by a vector-summation over consecutive 1-second pieces ($\mathbf{dt_i} \approx$ 1sec, Doppler-speed $\mathbf{ds_i}$, direction $\boldsymbol{\alpha_i}$ measured clockwise from the North-direction) of the whole trajectory resulting in the total projected distance $\mathbf{P_{DD}}$, the total time $\mathbf{T}$, and the average speed $\mathbf{S_{DD}}$ (compare Eq. 2):

$$\mathbf{P_{DD} = [(\Sigma dp_{ix})^2 + (\Sigma dp_{iy})^2]^{0.5}}$$
$$\text{with } \mathbf{dp_{ix} = ds_i} \sin \boldsymbol{(\alpha_i)}\ \mathbf{dt_i} \text{ and } \mathbf{dp_{iy} = ds_i} \cos \boldsymbol{(\alpha_i)}\ \mathbf{dt_i} \qquad \textbf{(6)}$$

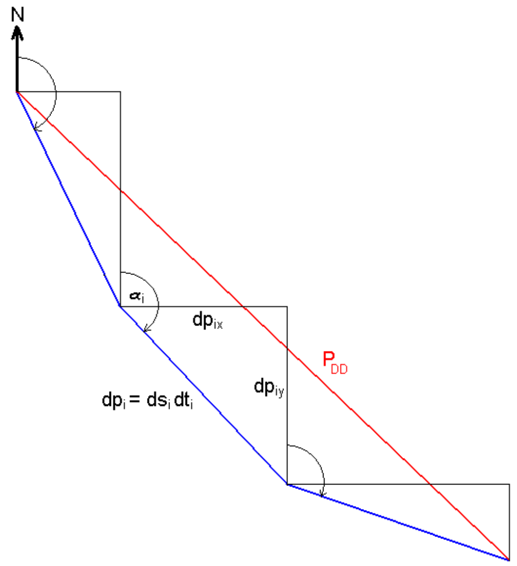

Fig. 10: Example of the vectorial sum of three path segments from measured Doppler-speeds $\mathbf{ds_i}$ and direction angles $\boldsymbol{\alpha_i}$.

By using the vectorial sum of the path-segments the projected travelled distance can be determined easily without using any less accurate positional information.

$$\mathbf{T = \Sigma dt_i} \qquad -> \qquad \mathbf{S_{DD} = P_{DD} / T}$$

Error Bounds

The accuracy of the Doppler-speed and the corresponding timestamps is very good (see above: <0.2knots per 1s segment). The vectorial sum of the segments is always smaller or equal than the scalar sum of the segments (if the segments are all on a straight line, both scalar and vectorial sums are equal, thus the overall accuracy is limited by the accuracy of the corresponding scalar (straight line) sum accuracy assuming error free angles. If the angles have errors, these errors will influence the length of the vectorial sum, however, the corresponding scalar sum still limits the projected length. Since the vectorial length for a given total time is always smaller than or equal to the accumulated length, the corresponding average projected speed is also always smaller than or equal to the average accumulated speed and thus limited by the latter measure.

Assuming that the mean Doppler-speed error spans the angle-range $\mathbf{d\alpha e_i} = \text{asin}(\mathbf{de_i}/\mathbf{ds_i})$, one can estimate the maximum angle error at a typical speed of $\mathbf{ds_i}$ = 40knots, $\mathbf{de_i}$ = 0.2knots as $\mathbf{d\alpha e_i}$ = 0.29 degrees corresponding to a relative projection length difference of 0.00125% with a mean value of 0.

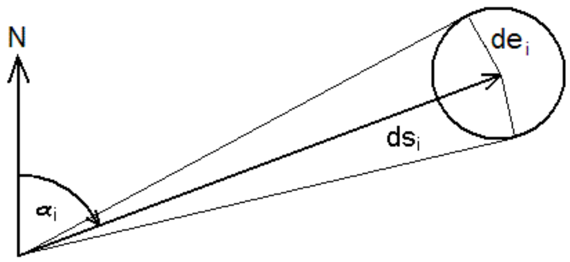

Fig. 11: Angle error estimation from measured Doppler-speed $\mathbf{ds_i}$, Doppler-speed error $\mathbf{de_i}$ and direction angle $\boldsymbol{\alpha_i}$.

The angle differences between neighbored segments are in general rather small (a few degrees maximum), thus the projected Doppler-speed errors are dominated by the accumulated Doppler-speed errors (in forward/backward direction) that are given by Eq. 4 and are thus in the same order as for the corresponding accumulated speed.

Conclusion

By using additional directional information and the corresponding Doppler-speeds projected Doppler-speed evaluations can be performed with the same accuracy as accumulated averaged Doppler-speeds over the same total run-time. Linear interpolation as described above to compute averaged speeds for a given projected distance (e.g. 500m) can be done with projected Doppler-speed as well. Thus highly accurate, projected distance speed measurements can be performed that can directly be compared to traditional video-timing measurements with available handheld consumer GPS devices like the GT-11 without using any less accurate positional data.

Reference

8. SiRF Binary Protocol Reference Manual

Appendix 2 (29. May 2007)

Error Propagation

In order to estimate the error propagation of the averaged projected Doppler-speed due to Doppler-speed- and -angle errors the partial derivatives of Eq. 6 have to be determined (time intervals $\mathbf{dt_i}$ are considered error-free):

$$\mathbf{P_{DD} = [X^2 + Y^2]^{0.5}} \quad \text{with} \quad \mathbf{X = \Sigma dp_{ix}} \quad \text{and} \quad \mathbf{Y = \Sigma dp_{iy}} \qquad \mathbf{(7)}$$

$$\mathbf{\delta X / \delta ds_i} = \sin\,(\alpha_i)\,\mathbf{dt_i} \quad \text{and} \quad \mathbf{\delta X / \delta \alpha_i = ds_i} \cos\,(\alpha_i)\,\mathbf{dt_i}$$

$$\mathbf{\delta Y / \delta ds_i} = \cos\,(\alpha_i)\,\mathbf{dt_i} \quad \text{and} \quad \mathbf{\delta Y / \delta \alpha_i = -ds_i} \sin\,(\alpha_i)\,\mathbf{dt_i} \qquad \mathbf{(8)}$$

For the total error of the accumulated distances $\Delta \mathbf{P_{DD}}$ follows:

$$\Delta \mathbf{P_{DD}}^2 = \mathbf{\Sigma[(\delta P_{DD} / \delta ds_i)^2\, \Delta ds_i^2 + (\delta P_{DD} / \delta \alpha_i)^2\, \Delta \alpha_i^2] =}$$
$$\mathbf{\Sigma[(X\, \delta X / \delta ds_i + Y\, \delta Y / \delta ds_i)^2\, \Delta ds_i^2 + (X\, \delta X / \delta \alpha_i + Y\, \delta Y / \delta \alpha_i)^2\, \Delta \alpha_i^2] / P_{DD}^2} \qquad \mathbf{(9)}$$

By a rotation (α_R) of the co-ordinate system (without loss of generality) such that the sum of all x-segments ($\mathbf{X = \Sigma dp_{ix}}$) is equal to 0, the expression in Eq. 9 can be simplified (the rotation does not change the total error of $\mathbf{P_{DD} = Y}$, α_i is replaced by $\beta_i = \alpha_i - \alpha_R$).

$$\Delta \mathbf{P_{DD}}^2 = \mathbf{\Sigma[(\delta Y / \delta ds_i)^2\, \Delta ds_i^2 + (\delta Y / \delta \alpha_i)^2\, \Delta \alpha_i^2] =}$$
$$\mathbf{\Sigma[}\cos^2\,(\beta_i)\,\mathbf{dt_i^2}\, \Delta \mathbf{ds_i^2 + ds_i^2} \sin^2\,(\beta_i)\,\mathbf{dt_i^2}\, \Delta \alpha_i^2] \qquad \mathbf{(10)}$$

If all Doppler-speed error $\Delta \mathbf{ds_i}$ are equal (= $\Delta \mathbf{ds}$), all angle errors $\Delta \alpha_i$ are equal (= $\Delta \alpha$), and all time intervals $\mathbf{dt_i}$ are equal (= $\mathbf{dt}$) Eq. 10 simplifies further to:

$$\Delta \mathbf{P_{DD}}^2 \approx \{\Delta \mathbf{ds}^2\, \Sigma \cos^2\,(\beta_i) + \Delta \alpha^2\, \mathbf{\Sigma[ds_i^2} \sin^2\,(\beta_i)]\}\, \mathbf{dt}^2 \qquad \mathbf{(11)}$$

$$\Delta \mathbf{P_{DD}} \approx \{\Delta \mathbf{ds}^2\, \Sigma \cos^2\,(\beta_i) + \Delta \alpha^2\, \mathbf{\Sigma[ds_i^2} \sin^2\,(\beta_i)]\}^{0.5}\, \mathbf{dt} \qquad \mathbf{(12)}$$

For the averaged speed error $\Delta \mathbf{S_{DD}}$ follows with $\Delta \mathbf{S_{DD}} = \Delta \mathbf{P_{DD} / (N\, dt)}$:

$$\Delta \mathbf{S_{DD}} \approx \{\Delta \mathbf{ds}^2\, \Sigma \cos^2\,(\beta_i) + \Delta \alpha^2\, \mathbf{\Sigma[ds_i^2} \sin^2\,(\beta_i)]\}^{0.5} / \mathbf{N} \qquad \mathbf{(13)}$$

A comparison to the error of accumulated Doppler-speed (special case with $\beta_i = 0$) without directional data shows the small influence of the angle-errors that can easily be calculated by Eq. 13

$$\Delta \mathbf{P_{DD}} \approx \Delta \mathbf{ds\, N^{0.5}\, dt} \qquad \Delta \mathbf{S_{DD}} \approx \Delta \mathbf{ds / N^{0.5}} \qquad \mathbf{(14)}$$

Example for a 500m run with N = 25 20m-segments ($\mathbf{ds_i}$ = 20m/s = 38.9knots), Doppler-speed-error $\Delta \mathbf{ds}$ = 0.2knots, angle-error $\Delta \alpha$ = 0.2° = 0.0035rad, β_i = (i - 13) 0.0175rad (bended run -12°…12°).

Error of accumulated Doppler-speed: $\Delta \mathbf{S_{DD}} \approx \Delta \mathbf{ds / N^{0.5} = 0.2 / 25^{0.5} = 0.2 / 5 = 0.04knots}$

Error of projected Doppler-speed ($\Sigma \cos^2\,(\beta_i) = 24.606$, $\Sigma[ds_i^2 \sin^2\,(\beta_i)] = 38.9^2{*}0.394$):

$$\Delta \mathbf{S_{DD}} \approx \{\Delta \mathbf{ds}^2\, \Sigma \cos^2\,(\beta_i) + \Delta \alpha^2\, \mathbf{\Sigma[ds_i^2} \sin^2\,(\beta_i)]\}^{0.5} / \mathbf{N} =$$
$$\mathbf{(0.2^2\; 24.606 + 0.0035^2\; 38.9^2\; 0.394)^{0.5}/25 = 0.0398knots}$$

This means that projected 'straight' Doppler-speed can be measured with about the same accuracy than accumulated 'curved' Doppler-speed.