On the relativistic Doppler Effects and high accuracy velocity determination using GPS

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Presented at GNSS 2004
The 2004 International Symposium on GNSS/GPS

Sydney, Australia
6–8 December 2004

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ABSTRACT

The NAVSTAR Global Positioning System (GPS) is a satellite-based radionavigation system designed for positioning, velocity determination and timing (PVT). The relative movement between a GPS receiver and a GPS satellite causes the received signal frequency to differ from its nominal frequency due to Doppler effects. This frequency difference, an observable in GPS measurements, is referred to as the Doppler shift. However, the Doppler shift is biased by the inherent error in the receiver clock. Since velocities of GPS satellites are known, the velocity of a user can be determined through observing four or more satellites, similar to GPS positioning.

When the GPS selective availability (SA) was activated, errors imposed on the ephemeris and satellite clocks significantly affected the accuracy of both GPS positioning and velocity determination. The removal of SA has enabled a significant improvement on the accuracy of both GPS satellite orbits and clocks. As a result, velocity accuracy at sub-centimeter per second level is achievable for standalone GPS users if all the errors associated with the Doppler observables are corrected for properly.

This paper overviews the developments in precise velocity determination and investigates all error sources associated with the Doppler measurement. The properties of errors in the Doppler shift are analysed and the methods to eliminate or mitigate these errors are discussed. The relativistic errors such as the orbit eccentricity and Sagnac effects are derived and formulated in easy forms for correction. Algorithms for modeling the atmosphere delay rates are also presented. A satellite Earth-Centered-Earth-Fixed (ECEF) velocity determination method is also proposed for real-time applications using the broadcast ephemeris. It is concluded that real-time velocities in millimeters per second level are achievable if the error correction schemes provided are used.

KEYWORDS: GPS velocity determination, Doppler shift, Doppler Effect, relativistic corrections, atmosphere delay rates

1. INTRODUCTION

The GPS System is designed for and operated by the US military as a radio navigation system to provide a high accuracy service of positioning, velocity determination and timing (PVT). Although the system has capabilities to provide precise real-time velocities, due to the deployment of Selective Availability (SA) which was an intentional accuracy degradation policy of the US government prior to May, 2000, and limitations of the receiver hardware, most methods were based on differential techniques to account for the errors introduced in the satellite orbits and onboard GPS clocks. With the SA off, ±0.2m/sec per axis (95%) accuracy is guaranteed by the GPS system (DoD 1996).

For precise velocity determination, GPS satellites are taken as space moving signal sources from which a GPS receiver senses the Doppler shifts that are generated due to the relative motions between the observed satellites and the receiver. The ground velocity of the receiver can then be determined if Doppler shifts can be measured from more than four GPS satellites.

Research groups at the Department of Geometrics Engineering, University of Calgary, Canada worked actively in the 1990's on precise velocity determination. Fenton and Townsend (1994) derived phase velocity, and phase acceleration from the carrier phase measurements of NovAtel GPS Card™ receivers, and ±0.28mm/s was achieved in a 7 km static baseline. Hebert et al. (1997) and Cannon et al. (1998) conducted an in-depth analysis on GPS velocity determination using a GPS signal simulator. Velocities with up to ±2mm/s 3D RMS accuracy were achieved under low dynamics (acceleration<0.5m/s^2). Szarmes et al. (1997) presented the results of a series of differential GPS tests conducted for the purpose of high accuracy aircraft velocity determination. Compared with velocities obtained through receiver generated raw Doppler measurements and the carrier phase derived 'precise' Doppler, it was concluded that under constant velocity or low acceleration conditions, the accuracy of the raw Doppler derived velocity estimates is at least as good as the velocity from the first-order central difference approximation of the carrier phase. Note that all the methods referred to above were based on differential techniques using reference receivers.

It is the removal of SA that allows a significant velocity accuracy improvement. Without SA, static velocity accuracy of ±3~5cm/s 3D RMS first appeared in standalone mode (Misra and Enge 2001). Zhang et al. (2003) conducted a comparison of real-time velocities obtained from an inexpensive code-only GPS receiver with the velocities from a Trimble 5700 RTK system and reported that ±3cm/s accuracy was achieved using the code-only GPS receiver in either static or dynamic mode. Zhang et al. (2003a) demonstrated that the same accuracy level had been achieved using an inexpensive 1 Hz GPS receiver as with a 10Hz sampling rate in standalone mode.

Graas and Soloview (2003) showed that sub-centimetre per second velocity accuracy is achievable whether in static or dynamic, stand-alone or relative mode; what really matters is the receiver quality. Serrano et al (2004) reported that by employing the first-order central difference approximation of the carrier phase, better than ±1cm/s (2-sigma) under high-multipath conditions can be achieved from a low-cost GPS receiver. Based upon post processing of the kinematic data, they reported that phase measurements were degraded in the moving environment and there was bias in both static and kinematic results.

Perhaps the highest velocity published so far in standalone mode is from Septentrio's PolaRx2® receiver; ±1.5mm/s horizontal and ±2.8mm/s vertical precision is stated in the product specifications (PolaRx2 2004). This dual frequency GPS receiver achieves such a high accuracy through introduction of a special scheme which accounts for the change rate of tropospheric delay (Simsky and Boon 2003).

The benefits of obtaining precise velocity from GPS in standalone mode are significant. Kinematic users may have a better dynamic model that could lead to a significant improvement on positioning; RTK users could be able to work in centimetre level for a certain time period when data links are down, and users who need attitude information might obtain it just using a single receiver (Farrell and Barth 1999).

It is well known that GPS observables have errors and biases that are characterized as having correlations spatially and temporally. Differential technique is the best approach to account for the correlated errors and biases; however, it is at the cost of introducing additional GPS receivers and losing useful information contained in one-way observations. In order to achieve high accuracy in standalone mode, all errors associated with the observations are required to be well modelled and be properly accounted for.

From the outset this paper relates a user velocity to the measured GPS Doppler shift, to formulate the precise observation model for velocity determination. By identifying all inherent significant error sources in the carrier phase observable, the corresponding errors associated with the Doppler shift observable are able to be analysed. Since the carrier phase measurement is the integrated Doppler measurements over time, and the inherent errors in carrier phase are well documented, (i.e. Kouba and Héroux 2001), the corrections for the Doppler measurement are then derived by getting the change rate of the raw errors in the carrier phase. The formulae derived include satellite clock rate correction, relativistic delay rate corrections, ionosphere delay rate and troposphere delay rate corrections. In addition, the satellite velocity calculation from the broadcast ephemeris is analysed for real-time applications.

It is concluded that a GPS user may be able to achieve millimetres per second level high accuracy in real time provided the errors and biases have been careful treated, and good quality Doppler shift measurements are available.

2. DOPPLER EFFECT VERSUS VELOCITY

Doppler Effect is the apparent change in frequency or wavelength of a wave that is perceived by an observer experiencing relative motion to the source of the waves. Because a satellite orbits the Earth at a high speed (approximately 3.8 Km/s), transmitting electromagnetic wave signals that propagate at the speed of light, Einstein's relativity theory applies to GPS observations. Accordingly the received signal frequency can be expressed as (Wells et al. 1987; Seeber 1993)

$$f_r = f_s \cdot \frac{(1 - \frac{\dot{\rho}_r}{c})}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

where

- $\dot{\rho}_r$ is the range rate of the receiver and satellite pair
- v is the velocity of the satellite
- c is the speed of light in vacuum

Equation (1) can be expanded using binomial series as

$$f_r = f_s \cdot (1 - \frac{\dot{\rho}_r}{c}) \cdot (1 + \frac{v^2}{2c^2} + \frac{v^4}{8c^4} + \cdots) \tag{2}$$

when the higher order terms are neglected, Doppler shift, i.e. the frequency difference between the received frequency and the transmitted frequency, becomes

$$D \equiv f_r - f_s = -\frac{\dot{\rho}_r}{c} f_s = -\frac{1}{\lambda} \cdot \dot{\rho}_r \tag{3}$$

Equation (3) relates a Doppler shift measurement to the GPS line-of-sight geometric range rate. When a GPS satellite approaching towards a user, the range rate is negative thus the Doppler shift is positive, while a departing satellite has a positive range velocity that yields a negative Doppler shift.

Among the neglected terms in equation (2), only the second-order term may have some numerical effects on the frequency reception. Seeber (1993) terms this as the “transversal” Doppler effect. The magnitude of this effect may be evaluated as follows

$$\Delta f_r \equiv f_s \cdot (1 - \frac{\dot{\rho}_r}{c}) \cdot \frac{v^2}{2c^2} = \frac{f_s v^2}{2c^2} - \frac{f_s \dot{\rho}_r v^2}{2c^3} \approx \frac{1}{2\lambda} \cdot \frac{v^2}{c} \tag{4}$$

3. ERROR ANALYSIS/MODELLING OF DOPPLER MEASUREMENTS

Most errors associated with the carrier phase measurement can be cancelled out through double difference scheme. However, in order to get a precise position in standalone mode, it is required to properly model and account for each error.

It is well known that the instantaneous frequency f is the derivation of the phase φ with respect to time, and the GPS phase observable φ is obtained through an integration of the frequency f over time. Since the errors imposed on the carrier phase measurement have been intensively investigated, especially in the category of precise point positioning (PPP), we can take advantages of the error models used in PPP to analyse the corresponding corrections for Doppler measurements. The methodology is simply to differentiate those delays in the carrier phase measurements with respect to time; the delay rate models can then be established for Doppler measurements.

We begin with the most precise observation equation for the carrier phase measurement given by Teunissen and Kleusberg (1998)

$$\lambda_i \varphi_{r,i}^s(t) = \left\| \vec{\rho}^s(t-\tau_r^s) + d\vec{\rho}^s(t-\tau_r^s)) - (\vec{\rho}_r(t) + d\vec{\rho}_r(t)) \right\| - dI_{r,i}^s + dT_r^s - \lambda_i \varphi_{r,i}^s(t_0) - \lambda_i N_{r,i}^s$$
$$c \cdot dt_r(t) - c \cdot dt^s(t-\tau_r^s) + dM_r^s + dR_r^s + \varepsilon_r^s \qquad (5)$$

where:

- i in the subscripts designates frequency band, '1' for L1 and '2 'for L2 respectively
- s superscript stands for satellite s
- r subscript represents receiver r
- t is the measuring epoch in GPS time
- $\|\bullet\|$ double vertical bars indicate the length (norm) of a vector
- $d\vec{\rho}^s$, $d\vec{\rho}_r$ are the satellite positional error and user site displacement respectively
- τ_r^s is the signal propagation time from satellite s to receiver r
- dI, dT, dM, dt_r, dt^s, dR are the ionospheric delay, tropospheric delay, multipath delay, receiver clock delay, satellite clock delay, and the relativistic error respectively
- $\varphi_{r,i}^s(t_0)$ is the initial phase biases for the receiver and satellite pair
- $N_{r,i}^s$ stands for the integer ambiguity

With the carrier phase observation equation (5) and equation (3), we have the Doppler observation equation scaled into range rate as follows

$$\lambda_i D_{r,i}^s(t) <=> \lambda_i \dot{\varphi}_{r,i}^s(t) = \left\| (\dot{\vec{\rho}}^s(t-\tau_r^s) + d\dot{\vec{\rho}}^s(t-\tau_r^s)) - (\dot{\vec{\rho}}_r(t) + d\dot{\vec{\rho}}_r(t)) \right\| - d\dot{I}_{r,i}^s + d\dot{T}_r^s +$$
$$c \cdot d\dot{t}_r(t) - c \cdot d\dot{t}^s(t-\tau_r^s) + d\dot{M}_r^s + d\dot{R}_r^s + \varepsilon_r^s \qquad (6)$$

where a dot over a variable denotes the first derivative of the variable with respect to time. Note that the Doppler measurement has a negative sign when the receiver-satellite pair is approaching each other and a positive sign when departing from each other.

It can be seen that all errors imposed on the carrier phase would affect the Doppler shift measurement accordingly, but through their change rates. Since errors such as those in the troposphere and ionosphere, change slowly with time, the Doppler measurement is cleaner than the carrier phase. The errors inherent in the carrier phase have been well documented. Following the error budgets presented by Kouba and Héroux (2001), we will analyse the error terms based on Equation (6).

3.1. Errors inside the Double Vertical Bars

3.1.1. Satellite velocity in ECEF

Real-time GPS users rely on the broadcast ephemeris to calculate satellite position which has been well documented in ICD-GPS-200c (ARINC 2000). With SA off, the positional accuracy is about ±1~5m. According to ICD-GPS-200c, the satellite orbital system (ICD system) is defined relative to the right ascending node rather than a 'nominal' system which is with respect to the perigee. Adopting the ICD system has some advantages. Firstly it only requires two rotations to transfer an orbital position into the ECEF system thus the algorithms are very effective. Secondly, the six harmonic perturbation parameters provided are with respect to the ascending node thus there saved one parameter, i.e. the change rate of the argument of perigee. This therefore alleviates the navigation payload. However, due to the lack of such a parameter, any analytical rotation method may only achieve centimetre per second level accuracy in the calculated ECEF satellite velocity. This will be discussed in a separate paper. Readers who have interests may test this by downloading a program named Skyplot under GPSTools at the NGS website (Hilla and Adams 2001).

Zhang et al. (2003a) proposed a position differential method that uses the first-order central difference approximation of Taylor series. Since the errors in the broadcast ECEF satellite position are not Gaussian white, but most likely behave as an integrated Gauss-Markov process with some systematic biases, this simple position differential scheme works well.

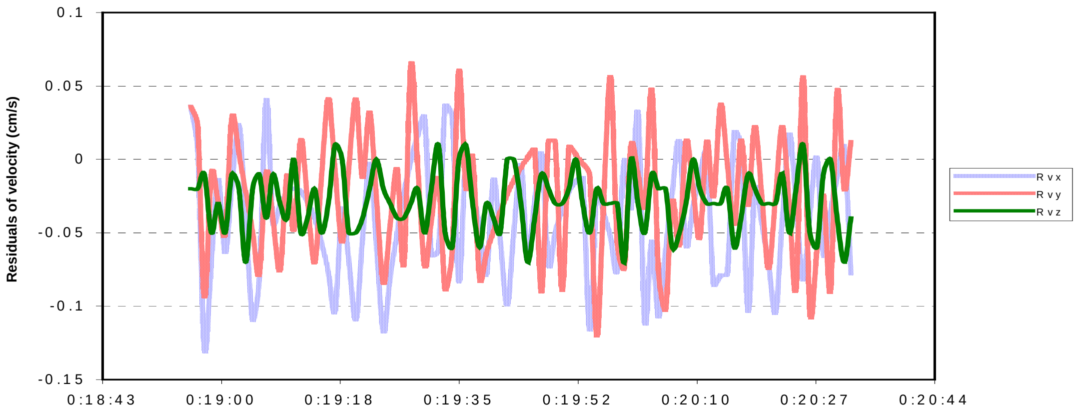

Figure 1: 3-axis residual vectors of the ECEF velocities by differentiation of the broadcast positions (PRN=07, 22:18:56~22:20:32, 08/20/2002, GPS time)

The precise ephemeris from the International GPS Service (IGS) is defined with reference to the centre of mass of a satellite; it has an offset when compared with the broadcast ephemeris that is based on the antenna phase centre. However velocities from both the IGS and the broadcast ephemeris can be directly compared with each other because the constant offset will be cancelled out in the derivation of velocity. Taking IGS velocities as "ground truth", the broadcast velocity precision is well within ±1mm/s per axis; this can be seen from Figure 1. For real-time users who use polynomial interpolation to calculate positions, it is proposed that the velocity can be analytically differentiated from the position coefficients.

It is suggested that ± 1mm/s per axis ECEF satellite velocity can be obtained if a satellite position differential scheme from the broadcast ephemeris is used.

3.1.2. Antenna phase wind-up effect

This effect involves both antennas in a receiver-satellite pair. The relative orientation of the satellite and receiver antennas may affect GPS range measurements because the GPS signal is a right circularly polarized radio wave (Wu et al. 1993). Any rotation of either receiver or satellite antenna around its bore axis would affect the carrier phase measurement. The maximum effect may reach one cycle. For a signal with an angular frequency ω and a receiver spinning rapidly with an angular frequency Ω which is parallel to the propagation direction of the signal, the antenna and signal electric field vector rotate in opposite directions. Hence the received frequency will be $\omega + \Omega$, which is an additional Doppler shift (Ashby 2003). However, due to the high orbital movement of GPS, this shift is too small for a GPS receiver to sense unless deliberately mounting the GPS receiver in a fast spinning environment.

Therefore it has no effect on Doppler measurements in most cases.

3.1.3. Receiver site displacement

The site displacement due to ocean load, air load, solid earth tide, and polar motion etc., changes relatively slowly and therefore they have little effect on the instantaneous Doppler measurements.

3.2. Atmospheric Delay Rate

3.2.1. Ionosphere

For a dual frequency geodetic GPS receiver capable of outputting Doppler shift measurements on both frequency bands, the ionosphere free linearized Doppler observable can be formed using the same coefficients applied in the ionosphere-free carrier phase observable, i.e. (p139, Rothacher et al. 1996)

$$D_{ion-free} \equiv \frac{f_1^2}{f_1^2 - f_2^2} D_1\lambda_1 - \frac{f_2^2}{f_1^2 - f_2^2} D_2\lambda_2 \approx 2.5457 D_1\lambda_1 - 1.5457 D_2\lambda_2 \tag{7}$$

For those dual frequency receivers which have Doppler measurements in L1 frequency only, such as the Trimble 5700 GPS receivers, the ionospheric delay in L1 can be precisely measured through the geometry-free carrier phase observable (Langley 1998). This delay, however, is biased by a constant due to the ambiguities, i.e.

$$\begin{aligned} dI_1 &= \frac{f_2^2}{f_1^2 - f_2^2}\left[(\lambda_1 N_1 - \lambda_2 N_2) + (\lambda_1\varphi_1 - \lambda_2\varphi_2)\right] + \varepsilon \approx 1.5456\left[(\lambda_1 N_1 - \lambda_2 N_2) + (\lambda_1\varphi_1 - \lambda_2\varphi_2)\right] \\ &= 1.5456 \cdot [L_g + (\lambda_1 N_1 - \lambda_2 N_2)] \end{aligned} \tag{8}$$

where L_g is the geometry-free linear combination of L1 and L2 carrier phase observable. Thus the ionospheric correction for Doppler measurements may be obtained by differentiating the above equation with respect to time. This leads to

$$d\dot{I}_1 = 1.5456 \cdot \dot{L}_g \tag{9}$$

Figure 2 shows a $1.5456L_g$ observable time series of 200 seconds, with the satellite elevation angle changing from 20.8º~19.6º. It can be clearly seen that the ionosphere delay in a one-way observation changes smoothly and linearly. There is about –5.0mm/s ionospheric rate correction for the Doppler shift measurements.

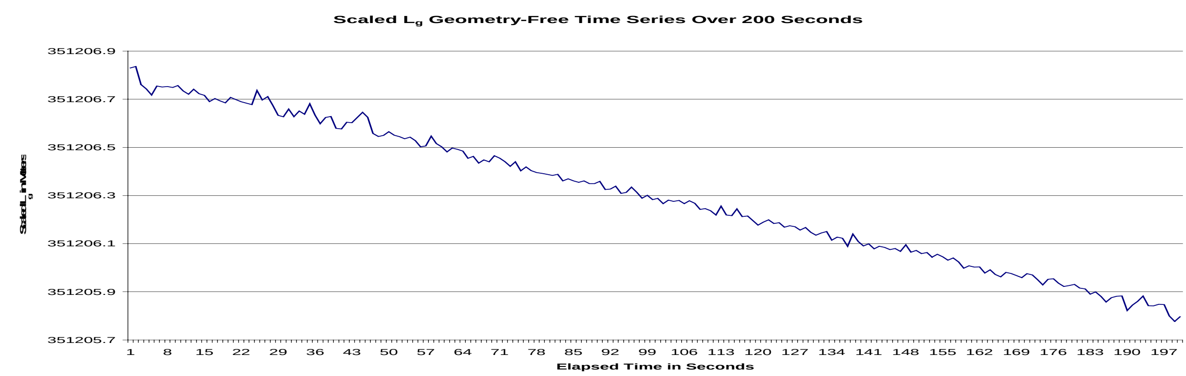

Figure 2: A 1.5456 L_g ‘observable’ series of 200 seconds (elevation 20.8º~19.6 º)

Therefore, the ionospheric correction is critical for millimetres per second level velocity determination. Appropriate correction methods for single frequency users are still under investigation. Without better alternatives, it is preferable to use the ionospheric delay rate derived from the Klobuchar's model (Klobuchar 1987; ARINC 2000).

3.2.2. Troposphere

Most textbooks suggest that the contribution of the troposphere to the Doppler measurement is so small that it can be neglected. However, according to recent study reported by Simsky and Boon (2003), not properly accounting for the tropospheric delay rate would result in nearly centimetre per second level noise and significant bias.

A tropospheric delay model for range measurement correction consists of a hydrostatic and a non-hydrostatic component. The delay can be modelled with a zenith delay model and a mapping function, in the form of

$$dT = dT_{dry}^{zenith} \cdot m_{dry} + dT_{wet}^{zenith} \cdot m_{wet} \tag{10}$$

The dry component contributes 90% of the total tropospheric delay while the wet component contributes only 10%. The tropospheric delays in the zenith direction, according to Kouba and Héroux (2001), vary in time by a relatively small amount, in the order of a few centimetres per hour thus can be viewed as a constant for Doppler measurement. Hence it is sufficient to choose a standard model such as the Saastamoinen model (Saastamoinen 1972) by using standard meteorology data to represent the zenith delays. Then what really matters is the adoption of mapping functions.

Ordinary mapping functions in simple cosecant E (elevation) forms are unable to reflect the tropospheric change rate because the elevation angle changes very slowly at a rate of ±0.1millirad/sec, due to the slow change of satellite geometry (Simsky and Boon 2003). This requires that the precise geodetic mapping functions should be used for the tropospheric delay rate modelling.

The global mapping function developed by Niell (1996) may be used to model the tropospheric delay rate. This model is precise even when the elevation angle is down to 3º~5º; however it is complicated in calculation. In kinematic applications, an elevation cut-off angle of 15º is normally set. Up to this elevation, other geodetic mapping functions such as Chao's mapping functions (Chao 1974) may be considered:

$$\begin{aligned} m_d(E) &= \frac{1}{\sin E + \dfrac{0.00143}{\tan E + 0.0445}} \\ m_w(E) &= \frac{1}{\sin E + \dfrac{0.00035}{\tan E + 0.017}} \end{aligned} \tag{11}$$

The merits of Chao's formulae are simplicity in form and independence of location and height. Since no meteorological data is required, these mapping functions are suitable for kinematic applications.

One may numerically get the tropospheric delay change rate after applying tropospheric correction to the range measurement, through a simple differentiator. That is, to differentiate the calculated tropospheric delays with respect to time

$$d\dot{T} = \frac{d}{dt}(dT) \tag{12}$$

This avoids the computation of the bulky expressions of the derivatives of the mapping functions, see Simsky and Boon (2003). As the tropospheric delay changes slowly, a low pass differentiator is sufficient for this purpose.

Figure 3 illustrates the tropospheric delay rate from Chao's model. It clearly shows the importance of troposphere modeling because even in higher elevation angles there are still contributions in millimeters per second level to the Doppler measurements. Comparing with the tropospheric delay rate from the model that Simsky and Boon used, (Figure 3 in the above reference), differences between the models are apparent, thus there is a trade-off in choosing tropospheric mapping functions.

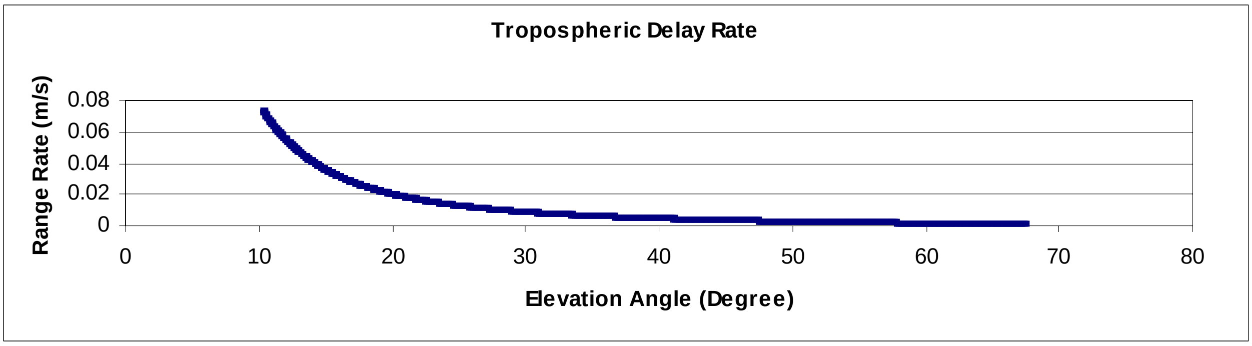

Figure 3: Tropospheric delay rate from Chao's model

3.3. Clock Rate Corrections

3.3.1. Receiver clock

Similar to navigational point positioning, the clock rate becomes the fourth unknown and is solved along with the receiver velocity unknowns if more than four Doppler measurements are observed. A GPS receiver clock is normally made of quartz crystal oscillator, thus the clock rate stability depends on the quality of receiver crystal oscillator. The errors introduced through the receiver oscillator affect the quality of measured Doppler shifts, and therefore degrade the velocity estimations.

With the advancement of technology, many oven-controlled-crystal-oscillators (OCXO) with superior short-term stability are in the market with low prices. Some OCXOs have better stability than atomic oscillators over short time, see Chandelier et al. (1999) and Ashby

(2003). Receivers equipped with such good oscillators will bring high quality Doppler measurements that eventually lead to ultra high precision velocity estimations.

3.3.2. Satellite clock

The algorithm to correct satellite clock errors is given in ICD-GPS-200c (ARINC 2000). Accordingly, a user corrects the time received from a satellite with the following equation in seconds

$$t = t^s - dt^s \tag{13}$$

where:

- t is the true GPS time in seconds
- t^s represents the effective PRN code phase time at the transmission epoch
- dt^s is the SV PRN code phase time offset in seconds

The satellite PRN code phase offset time is given by

$$dt^s = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2 \tag{14}$$

where:

- a_{f0}, a_{f1}, a_{f2} are the polynomial coefficients in sub-frame one of the navigation message
- t_{oc} represents the clock data reference time in GPS seconds

From Equation (14), the satellite clock rate correction for Doppler measurement can be expressed as

$$d\dot{t} = a_{f1} + 2 \cdot a_{f2}(t - t_{oc}) \tag{15}$$

Another error associated with the satellite clock is the satellite group delay T_{gd}, which is due to the satellite hardware bias. This would be cancelled out by differentiation with respect to time. Hence the satellite group delay has no effect on Doppler measurements.

3.4. Multipath

Thus far, we haven't observed significant multipath effects on our velocity determinations; even though some of our field trials were conducted on water surface that is thought to be a high multipath environment. Nor have others reported significant deterioration on velocity determination due to multipath. According to Serrano et al (2004), ±1cm/s velocity accuracy was achieved despite multipath-rich conditions.

Theoretically the multipath effect reaches a maximum of one quarter of a cycle for the phase observable, and changes periodically (Hofmann-Wellenhof et al. 2001). This may indicate that the multipath errors associated with the Doppler would have been averaged out, since the Doppler is the first derivative of the carrier phase. However, further research is needed before making a decisive conclusion.

3.5. Relativistic Corrections

The special and general relativistic effects on onboard GPS satellite clocks due to the motion in circular orbits have been accounted for by the GPS system. This is implemented by shifting the system fundamental frequency of 10.23 MHz to 10.229 999 999 543MHz, i.e. -0.0004567 Hz correction (Ashby and Spilker 1996). In the following relativistic corrections for satellite clocks, we will scale all of them into distance in metres. Therefore the following corrections given for the Doppler shift measurement are in the unit of meters per second.

3.5.1. Orbit eccentricity correction

The slight eccentricity of the satellite orbit causes a periodic relativistic effect on the satellite clocks. In ICD-GPS-200c, this relativistic correction is calculated and corrected for as an additional term in the clock corrections by GPS users (ARINC 2000). The scaled eccentricity correction can be expressed as follows

$$\delta S_{eccentricity} = \frac{2\sqrt{GM}}{c} e \cdot \sqrt{a} \cdot \sin E_k \tag{16}$$

where :

- GM=3.986005e14, is the Earth's universal gravitational parameter
- e represents the eccentricity of the satellite orbit
- a is the semi-major axis of the satellite orbit
- E_k is the orbital eccentric anomaly

The relativistic effect correction is normally obtained after solving the Keplerian equation. However, its equivalent form expressed in terms of satellite position and velocity (ibis)

$$\delta S_{eccentricity} = \frac{2}{c} \cdot \vec{r}^s \bullet \dot{\vec{r}}^s \tag{17}$$

can be more conveniently used to derive corrections for Doppler. Differentiating the above equation with respect to time leads to

$$\delta D_{eccentricity} = \frac{2}{c} \cdot [(\dot{\vec{r}}^s)^2 + \vec{r}^s \cdot \ddot{\vec{r}}^s] \tag{18}$$

where $\ddot{\vec{r}}^s$ is the satellite acceleration in ECEF which can be calculated according to the close formula given by Zhang et al (2003b) from the broadcast ephemeris. An alternative is to calculate it using the following formula

$$\delta D_{eccentricity} = \frac{2GM}{c} \left(\frac{1}{a_{orb}} - \frac{1}{\|\vec{r}^s\|}\right) \tag{19}$$

The contribution of the orbit eccentricity to Doppler measurements, according to Equation (18) may reach some centimetres per second thus must be corrected for.

3.5.2. Earth rotation correction (Sagnac effect)

The rotation of the earth during the GPS signal propagation period causes another relativistic error that is known as the Sagnac effect (Ashby and Spilker 1996; Leva et al. 1996) which causes the incoming signal to have an extra travelling distance.

The Sagnac effect correction given by Ashby and Spilker (1996) is

$$\delta S_{Sagnac} = \frac{2\Omega_e \cdot A}{c} \tag{20}$$

where $\Omega_e = (0,0,\omega_e)^T$ is the angular velocity vector of the Earth, and A is the shading area of the triangles swept out by an arrow with its tail at the earth centre and its head following the electromagnetic signals. The Sagnac correction for range measurements can be calculated by

$$\delta S_{Sagnac} = \frac{2\Omega_e^{\,T}}{c} \cdot \frac{\vec{r}_r \times \vec{r}^s}{2} = \frac{\omega_e}{c}[x^s(y^s - y_r) - y^s(x^s - x_r)] = \frac{\omega_e}{c}(y^s x_r - x^s y_r) \tag{21}$$

which coincides with the aberration correction given by Seeber (1993).We can then get the Sagnac correction for Doppler by differentiation with respect to time again

$$\delta D_{Sagnac} = \frac{\Omega_e^{\,T}}{c} \cdot \frac{d}{dt}(\vec{r}_r \times \vec{r}^s) = \frac{\omega_e}{c}[\dot{x}_r y^s - \dot{y}_r x^s + x_r \cdot \dot{y}^s - y_r \dot{x}^s] \tag{22}$$

This correction is generally very small, but the maximum may reach several millimetres per second and therefore requires consideration.

3.5.3. Secondary Relativistic Effects

Several additional relativistic effects should be taken into consideration when analysing the measurements from the Very Long Baseline Interferometry (VLBI), Satellite Laser Ranging (SLR), and GPS. These include the signal propagation delay, effect on geodetic distance, and effects of other solar system bodies. Ashby (2003) classifies these relativistic errors as secondary. The contributions from these secondary effects to the frequency shift would be very small and therefore can be neglected.

4. NUMERICAL ANALYSIS

Numerical analysis of each of the errors discussed has been carried out in this research, and the results are summarised in Table (1).

	Error Terms	Modelling in Doppler Measurements	Magnitude Estimated
Satellite Orbit	Broadcast Ephemeris	Yes	±1mm/s per axis
Satellite Clock	Satellite Clock Correction	Yes	negligible
	L1-L2 Correction	No	negligible
Relativity	Orbit Eccentricity	Yes	several cm/s
	Sagnac	Yes/No**	several mm/s
	Secondary Relativistic Effects	No	negligible
Atmosphere	Ionospheric Correction	Yes	mm/s to cm/s
	Tropospheric Correction	Yes	mm/s to cm/s
Receiver	Receiver Site Displacement	No	
	Receiver Clock	As an unknown to be estimated	

****:** whether or not to apply Sagnac correction depends on different treatments of the earth rotation

Table 1: Main error sources in Doppler measurements

It can be seen that the relativity is the major error source for the Doppler shift measurements; fortunately, it can be well modelled and corrected for. The atmosphere effects are relatively small, however, due to their unpredictable nature; they are hard to model and therefore critical to sub-centimetre per second level velocity determination.

5. CONCLUDING REMARKS

This paper overviews the development of precise velocity determination. Based on a highly accurate observation model and the error modelling methods used in precise point positioning, a comprehensive error analysis has been presented and many correction formulae derived. These corrections contribute accuracy improvement on ground velocity determination in either static or dynamic situations.

The discussion of the ECEF satellite velocity determination using the broadcast ephemeris allows a user to be able to determine precise velocity in real time. The proposed polynomial orbit interpolation scheme makes it feasible to get accurate velocities at high output rate.

Although this paper highlights real-time issues where the relativistic errors appear trivial, for post processing of data using the IGS precise ephemeris and clock product, the explicit relativistic correction formulae presented in this paper would benefit those who are interested in seeking the ultimate velocity accuracy that GPS can provide.

The ionospheric delay rate correction is provided for dual frequency GPS receivers, by either forming the ionosphere-free Doppler 'observable' or getting the first derivative of the geometry-free carrier phase 'observables'. More appropriate methods than using the broadcast model for single frequency users are still under investigation.

The tropospheric delay rate correction has been identified so as to find proper mapping functions that may best represent the change of tropospheric delay along signal profiles. This requires precise geodetic mapping functions to be used. Since both ionospheric and tropospheric delay rates contribute several millimetres per second level errors to Doppler measurements, and since their changes are hard to predict and to model, the atmosphere becomes the biggest error source degrading the velocity accuracy.

If all the errors inherent in Doppler measurement have been properly accounted for, millimetres per second levels of velocity accuracy is achievable in real time for standalone GPS users, and much higher accuracy can be achieved in post processing mode using IGS products. However, this depends on the quality of the available Doppler shift measurements, which will be discussed in a separate paper.

ACKNOWLEDGEMENTS: The research presented in this paper was partially supported by the Cooperative Research Centre (CRC) for microTechnology, Australia.

REFERENCES

ARINC (2000). ***Interface Control Document, Navstar GPS Space Segment/Navigation User Interfaces, ICD-GPS-200, Revision C***. Arinc Research Corporation. http://www.navcen.scg.mil/gps.

Ashby, N. (2003). ***Relativity in the Global Positioning System***. Living Rev. Relativity 6(Online Article http://www.livingreviews.org/Articles/Volume6/2003-ashby/).

Ashby, N. and J. J. Spilker (1996). ***Introduction to Relativistic Effects on the Global Positioning System***. B. W. Parkinson. Global Positioning System: Theory and Applications, AIAA, Inc. I: 623-698.

Cannon, M. E., G. Lachapelle, M. C. Szarmes, J. M. Hebert, J. Keith and S. Jokerst (1998). ***DGPS Kinematic Carrier Phase Signal Simulation Analysis for Precise Velocity and Position Determination***. Journal of the Institute of Navigation 44(2): 231-245.

Chandelier, V., G. Marotel, D. Thorax and C. Trialoup (1999). ***New Miniature Ultra-Stable Oscillators***. Proceedings of the 1999 joint meeting of the European frequency and time forum, and the IEEE international frequency control symposium, Besancon, France: 383-388.

Chao, C. C. (1974). ***The Tropospheric Calibration Model for Mariner Mars 1971***. Jet Propulsion Laboratory. Pasadena, CA, USA: 61-76.

DoD (1996). ***NAVSTAR GPS User Equipment Introduction***. Department of Defense, USA. www.spacecom.af.mil/usspace/gps_support/documents/gpsuser.pdf.

Farrell, J. A. and M. Barth (1999). ***The Global Positioning System & Inertial Navigation***, McGraw-Hill. ISBN: 0-07-022045-X.

Fenton, P. and B. Townsend (1994). ***NowAtel Communications Ltd,-What's New?*** Proceedings of KIS'94, Banff, AB, CA: 25-29.

Graas, F. V. and A. Soloview (2003). ***Precise Velocity Estimation Using a Stand-Alone GPS Receiver***. Proceedings of ION NTM 2003, Anaheim, California, USA.

Hebert, C. J., J. Keith, S. Ryan, M. Szarmes, G. Lachapelle and M. E. Canon (1997). ***DGPS Kinematic Carrier Phase Signal Simulation Analysis for Precise Aircraft Velocity Determination***. Proceedings of the ION GPS-97, Albuquerque, NM.

Hilla, S. and G. Adams (2001). ***The GPS Toolbox***. GPS Solutions 5(1): 65-67.

Hofmann-Wellenhof, B., H. Lichtenegger and J. Collins (2001). ***Global Positioning System Theory and Practice***, Springer-Verlag/Wien New York. ISBN: 3-211-83534-2.

Klobuchar, J. A. (1987). ***Ionospheric Time-delay Algorithm for Single-Frequency GPS User***. IEEE Transactions on Aerospace and Electronic Systems AES-23: 325-331.

Kouba, J. and P. Héroux (2001). ***Precise Point Positioning Using IGS Orbit and Clock Products***. GPS Solutions 5(2): 12-28.

Langley, R. B. (1998). ***Propagation of the GPS Singals***. T. P.J.G and A.Kleusberg. GPS for Geodesy, Springer: 111-150.

Leva, J. L., M. U. d. Haag and K. V. Dyke (1996). ***Performance of Standalone GPS***. E. D. Kaplan. Understanding GPS: Principles and Applications. Boston, London, Artech House: 237-320.

Misra, P. and P. Enge (2001). ***Global Positioning System: Signals, Measurements, and Performance***. Lincoln, Massachusetts, Ganga-Jamuna Press. ISBN: 0-9709544-0-9.

Niell, A. E. (1996). ***Global Mapping Functions for the Atmosphere Delay at Radio Wavelengths***. ftp://web.haystack.edu/pub/aen/nmf/NMF_JGR.pdf B2 101.

PolaRx2 (2004). ***PolaRx2 GPS Receiver Specifications***. Septentrio Satellite Navigation, 2004. www.septentrio.com.

Rothacher, M., L. Mervart, G. Beutler, E. Brockmann, S. Fankhauser, W. Gurtner, J. Johnson, S. Schaer, T. Springer and R. Weber (1996). ***Bernese GPS Software Version 4.0***.

Saastamoinen, J. (1972). ***Atmospheric Correction for the Troposphere and Stratosphere in Radio Ranging of Satellites***. Geophysical Monograph 15, American Geophysical Union, Washington, DC, 1972.

Seeber, G. (1993). ***Satellite Geodesy: Foundations, Methods, and Applications***. Berlin. New York, Walter de Gruyter. ISBN: 3-11-012753-9.

Serrano, L., D. Kim, R. B. Langley, K. Itani and M. Ueno (2004). ***A GPS Velocity Sensor: How***

Accurate Can It Be? A First Look. Proceedings of ION NTM 2004, San Diego, California, USA**:** 875-885.
Simsky, A. and F. Boon (2003). ***Carrier Phase and Doppler-based Algorithms for Real-Time Standalone Positioning***. Proceedings of GNSS 2003, The European Navigation Conference, Graz, Austria.
Szarmes, M., S. Ryan, G. Lachappelle and P. Fenton (1997). ***DGPS High Accuracy Aircraft Velocity Determination Using Doppler Measurements***. Proceedings of KIS-97, Banff, AB, Canada.
Teunissen, P. J. G. and A. Kleusberg (1998). ***GPS Observation Equations and Positioning Concepts***. P. J. G. Teunissen and A. Kleusberg. GPS for Geodesy, Springer**:** 187-230.
Wells, D. E., B. N. Beck, D. Delikaraoglou, A. Kleusberg, E. J. Krakiwsky, G. Lachapelle, R. B. Langley, M. Nakiboglu, K. P. Schwarz, J. M. Tranquilla and P. Vanicek (1987). ***Guide to GPS Positioning***. Fredericton, New Brunswick, Canada ISBN: 0-920-114-73-3.
Wu, J. T., S. C. Wu, G. A. Hajj, W. I. Bertiger and S. M. Lichten (1993). ***Effects of Antenna Orientation on GPS Carrier Phase***. Manuscripta Geodetica 18: 1647-1660**.**
Zhang, J., K. Zhang, R. Grenfell and R. Deakin (2003a). ***Realtime GPS Orbital Velocity and Acceleration Determination in ECEF System***. Proceedings of the ION GPS 2003, Portland, Oregon, USA**:** 1288-1296.
Zhang, J., K. Zhang, R. Grenfell, Y. Li and R. Deakin (2003b). ***On GPS Satellite Velocity and Acceleration Determination Using the Broadcast Ephemeris***. Proceedings of SatNav 2003, the 6th International symposium of Satellite Navigation Technology, Melbourne, VIC, Australia, Paper (85).
Zhang, K., R. Grenfell, R. Deakin, Y. Li and J. Zhang (2003). ***Current Development of a Low-Cost, High Output Rate RTK GPS Multisensor System for Rowers***. Proceedings of SatNav 2003, Melbourne, Australia, Paper (75).