**A**

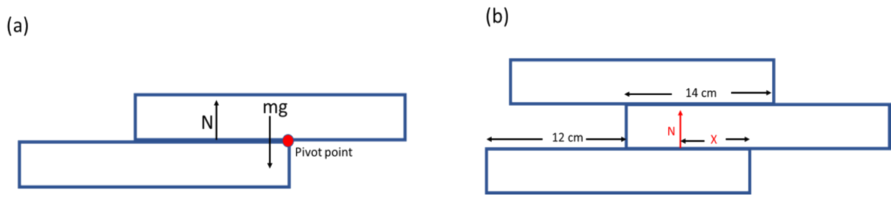
# 114 學年第一學期 普通物理第一次段考試題

[Benson Ch. 07 – 13] 2025/10/30, ***10:10AM – 12:30PM***

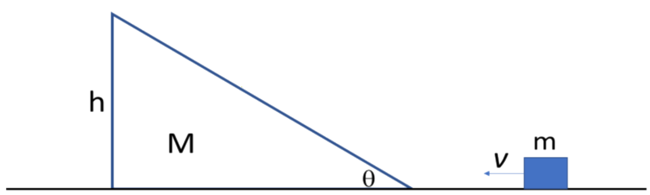
$G = 6.67\times10^{-11}$ N·m²/kg²; Moments of inertia: uniform disk (M, R) $I_{CM} = MR^2/2$, uniform ball (M, R) $I_{CM} = 2ML^2/5$; $g = 9.80$ m/s²

**(Detailed and elaborate process is necessary for each problem)**

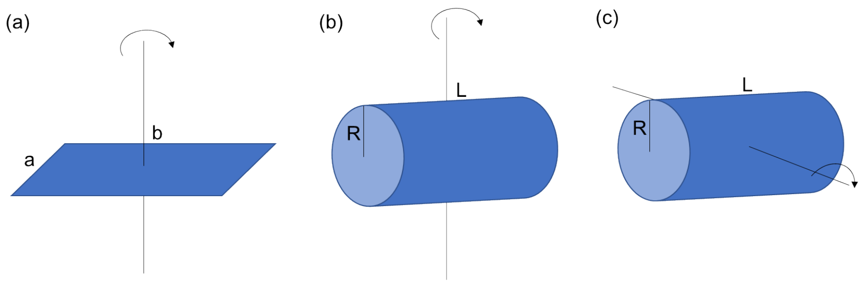
1. Mars moves in an elongated elliptical orbit around the Sun. At perihelion (近日點), Mars is $2.0665\times10^8$ km from the Sun with the speed 24.077 km/s. At aphelion (遠日點), it is $2.492\times10^8$ km from the Sun. (a) Find its speed at aphelion, (b) the semi-minor axis of the orbit, and (c) the period of the orbit. (d) The weight of the Sun. (e) The total energy of Mars. The mass of Mars is $6.39\times10^{23}$ kg. (2, 3, 4, 4, 6 points)
2. Derive the potential energy as well as the gravitational force on a particle of mass $m$ from a hollow sphere of inner radius $R_2$ and outer radius $R_1$, and mass $M$ in the region $r < R_2$, $R_2 < r < R_1$, $r > R_1$. The potential energy of a particle of mass $m$ inside and outside a spherical shell is –GmM/R and –GmM/r, respectively. (6, 6, 6 points)
3. (a) In the right figure (a), the upward contact force $N$ seems to exert more torque than the weight of the top board does with respect to the pivot point, but the two boards stay in static equilibrium, please explain why. (b) In the right figure (b), the three stacking boards are in static equilibrium, please derive the position ***x*,** which the effective upward contact force $N$ are exerted to. (5, 5 points)




4. The object of the mass $m$ and the wedge (斜面) of the mass $M$ are both on a **frictionless** surface. The object moves toward the wedge with a velocity $v$. (a) How much is the minimum velocity $v$ needed for the object to move to the top of the wedge? (b) In (a), when the object reaches the top of the bevel, both the object and the bevel have the same velocity in the horizontal direction, how much is the velocity $v_f$ of the object at that moment? (10, 10 points)

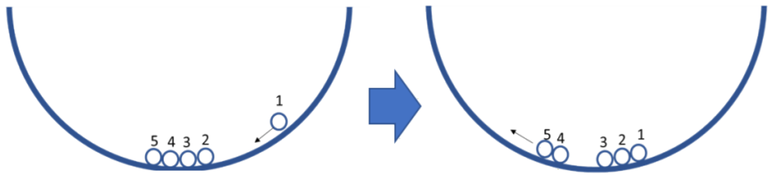



5. Please derive the moment of inertia for (a) a 2D rectangular sheet (b) a 3D cylinder rotating horizontally (c) 3D cylinder rotating vertically Each has mass $M$. (4, 8, 4 points)

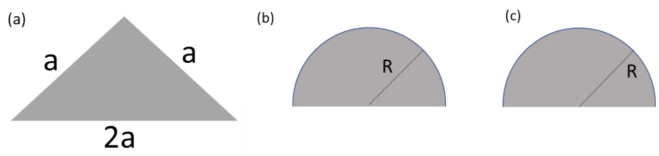



6. Given that the potential energy between a mass point ($m$) and a solid sphere ($M$) is –GmM/R, prove that the potential energy and the gravitational force between a solid sphere ($m$) and a solid sphere ($M$) are –GmM/R and –GmM/R². (6, 3 points).

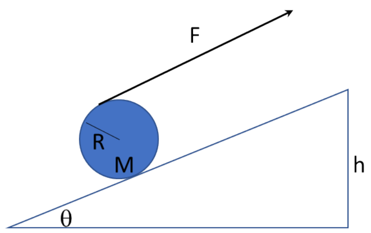
**7.** In the right figure, the balls move along the surface of a hemispherical bowel. (a) Are the angular momentums of the balls conversed? Why? (b) Assume the bottom of the bowel is flat. Prove that the situation in the right figure is impossible; when ball 1 is released, it stops after the collision and ball 4 and ball 5 move out together with the same speed. (c) If ball 4 and ball 5 are glued together, please obtain the final speed of ball 1, ball 4 and ball 5. (d) How high can the ball 4 and ball 5 reach together? The initial speed of ball 1 is $v$ and all the balls are identical with the same mass $m$. (e) Please explain why ball 2 and ball 3 stay still in spite of collision? (3, 5, 3, 3, 3 points)



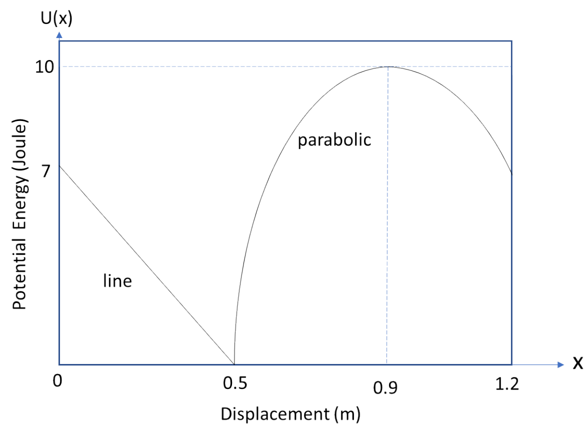

**8.** Please derive the $y$ (vertical) position of the center of the mass with respect to the bottom for (a) an isosceles triangle with the mass $M$, (b) a hemispherical shell with the mass $M$, and (c) a hemispherical solid with the mass $M$. (8, 4, 4 points)



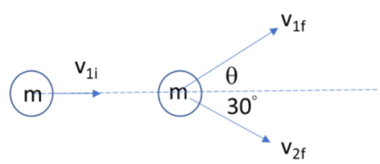

**9.** An applied force $F$ is exerted to a pulley (a disk) on an inclined surface. (a) How much is the least coefficient of friction, $u$, required to pull up the pully? (b) How much is friction force $f$ (including direction)? (c) How much is the acceleration of center of mass $a_{cm}$? (d) If the coefficient of friction is less than the required value in (a), how much time does it take for the pully to roll down from the top (initially at rest) to the bottom of the inclined surface? (e) How much is $v_{cm}$ at the bottom? (8, 4, 3, 4, 4 points)




**10.** An object with the mass $m$ = 2 kg is driven by a conservative force and the corresponding potential energy $U$(x) is shown in the right figure. At the starting point, $t$ = 0 s and $x$ = 0 m, the object has an initial speed $v$ = 1.4 m/sec. (a) How much is the max distance the object can move through? (b) How much is the force acting on the object when $x$ = 0.6 m? (magnitude and direction) (c) How much work is done by the conservative force after the object moves from $x$ = 0 to $x$ = 0. 6 m? (d) What is the time when object is at the position $x$ = 0.5 m? (e) From $x$ = 0 to $x$ = 0.5 m, how much impulse has been applied to the object? (f) If we want the object to move to $x$ = 1.2 m, what is the minimum initial speed of the object? (g) With the initial spend in (f), how much is the total energy when the object reaches $x$ = 1.2 m? (h) Drive the function x(t) from x = 0 to x = 1.2 m. (2, 2, 4, 4, 3, 3, 3,12 points)




**11.** A ball of mass m = 0.02 kg with the speed $v_{1i}$ = 0.1 m/s hits another identical ball at rest (the right figure). Please obtain (a) $v_{1f}$ (b) $v_{2f}$ and (c) θ. (d) If the collision time is 0.1 s, how much is the force between the two balls upon collision? (4, 4, 4, 6 points)



1. (a) Angular momentum is conserved so $mv_1 r_{min} = mv_2 r_{max}$, $24.077\times2.0665\times10^8 = v_2 \times 2.492\times10^8$

$v_2 = 19.97$ (km/s) $=19970$ (m/s)

(b) $r_{max} = 2.492\times10^8$ km, $r_{min} = 2.0665\times10^8$ km

The semi-major axis $a = (r_{max} + r_{min})/2 = 2.28\times10^8$ km
The focus length $c = 2.28\times10^8 - 2.0665\times10^8 = 2.135\times10^7$ km
The semi-minor axis $b = (a^2-c^2)^{1/2} = 2.27\times10^8$ km #

(c) From Kepler's second law, $\frac{dA}{dt} = \frac{|\vec{L}|}{2m}$ so $A = \frac{|\vec{L}|}{2m}t \Rightarrow t = \frac{2mA}{|\vec{L}|}$

For one cycle,

$T = \frac{2m\pi ab}{mv_1 r_{min}} = \frac{2\pi ab}{v_1 r_{min}} = 65358607.41376089677036258848775 7(s) = 756.47 \quad (year)$ # ($\pi$ab is the area of the ellipse). Actually, the mass of the Sun is not needed to get the period.

(d) Kepler's third law $T = \frac{2\pi a^{\frac{3}{2}}}{\sqrt{GM_{Sun}}} = 65358607.413760896770362588487757(s)$

$$M_{Sun} = \frac{4\pi^2}{G\cancel{M_{Mars}}T^2}a^3$$

$1.6 \times 10^{31}$ N

$$W_{Sun} = M_{Sun}g = \frac{4\pi^2}{G\cancel{M_{Mars}}T^2}a^3 g =$$ 請助教幫我把數字代進去算.

$-1.53 \times 10^{32}$ J

(e) $E = \frac{1}{2}m_{Mars}v^2 - G\frac{m_{Mars}M_{Sun}}{r} = \frac{1}{2}m_{Mars}v_{min}{}^2 - G\frac{m_{Mars}M_{Sun}}{r_{min}} =$ 請助教幫我把數字代進去算.

2.

(a) For the potential energy inside the hollow region of the solid sphere, $r < R_2$

$$U(r) = -Gm\int_{R_2}^{R_1}\frac{dM}{r} = -Gm\int_{R_2}^{R_1}4\pi r\rho dr = -Gm[2\pi\rho(R_1{}^2 - R_2{}^2)]$$

$$= -Gm[2\pi\frac{M}{\left(\frac{4}{3}\pi(R_1{}^3 - R_2{}^3)\right)}(R_1{}^2 - R_2{}^2)] = \frac{-3GmM(R_1+R_2)}{2(R_1{}^2 + R_1R_2 + R_2{}^2)}$$

For the potential energy in the region, $R_1 < r < R_2$,of the solid sphere,

$$U(r) = -Gm\frac{\left(\frac{\frac{4}{3}\pi(r^3 - R_2^{\ 3})}{\frac{4}{3}\pi(R_1^{\ 3} - R_2^{\ 3})}\right)M}{r} - Gm\int_r^{R_1}\frac{dM}{r} = -Gm\frac{\left(\frac{(r^3 - R_2^{\ 3})}{(R_1^{\ 3} - R_2^{\ 3})}\right)M}{r} - Gm\int_r^{R_1} 4\pi r\rho dr$$

$$= -Gm[\frac{\left(\frac{(r^3 - R_2^{\ 3})}{(R_1^{\ 3} - R_2^{\ 3})}\right)M}{r} - 2\pi\rho(R_1^{\ 2} - r^2)]$$

$$= -Gm[\frac{\left(\frac{(r^3 - R_2^{\ 3})}{(R_1^{\ 3} - R_2^{\ 3})}\right)M}{r} + 2\pi(\frac{M}{\left(\frac{4}{3}\pi(R_1^{\ 3} - R_2^{\ 3})\right)})(R_1^{\ 2} - r^2)]$$

$$= -GmM[\frac{\left(\frac{(r^3 - R_2^{\ 3})}{(R_1^{\ 3} - R_2^{\ 3})}\right)}{r} + \frac{3(R_1^{\ 2} - r^2)}{2(R_1^{\ 3} - R_2^{\ 3})}]$$

For the potential energy in the region, $r > R_1$,of the solid sphere,

$$U(r) = -Gm\int_0^M \frac{dM}{r} = -\frac{Gm}{r}\int_0^M dM = -\frac{GmM}{r}$$

For the gravitaional force in the region, $R_2 < r < R_1$,of the solid sphere

$$\mathrm{F}(r) = -\frac{dU}{dr} = -GmM\frac{d}{dr}[\frac{\left(\frac{(r^3 - R_2^{\ 3})}{(R_1^{\ 3} - R_2^{\ 3})}\right)}{r} + \frac{3(R_1^{\ 2} - r^2)}{2(R_1^{\ 3} - R_2^{\ 3})}]$$

$$= -GmM[\frac{2r}{R_1^{\ 3} - R_2^{\ 3}} + \frac{R_2^{\ 3}}{(R_1^{\ 3} - R_2^{\ 3})r^2} - \frac{3r}{R_1^{\ 3} - R_2^{\ 3}}]$$

$$= -GmM[\frac{R_2^{\ 3}}{(R_1^{\ 3} - R_2^{\ 3})r^2} - \frac{r}{R_1^{\ 3} - R_2^{\ 3}}]$$

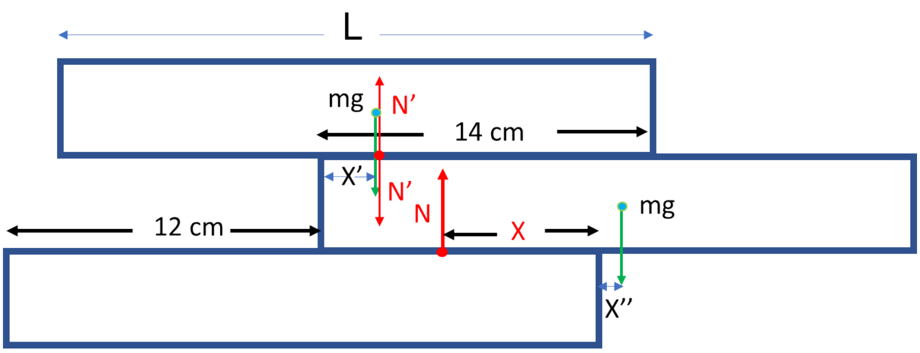
For the gravitaional force in the region, $r > R_1$,of the solid sphere,

$$\mathrm{F}(r) = -\frac{dU}{dr} = -Gm\frac{M}{r^2}$$

3.

(a) If the upward contact force N exerts on the top board evenly, the effective contact point should be in the middle of the distance between the left end of the top board and the pivot point. However, in reality, the contact force N is not applied to the top borad evenly; more magniture toward the pivot point, and less toward the left end. Eventually the postion of the effective contact point for the conatct force N is the same as the center of mass of the top board.

(b) Sorry, I forget to indicate the lenght of the board is $L$ in the problem. But you are smart so I believe you assigned one when doing this probelm.




Let the legnth of the board be $L$.

For the top board

$$N' = mg,\ x' = 14 - \frac{L}{2}$$

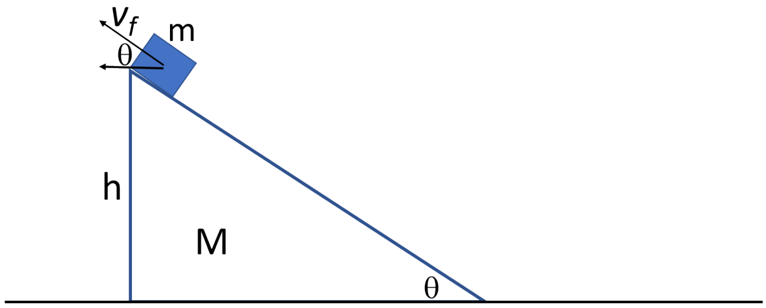
For the middle board

$$N' + mg = N,\ N = 2mg$$

$$Nx + (12 - \frac{L}{2})mg = (L - 12 - x')N' \Rightarrow 2mgx + (12 - \frac{L}{2})mg = (L - 12 - 14 + \frac{L}{2})mg$$

$$\Rightarrow 2x + (12 - \frac{L}{2}) = (L - 12 - 14 + \frac{L}{2}) \Rightarrow 2x = 2L - 38 \Rightarrow x = L - 19\ \#$$

4.




(a) (with p-conservation & E-conservation, you may get 4 point)

Momentum conservation in horizontal direction:

$$mv = (m+M)v_f \cos\theta$$ (4) ⇒ no cosθ (−2)

$$v_f = \frac{m}{(m+M)\cos\theta} v \text{...........(1)}$$

Energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}Mv_f{}^2\cos^2\theta + \frac{1}{2}mv_f{}^2 + mgh \text{........(2)}$$ (4) ⇒ no cosθ (−2)

Plugging (1) into (2)

$$\frac{1}{2}mv^2 = \frac{1}{2}v^2\left(\frac{m}{(m+M)\cos\theta}\right)^2 (M\cos^2\theta + m) + mgh$$

$$\frac{1}{2}\left[m - \left(\frac{m}{(m+M)\cos\theta}\right)^2 (M\cos^2\theta + m)\right]v^2 = mgh$$

$$v^2 = \frac{2mgh}{m - \left(\frac{m}{(m+M)\cos\theta}\right)^2 (M\cos^2\theta + m)}$$

$$v = \frac{\sqrt{2mgh}}{\sqrt{m - \left(\frac{m}{(m+M)\cos\theta}\right)^2 (M\cos^2\theta + m)}} \quad \#$$ (2)

(b)

$$v_f = \frac{m}{(m+M)\cos\theta} v = \frac{m}{(m+M)\cos\theta} \frac{\sqrt{2mgh}}{\sqrt{m - (\frac{m}{(m+M)\cos\theta})^2 (M\cos^2\theta + m)}}$$

(6)

no cosθ −2

$$= \frac{m\sqrt{2mgh}}{\sqrt{m(m+M)^2\cos^2\theta - m^2(M\cos^2\theta + m)}}$$

$$= \frac{m\sqrt{2gh}}{\sqrt{(m+M)^2\cos^2\theta - m(M\cos^2\theta + m)}} \#$$

(4)

5.

**(a)**

For a rod of the lengh $b$

$$I_{CM} = \int x^2 dm = \int_{-\frac{b}{2}}^{\frac{b}{2}} x^2 \lambda dx = \lambda \frac{b^3}{12} = \frac{Mb^2}{12} \qquad (\lambda = \frac{M}{b})$$

For a rod with the rotation axis away from CM by a distance $d$

$$I = I_{CM} + Md^2 = \frac{Mb^2}{12} + Md^2$$ 1

For a 2d sheet with length $b$ and width $a$.

$$I = \int dI = \int \frac{b^2}{12} dm + y^2 dm = \int_{-\frac{a}{2}}^{\frac{a}{2}} (\frac{b^2}{12} + y^2)\sigma b dy = \frac{M}{12} a^2 + \frac{M}{12} b^2 \qquad (\sigma = \frac{M}{ab}) \#$$

2 1

**(b)**

For a cylinder, the composite element is a retangular plate.

$$dI = \frac{dm}{12}\left((2R\sin\theta)^2 + L^2\right), \; dm = 2LR\sin\theta dy \rho$$ 2

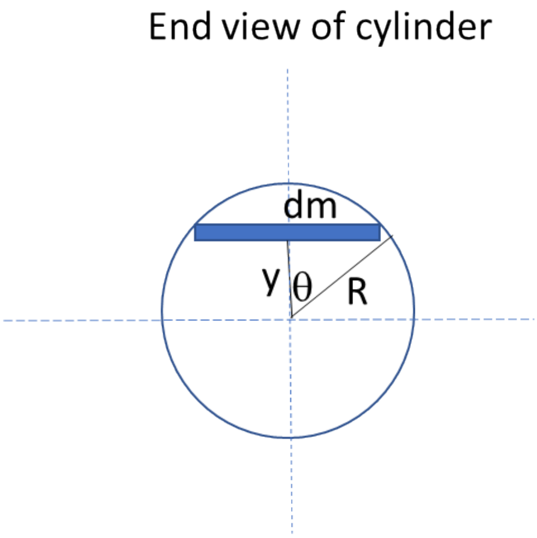
$$y = R\cos\theta, \; dy = -R\sin\theta d\theta$$

2

$$I = \int dI = -\frac{\rho L R^2}{6} \int_{\pi}^{0} (4R^2\sin^4\theta + L^2\sin^2\theta) d\theta = \frac{\rho L R^2}{6} (4R^2 g \frac{3}{8}\pi + L^2 g \frac{1}{2}\pi)$$ 2

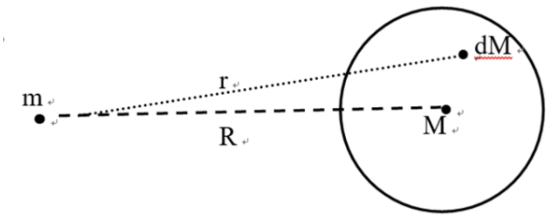
$$= \frac{M}{\pi R^2 L} L R^2 (\frac{1}{4} R^2 + \frac{1}{12} L^2) = \frac{1}{4} M R^2 + \frac{1}{12} M L^2 \; \# \quad (\rho = \frac{M}{\pi R^2 L})$$

2



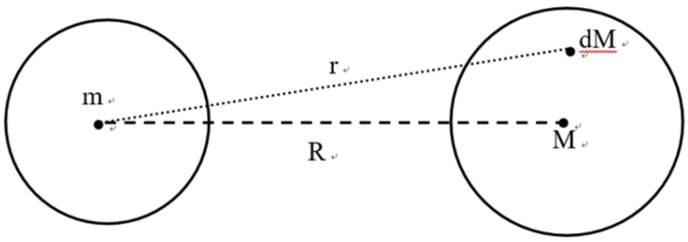

(c) 和 (b) 一 樣. 送分用. 4

**6.**




**以知條件：** 點和球的位能等於點和點的位能．

$$U=\int -G\frac{mdM}{r}=-G\frac{Mm}{R}$$




M 球和 m 球的位能可看成是 M 球上的一個小質點 dM 與 m 球的位能積起來
但根據**已知條件,** dM 點和 m 球的位能等於 dM 點和 m 點的位能, 所以積分變成是把 dM 點和 m 點的位能積起來, 因而變成 M 球和 m 點的位能而根據**已知條件**，這就是 M 點和 m 點的位能，故得證．

$$dU=-G\frac{mdM}{r}$$

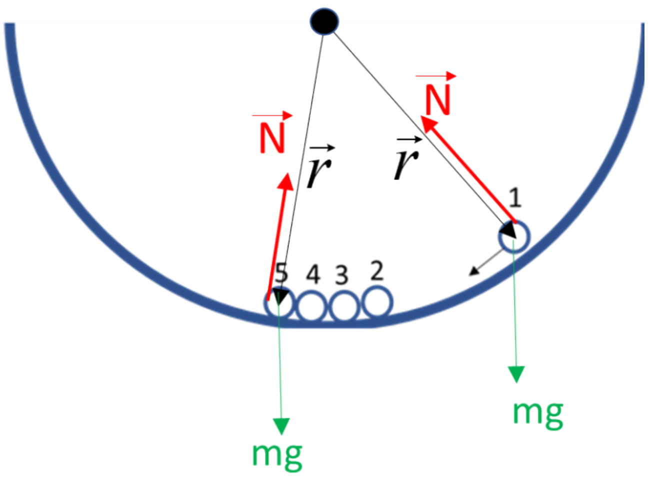
$$U=\int dU=\int -G\frac{mdM}{r}=-G\frac{mM}{R}\ \#$$

$$F=-\frac{dU}{dR}=-G\frac{mM}{R^2}\ \#$$

7.

(a) No, the angluar momentum is not conserved because as shown in the figure below, although the contact force $N$ is directed along the dierction of $-\vec{r}$ toward the commom center, the weight $m\vec{g}$ isn't so

$\vec{\tau} = \vec{r} \times (\vec{N} + m\vec{g}) = 0 + \vec{r} \times m\vec{g}$, $\frac{d\vec{L}}{dt} = \vec{\tau} \neq 0$ so $\vec{L} \neq$ constant. # 3




(b) The initial speed of ball 1 is $v$. The final speed of ball 1 is zero.

( Conservation of linear momentum)

$mv = mv_f + mv_f$, $v_f = \frac{1}{2}v$, the final speed of ball 4 and ball 5

( Conservation of energy)

before collision, for ball 1, $K_{1i} = \frac{1}{2}mv^2$, for ball 4 and ball 5, $K_{4i} = K_{5i} = 0$

$K_i = \frac{1}{2}mv^2$ 5

After collision, for ball 1, $K_{1f} = 0$, for ball 4 and ball 5, $K_{4f} = K_{5f} = \frac{1}{8}mv^2$

$K_f = \frac{1}{8}mv^2 + \frac{1}{8}mv^2 = \frac{1}{4}mv^2$

$K_f \neq K_i$, Energy dosn't conserve, so this can't happen.

(c) The initial speed of ball 1 is $v$. The final speed of ball 1 is $v_{1f}$.

( Conservation of linear momentum)

$mv = mv_{1f} + mv_{4f} + mv_{5f} = mv_{1f} + 2mv_{4f}\ (v_{4f} = v_{5f})$

$mv = mv_{1f} + 2mv_{4f}$......(1) 1

( Conservation of energy )

before collision, for ball 1, $K_{1i} = \frac{1}{2}mv^2$, for ball4 and ball 5, $K_{4i} = K_{5i} = 0$

$K_i = \frac{1}{2}mv^2$

after collision, for ball 1, $K_{1f} = \frac{1}{2}mv_{1f}^2$, for ball4 and ball 5, $K_{4f} = K_{5f} = \frac{1}{2}mv_{4f}^2$

$K_i = K_f$,

$\frac{1}{2}mv^2 = \frac{1}{2}mv_{1f}^2 + mv_{4f}^2$....(2) 1

Sovle (1) and (2), we get $v_{1f} = -\frac{1}{3}v,\ v_{4f} = v_{5f} = \frac{2}{3}v$ # 1

(d)

$\frac{1}{2}(2m)(\frac{2}{3}v)^2 = 2mgh,\ h = \frac{2}{9g}v^2$ # 3

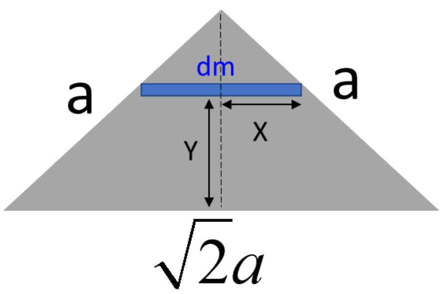
(e)

因為 ball 2 及 ball 3 很懶惰，他們在中間只想傳遞動量而不想增加動量．在最旁邊的球(ball 5)很可憐，因為右邊沒球可傳遞動量了，只好帶著動量跑了. 3

and ball 4

8.

(a)

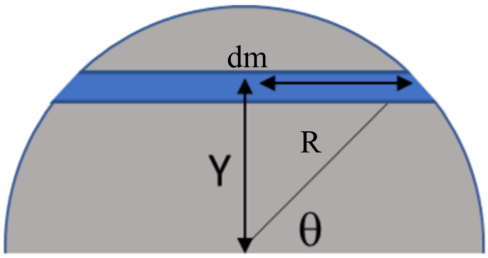



$y_{cm} = \frac{1}{M}\int y\,dm,\ dm = \sigma(2x)dy = \sigma(\sqrt{2}a - 2y)dy$ 2

$y_{cm} = \frac{\sigma}{M}(\sqrt{2}a\int_0^{\frac{\sqrt{2}}{2}a} y\,dy - 2\int_0^{\frac{\sqrt{2}}{2}a} y^2 dy) = \frac{\sigma}{M}(\frac{\sqrt{2}}{4}a^3 - \frac{\sqrt{2}}{6}a^3)$ 4

$= \frac{\sigma}{M}\frac{\sqrt{2}}{12}a^3 = \frac{(\frac{M}{\frac{1}{2}a^2})}{M}\frac{\sqrt{2}}{12}a^3 = \frac{\sqrt{2}}{6}a$ # 2

(b)




(2)

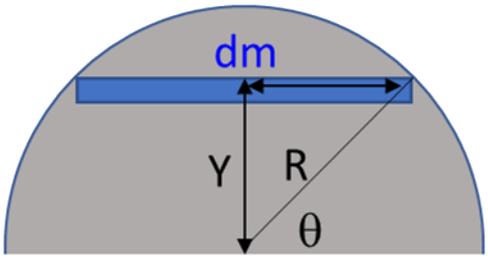
$$y_{cm} = \frac{1}{M}\int y\,dm,\ dm = \sigma(2\pi R\cos\theta)Rd\theta = 2\pi\sigma R^2\cos\theta d\theta,\ \text{y} = \text{R}\sin\theta$$

$$y_{cm} = (\frac{1}{M})2\pi\sigma R^3\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta d\theta = (\frac{1}{M})2\pi\sigma R^3(\frac{1}{2}\sin^2\theta|_0^{\frac{\pi}{2}})$$ (1)

$$= \pi\sigma R^3 = (\frac{1}{M})\pi(\frac{M}{2\pi R^2})R^3 = \frac{R}{2}\ \#$$ (1)

$\frac{1}{M}$

(c)




(2)

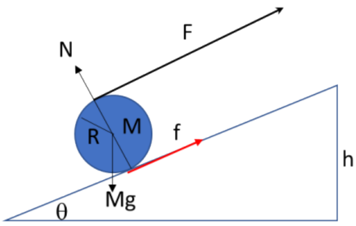
$$y_{cm} = \frac{1}{M}\int y\,dm,\ dm = \rho(R\cos\theta)^2\pi dy = \pi\rho R^3\cos^3\theta d\theta,\ \text{y} = \text{R}\sin\theta, dy =$$

$$y_{cm} = (\frac{1}{M})\pi\rho R^4\int_0^{\frac{\pi}{2}}\sin\theta\cos^3\theta d\theta = (\frac{1}{M})\pi\rho R^4(-\frac{1}{4}\cos^4\theta|_0^{\frac{\pi}{2}})$$ (1)

$$= \frac{1}{4M}\pi\rho R^4 = (\frac{1}{4M})\pi(\frac{M}{\frac{2}{3}\pi R^3})R^4 = \frac{3R}{8}\ \#$$ (1)

9.

(a)




$F + f - Mg\sin\theta = Ma_{cm}$......(1)

$\cancel{F(2R) = I\alpha = I\frac{a_{cm}}{R},\ F = \frac{a_{cm}I}{2R^2}}$.....(2) $\Rightarrow$ $F\cdot 2R - mg\sin\theta\cdot R = I\frac{a_{cm}}{R}$ } (4)

Plugging (2) into (1), we get $a_{cm} = \dfrac{Mg\sin\theta - f}{\frac{I}{2R^2} - M} = \dfrac{f - Mg\sin\theta}{M - \frac{I}{2R^2}}$ $\quad a_{cm} = \dfrac{f - \frac{1}{2}Mg\sin\theta}{\frac{1}{4}M}$

For a rolling disk , I $= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

so for $a_{cm} > 0,\ f - Mg\sin\theta > 0$

$f = \mu N,\ (N = Mg\cos\theta)$ $\quad a_{cm} > 0,\ f = \mu Mg\cos\theta$ (2)

so $f = \mu Mg\cos\theta > Mg\sin\theta$

$\mu > \tan\theta$ $\quad \Rightarrow\ \mu > \frac{1}{2}\tan\theta$

The least value of $\mu$ is $\tan\theta$ for pulling the pulley up. #

(b)

$$F + f - Mg\sin\theta = Ma_{cm} ......(1)$$

$$F(2R) = I\alpha = I\frac{a_{cm}}{R},\ F = = \frac{a_{cm}I}{2R^2} .....(2)$$

$$a_{cm} = = \frac{2R^2F}{I} ...(3)$$

Plugging (3)into (1), $(I = \frac{3}{2}MR^2)$

$\frac{1}{3}Mg\sin\theta + \frac{1}{3}F$

we get $f = Mg\sin\theta - \text{F} + \text{M} + \frac{2R^2F}{I} = \cancel{Mg\sin\theta + \frac{1}{3}F}$ #

(c) $a_{cm} = = \cancel{\frac{2R^2F}{I} = \frac{4}{3}\frac{F}{M}}$ #  $a_{cm} = \frac{4F - 2Mg\sin\theta}{3M}$

(d) 這一子題的觀念有點 tricky. 當摩擦力不足以幫拉力拉上滑輪時, 滑輪往下跑, 滑輪還是順時針轉, 但滑輪是純轉動而不是滾動. 這 case 有點像溜溜球往下跑, 即使拉力用力往上拉. 然而摩擦力還是有阻止轉動及下滑平移. 對, 更像上坡的車子輪胎磨平,抓地力不夠, 車子打滑下去.

with

$$Mg\sin\theta - F - f = Ma_{cm} ......(1)$$

$$(F - f)R = I\alpha = I\frac{a_{cm}}{R},\ f = F - \frac{a_{cm}I}{R^2} .....(2)$$

(d) $\frac{h}{\sin\theta} = \frac{1}{2}a_{cm}t^2$ (2)

Plugging (2)into (1), $(I = \frac{1}{2}MR^2$, pure rotation about the center of mass)

we get $a_{cm} = \frac{2(Mg\sin\theta - 2F)}{M}$

(e) with

The inclined length $\frac{h}{\sin\theta} = \frac{1}{2}a_{cm}t^2,\ t = \sqrt{\frac{2h}{a_{cm}\sin\theta}} = \sqrt{\frac{2hM}{2(Mg\sin\theta - 2F)\sin\theta}}$ #

$V = a_{cm}t$ (2)

Final speed at the bottom of inclined surface, $v = a_{cm}t = \sqrt{\frac{2ha_{cm}}{\sin\theta}} = \sqrt{\frac{4h(Mg\sin\theta - 2F)}{M\sin\theta}}$ #

10. (without unit −1, vector without direction −1)

(a) $E = \frac{1}{2}mv^2 + U(x) = 1.96 + 7 = 8.96$ (J)

Bcause the total energy 8.90 J is less the than the top of the potential energy 10 J, the max distance the object can move is ~~0.9~~ m. 0.771m

(b) $F = -\frac{dU}{dx}$, From x = 0.5 to x = 0.9, $U(x) = -62.5(x - 0.9)^2 + 10$

$$F(x) = -\frac{dU}{dx} = 125(x - 0.9),\ F(0.6) = 37.5 \text{ (N), in the direction of } +\text{x}.$$

(c)

$$U(x) = -62.5(x-0.9)^2 + 10,\ U(0.6) = -62.5(0.6-0.9)^2 + 10 = 4.375$$
$$U(0) = 7,\ W = -\Delta U = -(U(0.6) - U(0)) = 2.625 \text{ (J) \#}$$

(d)

$$U(x) = -\overset{14}{\cancel{1.4}}x + 7,\ F = -\frac{dU(x)}{dx} = \overset{14}{\cancel{1.4}} \text{ (N)}$$

From x = 0 to x = 0.5, $\overset{14}{\cancel{1.4}} = m\frac{d^2x}{dt^2} \Rightarrow \frac{d^2x}{dt^2} = \overset{7}{\cancel{0.7}} \Rightarrow x(t) = x_0 + v_0 t + \frac{1}{2}\times\overset{7}{\cancel{0.7}}t^2,\ x_0 = 0, v_0 = 1.4$

$$\Rightarrow x(t) = \underset{14}{\cancel{1.4}}t + \underset{35}{\cancel{0.35}}t^2,\ \text{when } x = \underset{0.5}{\cancel{5}},\ t = \cancel{2.276} \text{ (s) \#}\quad 0.2276\text{(s)}$$

(e) $F = -\frac{dU(x)}{dx} = \overset{14}{\cancel{1.4}}$ (N), Impulse $I = F\Delta t = \overset{14\times 0.2276}{\cancel{1.4\times 2.276}} = \underline{3.19}$ (N•s)

(f) Minimum total energy should be $E = 10$ (J). $E = \frac{1}{2}mv^2 + U(x) \Rightarrow 10 = \frac{1}{2}\times 2\times v^2 + 7 \Rightarrow v = \sqrt{3}$ (m/s) #

(g) $E = 10$ J , 送分用的.

(h) From x = 0 to x = 0.5, $x(t) = \sqrt{3}t + 0.35t^2$

寫到這就給 5 分

From x = 0.5 to x = 1.2,

$$F(x) = -\frac{dU}{dx} = 125(x-0.9)$$

$$125(x-0.9) = m\frac{d^2x}{dt^2}$$

$$\Rightarrow \frac{d^2x}{dt^2} = 62.5(x-0.9),\ \text{Solve for } x(t).$$

$$x(t) = A(e^{\sqrt{62.5}} + 0.9) + B(e^{-\sqrt{62.5}} + 0.9)$$

Conditions: When $t$ =2.276, $x$ =5 and $v = \sqrt{10}$ to get the constants $A$ and $B$.

有同學能寫到這就給 12 分 .

11.

Conservation of linear momentum:

$x$ component

$$mv_{1i} + 0 = mv_{1f}\cos\theta + mv_{2f}\cos 30^\circ$$

$$\Rightarrow v_{1i} = v_{1f}\cos\theta + v_{2f}\cos 30^\circ .......(1) \qquad 2$$

y component

$$0 = mv_{1f}\sin\theta - mv_{2f}\sin 30^\circ$$

$$\Rightarrow v_{2f} = 2\sin\theta v_{1f} .......(2) \qquad 2$$

Conservation of total energy

$$\frac{1}{2}mv_{1i}^{\ 2} = \frac{1}{2}mv_{1f}^{\ 2} + \frac{1}{2}mv_{2f}^{\ 2} \qquad 2$$

$$\Rightarrow v_{1i}^{\ 2} = v_{1f}^{\ 2} + v_{2f}^{\ 2} .............(3)$$

From (1)

$$v_{1i}^{\ 2} = v_{1f}^{\ 2}\cos^2\theta + v_{2f}^{\ 2}\cos^2 30^\circ + 2v_{1f}v_{2f}\cos\theta\cos 30^\circ ......(4) \qquad 1$$

Pluging (3) into (4)

$$v_{1f}^{\ 2} + v_{2f}^{\ 2} = v_{1f}^{\ 2}\cos^2\theta + v_{2f}^{\ 2}\cos^2 30^\circ + 2v_{1f}v_{2f}\cos\theta\cos 30^\circ$$

$$\Rightarrow v_{1f}^{\ 2}\sin^2\theta + v_{2f}^{\ 2}\sin^2 30^\circ = 2v_{1f}v_{2f}\cos\theta\cos 30^\circ .......(5) \qquad 1$$

Pluging (2) into (5)

$$v_{1f}^{\ 2}\sin^2\theta + 4\sin^2\theta v_{1f}^{\ 2}\sin^2 30^\circ = \overset{4}{\cancel{2}}\sin\theta v_{1f}^{\ 2}\cos\theta\cos 30^\circ \qquad 1$$

$$\Rightarrow \tan\theta = \cos 30^\circ = \sqrt{3},\ \ \theta = 60^\circ \quad \# \quad \text{The answer for (c)}$$

1

$$v_{1i} = v_{1f}\cos 60^\circ + v_{2f}\cos 30^\circ$$

$$v_{2f} = 2\sin 60^\circ v_{1f} = \sqrt{3}v_{1f}$$

$$v_{1i} = v_{1f}\cos 60^\circ + \sqrt{3}v_{1f}\cos 30^\circ$$

1

$$\Rightarrow v_{1f} = \frac{1}{\cos 60^\circ + +\sqrt{3}\cos 30^\circ}v_{1i} = \frac{1}{2}v_{1i} = 0.05\ \text{(m/s)}\ \#\ \text{The answer for (a)}$$

$$\Rightarrow v_{2f} = \sqrt{3}v_{1f} = \frac{\sqrt{3}}{2}v_{1i} = 0.0866\text{(m/s)}\ \#\ \text{The answer for (b)}$$

1

d

(d) For this problem, you can focus either on ball 1 or ball 2. Certainly, it is much eaiser to focus on the ball 2.

For ball 2, $F\Delta t = mv_{2f} - 0 = 0.02 \times 0.0866$ 6

$\Rightarrow 0.1F = 0.02 \times 0.0866,\ F = 0.01732$ (N)

The direction is 30 degree below the horizontal line. This is the force ball 2 received from ball 1. As for ball 1, acording to Newton's third law, the magnitude of the force is also 0.01732 (N) but the direction is opposite, that is, 150 dgree above the horizontal line.#