

$$\begin{pmatrix} 1 & 6 & 1 & -4 \\ 0 & -4 & -1 & 3 \\ -1 & 1 & 1 & -1 \\ 0 & 2 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 4 & 2 & 4 & 6 \\ 6 & 3 & 6 & 9 \\ 6 & 3 & 6 & 9 \\ 2 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 38 & 19 & 38 & 57 \\ -24 & -12 & -24 & -36 \\ 6 & 3 & 6 & 9 \\ 10 & 5 & 10 & 15 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 6 & 1 & -4 \\ 0 & -4 & -1 & 3 \\ -1 & 1 & 1 & -1 \\ 0 & 2 & 0 & -1 \end{pmatrix} \cdot -4 = \begin{pmatrix} -4 & -24 & -4 & 16 \\ 0 & 16 & 4 & -12 \\ 4 & -4 & -4 & 4 \\ 0 & -8 & 0 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 6 & 1 & -4 \\ 0 & -4 & -1 & 3 \\ -1 & 1 & 1 & -1 \\ 0 & 2 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 2 & 4 & 6 \\ 6 & 3 & 6 & 9 \\ 6 & 3 & 6 & 9 \\ 2 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 8 & 5 & 2 \\ 6 & -1 & 5 & 12 \\ 5 & 4 & 7 & 8 \\ 2 & 3 & 2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 6 & 1 & -4 & | & 1 & 0 & 0 & 0 \\ 0 & -4 & -1 & 3 & | & 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & -1 & | & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & | & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{matrix} W_1 \\ W_2 \\ W_3 + W_1 \\ W_4 \end{matrix} \begin{pmatrix} 1 & 6 & 1 & -4 & | & 1 & 0 & 0 & 0 \\ 0 & -4 & -1 & 3 & | & 0 & 1 & 0 & 0 \\ 0 & 7 & 2 & -5 & | & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & | & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{matrix} W_1 - 3W_4 \\ \frac{1}{2}W_4 \\ W_3 + -\frac{7}{2}W_4 \\ 2W_4 + W_2 \end{matrix} \begin{pmatrix} 1 & 0 & 1 & -1 & | & 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -\frac{1}{2} & | & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 2 & -\frac{3}{2} & | & 1 & 0 & 1 & -\frac{7}{2} \\ 0 & 0 & -1 & 1 & | & 0 & 1 & 0 & 2 \end{pmatrix}$$

$$= \begin{matrix} W_1 + W_4 \\ W_2 \\ -W_4 \\ 2W_4 + W_3 \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 0 & | & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & -\frac{1}{2} & | & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -1 & | & 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & \frac{1}{2} & | & 1 & 2 & 1 & \frac{1}{2} \end{pmatrix}$$

$$= \begin{matrix} W_1 \\ W_2 + W_4 \\ 2W_4 + W_3 \\ 2W_4 \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 0 & | & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & | & 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 & | & 2 & 3 & 2 & -1 \\ 0 & 0 & 0 & 1 & | & 2 & 4 & 2 & 1 \end{pmatrix}$$

$$\begin{aligned}
& \left(\begin{array}{cccc|cccc} 1 & 6 & 1 & -4 & 1 & 0 & 0 & 0 \\ 0 & -4 & -1 & 3 & 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 & 0 & 1 \end{array} \right) \\
&= \begin{array}{c} W_1 \\ W_2 \\ W_3 + W_1 \\ W_4 \end{array} \left(\begin{array}{cccc|cccc} 1 & 6 & 1 & -4 & 1 & 0 & 0 & 0 \\ 0 & -4 & -1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 7 & 2 & -5 & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 & 0 & 1 \end{array} \right) \\
&= \begin{array}{c} W_1 - 3W_4 \\ 1/2 W_4 \\ W_3 + -7/2 W_4 \\ 2W_4 + W_2 \end{array} \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & -1 & 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 2 & -3/2 & 1 & 0 & 1 & -7/2 \\ 0 & 0 & -1 & 1 & 0 & 1 & 0 & 2 \end{array} \right) \\
&= \begin{array}{c} W_1 + W_4 \\ W_2 \\ -W_4 \\ 2W_4 + W_3 \end{array} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & -1 & 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & 1/2 & 1 & 2 & 1 & 1/2 \end{array} \right) \\
&= \begin{array}{c} W_1 \\ W_2 + W_4 \\ 2W_4 + W_3 \\ 2W_4 \end{array} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 & 2 & 3 & 2 & -1 \\ 0 & 0 & 0 & 1 & 2 & 4 & 2 & 1 \end{array} \right)
\end{aligned}$$

$$\left(\begin{array}{cccc} 1 & 6 & 1 & -4 \\ 0 & -4 & -1 & 3 \\ -1 & 1 & 1 & -1 \\ 0 & 2 & 0 & -1 \end{array} \right) \cdot \left(\begin{array}{cccc} -1 & -1 & 0 & 1 \\ -1 & -2 & -1 & -1 \\ -2 & -3 & -2 & 1 \\ -2 & -4 & -2 & -1 \end{array} \right) = \left(\begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right)$$

$$\begin{aligned}
& \left(\begin{array}{cccc|cccc} 1 & 6 & 1 & -4 & 4 & 2 & 4 & 6 \\ 0 & -4 & -1 & 3 & 6 & 3 & 6 & 9 \\ -1 & 1 & 1 & -1 & 6 & 3 & 6 & 9 \\ 0 & 2 & 0 & -1 & 2 & 1 & 2 & 3 \end{array} \right) \\
&= \begin{array}{c} W_1 \\ W_2 \\ W_3 + W_1 \\ W_4 \end{array} \left(\begin{array}{cccc|cccc} 1 & 6 & 1 & -4 & 4 & 2 & 4 & 6 \\ 0 & -4 & -1 & 3 & 6 & 3 & 6 & 9 \\ 0 & 7 & 2 & -5 & 10 & 5 & 10 & 15 \\ 0 & 2 & 0 & -1 & 2 & 1 & 2 & 3 \end{array} \right) \\
&= \begin{array}{c} W_1 - 3W_4 \\ \frac{1}{2}W_4 \\ W_3 + -\frac{7}{2}W_4 \\ 2W_4 + W_2 \end{array} \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & -1 & -2 & -1 & -2 & -3 \\ 0 & 1 & 0 & -\frac{1}{2} & 1 & \frac{1}{2} & 1 & \frac{3}{2} \\ 0 & 0 & 2 & -\frac{3}{2} & 3 & \frac{3}{2} & 3 & \frac{9}{2} \\ 0 & 0 & -1 & 1 & 10 & 5 & 10 & 15 \end{array} \right) \\
&= \begin{array}{c} W_1 + W_4 \\ W_2 \\ -W_4 \\ 2W_4 + W_3 \end{array} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 8 & 4 & 8 & 12 \\ 0 & 1 & 0 & -\frac{1}{2} & 1 & \frac{1}{2} & 1 & \frac{3}{2} \\ 0 & 0 & 1 & -1 & -10 & -5 & -10 & -15 \\ 0 & 0 & 0 & \frac{1}{2} & 23 & \frac{23}{2} & 23 & \frac{69}{2} \end{array} \right) \\
&= \begin{array}{c} W_1 \\ W_2 + W_4 \\ 2W_4 + W_3 \\ 2W_4 \end{array} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 8 & 4 & 8 & 12 \\ 0 & 1 & 0 & 0 & 24 & 12 & 24 & 36 \\ 0 & 0 & 1 & 0 & 36 & 18 & 36 & 54 \\ 0 & 0 & 0 & 1 & 46 & 23 & 46 & 69 \end{array} \right)
\end{aligned}$$