Exercise 1:

The linear optimal predictor is:

$$(\alpha, \beta) = argminE[(y - a - bx)^2]$$

FOCs:

$$\beta = \frac{cos(x, y)}{var(x)}$$

$$\alpha = E[y] - \beta E[x]$$

$$cov(x, y) = E[xy] - E[x]E[y]$$

Since $Y_i = 1 - X_i^2 + \epsilon_i$ and X is distributed uniformly U(0,1)

$E[x] = \frac{1-0}{2}$
$E[x, y] = E[x(1 - x^2 + \epsilon)]$
$= 1 - E[x - x^3 + x\epsilon]$
$= E[x - x^3] + E[x\epsilon]$
$= \int_0^1 (x - x^3)dx + 0$
$= \frac{1}{2} - \frac{1}{4}$
$= \frac{1}{4}$
$E[Y] = E[1 - x^2 + \epsilon] = 1 - E[x^2] + 0$
$= 1 - \int_0^1 [x^2]dx = \frac{2}{3}$
$cov(x, y) = E[xy] - E[x]E[y] = \frac{1}{4} - \frac{1}{2}(\frac{2}{3}) = -\frac{1}{12}$

$$Var[x] = \frac{[a - b]^2}{12}$$

$$\beta = \frac{cos(x, y)}{var(x)} = \frac{-\frac{1}{12}}{\frac{1}{12}} = -1$$

$$\alpha = E[y] - \beta E[x] = \frac{2}{3} - (-1) * \frac{1}{2} = \frac{7}{6}$$