Econometrics Problem Set 1

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Exercise 4:

a:

$$f_Y(y,p) = p^y(1-p)^{1-y}$$
$$\ln_y(y,p) = (y)\ln(p) + (1-p)\ln(1-y)$$
$$S_n(p) = \frac{1}{n}\sum_{t=1}^{n} y_t \ln(p) + (1-y_t)\ln(1-p)$$

FOCs:

$$\frac{\partial \ln S_n(p)}{\partial (p)} = \frac{1}{n}\sum_{t=1}^{n}\frac{y_t - p}{p(1-p)} = 0$$
$$\hat{p} = \bar{y}$$

This is the Max Likelihood Estimator of p_0.

b:

$$g_t = D_p \ln f(y_t|p)dy_t = \int \frac{\partial \ln f_Y(y,p)}{\partial p} = \frac{y_t - p}{p(1-p)}$$
$$\xi_p[g_t(p)] = \int [D_p \ln f(y_t|p)] f(y_t|p) dy_t$$
$$= \int \frac{y_t - p}{p(1-p)} p^(y_t)(1-p)^{1-y_t} dy_t$$
$$= \int y_t p^{y_t - 1)}(1-p)^{-y_t} - p^(y_t(1-p)^{-y_t} dy_t$$
$$\int y_t p^{y_t-1}(1-p)^{-y_t} + (1-y_t)(1-p)^{-y_t} p^{y_t} dy_t$$
$$= \int \frac{\partial p^{y_t}(1-p)^{1-y_t}}{\partial p} dy_t$$

c:

$$\sqrt{n}D_pS_n(p) = \frac{1}{\sqrt{n}}\sum_{t=1}^{n}\frac{y_t - p}{p(1-p)} \longrightarrow^d \mathbb{N}(0, I_\infty(p_0))$$

$$I_\infty(p_0) = \lim_{n\longrightarrow\infty} Var(\frac{1}{\sqrt{n}}\sum_{t=1}^{n}\frac{y_t - p_0}{p_0(1-p_0)})$$

$$= \lim_{n\longrightarrow\infty} Var(\frac{1}{\sqrt{n}}\sum_{t=1}^{n}\frac{y_t}{p_0(1-p_0)})$$

$$= \frac{1}{p_0^2(1-p_0^2)}\lim_{n\longrightarrow\infty}\frac{1}{\sqrt{n}}\sum_{t=1}^{n} Var(y_t)$$

$$= \frac{1}{p_0(1-p_0)}$$

$J_n(p) := D^2S_n(p)$
$= \frac{\partial \frac{1}{n}\sum_{t=1}^{n}[\frac{y_t}{p(1-p)} - \frac{1}{1-p}]}{\partial p}$
$= \frac{1}{n}\sum_{t=1}^{n}[\frac{y_t(2p-1)}{p^2(1-p)^2} - \frac{1}{(1-p)^2}]$
$= \frac{1}{n}\sum_{t=1}^{n}[\frac{y_t(2p-1)-p^2}{p^2(1-p)^2}]$
$= \frac{[\frac{1}{n}\sum_{t=1}^{n} y_t](2p-1)-p^2]}{p^2(1-p)^2}$

$\mathrm{J}_\infty(p_0) := \frac{p_0(2p_0-1)-p_0^2}{p_0^2(1-p_0)^2} = \frac{p_0^2-p_0}{p_0^2(1-p_0)^2} = \frac{1}{p_0(p_0-1)}$

d:

$$\lim_{n\longrightarrow\infty} Var\sqrt{n}(\bar{y} - p_0) = \lim_{n\longrightarrow\infty} n\ Var(\bar{y})$$

$$= \lim_{n\longrightarrow\infty} n\ Var\frac{\sum_{t=1}^{n} y_t}{n}$$

$$= \frac{1}{n}\lim_{n\longrightarrow\infty}\sum_{t=1}^{n} Var(y_t)$$

$$= p_0(1-p_0)$$

$$J_\infty(p_0)^{-1}I_\infty(p_0)J_\infty(p_0)^{-1} = \lim_{n\longrightarrow\infty} n\ Var(\bar{y})$$

$$= \lim_{n\longrightarrow\infty} n\ Var\frac{\sum_{t=1}^{n} y_t}{n}$$

$$= \frac{1}{n}\lim_{n\longrightarrow\infty}\sum_{t=1}^{n} Var(y_t)$$

$$= p_0(1-p_0)$$

Exercise 5:

b:

$$f_Y(y) = \frac{e^{-\lambda}\lambda^y}{y!}$$

$$\ln[f_Y(y)] = -e^{\lambda} + \lambda y - \ln(y!)$$

$$S_n(\beta) = \frac{1}{n}\sum_{t=1}^{n}(-e^{\lambda} + \lambda y - \ln(y!))$$

$$\frac{\partial S_n(\beta)}{\partial \lambda} = \frac{1}{n}\sum_{t=1}^{n}(-e^{\lambda} + y_t) = 0$$

$$\bar{y} = e^{\tilde{\lambda}}$$

$$\tilde{\lambda} = \ln(\bar{y})$$