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Terminal Value, Accounting Numbers, and Inflation

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The question of how inflation affects the market values of publicly-held corporations is once again quite topical. Because many financial analysts and investors fear that inflation rates in European countries are moving back towards the high levels of the 1970s, they worry that stock prices may decline as they did then. Since changes in inflation rates affect manufacturing costs, capital expenditures, prices of goods sold, it seems obvious that inflation will also have an impact on corporate value. What is not obvious, however, is precisely how companies are affected by inflation and why some firms might be affected differently than others.

A number of fairly recent finance papers have also helped revive interest in the topic. For example, in a 2008 article published in this journal, Professors Michael Bradley of Duke University and Gregg Jarrell of the University of Rochester argued that the well-known Gordon-Shapiro (henceforth "GS") model for calculating terminal values does not properly account for the effects of inflation. Bradley and Jarrell propose to modify the growth factor in the standard GS model by adding an additional term to the nominal growth rate that reflects the positive effect of inflation on the value of existing assets.[1]

In this article, we support the original Gordon-Shapiro method for calculating terminal values by showing what we believe to be an oversight of the Bradley-Jarrell critique. We believe that the crux of our disagreement stems from fundamentally different assumptions about the effect of inflation on the capital investment needed to sustain a business. Although Bradley-Jarrell agree with us in principle that intrinsic value is the discounted value of future free cash flows, we find their assumptions about capital investment lead them to conclusions similar to those practitioners who attempt to value firms on the basis of discounted future accounting earnings.

The GS model was meant to be applied to free cash flows though it is very often (even if erroneously) also applied to accounting earnings by practitioners. The economic difference between accounting earnings that involve accruals and free cash flows is quite significant.

In this paper, we will demonstrate formally how our assumptions support GS and differ from Bradley and Jarrell. Before we proceed to the algebra, however, we begin with some insights on the matter of inflation and corporate values from a notably successful financial practitioner.

Warren Buffett on Inflation and Asset Values

In his 1984 Chairmen's Letter to the shareholders of Berkshire Hathaway, Warren Buffett made the following comments on corporate earnings and corporate balance sheets:

The first point to understand is that all earnings are not created equal. In many businesses—particularly those that have high asset/profit ratios—inflation causes some or all of the reported earnings to become ersatz. The ersatz portion—let's call these earnings "restricted"—cannot, if the business is to retain its economic position, be distributed as dividends. Were these earnings to be paid out, the business would lose ground in one or more of the following areas: its ability to maintain its unit volume of sales, its long-term competitive position, its financial strength. No matter how conservative its payout ratio, a company that consistently distributes restricted earnings is destined for oblivion unless equity capital is otherwise infused.

Restricted earnings are seldom valueless to owners, but they often must be discounted heavily. In effect, they are conscripted by the business, no matter how poor its economic potential. (This retention-no-matter-how-unattractive-the-return situation was communicated unwittingly in a marvelously ironic way by Consolidated Edison a decade ago. At the time, a punitive regulatory policy was a major factor causing the company's stock to sell as low as one-fourth of book value; i.e., every time a dollar of earnings was retained for reinvestment in the business, that dollar was transformed into only 25 cents of market value.

But, despite this gold-into-lead process, most earnings were reinvested in the business rather than paid to owners. Meanwhile, at construction and maintenance sites throughout New York, signs proudly proclaimed the corporate slogan, "Dig We Must").

1. See Bradley, Michael H. and Gregg A. Jarrell, 2008, "Expected Inflation and the Constant-Growth Valuation Model," *Journal of Applied Corporate Finance*, Vol. 20, pp. 66-78. For an earlier version that proposes adding the inflation rate to a company's "nominal production yield" (the ratio of nominal earnings to the firm's book value at the beginning of the period), see Michael H. Bradley and Gregg A. Jarrell, 2003, "Inflation and the Constant-Growth Valuation Model: A Clarification," Working Paper, Duke University and University of Rochester. In a similar vein, Jay Ritter and Richard Warr argue that accounting numbers should be adjusted for inflation based on empirical observations of firm value under inflation. See Ritter, Jay R. and Richard S. Warr, 2002, "The decline of inflation and the bull market of 1982–1999," *Journal of Financial and Quantitative Analysis*, Vol. 37, pp. 29–61.

A Tale of Two Companies

The Importance of Distinguishing between Restricted, or Accounting, Earnings and Unrestricted Earnings (Free Cash Flow)

Buffett's ability to identify firms with the potential to generate lots of unrestricted earnings is the cornerstone of his investing success. Such firms are able to expand revenues earnings and cash flows without having to invest proportionately more in capital to support that growth.

In his 1983 Chairman's Letter, Buffett offered the example of a company that "was fortunate enough to obtain some important television stations by original FCC grant" as an example of a firm able to generate high returns on little or no incremental tangible capital. If the population of a city grew and more people bought televisions, the franchise value of the TV station would increase without any additional capital investment.

In an Appendix to the same Chairman's Letter, Buffett described the ability to generate incremental returns with little or no incremental capital investment as "economic goodwill." Economic goodwill might occasionally also correspond to the accounting goodwill on the balance sheet, but often it bore no relation at all.

In this context, he explained why he found the economic characteristics of See's Candies, a California chocolate maker, to be so attractive that he acquired the firm for Berkshire Hathaway despite its being slightly "expensive" according to some commonly used price/earnings ratios:

[T]rue economic Goodwill tends to rise in nominal value proportionally with inflation. To illustrate how this works, let's contrast a See's kind of business with a more mundane business. When we purchased See's in 1972, it will be recalled, it was earning about $2 million on $8 million of net tangible assets. Let us assume that our hypothetical mundane business then had $2 million of earnings also, but needed $18 million in net tangible assets for normal operations. Earning only 11% on required tangible assets, that mundane business would possess little or no economic Goodwill.

A business like that, therefore, might well have sold for the value of its net tangible assets, or for $18 million. In contrast, we paid $25 million for See's, even though it had no more in earnings and less than half as much in "honest-to-God" assets. Could less really have been more, as our purchase price implied? The answer is "yes"—even if both businesses were expected to have flat unit volume—as long as you anticipated, as we did in 1972, a world of continuous inflation.

To understand why, imagine the effect that a doubling of the price level would subsequently have on the two businesses. Both would need to double their nominal earnings to $4 million to keep themselves even with inflation. This would seem to be no great trick: just sell the same number of units at double earlier prices and, assuming profit margins remain unchanged, profits also must double.

But, crucially, to bring that about, both businesses probably would have to double their nominal investment in net tangible assets, since that is the kind of economic requirement that inflation usually imposes on businesses, both good and bad. A doubling of dollar sales means correspondingly more dollars must be employed immediately in receivables and inventories. Dollars employed in fixed assets will respond more slowly to inflation, but probably just as surely. And all of this inflation-required investment will produce no improvement in rate of return. The motivation for this investment is the survival of the business, not the prosperity of the owner.

Remember, however, that See's had net tangible assets of only $8 million. So it would only have had to commit an additional $8 million to finance the capital needs imposed by inflation. The mundane business, meanwhile, had a burden over twice as large—a need for $18 million of additional capital.

After the dust had settled, the mundane business, now earning $4 million annually, might still be worth the value of its tangible assets, or $36 million. That means its owners would have gained only a dollar of nominal value for every new dollar invested. (This is the same dollar-for-dollar result they would have achieved if they had added money to a savings account.)

See's, however, also earning $4 million, might be worth $50 million if valued (as it logically would be) on the same basis as it was at the time of our purchase. So it would have gained $25 million in nominal value while the owners were putting up only $8 million in additional capital—over $3 of nominal value gained for each $1 invested.

Remember, even so, that the owners of the See's kind of business were forced by inflation to ante up $8 million in additional capital just to stay even in real profits. Any unleveraged business that requires some net tangible assets to operate (and almost all do) is hurt by inflation. Businesses needing little in the way of tangible assets simply are hurt the least.

And that fact, of course, has been hard for many people to grasp. For years the traditional wisdom—long on tradition, short on wisdom—held that inflation protection was best provided by businesses laden with natural resources, *plants and machinery, or other tangible assets ("In Goods We Trust"). It doesn't work that way. Asset-heavy businesses generally earn low rates of return—rates that often barely provide enough capital to fund the inflationary needs of the existing business, with nothing left over for real growth, for distribution to owners, or for acquisition of new businesses.*

In contrast, a disproportionate number of the great business fortunes built up during the inflationary years arose from ownership of operations that combined intangibles of lasting value with relatively minor requirements for tangible assets. In such cases, earnings have bounded upward in nominal dollars, and these dollars have been largely available for the acquisition of additional businesses. This phenomenon has been particularly evident in the communications business. That business has required little in the way of tangible investment—yet its franchises have endured. During inflation, [economic] Goodwill is the gift that keeps giving.

Restricted earnings need not concern us further in this dividend discussion. Let's turn to the much-more-valued unrestricted variety. These earnings may, with equal feasibility, be retained or distributed. In our opinion, management should choose whichever course makes greater sense for the owners of the business.

Our differences with Bradley and Jarrell reflect primarily the inability of their approach to distinguish between what Buffett calls "restricted" and "unrestricted" accounting earnings. Obviously, we would all prefer to own businesses generating unrestricted earnings. The reality, however, is that businesses whose earnings are at least to some degree restricted are far more common than those with unrestricted earnings. Most businesses simply have to reinvest a significant portion of their earnings in working and fixed capital just to remain in place. Inflation places an especially heavy burden on corporations with high capital requirements.

The distinction between restricted and unrestricted earnings and between accounting earnings and free cash flow is very relevant to everything that follows. The GS model was originally intended to value companies on the basis not of earnings, but rather their future dividend streams. And because residual dividends are equal roughly to free cash flow, or unrestricted earnings, the GS model was meant to value streams of free cash flow, not conventional accounting earnings. And for any companies requiring significant capital reinvestment, the difference between unrestricted and restricted earnings will be substantial.

In the remainder of this paper, we aim to clarify the link between inflation, nominal values, and real values when standard constant growth models are used to calculate terminal values. Modelling the firm as a set of investment projects, we derive nominal and real, inflation-adjusted valuation formula based on accounting figures. Our results show that the Bradley-Jarrell critique of the GS constant growth valuation formula is not justified, and that the impact of inflation on corporate values is fully captured by the GS model. There is no need to adjust either the growth factor or the return on investment for inflation in general, or in special situations.

A Model of the Firm

Projects and Cash Flows

Now let us consider a simple firm that can invest in one project in each period.[2] The capital investment for the project is represented as b. The projects then generate either *nominal* cash flows (denoted as C) in the manner $\left(b, C_1, \ldots, C_T\right)$, or, *real* cash flows (denoted as c) in the manner $(b, c_1, \ldots, c_T)$.[3]

Nominal and real cash flows are linked by the (constant) rate of inflation π, such that

$$C_t = c_t\left(1+\pi\right)^t \tag{1}$$

We denote the nominal internal rate of return by R and the real internal rate of return as $r = \left(R-\pi\right)/\left(1+\pi\right)$.[4]

Terminal value calculations are extremely sensitive to growth assumptions. We account for growth simply but directly through changes in the project's capital expenditures. This is consistent with a firm operating in a stable but competitive industry. So, growing population could result in growing demand and growth in a firm's capital investments. Real, or inflation-adjusted, growth w is captured by the change of the scale of the firm's representative investment between t and t+1; we assume the investment project to be "scalable," i.e. investment expenditure b and real cash flows c_t both change proportionally. Keeping inflation rate constant

2. For similar approaches see Gjesdal (2004) and Rajan/Reichelstein/Soliman (2007)

3. We follow the convention that capital letters denote nominal figures while normal letters relate to inflation-adjusted, real figures.

4. Note that for the initial cash flow of the representative project, the nominal value equals the real value. We also assume that there is a unique internal rate of return to each cash flow pattern. This holds logically if there is only one change of sign in the cash flow pattern, e.g. a negative cash outflow followed by a sequence of positive cash inflows.

Table 1 **The Key Variables**

Initial investment	b
Nominal cash inflows from the project at date t	C_t
Real cash inflows from the project at date t	c_t
Nominal cash flows	$(b, C_1, \ldots, C_T)$
Real cash flows	$(b, c_1, \ldots, c_T)$
Rate of inflation	π
Nominal internal rate of return	R
Real internal rate of return	$r = (R - \pi) / (1 + \pi)$
Relationship of real cash flows to nominal cash flows	$C_t = c_t (1 + \pi)^t$
Real, inflation-adjusted growth between t and t+1	w
Nominal growth	W
Relationship of nominal growth to real growth	$W = w+\pi+w\pi$
The firms' nominal cost of capital	K
Book value at time zero	BV_0
Real, inflation-adjusted earnings	inc
Nominal earnings	Inc
Relationship of real to nominal earnings	$inc_t = (Inc_t - \pi\, BV_{t-1}) (1+\pi)^{-1}$
Residual income (AKA economic profit or EVA)	RI

over time, the same holds for the nominal cash flows C_t. Thus nominal growth W will be a product of inflation and real growth, such that $W = w + \pi + w\pi$. Note that the internal rate of return IRR of the project does not change, if the size of the representative project changes. A constant internal rate of return seems to be a plausible assumption in particular for terminal value calculations (see Table 1 for a summary of the key variables).

When estimating cash flows for terminal value calculations, we have to consider not only the cash flows of all future projects, but also the cash flows from investments made after time 0 but before the point when terminal value is calculated.[5] The firm's overall cash flow at date 1, therefore, comes from capital expenditures on the project made at date 1 but also the cash flows from investments made in the last T – 1, adjusted for the constant growth rate. As expressed in Equation (2),

$$\begin{aligned} CF_1 &= -b(1+\pi)(1+w) \\ &\quad + c_1(1+\pi) + c_2(1+\pi)(1+w)^{-1} + \ldots \\ &\quad + c_T(1+\pi)(1+w)^{-T+1} \end{aligned} \tag{2}$$

The first term on the right-hand side of Equation 2 represents the capital expenditure of next year's representative project subject to both, growth and inflation. The second term on the right-hand side represents the cash inflow from last year's representative project, subject to inflation. Finally, the following terms represent the cash inflows from previous year's projects, subject to both inflation and growth. Note that we account for inflation by multiplying each cash flow with the inflation term $(1 + \pi)$. Similarly, at date 2, the nominal cash flow will be as follows:

$$\begin{aligned} CF_2 &= -b(1+\pi)^2(1+w)^2 \\ &\quad + c_1(1+\pi)^2(1+w) + c_2(1+\pi)^2 + \ldots \\ &\quad + c_T(1+\pi)^2(1+w)^{-T+2} \\ &= CF_1(1+\pi)^1(1+w)^1 \end{aligned} \tag{3}$$

The nominal cash flow at date 2 can be written as the nominal cash flow at date 1, subject to the annual growth and inflation rate. At date t we obtain

$$\begin{aligned} CF_t &= CF_1(1+\pi)^{t-1}(1+w)^{t-1} \\ &= CF_1(1+W)^{t-1} \end{aligned} \tag{4}$$

Using this cash flow pattern, the terminal value of the firm at date 0 is

5. We normalize the time line such that date 0 indicates the date, when terminal values are calculated. We do not consider the estimation of cash flows before date 0.

$$V_0 = \sum_{t=1}^{\infty} CF_t (1+K)^{-t} = \frac{CF_1}{K-W} \tag{5}$$

where K is the firm's nominal cost of capital. Equation (5) is the well-known constant-growth valuation formula, written in nominal values. Note that using real cash flows in combination with the real cost of capital and the real growth rate will result in exactly the same terminal value.

Accounting Earnings

Now we turn to accounting earnings. For simplicity's sake, we assume depreciation of capital is the only accrual. So, we define accounting income as

$$\begin{aligned} Inc_t^o &= C_t - d_t b \\ &= c_t (1+\pi)^t - d_t b \end{aligned} \tag{6}$$

where $d_t b^0$ is the depreciation expense in period t. We assume clean-surplus accounting in which the sum of the depreciation expenses equals the initial capital expenditure, i.e.,

$$\sum_{t=1}^{T} d_t = 1 .$$

Note that we use the nominal cash flows in our definition of accounting income and we use historical costs as the basis for depreciation expenses as it is common under most accounting standards in low inflation environments.[6]

The firm's total income at date 1 is determined by the last T period's projects, where cash flows and depreciation expenses are adjusted for the constant growth and the inflation rate as follows:

$$\begin{aligned} Inc_1 = &\left(c_1(1+\pi) - d_1 b\right) \\ &+\left(c_2(1+\pi)^2 - d_2 b\right)(1+W)^{-1} + \ldots \\ &+\left(c_T(1+\pi)^{-T} - d_T b\right)(1+W)^{-T+1} \end{aligned} \tag{7}$$

The first term on the right-hand side represents the effects from the last year's representative project on accounting income. While cash inflows are subject to inflation, depreciation charges are based on historical costs. The following terms represent the effect of previous year's projects on accounting income with all cash flows subject to inflation and growth. In terms of the representative project, accounting income at date 1 can be also written as

$$Inc_1 = \sum_{t=1}^{T} Inc_t^o (1+W)^{-t+1} \tag{8}$$

Total income at date 2 will be

$$\begin{aligned} Inc_2 = &\left(c_1(1+\pi) - d_1 b\right)(1+W) \\ &+\left(c_2(1+\pi)^2 - d_2 b\right) + \ldots \\ &+\left(c_T(1+\pi)^T - d_T b\right)(1+W)^{-T+2} \\ &= Inc_1(1+W) \end{aligned} \tag{9}$$

Similarly, income at date t will be

$$Inc_t = Inc_1 (1+W)^{t-1} \tag{10}$$

Hence, accounting income increases with the project's nominal growth rate. Accrual accounting is a nominal concept that uses nominal cash flows. It is important to note that this result holds for any given depreciation policy, as long as the clean surplus condition is going to hold.

Accounting Returns

Now we turn to the book values. At date 0, the book value BV_0 of all assets is the sum of all previous investments less accumulated depreciation, which is

$$\begin{aligned} BV_0 = &\, b + b(1+W)^{-1} + \ldots + b(1+W)^{-T+1} \\ &- d_1 b(1+W)^{-1} - (d_1 + d_2) b(1+W)^{-2} \\ &- \ldots - (d_1 + \ldots + d_{T-1}) b(1+W)^{-T+1} \end{aligned} \tag{11}$$

At date 1, book value is

$$\begin{aligned} BV_1 = &\, b(1+W) + b(1+W)^0 + \ldots + b(1+W)^{-T+2} \\ &- d_1 b(1+W)^{-0} - (d_1 + d_2) b(1+W)^{-1} \\ &- \ldots - (d_1 + \ldots + d_{T-1}) b(1+W)^{-T+2} \\ &= BV_0(1+W) \end{aligned} \tag{12}$$

At date t, book value is

$$BV_t = BV_0 (1+W)^t = BV_0 \left((1+w)(1+\pi)\right)^t \tag{13}$$

Using our growth assumption, book value increases at the nominal growth rate. The (real) change of the project's scale w and the inflation rate π annually increase the investment expenditure and thus the book value of the project currently realized. Assuming w and π to be constant over time,[7] the increase in investment expenditures in the preceding years yields the same effect on the remaining book value of the previous years' projects.

6. Both U.S.-GAAP and IFRS use historical costs as the basis for depreciation measurement.

7. Gjesdal (2004) analyzes changes of the growth rates over time.

Using our expressions for accounting income and the book value, we can calculate the return on investment at date t

$$\mathrm{RoI}_t = \frac{\mathrm{Inc}_t}{\mathrm{BV}_{t-1}} = \frac{\mathrm{Inc}_1}{\mathrm{BV}_0} \tag{14}$$

Our result (14) is critically different from the finding of Bradley and Jarrell (2003), which find the nominal return on investment under inflation to be[8]

$$\mathrm{RoI}_t = \frac{\mathrm{Inc}_t}{\mathrm{BV}_{t-1}} + \pi \tag{15}$$

We believe Bradley and Jarrell (15) have misspecified the real, inflation-adjusted earnings figure inc_t and its link with the nominal income Inc_t. In their equation (13), Bradley/Jarrell 2003 propose the following relation between nominal and real income:

$$\mathrm{Inc}_t = \mathrm{inc}_t(1+\pi)$$

This proposed relationship conflicts with the recognition, common in the accounting literature,[9] that maintaining the firm's asset base under inflation involves an additional cost that can be represented by an accrual whereby the inflation-adjusted income is stated as follows:

$$\mathrm{inc}_t = \left(\mathrm{Inc}_t - \pi\, \mathrm{BV}_{t-1}\right)(1+\pi)^{-1} \tag{16}$$

reflects the necessary reinvestment of earnings $\pi\, \mathrm{BV}_{t-1}$ to maintain the firm's asset base under inflation.[10] If we use formula (16) to represent the real, inflation-adjusted income, we also get the well-known Fisher relation to hold for return on investment: for t=1

$$\mathrm{inc}_1 = \mathrm{BV}_0 \text{ roi and roi} = \frac{\mathrm{inc}_1}{\mathrm{BV}_0} = \frac{\mathrm{RoI} - \pi}{1+\pi}$$

But if we instead followed Bradley/Jarrell's suggestion, we would get the following relation between nominal and inflation-adjusted, real return on investment:

$$\mathrm{roi} = \frac{\mathrm{RoI}}{1+\pi}$$

By contrast, we argue that it is necessary to recognize the cost accrual for inflation—which reveals the firm's nominal rate of return (RoI) be independent of inflation.

Residual Income Valuation

It is well known that valuation models based on residual income[11] should arrive at the same valuation as discounted cash flow valuations. Using our definitions of income numbers and book values, residual income in period t can be written as

$$\mathrm{RI}_t = \mathrm{Inc}_t - K \cdot \mathrm{BV}_{t-1} \tag{17}$$

Note that we use the nominal cost of capital in our definition of residual income. This is consistent with the definition of income on the basis of nominal numbers. Using our growth assumption, we can write residual income as

$$\mathrm{RI}_t = \mathrm{Inc}_1(1+W)^{t-1} - K \cdot \mathrm{BV}_0(1+W)^{t-1} = \mathrm{RI}_1(1+W)^{t-1}$$

Hence, residual income grows at the nominal growth rate as is the case for accounting income and the book value. The terminal value of the firm on the basis of residual income at date 0 is given by

$$V_0 = \mathrm{BV}_0 + \sum_{t=1}^{\infty} \mathrm{RI}_t (1+K)^{-t} = \mathrm{BV}_0 + \frac{\mathrm{RI}_1}{K-W} \tag{18}$$

which is equal to the terminal value that arises on the basis of discounted cash flows. This formulation is consistent with the historical cost formulation of a residual income valuation model under inflation, as derived by O'Hanlon and Peasnell (2004, p. 378). They show that BV_0 must be the book value at date 0, and historical cost residual income is calculated using depreciation expenses based on historical cost.

Inflation, Growth and the Gordon-Shapiro Model

The Gordon-Shapiro Model

The standard valuation model for terminal value calculation is the well-known Gordon-Shapiro model; in its nominal version the valuation equation is

$$V_0 = \frac{\mathrm{Inc}_1(1-q)}{K-W} \tag{19}$$

with q denoting the firm's retention rate to be applied on accounting income in the first period Inc_1.[12] The relation between retention rate and growth is usually established by linking the return on investment with the retained funds via W = q RoI, thus yielding the well-known relation

$$V_0 = \frac{\mathrm{Inc}_1\left(1 - \frac{W}{\mathrm{RoI}}\right)}{K-W} \tag{20}$$

Because of its simplicity and data availability, this formula is widespread used for terminal value calculations. As has been reviewed by Bradley and Jarrell (2008), most textbooks in corporate finance and corporate valuation propose the use of this formula albeit not justifying the use of this formula under inflationary environments.

8. Bradley/Jarrell (2003) Formula (30) on p. 11.
9. See, e.g., Scapens (1981). An early treatment can be found in Spear (1949).
10. Note that this relation is independent from the GAAP determining income and book values.
11. Also known as economic profit or EVA.©
12. In general retention rates are applied upon accounting income figures. In contrast to their 2003 paper Bradley/Jarrell 2008 relate retention rates to the firm's "net cash flow"; see Bradley/Jarrell (2008) p. 68.

Proposed Adjustments to the Gordon-Shapiro Valuation Formula with Inflation

Bradley and Jarrell (2003 and 2008) claim that this standard valuation model is based on an "erroneous specification" and thus does not fully capture the effect of inflation upon corporate value. The authors argue that in certain situations (20) does not account for the impact of inflation upon existing, already acquired assets of the firm. They propose to replace nominal growth W in the nominator of (20) by the following factor reflecting nominal growth under inflation:

$$W' = q \text{ RoI } (1-q)\, \pi \tag{21}$$

The second term $(1-q)\,\pi$ is argued to reflect the "increase in cash flow attributed to the increase in the nominal value of the firm's ... capital that solely results from inflation" (Bradley and Jarrell, 2008, p. 68).

An Analysis of the Gordon-Shapiro Model with Inflation

Based on our model of the firm, we will now analyze the constant growth valuation equation for the terminal value, when inflation is present. Starting point is the firm's current and future set of investment opportunities. We define w^* as the real and W^* as the nominal growth rate of the optimal investment expenditure of the representative project maximizing its current and future net present value.[13] Thus optimal investment in t + 1, $b^*_{t+1} = b^*_t (1+w^*)(1+\pi)$ is determined by the firm's production technology.[14]

As free cash flow is defined after (optimal) investment, optimal investment b^*_{t+1} determines future income and future free cash flows. If free cash flow is fully paid out to shareholders, retention is defined as the difference between income and free cash flow. Using (7) and (2), we can write retention in t = 1 as

$$RET_1 = Inc_1 - CF_1 \tag{22}$$
$$= b\left[(1+\pi)(1+w^*) - \sum_{j=1}^{n} d_j \left((1+\pi)(1+w^*)\right)^{-j+1}\right]$$

Since both Inc_t and CF_t grow by the nominal rate W^*, retention as the difference between the two grows by W^* as well.

As $\sum_{i=1}^{n} d_j = 1$ holds for any depreciation pattern, retention in period 1 is zero, if and only if

$$(1+w^*)(1+\pi) = 1 \tag{23}$$

Put it differently, the amount of cash retained in the business will be positive, if and only if nominal growth is greater than zero. Hence, the sign of the amount retained does not depend on the rate of inflation or the real growth rate alone. Both factors have to be taken into account simultaneously.

Thus the following statement can be made about the two factors driving the absolute amount retained:

- If $w^* > 0$, retention is positive, if and only if the rate of inflation exceeds a negative threshold level that depends on the real growth rate. If the firm's optimal investment program requires scaling up the inflation-adjusted investment expenditure, the positive net investment requires a positive retention. The first term in brackets of (22) reflects the immediate additional investment, the second term is the additional earnings impact caused by the expansion in the past periods. Note that as w^* represents the optimal, value maximizing scale, it may well be smaller than zero, if the firm is operating in an industry with shrinking margins.
- If $\pi > 0$, retention is positive, if and only if the real growth rate exceeds a negative threshold level, depending on the rate of inflation. In this case, an increase in capital expenditures for the optimal scale of the representative project would require the firm to retain some of its income in order to fund it.

Some cases deserve special interest:

1. Zero real growth, positive inflation: $w^* = 0$, $\pi > 0$. If there is no real growth in optimal investment size, but positive inflation, absolute retention in t=1 is

$$RET_1 = Inc_1 - CF_1$$
$$= b\left[(1+\pi) - \sum_{j=1}^{n} d_j \left((1+\pi)\right)^{-j+1}\right] \tag{24}$$

and thus greater than zero. This is not a surprise: if capital expenditures exceed depreciation due to inflation, then cash must be retained to fund the increased investment outlay. The retention rate will be greater than zero. Only in the case where GAAP would allow for immediate expensing, i.e., $d_0 = 1$, $d_t = 0 \forall t \neq 0$, is zero retention a feasible assumption. That is, when earnings are completely unrestricted in the Buffett sense. Nominal growth of income and cash flow W^* is exactly equal to inflation rate in this case: $W^* = \pi$. The appropriate formula for terminal value calculation is

$$V_0 = \frac{CF_1}{K - W^*} = \frac{Inc_1(1-q)}{K-\pi} \tag{25}$$

2. Zero inflation and positive real growth: $\pi = 0$ and $w^* > 0$. In the absence of inflation and with positive real growth $w^* > 0$ absolute retention is positive due to the increase in the optimal scale of the representative project. Terminal value

13. For a similar approach see Rajan/Reichelstein/Soliman (2007, p. 329). Gjesdal (2004, p.11) also notes that the optimal investment is the main factor driving free cash flows.

14. For instance one can think of b^*_{t+1} as being the investment into the representative project that yields the marginal rate of return to be equal to the firm's cost of capital. Clearly in this case the IRR of the representative project is to be interpreted as the average rate of return on the total amount invested.

calculation formula in this case is

$$V_0 = \frac{CF_1}{K - W^*} = \frac{Inc_1(1-q)}{K - w^*} \tag{26}$$

We will now show that in any case and for any combination of w* and π the correct nominal growth rate in the valuation equation is equal to the relation W* = q RoI as proposed in the GS model. We start by setting absolute retention equal to the increase in asset book value between t and t+1; from equation (13) we know that $BV_1 - BV_0 = W^* BV_0$. Thus retention in period 1 is $RET_1 = W^* BV_0$. Note that the increase in book value is purely caused by the combination of real growth in assets w* and inflation π. Additionally the impact of inflation is fully reflected in the increase in investment expenditure b (and the depreciation based on b) and thus book values reflect historic cost. There is no increase in book values due to appreciation. Second, absolute retention can be expressed by combining retention rate and accounting income in period 1: $RET_1 = q\ Inc_1$. Setting the two equations equal to each other, we get

$$BV_0\ W^* = q\ Inc_1 \Leftrightarrow W^* = q\frac{Inc_1}{BV_0} = q\ RoI \tag{27}$$

which is the well-known GS relation. Hence, the presence of inflation does not affect the GS relation.

The Gordon-Shapiro Model for Zero Investments

There is an important special case of the Gordon-Shapiro model, one in which the firm makes zero net investments. The zero net investment case is characterized by $RET_t = 0 \forall t$. According to equation (23) this requires the condition $(1 + w^*)(1 + \pi) = 1$ and thus a negative real growth in investments to compensate the positive inflation rate. When the absolute amount retained is zero, we obtain a zero retention rate and a zero nominal growth. It follows from (19) that the Gordon-Shapiro model for this special case simplifies to

$$V_0 = \frac{Inc_1}{K} \tag{28}$$

Clearly equation (28) is consistent with a real, inflation-adjusted valuation equation, if the condition $(1 + w^*)(1 + \pi) = 1$ applies. Inflation-adjusted valuation equation is

$$V_0 = \frac{cf_1}{k - w^*} \tag{29}$$

Now using the Fisher relation

$$k = \frac{1+K}{1+\pi} - 1$$

and combining it with the above condition

$$w^* = \frac{1}{1+\pi} - 1,$$

we find the correct nominal valuation equation again to be equal to the GS model

$$V_0 = \frac{cf_1}{\frac{K-\pi}{1+\pi} - \frac{-\pi}{1+\pi}} = \frac{CF_1}{K} \tag{30}$$

By defining nominal growth as $W' = q\ RoI + (1-q)\pi$, Bradley and Jarrell get different results from the Gordon-Shapiro model under inflation. We have shown here that under inflation, there is no need to adjust the Gordon-Shapiro model even in the special case of zero net investments.

Combining zero net investment with zero nominal growth requires a special combination of input factors: inflation and price adjusted growth of optimal investment exactly have to offset each other. However there is no need for an adjusted valuation formula.

The Gordon-Shapiro Model for Zero Net Present Value Investments

Bradley and Jarrell also criticize the GS model in a second special case where the firm makes zero net present value investments.[15] If the firm's projects are optimally scaled in real terms and the inflation rate is non-zero, then the zero NPV net investment case requires some additional assumptions: If for instance w* = 0 and thus the optimal scale of the representative project in real terms is constant, an inflation rate π > 0 requires a positive net investment and a positive retention. Because the project still has a positive NPV, the cash retention caused by inflationary increases in capital expenditure by definition cannot be a zero NPV investment. Thus the assumption of a zero NPV investment implies that optimal investment program requires a zero retention rate. Again combining a positive inflation rate π > 0 with an optimal retention rate of zero requires a negative real growth rate of

$$w^* = \frac{1}{1+\pi} - 1.$$

Suppose a firm expands beyond the optimal scale and has to retain and reinvest additional capital in the amount RET_1 at its cost of capital K in t=1; RET_1 thus represents the additional zero NPV investment. Again, with positive NPV investments, the cash retained reflects the nominal growth in

15. The literature usually restricts its attention to the analysis of net investments; Bradley and Jarrell refer to the case of total investments having a zero NPV. Our analysis also relates to the first case: if the firm would not have any positive net present value projects obviously the optimal investment program of the firm would not be defined.

assets: $RET_1 = W'*BV_0$. W′ denotes the nominal growth rate in assets under the sub-optimal investment program including the zero-NPV projects. To link cash retention and growth under the zero-NPV assumption requires an additional assumption ensuring that K = IRR = RoI is going to hold in any future period. Except for the trivial case of a one period lifetime of the representative project, the condition RoI = IRR requires a special depreciation policy of the firm. Following Gjesdal (Gjesdal 2004, p. 10), we assume the firm to follow a "IRR depreciation" for the representative investment that meets the requirement RoI_t = IRR for t=1, ..., n. Under this assumption period t=2 income is

$$\begin{aligned} \mathrm{Inc}_2 &= \mathrm{Inc}_1 + \mathrm{RoI} \cdot \mathrm{RET}_1 = \mathrm{Inc}_1 + \mathrm{Inc}_1 \cdot q \cdot K \\ &= \mathrm{Inc}_1 \left(1 + q \cdot K\right) \end{aligned} \tag{31}$$

Income growth between t=1 and t=2 is then

$$\frac{\mathrm{Inc}_2}{\mathrm{Inc}_1} - 1 = q \cdot K = W' \tag{32}$$

Thus finally the zero – NPV net investment case also yields the GS valuation equation as the correct result:

$$V_0 = \frac{\mathrm{Inc}_1 \left(1 - q\right)}{K - q\,K} = \frac{\mathrm{Inc}_1}{K} \tag{33}$$

Under our specific set of assumptions, even the zero net present value case is also consistent with the Gordon-Shapiro model. Thus there is no need to inflate existing assets in order to properly account for inflation in corporate valuation. As cash flows of the project are determined by factors outside the firm, any increase in book value caused by inflation (if recognized by the GAAP under consideration) will not affect the project's and the firms' cash flows.[16]

Conclusion

The Gordon-Shapiro model has been widely used to calculate terminal values in corporate practice for decades. The terminal value is highly sensitive to the growth parameter in the GS model. In response to recent critiques of the model, we argue that use of the GS leads to correct results when applied to corporate free cash flows and with the consistent use of nominal and real input parameters. At the same time, however, we recognize that the GS model can lead to errors when used with accounting figures such as net income rather than cash flows because the relationships between growth and accounting earnings and returns are not clear.

In sum, recent calls to adjust the standard Gordon-Shapiro model for inflation appear to overlook an important feature of the model and have considerable potential for error. By revealing the origins of the discrepancies arising out of the differences between cash flows and accounting earnings, our paper helps to clarify the use of an important valuation tool under inflationary environments.

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16. Unfortunately Bradley/Jarrell do not give any justification for their proposal to inflate existing assets. Their argument, the increase in book values yields an increase in cash flows (Bradley/Jarrell 2008 p. 67,68) in our eyes is not convincing: as the increase in the firm's capital stock is not caused by newly acquired assets but the increase of book value on *existing* assets neither the IRR on the firm's investments nor the firm's cash flows should be affected. In our model, cash flows are exogenous and thus not affected by any book value appreciation; thus there will be no increase in cash flows caused by this.

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