**Discount Rates and changing Cash Flows and Debt Levels under different Financing Policies**
Joy van der Veer, May 2020

## 1. Introduction

In valuing businesses, the valuator usually chooses between several applicable valuation approaches. In this respect also a decision is needed regarding the assumed financing policy. This means that the valuator should value based on a fixed ratio, a fixed debt or a mixed financing policy. In some cases, the value is based on an all equity assumption, which means that the financing policy is based on the premises that operations are only equity financed. Whatever the choice is, the presumption must be that the valuation outcome should be equal, regardless the executed valuation approach. This means that the valuation outcome based on the WACC-approach should be equal to that of the APV-approach, assuming the predetermined variables are equal for both approaches.

For residual values, this can be achieved without much complications and is therefore straightforward. We have the following different residual APV approaches available,

| | *Applicability* | | | | | | |
|---|---|---|---|---|---|---|---|
| **APV model** | **Cash flow no growth** | **Cash flow with growth** | **Debt no growth** | **Debt with growth** | **fixed debt** | **fixed ratio** | **mixed** |
| **Harris & Pringle** | yes | yes | yes | yes | no | yes | - |
| **M&M / Hamada** | yes | no/yes | yes | no | yes | no | - |
| **Myers** | no | yes | no | yes | yes | no | - |
| **Vd Veer / de Roon** | yes | yes | yes | yes | - | - | yes |

The fundamental difference between Fixed Debt and Fixed Ratio is that using the Fixed Debt proposition means that it is assumed that the annual tax shield is "fixed" and therefore is known at the beginning of each fiscal year, in contrast, using the fixed ratio implies that the tax shield is not known at the beginning of each fiscal year and depends on the cash flow expectations during the fiscal year and beyond, and most likely the development of net working capital[1].

[1] It can be assumed that higher free cash flows will increase net operational working capital and as a result a higher tax shield if working capital is partially financed with debt based on a fixed percentage (fixed ratio) of this working capital.

In case of a fixed ratio financing policy there is a strong correlation between expected free cash flows and tax shields, the linkage between the APV and the WACC approaches then is easy to establish. It becomes more difficult to define the linkage between the APV and the WACC approach assuming a fixed debt financing policy. This becomes even more difficult when we are valuing annually different expected cash flows and debt levels, as usually is the case in the forecast period.

In valuing businesses, valuators wants to establish a linkage between the APV and the WACC approach to check and assess outcomes. In this short paper this linkage between these valuation approaches is discussed. Section two provides a short summary regarding the residual APV-WACC linkage equations, seen from different financing policy assumptions. In the third (traditional approaches) and the fourth (mixed policy approaches) sections the linkages are covered regarding changing cash flows and debt levels. The last section provides closing thoughts.

## 2. Residual APV-WACC approaches

In this section, a short summary is given regarding the residual APV-WACC equations under different financing policies. For a more in depth and thorough analysis, the working paper "Discount Rates and Financing Policy" should be studied.

**<u>Expected Free Cash Flow without growth and fixed debt financing policy (M&M)</u>**

The firm value can be valued following the APV and WACC approach under the presumption that the outcomes must be equal,

$$V_L = \underbrace{\frac{E(FcF)}{k_u} + \frac{D_0 * Kd * Tc}{Kd} = \frac{E(FcF)}{k_u} + D_0 * Tc}_{\text{APV approach}} = \underbrace{\frac{E(FcF)}{wacc} = \frac{E(FcF)}{k_e * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D}{V_L}}}_{\text{Wacc approach}} \tag{1}$$

Based on this equation, and the cost of equity unlevered ($k_u$) is known, the cost of equity levered ($k_e$) can be determined using the following equation,

$$k_e = k_u + (k_u - Kd) * (1 - Tc) * \frac{D}{E} \tag{2}$$

**Expected Free Cash Flow with growth and fixed debt financing policy (M&M/Hamada)**

The firm value can be valued following the APV and WACC approach under the presumption that the outcomes must be equal,

$$V_L = \underbrace{\frac{E(FcF)}{k_u - g} + \frac{D_0 * Kd * Tc}{Kd} = \frac{E(FcF)}{k_u - g} + D_0 * Tc}_{\text{APV approach}} = \underbrace{\frac{E(FcF)}{wacc - g} = \frac{E(FcF)}{k_e * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D}{V_L} - g}}_{\text{Wacc approach}} \tag{3}$$

Based on this equation, and the cost of equity unlevered ($k_u$) is known, the cost of equity levered ($k_e$) can be determined using the following equation,

$$k_e = k_u + [k_u - Kd] * [1 - Tc] * \frac{D}{E} + g * \frac{D}{E} * Tc \tag{4}$$

An important implication executing this approach is that the debt-ratio will decrease year by year as time passes and will tend to zero in the limit.

**Expected Free Cash Flow with and without growth and fixed ratio financing policy (Harris & Pringle)**

Because the fixed ratio financing policy is at the core of this approach, there is no difference in executing the APV approach with or without growth. The option with cash flow growth is presented here.

The firm value can be valued following the APV and WACC approach under the presumption that the outcomes must be equal,

$$V_L = \underbrace{\frac{E(FcF)}{k_u - g} + \frac{D_0 * Kd * Tc}{k_u - g}}_{\text{APV approach}} = \underbrace{\frac{E(FcF)}{wacc - g} = \frac{E(FcF)}{k_e * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D}{V_L} - g}}_{\text{Wacc approach}} \tag{5}$$

Based on this equation, and the cost of equity unlevered ($k_u$) is known, the cost of equity levered ($k_e$) can be determined using the following equation,

$$k_e = k_u + [k_u - Kd] * \frac{D}{E} \tag{6}$$

This is a straight forward equation with the basic assumption that the debt ratio remains constant.

**<u>Expected Free Cash Flow and fixed debt with growth (Myers)</u>**

Myers' APV approach is an adjustment on the Hamada APV approach where Myers assumes that besides expected cash flows also debt should grow with the same growth rate as the expected cash flows, but under the basic assumption of fixed debt.

The firm value can be valued following the APV and WACC approach under the presumption that the outcomes must be equal,

$$V_L = \underbrace{\frac{E(FcF)}{k_u - g} + \frac{D_0 * Kd * Tc}{Kd - g}}_{\text{APV methode}} = \underbrace{\frac{E(FcF)}{wacc - g} = \frac{E(FcF)}{k_e * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D}{V_L} - g}}_{\text{Wacc methode}} \tag{7}$$

Based on this equation, and the cost of equity unlevered ($k_u$) is known, the cost of equity levered ($k_e$) can be determined using the following equation,

$$k_e = k_u + \left[k_u - Kd * \left[1 + Tc * \left[\frac{k_u - g}{kd - g} - 1\right]\right]\right] * \frac{D}{E} \tag{8}$$

Compared to the Harris & Pringel and Hamada leverage equation, this equation is more complicated due to the debt-growth rate under the fixed debt assumption.

The above expressed approaches are based on a fixed ratio or a fixed debt assumption. In many cases, it is difficult to decide which approach is applicable because there are often fixed debt and fixed ratio elements present[2]. A simple solution in this perspective is the mixed policy approach where we can incorporate fixed debt and fixed ratio components simultaneously. The only fundamental question is whether fixed debt should grow or remain constant. We can express the following two mixed policy approaches,

- Fixed debt without growth;
- Fixed debt with growth, which growth rate is equal to that of the cash flow.

**Fixed debt without growth**

The firm value can be valued following the APV and WACC approach under the presumption that the outcomes must be equal,

$$APV - variant: V_L = \frac{E(FcF)}{k_u - g} + \frac{D_f * Kd * Tc}{Kd} + \frac{D_R * Kd * Tc}{k_u - g} \tag{9}$$

[2] Often fixed assets are financed with long term “fixed” debt whereas net working capital investments are financed using roll-over banking facilities (short term debt). Many firms using long term debt as well as short term debt financing the business.

$$Wacc - variant: V_L = \frac{E(FcF)}{wacc - g} = \frac{E(FcF)}{k_e * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D_f}{V_L} + Kd * (1 - Tc) * \frac{D_R}{V_L}} \quad (10)$$

The Wacc and the cost of equity levered can be determined, based on equations (9) and (10), as follows,

$$wacc = k_u - \left[k_u * \frac{D_f}{V_L} + Kd * \frac{D_R}{V_L}\right] * Tc + g * Tc * \frac{D_f}{V_L} \quad (11)$$

and

$$k_e = k_u + [k_u - Kd] * \left[\frac{D_{TL} - Tc * D_f}{E}\right] + g * Tc * \frac{D_f}{E} \quad (12)$$

**Fixed debt with growth**

The firm value can be valued following the APV and WACC approach under the presumption that the outcomes must be equal,

$$APV - variant: V_L = \frac{E(FcF)}{k_u - g} + \frac{D_f * Kd * Tc}{Kd - g} + \frac{D_R * Kd * Tc}{k_u - g} \quad (13)$$

$$Wacc - variant: V_L = \frac{E(FcF)}{wacc - g} = \frac{E(FcF)}{k_e * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D_f}{V_L} + Kd * (1 - Tc) * \frac{D_R}{V_L}} \quad (14)$$

The Wacc and the cost of equity levered can be determined, based on equations (13) and (14), as follows,

$$wacc = k_u - Kd * Tc * \left[\frac{D_R}{V_L} + \left[\frac{k_u - g}{Kd - g}\right] * \frac{D_f}{V_L}\right] \quad (15)$$

and

$$k_e = k_u + [k_u - Kd] * \left[\frac{D_{TL}}{E}\right] + Kd * Tc * \frac{D_f}{E} * \left[1 - \frac{k_u - g}{Kd - g}\right] \quad (16)$$

Although it seems easier to execute a valuation approach based on fixed debt or fixed ratio, in many valuation cases the mixed policy approach is more realistic and should therefore be used.

In this section only the residual approaches are covered. They are straightforward and relatively easy to execute. It becomes more difficult when cash flows are changing in the forecast period. A check between APV and WACC is not straightforward any longer for most approaches. An exception is the Harris & Pringle (fixed ratio) approach. This approach remains relatively easy to execute. The next section will cover this linkage between the APV and WACC approaches in the forecast period.

## 3. Linkage traditional APV-WACC approaches in forecast period

In the previous section, I discussed the residual APV and WACC approaches broadly. These approaches are in general straightforward without any major complexities. The linkage between the APV and WACC approaches in the forecast period are a bit more complex, although this is the case when fixed debt is involved. If we use the fixed ratio assumption, the linkage is (again) straightforward and not complex. In this section first the fixed ratio approach is covered before going into the more complex approaches based on fixed debt policies. A simple example is used showing the linkage between the approaches under the presumption that the outcomes of APV and Wacc approaches should be equal. The following base case example is used for all covered approaches in this paragraph,

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ku | 10,00% | | | | | | | |
| Kd | 4,00% | | | | | | | |
| Tc | 25,00% | | | | | | | |
| Growth | 1,50% | | | | | | | |
| | | 0 | 1 | 2 | 3 | 4 | 5 | TV |
| E(FcF) | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 2.588,25 | 2.627,07 |
| Debt | | 5.000 | 4.750 | 4.500 | 4.250 | 4.000 | 3.750 | |
| Kd * Do | | | 200 | 190 | 180 | 170 | 160 | 150 |

Important observations regarding this example are that the expected free cash flows and annual debt levels are year-over-year not equal. In practice, this is often the case.

**<u>Harris & Pringle – Fixed Ratio approach</u>**

In the forecast period, we must link the APV and WACC approach for a single period (one year). This means that the firm value can be valued on the following APV and WACC approach under the presumption that the outcomes must be equal,

$$V_{L,t-1} = \frac{PV_{u,t} + E(FcF)_t}{1+k_u} + \frac{PV_{TS,t} + D_{t-1} * Kd * Tc}{1+k_u} = \frac{V_{L,t} + E(FcF)_t}{1+wacc} = \frac{V_{L,t} + E(FcF)_t}{1 + k_e * \frac{E_{t-1}}{V_{L,t-1}} + Kd * (1-Tc) * \frac{D_{t-1}}{V_{L,t-1}}} \quad (17)$$

Equation (17) can be rearranged as follows,

$$V_{L,t-1} = \frac{PV_{u,t} + E(FcF)_t}{1+k_u} + \frac{PV_{TS,t} + D_{t-1} * Kd * Tc}{1+k_u} \rightarrow V_{L,t-1} * (1+k_u) = PV_{u,t} + E(FcF)_t + PV_{TS,t} + D_{t-1} * Kd * Tc \rightarrow$$

$$(1+k_u) = \frac{PV_{u,t} + E(FcF)_t + PV_{TS,t}}{V_{L,t-1}} + \frac{D_{t-1}}{V_{L,t-1}} * Kd * Tc \rightarrow \frac{PV_{u,t} + E(FcF)_t + PV_{TS,t}}{V_{L,t-1}} = \mathbf{1 + \boldsymbol{Wacc}}$$

The Wacc and the cost of equity levered can be determined, based on equation (17), as follows,

$$wacc = k_u - Kd * Tc * \frac{D_{t-1}}{V_{L,t-1}} \quad (18)$$

and,

$$k_e = k_u + [k_u - Kd] * \frac{D_{t-1}}{E_{t-1}} \quad (19)$$

Equations (18) and (19) are equal to the equations for the residual Harris & Pringle approach.

Based on the base case example the linkage between the APV and the WACC approach can be shown as follows,

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Ku** | **10,00%** | | | | | | | |
| **Kd** | **4,00%** | | | | | | | |
| **Tc** | **25,00%** | | | | | | | |
| **Growth** | **1,50%** | | | | | | | |
| | | **0** | **1** | **2** | **3** | **4** | **5** | **TV** |
| **E(FcF)** | | | **975,00** | **1.275,00** | **2.500,00** | **2.550,00** | **2.588,25** | **2.627,07** |
| **Debt** | | 5.000 | 4.750 | 4.500 | 4.250 | 4.000 | 3.750 | |
| **Kd * Do** | | | 200 | 190 | 180 | 170 | 160 | 150 |
| **TS** | | | 50,00 | 47,50 | 45,00 | 42,50 | 40,00 | 37,50 |
| **E(FCF)+TS** | | | 1.025,00 | 1.322,50 | 2.545,00 | 2.592,50 | 2.628,25 | 2.664,57 |
| | | | | | | | 31.347,93 | |
| **Total Cash flow** | | | 1.025,00 | 1.322,50 | 2.545,00 | 2.592,50 | 33.976,18 | |
| **Value levered APV** | | **26.804,13** | | | | | | |
| **APV - WACC linkage** | | | | | | | | |
| **Value 1/1** | | | 26.804,13 | 28.459,55 | 29.983,00 | 30.436,30 | 30.887,43 | |
| **Wacc** | | | 2.630,41 | 2.798,45 | 2.953,30 | 3.001,13 | 3.048,74 | |
| **FcF** | | | (975,00) | (1.275,00) | (2.500,00) | (2.550,00) | (2.588,25) | |
| **Value 31/12** | | | 28.459,55 | 29.983,00 | 30.436,30 | 30.887,43 | 31.347,93 | |
| **D/E 1/1** | | | 0,23 | 0,20 | 0,18 | 0,16 | 0,15 | |
| **Ke** | **Eq. (19)** | | 11,38% | 11,20% | 11,06% | 10,97% | 10,89% | |
| **Wacc** | **Eq. (18)** | | 9,81% | 9,83% | 9,85% | 9,86% | 9,87% | |

## Fixed debt approaches

In the previous section, I discussed several residual value approaches seen from a fixed debt perspective. For the forecast period, they have all the same challenge. This can be expressed as follows,

$$V_{L,t-1} = \frac{PV_{u,t} + E(FcF)_t}{1 + k_u} + \frac{PV_{TS,t} + D_{t-1} * Kd * Tc}{1 + Kd} = \frac{V_{L,t} + E(FcF)_t}{1 + wacc} = \frac{V_{L,t} + E(FcF)_t}{1 + k_e * \frac{E_{t-1}}{V_{L,t-1}} + Kd * (1 - Tc) * \frac{D_{t-1}}{V_{L,t-1}}} \quad (20)$$

This means that the valuator must decide which residual value approach is applicable for the actual valuation case. If the valuator made up his or her mind, the linkage between the APV and WACC approach is equal for all residual value approaches based on the fixed debt assumption for the forecast period.

One important observation can be made regarding these fixed debt approaches. If debt levels are constant in the forecast period and beyond (fixed=fixed), equation (20) can be simplified as follows,

$$V_{L,t-1} = \frac{PV_{u,t} + E(FcF)_t}{1 + k_u} + D_{t=0} * Tc = \frac{V_{L,t} + E(FcF)_t}{1 + wacc} = \frac{V_{L,t} + E(FcF)_t}{1 + k_e * \frac{E_{t-1}}{V_{L,t-1}} + Kd * (1 - Tc) * \frac{D_{t-1}}{V_{L,t-1}}} \tag{21}$$

The debt level at valuation date (t=0) remains constant for all future years. Even dealing with changing expected cash flows, the treatment of this fixed debt approach for the forecast period is straightforward, as we can prove as follows,

$$V_{L,t-1} = \frac{PV_{u,t} + E(FcF)_t}{1 + k_u} + D_{t=0} * Tc \rightarrow 1 + k_u = \frac{PV_{u,t} + E(FcF)_t + D_{t=0} * Tc * (1 + k_u)}{V_{L,t-1}}$$

$$\rightarrow \frac{PV_{u,t} + E(FcF)_t + D_{t=0} * Tc}{V_{L,t-1}} + \frac{D_{t=0} * Tc * k_u}{V_{L,t-1}} \rightarrow 1 + Wacc = \frac{PV_{u,t} + E(FcF)_t + D_{t=0} * Tc}{V_{L,t-1}}$$

and,

$$wacc = k_u * \left(1 - Tc * \frac{D_{t=0}}{V_{L,t-1}}\right) \tag{22}$$

Based on equation (22) and using equation (21) the cost of equity levered can be written as follows,

$$k_e = k_u + [k_u - Kd] * [1 - Tc] * \frac{D_{t=0}}{E_{t-1}} \tag{23}$$

In literature equations (22) and (23) are the so-called Hamada-equations, with the implicit fixed debt assumption. Equation (22) can only be used in the forecast period setting. For the determination of the residual value, with the fixed debt assumption plus a constant cash flow growth rate, we must use the following Hamada equation,

$$Wacc = k_u - [k_u - g] * \frac{D}{V_L} * Tc \tag{24}$$

To show how to execute the Hamada-approach, I have adjusted the base case in a way that the debt-level at valuation date remains constant. Based on the adjusted base case example the linkage between the APV and the WACC approach can be shown as follows,

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ku | | 10,00% | | | | | | | | | |
| Kd | | 4,00% | | | | | | | | | |
| Tc | | 25,00% | | | | | | | | | |
| Growth | | 1,50% | | | | | | | | | |
| | | | **0** | **1** | **2** | **3** | **4** | **5** | **TV** | | |
| E(FcF) | | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 2.588,25 | 2.627,07 | | |
| Debt | | | 5.000 | 5.000 | 5.000 | 5.000 | 5.000 | 5.000 | | | |
| Kd * Do | | | | 200 | 200 | 200 | 200 | 200 | 200 | | |
| **APV** | | | | | | | | | | | |
| **E(FcF)** | | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 2.588,25 | 2.627,07 | | |
| **RV** | | | | | | | | 30.906,75 | | | |
| **Total E(fcf)** | | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 33.495,00 | | | |
| **Value unlevered** | | | 26.357,81 | | | | | | | | |
| **Value TS** | =D*Tc | | 1.250,00 | | | | | | | | |
| **Value levered** | | | 27.607,81 | | | | | | | | |
| **APV - WACC linkage** | | | | | | | | | | | |
| **Value 1/1** | | | | 27.607,81 | 29.268,60 | 30.795,45 | 31.250,00 | 31.700,00 | | | |
| **Wacc** | | | | 2.635,78 | 2.801,86 | 2.954,55 | 3.000,00 | 3.045,00 | | | |
| **FcF** | | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 2.588,25 | | **Wacc RV** | |
| **Value 31/12** | | | | 29.268,60 | 30.795,45 | 31.250,00 | 31.700,00 | 32.156,75 | 32.156,75 | 9,67% | **Eq. (24)** |
| | | | | | | | | | FcF / w-g | | |
| **D/E 1/1** | | | | **0,22** | **0,21** | **0,19** | **0,19** | **0,19** | | | |
| **Ke** | **Eq. (23)** | | | **11,00%** | **10,93%** | **10,87%** | **10,86%** | **10,84%** | | | |
| **Wacc** | **Eq. (22)** | | | **9,55%** | **9,57%** | **9,59%** | **9,60%** | **9,61%** | | | |

The eminent difference between the Wacc-equations (22) and (24) is that equation (22) is used on a year-over-year basis and equation (24) for the residual value based on a constant cash flow growth rate.

It becomes more complicated when debt-levels are not constant in the forecast period, which in practice is often the case. I start with the following basic equation,

$$V_{L,t-1} = \frac{PV_{u,t} + E(FcF)_t}{1 + k_u} + \frac{PV_{TS,t} + D_{t-1} * Kd * Tc}{1 + Kd} \tag{25}$$

Equation (25) can be rearranged as follows,

$$V_{L,t-1} = \frac{PV_{u,t} + E(FcF)_t}{1 + k_u} + \frac{PV_{TS,t} + D_{t-1} * Kd * Tc}{1 + Kd} \to V_{L,t-1} * (1 + k_u)$$
$$= PV_{u,t} + E(FcF)_t + \boldsymbol{PV_{TS,t}} - \boldsymbol{PV_{TS,t}} + \frac{\left(PV_{TS,t} + D_{t-1} * Kd * Tc\right) * (1 + k_u)}{1 + Kd} \to$$

$$(1 + k_u) = \frac{PV_{u,t} + E(FcF)_t + \boldsymbol{PV_{TS,t}}}{V_{L,t-1}} - \frac{\boldsymbol{PV_{TS,t}}}{V_{L,t-1}} + \left(\frac{PV_{TS,t} + TS_t}{V_{L,t-1}}\right) * \left(\frac{1 + k_u}{1 + kd}\right) \to \quad (1 + k_u)$$
$$= (1 + Wacc) - \frac{\boldsymbol{PV_{TS,t}}}{V_{L,t-1}} + \left(\frac{PV_{TS,t} + TS_t}{V_{L,t-1}}\right) * \left(\frac{1 + k_u}{1 + kd}\right) \to$$

$$Wacc = k_u - \left(\frac{PV_{TS,t} + TS_t}{V_{L,t-1}}\right) * \left(\frac{1 + k_u}{1 + kd}\right) + \frac{PV_{TS,t}}{V_{L,t-1}} \quad (26)$$

In essence of equation (26) lies the development of the present value of the expected tax shields, which are related to the development of the debt levels. The Wacc is besides the cost of equity unlevered, depended on the development of the expected debt-levels. This sounds logical because a higher leverage means a higher debt level and therefore higher tax shields, and consequently a lower Wacc and a higher firm value.

If we assume a fixed ratio (Harris & Pringle) or the Hamada (fixed debt and growing cash flows) proposition, we can determine the Wacc easily. Using the fixed ratio, we need a fixed debt-equity ratio to determine the Wacc (see equation (18)). If we want to execute the Hamada proposition, we need the debt-equity ratio at valuation date and a constant cash flow growth rate (using the residual value approach). If we have changing cash flows and changing debt-levers, the determination of the annual Wacc is much more complicated. If this is the case, it is easier to execute the APV approach. Knowing the annual values based on the APV approach, the linkage with the WACC approach can be established using equation (26).

The cost of equity levered can be determined using the equation (26) and (20) as follows,

$$k_e * \frac{E_{t-1}}{V_{L,t-1}} + Kd * (1 - Tc) * \frac{D_{t-1}}{V_{L,t-1}} = k_u - \left(\frac{PV_{TS,t} + TS_t}{V_{L,t-1}}\right) * \frac{(1 + k_u)}{(1 + kd)} + \frac{PV_{TS,t}}{V_{L,t-1}} \rightarrow$$

$$k_e * \frac{E_{t-1}}{E_{t-1}} + Kd * (1 - Tc) * \frac{D_{t-1}}{E_{t-1}} = k_u * \frac{PV_{TS,t}}{E_{t-1}} - \left(\frac{PV_{TS,t} + TS_t}{E_{t-1}}\right) * \frac{(1 + k_u)}{(1 + kd)} + \frac{PV_{TS,t}}{E_{t-1}} \rightarrow$$

$$k_e = k_u * \frac{PV_{TS,t}}{E_{t-1}} - \left(\frac{PV_{TS,t} + TS_t}{E_{t-1}}\right) * \frac{(1 + k_u)}{(1 + kd)} + \frac{PV_{TS,t}}{E_{t-1}} - kd * (1 - Tc) * \frac{D_{t-1}}{E_{t-1}} \rightarrow$$

$$k_e = k_u + k_u * \frac{D_{t-1}}{E_{t-1}} - \left(\frac{PV_{TS,t} + TS_t}{E_{t-1}}\right) * \frac{(1 + k_u)}{(1 + kd)} + \frac{PV_{TS,t}}{E_{t-1}} - kd * (1 - Tc) * \frac{D_{t-1}}{E_{t-1}} \rightarrow$$

$$k_e = k_u + [k_u - kd * (1 - Tc)] * \frac{D_{t-1}}{E_{t-1}} + \frac{PV_{TS,t} * \left[1 - \frac{(1 + k_u)}{(1 + kd)}\right] - TS_t * \frac{(1 + k_u)}{(1 + kd)}}{E_{t-1}} \quad (27)$$

Before we can execute this approach the valuator must decide which residual fixed debt valuation approach is applicable. The fundamental question is whether fixed debt should grow or remain constant.

We can express the following fixed debt policy approaches for the residual value,

- Fixed debt without growth (Hamada, see equation (4));
- Fixed debt with growth, which is equal to the cash flow growth rate (Myers, see equation (8)).

Based on the base case example and the fixed debt approach with changing cash flows and debt-levels, the linkage between the APV and WACC approach can be shown as follows,

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Ku | 10,00% | | | | | | | | |
| Kd | 4,00% | | | | | | | | |
| Tc | 25,00% | | | | | | | | |
| Growth | 1,50% | | | | | | | | |
| | | **0** | **1** | **2** | **3** | **4** | **5** | **TV** | |
| E(FcF) | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 2.588,25 | 2.627,07 | |
| Debt | | 5.000 | 4.750 | 4.500 | 4.250 | 4.000 | 3.750 | | |
| Kd * Do | | | 200 | 190 | 180 | 170 | 160 | 150 | |
| APV | | | | | | | | | |
| E(FcF) | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 2.588,25 | 2.627,07 | |
| | | | | | | | 30.906,75 | | |
| Total E(fcf) | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 33.495,00 | | |
| Value unlevered | | 26.357,81 | 28.018,60 | 29.545,45 | 30.000,00 | 30.450,00 | 30.906,75 | | |
| TaxShield | | | 50,00 | 47,50 | 45,00 | 42,50 | 40,00 | 37,50 | |
| | | | | | | | **937,50** | **Fixed Debt Hamada (!)** | |
| Total TS | | | 50,00 | 47,50 | 45,00 | 42,50 | 977,50 | | |
| Value TS | | 971,76 | 960,63 | 951,56 | 944,62 | 939,90 | 937,50 | | |
| **Value levered** | | **27.329,57** | **28.979,23** | **30.497,01** | **30.944,62** | **31.389,90** | **31.844,25** | | |
| Value 1/1 | | | 27.329,57 | 28.979,23 | 30.497,01 | 30.944,62 | 31.389,90 | | |
| Wacc | | | 2.624,65 | 2.792,78 | 2.947,61 | 2.995,28 | 3.042,60 | | |
| FcF | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 2.588,25 | **Wacc** | |
| Value 31/12 | | | 28.979,23 | 30.497,01 | 30.944,62 | 31.389,90 | 31.844,25 | **Eq. (28)** | **9,75%** |
| | | | | | | | | | **31.844,2500** |
| D/E 1/1 | | | 0,22 | 0,20 | 0,17 | 0,16 | 0,15 | | |
| **Ke** | **Eq. (27)** | | 11,08% | 10,94% | 10,82% | 10,74% | 10,67% | | |
| **Wacc** | **Eq. (26)** | | 9,604% | 9,637% | 9,665% | 9,680% | 9,693% | | |

In this example I used the fixed debt Hamada residual approach (no debt-level growth). To determine the Wacc for the residual value I used, based on equation (3), the following equation,

$$Wacc = k_u - (k_u - g) * Tc * \frac{D_{t-1}}{V_{t-1}} \qquad (28)$$

If the valuator chooses to use the Myers approach for the residual value, an adjustment is easily made. First, the residual tax shield value should be recalculated using the predetermined growth rate, followed by the adjustment of the Wacc for the residual value, based on equation (7). The approach for the forecast period remains the same.

The following table shows the residual value based on the Myers approach,

| | | |
|---|---|---|
| TaxShield Residual Value | 1.500,00 | Fixed Debt Myers |
| Value Levered | 32.406,75 | Residual Value |
| Wacc - Myers approach | 9,61% | Eq. (7) |
| Ke | 10,47% | Eq. (8) |
| Value Levered (check) | 32.406,75 | Residual Value |

In sum we can conclude that the Harris & Pringle approach is easy to execute in all valuation settings. It becomes more complicated when fixed debt is involved in a setting where a linkage between the APV and WACC approach must be established. If we use the fixed debt approach with changing debt-levels in the forecast period, the linkage between APV and WACC becomes even more complicated.

In the next section the linkage between APV and WACC is covered based on a mixed-financing-policy.

## 4. Linkage APV-WACC and mixed policies

The linkage between the APV and the WACC approach regarding the traditional approaches, related to the forecast period, is covered in the previous paragraph. Using traditional approaches, the choice for the valuator is binary, he chooses the fixed ratio or the fixed debt financing policy. In practice, however, it is often more likely to choose for a mixed financing policy. Fixed assets are often financed based on fixed debt, with a predetermined amortization scheme, in contrast, net working capital is often financed based on a business line of credit (revolving credit), mostly granted by a financial institution. Using revolving credits as financing instrument means in practice that there is (normally) a strong correlation between net working capital and the revolving credits or flexible debt. Due to this correlation we can also assume that there is also a strong relationship between the tax shields regarding the revolving credits and the cash flows, which basically implies a fixed ratio approach.

Valuing a business based on the WACC approach with mixed financing policies is difficult and hardly doable. It is more convenient to value based on the APV approach.

We can establish a linkage between these approaches (mixed financing policies) based on equations (17) and (20). Based on these equations, the following basic equations regarding the mixed policies for the forecast period can be presented,

$$APV - variant: V_{L,t-1} = \frac{PV_{u,t} + E(FcF)_t}{1 + k_u} + \frac{PV_{TS,f,t} + D_{f,t-1} * Kd * Tc}{1 + Kd} + \frac{PV_{TS,R,t} + D_{R,t-1} * Kd * Tc}{1 + k_u} \quad (29)$$

$$Wacc - variant: V_L = \frac{E(FcF)}{wacc - g} = \frac{E(FcF)}{k_e * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D_f}{V_L} + Kd * (1 - Tc) * \frac{D_R}{V_L}} \quad (14)$$

I assumed that Kd regarding fixed debt is equal to Kd regarding fixed ratio. In practice this will not often be the case, but the differences are mostly minor and therefore, seen from a valuation outcome perspective, negligible. Equations (29) and (14) must generate the same outcome, as stated before.

For the residual value I stated earlier that the fundamental question is whether fixed debt should grow or remain constant. Regarding the forecast period this question is not relevant. This means that the valuator must decide to base the residual value on the premises that debt should growth with a constant growth rate or remain fixed. Based on this residual value, we can determine the value at valuation date based on equation (29).

To show how this works, the following base case example is used,

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Ku | 10,00% | | | | | | |
| Kd | 4,00% | | | | | | |
| Tc | 25,00% | | | | | | |
| Growth | 1,50% | | | | | | |
| | | 0 | 1 | 2 | 3 | 4 | 5 | TV |
| E(FcF) | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 2.588,25 | 2.627,07 |
| Debt (Fixed) | | 4.000 | 3.750 | 3.500 | 3.250 | 3.000 | 2.500 | |
| Kd * Do | | | 160 | 150 | 140 | 130 | 120 | 100 |
| Debt ("Ratio") | | 2.000 | 2.100 | 2.400 | 2.600 | 2.639 | 2.679 | |
| Kd* Do | | | 80 | 84 | 96 | 104 | 106 | 107 |

Before this example can be executed, we first have to rewrite equation (29) to establish a suitable Wacc-equation which can be used valuing the year-over-year free cash flows in the forecast period.

Equation (29) can be rearranged as follows,

$$V_{L,t-1} = \frac{PV_{u,t} + E(FcF)_t}{1 + k_u} + \frac{PV_{TS,f,t} + D_{f,t-1} * Kd * Tc}{1 + Kd} + \frac{PV_{TS,R,t} + D_{R,t-1} * Kd * Tc}{1 + k_u} \rightarrow$$

$$V_{L,t-1} = \frac{PV_{u,t} + E(FcF)_t}{1 + k_u} + \frac{PV_{TS,f,t} + TS_{f,t}}{1 + Kd} + \frac{PV_{TS,R,t} + TS_{R,t}}{1 + k_u} \rightarrow$$

$$V_{L,t-1} * (1 + k_u) = \left[\boldsymbol{PV_{u,t} + E(FcF)_t + PV_{TS,R,t} + PV_{TS,f,t}}\right] - PV_{TS,f,t} + \left[PV_{TS,f,t} + TS_{f,t}\right] * \frac{(1 + k_u)}{(1 + Kd)} + TS_{R,t} \rightarrow$$

$$k_u = WACC - \frac{PV_{TS,f,t}}{V_{L,t-1}} + Kd * Tc * \frac{D_{R,t-1}}{V_{L,t-1}} + \frac{\left[PV_{TS,f,t} + TS_{f,t}\right]}{V_{L,t-1}} * \frac{(1 + k_u)}{(1 + Kd)} - \frac{PV_{TS,f,t}}{V_{L,t-1}} \rightarrow$$

$$\boldsymbol{WACC = k_u - Kd * Tc * \frac{D_{R,t-1}}{V_{L,t-1}} - \frac{\left[PV_{TS,f,t} + TS_{f,t}\right]}{V_{L,t-1}} * \frac{(1 + k_u)}{(1 + Kd)} + \frac{PV_{TS,f,t}}{V_{L,t-1}}} \qquad \boldsymbol{(30)}$$

Alike I discussed in the previous section regarding the fixed debt approach with changing debt-levels in the forecast period, it is easier to execute the APV approach first, using the mixed financing policy assumption, and subsequently, based on the APV outcome, establish the linkage between the APV and WACC approach based on equation (30). First the firm value based on the APV approach is determined and regarding the residual value it is assumed, besides a fixed ratio component, a fixed debt component without growth.

The APV outcome can be presented as follows,

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Ku | 10,00% | | | | | | | | |
| Kd | 4,00% | | | | | | | | |
| Tc | 25,00% | | | | | | | | |
| Growth | 1,50% | | | | | | | | |
| | | 0 | 1 | 2 | 3 | 4 | 5 | TV | |
| E(FcF) | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 2.588,25 | 2.627,07 | |
| Debt (Fixed) | | 4.000 | 3.750 | 3.500 | 3.250 | 3.000 | 2.500 | | |
| Kd * Do | | | 160 | 150 | 140 | 130 | 120 | 100 | |
| Debt ("Ratio") | | 2.000 | 2.100 | 2.400 | 2.600 | 2.704 | 2.812 | | |
| Kd* Do | | | 80 | 84 | 96 | 104 | 108 | 112 | |
| APV | | | | | | | | | |
| E(FcF) | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 2.588,25 | 2.627,07 | |
| | | | | | | | 30.906,75 | | |
| Total E(FcF) | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 33.495,00 | | |
| Value unlevered | | 26.357,81 | 28.018,60 | 29.545,45 | 30.000,00 | 30.450,00 | 30.906,75 | | |
| TaxShield Fixed | | | 40,00 | 37,50 | 35,00 | 32,50 | 30,00 | 25,00 | |
| | | | | | | | 625,00 | Fixed Debt no Growth | |
| | | | 40,00 | 37,50 | 35,00 | 32,50 | 655,00 | | |
| Value TS Fixed | | 670,39 | 657,21 | 645,99 | 636,83 | 629,81 | 625,00 | | |
| TaxShield Ratio | | | 20,00 | 21,00 | 24,00 | 26,00 | 27,04 | 28,12 | |
| | | | | | | | 330,84 | Fixed Ratio | |
| | | | 20,00 | 21,00 | 24,00 | 26,00 | 357,88 | | |
| Value TS Ratio | | 293,54 | 302,90 | 312,19 | 319,41 | 325,35 | 330,84 | | |
| Value Levered | | 27.321,75 | 28.978,70 | 30.503,64 | 30.956,24 | 31.405,16 | 31.862,59 | | |

When the firm value is determined, the linkage between the APV and the WACC approach can be establish using equation (30) as follows,

| | | 0 | 1 | 2 | 3 | 4 | 5 | TV | |
|---|---|---|---|---|---|---|---|---|---|
| Equity 1/1 | | | 78,06% | 79,83% | 80,68% | 81,13% | 81,86% | 82,14% | |
| Debt Fixed 1/1 | | | 14,63% | 12,93% | 11,46% | 10,49% | 9,54% | 7,83% | |
| Debt Ratio 1/1 | | | 7,31% | 7,24% | 7,86% | 8,39% | 8,60% | 10,03% | |
| Value 1/1 | | | 27.350,08 | 29.009,86 | 30.537,92 | 30.993,95 | 31.446,64 | 31.908,22 | |
| Wacc | | | 2.634,78 | 2.803,05 | 2.956,03 | 3.002,68 | 3.049,84 | | Wacc |
| FcF | | | 975,00 | 1.275,00 | 2.500,00 | 2.550,00 | 2.588,25 | Eq. (11) | 9,73% |
| Value 31/12 | | | 29.009,86 | 30.537,92 | 30.993,95 | 31.446,64 | 31.908,22 | | 31.908,22 |
| Wacc | Eq.(30) | | 9,63% | 9,66% | 9,68% | 9,69% | 9,70% | | |
| Ke | Eq. (31) | | 11,50% | 11,35% | 11,28% | 11,24% | 11,18% | | |
| | | *Wacc* | *9,63%* | *9,66%* | *9,68%* | *9,69%* | *9,70%* | | |

We can make some observations regarding this table. First of all, the valuator assumed a fixed ratio after the forecast period of approximately 10%. The future total debt-ratio (fixed and ratio) will tend towards 10% of the firm value because the fixed debt component is fixed without growth, meaning that the firm value will grow annually with a non-growing fixed debt component and a fixed debt ratio component of 10% of firm value. If this isnot a desirable setting, the valuator can choose to adjust the residual mixed financing policy approach in a way he assumes that fixed debt also will grow with the same growth rate. He should then use equation (15) for valuing the residual value.

Another observation is that this table is very transparent regarding the development of fixed debt and fixed (debt) ratio. These ratios can be used to compare this case with comparable cases, for example ratios among pure play companies. The valuator can, based on these ratios, assess whether a possible probability of financial distress is present.

I presented also the cost of equity levered in this table. I have used the following equation, based on equations (30) and (29), to determine the cost of equity,

$$k_e = k_u + (k_u - kd) * \frac{D_{R,t-1}}{E_{t-1}} + [k_u - kd * (1 - Tc)] * \frac{D_{f,t-1}}{E_{t-1}} + \frac{PV_{TS,f,t} * \left[1 - \frac{(1+k_u)}{(1+Kd)}\right] - TS_{f,t} * \frac{(1+k_u)}{(1+Kd)}}{E_{t-1}} \quad (31)$$

## 5. Closing Thoughts

The APV and WACC approach, both, are discussed in valuation textbooks and business schools. For valuators the question of which valuation approach to use has always come down to a comparison of alternatives with the best fit for the actual valuation case.

According to Luehrman (HBR, May-June 1997) the following arguments are in favor of the APV over WACC,

- APV always works when WACC does, and sometimes when WACC does not works;
- It requires fewer restrictive assumptions;
- APV is less prone to serious errors than WACC;
- APV's power lies in the added managerial relevant information it can provide;
- APV can help managers analyze not only how much an asset is worth but also where the value comes from.

These arguments are rock solid and undisputable. APV has also irrefutable limitations. A very important limitation is that most valuators, using APV, neglect the probability of financial distress associated with financial leverage, and they may ignore also other interesting financial side effects as well. In my opinion, only using APV is a too small basis for a solid and undisputable valuation. Therefore, I would advise to use APV and create a linkage with the WACC, based on what is discussed in this short paper. The advantages of APV over WACC does not need to be neglected in this way, and simultaneously, using the linkage with WACC, the limitations of APV can be countered. The strong sides of each approach can optimally be used. An important point of departure is to execute the linkage between APV and WACC consistently, otherwise wrong conclusions can be the result with (possibly) a disputable valuation outcome.

Establishing the linkage between APV and WACC with changing cash flow and debt-levels is a complex task, as I discussed in this paper. An extra complexity is due when the mixed financing policy is used, which is in most cases an applicable and a wanted policy. The valuator should remember that the APV and the WACC outcomes must be equal at all times. This requirement is an extra check which should be taken seriously in each valuation case.