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Mid Year Discounting and Seasonality Factors
By Michael Dobner, MBA, ASA, CBV

In the December 2001 issue of the BVR, Dr. Robert Trout asserted that Dr. Richard Duvall's suggestion of correcting the discounted cash flow formula to recognize that cash flow is obtained throughout the year and not at the end of the year, results in a reverse bias.

Dr. Duval went on to develop time factors that correct for the said bias, and he concluded that for present values with a long duration (over 20 years) the time factor approaches 1.0. Since most business valuations assume a long time horizon, this conclusion means that no correction is required. In other words, according to Dr. Trout, the time factor of 1.0 that is used for cash obtained at the end of the year is also appropriate for the assumption that cash flow is evenly distributed throughout the year.

The above conclusion is not intuitive and I believe that it is the result of a mathematical error in Dr. Duval calculations. In this article I will develop the mathematical basis for calculating time factors for evenly and unevenly distributed cash flow. In addition I will suggest simple proxies for calculating time factors.

The basic error made by Dr. Duval is in the calculation of a periodic discount rate from a given annual discount rate. In his article, Dr. Duval gives the example of a 12% annual discount rate. He then goes on to claim that in this case the monthly discount rate would be 1% (12%/12). However, Dr. Duval fails to take into consideration the effect of (monthly) compounding. A 1% monthly discount rate is equal to about 12.68% annual discount rate ($1.01^{12}-1$). Therefore a 12% annual discount rate is equivalent to about 0.95% monthly discount rate. This can be calculated using the following steps:

(a) $K_a = (1+ K_p)^n - 1$
(b) $Ln(1+ K_a) = Ln(1+ K_p)^n$
(c) $Ln(1+ K_p) = [Ln(1+ K_a)]/n$
(d) $e^{[Ln(1+ Kp)]} = e^{[Ln(1+Ka)]/n}$
Hence:
Formula 1:
$K_p = e^{[LN(1+Ka)/n]}-1$

Where:
K_p is the periodic (monthly, weekly or daily) discount rate
Ln is the natural log
e is the natural logarithmic base (approximately 2.718)
K_a is the annual discount rate
n is the number of periods during a year

On the basis of the above we can now develop a formula for the time factor required to correct for the fact that cash flow is obtained throughout the year.

Lets first assume that cash flow is obtained evenly throughout the year. To find the time factor that would equate the present value of the one-year cash flow to the present value of the periodic (e.g. daily, weekly, monthly) cash flow we need to solve the following equation:

$n*R/(1+ K_a)^F = R*[(1/K_p)-1/\{ K_p *(1+ K_p)^n\}]$

Where:
R is the periodic payment
F is the required time factor

The left side of the equation is the one time annual present value and the right side is the periodic present value and we need to solve for F.

Since R appears in both sides of the equation we can eliminate it and solve the equation using the following steps:

(a) $n /(1+ K_a)^F = [(1/K_p)-1/\{ K_p *(1+ K_p)^n\}]$
(b) $(1+ K_a)^F = n / [(1/K_p)- 1/\{ K_p *(1+ K_p)^n\}]$
(c) $Ln\{(1+ K_a)^F\} = Ln\{n / [(1/K_p)- 1/\{ K_p *(1+ K_p)^n\}]\}$
(d) $F* Ln(1+ K_a) = Ln\{n / [(1/K_p)- 1/\{ K_p *(1+ K_p)^n\}]\}$

Hence:

Formula 2:

$F= Ln\{n / [(1/K_p)- 1/\{ K_p *(1+ K_p)^n\}]\}/ Ln(1+ K_a)$

Table 1 calculates F for various scenarios, using Formula 2:

Table 1	**Number of periods during the year**		
	Daily	Weekly	Monthly
Annual Discount Rate	365	52	12
5%	0.4993	0.5076	0.5396
10%	0.4974	0.5056	0.5377
15%	0.4955	0.5038	0.5359
20%	0.4938	0.5020	0.5341
25%	0.4921	0.5003	0.5324
30%	0.4904	0.4987	0.5308
35%	0.4889	0.4971	0.5293

As can be seen from the Table 1 the value of the required time factor (F) increases with the number of periods and the discount rate. This happens because the increase in the number of periods and the increase in the discount rate result in a higher present value for the early months compared to the later months in the year and so the time factor shifts to the early months of the year (i.e. shifts towards 1.0). Fortunately the effect of both these factors is relatively small and the value of F hovers around 0.5, which is the factor most valuators have been using, thus confirming the reasonability of using such factor. However, it must be noted that when using extremely high discount rates the effect on F might be significant.

Would this conclusion hold for a multi period present value? To answer this question we need to calculate the time factor that would equate the present value of the multi year annual cash flow to the present value of the periodic cash flow (e.g. daily, weekly, monthly). To this end we should solve the following equation for F:

$n*R * [(1/K_a)-1/\{ K_a *(1+ K_a)^y\}]*(1+ K_a)^{(1-F)}=R*[(1/K_p)-1/\{ K_p *(1+ K_p)^n\}]$

Where:
y is the number of years to be present valued

Since R appears in both sides of the equation we can eliminate it and solve the equation using the following steps:

(a) $(1+ K_a)^{(1-F)}= [(1/K_p)-1/\{ K_p (1+ K_p)^{(n*y)}\}]/ \{n * [(1/K_a)-1/\{ K_a *(1+ K_a)^y\}]\}$
(b) $Ln\{(1+ K_a)^{(1-F)}\}=Ln\{[(1/K_p)-1/\{ K_p *(1+ K_p)^{(n*y)}\}]/ \{n * [(1/K_a)-1/\{ K_a *(1+ K_a)^y\}]\}\}$
(c) $1-F= Ln\{[(1/K_p)-1/\{ K_p *(1+ K_p)^{(n*y)}\}]/ \{n * [(1/K_a)-1/\{ K_a *(1+ K_a)^y\}]\}\}/ Ln(1+ K_a)$

Hence:

Formula 3:

$F=1- Ln\{[(1/K_p)-1/\{ K_p *(1+ K_p)^{(n*y)}\}]/ \{n * [(1/K_a)-1/\{ K_a *(1+ K_a)^y\}]\}\}/ Ln(1+ K_a)$

Table 2 calculates F for various scenarios, using Formula 3 under the assumption of 52 periods (i.e. weekly cash flow):

Table 2

Annual Discount Rate	Number of Years: 5	10	100
5%	0.5076	0.5076	0.5076
10%	0.5056	0.5056	0.5056
15%	0.5038	0.5038	0.5038
20%	0.5020	0.5020	0.5020
25%	0.5003	0.5003	0.5003
30%	0.4987	0.4987	0.4987
35%	0.4971	0.4971	0.4971

As can be seen from Table 2 the value of the required time factor (F) is not affected at all by the number of years being present valued. This conclusion is intuitive since there is no reason why the time factor for the first year would be any different then the time factor for any other year. Again the value of F hovers around 0.5.

The next natural question is how do we deal with cash flows that are not evenly distributed throughout the year? In other words what time factor should we use in case of companies with seasonality in their cash flows.

To develop a general formula for this case we need to solve for F in the following equation:

PV(periodic payment) = $n*R/(1+K_a)^F$

Since R appears in both sides of the equation (R is imbedded in the Present Value) we can eliminate it and solve the equation using the following steps:

(a) $(1+K_a)^F$ = n/ NPV(periodic payments through the year)
(b) $(1+K_a)^F$ = 1/ NPV(distribution of payments through the year)
(c) Ln[$(1+K_a)^F$] = Ln[1/ NPV(distribution of payments through the year)]
Hence:
Formula 4:
F = Ln[1/ NPV(distribution of payments through the year)]/ Ln($1+K_a$)

To calculate F in this case one should estimate the distribution of payments and then calculate the net present value of this distribution. For illustration purposes, lets assume that cash flow is obtained on a monthly basis (at the end of each month) and that the present value is calculated as of the beginning of a calendar year. Tables 3 and 4 calculate the time factors, using Formula 4, for 5 examples of monthly distribution of cash flow, under two different discount rates. In addition, a proxy time factor was derived by calculating the weighted average of the distribution (i.e. summing the product of each month number and its corresponding distribution and dividing the result by 12).

Table 3: Time Factors for various cash flow distributions
20% Annual Discount Rate

Cash Flow Distribution	Month 1	2	3	4	5	6	7	8	9	10	11	12	Time Factor per formula	Time Factor per weighted average
	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%	1.000	1.000
	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.083	0.083
	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	0.534	0.542
	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	30.0%	40.0%	0.806	0.813
	2.0%	2.0%	2.0%	2.0%	2.0%	12.0%	40.0%	30.0%	2.0%	2.0%	2.0%	2.0%	0.586	0.588

Table 4: Time Factors for various cash flow distributions
40% Annual Discount Rate

Cash Flow Distribution	Month 1	2	3	4	5	6	7	8	9	10	11	12	Time Factor per formula	Time Factor per weighted average
	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%	1.000	1.000
	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.083	0.083
	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	0.528	0.542
	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	30.0%	40.0%	0.800	0.813
	2.0%	2.0%	2.0%	2.0%	2.0%	12.0%	40.0%	30.0%	2.0%	2.0%	2.0%	2.0%	0.584	0.588

As can be seen the results are fairly intuitive**.** The proxy calculation is reasonably close to the accurate calculation using Formula 4.

Summary

I have shown that the calculation of a time factor is relatively insensitive to the number of cash flow periods during a year and the annual discount rates as long as the discount rates are not extremely high. In addition I have shown that the time factor is not affected by the number of years being present valued.

A valuator who wants to avoid the fancy mathematical calculations can reasonably use a time factor of 0.5 when an assumption of even distribution of cash flow is appropriate. When seasonality exists a reasonable approximation could be used by calculating the weighted average of the monthly distribution.

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