VOLUME 30 | NUMBER 1 | WINTER 2018

Journal of APPLIED CORPORATE FINANCE

In This Issue: **Corporate Finance Addresses Uncertainty**

Columbia Business School

An Improved Method for Valuing Mature Companies and Estimating Terminal Value

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Discounted cash flow (DCF) models have been the standard valuation tools taught in business schools for many decades.[1] Their basic underlying theory is uncontroversial in academia, the economic intuition behind them is straightforward, and the mathematics reassuringly simple. Nevertheless, in business practice, they are applied in a wide variety of inconsistent ways that often lead to very different valuation results.

Terminal Flaw

The challenge lies in the presumed infinite life ("going concern") of a business enterprise. The DCF calculations, therefore, are of cash flow perpetuities and not just finite-lived annuities. While theoretically elegant, DCF models implicitly assume analysts' ability to forecast future cash flows forever. Understandably, analysts focus their energy on the first few years of a forecast where they have confidence estimating, yet about 80% of the DCF value typically resides in the terminal period (see Figure 1) beyond the forecast horizon. This forces analysts to make forecast assumptions about the terminal period using critical yet simplistic metrics such as P/E or EV/EBITDA[2] to estimate terminal values or to embed a perpetual stream of excess profitability and value creation in the terminal period.

As stated by standard textbook author and NYU Finance Professor Aswath Damodaran,

using multiples to estimate terminal value, when those multiples are estimated from comparable firms, results in a dangerous mix of relative and discounted cash flow valuation. While there are advantages to relative valuation, a discounted cash flow valuation should provide you with an estimate of intrinsic value, not relative value. Consequently, the only consistent way of estimating terminal value in a discounted cash flow model to use either a liquidation value or stable growth model.[3]

Company Valuation in a Nutshell

A simple DCF valuation model can be derived for a mature company with a constant return on invested capital *ROIC*, invested capital growth rate *g*, and cost of capital *r*. Growth must be less than the cost of capital, and is usually set to the long-term inflation rate or risk-free rate. The derivation is provided in Appendix A.

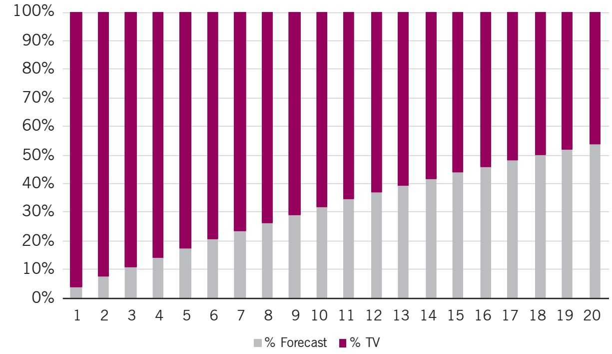
$$V = \frac{FCFF_1}{(r-g)} = \frac{NOPAT_1\left(1 - \frac{g}{ROIC_\infty}\right)}{(r-g)} \qquad (1)$$

The enterprise value is V and $NOPAT_1$ is next year's net operating profit after tax. The return on invested capital ROIC equals NOPAT divided by invested capital (IC) and is a popular measure of profitability.[4]

The numerator represents the free cash flow to the firm's capital providers (FCFF), which grows at a constant rate into perpetuity. The numerator also represents the surplus cash flow that can be distributed to the firm's capital providers (the term g/ROIC is the retention ratio). We have given ROIC

Figure 1 **Value split as a function of explicit forecast period**

The value of a growing free cash flow perpetuity was split into explicit and terminal components for each year into the future. Perpetual growth of 2% and a 6% cost of capital were assumed to generate the chart. A five-year forecast has 17% of its DCF value in the forecast window and 83% in the terminal period (year six to infinity).



1. The framework was first presented by John Burr Williams of Harvard Business School in 1938. See John B. Williams, *The Theory of Investment Value*, Harvard University Press, 1938.

2. Price/Earnings and Enterprise Value/ Earnings Before Interest, Taxes, Depreciation, and Amortization, respectively.

3. See Aswath Damodaran, *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset*, 3rd Edition, John Wiley & Sons, 2012.

4. See Damodaran (2012) and Tim Koller, Marc Goedhart, and David Wessels, *Valuation: Measuring and Managing the Value of Companies*, 5th Edition, John Wiley & Sons, 2010.

the subscript infinity to remind ourselves that this return on capital is forecast to last forever—it is not simply next year's return on capital. In their landmark 1961 paper,[5] future Nobel laureates Merton Miller and Franco Modigliani showed the equivalence between discounting future earnings and residual income streams. A subtle but important difference is that we will grow the asset base, not earnings. Operating profit and invested capital growth rates are equal when ROIC is constant.

This equation is the ubiquitous favorite for calculating terminal values in DCF models. Please note that it embeds a permanent stream of excess returns when the return on capital is greater than the cost of capital. You should think carefully when applying it.

Equation (1) can be reformulated into an enterprise value-to-book ratio V/IC_0 by substituting $NOPAT_1 = ROIC_\infty \times IC_0$ into equation (1) and then dividing both sides by IC_0.

$$\frac{V}{IC_0} = \frac{(ROIC_\infty - g)}{(r - g)} \tag{2}$$

This equation estimates the intrinsic value of the enterprise relative to its invested capital. The basic principles of shareholder value are evident upon inspection.

- When ROIC > r, value-to-book is greater than 1.0 and the intrinsic value is at a premium to book value. This company would be a "value creator."
- When ROIC < r, value-to-book is less than 1.0 and the intrinsic value is at a discount to book value. This company would be a "value destroyer."
- When ROIC = r, intrinsic value equals book value. Asset growth is irrelevant for this company and it should focus on improving its return on capital before growing.
- All things being equal, an increase in the return on capital is always positive for value creation.
- Growth is not always value enhancing. All things being equal, asset growth is additive for value creators but subtractive for value destroyers.

Factoring in Fade

Most DCF models assume a perpetual return on capital. An uncomplicated alternative to escape this unrealistic trap would be beneficial. The first step is to relax the usual assumption and introduce an adjustable fade rate called f.

This fade driver is a critical valuation component because it controls the speed at which the return on capital converges toward the cost of capital.[6] The central idea here is the competitive life-cycle framework of researcher Bartley Madden, where firms evolve through different profitability and growth stages.[7] A fade rate of 100% brings about immediate convergence, and a fade rate of 0% specifies no fade and perpetual excess profitability. You can think of the fade driver as a profitability attenuator. The fade parameter does not alter the invested capital growth rate, g, which is assumed to be constant.

Nobel laureate and University of Chicago Professor George Stigler wrote "There is no more important proposition in economic theory than that, under competition, the rate of return on investment tends toward equality in all industries."[8] Academic studies on the persistence of profitability find that corporate profitability is persistent and exhibits reversion to the mean. We document this effect for various industries.[9]

The notion that excess profits get competed away over time can be modeled by assuming that the spread of (ROIC – r) fades to zero and that economic profit EP dissipates. Economic profit is simply the spread multiplied by the beginning-of-year invested capital.[10]

$$EP_1 = (ROIC_1 - r) \times IC_0$$

Economic profit quantifies the monetary amount of value creation. When the return on capital equals the cost of capital, the economic profit is zero. Enterprise value can be expressed as a function of fade, invested capital growth and forward economic profit EP_1. The derivation of equations (3) and (4) is shown in Appendix B.[11]

$$V = IC_0 + \frac{EP_1}{[r - g + f]} = \frac{NOPAT_1\left(1 - \frac{(g - f)}{ROIC_1}\right)}{(r - g + f)} \tag{3}$$

$$\frac{V}{IC_0} = \frac{(ROIC_1 - g + f)}{(r - g + f)} \tag{4}$$

In equations (3) and (4), ROIC fades from a forward return of $ROIC_1$ to the cost of capital r at an exponential rate of f, which is illustrated in Figure 2. We reiterate that the invested capital growth rate is constant which means that the retention ratio increases as profitability decreases, leaving relatively less cash available for distribution to capital providers. You have the choice of thinking in terms of forward economic profit or NOPAT when applying equation (3).

5. Merton H. Miller and Franco Modigliani, "Dividend Policy, Growth and the Valuation of Shares," *Journal of Business,* Vol 34, No 4, pp 411-433, 1961
6. David A. Holland and Bryant A. Matthews, *Beyond Earnings: Applying the HOLT CFROI and Economic Profit Framework*, John Wiley & Sons, 2018.
7. Bartley J. Madden, *CFROI Valuation: A Total System Approach to Valuing the Firm*, Butterworth Heinemann, 1999.
8. George J. Stigler, *Capital and Rates of Return in Manufacturing Industries*, Princeton University Press, 1963.
9. See Chapter 9 of Holland and Matthews (2018).
10. See Koller et al. (2010)
11. The equation was simplified by dropping the cross-term $f \times g$. The exact expressions are:

$$V = IC_0 + \frac{EP_1}{[r-g+f+fg]} = \frac{NOPAT_1\left(1-\frac{(g-f-fg)}{ROIC_1}\right)}{(r-g+f+fg)} \text{ and } \frac{V}{IC_0} = \frac{(ROIC_1-g+f+fg)}{(r-g+f+fg)}$$

Figure 2 **Fade in profitability spread**

A forward spread of 15% is shown fading at different rates, e.g., ROIC1 = 25% and r = 10% for a forward spread of 15%. A fade rate of 0% results in a perpetual spread and a rate of 100% leads to an immediate jump to zero.

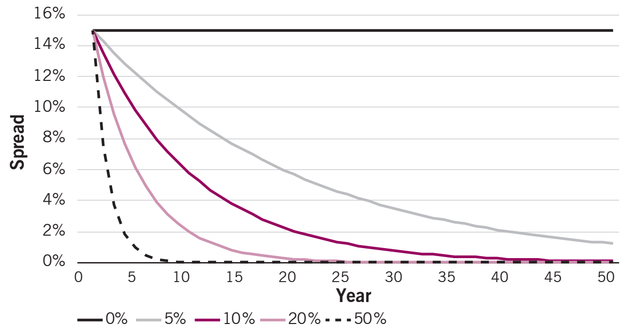


Figure 3 **Sensitivity of Enterprise Value/Invested Capital to fade and growth rates**

The sensitivity of the enterprise value-to-book ratio for different growth and fade rates when the cost of capital equals 10% and forward ROIC equals 25% (forward spread of 15%). Faster fade leads to sharp deterioration in value. Growth has diminishing influence on value as fade increases.

		Fade			
	1.3	0%	5%	10%	50%
	0%	2.5	2.0	1.8	1.3
Growth	2%	2.9	2.1	1.8	1.3
	4%	3.5	2.3	1.9	1.3
	6%	4.7	2.6	2.0	1.3

Companies that earn positive economic profits should trade at a premium to book while those that produce negative economic profits should trade at a discount to book. The present value of future economic profits is equivalent to the net present value (NPV) of the firm's existing and future projects. Note that the enterprise value-to-book ratio V/IC_0 equals one when ROIC equals the cost of capital. These results agree with common sense. Setting the fade rate to zero leads to the growing perpetuity formulation of equations (1) and (2). We can add a few more points to the principles of shareholder value by inspection of equation (4).

• All things being equal, if ROIC > r, then slower fade creates more value. Companies that can maintain high operating returns are worth more than companies whose return on capital fades quickly. Sturdy competitive moats are rewarded with larger price-to-book multiples. The relationship between moat strength and value can be assessed by altering the fade rate.

• All things being equal, if ROIC < r, then faster fade creates more value. This result makes perfect sense. Value destroyers that are able to rebound quickly to their cost of capital are worth more than those that cannot dig themselves out of the grave. Asset growth only exacerbates value destruction when incremental investments are in below cost of capital projects. The first law of holes is to stop digging when you're in one.

Adding fade as a value driver enhances the soundness of a forecast. It also offers greater flexibility in estimating the terminal value in a DCF valuation and testing its sensitivity to erosion in profitability over the long-term. It is simple to add a fade rate to any DCF model and this avoids embedding a perpetual spread into the terminal value. The sensitivity of value-to-book and fade is highlighted in Figure 3.

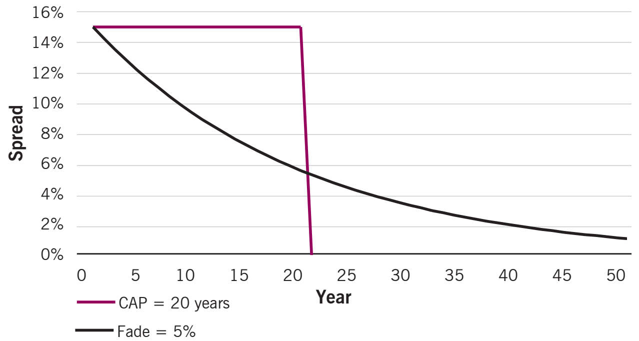
The Link between Fade and Competitive Advantage Period (CAP)

Using a fade factor leads to greater flexibility in DCF valuation models and helps in rapidly testing the sensitivity of value to the rate of excess spread decay. Few practitioners routinely think in terms of exponential fade rates. So, rather than fading the excess spread at a constant rate, let us assume that in each year the profitability spread has a probability p of jumping to zero and shutting-off forever. The spread is binomial at $(ROIC_1 - r)$ or zero: ON or permanently OFF. This phenomenon is tantamount to permanent economic disruption.[12] The expected spread in year n is:

12. See Chapter 5 of Holland and Matthews (2018) for a detailed discussion of fade and CAP. The basic logic is that a firm has a competitive advantage if its return on capital exceeds its cost of capital. The *magnitude* of the competitive advantage is the spread, and its *sustainability* is the number of years this spread can be maintained, i.e., the firm's competitive advantage period CAP. It should be intuitively clear that a firm's intrinsic value is intimately related to both dimensions. The term "permanent economic disruption" means that the spread jumps from a positive value to zero in a permanent and irreversible manner.

Figure 4 **Fade and expected CAP**

All things being equal, valuations will be equivalent whether it is assumed excess spread fades exponentially at a rate of f or shuts-off like a light with an expected CAP of 1/f. A comparison of excess spread decay at a fade rate of 5% and expected CAP of 20 years, i.e., 1/f, is diagrammed..



$$(Expected\ spread)_n = (ROIC_1 - r) \times (Probability\ of\ ON)_n + 0 \times (Probability\ of\ OFF)_n$$

$$(ROIC_n - r) = (ROIC_1 - r) \times (1 - p)^{n-1}$$

Value as a function of the probability of economic disruption p is shown in equation (5) and derived in Appendix C.

$$\frac{V}{IC_0} = \frac{(ROIC_1 - g + p)}{(r - g + p)} \quad (5)$$

Equations (4) and (5) are identical and prove that p and f are equal. In other words, the fade rate and probability of economic disruption serve identical purposes. Exponential fade and an irreversible disruption probability are mathematically indistinguishable. You can think of excess spread and economic profit as fading or being switched-off like a light. Similar to a light bulb failing forever once it burns out, economic profit is assumed to fall to zero once a company's competitive advantage has been extinguished. The expected "competitive advantage period" *E[CAP]* equals the expected life or sustainability of the excess spread.[13] This may prove more intuitive than exponential decay.

$$E[CAP] = \sum_{n=1} (1 - p)^{n-1} = \frac{1}{p}$$

An analytical solution for expected CAP in years equals *1/p*. Thus p=0% is a perpetuity while p=10% is an expected CAP of 10 years, and p=50% is an expected CAP of 2 years. We have shown that the fade rate and probability of spread permanently falling to zero are identical, and thus the inverse of the fade rate is the expected competitive advantage period or CAP.

While Merton Miller and Franco Modigliani were, in 1961, the first to approximate the value of a truncated stream of abnormal profits, Credit Suisse investment strategist Michael Mauboussin popularized the term CAP and its application in 1997.[14] In 2013, Brett Olsen used an iterative residual income approach to study the market-implied CAP, which he found to be about 7.8 years.[15] The same year, Credit Suisse HOLT researchers Bryant Matthews and David Holland showed that persistence of profitability differs from industry to industry, and that the base fade rate is about 10% per year.[16] Holland and Matthews explore practical applications of this pricing model to equities in their textbook.[17]

To be perfectly clear, in our framework CAP is not represented by a single value but rather by a geometric distribution of on/off lives with an expected value of *1/f*. This distribution is also known as the waiting-time distribution since it describes the length of time between catastrophic events, which is excess spread failure in our case. Figure 5 shows the cumulative probability of the excess spread being on or off for an expected CAP of 10 years. There is a 66% probability that the excess spread will be on in 5 years and a 34% probability it will have jumped to zero in 5 or fewer years. The probability of shutting off in 10 or fewer years is 61%. The probability of the excess spread existing in 20 years is 14% and still 5% in 30 years despite the expected CAP only being 10 years. We demonstrate the valuation equivalence in Appendix C.

13. The Competitive Advantage Period or CAP corresponds roughly to what investor Warren Buffett calls a "moat." That is, a structural advantage a firm has to earn economic rents.

14. Michael Mauboussin and Paul Johnson, "Competitive Advantage Period: The Neglected Value Driver," *Financial Management*, Vol 26, No 2, pp 67-74. 1997.

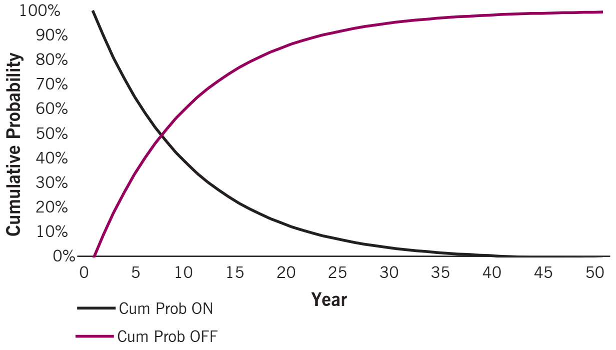
15. Brett C. Olsen, "Firms and the Competitive Advantage Period," *Journal of Investing*, Winter, pp 41-50, 2013.

16. Bryant Matthews and David A. Holland, "Modeling Persistence in Corporate Profits by Industry and Estimating a Company's Fair Price," *Credit Suisse HOLT Research*, October 2013.

17. See Chapters 4 and 5 of Holland and Matthews (2018).

Figure 5 **Cumulative probability of spread switch from ON to OFF**

The cumulative probability for the excess spread remaining on is graphed for an expected CAP of 10 years. Its complement is the cumulative probability that the excess spread will have jumped to zero and turned off. The distribution of on/off lives is geometric, also known as the waiting-time distribution.



The total expected CAP for a valuation employing a fade rate in the terminal period is simply the sum of the explicit forecast period in years and *1/f* for cases where the forecast ROIC exceeds the cost of capital. A ten-year forecast followed by a 5% fade rate equates to a total expected CAP of 30 years (10 + 1/0.05 = 30 years). For situations where the forward ROIC is less than the cost of capital, *1/f* equals the expected recovery period.

Intrinsic value is very sensitive to the fade rate assumption for value creators and this helps explain the risk premium for quality stocks.[18] The risk of owning quality stocks is that they lose their luster, economic moat, and competitive advantage. An adjustable fade rate provides an excellent means to value the effect of profitability attenuation in a DCF model. Our approach can be readily extended to other performance metrics such as ROE and CFROI. A simple pricing equation that includes a fade rate is a better way for valuing mature companies and estimating terminal value than a static DCF approach.

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18. David A. Holland and Bryant Matthews, "An Economic Foundation for Profitability and its Fade as Quality Risk Factors," *Credit Suisse HOLT Research*, April 2016.

Appendix

A. Enterprise Value as a Growing Perpetuity

The enterprise value V of a firm is the present value of its future free cash flows to the firm's capital providers *FCFF*. Free cash flows are discounted at the firm's cost of capital r.

$$V = \sum_{i=1}^{\infty} \frac{FCFF_i}{(1+r)^i}$$

Free cash flow equals net operating profit after tax *NOPAT* minus the change in invested capital ΔIC, which is the change in invested capital from the beginning to the end of the period. Return on invested capital *ROIC* equals NOPAT divided by the opening invested capital.

$$FCFF_i = NOPAT_i - \Delta IC_i = NOPAT_i - g_i \times IC_{i-1}$$

$$ROIC_i = \frac{NOPAT_i}{IC_{i-1}}$$

The change in invested capital is simply invested capital growth g multiplied by the opening invested capital. We can now write a general valuation equation in terms of the drivers ROIC and growth after some algebraic juggling.

$$V = \sum_{i=1}^{\infty} \frac{FCFF_i}{(1+r)^i} = \sum_{i=1}^{\infty} \frac{NOPAT_i - g_i \times IC_{i-1}}{(1+r)^i}$$
$$= \sum_{i=1}^{\infty} \frac{NOPAT_i \left(1 - \frac{g_i}{ROIC_i}\right)}{(1+r)^i}$$

If growth, ROIC and the cost of capital are assumed to remain constant, this expression simplifies to the growing perpetuity equation, which is equation (1) in the report.[19]

$$V = \sum_{i=1}^{\infty} \frac{NOPAT_1 \times (1+g)^{i-1} \left(1 - \frac{g}{ROIC_\infty}\right)}{(1+r)^i} =$$
$$\frac{NOPAT_1 \left(1 - \frac{g}{ROIC_\infty}\right)}{(r-g)}$$

This equation is very handy for terminal value calculations in a DCF model. Please note that it assumes a perpetual return on capital, which embeds a permanent stream of excess returns when the return on capital is greater than the cost of capital. You should think carefully when applying it.

When ROIC equals the cost of capital or growth equals zero, the equation simplifies further to a well-known perpetuity.

$$V = \frac{NOPAT_1}{r}$$

This equation generally represents a more conservative approach to valuing the terminal period.

B. Enterprise Value and Fade

We examined the valuation case where ROIC is constant. We will relax this condition and assume that while invested capital grows at a constant rate of g, the spread of $(ROIC - r)$ decays to zero, or mean-reverts at a fade rate f. The ROIC in the first forecast year is $ROIC_1$ and the initial economic profit EP_1 is $(ROIC_1 - r) \times IC_0$ where IC_0 is the opening enterprise book value. The economic profit decays to zero when the fade rate exceeds the growth rate, eliminating the unrealistic assumption of perpetual excess returns.

$$ROIC_i - r = (ROIC_1 - r)(1-f)^{i-1}$$

$$EP_i = (ROIC_i - r) \times IC_{i-1} = (ROIC_1 - r) \times (1-f)^{i-1} \times (1+g)^{i-1} \times IC_0$$

$$EP_i = EP_1 \times [(1-f) \times (1+g)]^{i-1}$$

$$V = IC_0 + \sum_{i=1}^{\infty} \frac{EP_i}{(1+r)^i}$$

$$V = IC_0 + \sum_{i=1}^{\infty} \frac{EP_1 \times [(1-f) \times (1+g)]^{i-1}}{(1+r)^i}$$

The last two equations state that a firm's enterprise value is equal to its book value plus the present value of future economic profit streams. This value is equivalent to the present value of future free cash flows to the firm's capital providers. When economic profits are zero, enterprise value should equal book value. Companies that earn positive economic profits should trade at a premium to book while those that produce negative economic profits should trade at a discount to book. The present value of future economic profits is equivalent to the net present value (NPV) of the firm's existing and future projects.

The enterprise value equation has the exact form of a growing perpetuity! Fade is essentially negative geometric growth, i.e., exponential decay. The analytical solution is simply:

$$V = IC_0 + \frac{EP_1}{[r - (1+g)(1-f) + 1]}$$

19. This growth series has a closed-form solution:

$$\sum_{i=1}^{\infty} \frac{(1+g)^{i-1}}{(1+r)^i} = \frac{1}{(r-g)}$$

For our purposes, the term *(1 + g)(1 – f)* can be approximated by *(1 + g – f)* which simplifies the math for intrinsic value. These are equations (3) and (4) in the report.

$$V \cong IC_0 + \frac{EP_1}{[r - g + f]}$$

$$V \cong IC_0 \times \frac{(ROIC_1 - g + f)}{(r - g + f)}$$

The NOPAT form of the equation can derived by simply substituting $IC_0 = \frac{NOPAT_1}{ROIC_1}$ into the last equation and a bit of algebraic juggling.

$$\frac{NOPAT_1\left(1 - \frac{(g-f)}{ROIC_1}\right)}{(r - g + f)}$$

The introduction of a fade driver makes these relationships far more realistic and useable. We can extend our treatment to ROE or other return metrics such as CFROI.

C. The Equivalence of Fade and CAP in Valuation

Instead of fading the excess spread at a constant rate, we will assume that each year the profitability spread has a probability p of being shut-off forever. The spread is binomial at $(ROIC_1 - r)$ or zero. The expected spread in years two, three and n years is thus:

$$(ROIC_2 - r) = (ROIC_1 - r) \times (1 - p) + p \times 0$$
$$(ROIC_3 - r) = (ROIC_2 - r) \times (1 - p) + p \times 0 =$$
$$(ROIC_1 - r) \times (1 - p)^2$$
$$(ROIC_n - r) = (ROIC_1 - r) \times (1 - p)^{n-1}$$

Eureka! This has the exact same form and solution as equation (4) and the fade equations derived in Appendix B.

We will show a worked example and focus on calculating the present value of future economic profits. Let's assume EP_1 equals \$1000, the fade rate is 50%, the invested capital growth rate is 4%, and the cost of capital is 10%.

$$PV\ of\ EP = \frac{EP_1}{[r - (1+g)(1-f) + 1]} =$$
$$\frac{\$1000}{[0.10 - (1 + 0.04)(1 - 0.5) + 1]} = \$1724$$

For the heck of it, we'll check how close the exact result is to the approximation.

$$PV\ of\ EP \cong \frac{EP_1}{[r - g + f]} = \frac{\$1000}{[0.10 - 0.04 + 0.50]} = \$1786$$

The error in the present value approximation is 3.6% which is reasonable for most valuations considering the lack of precision in estimating growth and fades rates. The error would be less for lower fade rates. A spreadsheet calculation would agree with the exact equation if we calculated 500 years' worth of present values (it would converge much sooner than 500 years).

$$PV\ of\ EP = \sum_{i=1}^{\infty} \frac{EP_1 \times [(1-f) \times (1+g)]^{i-1}}{(1+r)^i} = \$1000 \times \sum_{i=1}^{500} \frac{[(1 - 0.50) \times (1 + 0.04)]^{i-1}}{(1 + 0.10)^i}$$

$$PV\ of\ EP = \$1000 \times \left[\frac{(0.5 \times 1.04)^0}{(1.10)^1} + \frac{(0.5 \times 1.04)^1}{(1.10)^2} \cdots + \frac{(0.5 \times 1.04)^{499}}{(1.10)^{500}}\right]$$
$$= \$1724$$

Because the equations are the exact same, the numerical solutions would be the exact same if we assumed a dichotomous response of spread on or permanently off, and a failure rate of 50%. Fade rate and failure rate are mathematically equivalent.

Let's work out the failure probabilities and the present value of economic profits at each failure life. If the spread remains on, we'll denote it by "S" for success. If the spread shuts off to zero (and thus EP drops to zero), we'll denote it by "F" for failure. In this example, the conditional probability of failure is always 50%, so the probability of failing in year 3 *p(SSF)* is the probability of two years of success *p(SS)* multiplied by the conditional probability of failure *p(F|SS)*.

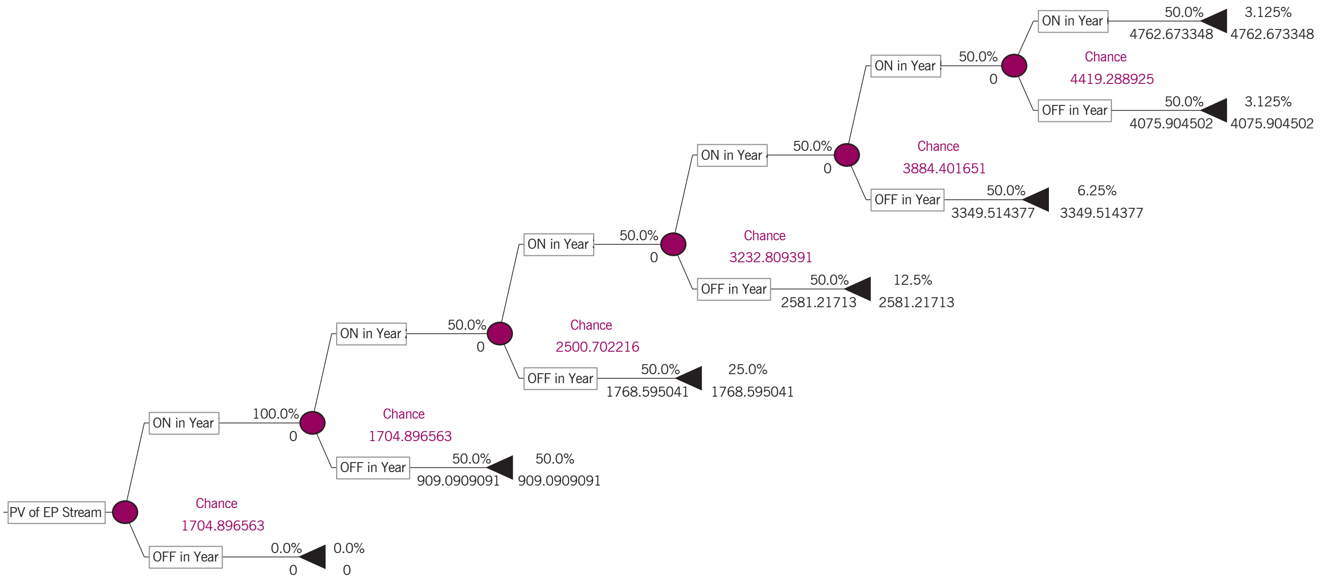
Shut off in year 1: p(S) = 100%; p(F) = 0%; PV(F) = 0
Shut off in year 2: p(SS) = $(0.5)^1$ = 50%; p(SF) = 50%; PV(SF) = \$909
Shut off in year 3: p(SSS) = $(0.5)^2$ = 25%; p(SSF) = 25%; PV(SSF) = \$1769
Shut off in year 4: p(SSSS) = $(0.5)^3$ = 12.5%; p(SSSF) = 12.5%; PV(SSSF) = \$2581

The expected value is the probability of failure in each year multiplied by the present of economic profits if failure occurs in that year.

PV = p(F) x PV(F) + p(SF) x PV(SF) + p(SSF) x PV(SSF) …
PV = 0 x 0 + 0.50 x \$909 + 0.25 x \$1769 + 0.125 x \$2581 + … = \$1724

The present value for a failure rate of 50%, or expected CAP of 2 years, is \$1724 which equals the value from the fade valuation. Note that PV(SSF) which is the value for an exact CAP of 2 years is \$1769. Expected CAP is a geometric random variable and thus an average which equals *1/f*. The valuations will not be equivalent for the same expected CAP and singular CAP because of discounting and growth. Here is what the probability tree would look like for the first six years. The truncated PV is \$1705, so despite the short expected CAP of 2 years, there is the possibility of survival beyond 6 years and additional value.

Figure 6 **Probability tree to illustrate the expected present value of future EP with a 50% annual probability of irreversible EP failure (EP =0). The tree was truncated after 6 years.**



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