TIASNIMBAS BUSINESS SCHOOL

A Market Beta Manual

How to interpret and use Market Beta's Working Paper

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In this paper a consistent leverage process is presented. How to unlever the equity Beta and how to use the Beta's in estimating the cost of equity is often been discussed among practitioners. This paper can also be seen as a first step in understanding the meaning of Beta's in relation toward their underlying assumptions regarding to financing and growth policies.

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A Market Beta Manual: how to interpret and use Market Beta's

1. Introduction

Many Business Valuators use the Beta to estimate the proper cost of equity or the cost of capital. Equity Beta's are observable in the market, they are called Market Beta's or published Beta's. Basically, the equity Beta's are levered Beta's, meaning, a degree of financial leverage is involved. In contrast, the unlevered Beta, also called the asset Beta, is not observable in the market and can only be determined by unlevering the equity Beta.

In this paper it is assumed that the fully diversified investor is estimating the cost of equity for assessing an investment. It is also assumed that the Capital Asset Pricing Model (CAPM), which includes the expected return-beta relationship, is used for this purpose. The CAPM can be expressed as follows,

$$E(R_i) = Rf + \beta_{e,i} * [E(R_m) - Rf] \qquad (1)$$

Where $E(R_i)$ is the expected investment rate of return, Rf the risk free rate, β_e the equity Beta and $E(R_m)$ the expected market rate of return.

How to unlever the equity Beta and how to use the Beta's in estimating the cost of equity, is the subject of this short paper.

2. Equity Beta estimation

The equity Beta is an estimate of the systematic risk of a stock, mostly a tradable security. It measures the volatility of an investment in a security relative to the return of the market as a whole (measured by an index[1]), also called the market portfolio. By definition, the market portfolio has a Beta of one. An investment with an equity Beta greater than one would be more risky than the market. In contrast, an investment with an equity Beta lower than one, would be less risky than the market.

The equity Beta of an investment is essentially determined both by the riskiness of the business operations as well as the financial leverage. The riskiness of the business operations is a function of the cost structure of the investment and is usually defined in terms of the relationship between fixed

[1] For example the AEX-index or the S&P 500, etc.

costs and EBIT[2]. High fixed costs relative to EBIT (high operating leverage) will also have higher variability in operating income. A simple example will give an idea how it works.

Table 1 : operational leverage example

		A		B
Sales		1.000		1.000
St. Deviation	**10%**		**10%**	
Gross Margin	**50%**	500	**25%**	250
Fixed Costs		300		50
Variable Costs	**10%**	100	**10%**	100
EBIT		**100**		**100**

Investments A and B have equally EBIT returns, although the cost structure is different.

The operational riskiness can be measured using the following simple equation[3],

$$\sigma_{Ebit,\%} = \sigma_{Sales,\%} * \left[1 + \frac{Fixed\ Costs}{EBIT}\right] \qquad (2)$$

For **investment A** the standard deviation can be determined as follows,

$$0{,}10 * \left[1 + \frac{300}{100}\right] = 40\%$$

And for **investment B**,

$$0{,}10 * \left[1 + \frac{50}{100}\right] = 15\%$$

Although the EBIT-returns are equal, the riskiness of investment A is substantial higher than that of investment B. Other things equal, the higher variance in operating income of investment A will lead to a higher Beta than the Beta of investment B.

Operating leverage affects equity Beta's, but it is difficult to measure the operating leverage of an investment, seen as an outside investor, due to the fact that fixed and variable costs are often aggregated in EBIT-statements.

Basically, an increase in financial leverage will increase the equity Beta of an investment. Higher leverage increases the variance in free cash flows to equity[4] and makes equity investments riskier. A simple example will show the idea behind the financial leverage effect.

[2] EBIT = Earnings before interest and Tax.
[3] If there are more distributions included in the EBIT-statement, the compounded standard deviation is needed, instead of (just) the sales standard deviation.

Table 2 : financial leverage example

		A		B
Free Cash Flow		80,00		80,00
Debt	**100,00**		**700,00**	
Kd = 5%		-5,00		-35,00
Taxshield (20%)		1,00		7,00
Debt installments		-		-
Cash flow to Equity		**76,00**		**52,00**

The financial leverage of investment B is much higher than that of investment A. If the free cash flow to equity drops to 28,-, the shareholder in investment B will receive no cash flow, in contrast, the shareholder in investment A will receive 24,-.

Other things equal, the higher variance in cash flow to equity of investment B will lead to a higher Beta than the Beta of investment A.

The conventional approach for estimating market equity Beta's used by most data services and financial analysts is based on historical returns analyses. For companies that have been publicly traded for a length of time, it is relatively straightforward to estimate equity Beta's. For these companies the historical returns are available and can be related to returns on an equity market index to get an equity Beta in the CAPM. In these estimated equity Beta's the operational and financial leverages are incorporated; ***the equity Beta is essentially a reflection of its operational and financial riskiness***.

A standard procedure for estimating equity Beta's is to regress excess stock returns (r_i) against excess market returns (r_m), using the following regression equation,

$$r_i = b_{0,i} + b_{1,i} * r_m + \varepsilon_i \qquad (3)$$

Where $\boldsymbol{b_0}$ is the intercept from the regression, $\boldsymbol{b_1}$ the slope of the regression (= equity beta) and ε_i is the error or disturbance term. The intercept provides a simple performance measure during the period of the regression, relative to the CAPM.

An example will show how an equity Beta can be estimated, based on historical returns. In this example two listed companies (AT&T and Boeing) will be analyzed and compared with the market returns (world index). An abstract of the data-set can be shown as follows,

[4] Sometimes called earnings per share

Table 3 : Abstract data-set AT&T, Boeing and the World index (1976 up to 1/6 2002)

	World		AT&T	Boeing
1-1-1976	0,0887		0,0808	0,1331
1-2-1976	-0,0060		0,0310	-0,0502
1-3-1976	0,0175		0,0131	0,0341
1-4-1976	-0,0037		0,0221	0,1256
1-5-1976	-0,0110		-0,0350	0,2117
1-6-1976	0,0287		0,0324	0,1207

(monthly returns)

Instead of using equation (3), which is theoretically better, in this analysis the following regression equation is used,

$$R_i = b_{0,i} + b_{1,i} * R_m + \varepsilon_i \qquad (4)$$

In equation (4) is not the excess return the object of analysis, but the return. If the risk free rate is constant, the outcome will not differ.

For the two companies the regression analyses gave the following results,

Table 4 : Regression analyses, AT&T and Boeing

Summary	AT&T			
Data for Regression			*Coëfficiënten*	*T- stat.*
Multiple correlation coëfficiënt R	0,38	**Intercept b0**	0,00	0,54
R-Square	0,14	**Variable b1**	0,74	7,31
Adj smallest square	0,14			
standard error	0,07			
Observations	318,00			

Summary	Boeing			
Data for Regression			*Coëfficiënten*	*T- stat.*
Multiple correlation coëfficiënt R	0,44	**Intercept b0**	0,01	1,87
R-Square	0,20	**Variable b1**	1,07	8,78
Adj smallest square	0,19			
standard error	0,08			
Observations	318,00			

Based on these analyses the equity Beta of AT&T is equal to 0,74 (significant with a t-value of 7,31) and of Boeing 1,07 (significant with a t-value of 8,78). Mostly, these Beta's are the Beta's for estimating the expected returns for these companies.

Based on these analyses, Boeing as an investment is more risky than the AT&T investment, which reflects in the equity Beta's.

Seen from a risk perspective, equation (3) can be rewritten as follows,

$$\sigma_i^2 = b_i^2 * \sigma_m^2 + \sigma_{\varepsilon,i}^2 \qquad (5)$$

Where

$$b^2 = \left(\frac{\rho * \sigma_i}{\sigma_m}\right)^2 \rightarrow \beta_e = \frac{\rho * \sigma_i}{\sigma_m} = \frac{Cov(R_i, R_m)}{\sigma_m^2} \tag{6}$$

Equation (5) can be rewritten, incorporating equation (6), as follows,

$$\sigma_i^2 = \sigma_i^2 * \rho^2 + \sigma_\varepsilon^2 \tag{7}$$

The **systematic risk** component of equation (7) can be expressed as follows,

$$\sigma_{sys,i}^2 = \sigma_i^2 * \rho^2 \tag{8}$$

Systematic risks are risks that are correlated with the market as whole (the economy), such as the possibility of a rise in interest rates.

The **specific risk** component, after rewriting equation (7), can be expressed as follows,

$$\sigma_{\varepsilon,i}^2 = \sigma_i^2 * (1 - \rho^2) \tag{9}$$

Specific risks are associated with events affecting earnings that are specific to the investment in question, such as the dependency of some managers.

The distinction between these two fundamental types of risk is important due to modern portfolio theory. This is because fully diversified investors do not need to bear specific risks, which offset each other and can be eliminated by holding a portfolio of diversified investments. Systematic risks, on the other hand, cannot be eliminated by diversification and must therefore be prized.

Using the data-set shown in table 2, the equity Beta's can be determined using equation (6) and must be equal to the regression-analyses outcome.

Table 5 : Equity Beta determination of AT&T and Boeing using equation (6)

Correlation		**0,38**	**0,44**
St.Dev. (m)	**3,87%**	**7,48%**	**9,32%**
St.Dev. (Y)	**13,39%**	**25,93%**	**32,29%**
Equity Beta		**0,74**	**1,07**
	World	**AT&T**	**Boeing**
1-1-1976	0,0887	0,0808	0,1331
1-2-1976	-0,0060	0,0310	-0,0502
1-3-1976	0,0175	0,0131	0,0341
1-4-1976	-0,0037	0,0221	0,1256
1-5-1976	-0,0110	-0,0350	0,2117

The standard deviation metric is used to express the total (return) risk. The Boeing standard deviation is significantly higher than the standard deviation of AT&T, meaning that the combination of operational and financial riskiness is higher at Boeing. The systematic risk of Boeing , which is equal to the correlation coefficient multiplied by the standard deviation, is as well higher than the systematic risk of AT&T, resulting in a higher Boeing equity Beta, which is in line with was elaborated above.

Operational and financial riskiness are incorporated in the standard deviation. The systematic risk is based on the standard deviation multiplied by a cyclicality metric, the historic observed correlation coefficient. If the standard deviation of investment return is extremely high, assuming a high operational and financial riskiness, but it is not or less correlated with the market as a whole, the equity Beta will not be large. The cyclicality metric, measured as the correlation between the investment and the market as a whole, in combination with the operational and financial riskiness, determine finely the equity Beta.

In sum it can be concluded that the following factors drive the equity Beta,

- Operational leverage;
- Financial leverage;
- Cyclicality.

Or expressed in equation (8),

$$\sigma_{sys,i}^2 = \sigma_i^2 * \rho_{i,m}^2$$

- ***Operational leverage***
- ***Financial leverage***

Cyclicality

An historical equity Beta for an investment determined on the mentioned basis is generally used as a proxy for the investments equity Beta in the future. To execute the regression analysis in a proper way, historical data are needed, and in the absence of such data it is difficult to quantify what the equity Beta for an investment should be. If historical data are not useful or the business-investment opportunity is not listed on any stock market, a peer-group of comparator listed companies can be selected as 'twin-securities' to use their average equity Beta, or if it is possible to select a listed company with excellent historic data and it matches perfectly with the business-investment opportunity, the equity Beta of that company can be used as well.

Nevertheless it is important that the Business Valuator should be aware that the past value of the equity Beta for an investment may not be a good guide to its future value, in almost all situations it is the future value of the equity Beta that is needed.

3. Measuring Beta's and Data Providers

According to Ogier, Rugman and Spicer[5] the historic equity Beta estimation presents similar challenges, these include:

- the choice of period over which to measure the equity Beta;
- the frequency and number of observations used;
- whether Bayesian adjustment techniques are beneficial or not;
- the choice of data provider;
- whether there is value in comparable or sector analysis.

Choice of period

In this sense there is a relatively simple trade-off involved. A longer period for historical beta measurement provides more data in order to maximize confidence in the statistical reliability (a low standard error), but the company and markets might have changed in its characteristics over the time period. Most data providers that calculate Beta's use a two- to five-year sample measurement or look-back period, where five years is the most common historical period on which the equity Beta estimate is based.

Frequency

Returns on listed securities are available on annual, monthly, weekly, daily, and even on intraday bases. Using daily returns will increase the number of observations but, normally, regressions using monthly information have lower standard errors than using either weekly of daily data. This is partly because daily returns (or intraday returns) exposes the estimation process to a significant bias in equity Beta estimates due to non-trading and non-normalities, or in other words, daily returns are likely to suffer from 'noise' such as measurement problems associated with thin trading.

Using monthly returns for estimating the equity Beta is commonly used by data-providers.

Bayesian adjustment[6]

Equity Beta's are normally measured from a regression of listed security returns against market returns and there is likely to be error in the estimate Beta obtained due to using a limited number of observations. A technique that can be used to offset this error is the Bayesian adjustment equation, which can be expressed as follows,

$$\beta_{adjusted} = \beta_{raw} * P + 1.0 * (1 - P) \qquad (10)$$

Where P is the measure of estimation error an 1.0 the Beta of the market portfolio.

This equation has been tested empirically by Mashall Blume, and he found that Beta's tend to revert toward their mean value, or the market value of one. According to Blume this means that high

[5] Ogier, Rugman and Spicer, The Real Cost of Capital (2004), page 51.

[6] The Bayesian adjustment is presented here pure from an informative point of view. This adjustment is subject to some discussions between academics and is challenged several times.

historical equity Beta's tend to over-estimate equity Beta's in future time periods, and in contrast, low historical equity Beta's tend to underestimate equity Beta's in future time periods.

Blume was able to develop the following Bayesian adjustment equation, based on performing analyses over different time periods[7],

$$\beta_{adjusted} = \beta_{raw} * 0{,}635 + 1.0 * 0{,}371 \qquad (11)$$

Equation (11) shows that the adjusted equity Beta (= prospective Beta) is equal to 0,635 times the raw equity Beta (= historical Beta) plus 0,371.

Most data providers utilize the Blume Bayesian adjustment equation[8] to adjust the equity Beta's.

Data Providers

Equity Beta's reported by different data providers for the same listed company can be very different because the use different time periods, different return intervals, different markets, and different post-regression Beta adjustments.

A number of data providers exist, including:

- Datastream;
- Value Line;
- Bloomberg;
- Barra;
- Damodaran;
- Morningstar – Ibbotson;
- Morningstar – Duff & Phelps;
- London Business School;
- Etc.

Industry Beta's

Equity Beta's for individual companies can often be unreliable. In such situation the Valuator would like to have a sampling of Beta's from many "Pure Play" listed securities when estimating the industry Equity Beta.

Annually, Damodaran publish a list with industry Beta's for valuation purposes. An abstract of such a list is shown in the following table.

[7] Ibbotson SBBI 2011 valuation yearbook, p.75
[8] There are other adjustment techniques available, such as Vasicek and Sum Beta, but are not covered in this paper due to the fact that they are not widely used in practice.

Table 6 : Abstract of the industry Beta list, provided by Damodaran

Damodaran Europe January 2011							
Industry Group	*Number of firms*	*Beta*	*D/E Ratio*	*Tax rate*	*Unlevered beta*	*Cash/Firm value*	*Unlevered beta corrected for cash*
Advertising	58	0,60	29,50%	17,84%	0,48	9,10%	0,53
Aerospace/Defense	32	0,67	38,18%	16,01%	0,51	14,10%	0,59
Air Transport	32	0,97	94,22%	13,38%	0,53	15,17%	0,63
Apparel	90	0,74	12,55%	13,77%	0,67	4,57%	0,70
Auto & Truck	18	1,09	127,52%	17,56%	0,53	13,86%	0,62
Auto Parts	**38**	**1,31**	**53,28%**	**18,38%**	**0,92**	**8,40%**	**1,00**
Bank	92	1,29	852,22%	19,02%	0,16	10,21%	0,18
Banks (Regional)	84	0,45	564,31%	16,73%	0,08	2,65%	0,08
Beverage	8	0,31	37,48%	20,16%	0,24	5,73%	0,25

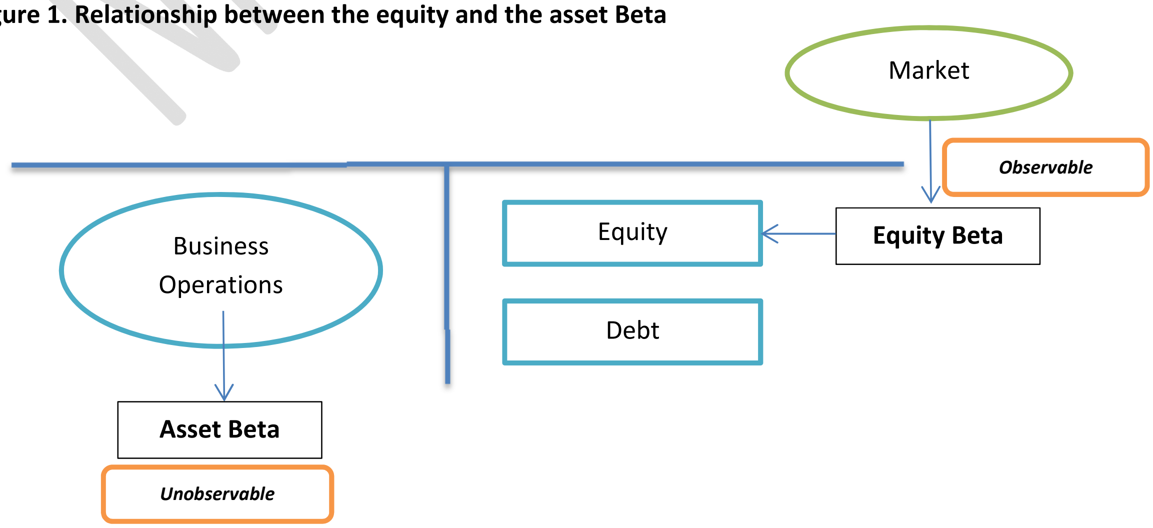
Many Business Valuators use these equity Beta's provided by Damodaran. The "Beta" column are the equity Beta's, and these are determined based on a monthly return interval with a look-back period of five years.

4. Unlevered and Re-levered Betas

Almost all published Beta's by the data providers are equity Beta's. When a company is financed only with equity, the regarding Beta can be qualified as an un-levered Beta or, often as notion used in practice, asset Beta. The asset Beta is without financial leverage influence, and is only based on the riskiness of the business operations and the cyclicality. The asset Beta should be a better measure for comparing businesses in the same industry for the estimation of the asset Beta and the expected return. A huge disadvantage of the asset Beta is, compared to the equity Beta, that asset Beta's are generally unobservable. Asset Beta's have to be determined from equity Beta's by adjusting for the financial leverage of the listed security (mostly companies) in question. To be clear, equity Beta's include financial risk, whereas asset Beta's do not.

To get a clear picture of the difference between the equity and asset Beta, the following figure shows the relationship between those two Beta's.

Figure 1. Relationship between the equity and the asset Beta



From figure 1 it is clear to notice that when no Debt is involved (no financial leverage), the observable Beta is equal to the asset Beta. The only difference between the equity Beta and the asset Beta is the incorporation of the riskiness of financial leverage. Equity Beta's are also referred to as levered Beta's, Beta's reflecting the leverage in the capital structure of the company. Removing the effect of financial leverage, or unlevering the equity Beta, results the effect of business risk and cyclicality only.

Most published equity Beta's are based on a relatively short look-back period. These Beta's are used as proxy for valuing investment opportunities. This valuation is based on an extremely long look-forward period, as expressed in the following Discounted Cash Flow equation (Wacc-method),

$$V_{L,t} = \sum_{i=1}^{\infty} \frac{FcF_i}{(1 + wacc)^i} \qquad (12)$$

Where V_L the levered value of the investment opportunity is, FCF_i the free cash flow and *wacc* the weighted average cost of capital.

The wacc can be espressed as follow,

$$wacc = (Rf + \beta_e * [E(R_m) - Rf]) * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D}{V_L} \qquad (13)$$

Where *Kd* is the cost of debt, *Rf* the risk free rate, E/V_L the equity-ratio and D/V_L the debt-ratio.

The cost of debt, in analogy of the cost of equity, can be expressed as follow,

$$Kd = Rf + \beta_d * [E(R_m) - Rf] \qquad (14)$$

The question is whether β_e is equal to the proxy equity Beta ($\hat{\beta}_e$), which is based on a relatively short look-back period. Besides the financial leverage effect, there are other effects why mostly the proxy equity Beta cannot be used. The proxy equity Beta must be seen as a starting point for estimating the proper equity Beta. Some effects which must be accounted for in estimating the equity Beta are:

- Financial Leverage (historical versus forecasted);
- Tax-rate (historical versus forecasted);
- Default probability;
- Cost of Debt (historical versus forecasted);
- Financing policy (historical versus forecasted).

Equity Beta providers, for example Damodaran[9], make assumptions regarding the process of un-levering the equity Beta's. Damodarans' important assumptions, using the look-back period of five years, can be expressed as follows[10],

- The default probability of the proxy company during a five years period is negligible small (look-back period);
- Due to a very low default probability for the five years period, the debt Beta is assumed to be zero, meaning that Kd is equal to Rf;
- In the short period of five years, no growth rate is assumed for the proxy company (growth is zero);
- Debt is in the five years period relatively fixed, implying a very low correlation with free cash flows of the proxy company;
- The average marginal tax-rate of the proxy company and the average D/E are used (five years period).

Most of these assumptions are conflicting with the assumptions underlying the forward-looking equity Beta, although they might be reasonable for the unlevering process. For example, the default probability is not negligible low for the look-forward period, and because of this Kd is (mostly) higher than Rf, implying a debt Beta greater than zero. It is possible that there is a correlation between the tax-shields and free cash flows, meaning that the financing policy is based on fixed D/E financing, but a fixed debt financing policy is also possible (case dependent).

This means that all equity Beta's must be unlevered[11] with the assumptions regarding the look-back period. The determined asset Beta must then be re-levered with the determined (look-forward) assumptions. Before unlevering the proxy equity Beta, the Valuator must assess whether the above mentioned assumptions apply to the proxy company.

In the unlevering and re-levering process the financing policy plays an important role. Although it is beyond the scope of this paper to elaborate in depth on this subject, a short explanation will be given.

Equation (12) can be rewritten as follows,

$$V_{L,t} = \sum_{i=1}^{\infty} \frac{FcF_i}{(1+wacc)^i} = \sum_{i=1}^{\infty} \frac{FcF_i}{(1+K_{eu})^i} + \sum_{i=1}^{\infty} \frac{Kd * D_{i-1} * Tc}{(1+K_{TS})^i} \qquad (15)$$

Seen from a perpetual perspective, equation (15) can be rewritten as follows,

$$V_{L,t} = \frac{FcF_{t+1}}{wacc} = \frac{FcF_{t+1}}{K_{eu}} + \frac{Kd * D_t * Tc}{K_{TS}} \qquad (16)$$

Where $Kd*D_t*T_c$ is equal to the Tax Shield, and K_{TS} the cost of tax shield.

[9] For more information, visit Damodaran's website.
[10] For more information, visit Damodaran's website.
[11] Unless the assumptions of the investment opportunity matches with the assumptions regarding the proxy company, but this is more an exception than a rule.

If K_{TS} equals K_{eu}, the implicit assumption is that the riskiness of the Tax Shields are equal to the riskiness of the free cash flows. This can only be the case when a fixed debt-ratio financing policy is assumed.

In contrast, if K_{TS} equals K_d, the implicit assumption is that no relationship exists between the riskiness of the Tax Shields and the riskiness of the free cash flows. This can only be the case when a fixed debt financing policy is assumed.

When no growth is involved and financing policy is based on fixed debt, the Hamada Beta unleverage equation is to be used and when the financing policy is based on fixed ratio, the Harris & Pringle Beta unleverage equation.

These unleverage equations can be expressed as follows[12],

Hamada Beta unleverage equation[13]

With no growth and Beta debt = 0:

$$\beta_a = \frac{\hat{\beta}_e}{\left[1 + \left(1 - \hat{T}c\right) * \frac{\widehat{D}}{E}\right]} \tag{17}$$

With no growth and Beta debt > 0:

$$\beta_a = \frac{\hat{\beta}_e}{\left[1 + \left(1 - \hat{T}c\right) * \frac{\widehat{D}}{E}\right]} + \hat{\beta}_d * \frac{\left(1 - \hat{T}c\right) * \frac{\widehat{D}}{E}}{\left[1 + \left(1 - \hat{T}c\right) * \frac{\widehat{D}}{E}\right]} \tag{18}$$

Harris & Pringle Beta unleverage equation

With Beta debt = 0[14]:

$$\beta_a = \frac{\hat{\beta}_e}{\left[1 + \frac{\widehat{D}}{E}\right]} \tag{19}$$

[12] The Miles & Ezzell equations are not shown in this paper, although in many textbooks these equations are discussed. Except for only the first period, the Miles & Ezzel and Harris & Pringle equations have the same financing policy assumption (fixed ratio), seen from a bigger picture, the difference between Miles & Ezzell and Harris & Pringle is small.
[13] When growth is involved, the Hamada Beta is not the correct unleverage equation. Myers adjusted the Hamada equation when growth is involved. In Appendix 1. this equation is presented.
[14] The equation will not differ when growth or no growth is involved.

With Beta debt > 0[15]:

$$\beta_a = \frac{\hat{\beta}_e + \hat{\beta}_d * \frac{\widehat{D}}{E}}{\left[1 + \frac{\widehat{D}}{E}\right]} \qquad (20)$$

In many textbooks[16] equation (17) is used for unlevering the proxy equity Beta's. They all assume that the amount of debt capital was fixed during the look-back period. Although this is unlikely, they assume that the amount of debt capital was "relatively" fixed, thus fixed must be taken not too literally. In contrast, some data providers, such as Morningstar and Duff & Phelps, use equation (20) or (19) for the un-levering process. They assume that there is a tendency that market value of debt capital remains at a constant percentage of equity capital, which is equivalent to stating that debt increases in proportion to increases in the net cash flow of the listed company in every period in the look-back period.

In short, the Valuator has to make a judgement between a financing policy in which there is an tendency toward a "fixed" market debt or toward a "fixed ratio". The decision is up to the Valuator for which approach he chooses. Before he can make a decision, he has to analyze the proxy, or Pure Play, company from which the equity Beta has been taken. His analysis should be focused on the type of business of the proxy company[17] for a possible financing policy direction. If the proxy company heavily rely on fixed asset investments (for example manufacturers) and those investments are financed mainly with long term debt, it can be assumed that a "fixed" market debt approach is appropriate (equation 17 or 18). On the other hand, if the proxy company heavily rely on net working capital investments (for example trading companies) with a focus on short term debt, the appropriate financing policy is probably based on "fixed ratio" (equation 19 or 20). As with many cases, the reality is not always clear, meaning that it is not always obvious which financing policy the Valuator, by definition, has to choose. A solid analysis and argumentation is needed to make a well thought decision.

After the asset Beta is determined, the valuator has to re-lever the asset Beta for the estimation of the proper equity Beta. For the re-levering process the valuator has to formulate new assumption, regarding the investment-opportunity, such as:

- Financing policy (fixed debt versus fixed ratio);
- Beta debt;
- Tax rate;
- D/E ratio (forecasted).

It is unlikely to assume (in the case of a look-forward proposition) that the Beta debt is equal to zero (Kd=Rf). In this case, the riskiness of the investment-opportunity equals the riskiness of a default security, such as a long term government bonds, which is unlikely unless the investment-opportunity is a government bond with a triple A status.

[15] The equation will not differ when growth or no growth is involved.
[16] Such as Damodaran, Hitchner, Brealy Myers & Allen, etc.
[17] Normally, this is also the type of business of the investment opportunity, which is the valuation object.

As with the unlevering process, the equations regarding the re-levering process can divided in two basic categories (Hamada versus Harris & Pringle).

The unleverage equations can be expressed as follows,

Hamada Beta re-leverage equation

With no growth and Beta debt > 0:

$$\beta_e = \beta_a * \left[1 + (1 - Tc) * \frac{D}{E}\right] - \beta_d * (1 - Tc) * \frac{D}{E} \qquad (21)$$

Harris & Pringle Beta re-leverage equation

With Beta debt > 0[18]:

$$\beta_e = \beta_a * \left[1 + \frac{D}{E}\right] - \beta_d * \frac{D}{E} \qquad (22)$$

Seen from a look-forward perspective, it is unlikely to assume that no growth is involved. The consequence of this is that equation (21) cannot be used in this re-leverage process. Myers adjusted the Hamada equation and created a new re-leverage equation based on the "fixed" market debt principle. This equation can be expressed as follow,

Myers Beta re-leverage equation

With growth and Beta debt > 0:

$$\beta_e = \beta_a * \left[1 + \frac{D}{E} * \left(1 - \frac{Kd * Tc}{Kd - g}\right)\right] - \beta_d * \frac{D}{E} * \left(1 - \frac{Kd * Tc}{Kd - g}\right) \qquad (23)$$

Where ***g*** is the growth rate.

After β_e is determined, this equity Beta can be used as an estimation for the determination of the cost of equity and cost of capital (wacc). For these purposes equation (1) and (13) can be used.

In the following paragraph some examples will show how to execute unleverage and re-leverage equations. In these valuation examples it is assumed that the Free Cash Flows are a level perpetuity – a constant free cash flow received annually in perpetuity.

[18] The equation will not differ when growth or no growth is involved.

5. Unleverage and Re-leverage process examples

5.1. Introduction

In this paragraph some examples of the unleverage (based on look-back assumptions) and the re-leverage (based on look-forward assumptions) beta process are presented. In the following matrix the presented examples are structured based on the basic assumptions regarding financing policy and growth.

	Unleverage and Re-leverage process	
Basic assumptions	**Fixed Debt**	**Fixed Ratio**
No Growth – Look back/Forward	**Par. 5.2**	**Par. 5.3**
No Growth – Look Back	**Par. 5.4**	**Par. 5.5**
With Growth – Look Forward		

In paragraph 5.6 the excel results of the examples are shown and in paragraph 5.7 an often used inconsistent approach is presented.

5.2. Fixed Debt – no growth

The first example deals with a valuation case where the financing policy is assumed to be "fixed" market debt and no growth is involved (look-back and look-forward period).

Table 7: case assumptions

Damodaran Europe					
Industry Group	***Number of firms***	***Beta***	***D/E Ratio***	***Tax rate***	***Unlevered beta***
Electrical Equipment	77	0,91	20,86%	15,36%	0,78
Look-back assumptions:					
- Fixed Debt;					
- Kd = Rf;					
- Growth = 0%					
- no Bayesian adjustment					
Look-forward assumptions:			**Market data:**		
- Fixed Debt;			**- Rf**	**2,50%**	
- Beta debt = 0,25;			**- MRP**	**6,50%**	
- Growth = 0%					
- FcF t=1	**1.000,00**				
- targe D/E	**0,35**				
- Taxrate	**20%**				

The valuation process, based on these assumptions, can be executed as follows,

Step 1. : un-levering the proxy equity Beta using the Hamada un-leverage equation (17),

$$\beta_a = \frac{\hat{\beta}_e}{\left[1 + (1 - Tc) * \frac{\hat{D}}{E}\right]} = \frac{0{,}91}{[1 + (1 - 0{,}1536) * 0{,}2086]} = 0{,}7734$$

Step 2.: re-levering the asset Beta using the Hamada re-leverage equation (21),

$$\beta_e = \beta_a * \left[1 + (1 - Tc) * \frac{D}{E}\right] - \beta_d * (1 - Tc) * \frac{D}{E}$$
$$= 0{,}7734 * [1 + (1 - 0{,}20) * 0{,}35] - 0{,}25 * 0{,}8 * 0{,}35 = 0{,}92$$

Step 3.: Estimating Cost of Equity (eq. 1), cost of Debt (eq. 14) and Cost of Capital (eq. 13),

$$E(R_i) = Rf + \beta_e * [E(R_m) - Rf] = 0{,}025 + 0{,}92 * 0{,}065 = 8{,}48\% \ (Kel)^{19}$$

$$Kd = Rf + \ \beta_d * [E(R_m) - Rf] = 0{,}025 + 0{,}25 * 0{,}065 = 4{,}13\%$$

For estimating the Cost of Capital (wacc) the Debt-ratio is needed, this can be determined as follows,

$$DR = \frac{\frac{D}{E}}{\left[1 + \frac{D}{E}\right]} = \frac{0{,}35}{1 + 0{,}35} = 0{,}2593 \qquad (24)$$

$$wacc = K_{el} * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D}{V_L} = 0{,}0848 * (1 - 0{,}2593) + 0{,}0413 * 0{,}8 * 0{,}2593$$
$$= 7{,}137\%$$

Step 4. Valuation (eq. 12)

$$V_{L,t=0} = \frac{FcF_1}{wacc} = \frac{1.000}{0{,}07137} = 14.011{,}49$$

This valuation result can be checked using the APV approach[20]. The APV approach can be expressed as follows,

$$V_{L,t=0} = \frac{FcF_1}{K_{eu}} + \frac{D_0 * Kd * Tc}{K_{TS}} \qquad (25)$$

Because the "fixed" market debt is due in this example, K_{TS} should be equal to K_d.

The K_{eu} can be determined as follows,

$$K_{eu} = Rf + \beta_a * [E(R_m) - Rf] = 0{,}025 + 0{,}7734 * 0{,}065 = 7{,}5274\%$$

Equation (25) can be used to check the valuation result according to the wacc-approach. This can be executed as follows,

$$V_{L,t=0} = \frac{FcF_1}{K_{eu}} + \frac{D_0 * Kd * Tc}{K_{TS}} = \frac{1.000}{0{,}075274} + \frac{14.011{,}49 * 0{,}2593 * 0{,}0413 * 0{,}2}{0{,}0413} = 14.011{,}44$$

In spite of some rounding differences, the valuation result based on the APV approach is equal to that of the wacc-approach, meaning that the valuation process is consistently executed[21].

[19] Kel = cost of equity levered, is equal to E(Ri).
[20] APV = Adjusted Present Value - $V_L = V_u + V_{TS}$

5.3. Fixed Ratio – no growth

This example deals with a valuation case where the financing policy is assumed to be “fixed ratio” and no growth is involved (look-back and look-forward period).

Table 8: case assumptions

Damodaran Europe					
Industry Group	***Number of firms***	***Beta***	***D/E Ratio***	***Tax rate***	***Unlevered beta***
Electrical Equipment	77	0,91	20,86%	15,36%	0,78
Look-back assumptions:					
- Fixed Ratio;					
- Kd = Rf;					
- Growth = 0%					
- no Bayesian adjustment					
Look-forward assumptions:			**Market data:**		
- Fixed Ratio;			**- Rf**	**2,50%**	
- Beta debt = 0,25;			**- MRP**	**6,50%**	
- Growth = 0%					
- FcF t=1	**1.000,00**				
- targe D/E	**0,35**				
- Taxrate	**20%**				

The only main difference, compared to the previous example, is the financing policy assumption. Instead of the “fixed” debt approach, in this example the “fixed ratio” is the financing policy assumption.

The valuation process, based on these assumptions, can be executed as follows,

Step 1. : un-levering the proxy equity Beta using the Harris & Pringle un-leverage equation (19),

$$\beta_a = \frac{\hat{\beta}_e}{\left[1 + \frac{\widehat{D}}{E}\right]} = \frac{0{,}91}{[1 + 0{,}2086]} = 0{,}7529$$

Step 2.: re-levering the asset Beta using the Harris & Pringle re-leverage equation (22),

$$\beta_e = \beta_a * \left[1 + \frac{D}{E}\right] - \beta_d * \frac{D}{E} = 0{,}7529 * [1 + 0{,}35] - 0{,}25 * 0{,}35 = \ 0{,}92897$$

Step 3.: Estimating Cost of Equity (eq. 1), cost of Debt (eq. 14) and Cost of Capital (eq. 13),

$$E(R_i) = Rf + \beta_e * [E(R_m) - Rf] = 0{,}025 + 0{,}92897 * 0{,}065 = 8{,}538\%\ (Kel)^{22}$$

$$Kd = Rf + \ \beta_d * [E(R_m) - Rf] = 0{,}025 + 0{,}25 * 0{,}065 = 4{,}13\%$$

For estimating the Cost of Capital (wacc) the Debt-ratio is needed, this can be determined as follows,

$$DR = \frac{\frac{D}{E}}{\left[1 + \frac{D}{E}\right]} = \frac{0{,}35}{1 + 0{,}35} = 0{,}2593 \qquad (24)$$

[21] If the valuator uses a spreadsheet model, no rounding effects occur.
[22] Kel = cost of equity levered, is equal to E(Ri).

$$wacc = K_{el} * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D}{V_L} = 0{,}0854 * (1 - 0{,}2593) + 0{,}0413 * 0{,}8 * 0{,}2593 = 7{,}18\%$$

Step 4. Valuation (eq. 12)

$$V_{L,t=0} = \frac{FcF_1}{wacc} = \frac{1.000}{0{,}0718} = 13.927{,}58$$

This valuation result can be checked using the APV approach[23]. The APV approach can be expressed as follows,

$$V_{L,t=0} = \frac{FcF_1}{K_{eu}} + \frac{D_0 * Kd * Tc}{K_{TS}} \qquad (25)$$

Because the "fixed ratio" is due in this example, K_{TS} should be equal to K_{eu}.

The K_{eu} can be determined as follows,

$$K_{eu} = Rf + \beta_a * [E(R_m) - Rf] = 0{,}025 + 0{,}7529 * 0{,}065 = 7{,}3939\%$$

Equation (25) can be used to check the valuation result according to the wacc-approach. This can be executed as follows,

$$V_{L,t=0} = \frac{FcF_1}{K_{eu}} + \frac{D_0 * Kd * Tc}{K_{TS}} = \frac{1.000}{0{,}073939} + \frac{13.927{,}58 * 0{,}2593 * 0{,}0413 * 0{,}2}{0{,}073939} = 13.928{,}11$$

In spite of some rounding differences, the valuation result based on the APV approach is equal to that of the wacc-approach, meaning that the valuation process is consistently executed.

5.4. Fixed Debt – no growth (look-back) – growth (look-forward)

This example deals with a valuation case where the financing policy is assumed to be "fixed" market debt and no growth for the look-back period but there is assumed a growth rate for the look-forward period.

In this example the proxy equity Beta must unlevered using Hamada equation (17). But, due to the fact that growth is involved in the look-forward period, the Hamada re-leverage equation (21) is not suitable. In this case the Myers re-leverage equation (23) must be used to execute the re-leverage process.

[23] APV = Adjusted Present Value - $V_L = V_u + V_{TS}$

Table 9: case assumptions

Damodaran Europe					
Industry Group	*Number of firms*	*Beta*	*D/E Ratio*	*Tax rate*	*Unlevered beta*
Electrical Equipment	77	0,91	20,86%	15,36%	0,78
Look-back assumptions:					
- Fixed Debt;					
- Kd = Rf;					
- Growth = 0%					
- no Bayesian adjustment					
Look-forward assumptions:			Market data:		
- Fixed Debt;			- Rf	2,50%	
- Beta debt = 0,25;			- MRP	6,50%	
- Growth = 2%					
- FcF t=1	1.000,00				
- targe D/E	0,35				
- Taxrate	20%				

<u>The valuation process, based on these assumptions, can be executed as follows,</u>

Step 1. : un-levering the proxy equity Beta using the Hamada un-leverage equation (17),

$$\beta_a = \frac{\hat{\beta}_e}{\left[1 + (1 - Tc) * \frac{\hat{D}}{E}\right]} = \frac{0{,}91}{[1 + (1 - 0{,}1536) * 0{,}2086]} = 0{,}7734$$

Step 2.: re-levering the asset Beta using the Myers re-leverage equation (23),

$$\beta_e = \beta_a * \left[1 + \frac{D}{E} * \left(1 - \frac{Kd * Tc}{Kd - g}\right)\right] - \beta_d * \frac{D}{E} * \left(1 - \frac{Kd * Tc}{Kd - g}\right)$$
$$= 0{,}7734 * \left[1 + 0{,}35 * \left(1 - \frac{0{,}0413 * 0{,}20}{0{,}0413 - 0{,}02}\right)\right] - 0{,}25 * 0{,}35 * \left(1 - \frac{0{,}0413 * 0{,}20}{0{,}0413 - 0{,}02}\right) = 0{,}8855$$

Step 3.: Estimating Cost of Equity (eq. 1), cost of Debt (eq. 14) and Cost of Capital (eq. 13),

$$E(R_i) = Rf + \beta_e * [E(R_m) - Rf] = 0{,}025 + 0{,}8855 * 0{,}065 = 8{,}26\% \; (Kel)$$ [24]

$$Kd = Rf + \beta_d * [E(R_m) - Rf] = 0{,}025 + 0{,}25 * 0{,}065 = 4{,}13\%$$

For estimating the Cost of Capital (wacc) the Debt-ratio is needed, this can be determined as follows,

$$DR = \frac{\frac{D}{E}}{\left[1 + \frac{D}{E}\right]} = \frac{0{,}35}{1 + 0{,}35} = 0{,}2593 \qquad (24)$$

$$wacc = K_{el} * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D}{V_L} = 0{,}0826 * (1 - 0{,}2593) + 0{,}0413 * 0{,}8 * 0{,}2593 = 6{,}97\%$$

[24] Kel = cost of equity levered, is equal to E(Ri).

Step 4. Valuation (eq. 12)

$$V_{L,t=0} = \frac{FcF_1}{wacc - g} = \frac{1.000}{0,06971 - 0,02} = 20.116,68$$

This valuation result can be checked using the APV approach[25]. The APV approach can be expressed as follows,

$$V_{L,t=0} = \frac{FcF_1}{K_{eu} - g} + \frac{D_0 * Kd * Tc}{K_{TS} - g} \qquad (26)$$

Because the "fixed" market debt is due in this example, K_{TS} should be equal to K_d.

The K_{eu} can be determined as follows,

$$K_{eu} = Rf + \beta_a * [E(R_m) - Rf] = 0,025 + 0,7734 * 0,065 = 7,527\%$$

Equation (25) can be used to check the valuation result according to the wacc-approach. This can be executed as follows,

$$V_{L,t=0} = \frac{FcF_1}{K_{eu} - g} + \frac{D_0 * Kd * Tc}{K_{TS} - g} = \frac{1.000}{0,05527} + \frac{20.116,68 * 0,2593 * 0,0413 * 0,2}{0,0213} = 14.115,83$$

In spite of some rounding differences, the valuation result based on the APV approach is equal to that of the wacc-approach, meaning that the valuation process is consistently executed.

5.5. Fixed Ratio – no growth (look-back) – growth (look-forward)

This example deals with a valuation case where the financing policy is assumed to be "fixed ratio" and no growth for the look-back period but there is assumed a growth rate for the look-forward period.

In this example the first three steps of the process can be skipped because they are equal to the steps in paragraph 5.2, unless the original example description changes (see equation 19 and 20).

Step 4. Valuation (eq. 12)

$$V_{L,t=0} = \frac{FcF_1}{wacc - g} = \frac{1.000}{0,0718 - 0,02} = 19.305,02$$

This valuation result can be checked using the APV approach[26]. The APV approach can be expressed as follows,

$$V_{L,t=0} = \frac{FcF_1}{K_{eu} - g} + \frac{D_0 * Kd * Tc}{K_{TS} - g} \qquad (25)$$

Because the "fixed ratio" is due in this example, K_{TS} should be equal to K_{eu}.

[25] APV = Adjusted Present Value - $V_L = V_u + V_{TS}$

[26] APV = Adjusted Present Value - $V_L = V_u + V_{TS}$

The K_{eu} can be determined as follows,

$$K_{eu} = Rf + \beta_a * [E(R_m) - Rf] = 0{,}025 + 0{,}7529 * 0{,}065 = 7{,}3939\%$$

Equation (25) can be used to check the valuation result according to the wacc-approach. This can be executed as follows,

$$V_{L,t=0} = \frac{FcF_1}{K_{eu} - g} + \frac{D_0 * Kd * Tc}{K_{TS} - g} = \frac{1.000}{0{,}053939} + \frac{19.305{,}02 * 0{,}2593 * 0{,}0413 * 0{,}2}{0{,}053939} = 19.306{,}02$$

In spite of some rounding differences, the valuation result based on the APV approach is equal to that of the wacc-approach, meaning that the valuation process is consistently executed.

5.6. Excel results

The discussed examples are also executed in Excel. The results can be presented in the following table.

Table 10: excel results

	Examples Paragraph 5.2	5.3	5.4	5.5
Financing Policy	fixed D.	fixed R.	fixed D.	fixed R.
Growth look-forward	no	no	yes	yes
Asset Beta	0,77	0,75	0,77	0,75
Equity Beta	0,92	0,93	0,89	0,93
Kel	8,48%	8,54%	8,26%	8,54%
wacc	7,14%	7,18%	6,97%	7,18%
D/V	25,93%	25,93%	25,93%	25,93%
Kd	4,13%	4,13%	4,13%	4,13%
Value levered	14.011,27	13.927,16	20.116,40	19.304,22
Debt	3.632,55	3.610,75	5.215,36	5.004,80
Keu	7,53%	7,39%	7,53%	7,39%
Value unlevered	13.284,76	13.524,29	18.091,61	18.538,77
Taxshield Value	726,51	402,87	2.024,79	765,46
Value levered	14.011,27	13.927,16	20.116,40	19.304,22
Wacc -/- APV	-	-	-	-

In contrast with the previous paragraphs, there are no rounding differences detected.

5.7. Inconsistent Approaches

Some practitioners use the proxy equity Beta as estimate for the cost of equity. They assume that the proper D/E and marginal tax rate are the same as those from the proxy company (or industry). Mostly they disregard the debt Beta although the expected cost of debt exceeds the risk free rate. The result of this approach is an inaccurate valuation outcome. Using the example description of paragraph 5.2, this inconsistent approach can be presented as follows.

<u>The valuation process, based on these assumptions, can be executed as follows,</u>

Step 1. : un-levering the proxy equity Beta using the Hamada un-leverage equation (17),

$$\beta_a = \frac{\hat{\beta}_e}{\left[1 + (1 - Tc) * \frac{\hat{D}}{E}\right]} = \frac{0{,}91}{[1 + (1 - 0{,}1536) * 0{,}2086]} = 0{,}7734$$

Step 2.: re-levering the asset Beta using the Hamada re-leverage equation (21),

$$\beta_e = \beta_a * \left[1 + (1 - Tc) * \frac{D}{E}\right] = 0{,}7734 * [1 + (1 - 0{,}1536) * 0{,}2086] = 0{,}91$$

Step 3.: Estimating Cost of Equity (eq. 1), cost of Debt (eq. 14) and Cost of Capital (eq. 13),

$$E(R_i) = Rf + \beta_e * [E(R_m) - Rf] = 0{,}025 + 0{,}91 * 0{,}065 = 8{,}42\%\ (Kel)^{27}$$

$$Kd = Rf + \beta_d * [E(R_m) - Rf] = 0{,}025 + 0{,}25 * 0{,}065 = 4{,}13\%$$

For estimating the Cost of Capital (wacc) the Debt-ratio is needed, this can be determined as follows,

$$DR = \frac{\frac{D}{E}}{\left[1 + \frac{D}{E}\right]} = \frac{0{,}2086}{1 + 0{,}2086} = 0{,}1726 \qquad (24)$$

$$wacc = K_{el} * \frac{E}{V_L} + Kd * (1 - Tc) * \frac{D}{V_L} = 0{,}0842 * (1 - 0{,}1726) + 0{,}0413 * 0{,}8464 * 0{,}1726$$
$$= 7{,}57\%$$

Step 4. Valuation (eq. 12)

$$V_{L,t=0} = \frac{FcF_1}{wacc} = \frac{1.000}{0{,}0757} = 13.208{,}89$$

This valuation result can be checked using the APV approach[28]. The APV approach can be expressed as follows,

$$V_{L,t=0} = \frac{FcF_1}{K_{eu}} + \frac{D_0 * Kd * Tc}{K_{TS}} \qquad (25)$$

Because the “fixed” market debt is due in this example, K_{TS} should be equal to K_d.

The K_{eu} can be determined as follows,

$$K_{eu} = Rf + \beta_a * [E(R_m) - Rf] = 0{,}025 + 0{,}7734 * 0{,}065 = 7{,}5274\%$$

Equation (25) can be used to check the valuation result according to the wacc-approach. This can be executed as follows,

[27] Kel = cost of equity levered, is equal to E(Ri).

[28] APV = Adjusted Present Value - $V_L = V_u + V_{TS}$

$$V_{L,t=0} = \frac{FcF_1}{K_{eu}} + \frac{D_0 * Kd * Tc}{K_{TS}} = \frac{1.000}{0,075274} + \frac{13.209,89 * 0,1726 * 0,0413 * 0,1536}{0,0413} = 13.635,01$$

Although the difference is not significant large in this example, the wacc valuation outcome is inaccurate, based on an inconsistent unleverage and re-leverage process.

Many data providers assume that in a short Beta analysis period (for example five years) the growth rate for the proxy companies is close to zero (or zero). In other words, they are assuming that those companies are, over a short period of time, non-growing companies. For fixed debt ratio companies this assumption has no impact due to the fact that for the growth and no-growth assumption the same unleverage equation can be used (equation 19 and 20). In contrast, for fixed debt companies the impact of the wrong growth assumption can be significant. The two following unleverage Beta equations will show where they differ when the growth or no growth assumption is involved (Beta debt = 0),

$$\beta_a = \frac{\hat{\beta}_e}{\left[1 + \frac{\hat{D}}{E} * \left(1 - \hat{T}c * \frac{\hat{R}f}{(\hat{R}f - \hat{g})}\right)\right]} \rightarrow g > 0\% \quad \boldsymbol{versus} \quad \beta_a = \frac{\hat{\beta}_e}{\left(1 + \frac{\hat{D}}{E} * (1 - \hat{T}c)\right)} \rightarrow g = 0\%$$

It is obvious that when the growth rate increases, using the first equation, the asset Beta will increase as well[29]. A high realized (look-back) growth-rate can cause a significant different asset Beta compared to a determined asset Beta based on the no-growth assumption, as starting point in the valuation process. Just assuming that no growth is involved, without any executed analysis, can lead to an inconsistent valuation outcome.

The important point is that the valuator should always consider carefully what scenario was relevant when analyzing the historical Beta's (fixed debt level or ratio, growth of no growth, what is the Beta of debt) and what look-forward scenario is relevant for the valuation case.

6. Some closing thoughts

In practice many valuators are not aware how to unlever and re-lever Beta's in a consistent way. Mostly, they don't know what the basic underlying assumptions are. The result of this lack of knowledge is an inconsistent leverage process with an inaccurate valuation outcome.

In this paper the proper and consistent leverage process is presented. This paper can be seen as a basic Beta leverage process manual. It is a first step in understanding the meaning of Beta's in relation toward financing en growth policies defined by companies. Although these basic assumptions are not always clear, in the valuation process the valuator has to determine, based on well executed analysis, what the proper and suitable assumptions are before entering into the leverage process.

[29] Other things remaining equal.

Beyond the presented basic and consistent Beta unleverage and re-leverage approach, much more research is needed to clarify underlying uncertainties such as mixed financing policies in relation to the observed and re-levered Beta's. Many companies don't have a financing policy based on one of the two extreme policies discussed in this paper. They often have more or less mixed policies. A (more or less) fixed debt policy regarding fixed assets investments and a (more or less) fixed ratio regarding net working capital investments.

Due to additional research, the leverage process will be further fine-tuned resulting in a much better understanding of the Beta fundamentals.

Appendix 1: Summary Leverage Equations

Growth = 0%

	FIXED DEBT		FIXED RATIO	
	Kd = Rf	Kd > Rf	Kd = Rf	Kd > Rf
Unlevering Equity Beta	$\beta_a = \dfrac{\hat{\beta}_e}{\left(1 + (1 - \hat{T}c) * \dfrac{\widehat{D}}{E}\right)}$	$\beta_a = \dfrac{\hat{\beta}_e + \hat{\beta}_d * \dfrac{\widehat{D}}{E}}{\left(1 + (1 - \hat{T}c) * \dfrac{\widehat{D}}{E}\right)}$	$\beta_a = \dfrac{\hat{\beta}_e}{\left(1 + \dfrac{\widehat{D}}{E}\right)}$	$\beta_a = \dfrac{\hat{\beta}_e + \hat{\beta}_d * \dfrac{\widehat{D}}{E}}{\left(1 + \dfrac{\widehat{D}}{E}\right)}$
Relevering Asset Beta	$\beta_e = \beta_a * \left(1 + (1 - Tc) * \dfrac{D}{E}\right)$	$\beta_e = \beta_a * \left(1 + (1 - Tc) * \dfrac{D}{E}\right) - \beta_d * (1 - Tc) * \dfrac{D}{E}$	$\beta_e = \beta_a * \left(1 + \dfrac{D}{E}\right)$	$\beta_e = \beta_a * \left(1 + \dfrac{D}{E}\right) - \beta_d * \dfrac{D}{E}$

Growth > 0%

	FIXED DEBT		FIXED RATIO	
	Kd = Rf	Kd > Rf	Kd = Rf	Kd > Rf
Unlevering Equity Beta	$\beta_a = \dfrac{\hat{\beta}_e}{\left[1 + \dfrac{\widehat{D}}{E} * \left(1 - \dfrac{\widehat{R}f * \widehat{T}c}{\widehat{R}f - \hat{g}}\right)\right]}$	$\beta_a = \dfrac{\hat{\beta}_e + \hat{\beta}_d * \dfrac{\widehat{D}}{E} * \left[1 - \dfrac{\widehat{K}d * \widehat{T}c}{\widehat{K}d - \hat{g}}\right]}{\left[1 + \dfrac{\widehat{D}}{E} * \left(1 - \dfrac{\widehat{K}d * \widehat{T}c}{\widehat{K}d - \hat{g}}\right)\right]}$	$\beta_a = \dfrac{\hat{\beta}_e}{\left(1 + \dfrac{\widehat{D}}{E}\right)}$	$\beta_a = \dfrac{\hat{\beta}_e + \hat{\beta}_d * \dfrac{\widehat{D}}{E}}{\left(1 + \dfrac{\widehat{D}}{E}\right)}$
Relevering Asset Beta	$\beta_e = \beta_a * \left[1 + \dfrac{\widehat{D}}{E} * \left(1 - \dfrac{\widehat{R}f * \widehat{T}c}{\widehat{R}f - \hat{g}}\right)\right]$	$\beta_e = \beta_a * \left(1 + \dfrac{D}{E} * \left(1 - \dfrac{Kd * Tc}{Kd - g}\right)\right) - \beta_d * \dfrac{D}{E} * \left(1 - \dfrac{Kd * Tc}{Kd - g}\right)$	$\beta_e = \beta_a * \left(1 + \dfrac{D}{E}\right)$	$\beta_e = \beta_a * \left(1 + \dfrac{D}{E}\right) - \beta_d * \dfrac{D}{E}$

Appendix 2: Summary Leverage Process