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Valuing the Debt Tax Shield

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VALUING THE DEBT TAX SHIELD

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EXECUTIVE SUMMARY

The tax shield from debt represents a significant proportion of total value for many companies, projects, and transactions. Accurate valuation of the debt tax shield is of more importance than ever as leverage is now commonly used as a source of value added, and there is growing competition in buying assets. Changes in tax rules and more international transactions also make it important to understand how to value debt tax shields under different tax regimes.

The increased practical importance of accurate valuation of the debt tax shield has been paralleled by debates among researchers about how to do it. The different approaches have large effects on estimated values. In this article, we review these approaches and show their implications for practical debt tax shield valuation. The issue we stress is that each method relies on a few basic assumptions, primarily about the risk of debt tax shields. Picking the most appropriate assumption in any particular situation and then using only those procedures that are consistent with that assumption is the key to good valuation of debt tax shields. Using inconsistent procedures can lead to large errors.

The structure of the article is as follows: We start by giving a brief overview of the theory. Then we review the tensions that exist in the “how to value tax shields” literature. Next, we discuss the practical implications of the various approaches and show by way of an example the large errors that can arise if incorrect and inconsistent valuation

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methods are used. Finally, we offer our views as to which methods and assumptions are most appropriate in various real world economic settings.

INTRODUCTION

The tax shield from debt represents a significant proportion of total value for many companies, projects, and transactions. Its potential size can be seen by considering a company with a 30% debt-to-capital ratio and a corporate tax rate of 40%. One approach to valuing the debt tax shield is simply to multiply the amount of debt by the tax rate, in which case the debt tax shield would be seen as contributing 12% of total value. And if the leverage ratio were doubled, the debt tax shield could be shown to contribute almost a quarter of the value of the company.

Three developments have increased the importance of accurate valuation of the debt tax shield in recent years. First, to a greater extent than in any period since the restructurings of the 1980s, leverage is commonly used as a potential source of value added in investment decisions. Second, growing competition in buying assets means that accuracy in valuation can give potential buyers a competitive edge in many transactions. Finally, changes in tax rules, such as those brought about by the Bush tax cut of 2003, along with the rising frequency of international transactions, make it important to understand how to value debt tax shields under different tax regimes.

The rise in the importance of accurate valuation of the debt tax shield has been paralleled by debates among researchers about how to do it. Among the issues being debated are the following:

- Does the value of the debt tax shield reflect the full corporate tax rate or a lower rate?
- Does the value of the tax shield differ in tax regimes that favor dividends?

-Should you value the tax shield by using adjusted present values or by adjusting the discount rate?

-If you use adjusted present value, how should you obtain the discount rate?

-What formula should be used for the present value of the tax shield?

These debates have considerable practical importance. As we show later, how these issues get decided can have large effects on the estimated value of the debt tax shield.

In this article, we review these debates and show their implications for debt tax shield valuation. First, we give a brief overview of the theory. Then we review the tensions that exist in the “how to value tax shields” literature. Next, we discuss the practical implications of the various approaches. Finally, we offer our views as to which methods and assumptions are most appropriate in various real world economic settings, along with an example that illustrates the likely consequences of choosing a given approach.

Before embarking on the review, we emphasize that the issues we discuss here are different from the familiar debates about optimal capital structure. The question of how to value the debt tax shield is important, regardless of one’s view of the value-maximizing leverage ratio. The two issues are related in the sense that both depend on judgments about the value of the tax saving from debt. However, anyone involved in valuation needs to decide how to value debt tax shields, regardless of their view of optimal capital structure. In this article, we discuss how to value the tax shield *given* a company’s chosen debt policy; we do not address whether this policy is optimal or what a value-maximizing policy would be.²

OVERVIEW OF THEORY

All valuations that attempt to capture the value of the tax savings from interest can be represented by the following *adjusted present value* (APV) formula:

$$V_L = E + D = V_U + PVTS, \quad (1)$$

where V_L is the value of the levered company (the firm's "enterprise value," as it is often called); E is the market value of equity; D is the market value of debt, V_U is the value of the same company without any leverage (the unlevered company); and $PVTS$ is the present value of the tax saving from debt.

To use the APV approach, however, one must first calculate the firm's unlevered value. This means discounting the firm's operating free cash flow at the "unlevered" discount rate, and then making a separate calculation of the present value of the debt tax shield. The operating free cash flow is the cash flow after tax but before interest. All other standard methods for including the debt tax shield in valuation are derived from the APV equation. Although they appear different, their results will be consistent with those provided by APV, *as long as* one can satisfy a particular set of assumptions in each case.

The two most commonly used methods for valuing debt shields are the WACC approach and the *capital cash flow* approach. The WACC approach estimates the firm's levered value (V_L) directly by discounting its operating free cash flow at the weighted average cost of capital (WACC). In contrast to the APV approach, this method effectively captures the value of the debt tax shield in the (lower) discount rate, and not as a separate component of value. The capital cash flow approach discounts the sum of operating free cash flow and the tax saving from interest at the unlevered cost of capital (which is higher than WACC). This method puts the adjustment for the debt tax shield

²An excellent review of the broader issues can be found in John Graham, "Taxes and Corporate Finance: A Review," *Review of Financial Studies*, Vol. 16 (2003), pp. 1075-1129.

into the cash flows rather than making it a separate component of value or including it in the discount rate.³

Review of cash flow definitions***Operating free cash flow (FCF):***

Free cash flow after tax, assuming that the company is financed entirely with equity. This is net of capex and tax, but not depreciation and interest. The tax is calculated without the interest tax deduction.

Capital cash flow (CCF):

Free cash flow available to the combination of debt and equity holders.

The relationship:

$CCF = FCF + \text{Tax saved from interest}$

If the APV approach is used, one way of calculating the present value of the tax shield, PVTS, is to multiply the corporate tax rate, T_C , by the market value of debt:

$$PVTS = T_C D \quad (2)$$

But use of this formula is based on two strong assumptions: (1) the company is expected to be profitable enough to pay the full corporate tax rate every year in perpetuity; and (2) the amount of debt is fixed (forever) at the level D . This approach is sometimes called the Miller-Modigliani (M&M) approach because it was first developed by Merton Miller and Franco Modigliani in a 1963 paper.⁴ The use of the M&M formula, (2), for the debt tax shield is a special case of the APV approach that makes somewhat restrictive assumptions about the level and risk of the debt tax shield. In particular, use of this approach effectively assumes that the company is not expected to grow, which of course limits its range of application in ways that we discuss later.

Use of the WACC and capital cash flow approaches are based on somewhat different assumptions about the company's expected future tax status and debt policy--

³ A detailed explanation of the capital cash flow approach is provided by Richard Ruback in his article, "Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows," *Financial Management* Vol. 31 (2002) pp. 85-103.

assumptions that, as we will show, affect the level and risk of the future tax saving from interest and hence the value of the debt tax shield. The WACC approach assumes that debt is a *constant proportion* of company value instead of the *fixed amount* of debt assumed by M&M.⁵ Given this assumption, the WACC approach involves calculating the weighted average cost of capital as follows:

$$\text{WACC} = (E/V_L) * K_E + (D/V_L) * K_D * (1 - T_c), \quad (3)$$

where K_E is the cost of equity and K_D is the cost of debt. The operating free cash flow of the company is discounted at the WACC to estimate directly the value of the leveraged company (V_L), which includes the value of the debt tax shields. As mentioned earlier, the WACC approach effectively incorporates the value of the debt tax shield through use of a lower discount rate. By so doing, the WACC method estimates the value of the levered company in a one-step valuation, rather than separately estimating the unlevered value and the present value of debt tax shields. As we show later, this approach implies a particular value for PVTS but does not explicitly calculate it.

The capital cash flow approach assumes that the risk of the interest tax shield is the same as the risk of the operating free cash flow. This can arise for two reasons. One is because the amount of debt and interest will be proportional to the future value of the company. This is the assumption that underlies the WACC approach, and is best dealt with using that method. The other reason, which the proponents of the capital cash flow approach use as a justification, is that the future tax savings from interest depend on the level of future operating income. If the company may not have enough operating income to pay tax, then interest cannot be used to save tax immediately. In this case, the risk of

⁴ Franco Modigliani and Merton Miller “Corporate Income Taxes and the Cost of Capital: A Correction” *American Economic Review* Vol. 53 (1963), pp. 433-443.

⁵ This is sometimes called the Miles-Ezzell approach because they were the first to show the relationship between valuation using the WACC and the capital structure assumption of constant proportional leverage. See James Miles and John Ezzell, “The Weighed Average Cost of Capital, Perfect Capital Markets, and Project Life: A Clarification,” *Journal of Financial and Quantitative Analysis* Vol. 15 (1980), pp. 719-730.

the interest tax savings from debt will be higher than the risk of the interest charges. For such cases, the assumption that the risk of the interest tax savings is equal to the risk of the operating cash flows is advocated as a simple approximation.

With this assumption, the tax savings from interest and the operating free cash flows have the same levels of risk and can be combined into a single measure called capital cash flow. As already noted, this should be discounted at the unlevered cost of capital. The resulting present value is an estimate of the levered value because the debt tax shield has been included in the cash flows.

To sum up, then, there are three methods of incorporating the debt tax shield into a valuation:

Method 1: APV: Discount the operating free cash flow at K_U to give V_U . Add PVTS to give V_L .

Method 2: WACC: Discount the operating free cash at WACC to give V_L .

Method 3: Capital cash flow: Discount the capital cash flow at K_U to give V_L .

The most general of these is the APV method; it can accommodate any assumption about the debt policy of the company and the risk of debt tax shields. But even if the APV method is used, the choice of a particular method of estimating the unlevered discount rate implies a particular debt policy, and the use of a particular formula for PVTS also implies a particular debt policy. For example, the use of M&M formula (2) implies very strong assumptions about debt policy. Like use of the WACC method, it assumes that debt is a constant proportion of value. The capital cash flow method assumes that debt tax shields have the same level of risk as the operations of the company. The important point here is that given the appropriate assumptions, one can calculate the leveraged value of the company using either of these two methods in a way that is consistent with the basic APV formula.

The tax treatment of investors. Each of the three approaches discussed so far assumes that the present value of a dollar of tax saved by the company is fully reflected in shareholder value. But, in addition to arguments about the validity of each of these methods, there is also some disagreement as to whether the tax rate that should be used in calculating the value of the debt tax shield should be lower than the corporate tax rate because of taxes incurred by investors. The standard way to deal with this issue has been to define a *net tax saving* variable, T^* , that reflects the tax treatment of the investors who hold the company's debt and equity as follows:

$$(1-T^*) = (1-T_C)[(1-T_{PE})/(1-T_{PD})], \quad (4)$$

where T_{PE} is the marginal tax rate of the investors who determine the company's cost of equity, and T_{PD} is the tax rate at the margin of the investors who determine the company's cost of debt.

As can be seen from this equation, if the tax treatment of debt and equity is the same, then the net tax saving variable, T^* , is equal to the full corporate tax rate, and all the valuation formulas discussed above apply. But if the tax treatment of equity is more favorable than the tax treatment of debt, then T^* will be lower than the full corporate tax rate and the valuation formulas should be adjusted accordingly. Specifically, the value of the debt tax shield should be calculated using the lower *net* tax saving rate, rather than the full corporate tax rate. For instance, in that case equation (2) should be:

$$PVT_S = T^*D, \quad (5)$$

which yields a lower value for the debt tax shield. A value of T^* lower than the corporate tax rate would also affect the calculation of the cost of capital, which we discuss below.

This completes our brief review of the theory. We now summarize several debates among researchers that are relevant to the practical valuation of debt tax shields. These

concern the size of the net tax saving, T^* , and what formulas to use when valuing the debt tax shield.

DEBATES ABOUT THE SIZE OF T^*

Debate 1: How big was the value of debt tax shields in the U.S. prior to 2003?

Because the investor tax rates that enter the expression for T^* cannot be observed directly, the debate as to whether investor taxes reduce the value of the debt tax shield is an empirical one. The only studies that provide explicit estimates of T^* are based on regressions of the value of levered companies on their amounts of debt, while controlling for other variables that are assumed to affect their unlevered values.⁶

These regressions involve considerable econometric difficulties because the controls for the unlevered value are difficult to implement. However, the most sophisticated test for the U.S. to date, by Deen Kemsley and Doron Nissim, finds that the value of T^* for the U.S. prior to the 2003 tax changes was close to the then full corporate tax rate of 40%. Thus the best evidence concludes that, in a classical tax system where taxes on dividends and interest are the same, T^* is roughly equal to T_C . So the empirical evidence suggests that standard methods of valuing the debt tax shield are correct under a classical tax system.

Debate 2: Is the value of the debt tax shield lower in tax systems that favor dividends?

The evidence just cited concerned the U.S. prior to 2003, with its classical tax system in which dividends and interest were taxed at the same rate. In the U.S. since 2003, and in many countries with “imputation” tax systems as well, the investor tax on

⁶ Two important papers are Eugene Fama and Kenneth French, “Taxes, Financing and Company Value,” *Journal of Finance*, Vol. 53 (1998), pp. 819-843; and Deen Kemsley and Doron Nissim, “Valuation of the Debt Tax Shield,” *Journal of Finance* Vol. 57 (2002), pp. 2045-2073. The latter paper corrects an econometric problem in the former.

dividends is lower than the investor tax on interest. The U.S. tax system now has a lower tax rate for dividends than for interest, and imputation systems have the same effect by allowing investors to claim back part of the corporate tax when dividends are distributed. This creates the possibility that T^* may be lower than T_C because the tax treatment of equity returns is more favorable than the tax treatment of interest for the same investor.

There have been several attempts to test whether the value of the debt tax shield is smaller in countries with tax systems that favor equity over debt. Equation (4) says that when the equity tax rate is lower than the tax rate on interest, the net tax saving, T^* , is smaller than the corporate tax rate. There are some indirect indications that this may be the case in tax systems that favor dividends. Evidence from the U.K. before and after its switch from an imputation system to a classical tax system in 1997 shows that the tax rate on dividends implied by ex-dividend day share price moves was lower when dividends had a favored tax treatment.⁷ Evidence from the U.S. since the Bush tax cut of 2003 suggests that dividends have increased as a result of that change, as would be expected if their more favorable tax treatment increased their value and reduced the relative value of the debt tax saving.⁸

Such indirect evidence suggests that leverage may be less valuable in tax systems that favor dividends, but does not provide evidence on the value of T^* that should be used in valuations. In some cases where the tax system currently favors dividends over interest, such as the U.S. at the present time, that tax regime may not be expected to continue. Valuing the debt tax shield for a company involves predicting the net tax saving from interest for the indefinite future. The use of a net tax saving significantly less than the full corporate tax rate requires that the tax system favor equity returns--for

⁷Leonie Bell and Tim Jenkinson, "New Evidence of the Impact of Dividend Taxation and on the Identity of the Marginal Investor," *Journal of Finance*, Vol. 57 (2002), pp. 1321-1346.

⁸Brandon Julio and David Ikenberry, "Reappearing Dividends," *Journal of Applied Corporate Finance* Vol. 16 (2004), pp. 89-100.

example, due to a lower tax on dividends than interest--and that this be expected to continue. The one case where this clearly applies is in those countries that have had imputation tax systems for a long time, such as Australia, France, and Germany. This was also the case in the U.K. prior to the unexpected change in the tax system in 1997. We discuss later how to incorporate imputation taxes into valuations.

Debate 3: Is T^* lower for some companies and transactions?

Even in classical tax regimes that do not favor dividends over interest, such as the U.S. prior to 2003, there will be companies or transactions for which the debt tax shield has lower value because there is not enough taxable income to use the full interest tax deduction to save taxes. In such cases, the estimate of the value of the debt tax shield should be the present value of the actual taxes expected to be saved, an amount that will be less than the interest charge multiplied by the corporate tax rate. One way to include this in the valuation is to estimate the expected tax saving from interest using a method such as simulation.⁹ An alternative is to use the capital cash flow method, which assumes that the tax saving has the same level of risk as the cash flows. The first of these two methods is likely to be more accurate because the capital cash flow approach assumes that the risk of the interest tax saving is the same as the risk of the operating cash flows, which is only an approximation.

Another issue is whether companies that do not distribute their free cash flow as dividends have a lower tax rate on equity, and a lower value of T^* as a consequence. If equity returns are received in the form of capital gains, the ability to defer or avoid taxation on capital gains may result in a low effective tax rate on equity. This has given

⁹ John Graham shows one way to do this in "How Big are the Tax Benefits of Debt?," *Journal of Finance* Vol. 55 (2000), pp. 1901-1941. This paper also shows that most companies stop short of the leverage that would give rise to the problem of exhausting their taxable income with tax deductions. Dan Dhaliwal, Kaye Newberry, and Connie Weaver show that tax credits that reduce the net corporate tax rate have the predicted effect of reducing the use of leverage in transactions in "Corporate Taxes and Financing Methods for Taxable Acquisitions," *Contemporary Accounting Research* Vol. 22 (2005), pp 1-30.

rise to considerable controversy, with some studies claiming that there is no such effect, and that retained earnings result in taxation of investors that is ultimately the same as dividends.¹⁰ On the other side are studies whose results suggest that the effect of deferring capital gains results in a considerable reduction in shareholder taxes that has the effect of increasing shareholder value.¹¹ This debate is ongoing, and difficult to resolve because of the considerable econometric difficulties. The balance of the evidence, however, appears to lie with those who claim that capital gains are taxed at a lower effective rate than dividends, and that this effect feeds through to shareholder value. The implication is that companies that deliver a large proportion of their shareholder returns in the form of capital gains have a smaller tax benefit to borrowing, which should in turn be reflected in a lower value of T^* . However, few of the studies in this literature gives an empirical estimate of how one should adjust the estimate of T^* to reflect this effect.¹²

What T^* Should You Use?

The standard methods of valuing the debt tax shield assume that T^* is equal to T_C . The best empirical evidence suggests that this is a good assumption as long as (a) the tax system is a classical system and does not explicitly favor dividends over interest, (b) the company or transaction being valued will be able to use all its future interest charges to save tax, and (c) the operating free cash flow will be distributed as dividends. If the tax

¹⁰ On this side of the debate is a series of papers beginning with Trevor Harris and Deen Kemsley, "Dividend Taxation and Company Value: New Evidence," *Journal of Accounting Research* Vol. 37 (1999), pp. 275-291.

¹¹ Dan Dhaliwal, Merle Erickson, Mary Frank, and Monica Banyi, "Are Shareholder Dividend Taxes on Corporate Retained Earnings Impounded in Equity Prices," *Journal of Accounting and Economics* Vol. 35 (2003), pp. 179-200. These authors provide a summary of the debate and explain the econometric problems in measuring the effect.

¹² The exceptions are John Graham's (2000) paper, referred to above, and Rick Green and Burton Hollifield, "The Personal Tax Advantages of Equity," *Journal of Financial Economics* Vol. 67 (2003), pp. 175-216. These papers provide estimates of the average value impact of the capital gains effect based upon tax rules, rather than empirical evidence from equity prices. The difficulty with using these results is that equity prices and valuations should reflect market expectations of future tax regimes, and this cannot be observed directly from past tax rules.

system explicitly favors dividends, and this is expected to continue indefinitely, then the value of T^* should be lower. The most common situation where this occurs is the case of imputation tax systems. Such systems explicitly favor dividends over interest by using the imputation tax rate, T_I , to lower the effective tax rate on dividends. Assuming that the dividend payout is 100%, the formula for T^* is:

$$(1-T^*) = (1-T_C)/(1-T_I) \quad (6)$$

which can be approximated as:

$$(1-T^*) = (1-T_C)[(1-T_{PD}+T_I)/(1-T_{PD})] \quad (7)$$

This approximation illustrates that one way to deal with an imputation system is to set the investor tax rate on equity equal to the tax rate on debt minus the imputation tax rate.¹³

This was one standard approach used in the U.K. prior to its move away from an imputation tax system in 1997.¹⁴

Although the U.S. system also currently favors dividends, there is some uncertainty as to whether this will continue. No one has yet tested whether U.S. equity prices reflect the assumption that this tax system is permanent or temporary. If one believes it is permanent, then the lower tax rate on dividends should be reflected in T^* in a similar way to the imputation tax. If one believes that the U.S. tax system will soon return to a standard classical system, then such an adjustment may be unnecessary, given that a small proportion of the value of tax shields comes from the next few years.

If the company or transaction may not be able to use all of its interest charges to save tax either now or in the future, the debt tax shield will reflect a tax rate lower than the full corporate tax rate. The way to incorporate this into a valuation is either to explicitly forecast the expected tax savings from interest, including the effect of tax

¹³ We provide a thorough discussion of imputation tax systems in Ian Cooper and Kjell Nyborg, "Discount Rates and Tax", London Business School working paper, 2004.

exhaustion, or to use the capital cash flow method, which we discuss below. The capital cash flow method makes a particular assumption about the risk and expected value of the debt tax shield.

Finally, if the company is not distributing all its operating free cash flow as dividends, then the effective tax rate on equity returns is lower, and so is T^* . However, estimating T^* in this case is difficult because it involves an assumption about the effective tax on capital gains and how it affects equity prices.¹⁵ This, as already noted, is an area of great current controversy. The lack of a robust method for making this adjustment may explain why it is not commonly done.

Should you use APV, WACC, or capital cash flow?

The choice between the three methods of incorporating the debt tax shield into a valuation appears at first sight to be important. In one sense it is, because the three methods can give very different values. Nevertheless, they are entirely consistent with one another when the assumptions that each method makes about the company's tax status and future leverage policy are met. Table 1 summarizes these assumptions and indicates (with an \checkmark) which method is likely to work best given a particular set of assumptions, and which methods (marked with X) are inconsistent with the particular assumptions.

¹⁴ The use of a lower value of T^* also has implications for how one estimates discount rates, which we discuss in Ian Cooper and Kjell Nyborg, "Discount Rates and Tax," London Business School working paper (2004).

¹⁵ The method in John Graham's (2000) paper can be used, with the caveats noted in the previous footnote.

Table 1: Which valuation methods are consistent with which assumptions?

ASSUMPTIONS			METHODS		
Assumption name	Leverage policy	Future tax status	APV	WACC	Capital CF
WACC	Debt proportional to value	Tax-paying	PVTS is interest tax shields discounted at K_U ¹⁶	√	Can be used, but WACC is easier
MM	Constant amount of debt	Tax-paying	√ $PVTS = T \cdot D$	Only if the value of the company is not expected to change	X
Capital cash flow	Any policy	Risk of tax savings equal to operating free cash flow	PVTS is interest tax shields discounted at K_U	X	√
Extended MM	Any policy with fixed future debt amounts	Tax-paying	√ PVTS is interest tax shields discounted at K_D	X	X

The table shows that APV can always be used provided it is applied consistently. However, there are two cases where it clearly makes sense to use the other methods. One is when the company can be assumed to use a constant proportion of leverage. In that case, the WACC method is the simplest. The other is where the future tax saving is risky because, for example, the company may become non-taxpaying. In that case, if one is willing to assume that the risk of the interest tax shield is equal to the risk of the operating

free cash flows, the capital cash flow method may be used. But if neither of these specialized methods is appropriate, then the correct approach is the explicit APV approach, using a discount rate for the debt tax shields that correctly reflects their risk. Such an APV approach would be recommended, for example, for most leveraged buyouts, where both leverage ratios and thus the risk associated with tax shields are expected to fall over time.

The real issue in this area of debt tax shield valuation is not which method is the “best” in any absolute sense, but rather which set of assumptions is the most reasonable. In making this decision, it is important to realize that none of the assumptions will be fulfilled exactly by any company. What is needed is the set of assumptions that is the closest to the long-run leverage policy of the company being valued.

Evaluated in this way, the best assumption for most companies is the WACC assumption of a constant proportion of debt. But different assumptions may be appropriate in particular cases. For example, for companies that have a high probability of being non-taxpayers at some point, the capital cash flow method may be more appropriate. But even in such cases, it may be better to explicitly model the expected future tax savings from interest, as discussed in the previous section.

Debates About The Correct Formula For PVTs

Implementation of the WACC method does not require an explicit formula for PVTs because it calculates the total value of the leveraged company directly. In general, there is no simple formula that gives the value of PVTs with the WACC assumptions. However, in the important case of a constant growth company, there is such a formula. In

¹⁶ With annual rebalancing, the first tax shield is discounted at K_D and subsequent discounting is at K_U . With continuous rebalancing, all discounting is at K_U .

a recent study published in the *Journal of Financial Economics*, we showed that the formula for the present value of tax shields is as follows:¹⁷

$$PVTS = T_C D K_D / (K_U - g) \quad (8)$$

The set of cash flows in this formula begins with $D * K_D * T$, the tax saved by interest in the first period. This amount then grows at the rate g per annum, as does the entire company. Future debt levels and tax savings are assumed to be proportional to the value of the operations of the company. Because we assume the WACC assumption of a constant leverage ratio, the risk of the future tax saving from interest is equivalent to the rate K_U . So PVTS is equal to the value of a growing perpetuity that starts with $D * K_D * T$, grows at g , and is discounted at K_U .

The formula illustrates an important feature of the value of debt tax shields with the WACC assumption. This feature becomes clearest if the growth rate is set to zero, in which that case the value of the tax shields is as follows:

$$PVTS(WACC, \text{zero growth}) = T_C D K_D / K_U \quad (9)$$

This value is much less than the M&M value of $T_C D$, which comes from valuing a constant perpetual stream of tax savings from interest, $T_C D K_D$, at the (lower) discount rate appropriate to debt, K_D . The cash flow is discounted at the rate appropriate to debt because, under the M&M assumptions, the interest tax savings have the same level of risk as the debt. In contrast, under the WACC assumptions, debt tax shields are more risky because they vary with the success of the company. And the use of the resulting higher discount rate (K_U in equation (9)) can lead to a considerable reduction in the value of the tax shield.

Some studies have argued that the value of the debt tax shield is much higher than that given by our formula in equation (8).¹⁸ They claim that the correct formula is

¹⁷ See Ian Cooper and Kjell Nyborg, "The Value of Tax Shields IS Equal to the Present Value of Tax

$$PVTs = T_c D K_U / (K_U - g) \quad (10)$$

The difference is that the unlevered cost of capital, K_U , appears in the numerator instead of the cost of debt, K_D . For many companies, K_U may be as much as twice K_D , so this approach appears to double the size of the debt tax shield, which would have a major effect on valuation. For instance, consider a company with debt of \$100 million, an unlevered cost of capital of 8%, a cost of debt of 4%, a tax rate of 40%, and a growth rate of 3%. Whereas our formula (equation (8)) implies that PVTs is \$32 million, equation (10) implies it is \$64 million.

As we argued in our recent *JFE* paper, the higher values from equation (10) are the result of mixing the constant proportion leverage assumption that underlies the WACC with equations that are meant to be used only under the M&M assumptions of a constant amount of debt and zero growth. But for the arguably more representative situation in which a company is expected to experience constant growth and leverage, equation (8) is likely to provide the most reliable result. The size of the discrepancy from using inconsistent assumptions shows the importance of understanding the assumptions about debt policy that underlie the various methods of valuing debt tax shields and using them in a consistent way.

This has led to further debate about whether there is a leverage policy for a growing company that can give a value for the debt tax shield much larger than given by our formula. As yet, no one has come up with a convincing result, but the work is ongoing.

Shields,” *Journal of Financial Economics* Vol. 81 (2006), pp. 215-225.

¹⁸ This claim has been made by Pablo Fernandez in a series of papers, including “The Value of Tax Shields Is NOT Equal to the Present Value of Tax Shields,” *Journal of Financial Economics* Vol. 73 (2004), pp.

How To Relever Discount Rates

Both the capital cash flow method and the APV method require an unlevered discount rate, K_U , for their implementation. The unlevered discount rate is not usually observable. What can be observed are the betas and required rates of return of the levered equities of companies. Therefore, one needs a method of estimating the unlevered discount rate from these betas. If the WACC method is used, the discount rate may need to be relevered to a different debt level before it is used. Therefore, all methods need a way of adjusting either discount rates or betas for the amount of leverage. There are two competing ways of doing this.

With the WACC assumption of a constant proportion of equity, the unlevered beta can be derived from equity and debt betas using the following formula:¹⁹

$$\beta_U = (E/V_L)\beta_E + (D/V_L)\beta_D \quad (11)$$

The relationship between the WACC and K_U is given by:

$$K_U = WACC + T_C K_D (D/V_L) \quad (12)$$

But with the M&M assumptions of a constant amount of debt, the equivalent equations are different. Starting from equity and debt betas, the unlevered beta would be derived as follows:²⁰

$$\beta_U = (E/(V_L - T_C D))\beta_E + (D/(V_L - T_C D))\beta_D (1 - T_C) \quad (13)$$

The relationship between the WACC and K_U is given by:

$$K_U = WACC / (1 - T_C (D/V_L)) \quad (14)$$

These formulas for beta assume that debt is risky. Some readers may be more familiar with simpler formulas that assume the beta of debt is zero.

145-165. An earlier version of the same claim is made in Laurence Booth "Finding value where none exists: Pitfalls in using adjusted present value", *Journal of Applied Corporate Finance* Vol. 15 (2002), pp. 8-17.

¹⁹ This approach is used in Richard Brealey and Stewart Myers, *Principles of Corporate Finance*, McGraw-Hill, New York., 2006. This formula, as well as (12), assumes continuous rebalancing of the debt.

²⁰ This is the approach used by a number of scholars, including Mark Grinblatt and Sheridan Titman, *Financial Markets and Corporate Strategy*, McGraw-Hill, 2002. They note that it is a special case.

The two different approaches to unleveraging betas and discount rates give very different results. Suppose that we observe a WACC of 8% for a company that is levered with 30% debt. If the corporate tax rate is 40% and the cost of debt is 4%, equation (12) based on the WACC assumption implies that the unlevered discount rate is 8.5%. But, using the M&M assumption, equation (14) implies that it is 9.1%. This difference is large enough to lead to significant differences in valuation. The reason for the difference is that the two methods assume different levels of risk in the debt tax shields, which in turn affects the relationship between levered and unlevered discount rates.

The betas that will be used as the basis for the discount rate are the equity betas of companies in the same industry as the investment. These reflect whatever leverage policy the stock market assumed for the companies during the period the beta was estimated. Although neither assumption is likely to be an exact reflection of market beliefs, the assumptions underlying the M&M equations are much less realistic than those underlying the WACC. Thus the WACC equations (11) and (12) are more likely to be more appropriate in most cases than the M&M equations (13) and (14).²¹

AN EXAMPLE

Here we offer an example that illustrates the large differences in valuations that can result from using different methods. The first step involves calculating the cost of capital for a project using financial data from comparable companies. This is done in Table 2. For simplicity, we use only one comparable and ignore personal taxes.

If we assume that the comparable company follows a constant debt-to-value leverage policy, then equation (11) is the appropriate formula to use in calculating the

²¹ We assume here that the capital asset pricing model is used to set discount rates. Similar issues arise if other methods are used. When the CAPM is used, it is also important to make sure that the treatment of the riskless rate is consistent with the assumption about T^* . We discuss this in Ian Cooper and Kjell Nyborg, "Discount Rates and Tax," London Business School working paper (2004).

unlevered asset beta. This is what we referred to above as the WACC method (since it is consistent with the use of WACC in valuation).²² As shown in the table, this gives an estimated unlevered asset beta of 0.66 and an unlevered cost of capital of 8.15%.

On the other hand, if the comparable firm is assumed to have a fixed amount of debt in perpetuity, we need to use equation (13) to calculate the unlevered asset beta. Thus, the estimate of the unlevered asset beta increases to 0.75 and the estimated unlevered cost of capital increases to 8.49%. This increase arises from the M&M method's assumption that the tax shield is safer than what is implicit in the WACC method. As a result, more of the observed risk reflected in the equity beta must be due to the operating assets.

Table 2: Estimation of Cost of Capital

	Comparable Firm			Method	
				WACC	MM
	Equity	Debt	Levered Assets	Capital Cash Flow	Unlevered Assets
Value (mill)	10,000	6,000	16,000	Depends on growth rate	13,900
Beta	1	0.1		0.66	0.75
Rate (K)	9.50%	5.90%		8.15%	8.49%
Fraction of assets	62.5%	37.5%			
WACC			7.38%		

	Riskfree rate	Risk Premium
Input for CAPM	5.50%	4%
Corporate tax rate	35%	

The difference in estimated cost of capital of 34 basis points leads to significant differences in valuation. Additional differences can arise in the next step when we attempt to value the project.

Let us assume that the project to be valued requires an initial investment of 100 million. In year 1, it will return an after-tax cash flow, under all equity financing, of 7.5

²² Note that equation (12) is, strictly speaking, consistent with the WACC assumptions only if leverage is continuously rebalanced. If leverage is rebalanced only periodically, say once a year, a small adjustment is

million. This will grow by 1% in perpetuity. We assume that the project supports an initial debt level of 60 million. This will give rise to a different leverage ratio than that of the comparable firm. Finally, we assume that the cost of debt relevant for the project is 6.10%. The values provided by the different valuation methods are provided in Table 3.

Table 3: Valuation of Project

	Cost of investment	Cash flow year 1	Growth rate
	100	7.5	1%

	Method for Project Valuation		
	WACC	MM	Capital cash flow with constant debt
Initial debt level	60	60	60
Cost of debt	6.10%	6.10%	6.10%

Comparable based on WACC method			
KU	8.15%	8.15%	8.15%
NPV (unlevered)	4.90	4.90	4.90
PVTS	17.92	21.00	15.72
APV	22.81	25.90	20.61
Debt to value ratio	48.86%	47.66%	49.75%

MM method			
KU	8.49%	8.49%	8.49%
NPV (unlevered)	0.13	0.13	0.13
PVTS	17.10	21.00	15.09
APV	17.24	21.13	15.22
Debt to value ratio	51.18%	49.53%	52.07%

Difference in APV	5.57	4.76	5.39
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The highest and the lowest valuations are highlighted in italic bold. The table illustrates that the range of estimated values goes from 15.22 million (when the M&M method is used to estimate the unlevered cost of equity and the capital cash flow method used to value the project) to 25.90 million (when the WACC method is used to estimate the unlevered cost of equity and the M&M method to value the project).

necessary, which we ignore here. It is always consistent with the capital cash flow assumptions.

Using the WACC method to estimate the unlevered cost of capital provides higher valuation estimates than using the M&M method since it gives rise to a lower estimate of the cost of capital. The differences in valuations due to different cost of capital estimates range from 4.76 million (when the M&M method is used to estimate the project value) to 5.57 million (when the WACC method is used). The large differences in valuation estimates illustrate the importance of using the most appropriate method in the cost of capital estimation as well as in the final valuation step itself.

SUMMARY

Having reviewed the main areas of debate and tension in debt tax shield valuation, we now provide a summary.

Alternative assumptions about debt policy. There are four alternative assumptions about debt policy that can be used in the valuation of debt tax shields:

1. Operating cash flow is a risky flat perpetuity combined with a constant amount of debt (M&M).
2. Constant proportional market-value leverage (WACC).
3. Any arbitrary non-constant leverage policy, with tax savings from debt that have the same risk as the operating free cash flows (capital cash flow).
4. Any arbitrary non-constant leverage policy, with tax savings from debt that have the same risk as the debt (extended M&M).

Each has different implications for how to calculate the value of debt tax shields and how to unlever and relever discount rates. In Table 1 above, we summarize which methods of valuing debt tax shields are consistent with which assumptions.

Which leverage assumption should you use? The different leverage assumptions give different values for the debt tax shield. They imply different unlevered and

relevered discount rates. The discount rates and values that one should use are those that reflect the actual leverage policy that a company is expected to pursue. In valuations of stable companies, the best assumption is likely to be the constant proportional leverage that underlies the WACC method. But when valuing companies in transition or specific transactions or projects, one should generally assume that leverage will change over time—and the WACC approach should probably not be used. In such cases, the interest tax shields must be forecast year by year. If these tax shields are expected to have the same risk as the operating cash flows, one can use either the capital cash flow method or the APV method with the tax shields discounted at the unlevered discount rate. If the amount of debt is not expected to vary with the success of the investment, then APV may be used with the tax shields discounted at the debt rate. Rarely is the M&M assumption of a constant amount of debt likely to be the correct one.

Unlevering or relevering discount rates. Discount rates must be relevered in all cases except those in which the asset being valued will have the same leverage as the comparable company and the WACC method is used. In all other cases, a relevering formula must be used either to obtain the unlevered discount rate or to relever the WACC. There are two sets of formulas, one based on the WACC assumption of constant proportional leverage and the other on the M&M assumption of a fixed amount of debt. The choice of which to use depends on the leverage policy that stock prices reflected during the period of observation of the equity beta. In general, this is likely to have been closer to the WACC assumption than to the M&M assumption.

What T^ ?* The standard methods of valuing the debt tax shield assume that T^* is equal to T_C . The best empirical evidence suggests that this is a good assumption provided (a) the tax system is a classical system and does not explicitly favor dividends over interest, (b) the company or transaction being valued will be able to use all its future

interest charges to save tax, and (c) the operating free cash flow will be distributed as dividends.

If the tax system explicitly favors dividends, and this is expected to continue indefinitely, then the value of T^* is lower. The most common situation where this is the case is in imputation tax systems, in which case the tax rate on equity could be approximated by the tax rate on debt minus the imputation tax rate. Although the U.S. system also currently favors dividends, there is some uncertainty about whether this will continue. If one believes that the current system is likely to be permanent, then the lower tax rate on dividends should be reflected in T^* in a similar way to the imputation tax. But if one believes that the U.S. tax system will soon return to a standard classical system, then such an adjustment is unnecessary.

If the company or transaction may not be able to use all of its interest charges to save tax either now or in the future, the debt tax shield should reflect a tax rate lower than the full corporate tax rate. This lower rate can be incorporated into a valuation either by explicitly forecasting the expected tax savings from interest including the effect of tax exhaustion, or by using the capital cash flow method.

If the company is not distributing all its operating free cash flow as dividends, then the effective tax rate on equity returns is lower, and T^* may also be lower. Nevertheless, there is no agreed-upon method for making this adjustment, which may explain why it is rarely done.

THE IMPORTANCE OF CONSISTENCY

We have shown that there are several key decisions in valuing debt tax shields that can have major effects on the resulting values. Some are obvious, such as the size of the net tax saving variable, T^* , and the use of different valuation methods. Others are more

subtle, such as the use of the M&M rather than the WACC formulas for relevering discount rates, and the different formulas for the value of PVTs for a constant growth company.

In our opinion, the key to valuing tax shields is consistency. First, the method used should be consistent with the actual debt policy of the company being valued. Second, the relevering formula should be consistent with the debt policies of the companies whose equity betas are used to estimate discount rates. Third, different methods that assume different assumptions should not be mixed in the same valuation.

The danger of not reflecting the actual leverage policy in valuations was first pointed out by Stewart Myers, who showed that using an incorrect assumption about the leverage policy that will be pursued can have a large effect on the value of the debt tax shields.²³ The choice between the two possible relevering formulas can be equally important. Both approaches persist in part because the formulas are often used to unlever a discount rate and then relever it back to a leverage ratio similar to where it started. In that application, it doesn't much matter which approach one uses as long as one uses the same approach to unlever as to relever the rate. In other cases, where the unlevered rate itself is being used in a valuation, it does matter which approach one uses, and we have shown that the difference is significant. As we noted earlier, the WACC formulas for relevering are usually more likely to be accurate than the M&M formulas.

The importance of the third kind of consistency—the use of consistent assumptions about leverage policy throughout all stages of a valuation—is less widely appreciated. The ability of some studies to derive enormous values for debt tax shields for constant growth companies is attributable to this kind of inconsistency. The problem arises from inconsistent assumptions about capital structure that can be hidden within

apparently standard valuation equations. If valuations based on different assumptions are mixed, the error in the estimate of the value of the debt tax shield can be large. PVTs is the difference between the value of the levered company and the value of the unlevered company. If the two values are estimated inconsistently, relatively small errors in V_L or V_U can become a large error in PVTs.²⁴

In sum, inconsistency can have a significant effect on the valuation of debt tax shields. Significant misvaluation of PVTs can arise from internally inconsistent assumptions. It is difficult to predict how these errors will arise, and it is relatively simple to use internally consistent methods. So, in any individual valuation, it is worth picking one of the four assumptions about capital structure discussed in this note and sticking to it.

²³ Stewart Myers, "Interactions of Corporate Financing and Investment Decisions: Implications for Capital Budgeting," *Journal of Finance* Vol. 29 (1974), pp. 1-25.

²⁴ The problem is similar to the well-known example of the Cadillac and the Movie Star given in Brealey and Myers.