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Inflation and the Constant-Growth Valuation Model: A Clarification

Abstract

We examine the effects of inflation on the standard, Constant-Growth valuation model found throughout the finance literature. We find that the presence of inflation vitiates the generally accepted expression of this model for the value of a firm that either makes no new investments or invests only in zero net present value projects. If expected inflation is positive, the generally accepted and widely used expression for the value of the firm under either of these two conditions seriously understates the true value of the firm, even at modest levels of inflation. For example, assuming zero net new investments, a real interest rate of 6% and a rate of inflation of 2%, the commonly accepted expression understates the true value of the firm by 25%. We also examine the effects of inflation on the firm's weighted-average cost of capital (WACC), which is an important parameter in the Constant-Growth model. We find that the popular WACC equation developed by Modigliani and Miller is not inflation-neutral when stated in nominal terms. Specifically, when expected inflation and corporate tax rates are positive, the nominal M&M WACC understates the firm's true nominal WACC by a non-trivial amount. We show how to adjust the standard M&M formula to correct for this understatement. In contrast to the M&M model, we find that the WACC equation developed by Miles & Ezzel is inflation-neutral when stated in nominal terms, and thus, there is no need to adjust the equation in the presence of positive expected inflation. We conclude the paper by documenting the widespread misapplication of the Constant-Growth model under conditions of inflation found throughout the finance literature and in the practical application of corporate valuation techniques.

I. Introduction

This paper examines the effects of inflation on the Constant-Growth valuation model. This model is taught in all top-tier business schools and is used widely throughout the financial community. It is found in virtually all graduate-level corporate finance textbooks and valuation manuals. However, our review of this extensive literature has uncovered no reference that systematically analyzes the effects of inflation on this model.[1] As we will show, the failure to account properly for the effects of inflation leads to a misspecification of this model for the value of a firm that either makes no new investments or invests only in zero net present value projects. We will show that the error generated by this misspecification is significant, even at moderate levels of inflation.

It is common throughout the finance literature to express the value of a firm that either makes no net new investments, or invests only in zero net present value projects as a simple perpetuity of next period's free cash flows, with the capitalization rate being the firm's nominal cost of capital. The use of the (zero-growth) perpetuity formula is typically justified by noting that: (1) with zero net new investments, there will be no growth; and (2) growth through the acceptance of zero NPV projects does not affect value. Although this logic may seem compelling, we show that this Zero-Nominal-Growth model is based on an erroneous specification of the nominal growth in cash flows

[1] The popular valuation texts, Rappaport (1998), Copeland, Koller and Murin (1994) and Cornell (1993) all discuss various aspects of the effects of inflation on the valuation process. However, none develops the effects of inflation on the Constant-Growth Model from first principles, as we do in this paper. Rather, they impose intuitive, yet often incorrect, conditions on the standard valuation models. Our analysis most closely resembles that of Rappaport. On page 47 he presents a formula for a "perpetuity with inflation," which is in fact a major result derived in this paper. However, in the previous pages he argues that nominal growth is irrelevant to valuation if the firm's return on investment is equal to its cost of capital. This is a true statement. However, this fact does not justify the perpetuity model that he develops and defends on pages 41-44 under conditions of inflation. Copeland et.al. p. 283 write down an equation that is close to Rappaport's and state, "This formula assumes that earnings in the continuing-value period will grow at some rate, most often the inflation rate. The conclusion is then drawn that earnings should be discounted at the real WACC, rather than the nominal WACC." They then go on to state that, in general, this model is not appropriate "because it assumes that NOPLAT (Net Operating Profits After Taxes) can grow without any incremental capital investment. This is very unlikely (or impossible), because any growth will probably require additional working capital and fixed assets." Although this logic applies in real terms, it ignores the fact that in nominal terms, fixed and working capital will grow at the rate of inflation. Finally, Cornell (1993, p. 147 Equation (6-1)) stresses that the appropriate growth rate to be used in this model is the nominal growth rate, which is comprised of the real growth rate and the rate of inflation. However, as we show subsequently, he misstates the relation between the nominal growth rate and the nominal return on investment in applying inflation to the Constant-Growth Model. p. 156.

in the presence of inflation. Specifically, the generally accepted expression for the value of a "zero-growth" or a "zero-net-present value" firm, as presented throughout the finance literature, ignores the effects of inflation on the firm's initial total invested capital. In the traditional Constant-Growth model, if there is no new investment, there is no growth. However, this conclusion ignores the fact that the value of the initial invested capital will grow with inflation and, with a constant real return on invested capital, the firm's (nominal) free cash flows stemming from those investments will grow at the same rate. Thus, the generally accepted expression understates the true value of the firm, and the understatement is positively related to the rate of inflation.

We develop the correct expression for the nominal growth in cash flows in the presence of inflation and show that the correct model assuming either zero investments or zero net present value investments is the traditional Constant-Growth model, with the growth rate set equal to the rate of inflation. We also show that this (correct) expression for the nominal growth term yields a valuation model that is independent of inflation.[2]

The second contribution of this paper to the valuation literature is to analyze the effects of inflation on a firm's WACC. We find that the WACC model developed by Modigliani and Miller (M&M) is incorrect as presented in the finance literature, if inflation is positive. As we demonstrate subsequently, plugging nominal values into the M&M WACC equation results in an underestimate of the firm's WACC, which results in an over-valuation of the firm. We develop a correction factor that, when added to the nominal M&M WACC, yields the firm's true nominal WACC according to the M&M assumptions. We demonstrate however, that the M&M model does hold in real terms, provided that the firm makes no net new investments. In contrast, the WACC model developed by Miles and Ezzell is correct when the parameters of the model are stated in nominal terms, but understates the value of the firm if the parameters are stated in real terms.

The remainder of this paper is organized as follows. In the next section we develop the Constant-Growth model under conditions of inflation. In Section III we derive the expressions for the value of the firm assuming (1) zero investments and (2) investments

[2] If by "inflation" we mean a proportionate increase in all nominal interest rates and the prices of all goods and services, then expected inflation should have no effect on present values.

only in zero net present value projects. In Section IV we replicate the development of the model under these conditions as it appears in the literature. We prove that these formulations are based on an erroneous expression for the nominal growth in cash flows under conditions of inflation. In Section V we provide estimates of the magnitude of the errors generated by the traditional model, which are significant at historical rates of inflation. In Section VI we examine the effects of inflation on the firm's Weighted Average Cost of Capital (WACC), since the literature argues that the appropriate discount factor to be used in the Constant-Growth model is the firm's WACC.[3] We conclude the paper with several examples of the misapplication of the Constant-Growth model under conditions of inflation found throughout the finance and valuation literatures. We point out that certain formulas in the two most popular commercial valuation models in use today, Alcar's *Value Planner* and Stern / Stewart's *EVA*, are incorrect if inflation is not zero. Finally, we discuss the possibility that the confusion in the literature over the effects of inflation on the Constant-Growth model had a material effect on the decision rendered in perhaps the most important case ever to have been tried in the Delaware Chancery Court involving corporate valuation (*Cede & Co. v. Technicolor, Inc.*, 1990).

II. The Constant-Nominal-Growth Model

The most widely accepted valuation model in finance is the Constant-Nominal-Growth model:

$$V_0 = \frac{C_1}{W - G} \tag{1}$$

where V_0 is the (present) value of the firm, C_1 is its expected net cash flow in the first future period, W is the firm's nominal cost of capital, and G is the projected nominal growth rate of the firm's future net cash flows.[4] This model, which can be found in

[3] "The appropriate rate for discounting the company's cash flow stream is the weighted average of the costs of debt and equity capital," Rappaport, page 37. Also see Brealey and Myers (2000), Chapter 19, Copeland et.al., Chapter 8 and Cornell, Chapter 7.

[4] According to Brealey and Myers, p. 67, this formula was first developed in 1938 by J.B. Williams and rediscovered in 1956 by M.J. Gordon and E. Shapiro. It is often called the Gordon Growth Model.

virtually all finance textbooks, is always written in nominal terms, as is Equation (1) above.[5]

Our analysis of the effects of inflation on this model begins with a derivation of an expression for the firm's nominal net cash flows (C) in terms of observable accounting numbers. We then derive an expression for the growth in nominal cash flows (G) under the assumptions of the standard Constant-Growth model. We defer our discussion of the appropriate discount rate to be use in this model until we discuss the firm's WACC in Section VI. For present purposes, suffice it to say that in the subsequent analysis, we assume that the firm's nominal cost of capital (W) is defined by the Fisher Equation:

$$W = w + \Pi + w\Pi \tag{2}$$

where w is the firm's real cost of capital and Π is the rate of inflation.[6]

A. Net Cash Flows

Following the literature, we define net cash flows as all cash flows in excess of what are required to fund the firm's operations. Net cash flows in any period t can be written as:

$$C_t = EBIAT_t + DEP_t - CAPX_t - \Delta WC_t \tag{3}$$

Equation (3) states that net cash flow in any period t is equal to the firm's EBIAT (Earnings Before Interest After Taxes, also referred to in the literature as NOPAT for Net Operating Profit After Taxes), plus book (accounting) depreciation (DEP), minus any payments for capital equipment (CAPX) and incremental investments in working capital (ΔWC). Book depreciation is added back to EBIAT because depreciation is a non-cash expense that is subtracted from EBIT to determine taxable income.[7]

We assume that capital expenditures (CAPX) can be divided into two components: replacement expenditures (RE) and net new investments (NNI):

$$CAPX_t = RE_t + NNI_t \tag{4}$$

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[5] Brealey and Myers, p. 67, Ross, Westerfield and Jaffee, p. 128, Grinblatt and Titman, p. 832, Rao, p. 403, Bodie and Merton, p. 123, Emery and Finnerty, p. 149, Shapiro and Balbirer, p. 159, Benninga and Sarig, p. 9, Van Horn, pp. 30-31, Martin, Petty, Keown and Scott, p.123.

[6] Irving Fisher (1930). Also see Brealey and Myers, pages 670-675.

[7] In fact the firm's earnings should be adjusted for all accruals (non-cash costs and revenues).

Replacement expenditures are defined as expenditures that are necessary to offset exactly the economic depreciation of the firm's capital stock over the period. Net new investments represent outlays over and above replacement expenditures. We make the standard assumption in the valuation literature that accounting depreciation (DEP) is equal to the firm's economic depreciation, which is equal to its replacement expenditures (RE). Thus, Equation (4) becomes:

$$CAPX_t = DEP_t + NNI_t \tag{5}$$

or

$$DEP_t - CAPX_t = -NNI_t \tag{6}$$

Equation (6) allows us to rewrite Equation (3) as Equation (7) below:

$$C_t = EBIAT_t - NNI_t - \Delta WC_t \tag{7}$$

Define k (the plowback ratio) as net new investment and incremental investment in working capital (NNI + ΔWC) over and above replacement expenditures, expressed as a percentage of earnings before interest and after taxes, EBIAT.

$$k = (NNI + \Delta WC) / EBIAT \tag{8}$$

Equivalently,

$$NNI + \Delta WC = EBIAT * k \tag{9}$$

Note that k = 0 does not mean that the firm is not re-investing at all. Rather, k = 0 means that the firm's reinvestment is equal to its economic deprecation over the period, so that the firm's capital stock and working capital remain constant in real terms.

Substituting Equation (9) into Equation (7) converts observable accounting numbers into net cash flows C:

$$C_t = EBIAT_t \ (1\text{-}k) \tag{10}$$

or simply

$$C_t = E_t \ (1\text{-}k) \tag{11}$$

where E_t is equal to the firm's EBIAT in period t.

B. The Real Return on Investment

The real return on investment, r, over any arbitrary period t is equal to

$$r = \frac{e_t}{K_{t-1}} \tag{12}$$

where e_t is the firm's real earnings (EBIAT stated in period t-1 dollars) in period t and K_{t-1}, is its total invested capital at the beginning of the period.[8] Note that we have adopted the convention of expressing real variables as lower case letters and nominal variables as capital letters.

Defining Π as the rate of inflation between period t-1 and t, real earnings in t can be written as

$$e_t = \frac{E_t}{1+\Pi} \tag{13}$$

where E_t is the firm's nominal earnings in period t. Given Equation (13), Equation (12) can be rewritten as

$$r = \frac{E_t}{(1+\Pi)\ K_{t-1}} \tag{14}$$

We rely on this expression for the real return on investment in the next sub-section.

C. The Growth in Nominal Earnings

Rearranging Equation (14) yields:

$$E_t = K_{t-1}\ r\ (1 + \Pi) \tag{15}$$

Since all rates of return and all growth rates are constant through time in the Constant-Growth model, there is no need to subscript these variables. Thus, $\Pi = \Pi_1 = \Pi_2 = \Pi_t$ etc. Of course, the same is true for r, the real return on investment, and all other interest rates.

Equation (15) states that nominal earnings in period t are equal to the nominal value of total invested capital at the beginning of the period times a constant factor, $r\ (1+\Pi)$. As the first step in developing an expression for the growth rate of earnings, we "difference" Equation (15)

$$E_{t+1} - E_t = (K_t - K_{t-1})\ r\ (1 + \Pi) \tag{16}$$

[8] Equation (12) appears in Copeland et.al. (1994), p.155, and it is termed the return on invested capital (ROIC). Consistent with the entire valuation literature, the authors do not specify whether the parameters represent real or nominal values. However, given the context, the parameters are clearly nominal.

Next we relate the firm's total invested capital at the beginning of the current period (t) to the total invested capital at the beginning of the previous period (t-1), the rate of inflation and the amount of depreciation over the period and the incremental expenditures on fixed and working capital made at time t. Thus, Equation (17) below expresses the firm's total invested capital at the beginning of any period t, K_t, as the value at the beginning of the previous period, K_{t-1}, less economic depreciation for the period, DEP_{t-1}, times one plus the inflation rate, $(1 + \Pi)$, plus the capital expenditures, CAPX, and investments in working capital, ΔWC, made at the beginning of the current period, t.

$$K_t = (K_{t-1} - DEP_{t-1})\ (1 + \Pi) + CAPX_t + \Delta WC_t \tag{17}$$

The first term in Equation (17) represents the value of the firm's total invested capital in terms of period t-1 dollars after accounting for (economic) depreciation over the period. Multiplying by $(1 + \Pi)$ converts this value into period t dollars. Adding current expenditures on fixed and working capital gives the nominal value of the firm's total invested capital (capital stock plus working capital) at the beginning of period t.

Separating capital expenditures into replacement expenditures RE_t and net new investment NNI_t, and assuming that replacement expenditures RE_t are equal to economic depreciation $DEP_{t-1}(1 + \Pi)$ over the period, Equation (17) can be rewritten as:

$$K_t = K_{t-1}\ (1 + \Pi) + NNI_t + \Delta WC_t \tag{18}$$

Substituting K_t from (18) into (16) yields:

$$E_{t+1} - E_t = [\,K_{t-1}(1 + \Pi) + NNI_t + \Delta WC_t - K_{t-1}\,]\quad r(1 + \Pi) \tag{19}$$

Note that the first variable and the last variable in the bracketed term of this equation is K_{t-1}. Expanding Equation (19) and canceling out the K_{t-1} terms, yields:

$$E_{t+1} - E_t = \Pi\quad r(1 + \Pi)\ K_{t-1} + (NNI_t + \Delta WC_t)\ r(1 + \Pi) \tag{20}$$

Dividing both sides of (20) by E_t defines the nominal growth in earnings and, equivalently, the nominal growth in net cash flows from t to t+1:

$$G \equiv \frac{E_{t+1} - E_t}{E_t} = \frac{E_{t+1}(1-k) - E_t(1-k)}{E_t(1-k)} = \frac{C_{t+1} - C_t}{C_t} \tag{21}$$

Substituting Equation (20) into the numerator of Equation (21) yields

$$G = \frac{\Pi\ r(1 + \Pi)K_{t-1}}{E_t} + \frac{(NNI_t + \Delta WC_t)\ r(1 + \Pi)}{E_t} \tag{22}$$

Given:

$E_t = r \; (1 + \Pi) \; K_{t-1}$, from Equation (15), and $k = \dfrac{(NNI_t + \Delta WC_t)}{E_t}$ from

Equation (8), Equation (22) becomes:

$$G = \Pi + kr(1 + \Pi) = \Pi + kr + kr\Pi \tag{23}$$

Define g as the <u>real</u> growth in earnings. Since G = g when Π = 0, it follows from Equation (23) that:

$$g = kr \tag{24}$$

Equation (24) states that real growth in earnings g is equal to the plowback ratio k times the real return on investment r.

To obtain the equation for nominal growth (G) in terms of nominal returns (R) and inflation (Π) add the quantity (+kΠ – kΠ) to Equation (23)

$$G = \Pi + kr + kr\Pi + k\Pi - k\Pi \tag{25}$$

collect terms

$$G = k\,(r + \Pi + r\Pi) + (1 - k)\Pi \tag{26}$$

and substitute R = r + Π + rΠ from the Fisher Equation for the nominal return to investment:

$$G = kR + (1 - k)\Pi \tag{27}$$

The first term in Equation (27) represents the growth in the firm's cash flows from new investments. The second term represents the increase in cash flows due to the increase in the nominal value of the firm's fixed and working capital due to inflation. Thus, there are two forces that account for the growth in nominal cash flows. First is the increase in earnings due to the nominal return on new investments. Second is the increase in nominal earnings due to the fact that the nominal value of the firm's capital stock is higher by the rate of inflation. As we demonstrate subsequently, it is this second term that is ignored in the existing finance literature. Note that if k = 0, then G = Π, and all growth will be due to the nominal growth in the firm's <u>initial</u> fixed and working capital due to inflation. If k > 0, then the inflationary growth of the firm's invested capital will be augmented by the nominal growth from new investments.

D. The Nominal Return on Investment

It is instructive to pause and note a relation between the nominal return on investment (R) and the rate of inflation (Π) beyond the simple Fisher Equation. From the Fisher Equation,

$$R = (1 + r)(1 + \Pi) - 1 = \Pi + r + r\Pi = r(1 + \Pi) + \Pi \qquad (28)$$

and from Equation (14),

$$r(1 + \Pi) = \frac{E_t}{K_{t-1}} \equiv \text{"Nominal Production Yield"} \qquad (29)$$

Substituting (29) into (28) yields:

$$R = \frac{E_t}{K_{t-1}} + \Pi \qquad (30)$$

Equation (30) states that the nominal return on investment is comprised of two parts. The first, defined in Equation (29), can be thought of as the "nominal production yield." The nominal production yield is the component of the nominal return represented by the nominal cash flows generated by the firm's assets at the beginning of the period, K_{t-1}. The nominal production yield can be likened to the dividend yield in calculating the return to a share of common stock. In fact if k=0, then E_t would be the dividend for an all equity firm. It represents the cash flow component of the nominal return. The second component of the nominal return to investment is simply the rate of inflation. The addition of the rate of inflation represents the "capital gain" on all the firm's existing assets (its capital stock plus working capital) over the period due to inflation. Like a capital gain, the inflation return would only be realized if the assets of the firm, or the claims thereto, were liquidated.

E. The Nominal Return to Stockholders

It is also instructive to distinguish between the nominal return on investment (R) and the nominal return to the firm's stockholders in the Constant-Growth model. The return to the stockholders of an all equity firm over any arbitrary period t, S_t, is equal to:

$$S_t = \frac{V_t + C_t - V_{t-1}}{V_{t-1}} \qquad (31)$$

where, V_t is the value of the firm at t and C_t is the dividend paid in period t.[9] Substituting Equation (1) for V_t and Equations (7) and (11) for C_t into Equation (31) we have:

$$S_t = \frac{\dfrac{E_{t+1}\ (1 - k)}{W - G} + E_t - NNI_t - \Delta WC_t - \dfrac{E_t\ (1 - k)}{W - G}}{\dfrac{E_t\ (1 - k)}{W - G}} \tag{32}$$

Multiplying through by (W - G) / (W - G)

$$S_t = \frac{(E_{t+1} - E_t)(1 - k) + (W - G)(E_t - NNI_t - \Delta WC_t)}{E_t\ (1 - k)} \tag{33}$$

$$S_t = G + \frac{(W - G)\ E_t\ (1 - k)}{E_t\ (1 - k)} = W \tag{34}$$

Thus, the per period return to the firm's stockholders over the perpetuity period is equal to the firm's cost of capital, regardless of the values of the other parameters of the valuation formula. Since the NPV of all future investments would be capitalized into the firm's stock price at any point in time, the per period return to the firm's stockholders throughout the perpetuity period will be W, the firm's nominal cost of capital. And this will be so, regardless of the value of R. Of course if all of the firm's projects have a zero NPV, then $R_t = W_t = S_t$. Note that since $W = w + \Pi + w\Pi$ the nominal return to stockholders is positively related to the rate of inflation.

III. The Zero-Real-Growth Model

We now turn to the derivation of the expressions for the value of the firm assuming first zero investments, and then investments only in zero NPV projects. As it turns out, these two assumptions yield the same valuation expression. We refer to this valuation equation as the Zero-Real-Growth model, since the expression is equivalent to assuming that g, the real growth in cash flows, is equal to zero.

[9] If the firm is levered, the market value of debt must be subtracted from V to arrive at the value of equity.

A. Zero Investments

The value of the firm, according to the Constant-Growth model, assuming zero net new investment, is found by substituting G from Equation (25) and C from Equation (11) into Equation (1) and setting k, the reinvestment rate equal to zero:

$$V_0 = \frac{E_1(1-k)}{W-(kR+(1-k)\Pi)} \tag{35}$$

$$V_0 = \frac{E_1}{W-\Pi} \tag{36}$$

Intuitively, even though the firm makes no net new investments, it still makes replacement investments sufficient to offset its economic depreciation, so that the <u>real</u> output of the firm must be constant over time. Thus, future nominal cash flows (revenues, costs and profits) emanating from a constant <u>real</u> output logically must be projected to grow at the rate of inflation, as it is with the Zero-Real-Growth model given by Equation (36).

Since $W = w + \Pi + w\Pi$, it follows that $W - \Pi = w(1+\Pi)$. Substituting this expression and the expression for nominal earnings into Equation (36) yields:

$$V_0 = \frac{e_1(1+\Pi)}{w(1+\Pi)} = \frac{e_1}{w} \tag{37}$$

which justifies our designation of this model as the Zero-<u>Real</u>-Growth model.[10] Note that this model is inflation neutral. Equation (37) is expressed exclusively in real terms.

B. Zero Net Present Value Investments

The Constant-Growth model, under the assumption of zero net present value investments, is found by substituting G from Equation (25) and C from Equation (11) into Equation (1) and setting W = R:

$$V_0 = \frac{E_1(1-k)}{W-kR-(1-k)\Pi} = \frac{E_1(1-k)}{W(1-k)-(1-k)\Pi} \tag{38}$$

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[10] The <u>Constant</u>-Real-Growth model is equal to $V_0 = \frac{e_1}{w-g}$ and, if g = 0, then $V_0 = \frac{e_1}{w}$.

Thus, the expression for the value of the firm when r = w (or equivalently R = W) is simply:

$$V_0 = \frac{E_1}{W - \Pi} = \frac{e_1(1+\Pi)}{w(1+\Pi)} = \frac{e_1}{w} \tag{39}$$

which, again, is the Zero-Real-Growth model.[11]

Note that real growth g is not necessarily zero under the zero NPV assumption. Indeed, since g = kr, if r = w > 0, then real growth g will be positive unless k is zero. But any positive real growth that may be projected will not increase the value of the firm because all future investments have zero NPV (R = W and r = w).

IV. The Existing Literature

In this section we replicate the literature's development of the expressions for the value of the firm assuming (1) zero net new investments and (2) investments only in zero net present value projects. We begin with the former.

A. Zero Net New Investment

The expression for the value of a firm that makes no net new investments found in the literature is based on an erroneous expression for the growth in cash flows. Although it is incorrect, it is nevertheless common to define the nominal growth in net cash flows as:

$$G = kR \tag{40}$$

where R is the firm's nominal return on investment.[12]

[11] This equation (called the "perpetuity with inflation method") appears in Creating Shareholder Value, by Al Rappaport, Revised and Expanded Edition (1994), pages 44-47, without derivation. In the 1986 original version of the text, there is no mention of the perpetuity with inflation method. The author calls this "a variant of the perpetuity model" which produces "consistently higher values" than the perpetuity method, "in the presence of inflation". But, in the preceding five-page presentation of his 1994 edition, Rappaport argues that the standard perpetuity formula (Zero-Nominal Growth) should be used to estimate the firm's "Residual Value" when assuming zero NPV investments even with non-zero projected inflation rates. For mathematical support, the author cites the text Valuation, by Copeland, Koller and Murin, 1994. However, this equation appears nowhere in this reference. Indeed, we cite Copeland et. al. for making the same mistakes as the rest of the literature in applying (deriving) the Zero-Nominal Growth model. Nonetheless, Rappaport is the only reference to a Zero-Real Growth formula that we have found.

Recall that according to Equation (25), the correct expression for the growth in net cash flows is:

$$G = kR + (1 - k)\Pi \qquad (41)$$

As stated above, it is the second term in this equation that is ignored in the literature.

To derive what the literature describes as the zero-investment formula, simply substitute Equations (11) and (40) into Equation (1)

$$V_0 = \frac{E_1(1-k)}{W-kR} \qquad (42)$$

and set k, the reinvestment rate, equal to zero:

$$V_0 = \frac{E_1}{W} \qquad (43)$$

Throughout the finance literature Equation (43), which is the ratio of the firm's projected earnings (stated in next period's dollars) to the firm's nominal cost of capital, is defined as the "no-growth" value of the firm. For example, Brealey and Myers (2000), pp. 72-73, describe Equation (43) as the value of a firm "that does not grow at all. It does not plow back any earnings and simply produces a constant stream of dividends." Ross, Westerfield and Jaffee (1993), pp. 130-131, state that the above expression "is the value of the firm if it rested on its laurels, that is, if it simply distributed all earnings to the

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[12] Applying the Gordon Growth Model for the valuation of common stock, Brealey and Myers write on page 67: $P_0 = \frac{DIV_1}{r - g}$, where P_0 is the price of the stock at time 0, DIV_1 is the (expected) dividend in period, *r* is the firm's capitalization rate and *g* is the growth in dividends. It is important to note that B&M do not indicate whether *r* or *g* is stated in real or nominal terms. However, since DIV_1 is presumably a nominal number, stated in period 1 dollars, we must presume that both *r* and *g* are nominal values. Instead of deriving an expression for g, the nominal growth in earnings, they simply assert on page 68:

> " If Pinnacle earns 10 percent of book equity and reinvests 53 percent of that, then book equity will increase by .53 x .10 = .053, or 5.3 percent. **Earnings and dividends per share will also increase by 5.3%:**
>
> Dividend growth rate = ***g*** = plowback ratio * ROE."

(*Emphasis added.*) Again, on page 69 they write: "Dividend growth rate = plowback ratio x ROE." Other references that assert that G=kR include: Ross, Westerfield and Jaffee, p. 128, Grinblatt and Titman, p. 832, Rao, p. 403, Bodie and Merton, p. 123, Emery and Finnerty, p. 149, Shapiro and Balbirer, p. 159, Benninga and Sarig, p. 9, Van Horn, pp. 30-31, Martin, Petty, Keown and Scott, p.123. It is interesting to note that none of these authorities feels compelled to prove this relation. They all rely on it as though it is self-evident. Not only is this expression for the nominal growth rate not self-evident, it is incorrect as we demonstrate by the derivation of Equation (27) in the text.

stockholders." Martin, Petty, Keown and Scott, p. 123, describe Equation (43) as "the present value of a constant stream of earnings generated from the firm's assets already in place," Rao (1995), p. 411, describes Equation (43) as "the value of the firm if it chooses to pay out 100% of its earnings as dividends, thereby forgoing its growth options." And finally, Shapiro and Balbirer (2000) state that Equation (43) "represents the value of a no-growth firm that pays out all of its earnings in dividends. It is essentially the value of the assets in place."

While it may seem perfectly logical that with no new investments a firm would generate a constant stream of cash flows into perpetuity, this reasoning ignores the effects of inflation on the firm's initial invested capital at time t=0. If the firm's investments keep up with obsolescence and economic depreciation, the firm's total invested capital stock will increase at the rate of inflation and, with a constant real return on investment, the firm's cash flows will also increase at the rate of inflation. Put simply, one cannot express the value of the firm as a perpetuity in nominal terms – Equation (43) - when inflation is positive. As demonstrated by Equation (36) in Section III.A, the presence of inflation requires that the value of a zero-investment firm be expressed as a <u>growing</u> perpetuity, with the growth rate set equal to the rate of inflation.

We label this expression the Zero-Nominal-Growth model, since it is equal to Equation (1) with G, the nominal growth rate, set equal to zero. We also use the term to emphasize the internal inconsistency in deriving this expression. A zero-nominal-growth model in a world of inflation is somewhat of an oxymoron. Zero nominal growth implies a model with real, as opposed to nominal parameters. Indeed, as inflation is the only distinction between real and nominal values, what does it mean to have inflation and zero nominal growth? In general, you can't have both. Only under the pathological and mostly irrelevant case of a negative real growth rate, that just offsets a positive rate of inflation, can you have zero nominal growth and positive inflation.

To see this, substitute Equation (24) into Equation (23), which yields the familiar Fisher Equation relating nominal growth G to real growth g and inflation Π.

$$G = \Pi + g(1 + \Pi) = g + \Pi + g\Pi. \tag{44}$$

Equation (44) requires that for G to be zero and inflation positive, real growth (g) must equal $-\frac{\Pi}{1+\Pi}$. Thus, if inflation is positive, the Zero-Nominal-Growth model is actually a negative-real-growth model.[13] In essence, the Zero-Nominal-Growth model describes a wasting or declining asset, because the real output of the firm would be constantly declining into perpetuity.

The implication of negative real growth into perpetuity renders the Zero-Nominal-Growth model useless for most real-world valuation applications, where expected inflation is normally positive. In addition, the theoretical conditions under which real growth can be negative in a perpetuity model are unlikely to occur. As Equation (24) indicates, a negative real growth rate implies that either the real return on investment (r) or the plowback ratio (k) is negative into perpetuity. Since the Constant-Growth model is forward looking, firms cannot be expected to accept a project with an ex ante negative return. Thus, a negative real growth rate must be due to a negative k, which represents a steady liquidation of the firm over time. Obviously, this condition is not appropriate for most corporate valuation applications.

B. Zero Net Present Value Investments

The derivation of the expression for the value of a firm facing only zero net present value projects found throughout the finance literature relies on the same erroneous expression for the growth in nominal cash flows given in Equation (40). Zero net present value implies that W = R. Substituting Equations (11) and (40) into Equation (1) and imposing this zero-NPV condition also yields the Zero-Nominal-Growth formula: [14]

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[13] No economic justification for negative real growth is offered in the literature, by our search. The only source that recognizes this point is Rappaport's Creating Shareholder Value, (1998), page 45, which states that "In the standard perpetuity model…the cash flows are level in nominal terms but decrease each year in real terms, that is, nominal less expected inflation." Rappaport's standard perpetuity model is what we call the Zero-Nominal Growth model.

[14] Copeland, Koller and Murrin (1994), pp. 282-283, argue that Equation (45) is the value of the firm assuming that the firm faces only zero NPV projects. Rappaport (1998) p.42 also asserts that Equation (45) is the value of the firm if all of its projects have zero NPV. Copeland, et. al., pp. 513-515, derive the relation (stated in our notation) $C_t = E_t (1 - k)$. They then substitute G/R and write $C_t = E_t (1 - G/R)$. From this equation they follow the derivation as outlined in the text. Thus, these authors erroneously assume, like Brealey and Myers, that k = G/R, when in fact the correct relation is k = g/r. This substitution leads them to Equation (45). It is important to note that Copeland et. al. do not specify whether their variables are nominal or real. The clear implication though, is that they are all nominal values. Cornell

$$V_0 = \frac{E_1(1-k)}{W-kW} = \frac{E_1(1-k)}{W(1-k)} = \frac{E_1}{W} \tag{45}$$

Once again it may seem logical that since investments in zero net present value projects do not create value, G should not appear in Equation (45). Intuitively, the discounted value of earnings from additional investments just equals the present value of the new investments assuming the return on investment equals the cost of capital. Thus, value is not created by these additional investments. And of course this logic is correct. [15] However, with positive inflation, the value of the firm's initial invested capital will grow at the rate of inflation and, assuming a constant real rate of return, the nominal earnings from these existing investments (the numerator in Equation (45)) will grow at the rate of inflation as well. Equation (45) ignores this growth. Thus, the correct expression for the value of the firm that invests only in zero net present value projects also must be a growing perpetuity with the growth rate equal to the rate of inflation, as we demonstrate in Section III.B. Since Equation (45) can be obtained by setting G to zero in Equation (1), we refer to this expression as the Zero-Nominal Growth model as well, even though we realize that the development of this model in the literature is not based on the assumption of zero nominal growth.

C. The General Model

We have focused our criticism of the literature's treatment of the Constant-Growth model on the special cases of zero investments and zero net present value investments. However, our criticisms also apply to the general model regarding the estimation of G,

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(1993, p. 156) makes the same error. He explicitly states that G is the nominal rate of growth in free cash flows and distinguishes it from g, which he earlier defines on page 148 as the "long-run growth in real returns." Nevertheless, Cornell assumes that k=G/R. Weston et. al. (1998, p. 186) make the same, erroneous substitution. Rappaport (1998, pp. 40-44) rationalizes Equation (45) verbally, making the same arguments that are embodied in the algebra above.

[15] Copeland, et. al. state on page 283, "The growth term has disappeared from the equation. This does not mean that the nominal growth in NOPLAT (EBIAT, in our notation) will be zero. It means that growth will add nothing to value, because the return associated with growth just equals the cost of capital. This formula is sometimes interpreted as implying zero growth (not even with inflation), even though this is clearly not the case." Rappaport states on page 43 of his book, "Keep in mind that the perpetuity method for estimating residual value is *not* based on the assumption that all future cash flows will actually be identical. It simply reflects the fact that the cash flows resulting from future investments will not affect the value of the firm because the overall rate of return earned on those investments is equal to the cost of capital."

the nominal growth term. As developed above, the expression for the growth in nominal earnings that is found throughout the literature is

$$G = kR \tag{46}$$

whereas the correct expression for the nominal growth term is

$$G = kR + (1-k)\Pi \tag{47}$$

To the extent that researchers and practitioners use the former expression to calculate G, they are understating the true value of the firm. Thus, our criticism of the Constant-Growth model, as it is presented in the literature, goes beyond the two special cases of zero NPV and zero investments. Our criticisms also apply to the general model, provided the growth term is calculated according to Equation (46) instead of Equation (47).

V. The Error Rate of the Zero-Nominal-Growth Model under Inflation

As noted above, the Zero-Real Growth model is inflation-neutral, since the model can be written exclusively in real terms, as in Equation (39). However, somewhat ironically, the Zero-Nominal-Growth model is not inflation-neutral. As developed above in Equation (45), the Zero-Nominal-Growth model is:

$$V_0 = \frac{E_1}{W} \tag{48}$$

Substituting real for nominal terms yields,

$$V_0 = \frac{e_1(1+\Pi)}{w + \Pi + w\Pi} \tag{49}$$

and taking the derivative with respect to Π yields,

$$\frac{\partial V_0}{\partial \Pi} = \frac{-e_1}{(w + \Pi + w\Pi)^2} < 0 \tag{50}$$

Equation (50) above indicates that V_0 is negatively related to the rate of inflation, and Equation (51) below indicates that this inflation-induced underestimate is greater the lower is the real cost of capital w:

$$\frac{\partial^2 V_0}{\partial \Pi \partial w} = \frac{e_1(1+w)}{(w + \Pi + w\Pi)^4} > 0 \tag{51}$$

The negative sign of the derivative in Equation (50) illustrates the paradoxical implication of the Zero-Nominal Growth model that the value of the firm declines as projected inflation Π increases, which is clearly an undesirable feature of any valuation model. If by inflation we mean a general increase in the price level, affecting costs, revenues and interest rates proportinately, then the expected rate of inflation should have no real effect on present values. It should be noted that Equation (50) implies a substantial decline in the present value V_0 for even modest increases in projected inflation.

The amount by which the Zero-Nominal-Growth model understates the correct present value can be readily calculated as a function of the rate of inflation and the real cost of capital. To illustrate, assume $E_1 = K_0\ r\ (1+\Pi)$ and that w = r. Under the assumptions of the Zero-Real-Growth model, the value of the firm would be

$$V_0 = \frac{K_0\ r\ (1+\Pi)}{W - \Pi}.$$

Since $W = w + \Pi + w\,\Pi = \Pi + w(1+\Pi)$, it follows that $W - \Pi = w(1+\Pi)$. Substituting this expression into the denominator in the equation above and setting w = r,

$$V_0 = \frac{K_0\ r\ (1+\Pi)}{r\ (1+\Pi)} = K_0$$

Thus, the value of the firm would just equal the initial investment, since the project has a zero net present value (r = w). However, under the Zero-Nominal-Growth model, the value of the firm would be:

$$V_0 = \frac{K_0\ r\ (1+\Pi)}{W}.$$

The difference in the values generated by these two equations is a measure of the under-estimate of the Zero-Nominal-Growth model. Moreover, since the correct value is \$100, this difference is also a percentage error.

Figure 1 plots the value of the underestimate (error rate) generated by the traditional model for various levels of inflation assuming K_0 = \$100 and r = w = 7%. Thus, even with a modest 3% inflation rate, the Zero-Nominal-Growth model understates the present value of the firm, assuming either zero plowback investment (k=0) or zero NPV (R = W), by approximately 30%. With a 6% projected inflation rate, the Zero-Nominal-Growth model understates the present value by nearly 50%. Generally, these would be considered unacceptably high error rates in most corporate valuation applications.

Figure 1

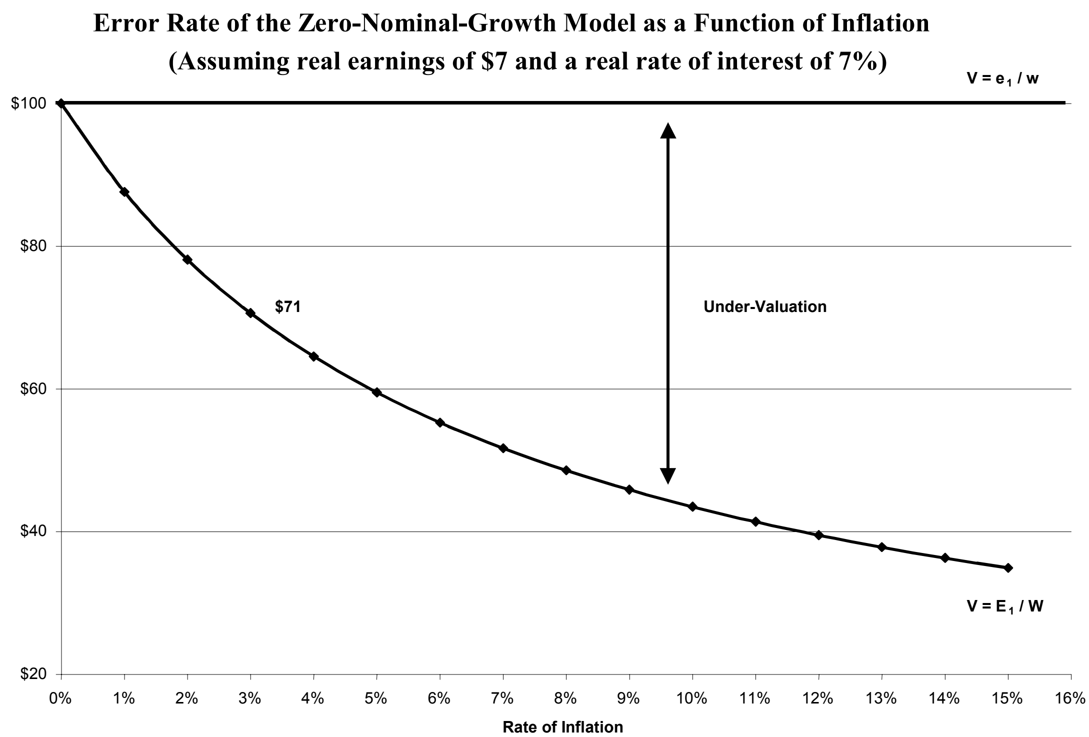

Table 1 generalizes the error rate for various levels of r = w. The difference between the entries in the table and $100 is the error estimated by the zero-nominal-growth model. To illustrate, with inflation of 4% and a real cost of capital of 4%, the zero-nominal-growth value is $51. This generates an “error” of $49, which is almost 50% of the true value.

Table 1

The Value of a $100 Firm According to the Zero-Nominal-Growth Model

Rate of Inflation (Π)

Real Cost of Capital (w) = Real Return on Investment (r)	0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%
1%	100	50	34	26	21	17	15	13	12	11	10	9	9	8	8	7
2%	100	67	50	41	34	30	26	23	21	19	18	17	16	15	14	13
3%	100	75	60	51	44	39	35	31	29	27	25	23	22	21	20	19
4%	100	80	67	58	$\boxed{51}$	46	41	38	35	33	31	29	27	26	25	23
5%	100	83	72	63	57	51	47	43	40	38	35	34	32	30	29	28
6%	100	86	75	67	61	56	51	48	45	42	40	38	36	34	33	32
7%	100	88	78	71	65	60	55	52	49	46	44	41	40	38	36	35
8%	100	89	80	73	68	63	59	55	52	49	47	45	43	41	39	38
9%	100	90	82	76	70	65	61	58	55	52	50	48	46	44	42	41
10%	100	91	84	77	72	68	64	60	57	55	52	50	48	47	45	43
11%	100	92	85	79	74	70	66	63	60	57	55	53	51	49	47	46
12%	100	92	86	80	76	72	68	65	62	59	57	55	53	51	49	48
13%	100	93	87	82	77	73	70	67	64	61	59	57	55	53	51	50
14%	100	93	88	83	78	75	71	68	65	63	61	59	57	55	53	52
15%	100	94	88	84	80	76	73	70	67	64	62	60	58	57	55	53

VI. Inflation and the WACC Approach to Valuation

In this section we analyze the effects of inflation on the so-called weighted-average-cost of capital (WACC) approach to valuation. All of the leading finance textbooks and valuation manuals suggest that the WACC should be used as the discount factor in the Constant-Growth model.[16] The WACC methodology is designed to account for the increase in the firm's net cash flows due to the tax deductibility of interest payments. Since the interest paid on debt is tax deductible at the corporate level, a debt-for-equity swap would decrease a firm's taxes and increase its market value, *ceteris paribus*.

There are two basic approaches to valuing the interest tax shields from debt financing. The first, called the Adjusted Present Value (APV) method, recognizes that the value of a levered firm (V_L) is equal to the value of the firm if it were unlevered (V_U) plus the present value of the interest tax shields (PVITS)

$$V_L \;=\; V_U \;+\; \text{PVITS} \qquad (52)$$

In general, the interest tax shield per period can be written as:

[16] See citations in footnote 3.

$$t \ W_D \ D_{t-1} \tag{53}$$

where t is the corporate tax rate, W_D is the cost of debt and D_{t-1} is the amount of debt outstanding at the beginning of the period.[17]

The insight behind the WACC methodology is that under certain circumstances, the value of a levered firm can be found directly by discounting the firm's net cash flows by its weighted-average cost of capital:

$$WACC = \frac{D}{V} (1 - t) W_D + \frac{E}{V} W_E \tag{54}$$

where D is the market value of debt, E is the market of equity, V is the market value of the firm (V=D+E), W_D is the cost of the firm's debt, W_E is the cost of (required rate of return on) its equity and t is the firm's marginal tax rate.

Because of the tax subsidy to debt financing, the value of the firm increases with increases in debt. Intuitively, as the firm substitutes debt for equity, a greater weight is placed on the first term of Equation (54). Since $t > 0$ and $W_D < W_E$, the greater the debt to value ratio, the lower the WACC and, the lower the WACC, the higher the value of the firm.[18] As mentioned above, under certain circumstances, the <u>increase</u> in the value of the firm caused by discounting the firm's net cash flows at a lower WACC is exactly equivalent to the present value of the debt tax shields created by the increase in debt financing.

There are two basic WACC approaches. The first is due to Modigliani and Miller, 1963 (M&M) and the second is due to Miles and Ezzell, 1980 (M&E). Each makes a different assumption regarding the firm's debt policy. Based on this assumption each model provides an expression for the value of levered equity. Holding the cost of debt constant, each then derives a WACC according to Equation (54).

As we will see, the M&M model (WACC formula) only holds under the strict assumptions of the Zero-Nominal-Growth model (zero inflation and zero net new investments). In contrast, the M&E WACC formula holds in nominal terms, but the

---

[17] If the bond is selling at par, the cost of debt will be the bond's coupon rate.

[18] Of course, according to M&M's Proposition I, if markets are perfect and taxes are zero, then w_E will adjust to offset exactly the change in weights leaving wacc unchanged.

formula under-states the value of the firm if the parameters of the model are stated in real terms.

A. Modigliani and Miller[19]

Under the M&M model, the firm's cash flows and the amount of debt outstanding are assumed constant. The interest tax shield in Equation (53) therefore is a simple perpetuity. Assuming that the interest payments are as risky as the debt itself, the value of this perpetuity is:

$$PVITS = (t\, W_D\, D) / W_D = t\, D. \tag{55}$$

Thus, substituting (55) into (52), under M&M,

$$V_L = V_U + t\, D. \tag{56}$$

The M&M WACC model posits a relation between the firm's cost of equity capital and leverage. Specifically, the cost of equity to a levered firm is equal to the cost of capital if the firm were unlevered, plus the difference between the unlevered cost of capital and the (constant) cost of debt times the firm's debt to equity ratio. Although this relation (formula) can be found in most corporate finance textbooks, like the Constant-Growth model in general, the authors never state whether this relation holds for nominal or real variables. Unfortunately, this silence and the context in which the M&M model is typically presented in the literature clearly imply that the relation holds in nominal terms. As we show below, this is not correct.

The fact that the M&M model assumes fixed cash flows and a fixed amount of debt outstanding immediately raises doubts that the model holds in nominal terms. As we will see, this intuition is correct. Since the M&M Model is based on the assumption of constant cash flows, it follows that it is only relevant if $G = g + \Pi + g\,\Pi = 0$. Thus, the M&M model is only correct if the rate of inflation and the real rate of growth are both equal to zero. We have previously labeled this the Zero-Nominal-Growth Model.

[19] This analysis follows the presentation of the M&M WACC model in Brealey and Meyers pp. 517-546.

1. M&M - Real WACC Analysis

According to the M&M model,[20] the (real) cost of equity is equal to:

$$w_E = w_U + (w_U - w_D)\frac{D}{E} \tag{57}$$

Substituting Equation (57) into Equation (54) and solving for the real weighted average cost of capital yields:

$$wacc^{M\&M} = w_U(1 - tL) \tag{58}$$

where L is the firm's debt to value ratio.

Now, according to the M&M model:

$$V_L = \frac{c}{wacc^{M\&M}} = V_U + t\,D \tag{59}$$

To see that Equation (57) holds, substitute Equation (58) for $wacc^{M\&M}$:

$$V_L = \frac{c}{w_U\ (1 - t\,L)} \tag{60}$$

Noting that $\frac{c}{w_U} = V_U$ and $\frac{D}{V_L} = L$,

$$V_L = \frac{V_U}{(1 - t\,L)} \tag{61}$$

$$\frac{V_U}{1 - \frac{t\,D}{V_L}} = \frac{V_U}{\frac{V_L - t\,D}{V_L}} = \frac{V_L * V_U}{V_L - t\,D} = \frac{V_L * V_U}{V_U} = V_L \tag{62}$$

Combining the above results, the value of a levered firm, assuming zero nominal growth, can be written as either Equation (56) or Equation (59), which implies

$$V_L = V_U + t\,D = \frac{c}{wacc^{M\&M}}\ . \tag{63}$$

Thus, the M&M model holds in real terms.

2. M&M - Nominal WACC Analysis

The <u>logic</u> of the M&M model is not affected by inflation. The value of the levered firm is still equal to the value of the unlevered firm plus the quantity t D. In order to hold constant D, the amount of debt outstanding, the firm will have to adjust its nominal

[20] M&M 1963.

coupon to the prevailing nominal rate. Thus, if inflation increases, both the firm's nominal coupon rate and its nominal discount rate on its debt will increase proportionately, leaving the present value of the interest tax shields constant at $t\,D$.

However, the M&M WACC formula does <u>not</u> yield the correct value when the parameters are stated in nominal terms. Under the Zero-Real-Growth model, the value of the levered firm is:

$$V_L = \frac{C_1}{WACC - \Pi} \tag{64}$$

The M&M WACC formula, stated in nominal terms, is

$$WACC^{M\&M} = W_U(1 - tL) \tag{65}$$

Now, the proposition to be demonstrated is:

$$V_L^{M\&M} = \frac{C_1}{WACC^{M\&M} - \Pi} \neq \frac{c_1}{wacc^{M\&M}} = V_L \tag{66}$$

where $V_L^{M\&M}$ is the value of the levered firm according to the M&M nominal WACC formula- Equation (65). Substituting Equation (65) into Equation (64) yields:

$$V_L^{M\&M} = \frac{C_1}{W_U(1 - tL) - \Pi} \tag{67}$$

Substituting $W_U = w_U + \Pi + w_U\Pi$

$$V_L^{M\&M} = \frac{C_1}{(w_U + \Pi + w_U\Pi)(1 - tL) - \Pi} \tag{68}$$

Expanding and substituting $wacc^{M\&M} = w_U(1 \text{ - } tL)$ yields:

$$V_L^{M\&M} = \frac{c_1(1+\Pi)}{wacc^{M\&M} + \Pi + \Pi wacc^{M\&M} - \Pi - \Pi tL} = \frac{c_1(1+\Pi)}{wacc^{M\&M}(1+\Pi)\boxed{-\Pi tL}} \tag{69}$$

It is clear by inspection that $V_L^{M\&M}$ in Equation (69) is equal to V_L in Equation (66) only if the second term in the denominator of Equation (69) (the product of Π, t and L) is zero. Under this <u>highly unlikely</u> situation, Equation (69) reduces to

$$V_L^{M\&M} = \frac{c_1(1+\Pi)}{wacc^{M\&M}(1+\Pi)} = V_L \tag{70}$$

In other words, the M&M formula only yields the correct valuation if the inflation rate Π, or the tax rate t, or the leverage ratio L is equal to zero. If all of these parameters are positive, which is most likely the case, then Equation (69) will over-state the value of the levered firm.[21]

In the Appendix we demonstrate that the error generated by the nominal version of the M&M WACC equation is positively related to the rate of inflation and that the error is greater, the greater the firm's leverage ratio.[22] Intuitively, the nominal M&M model assumes correctly that an increase in inflation will increase the firm's WACC. However, the model incorrectly assumes that the government will absorb the amount t L of the increase. In fact, an increase in inflation will cause a proportionate increase in both the firm's coupon rate and the cost of debt, leaving the value of the tax shields and hence, the value of the firm, unaffected.

Table 2 below illustrates the economic magnitude of the error generated by the nominal M&M model. Each entry in the table gives the percentage over-valuation caused by using the nominal M&M WACC model assuming a tax rate of 40%, a real cost of capital and a real return on investment of 10%, and the indicated combination of the rate of inflation and the firm's leverage ratio. As the entries indicate, the error is economically significant even at modest levels of inflation. For example, a 40% leverage ratio and a 5% rate of inflation results in a 10% over-valuation of the firm applying the M&M model in nominal terms.[23]

[21] Although not shown, it should be noted that the nominal version of the M&M model is also vitiated if the real growth term is positive. Intuitively, the M&M model is incompatible with growth of any sort, be it from inflation or new investment.

[22] See Propositions 1 and 2 in the Appendix.

[23] Although not shown, the over-estimate is higher the lower the real cost of capital and the greater the tax rate.

Table 2

Percentage Over-Valuation Using Nominal M&M

Rate of Inflation

Debt to Value Ratio	0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
10%	0.0%	0.4%	0.8%	1.2%	1.6%	2.0%	2.4%	2.8%	3.2%	3.6%	3.9%	4.3%	4.7%
20%	0.0%	0.9%	1.7%	2.6%	3.5%	4.3%	5.2%	6.0%	6.9%	7.7%	8.6%	9.4%	10.3%
25%	0.0%	1.1%	2.2%	3.3%	4.5%	5.6%	6.7%	7.8%	9.0%	10.1%	11.2%	12.4%	13.5%
30%	0.0%	1.4%	2.7%	4.1%	5.5%	6.9%	8.4%	9.8%	11.2%	12.7%	14.2%	15.6%	17.1%
35%	0.0%	1.6%	3.3%	5.0%	6.7%	8.4%	10.1%	11.9%	13.7%	15.5%	17.4%	19.2%	21.1%
40%	0.0%	1.9%	3.9%	5.9%	7.9%	10.0%	12.1%	14.2%	16.4%	18.7%	20.9%	23.3%	25.6%
45%	0.0%	2.2%	4.5%	6.8%	9.2%	11.7%	14.2%	16.8%	19.4%	22.1%	24.9%	27.8%	30.8%
50%	0.0%	2.5%	5.2%	7.9%	10.6%	13.5%	16.5%	19.6%	22.7%	26.0%	29.4%	32.9%	36.6%
55%	0.0%	2.9%	5.9%	9.0%	12.2%	15.5%	19.0%	22.6%	26.4%	30.4%	34.5%	38.8%	43.3%
60%	0.0%	3.2%	6.6%	10.1%	13.8%	17.7%	21.8%	26.0%	30.5%	35.3%	40.3%	45.5%	51.1%
65%	0.0%	3.6%	7.4%	11.4%	15.6%	20.1%	24.8%	29.8%	35.2%	40.9%	46.9%	53.4%	60.4%
70%	0.0%	4.0%	8.3%	12.8%	17.6%	22.7%	28.2%	34.1%	40.5%	47.3%	54.7%	62.7%	71.4%
75%	0.0%	4.4%	9.2%	14.3%	19.7%	25.6%	32.0%	39.0%	46.5%	54.8%	63.8%	73.8%	84.9%
80%	0.0%	4.9%	10.2%	15.9%	22.1%	28.9%	36.3%	44.5%	53.5%	63.5%	74.8%	87.4%	102%

The M&M Model can be expressed in nominal terms, provided $WACC^{M\&M}$ is defined using the Fisher Equation:

$$WACC \equiv (1+wacc^{M\&M})*(1+\Pi)-1 \tag{71}$$

and not by Equation (65). However Equation (65), not Equation (71) is the WACC according to the M&M assumptions.

Another way to see the incompatibility of the nominal version of the M&M WACC with the Constant-<u>Nominal</u>-Growth model is to note that the former is incompatible with the Fisher Equation. According to the Fisher Equation:

$$WACC^{M\&M} = (1+wacc^{M\&M})(1+\Pi)-1 \tag{72}$$

However, according to the M&M model, Equation (72) does not hold:

$$\mathrm{WACC}^{\mathrm{M\&M}} = \mathrm{W_U}(1-\mathrm{tL}) \tag{73}$$

$$\mathrm{WACC}^{\mathrm{M\&M}} = [\mathrm{w}(1+\Pi)+\Pi][1-\mathrm{tL}] \tag{74}$$

$$\mathrm{WACC}^{\mathrm{M\&M}} = \mathrm{wacc}^{\mathrm{M\&M}} + \Pi + \Pi\mathrm{wacc}^{\mathrm{M\&M}} - \Pi\mathrm{tL} \tag{75}$$

$$\boxed{\mathrm{WACC}^{\mathrm{M\&M}} = (1+\mathrm{wacc}^{\mathrm{M\&M}})(1+\Pi)+-1} - \Pi\mathrm{tL} \tag{76}$$

The last term in Equation (76) (-ΠtL) is the underestimate of the firm's nominal weighted-average cost of capital according to the M&M model. Note that Equation (76) shows that it is easy to correct the nominal WACC formula so that it is consistent with the Fisher Equation. One can simply add the amount (ΠtL) to the computed WACC using the standard M&M nominal WACC formula. Each of the three variables that comprise this corrective "patch" are known in most applications--the (assumed) expected inflation rate (Π), the corporate tax rate (t), and the firm's leverage ratio (L). Adding the quantity (ΠtL) to the standard M&M WACC yields the correct nominal WACC, which is inflation-neutral.

B. Miles and Ezzell

The WACC methodology due to Miles and Ezzell (M&E) does hold in nominal terms. This model assumes that the ratio of debt to value remains constant through time. Thus, if the nominal value of the firm grows with inflation, so does the amount of debt outstanding and hence, so does the per period interest tax shield. All else equal, the value of a levered firm (the market value of its outstanding securities) is positively related to the rate of inflation. The value of the firm is not inflation neutral under the M&E model. Since the value of the firm's tax shields increases with inflation, the M&E model is not correct if the parameters are stated in real terms.

1. M&E – Nominal WACC Analysis

According to the M&E assumptions,[24] the present value of the firm's interest tax shield is equal to:

[24] M&E 1980.

$$PVITS = \sum_{t=1}^{\infty} \frac{t\, W_D\, L\, V_L\, (1 + G)^{t-1}}{(1 + W_D)(1 + W_U)^{t-1}} \tag{77}$$

The numerator of Equation (77) is the interest tax shield in period t - the tax rate times the interest rate times the debt to value ratio times the value of the firm at the beginning of the first period times the appropriate growth factor. Note that the interest tax shield is based on the beginning value of the firm. The denominator of Equation (77) reflects the fact that under the M&E assumptions, the amount of debt in the first period is known and therefore the interest tax shield is discounted at the rate on debt, W_D. Thereafter, the amount of debt outstanding depends on the value of the firm. Since L is constant, the firm issues debt when the value of the firm rises and retires debt when the market value falls. Thus, the <u>size</u> of the interest tax shield rises and falls with the value of the firm. Consequently, the interest tax shield is as risky as the firm itself, which accounts for discounting the per period tax shield at the unlevered cost of capital, W_U, from period t=2 onward.

For purposes that will become clear shortly, we isolate the first year's tax shield

$$PVITS = \frac{t\, W_D\, L\, V_L}{1 + W_D} + \sum_{t=2}^{\infty} \frac{t\, W_D\, L\, V_L\, (1 + G)^{t-1}}{(1 + W_D)(1 + W_U)^{t-1}} \tag{78}$$

where, V_L is the value of the levered firm in period 0. Adjusting the time subscripts in the second term:

$$PVITS = \frac{t\, W_D\, L\, V_L}{1 + W_D} + \sum_{t=1}^{\infty} \frac{t\, W_D\, L\, V_L\, (1 + G)^{t}}{(1 + W_D)(1 + W_U)^{t}} \tag{79}$$

Define Q as:

$$Q = \frac{t\, W_D\, L\, V_L}{1 + W_D} \tag{80}$$

Substituting (80) into (79) and solving for the value of a growing perpetuity yields:

$$PVITS = Q + \frac{Q\,(1+G)}{W_U - G} = \frac{Q\,(1+W_U)}{W_U - G} \tag{81}$$

Recall that the value of a levered firm is equal to the value of an unlevered firm plus the present value of interest tax shields (PVITS).[25] Substituting Equation (81) into Equation (52):

$$V_L = V_U + \frac{Q\ (1+W_U)}{W_U - G} \tag{82}$$

We now demonstrate that the WACC model developed by M&E yields this same expression. Under the M&E assumptions, the firm's weighted average cost of capital can be written as:

$$WACC^{M\&E} = W_U - \frac{t\, W_D\, L\, (1 + W_U)}{1 + W_D} \tag{83}$$

Substituting Q yields:

$$WACC^{M\&E} = W_U - \frac{Q\, (1 + W_U)}{V_L} \tag{84}$$

Substituting $WACC^{M\&E}$ into the basic valuation formula of Equation (1) yields:

$$V_L = \frac{C_1}{W_U - \frac{Q\,(1 + W_U)}{V_L} - G} \tag{85}$$

Multiplying through by the denominator yields:

$$V_L\, W_U - Q\,(1 + W_U) - V_L\, G = C_1 \tag{86}$$

$$V_L\,(W_U - G) - Q\,(1 + W_U) = C_1 \tag{87}$$

$$V_L\,(W_U - G) = C_1 + Q\,(1 + W_U) \tag{88}$$

Dividing through by (W_U - G) yields:

$$V_L = \frac{C_1}{(W_U - G)} + \frac{Q\,(1 + W_U)}{(W_U - G)} \tag{89}$$

$$V_L = V_U + \frac{Q\,(1 + W_U)}{(W_U - G)} = V_L = \frac{C_1}{WACC^{M\&E} - G} \tag{90}$$

Thus, Equation (83) equals Equation (82), which implies that the M&E model is correct in nominal terms.

[25] See Equation (52).

Before turning to the analysis of the M&E WACC model in real terms, we pause to note that the nominal M&E model implies a positive relation between WACC and inflation[26]

$$\frac{\partial \mathrm{WACC}^{\mathrm{M\&E}}}{\partial \Pi} \geq 0. \tag{91}$$

as well as a positive relation between the value of a levered firm and inflation.[27]

$$\frac{\partial \mathrm{V}_{\mathrm{L}}^{\mathrm{M\&E}}}{\partial \Pi} \geq 0. \tag{92}$$

2. M&E – Real WACC Analysis

The M&E model also holds in real terms, provided the real WACC is derived from the nominal WACC and the Fisher Equation:

$$\mathrm{wacc}^{\mathrm{M\&E}} \equiv \frac{1+\mathrm{WACC}^{\mathrm{M\&E}}}{1+\Pi} - 1 \tag{93}$$

$$\mathrm{WACC}^{\mathrm{M\&E}} = (1+\mathrm{wacc}^{\mathrm{M\&E}})(1+\Pi) - 1 \tag{94}$$

Substituting real for nominal values in the basic valuation formula, which is repeated in Equation (90) above:

$$\mathrm{V}_{\mathrm{L}} = \frac{\mathrm{c}_1(1+\Pi)}{(1+\mathrm{wacc}^{\mathrm{M\&E}})(1+\Pi) - 1 - (1+\mathrm{g})(1+\Pi) - 1} \tag{95}$$

$$\mathrm{V}_{\mathrm{L}} = \frac{\mathrm{c}_1}{\mathrm{wacc}^{\mathrm{M\&E}} - \mathrm{g}} \tag{96}$$

Note that defining $\mathrm{wacc}^{\mathrm{M\&E}}$ as we do in Equation (93), imposes a negative relation between inflation and the real WACC as implied by the M&E model. As we show in the Appendix,[28]

[26] See Proposition 3 in the Appendix.

[27] See Proposition 4 in the Appendix.

[28] See Proposition 5 in the Appendix.

$$\frac{\partial \text{wacc}^{\text{M\&E}}}{\partial \Pi} < 0 \tag{97}$$

The negative relation between the real WACC and inflation under the M&E assumptions means that the M&E WACC formula does not hold in real terms. Put simply, you cannot write $\text{wacc}^{\text{M\&E}}$ in real terms as in Equation (98).

$$\text{wacc}^{\text{M\&E}} \neq w_U - \frac{t_x \; w_D \; L \; (\; 1 \; + \; w_U \;)}{1 + w_D} \tag{98}$$

Since each of the right-hand-side variables of this inequality is stated in real terms, none is affected by inflation. This implies that the value of a firm given by the M&E model stated in real terms understates the true value of the firm if leverage and inflation are positive.

C. Summary

The intuition behind the results in this Section can be seen by thinking of the tax shields from debt financing as net cash flows. The M&M Model is based on the assumption of constant cash flows and, as we have shown, the only variant of the Constant-Growth model with constant cash flows is the Zero-Nominal-Growth model. Thus, the M&M WACC methodology is only literally correct if both inflation and investment rates are zero. The M&M model can be expressed in nominal terms, provided the real WACC is calculated first according to the M&M formula and then the nominal WACC calculated from the Fisher Equation. Alternatively, the nominal version of the M&M model can be found by first calculating the WACC in nominal terms, and then adding the correction factor ΠtL.

Recall that the M&E model assumes a constant ratio of debt to firm value. Since the value of the firm, the amount of debt and therefore, the amount of the interest tax shield, all grow proportional to the rate of inflation, the M&E Model is correct when stated in nominal terms. The M&E Model can be stated in real terms, provided the nominal WACC is calculated according to the M&E formula and the real WACC calculated from the Fisher Equation.

VI. Implications for Practical Applications

In this section we highlight some of the misuses of the Zero-Nominal-Growth model in actual practice. It is always difficult to go from theory to practice. This task is even more daunting if the theory is based on unrealistic assumptions – like zero inflation. Inflation has averaged approximately only 3% per year in the United States over the past 10-15 years.[29] However, we have demonstrated that the valuation errors induced through the inappropriate use of the Zero-Nominal-Growth model can be quite substantial, even at modest levels of inflation. In other countries where the real rate is relatively low and the inflation rate is in double digits, the error could be several times the true value of the firm.

In the following subsections we discuss specific examples in which academicians and practitioners have inappropriately relied on the Zero-Nominal-Growth model to value a firm with zero-plowback or zero-NPV investments. In all of these examples, the Zero-Real-Growth model should be used instead of the Zero-Nominal-Growth model.

A. The Present Value of Continuing Operations (Zero Plowback)

The Zero-Nominal-Growth model is widely taught in business schools and used by practitioners to compute the present value of continuing operations (PVCO)[30]. In this application, the current market value of a firm is thought of as the present value of the firm's continuing operations (or assets in place) plus the present value of its future growth opportunities (PVGO), as shown below.

$$V_0 = PVCO + PVGO \tag{99}$$

$$V_0 = \frac{E_1}{W} + PVGO \tag{100}$$

[29] Statistical Abstracts of the United States, 2001.

[30] This is also called the present value of current operations and the present value of assets in place, which are just different names for the same concept. We refer to these as PVCO, because they all assume zero plowback investment and zero nominal growth.

To estimate the present value of continuing operations (PVCO), it is conventional to use the Zero-Nominal Growth model, which can be seen as the first term in Equation (100)above. The Zero-Nominal Growth model is used for PVCO because it is thought that assuming zero new investment, or zero plowback, results in zero nominal growth in future nominal net cash flows.[31] However, as we have shown in Section III, the correct formula for PVCO, when inflation is positive, is:

$$V_0 = \frac{E_1}{W - \Pi} \tag{101}$$

B. The Residual Value (Zero Net-Present-Value Investments)

The Zero-Nominal-Growth model,

$$V_0 = \frac{E_1}{W} \tag{102}$$

is also widely used to estimate the Residual Value of the firm, assuming that all of the firm's future investment projects have zero NPV.[32] This calculation is typically an important component in any discounted-cash-flow (DCF) analysis.

The DCF method of valuation requires detailed forecasts of cash flows over a multi-year forecast period.[33] It also requires an estimate of the "Residual Value," or "Terminal Value", which, in theory, describes a firm that is in long-run competitive equilibrium and can no longer expect to earn supra-competitive returns on its future

[31] Myers (1977) was perhaps the first to conceptualize the value of the firm as the sum of the value of its "assets in place" and "the present value of growth opportunities." As discussed previously in the text, Brealey and Myers (2000), pp. 72-73, the first term in Equation (44) - as the value of a firm "that does not grow at all. It does not plow back any earnings and simply produces a constant stream of dividends." After writing Equation (44), Ross, Westerfield and Jaffee (1993), pp. 130-131, state "The first term is the value of the firm if it rested on its laurels, that is, if it simply distributed all earnings to the stockholders." Rao (1995), p. 411, describes the first term as "the value of the firm if it chooses to pay out 100% of its earnings as dividends, thereby forgoing its growth options." And finally, Shapiro and Balbirer (2000) state: "The first term represents the value of a no-growth firm that pays out all of its earnings in dividends. It is essentially the value of the assets in place."

[32] Copeland, Koller and Murrin (1994), pp. 282-283, argue that Equation (44) is the value of the firm assuming that the firm faces only zero NPV projects. Rappaport (1998) p.42 also asserts that Equation (44) is the value of the firm if all of its projects have zero NPV.

[33] Rappaport and others refer to the forecast period as the period of Value Growth Duration, which is "management's best estimate of the number of years that investments can be expected to yield rates of return greater than the cost of capital." Rappaport (1998), p. 77.

investments[34]. Thus, all future investments in the perpetuity period have returns equal to the associated costs of capital, rendering them zero net-present-value (NPV) investments. But, again, as we have shown the zero-NPV model is given by Equation (39) and not Equation (46).

C. Alcar Valuation Software

The Alcar valuation software sold by Alcar/LEK has been licensed by thousands of firms and is widely used to compute DCF valuations of businesses. To compute the Residual Value, the value of the firm beyond the forecast period, the Alcar software recommends the Perpetuity for Shareholder Value Method, which assumes that the firm will provide a level stream of cash flows to its stockholders forever. The formula is:

$$\text{Residual Value} = \frac{\text{Operating Profit (1-Tax Rate)}}{\text{Long-term Cost of Capital}}$$

where all values are stated in nominal terms. Alcar's perpetuity formula is based on the assumption that "future investments will earn exactly the firm's long-term cost of capital or, in other words, the NPV of any new investment after the forecast period will be zero."[35]

But, as we have shown, this perpetuity formula significantly understates the correct Residual Value, assuming zero NPV investments, and the magnitude of this under-valuation is directly proportional to the projected inflation rate. Thus, if Alcar users are projecting positive inflation over the perpetuity period, then they should use the Zero-Real Growth formula to compute the Residual Value assuming zero NPV investments. If they follow Alcar's recommended Perpetuity for Shareholder Value Method, the Residual Value, and therefore the Total Corporate Value, will be significantly understated assuming a normal range for projected inflation. (See Section V.)

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[34] Supra-competitive returns can be logically assumed over the forecast period. But, in a competitive environment, entry by competitors and substitute products eventually eliminate these opportunities.

[35] Alcar Online Instruction Manual.

Users of Alcar's valuation software can avoid this error by choosing an alternative method that the software provides, called the Growth in Perpetuity Shareholder Value Method. The user should simply set the nominal growth rate equal to the projected inflation rate and the resulting Residual Value will accurately account for inflation, assuming that the firm has only zero NPV investments in the perpetuity period.[36, 37]

D. Economic Value Added (EVA)

EVA, or economic value added, is a version of the Constant-Growth Model popularized by the consulting firm of Stern/Stewart (S/S). In Chapter 8 of his book,[38] Bennett Stewart demonstrates that this valuation methodology conceptually is equivalent to the traditional discounted cash flow model.[39] Like the other popular valuation manuals and textbooks, S/S do not distinguish between real and nominal values in discussing their model. We assume, therefore, that the parameters of their model are stated in nominal dollars. Consequently, their methodology suffers from the same criticisms that we have lodged against the traditional valuation literature when it comes to dealing with inflation.

As explained in their valuation manual, S/S's model calculates a series of economic profit measures for each period over a "forecast period." Specifically, EVA for any period is calculated as the product of the amount of <u>nominal</u> investment at the beginning

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[36] To be perfectly correct, one should also adjust the numerator (multiply by $1+\Pi$) for any increase in the expected rate of inflation.

[37] Alcar's manual criticizes the Growth in Perpetuity Method, saying that "the method makes no assumption about the economic return on the investment required for growth." But, if nominal growth is set equal to the projected inflation rate, then real growth equals zero, reflecting the assumption that the firm's return on investment precisely equals its cost of capital in the perpetuity period. Alcar's manual goes on to assert that, under this Growth in Perpetuity Method, "the net present value of the growth in perpetuity can yield a value less than, equal to or greater than that of the Perpetuity Method (where the economic assumption of growth yielding NPV = 0 is invoked)." This is incorrect, on two counts. The Growth in Perpetuity Method will always yield a NPV that is greater than that of the Perpetuity Method, assuming that projected inflation is positive. Only if inflation is zero will the Perpetuity Method yield the same, correct NPV as does the Growth in Perpetuity Method where the inflation rate equals the growth rate. Further, the Growth in Perpetuity Method, with growth set equal to the projected inflation rate, correctly reflects the economic assumption that NPV = 0, and the Perpetuity Method does not. The Perpetuity Method yields a NPV that is negatively related to the projected inflation rate, so that NPV < 0 if inflation is greater than zero, in contrast to the assertion in Alcar's manual.

[38] Stewart, <u>The Quest for Value</u>, 1999.

[39] "The EVA approach is entirely equivalent to a procedure for discounting FCF," p. 320.

of the period, and the difference between the nominal return on investment and the firm's nominal cost of capital. S/S refer to this difference as the "Gap."

According to the S/S model, the value of a firm is the discounted value of each period's EVA plus the beginning total, nominal investment. EVA is forecasted for each period in the forecast period. Beyond the forecast period, the model, as presented in the text, assumes that the EVA per period is constant into perpetuity.[40] The problem with this formulation is that, if inflation is positive, then firm's nominal capital stock, which is the "driving force" behind the EVA model, must also increase with the rate of inflation. This will in turn increase the EVA through the perpetuity period. It should be noted that the under-valuation generated by the EVA method is significantly less than that generated by the standard Zero-Nominal-Growth formulation. Since EVA is based on the net present value in the perpetuity period, as opposed to the present value, the under-valuation is less severe.

E. *Cinerama v. Technicolor*

When it was decided in 1990, this prominent case in the Delaware Court of Chancery defined the contours of what expert testimony would be admissible in Delaware courts regarding the valuation of a corporation for appraisal and other litigation in which corporation valuation is an issue. It was the first major opinion after *Wineberger v. UOP*[41] in which the Court abandoned the so-called Delaware Block method. This valuation technique involved assigning weights to various measures of a firm's worth (sales, book value, liquidation value, accounting ratios) and calculating a weighted-average according to some pre-specified formula. In *Wineberger v. UOP* the Court, for the first time recognized discounted cash flow as a legitimate valuation technique. In dicta the Court opined that the Court should allow evidence based on techniques that are generally used in the discipline. In *Technicolor*, the Court takes great pains to carefully

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[40] "Applying this constant spread (*the "Gap"*) to the opening capital balance produces an EVA that is positive and growing at the rate of capital formation. This pattern of EVA growth continues through 1994, when it assumed that T is reached (*the last forecast period*). After that, new capital investment will earn a return identical to the cost of capital. EVA will not change; it will remain. The present value of the EVA as a perpetuity is estimated by using the (discount factor) used in the FCF approach," p.320.

[41] Weinberger v. UOP, Inc., 457 A.2d 701 (Del. Supr. 1983).

explain the various components of the discounted cash flow methodology and thus provides a road map for future litigants and expert witnesses involved in valuation proceedings.

The textbook-like opinion, written by the then Chancellor of the Delaware Chancery Court William Allen, was released on October 19, 1990.[42] In that opinion, Judge Allen provided a judicial appraisal of the fair value of Technicolor, which was purchased for $23 per share cash in January 1983.[43] The evidence in this case was "structured around the elaborate testimony of dueling experts", in Judge Allen's words. Both experts relied on a discounted cash flow analysis, although they came to "distressingly wide" disagreement regarding Technicolor's statutory fair value. The Plaintiff's witness opined that Technicolor's fair value as of January 24, 1983 was $62.75 per share, and that the $23 merger consideration was woefully inadequate. Defendant's expert witness opined that Technicolor's fair value at the same time was only $13.14 per share, and that $23 was more than fair. Judge Allen, after receiving this testimony, decided that Technicolor was worth $21.60 per share, largely siding with the Defendants, and ruled that the $23 was fair consideration.

Although Plaintiffs won the battle of the expert witnesses, they did so using an inappropriate valuation model - the Zero-Nominal-Growth model - to compute Technicolor's Residual Value, even though both experts projected significantly positive rates of inflation in the perpetuity period. Plaintiff argued convincingly that in the perpetuity period, Technicolor could not expect to earn returns greater than its cost of capital because of the inevitable competition from other firms. In such a competitive long-run equilibrium, the Plaintiff argued, growth will not add any value, so the growth term should not appear in the denominator of the present value formula.

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[42] *Cede & Co. v. Technicolor, Inc.*, Del. Ch., C.A. No. 7129, Allen, C. (Oct. 19, 1990). Much to the consternation of former Chancellor Allen, this case has not yet been settled - seventeen years after the first complaint was filed and four years after The Chancellor has left the bench. The Chancellor wrote four opinions in this case and all were either reversed outright or reversed in part and remanded by the Delaware Supreme Court. Chancellor Allen's succor Chancellor Chandler has written three opinions on the case and all of them have either been reversed or remanded back to the Chancery Court by the Delaware Supreme Court.

[43] The setting of the case is an appraisal proceeding in which the plaintiffs, Cinerama, dissented from a cash-out merger of Technicolor into MacAndrews & Forbes. Under Delaware law, stockholders have the

Judge Allen wisely agreed with Plaintiff's first argument, calling it "an application of elementary notions of neo-classical economics: profits above the cost of capital in an industry will attract competitors, who will over some time period drive returns down to the point at which returns equal the cost of capital." But, Judge Allen erred when he also accepted Plaintiff's view that the Residual Value using the Perpetuity Method was inflation neutral. Relying on the Plaintiff's testimony, Judge Allen wrote, "In estimating residual value, Plaintiff's witness, capitalizes a constant (the last forecast year) cash flow, not a perpetually growing one. He asserts that this is consistent with an inflating (or deflating) future world because he posits that whatever the value of money and indeed whatever the size of the company's cash flows, the most reasonable assumption about the future is that there will be a future time at which the firm will not earn returns in excess of its cost of capital. That is if, after that point, one posits increases [*in*] cash flows, due to inflation (or decreases due to deflation) his model stipulates off-setting increases (or decreases) in the firms overall cost of capital."

This is incorrect, as we have shown. Plaintiff's use of the Zero-Nominal-Growth model and the Court's reliance on it is inappropriate when inflation is positive. Interestingly, the Defendant's expert witness actually used the correct approach, subtracting his projected 5% inflation rate for the perpetuity period from the nominal cost of capital in the denominator. He effectively used the Zero-Real Growth model, which we have shown correctly accounts for the future positive inflation (or deflation). But, the Judge disagreed, saying that this assumption was not consistent with his conclusion about competitive equilibrium in the perpetuity period. Judge Allen wrote, "The result--and this is the practical gist of this theoretical difference between the experts--is that Plaintiffs assume that Technicolor's net profits (along with all other aspects of its cash flow) and its value will increase every year in perpetuity, while Defendants assume that there will come a time when, while it may make profits, Technicolor will not be increasing in value."

The fact that Judge Allen was mislead by the economic experts into relying on the Zero-Nominal Growth model to estimate the Residual Value of Technicolor's shares may

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right to dissent from a cash-out merger and seek an appraisal by the court to determine the "fair value" of the stock.

well have had a material effect on the outcome of this litigation. Judge Allen states "This 5% growth assumption adds very substantial value to the discounted present value of a share of Technicolor stock. That assumption alone contributes $16.56 in per share value…" holding all other variables constant. Adding this amount to Judge Allen's $21.60 fair value equals a "corrected fair value" of over $38 per Technicolor share, which is $15 (65%) above the $23 per share merger consideration. Presumably, had Judge Allen found that Technicolor's fair value was $38 per share, he would have ruled that $23 was inadequate, in contrast to his actual ruling that $23 constituted fair value to Petitioners.

We should note that this case raised a number of important issues in the appraisal process[44] and generated a flood of litigation. Indeed, not all of the issues in the case have been settled at the writing of this paper (2002) - including the "fair value" of Technicolor stock as of January 1983. We should also point out, however, that none of this litigation had anything to do whatsoever with the propriety of the Residual Value models used by the expert witnesses. Both expert witnesses and the Judge agreed that if the firm had only zero NPV value projects available to it, the proper way to compute the Residual Value was to use the Zero-Nominal Growth model.[45] Of course, it is the thesis of this paper that the proper model to use when valuing firms with only zero NPV projects is the Zero-Real Growth model, or equivalently, the Nominal-Growth model, where growth is the rate of inflation.

[44] Among the non-valuation issues raised by this seminal case are: the extent to which a court can defer its judgment to expert witnesses; whether dissenters in an appraisal (squeeze-out merger) proceeding have rights to discovery of information subsequent to the merger date; whether dissenters in an appraisal proceeding give up their right as stockholders to recoup losses arising out of a breach of duty by the firm's management; whether plaintiffs can pursue both claims simultaneously and, ultimately, be awarded separate damage claims on each; whether dissenters in an appraisal hearing have claims to any "synergies" created by the acquiring firm; whether the appraised value should be based exclusively on the pre-merger business plan of the acquired firm.

[45] Defendant apparently did not realize that his 5% growth rate for expected inflation was perfectly consistent with Plaintiff's zero NPV assumption. Defendant apparently presumed that the 5% growth rate implied that the firm's return on investment was greater than its cost of capital in the perpetuity period.

VIII. Conclusion

According to virtually all graduate-finance textbooks, the present value of a firm that makes no new investments into perpetuity is simply this year's net cash flows divided by the firm's nominal cost of capital. This formula implies zero nominal growth of the firm's future net cash flows into perpetuity. According to the same finance textbooks, this same "Zero-Nominal-Growth" formula also purportedly provides the present value of a firm that has only zero Net Present Value (NPV) projects available to it into perpetuity. Although widely-accepted and taught, we show that in both cases (no new investments or zero NPV investments) this Zero-Nominal-Growth formula is incorrect if expected inflation is non-zero. Specifically, if expected inflation is as low as 2%, this formula understates the present value of the firm by approximately 25%, assuming a 6% real cost of capital.

Our analysis illustrates that the Zero-Nominal-Growth formula is incorrect in these two popular special cases when inflation is positive. If expected inflation is positive, zero nominal growth in future net cash flows implies that real (constant-dollar) future net cash flows must be negative. But, negative real growth is patently inconsistent with the special conditions to which the formula is being applied. The real future net cash flows of the firm must be constant, not negative, if the firm invests exactly enough to replace economic depreciation, by definition. In such a circumstance, the real productive capacity of the firm, and thus its real output per period, must remain constant into perpetuity. If expected inflation is positive, then the firm's nominal net cash flows must grow by the rate of inflation, not zero, in order to keep real net cash flow constant into perpetuity. Failing to account for this inflation-induced growth, as one does when using the Zero-Nominal-Growth formula, can result in a significant underestimate of the present value of such a firm.

In the case of a firm having only zero NPV projects, a negative real growth rate of future net cash flows requires the firm to take on negative NPV projects in the future, which of course is inconsistent with the starting assumption that the firm has only zero (not negative) NPV projects from which to choose in the future. Again, the nominal growth rate of future net cash flows must (roughly) equal the expected inflation rate to correctly compute the present value of such a firm. As we show, use of the Zero-

Nominal-Growth formula as universally recommended leads to material errors in applications with positive expected inflation.

Only if one assumes that expected inflation is zero into perpetuity can the Zero-Nominal-Growth formula be correctly used in these two popular cases. None of the finance textbooks discloses this inflation-based qualification, even though it would appear to be a critical barrier to using the Zero-Nominal-Growth model that they espouse in any real-world application. This is because the expected inflation rate is quite generally assumed to be positive, not zero, in valuation applications. Indeed, it is obvious from the textbooks that our paper is the first to systematically investigate the effects of expected inflation on these particular widely used valuation formulas. Here, we prove mathematically that these two special conditions--zero new investments and zero NPV projects--require what we call the Zero-Real-Growth formula, and that unless expected inflation is assumed to be zero, the Zero-Nominal-Growth model will produce material valuation errors. It is our hope that the finance textbooks make this correction in their next editions.

In the second part of the paper we show that the weighted average cost of capital formula developed by Modigliani and Miller, which too is universally used in valuation practice, is also incorrect in nominal terms when expected inflation is positive. Although these two giants of the field probably understood that their formula was not inflation neutral, we also show exactly how to correct the nominal weighted average cost of capital formula so that it is inflation neutral. This formulaic "patch" involves adding a term that is the product of expected rate of inflation, the firm's leverage ratio, and the corporate tax rate. We prove mathematically that if expected inflation is positive, as is normally the case in most all applications, the M&M weighted average cost of capital will be too low, resulting in an overvaluation, unless our recommended patch is used. Interestingly, we also show mathematically that the cost of capital formula developed by Miles and Ezzel, which is nowhere near as popular in valuation practice as the M&M formula, is inflation neutral in nominal terms and thus needs no correction when expected inflation is positive. We also hope that the authors of finance textbooks correct the M&M weighted-average-cost-capital formula by adding our patch when applying the formula to cases where expected inflation is not zero.

APPENDIX

Proposition 1: $\dfrac{\partial \mathbf{V_L}^{\mathbf{M\&M}}}{\partial \boldsymbol{\Pi}} > \mathbf{0}$

$$V_L^{M\&M} \quad = \quad \frac{c_1(1+\Pi)}{wacc^{M\&M}(1+\Pi) - \Pi tL} \tag{1}$$

$$\frac{\partial V_L}{\partial \Pi} = \frac{\left[wacc^{M\&M}(1+\Pi) - \Pi tL\right]c_1 - c_1(1+\Pi)\left[wacc^{M\&M} - tL\right]}{(+)} \tag{2}$$

$$\frac{\partial V_L}{\partial \Pi} = \frac{c_1 wacc^{M\&M}(1+\Pi) - c_1\Pi tL - c_1 wacc^{M\&M}(1+\Pi) + c_1\Pi tL + c_1 tL}{(+)} \tag{3}$$

$$\frac{\partial V_L}{\partial \Pi} = \frac{c_1 tL}{(+)} > 0 \tag{4}$$

Proposition 2: $\dfrac{\partial^2 \mathbf{V_L^{M\&M}}}{\partial \boldsymbol{\Pi} \partial \mathbf{L}} > \mathbf{0}$

$$\frac{\partial V_L}{\partial \Pi} = \frac{c_1 tL}{(+)} \tag{1}$$

$$\frac{\partial^2 V_L}{\partial \Pi \partial L} = \frac{c_1 t}{(+)} > 0 \tag{2}$$

Proposition 3: $\dfrac{\partial \mathbf{WACC^{M\&E}}}{\partial \boldsymbol{\Pi}} > \mathbf{0}$

$$\mathrm{WACC^{M\&E}} = W_U - tLq(1 + W_U) \tag{3}$$

$$W_U = w_U + \Pi(1 + w_U) \tag{4}$$

$$\frac{\partial W_U}{\partial \Pi} = 1 + w_U \tag{5}$$

$$q \equiv \frac{W_D}{1 + W_D} = \frac{w_D + \Pi(1 + w_D)}{1 + w_D + \Pi(1 + w_D)} \tag{6}$$

$$\frac{\partial q}{\partial \Pi} = \frac{[1 + w_D + \Pi(1 + w_D)][1 + w_D] - [w_D + \Pi(1 + w_D)][1 + w_D]}{(1 + W_D)^2} \tag{7}$$

$$= \frac{1 + w_D}{(1 + W_D)^2} \tag{8}$$

$$\frac{\partial \mathrm{WACC}}{\partial \Pi} = \frac{\partial W_U}{\partial \Pi} - tL\left[\frac{\partial q}{\partial \Pi}(1 + W_U) + \frac{\partial W_U}{\partial \Pi} q\right] \tag{9}$$

$$\frac{\partial \mathrm{WACC}}{\partial \Pi} = \left(1 + w_U\right) - tL\left[\left(\frac{1 + w_D}{(1 + W_D)^2}\right)(1 + W_U) + \left(1 + w_U\right)\left(\frac{W_D}{1 + W_D}\right)\right] \tag{10}$$

$$\frac{\partial \mathrm{WACC}}{\partial \Pi} = \left(1 + w_U\right) - tL\left[\left(\frac{(1 + w_D)(1 + \Pi)}{(1 + W_D)^2}\right)\left(\frac{(1 + W_U)}{(1 + \Pi)}\right) + \left(\frac{W_D}{1 + W_D}\right)\left(1 + w_U\right)\right] \tag{11}$$

$$\frac{\partial \mathrm{WACC}}{\partial \Pi} = \left(1 + w_U\right) - tL\left[\left(\frac{1 + W_D}{(1 + W_D)^2}\right)\left(1 + w_U\right) + \left(\frac{W_D}{1 + W_D}\right)\left(1 + w_U\right)\right] \tag{12}$$

$$\frac{\partial \mathrm{WACC}}{\partial \Pi} = \left(1 + w_U\right) - tL(1 + w_U)\left[1\right] = (1 - tL)(1 + w_U) > 0 \tag{13}$$

Proposition 4: $\frac{\partial \mathbf{V}_{\mathbf{L}}^{\mathbf{M\&E}}}{\partial \mathbf{\Pi}} > \mathbf{0}$

$$V_L^{M\&E} = \frac{C_1}{WACC^{M\&E} - G} \quad (1)$$

$$\frac{\partial V_L}{\partial \Pi} = \frac{\left[WACC^{M\&E} - G\right]c_1 - c_1(1+\Pi)\left[\frac{\partial WACC^{M\&E}}{\partial \Pi} - \frac{\partial G}{\partial \Pi}\right]}{(+)} \quad (2)$$

$$\frac{\partial V_L}{\partial \Pi} = \frac{\left[WACC^{M\&E} - G\right]c_1 - c_1(1+\Pi)\left[(1-tL)(1+w_U)-(1+g)\right]}{(+)} \quad (3)$$

$$\frac{\partial V_L}{\partial \Pi} = \frac{\left[WACC^{M\&E} - G\right]c_1 - c_1\left[(1-tL)(1+\Pi)(1+w_U)-(1+\Pi)(1+g)\right]}{(+)} \quad (4)$$

$$\frac{\partial V_L}{\partial \Pi} = \frac{\left[WACC^{M\&E} - G\right]c_1 - c_1\left[(1-tL)(1+W_U)-(1+G)\right]}{(+)} \quad (5)$$

$$\frac{\partial V_L}{\partial \Pi} = c_1\left[\frac{WACC^{M\&E} - G - (1-tL)(1+W_U)+1+G}{(+)}\right] \quad (6)$$

$$\frac{\partial V_L}{\partial \Pi} = c_1\left[\frac{W_U - tLq(1+W_U) - W_U + tL(1+W_U)}{(+)}\right] \quad (7)$$

$$\frac{\partial V_L}{\partial \Pi} = c_1\left[\frac{-tLq(1+W_U) + tL(1+W_U)}{(+)}\right] \quad (8)$$

$$\frac{\partial V_L}{\partial \Pi} = c_1\left[\frac{(1-q)\,(tL)(1+W_U)}{(+)}\right] > 0 \quad (9)$$

Proposition 5: $\dfrac{\partial \textbf{wacc}^{\textbf{M\&E}}}{\partial \boldsymbol{\Pi}} < \textbf{0}$

$$\text{wacc}^{\text{M\&E}} \equiv \frac{1+\text{WACC}^{\text{M\&E}}}{1+\Pi} - 1 \tag{1}$$

$$\frac{\partial \text{wacc}^{\text{M\&E}}}{\partial \Pi} = \frac{(1+\Pi)\left(\dfrac{\partial \text{WACC}^{\text{M\&E}}}{\partial \Pi}\right) - \left(1+\text{WACC}^{\text{M\&E}}\right)}{(+)} \tag{2}$$

$$\frac{\partial \text{wacc}^{\text{M\&E}}}{\partial \Pi} = \frac{(1+\Pi)(1-\text{tL})(1+\text{w}_{\text{U}}) - \left(1+\text{WACC}^{\text{M\&E}}\right)}{(+)} \tag{3}$$

$$\frac{\partial \text{wacc}^{\text{M\&E}}}{\partial \Pi} = \frac{(1-\text{tL})(1+\text{W}_{\text{U}}) - \left(1+\text{W}_{\text{U}} - \text{tqL}(1+\text{W}_{\text{U}})\right)}{(+)} \tag{4}$$

$$\frac{\partial \text{wacc}^{\text{M\&E}}}{\partial \Pi} = \frac{\left[1+\text{W}_{\text{U}} - \text{tL} - \text{tLW}_{\text{U}}\right] - \left[1+\text{W}_{\text{U}} - \text{tLq} - \text{tLqW}_{\text{U}}\right]}{(+)} \tag{5}$$

$$\frac{\partial \text{wacc}^{\text{M\&E}}}{\partial \Pi} = \frac{-\text{tL}(1+\text{q})(1+\text{W}_{\text{U}})}{(+)} < 0 \tag{6}$$

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