DEBT TAX SHIELDS VALUATION: A PRACTITIONERS APPROACH

1. Introduction

The interest tax shields can be properly valued by either adjusting the discount rate to reflect the net interest tax shield, or by adding the present value of interest tax shields to the all-equity-financed value of the firm.

In the literature the importance of accurate debt tax shields valuation is acknowledged but at the same time has been paralleled by discussions about how to do it. According to Cooper and Nyborg (2007) the issues being discussed are the following:

- Does the value of the debt tax shield reflect the full corporate tax rate or a lower rate (marginal tax rate)?
- Does the value of the tax shield differ in tax regimes that favour dividends?
- Should you value the tax shield by using adjusted present values or by adjusting the discount rate?
- If you use adjusted present value, how should you obtain the tax shield discount rate?
- What formula should be used to calculate financial distress costs?

This paper covers all issues related to the questions mentioned above. We ignore the effects of personal taxes, assuming that debt and equity income are not exposed to different personal tax rates.

Section 2 reviews the literature on tax shield valuation, followed in section 3 with a clarification on tax treatment and consistent valuation methods. Section 4 covers essential topics in valuation such as the impact of growth, interest tax shields, financing choices, and the issue of financial distress. Section 5 develops a comprehensive valuation approach, and the last section provides some concluding remarks.

2. Literature review

The impact of leverage on the value of a firm and its cost of capital has been discussed extensively since the seminal (classical) works of Modigliani and Miller (MM) (1958, 1963), however, there is (still) no consensus regarding the correct approach to value tax shields. We first present different approaches based on a preset level of debt before presenting the approaches based on a constant leverage ratio.

Due to the lack of consensus in the financial literature about the correct tax shield discount rate to be used, Fernandez (2004) claims *the value of tax shields is not equal to the present value of tax shields*. We show that Fernandez's claim is based on inconsistent assumptions.

For simplicity, the approaches presented in this section assume that there exists only equity and debt financing and disregard any financial distress factors.

Debt tax shields valuation literature: a preset level of debt

In 1963 MM published an article about how corporate tax (t_c) interact with the firm's financing choices, showing that the value of a leveraged firm (V_L) is equivalent to the value of an otherwise identical but unlevered firm (V_U) plus the value of the debt-related tax savings (*VTS*), therefore:

$$V_L = V_u + VTS \tag{1}$$

MM's leverage assumption is a constant amount of debt combined with the assumption that the free cash flow is a risky perpetuity. They also assumed that the growth rate of free cash flow (*FCF)* is zero and that the appropriate discount rate for the tax shield of debt is the cost of debt (K_d). A consequence of the leverage assumption is that the risk of the interest tax shield is the same as the risk of the underlying debt, so the tax shield should be discounted at the cost of debt. The identities that are consistent with the assumption of a constant amount of debt and free cash flow that are called risky perpetuity:

Starting with the value of the levered firm:

$$V_L = V_u + VTS = \frac{FCF_0}{K_u} + \frac{K_d t_c D}{K_d} = \frac{FCF_0}{K_u} + t_c D \tag{2}$$

K_u = unlevered cost of capital (= cost of equity unlevered)

D = market value of debt

FCF_0 = Actual Free Cash Flow

The unlevered cost of capital is related to the cost of debt and equity levered (K_e) by:

$$K_u = \left(\frac{E}{(V_L - t_c D)}\right) * K_e + \left(\frac{E}{(V_L - t_c D)}\right) * K_d * (1 - t_c) \tag{3}$$

$$K_e = K_u + (K_u - K_d) * (1 - t_c)\frac{D}{E} \tag{4}$$

E = market value of equity

The relationship between the weighted average cost of capital (WACC) and K_u is given by:

$$WACC = K_u * (1 - t_c\left(\frac{D}{V_L}\right)) \tag{5}$$

The identities do not hold if either of the mentioned assumptions is not true, in particular if the free cash flow is expected to grow.

The adjusted present value (APV) approach (see eq. 1) was first explicitly presented in Myers' (1974) article with the purpose to present a general approach for analysis of the interactions of corporate financing and investment decisions, and to derive the approach's implications for capital investment decisions. The APV is built on the presumption that it is easier and more precise to formulate the valuation impact in absolute terms rather than in proportional terms. The idea is that management does not state target debt as a ratio of market value but in preset value terms.

In the (basic) APV approach, we basically estimate the value of the firm in two steps. The first step in this approach is the estimation of the value of the unlevered firm. This can be accomplished by discounting the expected free cash flow to the firm at the unlevered cost of equity or unlevered cost of capital (K_u). In the special case where free cash flows grow (expected growth rate = g) at a constant rate in perpetuity, the value of the unlevered firm can easily computed as follows:

$$V_u = \frac{FCF_0 * (1+g)}{K_u - g} \tag{6}$$

The unlevered firm can be valued using any set of growth assumptions we believe are reasonable.

The second step in this approach is the calculation of the expected tax benefit from a given level of debt. The debt tax shields valuation is a function of the tax rate of the firm and is discounted to reflect the riskiness of the expected cash flow (tax shield on interest). In the Myers APV approach, the tax rate and preset debt were viewed as constants and therefore the pre-tax cost of debt was used as the discount rate leading to a simplification of the debt tax shield value (*VTS*):

$$VTS = \frac{K_d t_c D}{K_d} = t_c D \tag{7}$$

This Myers VTS approach is equal to the MM VTS approach (see eq. 2). This means that the Myers APV approach assumes that the amount of leverage is set to a particular level and then not revised in light of later developments, generates a tax saving from interest that does not vary as the value of the firm varies. There is no correlation between the free cash flows and the tax shields. In the literature this is called the MM financing (leverage) policy of a constant amount of debt (Cooper and Nyborg, 2006).

Ehrhard and Daves (1999) adjusted the original Myers APV approach by extending the VTS formula with the expected growth factor. Therefore the adjusted VTS formula can be presented as follows:

$$VTS = \frac{K_d t_c D}{K_d - g} \tag{8}$$

Based on this equation the adjusted Myers' APV approach for the levered firm value can be set as follows:

$$V_L = \frac{FCF_0 * (1+g)}{K_u - g} + \frac{K_d t_c D}{K_d - g} \tag{9}$$

This levered firm value approach can be linked to the following WACC approach:

$$V_L = \frac{FCF_0 * (1+g)}{WACC - g} \tag{10}$$

Where the WACC must be computed as follows:

$$WACC = K_u - (\frac{K_u - g}{K_d - g}) K_d t_c \frac{D}{V_L} \tag{11}$$

The assumption of the MM financing policy of a preset amount of debt is, according to Ehrhard and Daves, still existing although the correlation between the free cash flows and the tax shields seems to be very strong compared to the Myers VTS approach (compare eq. 8 with eq.7). It is questionable whether this assumed leverage policy is in line with the assumption that the tax shields are as risky as debt. This can lead to inconsistency when the tax shields are riskier than in the Myers VTS approach, but discounted at the same rate; the cost of debt.

More in line with the Myers APV approach is an arbitrary leverage policy, with tax savings that have the same risk as debt. In this case it is not assumed that debt is a constant amount. The amount of debt in each future period is known, and the VTS is equal to the present value of the known amounts of tax saving from interest in each future period, discounted at the cost of debt.

In our opinion, a more suitable VTS formula based on an arbitrary leverage policy is:

$$VTS = \sum_{t=1}^{T} \frac{K_d t_c D_{t-1}^*}{(1+K_d)^t} \qquad (12)$$

D_{t-1}^* = an arbitrary amount of debt, fixed at t-1.

Luehrman (1997) presents a model where the amount of debt level, rather than remaining fixed as in the Myers and the MM approach, changes over time as a fraction of book value. Luehrman (1997) elaborates as follows about the appropriate tax shields discount rate:

"Academics agree that tax shields, like any other future cash flow, should be discounted at an "appropriate" risk-adjusted rate - that is, a rate that reflects riskiness..... A common expedient is to use the cost of debt as a discount rate, on the theory that tax shields are about as uncertain as principal and interest payments. Of course, there may come a time when you can afford to make your interest payments but can't use the tax shields. This suggest that tax shields are a bit more uncertain and so deserve a somewhat higher discount rate."

According to Luehrman there will be firms for which the debt tax shield has lower value because there is not enough taxable income to use the full interest tax deduction to save taxes. A possible way to include this in the valuation model is to estimate the expected tax saving from interest using Monte-Carlo simulation software, to determine the appropriate debt tax shields discount rate (K_{TS}) (see section 3).

Luerhman's approach is based on an arbitrary leverage policy assumption, with tax savings from debt that have a "somewhat higher" or the same risk as the debt, which means a "somewhat higher" or the same cost of debt discount rate. This means that Luerhman argued that the appropriate discount rate for the debt tax shields should be the same as the cost of debt (K_d) or between the cost of debt and the unlevered cost of capital (K_u). Luerhman's approach is reasonable and endorsed by professionals in the field, the existing literature does not actually show how an arbitrary discount rate for the tax shield affects the value of the firm, the cost of capital, the unlevered cost of capital, or systematic risk (Ehrhard and Daves, 1999).

In the Luerhman's approach where free cash flows grow (expected growth rate = g) at a constant rate in perpetuity, the value of the levered firm can easily computed as follows:

$$V_L = \frac{FCF_0 * (1+g)}{K_u - g} + \underbrace{\frac{K_d t_c D}{K_{TS} - g}}_{=\text{VTS}} \tag{13}$$

Based on equation 10 the WACC formula can be formulated as follows:

$$WACC = K_u - t_c \left(\frac{D}{V_L} \right) K_d \left(\frac{K_u - g}{K_{TS} - g} \right) \tag{14}$$

The MM, Myers and Luerhman approaches departs with the assumption that there is a preset level of debt and that the risk of tax savings from interest is, in principle, the same as the debt. The extended Myers VTS approach involves making a specific forecast of future debt levels and deriving from these the tax saving from interest in each future with the assumption that these tax savings are as risky as the debt, and thus discounted at the cost of debt.

Debt tax shields valuation literature: a constant leverage ratio

In contrast with MM and Myers, Miles and Ezzel (ME) (1980, 1985) argued that the tax shield is riskier than the debt itself. If a firm maintains a constant leverage ratio, then the amount of debt will depend on the value of the firm, and thus the tax shields are as risky as the underlying operational assets, and should be discounted at the unlevered cost of capital. ME analyzed this issue in discrete time, and they explicit assumed that the tax shield (only) for the first year should be discounted at the cost of debt and that the subsequent tax shields should be discounted at the unlevered cost of capital.

The ME identities that are consistent with the assumption of a constant leverage ratio and where free cash flows grow at a constant rate in perpetuity are:

Starting with the value of the levered firm:

$$V_L = \frac{FCF_0 * (1+g)}{K_u - g} + \underbrace{\frac{K_d t_c D}{K_u - g} * \frac{(1+K_u)}{(1+K_d)}}_{=\text{VTS}} \tag{15}$$

The relationship between the WACC and K_u is given by:

$$WACC = K_u - t_c K_d \left(\frac{D}{V}\right)\left(\frac{1+K_u}{1+K_d}\right) \tag{16}$$

These ME identities look similar to the Myers identities. The crucial difference between ME and Myers (and MM) approaches lies in the level of risk of the tax savings from interest. In the Myers approach the debt level is preset, so the tax shields have the same risk as debt. In the ME approach, the tax shields have a level of risk equal to the free cash flows based on the assumption that the future amount of debt and interest is tied to future cash flows.

In line with ME but with a small adjustment, Harris and Pringle (HP) (1985) argued that all tax shields, including the first year, should be discounted at the unlevered cost of capital, and they provided an analysis based on that. They elaborate on this as follows:

"If a project's cash flows were continuous and debt levels adjusted instantaneously (implying a continues rather than discrete model), ME's approach would be equivalent to this assumption. Given discrete cash flows and debt level adjustment, this assumption (K_0 as a discount rate for all tax shields) reduces, although only very slightly, the value of interest tax shields consistent with the effect of uncertainty about paying positions."

The value of the levered firm based on the HP approach:

$$V_L = \frac{FCF_0 * (1+g)}{K_u - g} + \underbrace{\frac{K_d t_c D}{K_u - g}}_{= \text{VTS}} \tag{17}$$

The relationship between the WACC and K_u is given by:

$$WACC = K_u - t_c K_d \left(\frac{D}{V}\right) \tag{18}$$

The difference in the VTS between the ME and HP approach, which is $(1+K_u)/(1+K_d)$, applied to the classical Gordon-Shapiro's formula which reflects the fact that the debt tax shields are fixed one period before receipt and it vanishes in a continuous time setting, as the length of time periods goes to zero (Barbi, 2007).

The HP approach is very popular with practitioners because of the simplicity of the structure, although, maybe, the ME would be theoretical more in line with the conceptual ideas of valuation when they valuate in a discrete time setting. The VTS difference between ME and HP is minor and from a total value perspective even negligible.

Besides ME and HP, also Sick (1990) and Ruback (2002) claims that discounting the interest tax shields at the cost of debt is not correct, because these flows accrue to equity investors and hence must reflect an adequate degree of risk.

Ruback (2002) has proposed the Capital Cash Flow (CCF) approach with the claim (p.21) that "in many instances the CCF method is substantially easier to apply and, as a result, is less prone to error". Basically this claim overstates the advantages of the CCF approach and we will show that this approach offers no (extra) advantages over the traditional approaches such as the WACC and the APV.

Ruback elaborates that the MM and Myers APV's assumption that debt tax shields have the same risk as the debt, and thus the cost of debt as discount rate, may be justifiable for a preset level of debt but that it is more reasonable to assume that interest tax savings share the same risk as the free cash flows and operational assets, when debt is expected to change over time. This means that when the debt raised each year is such that the debt ratio stays constant, the HP approach and the CCF approach yield identical results.

According to Ruback (2002, p.8), the Capital Cash Flows can be calculated as follows:

CCF = Free Cash Flow + Interest Tax shields (19)

The value of the levered firm based on the CCF approach where capital cash flows grow at a constant rate in perpetuity:

$$V_L = \frac{CCF_0 * (1+g)}{K_u - g} \tag{20}$$

In this respect the CCF approach is not "easier to apply" than for example the HP approach.

The CCF becomes different whenever debt is forecasted in levels, instead of a constant leverage ratio. Given the assumption that the interest tax shields have the same level of risk as the free cash flows, they may be discounted at the unlevered cost of capital. This means that the standard equity cash flow with a constant equity discount rate (K_e) may not be used, because the leverage ratio varies over time!

Based on eq. (20) and (19) and taking into account the forecasted debt levels, the appropriate value of the levered firm formula is:

$$V_L = \sum_{t=1}^{T} \frac{FCF + K_d t_c D_{t-1}^*}{(1+K_u)^t} \tag{21}$$

D_{t-1}^* = an arbitrary amount of debt, fixed at t-1.

Based on eq. 18 the VTS can be computed as follows:

$$VTS = \sum_{t=1}^{T} \frac{K_d t_c D_{t-1}^*}{(1+K_u)^t} \tag{22}$$

When the CCF approach is used with forecasted debt levels, the difference between this approach and the Myers APV approach with an arbitrary leverage policy and also Luerhman's approach is just only the interest tax shield discount rate (K_u or K_d). The views about the underlying assumption regarding the interest tax shields level of risk in relation to the operational assets must be radically opposed. In our view the proper assumption is case dependency based. When the forecasted debt levels in a valuation case are more related to the development of the expected free cash flows, K_u should be the appropriate discount rate. When this relation is not significant, $K_d(+)$ should be used. A wrong valuation case judgment about the underlying premise can lead to inconsistency where an over- or undervaluation can be the consequence.

The CCF approach is, thus, not easier to apply and is not less prone to error compared to other approaches!

The significant difference between the approaches with a preset level of debt (MM, Myers and Luerhman) compared to the constant leverage ratio approaches (ME, HP and CCP) is the assumed different levels of risk in the interest tax shields. It is essential to understand the assumptions about debt policy that underlie the various methods of valuing interest tax shields and using them in a consistent way.

Fernandez: a different view

Fernandez's (2004) argument is that the problem at stake here is that the riskiness of the interest tax shields is difficult to evaluate because it represents the difference in the present value of two tax flows with different risk. These flows are the taxes paid by the unlevered company (G_u) and those paid by the levered company (G_L). In Fernandez's view "discounted value of tax shields" in itself is senseless! In his opinion the value of the interest tax shields is the difference between the present values of two separate cash flows, each with its own risk.

The VTS is defined as the difference between the levered firm value and the unlevered firm value: $E + D = V_u + VTS$

The value of interest tax shields defines the increase in the firm's value as a result of the tax saving obtained by payments of interest and therefore, according to Fernandez, the general valuation formula can be restated as follows:

$$V_u + G_u = E + D + G_L \tag{23}$$

This means that the VTS is the difference between two separate tax flows, each with its own riks: $VTS = G_u - G_L$ (24)

As a corollary, the value generating by installing debt financing could not be calculated simply as the summation of the discounted interest tax benefits. Since the direct and indirect formulas of interest tax shields must offer the same outcome, the fact that this not occur in Fernandez's view, necessarily implies either market incompleteness or some other (unknown) basic assumptions.

Fernandez's result is (more or less) implicitly in line with some other studies. Although having different research questions, the analyses of Lewellen and Emery (1986), Booth (2002, 2007) and Massari, Roncaglio and Zanetti (2007) find exactly the same valuation formula as Fernandez.

Fernandez derives a value for the tax saving under a constant debt to value ratio (ME / HP financing strategy) of:

$$VTS = \frac{K_u t_c D}{K_u - g} \tag{25}$$

Fernandez elaborates about his VTS formula:

".....the value of tax shields is not the present value of tax shields due to payment of interest but rather the difference between G_u and G_L, which represent the present value of two cash flows with different risk. The appropriate way to do an adjusted present value analysis with a growing perpetuity is to calculate the VTS as the present value of D.T.Ku (not the interest tax shield) discounted at the unlevered cost of equity (K_u)......."

Fieten et al. (2005) Arzac and Glosten (2005), Cooper and Nyborg (2006, 2007) and Theis (2009) took an open action against this apparent violation of the value-additivity principle. They claim that Fernandez's approach directly arises from mixing two different leverage policies, that of a preset level of debt (MM and Myers) and a constant leverage ratio (ME and HP). Once the debt policy is treated consistently, VTS is the present value of the future interest tax shields. This means that the expected equality between the direct and indirect approach is achieved.

The mistake in Fernandez's derivations arises from confusing these debt policies. Examples of some inconsistencies in Fernandez's analysis can be shown as follows:

(1)

Between his eq. 37 and 38 (Fernandez, 2004) he states that "if (E/D) is constant, the left-hand side of Eq. (36) does not depend on growth (g) because for any growth rate (E/D), ku, Kd, and Ke are constant". In this sense he departs from a ME/HP debt policy approach. He then states that based on this that for g=0, VTS = t_cD. This is not consistent! When the ME/HP debt policy approach is the assumption, the VTS (with g=0) must be stated as follows:

$$VTS = \frac{K_d t_c D}{K_u} \quad \text{(HP)} \tag{26}$$

Instead of:

$$VTS = \frac{K_d t_c D}{K_d} = t_c D \quad \text{(MM)} \tag{27}$$

By substituting eq. (26) into his eq. (37), we get a correction of his eq. (37). Substracting his eq. (38) from the correct eq. (37), we get the following VTS formula:

$$VTS = \frac{K_d t_c D}{K_u - g} \tag{28}$$

This formula is equal to the HP VTS formula.

(2)

Between his eq. 32 and 33 (Fernandez, 2004) he states that "as ECF=E*(Ke – g), (32) can be rewritten as ….". The cost of equity levered (K_e) is based on the following Myers K_e formula:

$$K_e = K_u + (K_u - K_d) * \left(1 - \left(\frac{K_d}{K_d - g}\right) * t_c\right) * \left(\frac{D}{E}\right) \tag{29}$$

By substituting his eq. (28) into his eq. (35), he get eq. (21), which equals the MM K_e formula:

$$K_e = K_u + (K_u - K_d) * (1 - t_c) * \left(\frac{D}{E}\right) \tag{30}$$

In his analysis Fernandez (2004) uses two different relevering approaches (K_e) which lead to inconsistency.

Although Fernandez (2004) makes some interesting points in his challenging study, his final result, eq. (25), is incorrect.

Based on the literature review we can conclude that:

- The VTS approach should be consistent with the actual and advisable financing policy of the firm;

- The relevering approach should be consistent with the actual and advisable financing policy of the firm;
- Different approaches that make different assumptions should not be mixed, while valuing tax shields.

3. A clarification: tax treatment and consistent valuation methods

Each of the approaches discussed assumes that the present value of a dollar of tax saved by the firm is fully reflected in shareholders value and in most cases assumes that there will be enough taxable income to use the full interest tax deductions to save taxes, but is this reality?

Next we summarize several debates among researchers that are relevant to the valuation practice. In this perspective we will also discuss the available (consistent) valuation methods which can be used regarding all aspects of net tax rate saving and tax shield valuation.

Debt and the corporate marginal tax rate

According to Shevlin (1990) corporate tax law treats gains and losses asymmetrically by taxing income for the current period at statutory (corporate) rates (tc) only when positive. Losses may be carried back (1 year in the Netherlands) to obtain refunds of taxes paid in prior years or carried forward (9 years) to be offset against future taxes payable. In the situation where the firm faces losses, the estimate of the value of the debt tax shield should be the present value of the actual taxes expected to be saved, an amount that will be less than the interest costs multiplied by the statutory (corporate) rate. Because of this asymmetric treatment, losses and gains from other years have the potential to reduce current tax rates, in the literature called "marginal tax rate". Graham (1996) defines the marginal tax rate (t_{mtr}) as the present value of current and expected future taxes paid on an additional dollar of income earned today. The marginal tax rate calculation incorporates the effects of all kinds of tax regulations such as tax deductions and tax credits.

It is strange that in the tax shield valuation literature the marginal tax rate almost never explicitly is covered. Instead, the corporate (statutory) tax rate or proxies are used to calculate the interest tax shields although these rates are at best indirect and can be misleading. According to Graham (1996), this could explain why most financial research fails to find that tax considerations are an important factor in corporate decisions.

Shevlin (1990) examined t_{mtr} estimation and illustrated the impact of different tax proxies on the measurement of the t_{mtr} of a firm. He demonstrates that net operating loss (NOL) rules can have a major effect on the values of the t_{mtr} and shows that a simulated tax proxy constructed from firm data performs better than simpler proxies as measures of expected t_{mtr}. This is in line with Cooper and Nyborg (2007) suggestion of estimation the t_{mtr} besides a less convincing alternative to use the CCF approach, which assumes that the tax shields have the same level of risk as the cash flow. Simulated tax proxy is likely to be more accurate because the CCF approach assumes that the risk of the interest tax shield is the same as the risk of the operating assets, which is only a proxy.

To forecast taxable income, we use Shevlin's (1990) main model which states that firm *i*'s taxable income follows a random walk with drift:

$$\Delta TI_{it} = \mu_i + \varepsilon_{it} \tag{31}$$

ΔTI_{it} = is the first difference in taxable income;

μ = the mean of change in taxable income for firm *i*;

= is distributed normally with mean zero and variance equal to that of ΔTI_{it}, both μ and σ^2 are exogenous and therefore known. The choice of a random walk allows a tax series in the style of Shevlin (1990) and Graham (1996) to be constructed. The marginal tax rates are modeled on the basis of a taxable income stream, a discount rate, tax rate and tax regulations.

Based on a simple constructed simulation model, we will show that changing variances have an impact on the marginal tax rate.

Table 1 presents the results based on the assumptions outlined in appendix 1.

Table 1. Result MTR simulation model							
		discount rate					
		5%					
Mean	st.dev.	MTR					
1000	10	35%					
1000	500	34%					
1000	1000	30%					
1000	2000	22%					discount rate
							5%
		NDTS					
Mean	st.dev.	y1	y2	y3	y4	y5	MTR
1000	10	-100	-100	-100	-100	-100	35%
1000	500	-100	-100	-100	-100	-100	34%
1000	1000	-100	-100	-100	-100	-100	28%
1000	2000	-100	-100	-100	-100	-100	21%

NDTS = Not Debt Tax Shields

In line with the results of Shevlin (1990), Graham (1996) and Graham and Mills (2007), the results of this simple model show that when the volatility of the NOI increases, the t_{mtr} decreases and the effect of the NDTS on the t_{mtr} is not significant. The differences in this example between case #1 with a t_{mtr} of 35% (=t_c) and case #4 with a t_{mtr} of 22% is significant large allowing the conclusion that when in case #4 the t_c is used instead of t_{mtr}, the VTS would be overvalued substantially.

It is not always obvious that the lower t_{mtr} is due to the financing policy. It is also possible that a part of the lower t_{mtr} is caused by the asymmetrically tax treatment of operational (EBIT) gains and losses. When this is the case, then one should be careful using the t_{mtr} to calculate the VTS.

Several scenarios are presented in this simple example to illustrate why the marginal tax rate may differ from low (even sometimes zero) up to the maximum top statutory tax rate. The difference arises mainly due to the NOL-rules allowing carry back or carry forward of losses.

Probably a more accurate approach to include asymmetrically tax treatment of gains and losses into the valuation practice is to estimate the expected tax saving from interest using simulation models. This approach is based on Fernandez's basic idea that the VTS is the difference between two separate tax flows (eq. (24)).

The following general relationship is the fundamental principle in this approach:

$$VTS_{dynamic:t} = \sum_{i=t+1}^{\infty} \frac{E^P\left[TP_{u:i} - TP_{L:i} \middle| i_t\right]}{\left(1+k_{ts}\right)^{i-t}} \tag{32}$$

$VTS_{dynamic:t}$ = VTS based on a simulation model, including asymmetrical tax treatment of gains and losses (carry back and carry forward treatment)

E^P [.] = the expectation operator computed under the real world probability P

$TP_{u:i}$ $TP_{L:i}$ = Tax paid in an unlevered setting versus Tax paid in a levered setting

i_t = Information available at time t, including tax regulations.

Under the expectation operator computed under the real world probability (E^P [.]) we choose a probability distribution for each of the most critical variables (mostly key value drivers). There are a number of choices here, ranging from discrete probability distributions to continuous distributions. While no distribution will provide a perfect fit, the distribution that best fits the data should be used. With the use of IF-statements we can incorporate the asymmetrical tax treatment.

Each simulation will generate a value "tax paid", in both a levered setting and in an unlevered setting. The average across all simulated values will be the value of the "tax paid" ($TP_{u:i}$ vs. $TP_{L:i}$).

The idea is that the VTS is the difference between the total tax paid in a levered setting and unlevered setting, including the asymmetrical tax treatment, discounted at the appropriate discount rate, which is financing policy depended.

There is an one-to-one correspondence between the real world probability P and the risk-neutral probability Q, eq. (31) which can be rewritten as follows (Barbi, 2007):

$$VTS_{dynamic:t} = \sum_{i=t+1}^{\infty} \frac{E^Q\left[TP_{u:i} - TP_{L:i} \middle| i_t\right]}{\left(1 + k_{rfr}\right)^{i-t}} \tag{33}$$

E^Q [.] = the expectation operator computed under the risk neutral probability P

k_{rfr} = Risk Free Rate

The advantage of using the risk-neutral probability approach is that all kinds of analyses can be conducted with k_{rfr} as appropriate discount rate. However, the difficulty of first deciding which financing policy is relevant, is still the main point of departure. This is an important decision before the real world probability can be transformed into the risk-neutral probability.

Based on this analysis the conclusion can be drawn that the calculation of the marginal tax rate is (indeed) an important issue in valuing interest tax shields.

Consistent valuation methods

Several valuation approaches can be used to incorporate the effect of debt and tax shields when valuing the equity of a firm. Four are relatively common as well as in theory as in practice:

- Discount expected equity free cash flow (EFCF) at the cost of equity;
- Discount expected operating free cash flow at the WACC and then subtract the market value of debt;
- Value the firm using an APV approach and then subtract the market value of debt;
- Discount the expected CCF at the unlevered cost of capital and then subtract the value of debt.

In general the value of a levered firm equals the unlevered value of the firm plus the present value of the tax shields (VTS). All consistent valuation approaches should start from a clear and justifiable assumption about financing policy. Basically, all valuation approaches are derived from eq. (1). They differ only in the financing policy assumption and tax status of the firm and, consequently, the level and risk of the future tax savings resulting from the chosen financing policy.

The operating free cash flow (FCF), the capital cash flow (CCF) and the free cash flow to equity holders (ECF) are defined in the usual fashion, we have

$$FCF_i = EBIT_i(1-t_c) - C_i \tag{34}$$

and

$$CCF_i = EBIT_i(1-t_c) - C_i + (K_d D_{i-1} t_c) \tag{35}$$

and

$$ECF_i = (EBIT_i - K_d D_{i-1} t_c)(1-t_c) - C_i + \Delta D_i \tag{36}$$

Where C_i is the net Capex (depreciation and amortization included) plus the change in net working capital and $\Delta D_i = D_i - D_{i-1}$ is the change in the level of debt.

The required discount rates are derived using the CAPM from estimates of the risk free rate, equity beta, debt beta, and market risk premium (MRP):

$$K_e = K_{rfr} + \beta_e * MRP \tag{37}$$

$$K_d = K_{rfr} + \beta_d * MRP \tag{38}$$

An alternative for using the CAPM is the typical "build-up model" for estimating the cost of common equity capital and consists of two components (Pratt, 2002:57):

- The risk free rate;
- A premium for risk, including a general equity risk premium, a size premium and a company-specific risk premium.

The weighted average cost of capital is also defined in the usual fashion:

$$WACC = \left(\frac{E}{V_L}\right) * K_e + \left(\frac{D}{V_L}\right) * K_d(1-t_c) \tag{39}$$

According to Cooper and Nyborg (2006), four main assumptions about financing policy are used:

- A preset, constant amount of debt (MM/Myers);
- A constant (market) leverage ratio (ME/HP);
- An arbitrary non-constant financing policy, with tax savings from debt that have the same risk as the free cash flows (CCF/extended HP);
- An arbitrary non-constant financing policy, with tax savings from debt that have the same risk as the debt (extended Myers/Luerhman).

The last financing policy is probably the most attractive, in that it gives the highest VTS (compare eq. (12) and eq. 17). The assumption underling eq. (12) can be an intrinsically implausible when the expected level of debt is assumed to grow in line with the cash flows of the firm with relatively riskless tax shields. It is unlikely that this combination is realistic and practical. This example illustrates the hazard of simply assuming an arbitrary financing policy without examining its realism, such as probably done by many practitioners.

Table 2: Valuation models and consistency

		Methods:			
Assumption:		**WACC**	**Equity method**	**APV**	**CCF**
ME/HP	a constant leverage ratio	V	V	V	V
MM/Myers	a preset level of debt	V*	V*	V	****
Extented HP/CCF	arbitrary non-constant / fcf	**	***	V	V
Extented Myers/Luehrman	arbitrary non-constant / debt	**	***	V	****

* The WACC and equity methods may be used, but recall that MM/Myers requires a flat perpetual cash flows in these cases and that some of the formulas differ from the HP case.

** The WACC is not constant

*** The equity discount rate is not constant

**** The correct discount rate is not equal to Ku

Source: Cooper and Nyborg (2006)

Table 2 summarizes which methods are consistent with which assumptions. In all cases the APV valuation approach may be applied, if used properly. Each of the other approaches has shortcomings under some of the assumptions.

It has been widely reported that one can make significant valuation errors by assuming a leverage policy that is incorrect. It is important to choose the leverage assumption that most closely approximates reality.

4. Essential topics in valuation

In this section we combine the impact of both growth and the interest tax shield on the cost of capital, the cost of equity, and systematic risk. In line with this, we also cover the influence of tax shields on financing choices and the impact on the costs of financial distress. These are essential topics which are inseparable linked to interest tax shields valuation.

Impact of growth and interest tax shields

Analyzing the impact of growth on interest tax shields, we start with the expression for firm value shown in eq. (13). All other VTS expressions can easily be found by specifying specific values of K_{TS} and g and simplifying eq. (10).

Based on eq. (13):

$$\frac{FCF_0}{WACC - g} = V_u + VTS \tag{40}$$

so that

$$WACC = K_u - \left(\frac{K_u - g}{K_{TS} - g}\right) K_d t_c W_D \tag{41}$$

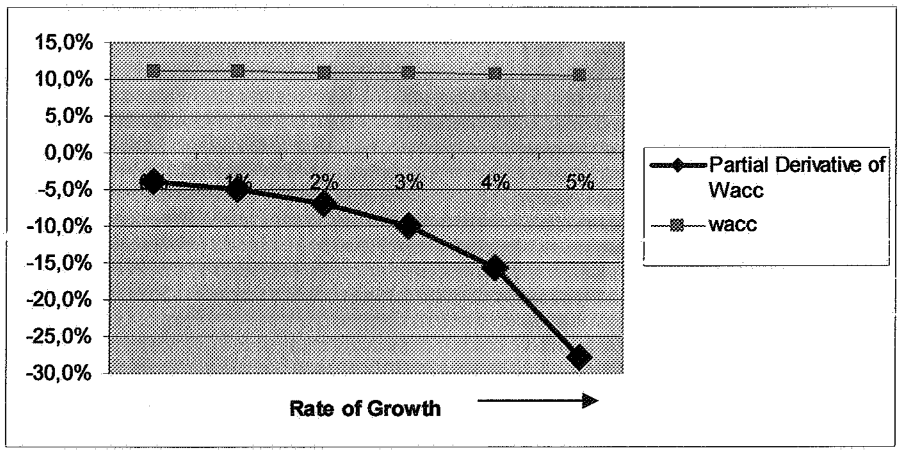
To show the differences between WACC's based on alternative financing choices, we use the partial derivative of eq. (41) with respect to g:

$$WACC\,'(g) = -K_d t_c W_D \left(\frac{K_u - K_{TS}}{(K_{TS} - g)^2} \right) \tag{42}$$

When holding W_D constant, this derivative is zero when $K_{TS}=K_u$, which is the case for the HP approach. For $K_{TS}<K_u$, it is negative, which means that the cost of capital is a decreasing function of the growth rate in the approaches where $K_{TS}<K_u$, such as the Luerhman's approach and the adjusted Myers' approach (see eq. 8), but the HP approach for the cost of capital is independent of growth. This means that growth does not affect the HP cost of capital.

Figure 1 shows the partial derivative when $K_{TS} \neq K_u$ and holding K_{TS}, K_U, W_D, t_c and K_d constant, with an increasing rate of growth. This figure shows also the WACC based on equation 41.

Figure 1: Partial derivative of WACC with an increasing rate of growth



The fundamental question to be asked analyzing this figure is if it is realistic to assume that K_{TS} remains constant when the rate of growth increases? To give an answer, a priori, we divide the growth rate into the following 3 parts:

- $g \leq$ inflation rate (i)
- $i < g \leq$ inflation rate plus average real economic growth (ecgr)
- $g > i +$ ecgr.

If the growth rate is equal to or lower than the inflation rate and the volatility of the FCF's is low, then the assumption that the risk of tax interest savings is, in principle the same as the debt, justifiable. When the growth rate is low but the volatility is high, then there is a higher probability that the level of debt will be more in line with the value of the firm, based on the free cash flows. In this case it is less justifiable to assume that there is a linkage between tax shield risk and debt risk. If the growth rate exceeds the inflation rate in combination with a low volatility, the assumption of the linkage between tax shields and debt is less justifiable than in the first situation but with solid arguments still be reasonable. It is hardly to come up with solid arguments to justify the tax shield linkage with debt when there is a higher volatility. It is not realistic to assume this linkage when the growth rate exceeds i + ecgr. Based on this reasoning, this means that there must be some kind of relationship between growth rate and K_{TS} in combination with cash flow volatility.

As far as we know, there is no empirical evidence to support this hypothesis, but still it sounds reasonable and logical. More research is needed to clarify this issue.

Based on eq. (41) and given a constant tax rate, cost of debt, debt ratio, unlevered cost of capital, the HP approach leads to the highest cost of capital and the adjusted Myers' approach (eq. 8) the lowest. This is not surprising, since tax shields with the HP approach are discounted at the highest rate ($K_u > K_d$).

Based on the analysis of the cost of capital, many of the results for the cost of capital also apply to the levered cost of equity and systematic risk.

Leverage, the cost of equity, and systematic risk formulas can be derived as follows:

Levered cost of equity

$$K_e = K_u + \left[K_u \left(1 - \frac{K_d t_c}{K_{TS} - g} \right) - K_d \left(1 - \frac{K_{TS} t_c}{K_{TS} - g} \right) \right] \frac{D}{E} \qquad (43)$$

Systematic risk

$$\beta_L = \beta_u \left(1 + \frac{D}{E} \right) - \beta_d \frac{D}{E} - \left(\beta_u - \beta_{TS} \right) \left(\frac{K_d t_c}{K_{TS} - g} \right) \frac{D}{E} \qquad (44)$$

When $K_{TS} = K_u$ (HP approach), then eq. (43) and eq. (44) can be rearranged as follows:

Levered cost of equity

$$K_e = K_u + (K_u - K_d) \frac{D}{E} \qquad (45)$$

Systematic risk

$$\beta_L = \beta_u \left(1 + \frac{D}{E} \right) - \beta_d \frac{D}{E} \qquad (46)$$

Eq. (45) and Eq. (46) are the same as the original results of MM (1958) in which they ignore taxes. The reason for this can be seen by comparing the standard WACC expression to that of eq. (41):

$$WACC = (1 - W_D) K_e + W_D K_d - W_D K_d t_c \qquad (47)$$

Versus

$$WACC = K_u - \left(\frac{K_u - g}{K_{TS} - g}\right) K_d t_c W_D \tag{48}$$

When $K_{TS} = K_u$ (HP approach) and rearranging eq. (47) and (41) for K_e, than the outcome is equal to that of eq. (45). If $K_{TS} < K_u$, the resulting levered cost of equity is a function of g, K_{TS} and t_c.

When we closely examine the adjusted Myers VTS approach then we can show that if g is sufficiently large the levered cost of equity is less than the unlevered cost of equity. This happens when $K_d(1\text{-}t_c) - g < 0$. In this case the interest tax shields adds so much value to the firm relative to the value of the firm's other cash flows that the levered cost of equity goes down. If $K_{TS} < K_u$, then larger growth rates increases the magnitude of *VTS*, relative to V_u, driving K_e, below K_u for sufficiently large g. Table 3 shows the difference between the adjusted Myers approach and the HP approach regarding the determination of the levered cost of equity.

Table 2: ***Adjusted Myers versus HP approach; levered cost of equity***

Adjusted Myers Approach					
Ku	10%				
Kd	6%				
Tc	25%				
Growth	2,0%	3,0%	4,0%	**4,6%**	4.6%
Wd	20%				
We	80%				
Ke	10.6%	10,5%	10,3%	**10,0%**	9,9%
	$K_e = K_u + (K_u - K_d)(1 - \left(\frac{K_d}{(K_d - g)}\right) t_c)\left(\frac{D}{E}\right)$				
HP Approach					
Growth	2,0%	3,0%	4,0%	**4,5%**	4.6%
Ke	11%	11%	11%	**11%**	11%
	$K_e = K_u + (K_u - K_d)\left(\frac{D}{E}\right)$				

The reason for the K_e difference is that under the HP approach the tax shields are discounted at the same rate as the free cash flows, which means that values of g cannot change the relative magnitudes of V_u and VTS.

As discussed before, when the growth rate exceeds the inflation rate, the assumption of the linkage between tax shields and debt is less justifiable. Based on this, it is to believe that with sufficiently large growth rates a rate higher than the cost of debt should be used.

Financing choices and Financial distress

According to Almeida and Philippon (2007) the main benefits of debt are the interest tax shields and, in the case of mature companies with limited growth opportunities, the reduction in the so-called "agency costs of free cash flow". The downside of higher leverage is the increase in the expected cost of financial distress.

According to Korteweg (2007) theories of optimal capital structure typically explain management choice of debt versus equity financing by a trade-off. Companies choose a debt ratio that optimally weights the benefits of interest tax shields (including agency benefits) against the costs of debt, which includes the direct costs of bankruptcy and indirect costs such as damage to the firms reputation, the loss of key employees and customers, and the loss of value from forgone investment opportunities (Nageswara and Ramachandran, 2007). Chava and Roberts (2008) show that capital investment declines sharply following a struggling and not succeeding to meet all of the financial obligations (covenant violation), when creditors use the threat of acceleration the debt to intervene in management. This could mean that good investment opportunities are neglected because of insufficient capital with the possibility that high-quality employees are leaving and managers who are distracted from running the business because of financing issues. Such indirect costs are believed to be important (Opler and Titman (1994) and Shleifer and Vishny (1992)). Andrade and Kaplan (1998) estimate losses in value given financial distress on the order of 10% to 23% of pre-distress firm value based on a sample of highly leveraged firms. They conclude further that the costs of financial distress seem low from an ex ante perspective which trade off expected costs of financial distress against the tax and incentive benefits of debt, and even lower when firms do not experience an economic shock. In line with this, Ofek (1992) results show that highly leveraged companies are more likely than their less leveraged counterparts to respond operationally to short-term distress. In contrast, Almeida and Philippon (2007) show that the existing literature substantially underestimates the magnitude of ex-ante distress costs. This is in support with the ideas of Korteweg (2007), Damodaran (2006) and Molina (2005).

The evidence presented in Opler and Titman's (1994) study indicates that there is a positive relationship between financial condition and company performance in economic downturns. They also show that the relation between leverage and performance tends to be more pronounced for those companies with significant R&D expenditures and for those in more concentrated industries. Based on their results they presented the following useful table.

Table 3: ***Potential causes of performance declines by highly leveraged firms in periods of economic distress***

	manager driven	**customer driven**	**competitor driven**
explanation	efficiently downsize by cutting poorly performing assets	customers and stakeholders abandon the firm	competitors reduce prices to gain market share
expl. Loss of Sales rev	yes	yes	yes
expl. Decline in value	no	yes	yes
other predicitons	decline may be related to firm size *1)	decline worse for firms with specialized products (R&D)	decline worse in concentrated industries

*1) Small firms may be more financially vulnerable, and may thus be more subject to customer-driven and competitor driven sales losses. Alternatively, larger firms may benefit the most from the discipline of financial distress and may be more subject to manager-driven sales reduction (with no decline in firm value). ***source: Opler and Titman (1994)***

Opler and Titman's (1994) results are similar with the ideas of Nageswara and Ramachandran (2007). They show that during an economic downturn smaller firms face higher financial distress costs in the form of reducing consumer base, costly credit, etc. They conclude: "Increased debt worsens the situation for smaller firms!".

What is in Opler and Titman's (1994) article meant by "small firms"? These are small listed companies which have sales of less than $100 million. Companies with a sales level of less than (for example) $ 50 million should be even more financially vulnerable, based on Opler and Titman's (1994) argumentation.

There is a relationship between business risk and financing policy (Brigham *et al*, 1999: 585). Business risk is defined as, in a stand alone sense with no debt, a function of the uncertainty inherent in projections of a company's future return on invested capital (ROIC). This means that the business risk of a leverage-free company can be measured by the standard deviation of its return on equity (ROE), σ_{ROE}. Business risk varies from industry to industry and among companies in a given industry (see appendix 2).

Some examples from industries with traditionally low business risk are food companies, grocery retailers, wholesalers, while cyclical manufacturing industries such as automobile and steel related, as well as many small start-up companies, are regarded as having especially high business risk.

There is, to some extent, a relationship between the results of Opler & Titman (1994) and the given view about business risk (examples listed). The greater the use of debt, and the larger the fixed interest costs, the greater the probability that a decline in earnings, especially companies with a high business risk, will lead to financial distress, hence the higher the probability that costs associated with financial distress will be incurred. Bradley *et al* (1984) show that the relationship between earnings volatility and leverage is significant and negative.

Knowing that financial distress is an essential topic in tax shields valuation (Almeida and Philippon (2007) and Damodaran (2006)), the challenge is how to determine the net present value of financial distress costs, given the uncertainty about when and if they might arise. Almeida and Philippon (2007) develop a formal model to calculate financial distress costs. The basic idea is to use the risk-neutral probability of default, ***q***, to perform the risk adjusted valuation of these costs. This risk-neutral probability of default can be formulated as follows:

$$q = \frac{y - K_{rfr}}{(1+y)(1-\rho)} \tag{49}$$

y = bond's / debt's yield (bond priced at par)

ρ = bond's / debt's recovery rate

An approximation of the present value of financial distress costs based on eq. (49) is:

$$\phi = \frac{q}{q + K_{rfr}} * \varphi \tag{50}$$

ϕ = PV of financial distress costs

φ = Costs of financial distress when they occur, which can be thought of as a % of the current value of the firm.

Andrade and Kaplan (1998) estimated the total loss in value attributable to financial distress in the range of 10-23% of pre-distress firm value (= φ). In the event of default, creditors receive a fraction ρ of the loan. Almeida and Philippon (2007) use an average historical recovery rate, which is common in the literature, of 41%. When debt is asset backed, this recovery rate could be a higher percentage (Shleifer and Vishny (1992)).

Important in this perspective are the tax benefits of debt compared to the costs of financial distress. Table 4 shows the Almeida and Philippon results in a more compressed way (their table VI panel A).

Table 4. Tax benefits minus cost of distress

	Tax benefits minus cost of distress			
Credit rating	**risk-adjusted** φ			
	16,5%	**10,0%**	**23,0%**	
AAA	0,15%	0,28%	0,02%	
AA	0,67%	1,40%	-0,05%	
A	0,56%	2,07%	-0,95%	
BBB	0,65%	2,43%	-1,14%	
BB	0,41%	3,09%	-2,28%	
B	-0,59%	3,17%	-4,35%	
		Source: Almeida and Philippon		

Based on table 4 it is clear that the gains from levering up are higher when the cost of financial distress (φ) get to 10%, however, for values of φ closer to 23%, the marginal distress costs are uniformly higher than the marginal tax benefits.

The overall conclusion is that when a company's φ is closer to 23%, there is no benefit in leverage. The company's φ is closer to 23%, based on the views presented in this section, when companies have (for example) significant R&D expenditures, companies with a high growth potential (high present value of growth options) and for those in more concentrated industries. Also smaller companies will have a φ closer to 23%. It will be no surprise that a greater part of those companies also have a high business risk profile.

Although more research is needed to clarify this trade-off issue between tax benefits and cost of distress, the insides presented by Almeida and Philippon, Opler and Titman but also Korteweg give inspiring confidence.

5. A comprehensive valuation approach

Tax valuation can not be seen separately from the total value of a firm with all its essential issues. In this section we link together all the issues discussed in the previous sections and we conclude with a comprehensive valuation approach, suitable for practitioners.

The current Dutch valuation practice

The most common multistage approach in practice is the two-stage DCF model that projects free cash flows for a finite number of periods, usually somewhere between 2 and 7 years, and then assuming a terminal value at the end of the discrete projection period. The APV and WACC approaches are popular in the Dutch practice.

The formula for the WACC approach in a multistage, using the constant growth factor, can be generalized as follow:

$$MV_t = \frac{E^p[FCF_1|i_t]}{(1+k_{wacc})^1} + \frac{E^p[FCF_2|i_t]}{(1+k_{wacc})^2} + ... + \frac{E^p[FCF_n|i_t]}{(1+k_{wacc})^n} + \frac{\frac{E^p[FCF_n|i_t](1+g)}{k_{wacc}-g}}{(1+k_{wacc})^n} \quad (51)$$

MV_t = market value of the firm at t

K_{wacc} = cost of capital firm

n = number of periods in the discrete projection period

A detailed discrete projection period is used and should be long enough so that the firm reaches a steady state by the end of the first stage (n). The free cash flow from the second stage is valued using a continuing-value formula (the last term in eq. 51). In the Dutch practice the growth factor is mostly equal to the inflation rate.

When this approach is conducted, the implicit assumptions are that there is a constant leverage ratio and that the tax shields are as risky as the underlying operational assets, and therefore the appropriate discount rate is K_u. These assumptions are in line with the assumptions used in the HP approach.

The formula for the APV approach in a multistage, using the constant growth factor, can be generalized in two steps as follow:

Step 1. : Market Value Unlevered

$$MV_{t|u} = \frac{E^p[FCF_1|i_t]}{(1+K_u)^1} + \frac{E^p[FCF_2|i_t]}{(1+K_u)^2} + ... + \frac{E^p[FCF_n|i_t]}{(1+K_u)^n} + \frac{\frac{E^p[FCF_n|i_t](1+g)}{K_u-g}}{(1+K_u)^n} \quad (52)$$

Step 2.: Value Tax Shields

$$VTS_t = \frac{[\ D_t t_c K_d | i_t\ ]}{(1+K_{ts})^1} + \frac{E^p[\ D_{t+1} t_c K_d | i_t\ ]}{(1+K_{ts})^2} + ... + \frac{E^p[\ D_{n-1} t_c K_d | i_t\ ]}{(1+K_{ts})^n} + \frac{\frac{E^p[\ D_n t_c K_d | i_t\ ]}{K_{ts} - g}}{(1+K_{ts})^n} \qquad (53)$$

Step 1 + Step 2 = Market value of the firm at t

A frequently used APV approach is the model based on the extended HP approach for the detailed discrete projection period and the HP approach for the terminal value. This means that for the projection period the financing policy assumption is based on an arbitrary non-constant with tax savings from debt that have the same risk as the free cash flow. For the terminal value this assumption is based on a constant leverage ratio. Mixing those financing policies is not per se wrong in this setting, but without a solid argumentation about risk equivalence between tax shields and cash flow, this could lead towards inconsistency and as a result an undervaluation of VTS.

Costs of distress are seldom calculated and rarely been a part of the valuation model. When the HP approach is used with a constant leverage ratio based on industry average, the probability of default is low and therefore the present value of distress costs would be negligible (see eq. (49) with a low debt yield and probably a higher recovery rate). This is not the case when the APV approach is used. This approach is often used to value high leverage transactions. In this case it is very important to be aware of the possibility of default and hence consequently, to estimate the total loss in value attributable to financial distress. When, in this setting, the distress costs are not a part of the valuation process, this can lead to an inconsistency and as a result an overvaluation.

A comprehensive practitioners approach

To build a comprehensive practitioners approach it is important to look at all issues discussed in this and previous sections. This means that the following issues must be incorporated:

- Financing policy and financial distress;
- Asymmetrical tax treatment of gains and losses;
- Consistent valuation methods with growth issues included.

We start with the valuation model which can be used to incorporate the effect of interest tax shields and the costs of financial distress when valuing the equity of a firm. We use the following APV model in this approach:

$$V_L = V_u + VTS + CFD \qquad (54)$$

where VTS is based on eq.(32):

$$VTS_{dynamic:t} = \sum_{i=t+1}^{\infty} \frac{E^P\left[TP_{u:i} - TP_{L:i} \middle| i_t\right]}{\left(1+K_{ts}\right)^{i-t}} \qquad (32)$$

and where CFD is based on eq. (50):

$$\phi = \frac{q}{q + K_{rfr}} * \varphi \qquad (50)$$

CFD = Costs of Financial Distress

Because the asymmetrical tax treatment has already been incorporated in eq. (32), the three remaining questions are:

- What is the appropriate discount rate for the tax shields?
- How high are the costs of financial distress when they occur (φ)?
- How high is the risk-neutral probability of default (q)?

These questions are linked with each other. Although it is not always clear how to answer these questions, we use a qualitative approach to explain how an answer can be given as a basis for using the valuation model (eq. (54)).

Based on previous empirical studies, mostly by financial economists, we constructed a table (see appendix 3) where an association is established between the level of ex post financial costs estimation and the characteristics of companies being valued following a suggestion for the appropriate tax shields discount rate. For example, when a company has steady and predictable cash flows with a low business risk, a low Tobin's q and redeployable liquid assets, the ex post financial distress costs estimation will be low and therefore it is realistic to assume that the interest tax shields have the same level of risk as debt with the appropriate discount rate equal to K_d. In contrast, when a company has volatile cash flows with a high business risk, a high Tobin's q and growth and cyclical assets, these financial costs will be high and therefore it is realistic to assume that in this dynamic setting leverage is depended to cash flow developments which imply a risk equivalence between interest tax shields and cash flows. The appropriate interest tax shields discount rate, in this example, equals K_u. It is not always clear whether the company is an example of the latter or the former. In this case the practitioner must judge whether the tendency is more related towards the first or second example, with as consequence to use K_d (K_d+) or K_u.

If the actual and assumed leverage is higher than industry average, an explicit analysis of the costs of financial distress is needed, based on the appropriate approach presented by Almeida and Philippon (2007). Based on table 4 it is clear that companies with high ex post financial distress costs estimation will not benefit (although it is not likely) from leverage (except for

AAA debt), this means that higher leverage, higher than industry average, will probably lead towards a lower value (interest tax benefits < costs of financial distress) than the unlevered value of the company.

The table presented in appendix 3 is a useful tool for the practitioner when he/she is valuing a business case, seen from the perspective that he/she must use consistent assumptions as input for conducting the appropriate valuation model.

6. Conclusion

There are several key decisions in valuing debt tax shields that can have major effects on the resulting values. The key point to valuing tax shields is consistency. The method used should be consistent with the actual debt policy and the relevering formula should be consistent with the debt policies of the companies.

In this paper we covered all relevant issues for conducting a comprehensive valuation approach. There are still remaining questions to be answered, such as questions related to capital structure issues, related to financial distress issues and related to the risk-neutral approach, as discussed in section 3. We look forward to future research addressing these questions.

References

Almeida, H. and Philippon, T. (2007) The Risk-Adjusted Cost of Financial Distress *Journal of Finance* 66: 2257-2586.

Andrade, G. and Kaplan, S.N. (1998) How Costly is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions that Became Distressed *Journal of Finance* 43(5): 1443-1493.

Arzac, E.R. and Glosten, L.R. (2005) A reconsideration of tax shields valuation *European Financial Management* 11: 453-461.

Barbi, M. (2007) A Marginale Approach to Compute the Value of Debt Tax Shields *Working paper, University of Bologna* 2007: 1-17.
[example taken from http://ssrn.com/abstract=1317249]

Booth, L. (2002) Finding value where none exist: Pitfalls in using adjusted present value *Journal of applied corporate finance* 15(1):8-17.

Booth, L. (2007) Capital Cash Flows, APV and Valuation *European Financial Management* 13(1):29-48.

Bradley, M., Jarrell, G. and Kim, E. (1984) On the existence of an optimal capital structure *Journal of Finance* 39:857-878.

Brigham, E.F., Gapenski, L.C. and Ehrhardt, M.C. (1999) *Financial Management: Theory and Practice* The Dryden Press: Fort Worth 9th edtn.

Chava, S. and Roberts, M.R. (2008) How Does Financing Impact Investment? The role of Debt Covenants *Journal of Finance* 63:2085-2121.

Cooper, I.A. and Nyborg, K.G. (2006) The value of tax shields IS equal to the present value of tax shields *Journal of Financial Economics* 81(2006): 215-225.

Cooper, I. and Nyborg K.G. (2007) Valuing the Debt Tax Shield *Journal of Applied Corporate Finance* 19: 50-59.

Damodaran, A. (2006) *Damdaran on Valuation* Wiley Finance: Hoboken, 2nd edtn.

Ehrhardt, M.C. and Daves, P.R. (1999) The Adjusted Present Value: The Combined Impact of Growth and the Tax Shield of Debt on the Cost of Capital and Systematic Risk *Working paper, Finance Department SMC 424, University of Tennessee* 1999:1-27. [example taken from http://bus.utk.edu/finance/wp/apva.pdf]

Fernandez, P. (2004) The value of tax shields is NOT equal to the present value of tax shields *Journal of Financial Economics* 73:145-165.

Fieten, P., Kruschwitz, L., Laitenberger, J., Löffler, A., Tham, J., Vélez-Pareja, I., and N. Wonder (2005) Comment on: The value of tax shields is NOT equal to the present value of tax shields *Quarterly Review of Economics and Finance* 45(1):184-187.

Graham, J.R. (1996b) Proxies for the corporate marginal tax rate *Journal of Financial Economics* 42(1996): 187-221.

Graham, J.R. and Mills, L.F. (2007) Using tax return data to simulate corporate marginal tax rates *Working paper, Duke University, Fuqua School of Business* 2007: 1-58. [example taken from http://ssrn.com/abstract=959245]

Harris, R.S. and Pringle, J.J. (1985) Risk-Adjusted discount rates – extension from the average-risk case *The Journal of Financial Research* 8(3):237-244.

Korteweg, A. (2007) The Costs of Financial Distress Across Industries *Working paper, Stanford Graduate School of Business* 2007: 1-59.
[example taken from http://ssrn.com/abstract=945425]

Lewellen, W.G. and Douglas, R.E. (1986) Corporate Debt Management and the Value of the Firm *The Journal of Financial and Quantitative Analysis* 21(4): 415-426.

Luehrman, T. (1997) Using APV: A better tool for valuing operations *Harvard Business Review* may-june 1997:3-10.

Massari, M., Roncaglio, F., Zanetti, L. (2007) On the Equivalence between APV and the *wacc* Approach in a Growing Leveraged Firm *European Financial Management* 2007:1-11.

Miles, J.A. and Ezzell, J.R. (1980) The Weighted Average Cost of Capital, Perfect Capital Markets, and Project Life: A Clarification *The Journal of Financial and Quantitative Analysis* 15(3):719-730.

Miles, J.A. and Ezzel, J.R. (1985) Reformulating Tax Shield Valuation: A Note *Journal of Finance* 40(5): 1485-1492.

Modigliani, F. and Miller, M. (1963) Corporate Income Taxes and the Cost of Capital: A Correction *American Economic Review* 53(june):433-443.

Modigliani, F. and Miller, M. (1958) The Cost of Capital, Corporation Finance, and the Theory of Investment *American Economic Review* (june):261-297.

Molina, C.A. (2005) Are Firms Underleveraged? An Examination of the Effect of Leverage on Default Probabilities *Journal of Finance* 60(3): 1427-1459.

Myers, S. (1974) Interactions in Corporate Financing and Investment Decisions-Implications for Capital Budgeting *Journal of Finance* 29 (March):1-25.

Nagesware Rao, S.V.D. and Ramachandran, V.S. (2007) Capital Structure, Industry Pricing, and Firm Performance *Proceedings of the 13th Asia Pacific Management Conference, Melbourne, Australia.*

Ofek, E. (1993) Capital structure and firm response to poor performance *Journal of Financial Economics* 34(1993): 3-30.

Opler, T.C. and Titman, S. (1994) Financial Distress and Corporate Performance *Journal of Finance* 49(3): 1015-1040.

Pratt, S.P. (2002) *Cost of Capital: Estimation and Application* Wiley Finance: Hoboken 2nd edtn.

Ruback, R.S. (2002) Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows *Financial Management* summer 2002:5-30.

Shevlin, T. (1990) Estimating Corporate Marginal Tax Rates with Asymmetric Tax Treatment of Gains and Losses *The Journal of the American Taxation Association* Spring (1990): 51-67.

Shleifer, A. and Vishny, R.W. (1992) Liquidation Values and Debt Capacity: A Market Equilibrium Approach *Journal of Finance* 47(4): 1343-1366.

Sick, G.A. (1990) Tax-Adjusted Discount Rates *Management Science* 36(12):1432-1450.

Theis, C. (2009) What is the Value of the Tax Shield of Debt? *Working paper, University Trier, Germany* 2009: 1-58
[example taken from wiwischriftenriehe.uni-trier.de]

Appendix 1. Shevlin's model

Estimates of future taxable income (TI) realizations are required, together with knowledge of the firm's current net operating losses (NOL) positions, to estimate the present value of a change in tax payable as a result of earning an extra dollar of TI in the current period. A probability distribution over the firm's future TI is required, necessitating some assumptions about the form and parameters of that distribution. The marginal tax rate depends not only on the amount of future TI but also its time path, given current Dutch tax regulations that allow a three-year carryback and a 9 year carryforward of tax losses.

The Monte Carlo simulation procedure to generate project cash flows in their analyses of the effect of tax law asymmetries.

Assumptions:

Taxable Income	0
Mean delta	1.000
St.Dev.	2.000
Rate of Return	10%

The tax bill is calculated using the entire corporate tax schedule. Next, a dollar is added to TI (using a simtable in the simulation model) and the present value of the tax bill is recalculated. The difference between tax bill 1 and tax bill 2 represents the present value of taxes owed on an extra dollar of income earned by the firm. This provides a single expected Marginal Tax Rate for a single firm-year.

Appendix 2 Business risk factors

Business risk depends on a number of factors, the two most important factors are listed below:

- Operating leverage;
- Sales concentration.

Operating leverage

If a high percentage of costs are fixed then the company is exposed to a relatively high degree of business risk. Even a small decline in sales can lead to a large decline in ROE. This means that when other things held constant, the higher a company's fixed costs, the greater its business risk.

Sales concentration

When 20% of the company's customers base is good for 80% of the sales, we can speak about sales concentration. When one of those customers leaves the company, it will have a large impact on ROE.

Appendix 3 Ku, Kd or Kts table – A practitioners approach

	Andrade and Kaplan (1998) **ex post financial distress costs estimation (φ)?** range		
	0%-10%	> 10% - 16,5%	>16,5% - 23%
Opler and Titman (1994) expl. decline in value	manager driven (no decline)	- customer driven - small companies - competitor driven	
Brigham *et al* (1999) Business Risk σ - ebit	low	medium	high
PVGO / growth	low	medium	high
Opler and Titman (1993) LBO firm characteristics	- steady/predictable cash flows - strong, defensible market position - limited working capital requirements - minimal future capital requirements - heavy asset base - divestible assets - strong MT - synergy opportunities	Less evident	
Korteweg (2009) Net benefits are higher for:	- highly profitable firms - with low depreciation - stable profits - low market-to-book ratios - during economic expansion		"net benefits increase in leverage for low debt firms but decrease when leverage becomes high. .... There is some optimal capital structure that maximizes the benefits net of costs of debt financing. In a dynamic setting, this is the leverage ratio that firms will choose on refinancing."
Balakrishnan and Fox (1993) firm specific assets & leverage			more specialized assets
	"a firm's leverage should be positively related to the redeployability of its existing assets"		"a firm's leverage will be negatively related to its investments in intangible, firm-specific assets"
	"a firm's leverage will be positively related to its investments in tangible assets"		
	"a firm's leverage will be positively related to its investment in reputational assets that signal commitment to the product market in which it competes"		
Lehn, Netter and Poulsen (1990)			
Tobin's q = the ratio of the firm's market value to the replacement value of its assets	low ***q*** firms - typically have more collateralizable assets and lower growth opportunities		high ***q*** firms - typically have less collateralizable assets and greater growth opportunities
Shleifer and Vishny (1992)	Redeployable (liquid) assets are good candidates for debt finance, when also redeployable outside the industry		Growth and cyclical assets: - poor candidates for debt financing - extremely illiquid in recession.
	↓		↓
appropriate tax shields discountrate	***Kd is a reasonable assumption***	K_{TS}	***Ku is a reasonable assumption***
	if leverage is > than industry average ↓ **CFD**	**if leverage is > than industry average** ↓ **CFD**	**if leverage is > than industry average** ↓ **CFD**