(UN)LEVERING COST OF CAPITAL:
A SUMMARY OF LEVERAGE EQUATIONS AND REARRANGEMENTS

Joy van der Veer
March 2011

This note briefly presents a summary of the most important leverage equations and rearrangements of those equations from $\mathbf{K_u}$ (return equity unleveraged) to $\mathbf{K_e}$ (return equity levered) and reversed, based on the correct financing policy.

<u>Basic financing policy assumptions</u>:

Harris & Pringle / Compressed APV	: A constant debt ratio	(Growth = 0 & > 0)
Modigliani & Miller / Hamada	: Preset level of debt	(Growth = 0)
Myers	: Preset level of debt	(Growth > 0)

1. <u>Harris & Pringle / Compressed APV</u>

(1.1) The APV - valuation approach, seen from the H&P/Compressed APV perspective (without growth), can be presented as follow:

Step 1. **V_U**	$\dfrac{fcf_t}{K_u}$	(Equity)	$K_e * \dfrac{E}{TV}$
	+		+
Step 2. **TS**	$\dfrac{D*T_c*K_d}{K_u}$	(Debt)	$K_d * \dfrac{D}{TV} * (1-T_c)$
			= ***WACC ($*_1$)***

= **Total Value (TV)**   **TV =** $\dfrac{fcf_t}{wacc}$

$$\textbf{(A)} \quad TV = \frac{fcf_t}{K_u} + \frac{D*T_c*K_d}{K_u} \;\; \textbf{(1)} \Longleftrightarrow \frac{TV}{TV} = \frac{fcf_t}{TV} * \frac{1}{K_u} + \frac{D*K_d*T_c}{TV} * \frac{1}{K_u}$$

$$\Longleftrightarrow K_u = \frac{fcf_t}{TV} + \frac{D*K_d*T_c}{TV} \Longleftrightarrow \frac{fcf_t}{TV} = wacc = K_u - \frac{D*K_d*T_c}{TV} \;\; \textbf{(2)}$$

Based on equation (2) and the Wacc equation ***($*_1$)***, the leverage equation can be rearranged as follows:

(B) $K_u - K_d * T_c * \frac{D}{TV} = K_e * \frac{E}{TV} + K_d * \frac{D}{TV} * (1 - T_c)$ {***wacc = wacc***}

$$\Longleftrightarrow \quad K_e * \frac{E}{TV} = K_u - K_d * T_c * \frac{D}{TV} - K_d * \frac{D}{TV} * (1 - T_c) \quad \textbf{(3)}$$

$$\Longleftrightarrow \quad K_e * \frac{E}{TV} = K_u - K_d * \frac{D}{TV} * (T_c + (1 - T_c)) \quad \Longleftrightarrow \quad K_e * \frac{E}{TV} = K_u - K_d * \frac{D}{TV}$$

$$\Longleftrightarrow \quad K_e = \frac{\left(K_u - K_d * \frac{D}{TV}\right)}{\frac{E}{TV}} \quad \Longleftrightarrow \quad K_e = \frac{\left(K_u - K_d * \frac{D}{TV}\right) * TV}{E}$$

$$\Longleftrightarrow \quad K_e = \frac{K_u * E}{E} + \frac{K_u * D}{E} - \frac{K_d * D}{E} \quad \Longleftrightarrow \quad \boxed{K_e = K_u + (K_u - K_d) * \frac{D}{E}} \quad \textbf{(4)}$$

(1.2) The APV **-** valuation approach, seen from the H&P/Compressed APV perspective with growth (g), can be presented as follow:

Step 1.	**V_U**	$\frac{fcf_t * (1 + g)}{K_u - g}$	(Equity)	$K_e * \frac{E}{TV}$
		+		+
Step 2.	**TS**	$\frac{D * T_c * K_d}{K_u - g}$	(Debt)	$K_d * \frac{D}{TV} * (1 - T_c)$
				= ***WACC (*$_1$)***
		= **Total Value (TV)**	**TV =**	$\frac{fcf_t * (1 + g)}{wacc - g}$

With growth involved, equation (2) can be rearranged as follow:

$\frac{fcf_t * (1 + g)}{TV} = wacc - g = K_u - g - \frac{D * K_d * T_c}{TV}$ which is equal to equation 2, meaning that

incorporating growth has no influence on the leverage equation (**4**)!

Leverage equation <u>with growth</u>: $\boxed{K_e = K_u + (K_u - K_d) * \frac{D}{E}}$ (**4**)

2. Modigliani & Miller / Hamada

(2.1) The APV - valuation approach, seen from the MM/Hamada APV perspective (without growth), can be presented as follow:

Step 1.	$\mathbf{V_U}$	$\frac{fcf_t}{K_u}$	(Equity)	$K_e * \frac{E}{TV}$
		+		+
Step 2.	**TS**	$\frac{D * T_c * K_d}{K_d}$	(Debt)	$K_d * \frac{D}{TV} * (1 - T_c)$
				= ***WACC*** $(*_1)$

= **Total Value (TV)** **TV =** $\frac{fcf_t}{wacc}$

(A) $TV = \frac{fcf_t}{K_u} + \frac{D * T_c * K_d}{K_d}$ (**5**) $\Longleftrightarrow$ $\frac{TV}{TV} = \frac{fcf_t}{TV} * \frac{1}{K_u} + \frac{D * T_c}{TV}$

$\Longleftrightarrow$ $K_u = \frac{fcf_t}{TV} + K_u * T_c * \frac{D}{TV}$ $\Longleftrightarrow$ $\frac{fcf_t}{TV} = wacc = K_u * (1 - T_c * \frac{D}{TV})$ (**6**)

Based on equation (**6**) and the Wacc equation ***$(*_1)$***, the leverage equation can be rearranged as follow:

(B) $K_u * (1 - T_c * \frac{D}{TV}) = K_e * \frac{E}{TV} + K_d * \frac{D}{TV} * (1 - T_c)$ {***wacc = wacc***}

$$\Longleftrightarrow \quad K_e * \frac{E}{TV} = K_u - K_u * T_c * \frac{D}{TV} - K_d * \frac{D}{TV} * (1 - T_c)$$

$$\Longleftrightarrow \quad K_e * \frac{E}{TV} * \frac{TV}{E} = K_u * \frac{TV}{E} - K_u * T_c * \frac{D}{E} - K_d * \frac{D}{E} * (1 - T_c)$$

$$\Longleftrightarrow \quad K_e = \frac{K_u * E}{E} + \frac{K_u * D}{E} - K_u * T_c \frac{D}{E} - K_d * \frac{D}{E} - K_d * \frac{D}{E} * T_c$$

$$\Longleftrightarrow \quad K_e = K_u + \frac{D}{E}\left(K_u - K_u * T_c - K_d - K_d * T_c\right)$$

$$\Longleftrightarrow \qquad \boxed{K_e = K_u + (K_u - K_d) * (1 - T_c) * \frac{D}{E}} \quad (\mathbf{7})$$

In textbooks equation (7) is often called “the financial leverage formula”.

3. Myers

(3.1) The APV - valuation approach, seen from the Myers’ perspective (growth > 0), can be presented as follow:

Step 1.	$\mathbf{V_U}$	$\frac{fcf_t * (1+g)}{K_u - g}$	(Equity)	$K_e * \frac{E}{TV}$
		+		+
Step 2.	**TS**	$\frac{D * T_c * K_d}{K_d - g}$	(Debt)	$K_d * \frac{D}{TV} * (1 - T_c)$
				= ***WACC (*$_1$)***

= **Total Value (TV)** **TV =** $\frac{fcf_t * (1+g)}{wacc - g}$

(A) $$TV = \frac{fcf_t * (1+g)}{K_u - g} + \frac{D * T_c * K_d}{K_d - g} \quad (\mathbf{8}) \qquad \Longleftrightarrow$$

$$\frac{TV}{TV} = \frac{fcf_t * (1+g)}{TV} * \frac{1}{K_u - g} + \frac{D * T_c * K_d}{TV} * \frac{1}{K_d - g} \qquad \Longleftrightarrow$$

$$\frac{fcf_t * (1+g)}{TV} * \frac{1}{K_u - g} = 1 - \frac{D * T_c * K_d}{TV} * \frac{1}{K_d - g} \qquad \Longleftrightarrow$$

$$\frac{fcf_t * (1+g)}{TV} = wacc - g = (K_u - g) - \frac{D * T_c * K_d}{TV} * \frac{K_u - g}{K_d - g} \qquad \Longleftrightarrow$$

$$wacc = K_u - \left(\frac{K_u - g}{K_d - g}\right) * K_d * T_c * \frac{D}{TV} \quad (\mathbf{9})$$

Based on equation (**9**) and the Wacc equation ***(*$_1$)***, the leverage equation can be rearranged as follow:

(B) $$K_u - \left(\frac{K_u - g}{K_d - g}\right) * K_d * T_c * \frac{D}{TV} = K_e * \frac{E}{TV} + K_d * \frac{D}{TV} * (1 - T_c) \qquad \{\boldsymbol{wacc = wacc}\}$$

$$\Longleftrightarrow \quad K_e * \frac{E}{TV} = K_u - \left(\frac{K_u - g}{K_d - g}\right) * K_d * T_c * \frac{D}{TV} - K_d * \frac{D}{TV} * (1 - T_c)$$

$$\Longleftrightarrow \quad K_e = K_u + K_u * \frac{D}{E} - \left(\frac{K_u - g}{K_d - g}\right) * K_d * T_c * \frac{D}{E} - K_d * \frac{D}{E} * (1 - T_c)$$

$$\Longleftrightarrow \quad K_e = K_u + K_u * \frac{D}{E} - K_d * \frac{D}{E} * (\left(\frac{K_u - g}{K_d - g}\right) * T_c + 1 - T_c)$$

$$\Longleftrightarrow \quad \boxed{K_e = K_u + \left(K_u - K_d * \left(1 + T_c * \left(\frac{K_u - g}{K_d - g} - 1\right)\right)\right) * \frac{D}{E}} \quad \textbf{(10)}$$

4. General Wacc and Cost of Equity Levered equations

(4.1) Based on Myers' Wacc equation (9) and Cost of Equity Levered equation (10) a general equation can be arranged which can be used for all approaches under the appropriate financing policy assumptions.

$$wacc = K_u - (\frac{K_u - g}{K_{ts} - g}) * K_d * T_c * \frac{D}{TV} \quad \textbf{(11)}$$

and

$$K_e = K_u + \left(K_u - K_d * \left(1 + T_c * \left(\frac{K_u - g}{K_{ts} - g} - 1\right)\right)\right) * \frac{D}{E} \quad \textbf{(12)}$$

A financing policy based on a constant debt ratio with a growth rate > 0 (***Harris & Pringle***), K_{ts} is equal to K_u, the following rearrangement can be presented:

$$wacc = K_u - (\frac{K_u - g}{K_u - g}) * K_d * T_c * \frac{D}{TV}$$, which is equal to equation (2), and

$$K_e = K_u + \left(K_u - K_d * \left(1 + T_c * \left(\frac{K_u - g}{K_u - g} - 1\right)\right)\right) * \frac{D}{E}$$, which is equal to equation (4).

A financing policy based on a preset level of debt with a growth rate = 0 (***Modigliani & Miller / Hamada***) K_{ts} is equal to K_d and g=0, the following rearrangement can be presented:

$$wacc = K_u - (\frac{K_u - 0}{K_d - 0}) * K_d * T_c * \frac{D}{TV}$$, which is equal to equation (6), and

$$K_e = K_u + \left(K_u - K_d * \left(1 + T_c * \left(\frac{K_u - 0}{K_d - 0} - 1 \right) \right) \right) * \frac{D}{E}$$, which is equal to equation (7).

A financing policy based on a preset level of debt with a growth rate > 0 (***Myers***) K_{ts} is equal to K_d and g>0, the following rearrangement can be presented:

$$wacc = K_u - \left(\frac{K_u - g}{K_d - g}\right) * K_d * T_c * \frac{D}{TV}$$, which is equal to equation (9), and

$$K_e = K_u + \left(K_u - K_d * \left(1 + T_c * \left(\frac{K_u - g}{K_d - g} - 1 \right) \right) \right) * \frac{D}{E}$$, which is equal to equation (10).

(4.2) In this section I will show examples based on the discussed approaches.

A. Harris & Pringle

Ku	12,0%
Kd	5,0%
g	2,0%
T	25,0%
D	50,0%
E	50,0%
K_{ts}	12,0%
FcF t=0	100,00
Wacc	11,38%

Harris & Pringle

$$wacc = K_u - \left(\frac{K_u - g}{K_u - g}\right) * K_d * T_c * \frac{D}{TV}$$

$$K_e = K_u + \left(K_u - K_d * \left(1 + T_c * \left(\frac{K_u - g}{K_u - g} - 1 \right) \right) \right) * \frac{D}{E}$$

											121,90	10
											1.326,27	RW
	0	1	2	3	4	5	6	7	8	9	10	
fcf	**100,00**	**102,00**	**104,04**	**106,12**	**108,24**	**110,41**	**112,62**	**114,87**	**117,17**	**119,51**	**1.448,17**	
Value	1.088,0	1.088,0	1.109,8	1.132,0	1.154,6	1.177,7	1.201,2	1.225,3	1.249,8	1.274,8	1.300,3	
wacc		123,76	126,24	128,76	131,34	133,96	136,64	139,37	142,16	145,00	147,90	
fcf		(102,00)	(104,04)	(106,12)	(108,24)	(110,41)	(112,62)	(114,87)	(117,17)	(119,51)	(1.448,17)	
value end		1.109,8	1.132,0	1.154,6	1.177,7	1.201,2	1.225,3	1.249,8	1.274,8	1.300,3	-	
Equity												
1-jan		544,00	554,88	565,98	577,30	588,84	600,62	612,63	624,89	637,38	650,13	
Ke		103,36	105,43	107,54	109,69	111,88	114,12	116,40	118,73	121,10	123,52	
cash to equity		(92,48)	(94,33)	(96,22)	(98,14)	(100,10)	(102,11)	(104,15)	(106,23)	(108,36)	(773,66)	
31-dec		554,88	565,98	577,30	588,84	600,62	612,63	624,89	637,38	650,13	-	
	Ke	**19,00%**	**19,00%**	**19,00%**	**19,00%**	**19,00%**	**19,00%**	**19,00%**	**19,00%**	**19,00%**	**19,00%**	
Debt												
1-jan		544,00	554,88	565,98	577,30	588,84	600,62	612,63	624,89	637,38	650,13	
Kd (after Tax)		20,40	20,81	21,22	21,65	22,08	22,52	22,97	23,43	23,90	24,38	
instalments		(9,52)	(9,71)	(9,90)	(10,10)	(10,30)	(10,51)	(10,72)	(10,94)	(11,15)	(674,51)	
31-dec		554,88	565,98	577,30	588,84	600,62	612,63	624,89	637,38	650,13	-	

B. Modigliani & Miller

Ku	12,0%
Kd	5,0%
g	0,0%
T	25,0%
D	50,0%
E	50,0%
K_{ts}	5,0%
FcF t=0	100,00
Wacc	10,50%

M&M and Hamada

$$wacc = K_u - \left(\frac{K_u - 0}{K_d - 0}\right) * K_d * T_c * \frac{D}{TV}$$

$$K_e = K_u + \left(K_u - K_d * \left(1 + T_c * \left(\frac{K_u - 0}{K_d - 0} - 1 \right) \right) \right) * \frac{D}{E}$$

											100,00	10
											952,38	RW
	0	1	2	3	4	5	6	7	8	9	10	
fcf	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**100,00**	**1.052,38**	
Value	952,4	952,4	952,4	952,4	952,4	952,4	952,4	952,4	952,4	952,4	952,4	
wacc		100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	
fcf		(100,00)	(100,00)	(100,00)	(100,00)	(100,00)	(100,00)	(100,00)	(100,00)	(100,00)	(1.052,38)	
value end		952,4	952,4	952,4	952,4	952,4	952,4	952,4	952,4	952,4	-	
Equity												
1-jan		476,19	476,19	476,19	476,19	476,19	476,19	476,19	476,19	476,19	476,19	
Ke		82,14	82,14	82,14	82,14	82,14	82,14	82,14	82,14	82,14	82,14	
cash to equity		(82,14)	(82,14)	(82,14)	(82,14)	(82,14)	(82,14)	(82,14)	(82,14)	(82,14)	(558,33)	
31-dec		476,19	476,19	476,19	476,19	476,19	476,19	476,19	476,19	476,19	-	
	Ke	**17,25%**	**17,25%**	**17,25%**	**17,25%**	**17,25%**	**17,25%**	**17,25%**	**17,25%**	**17,25%**	**17,25%**	
Debt												
1-jan		476,19	476,19	476,19	476,19	476,19	476,19	476,19	476,19	476,19	476,19	
Kd (after Tax)		17,86	17,86	17,86	17,86	17,86	17,86	17,86	17,86	17,86	17,86	
instalments		(17,86)	(17,86)	(17,86)	(17,86)	(17,86)	(17,86)	(17,86)	(17,86)	(17,86)	(494,05)	
31-dec		476,19	476,19	476,19	476,19	476,19	476,19	476,19	476,19	476,19	-	

B. Myers

Ku	12,0%
Kd	5,0%
g	2,0%
T	25,0%
D	50,0%
E	50,0%
K_{ts}	5,0%
FcF t=0	100,00
Wacc	9,92%

Myers

$$wacc = K_u - \left(\frac{K_u - g}{K_d - g}\right) * K_d * T_c * \frac{D}{TV}$$

$$K_e = K_u + \left(K_u - K_d * \left(1 + T_c * \left(\frac{K_u - g}{K_d - g} - 1\right)\right)\right) * \frac{D}{E}$$

	0	1	2	3	4	5	6	7	8	9	10	
											121,90	10
											1.570,58	RW
fcf	**100,00**	**102,00**	**104,04**	**106,12**	**108,24**	**110,41**	**112,62**	**114,87**	**117,17**	**119,51**	**1.692,48**	
Value	1.288,4	1.288,4	1.314,2	1.340,5	1.367,3	1.394,6	1.422,5	1.451,0	1.480,0	1.509,6	1.539,8	
wacc		127,77	130,32	132,93	135,59	138,30	141,07	143,89	146,77	149,70	152,70	
fcf		(102,00)	(104,04)	(106,12)	(108,24)	(110,41)	(112,62)	(114,87)	(117,17)	(119,51)	(1.692,48)	
value end		1.314,2	1.340,5	1.367,3	1.394,6	1.422,5	1.451,0	1.480,0	1.509,6	1.539,8	-	
Equity												
1-jan		644,21	657,09	670,24	683,64	697,31	711,26	725,49	740,00	754,80	769,89	
Ke		103,61	105,68	107,80	109,95	112,15	114,39	116,68	119,02	121,40	123,82	
cash to equity		(90,73)	(92,54)	(94,39)	(96,28)	(98,21)	(100,17)	(102,17)	(104,22)	(106,30)	(893,72)	
31-dec		657,09	670,24	683,64	697,31	711,26	725,49	740,00	754,80	769,89	-	
	Ke	**16,08%**	**16,08%**	**16,08%**	**16,08%**	**16,08%**	**16,08%**	**16,08%**	**16,08%**	**16,08%**	**16,08%**	
Debt												
1-jan		644,21	657,09	670,24	683,64	697,31	711,26	725,49	740,00	754,80	769,89	
Kd (after Tax)		24,16	24,64	25,13	25,64	26,15	26,67	27,21	27,75	28,30	28,87	
instalments		(11,27)	(11,50)	(11,73)	(11,96)	(12,20)	(12,45)	(12,70)	(12,95)	(13,21)	(798,76)	
31-dec		657,09	670,24	683,64	697,31	711,26	725,49	740,00	754,80	769,89	-	

Based on the presented examples, it is evident that Myers' approach shows the highest value which is due to a lower tax shields discount rate (Kd in stead of Ku). This lower discount rate is due to the financing policy assumption based on a preset level of debt, meaning that the correlation between the free cash flows and the tax shields is substantial lower than one.

5. Consistent valuation approaches

According to Cooper and Nyborg (2006), four main assumptions about financing policy are used:

- A preset, constant amount of debt (MM/Myers);
- A constant (market) leverage ratio (Harris & Pringle);
- An arbitrary non-constant financing policy, with tax savings from debt that have the same risk as the free cash flows (extended Harris & Pringle);
- An arbitrary non-constant financing policy, with tax savings from debt that have the same risk as the debt (extended Myers).

The following table summarizes which methods are consistent with which assumptions. In all cases the APV valuation approach may be applied, if used properly. Each of the other approaches has shortcomings under some of the assumptions.

		Methods:			
Assumption:		**WACC**	**Equity method**	**APV**	**CCF**
ME/HP	a constant leverage ratio	V	V	V	V
MM/Myers	a preset level of debt	V*	V*	V	****
Extented HP/CCF	arbitrary non-constant / fcf	**	***	V	V
Extented Myers/Luehrman	arbitrary non-constant / debt	**	***	V	****

* The WACC and equity methods may be used, but recall that MM/Myers requires a flat perpetual cash flows in these cases and that some of the formulas differ from the HP case.
** The WACC is not constant
*** The equity discount rate is not constant
**** The correct discount rate is not equal to Ku

Source: Cooper and Nyborg (2006)

ME = Miles and Ezzel CCF = Capital Cash Flow

6. Cost of Equity unlevered (K_u)

In academic literature as well as in practice, two mainstream cost of capital approaches are used. The first kind of models are based on the Capital Asset Pricing Model (CAPM) and the second kind of models on the Build-up Model (BU). Capital assets pricing models are mainly used for the valuation of listed and large firms whereas build-up models are mostly used to value non-listed firms, although there are specific developed models such as the BDO and KPMG model, based on the CAPM principle, which are (to a certain extend) suitable for valuing non-listed firms.

The CAPM and the BU are explanatory models. These take hypotheses of equity investor behaviour which are interpreted in formula which are then populated with numbers estimated statistically in a positivistic way from market data to calculate the cost of equity. Two important data analysis resources are Duff & Phelps and Damodaran. Other explanatory models are Arbitrage Pricing Theory (APT), and the Fama-French Three Factor model. Explanatory models normally are all based on the hypothesis that equity investors hold diversified portfolios of equity investments and therefore only require returns for systematic risk. For entrepreneurs (investors in a single venture), we have to count in for the effects of under-diversification and human capital investment by estimating total risk, meaning systematic risk plus un-systematic risk, also called specific firm risk. In this note I will only use models based for non-listed firms, meaning models which include specific risk elements.

To estimate the cost of equity un-levered we can use the CAPM and the BU model.

(6.1) Build-up model

The cost of equity un-levered can be estimated by the Build-up model as:

$$E(R_{eu,i}) = R_f + RP_m + RP_s \pm RP_u \qquad \textbf{(13)}$$

Where: $E(R_{eu,i})$ = Ku = Expected (market required) rate of return on security *i*
Rf = Rate of return available on a risk-free security
RP_m = General expected equity risk premium for the "market"
RP_s = Risk premium for smaller size
RP_u = Risk premium attributable to the specific firm or to the industry.

It is beyond the scope of this note to discuss the development of each of these four components. See for more detailed information "Cost of Capital: applications and examples" (Pratt and Grabiwski, 2010) and/or "Risk Premium Report" (Duff & Phelps, 2010).

(6.2) CAPM

The cost of equity un-levered can be estimated by the expanded CAPM formula as:

$$E(R_{eu,i}) = R_f + \beta_u * (RP_m) + RP_s \pm RP_u \qquad \textbf{(14)}$$

Where: $E(R_{eu,i})$ = Ku = Expected un-levered rate of return on security *i*
Rf = Rate of return available on a risk-free security
β_u = Beta un-levered
RP_m = General expected equity risk premium for the "market"
RP_s = Risk premium for smaller size
RP_u = Risk premium attributable to the specific firm or to the industry.

It is beyond the scope of this note to discuss the development of each of these four components. See for more detailed information "Cost of Capital: applications and examples" (Pratt and Grabiwski, 2010) and/or "Risk Premium Report" (Duff & Phelps, 2010).

Many practitioners and academics start searching for a suitable **levered** beta due to the fact that the un-levered beta is not observable in market data. Then they use a leverage formula to un-lever this beta. In section 7 I will discus how to un-lever betas in a proper and consequent way.

(6.3) Build-up versus CAPM

The expected un-levered rate of return on security *i* estimated by the expanded CAPM formula and the BU model must be equal due to the fact that it is impossible that there are two different rate of returns for exactly the same un-levered valuation object, meaning that:

$$R_f + \beta_u * (RP_m) + RP_{s,u,CAPM} = R_f + RP_m + RP_{s,u,BU}$$

and,

$$RP_{s,u,BU} = RP_m * (\beta_u - 1) + RP_{s,u,CAPM} \qquad \textbf{(15)}$$

If the beta un-levered is equal to 1, the $RP_{s,u,BU}$ is equal to $RP_{s,u,CAPM}$. In all other circumstances the Risk Premiums are not equal.

If $\beta_u > 1$ $\quad RP_{s,u,BU} > RP_{s,u,CAPM}$ and,
If $\beta_u < 1$ $\quad RP_{s,u,BU} < RP_{s,u,CAPM}$

7. Un-leveraging and Leveraging Equity Betas

Published betas for listed firms typically reflect the leverage of each respective firm at market values. These levered betas reflecting the leverage in the firm's capital structure. These betas incorporate two risk factors that bear on systematic risk:

- Business risk, and
- Financial risk.

Removing the effect of financial leverage leaves the effect of business risk only (unlevered beta = asset beta). Asset beta is the beta that would be expected were the company financed only with equity.

If the leverage of a private (non-listed) firm is subject to valuation differs significantly from the leverage of the guideline listed firms selected for analysis, or if the debt levels of these guideline firms differ significantly from one another, it is desirable to remove the effect that leverage has on the betas before using them as a proxy to estimate the beta of the subject firm.

This adjustment is performed in three steps (Pratt, 2010):

Step 1. Compute and un-lever the beta for each of the guideline listed firm;
Step 2. Decide where the risk would fall for the subject firm relative to the guideline firms, assuming all had 100% equity capital structures;
Step 3. Lever the beta for the subject firm based on one or more assumed capital structures.

Betas provided by data-providers such as Datastream, Morningstar, Damodaran and Bloomberg are derived directly from historical market information on equities and are therefore known as levered betas or also called ***equity betas***. Asset betas, on the other hand, are unobservable. They reflect only the operational risk of the underlying operational assets. Asset betas have to be determined from equity betas by adjusting for the leverage of the firm in question.

The relationship between asset betas and equity betas are based on the basis financing policy assumptions. In this sense I will show the asset beta equations based on the correct financing policy.

Basic financing policy assumptions:

Harris & Pringle / Compressed APV	: A constant debt ratio	(Growth = 0 & > 0)
Modigliani & Miller / Hamada	: Preset level of debt	(Growth = 0)
Myers	: Preset level of debt	(Growth > 0)

In determining the asset beta I will depart from the basic CAPM equation (without specific risk premiums), which can be presented as follows:

$$E(R_{eu,i}) = K_u = R_f + \beta_u * (RP_m) \qquad \textbf{(16)}$$

Based on this I can rearrange K_u, K_e and K_d as follows:

$$K_e = R_f + \beta_e (RP_m)$$
$$K_u = R_f + \beta_u (RP_m)$$
$$K_d = R_f + \beta_d (RP_m) .$$

Like asset betas, debt betas are also generally unobservable. In practice and in the academic literature a range is used between 0,15 and 0,25 (mostly 0,20). The size of debt beta, in general, depends on several parameters, such as size of the firm, collaterals, leverage, cash flow forecast, etc. If an interest rate for a specific firm is available, the debt beta can be calculated. For example, if a lender is willing to lend money at a 5% rate, the risk free rate is 3,75% and the equity risk premium is 6%, the debt beta can be calculated as follows:

$$\beta_d = \frac{K_d - R_f}{RP_m} = \frac{5\% - 3{,}75\%}{6\%} = 0{,}208$$

(7.1) Harris & Pringle: A constant debt ratio

As a starting point equation (4) will be used, which can be rearranged as follows:

$$R_f + \beta_e (RP_m) = R_f + \beta_u (RP_m) + \left(\left(R_f + \beta_u (RP_m)\right) - \left(R_f + \beta_d (RP_m)\right)\right) * \frac{D}{E} \quad \Longleftrightarrow$$

$$\beta_e (RP_m) = \beta_u (RP_m) + \left(\left(\beta_u (RP_m)\right) - \left(\beta_d (RP_m)\right)\right) * \frac{D}{E} \quad \Longleftrightarrow$$

$$\beta_e = \beta_u + \beta_u * \frac{D}{E} - \beta_d * \frac{D}{E} \quad \Longleftrightarrow \quad \beta_e = \beta_u * (1 + \frac{D}{E}) - \beta_d * \frac{D}{E} \quad \Longleftrightarrow$$

$$\beta_u * \left(1 + \frac{D}{E}\right) = \beta_e + \beta_d * \frac{D}{E} \quad \Longleftrightarrow \quad \beta_u = \frac{\beta_e + \beta_d * \frac{D}{E}}{\left(1 + \frac{D}{E}\right)} \quad \Longleftrightarrow \quad \beta_u = \frac{\frac{E}{TV} * (\beta_e + \beta_d * \frac{D}{E})}{\frac{E}{TV}\left(1 + \frac{D}{E}\right)}$$

$$\Longleftrightarrow \quad \boxed{\beta_u = \beta_e * \frac{E}{TV} + \beta_d * \frac{D}{TV}} \qquad \textbf{(17)}$$

(7.2) M&M and Hamada: Preset level of debt (growth = 0)

As a starting point equation (7) will be used, which can be rearranged as follows:

$$R_f + \beta_e (RP_m) = R_f + \beta_u (RP_m) + \left(\left(R_f + \beta_u (RP_m)\right) - \left(R_f + \beta_d (RP_m)\right)\right) * \left(1 - T_c\right) * \frac{D}{E} \quad \Longleftrightarrow$$

$$\beta_e(RP_m) = \beta_u(RP_m) + \left((\beta_u(RP_m)) - (\beta_d(RP_m))\right) * (1 - T_c) * \frac{D}{E} \quad \Longleftrightarrow$$

$$\beta_e = \beta_u + \beta_u * (1 - T_c) * \frac{D}{E} - \beta_d * (1 - T_c) * \frac{D}{E} \quad \Longleftrightarrow$$

$$\beta_e = \beta_u * (1 + (1 - T_c) * \frac{D}{E}) - \beta_d * (1 - T_c) * \frac{D}{E} \quad \Longleftrightarrow$$

$$\beta_u * \left(1 + (1 - T_c) * \frac{D}{E}\right) = \beta_e + \beta_d * (1 - T_c) * \frac{D}{E} \quad \Longleftrightarrow$$

$$\Longleftrightarrow \quad \boxed{\beta_u = \frac{\beta_e}{\left(1 + (1 - T_c) * \frac{D}{E}\right)} + \beta_d * \frac{(1 - T_c) * \frac{D}{E}}{\left(1 + (1 - T_c) * \frac{D}{E}\right)}} \tag{18}$$

(7.3) Myers: Preset level of debt (growth > 0)

As a starting point equation (10) will be used, which can be rearranged as follows:

$$R_f + \beta_e * RP_m = (R_f + \beta_u * RP_m) + \left[(R_f + \beta_u * RP_m) - (R_f + \beta_d * RP_m) * \left(1 + T_c * \left(\frac{K_u - g}{K_d - g} - 1\right)\right)\right] * \frac{D}{E}$$

After rearranging (see appendix 1.) the following equation can be presented:

$$\boxed{\beta_u = \frac{\beta_e + \beta_d * \frac{D}{E} * \left(1 - \frac{Kd * T_c}{K_d - g}\right)}{\left(1 + \frac{D}{E} * \left(1 - \frac{K_d * T_c}{K_d - g}\right)\right)}} \tag{19}$$

(7.4) A general approach

Based on equation (19) a general un-leverage formula can be arranged which can be used for all approaches under the appropriate financing policy assumption.

$$\beta_u = \frac{\beta_e + \beta_d * \frac{D}{E} - \beta_{ts} * \frac{D}{E} * \left(1 - \frac{Kd * T_c}{K_d - g}\right)}{\left(1 + \frac{D}{E} * \left(1 - \frac{K_d * T_c}{K_{ts} - g}\right)\right)} \qquad \textbf{(20)}$$

This equation (20) can not be used if $K_{ts} = K_u$ and $\beta_{ts} = \beta_u$. This equation must be rearranged as follows:

$$\beta_e = \beta_u * \left(1 + \frac{D}{E}\right) - \beta_d * \frac{D}{E} - (\beta_u - \beta_{ts}) * \frac{K_d * T_c}{K_{ts} - g} * \frac{D}{E} \longleftarrow \beta_u - \beta_{ts} = 0 \Longleftrightarrow$$

$$\beta_u = \frac{\beta_e}{\left(1 + \frac{D}{E}\right)} + \frac{\beta_d * \frac{D}{E}}{\left(1 + \frac{D}{E}\right)}$$ which is equal to equation (17).

8. Practitioners approaches

First I will show some different asset beta formulas, which are used by several well-known academics. Then I will give some comment and show some possible errors in these formulas and suggest some ideas for further research on this topic.

(8.1) Tim Ogier

$$\beta_u = \frac{\beta_e}{\left(1 + \frac{D}{E}\right)}$$

(8.2) Aswath Damodaran

$$\beta_u = \frac{\beta_e}{\left(1 + \frac{D}{E} * (1 - T_c)\right)}$$

(8.3) Shannon Pratt

$$\beta_u = \frac{\beta_e}{\left(1 + \frac{D}{E} * (1 - T_c)\right)}$$

(8.4) Duff & Phelps, LLC RP report 2010

$$\beta_u = \beta_e * \frac{E}{TV} + \beta_d * \frac{D}{TV}$$

(8.5) Comments

Tim Ogier and Duff & Phelps use the Harris & Pringle assumption of a constant debt ratio or an arbitrary non-constant financing policy, with tax savings from debt that have the same risk as the free cash flows. In contrast, Shannon Pratt and Aswath Damodaran took an opposite stance in which they assume a M&M/Hamada position of a preset level of debt or an arbitrary non-constant financing policy, with tax savings from debt that have the same risk as the debt. Only Duff & Phelps calculates with a debt beta. All the others don't use a debt beta, in which they assume that the debt beta is zero, meaning that the interest rate offered by lenders equals the risk free rate. Shannon Pratt and Aswath Damodaran assume that the growth rate equals to zero, meaning that in their data analysis no growth of cash flows is to be expected.

Based on this analysis three central questions can be formulated:

- Why do different practitioners use different financing policy assumptions (consequently)?
- Why do some practitioners calculates with a zero debt beta?
- Why do some practitioners calculates with a zero growth rate?

I will briefly address these questions, knowing that more research is needed to be more precise in analyzing and addressing these topics.

Zero Debt Beta

The implicit assumption with a zero debt beta is that the debt interest rate is risk free and therefore is equal to the risk free rate. In general, there are two risk associated with debt:

- Interest rate risk;
- Default risk (credit risk).

To calculate the debt beta, the following formula is used:

$$\beta_d = \frac{\rho * \sigma_d}{\sigma_m} \qquad \textbf{(21)}$$

The debt beta is equal to zero if the correlation coefficient ($\rho_{d,m}$) is zero or the standard deviation of debt (σ_d) is zero.

Interest rate risk is associated with general changes in long-term interest rates. All long-term debt (even it is default free) will see its value move inversely with long-term rates (yields). Asset values also tend to move inversely with long-term interest rates, meaning that long-term debt tends to have a positive beta, even it is default free. Risk free debt (default free and no interest rate risk) is necessarily short-term, because it cannot have its value altered by changes in long-term interest rates. In this sense it seems that short term default free debt tends to have a beta near zero and therefore can also be regarded as risk free debt.

Credit risk is associated with the possibility of default. Credit risk (default) is defined as the failure of a borrower to meet a contractual obligation to a lender. Consequently, claims on a borrower who is likely to default are subject to credit risk. A credit rating is a judgement on a firm's ability to meet its obligations. A rating measures the probability that an issuer will default.

Moody's Investor Service and Standard & Poor's provide ratings of the default risk of corporate bonds. Moody's designates the highest-grade bonds by the letters Aaa (S&P AAA), followed by Aa (AA), A (A), Baa (BBB), etc. These ratings are measures of relative risk, rather than of absolute risk. The spread between corporate and government bonds is, in general, too large to be explained only by expected default rates and must therefore reflect an additional risk premium. To isolate the component of the spread that is attributable to default risk, the part of the spread that is not likely to reflect default risk can be inferred from the spreads between short maturity AAA corporate bonds and Treasuries. It is unlikely that a 1-year maturity AAA corporate bond will ever default (Almeida and Philippon, 2007).

The next table will show the spread data used in Almeida and Philippon's study.

The Spread Data

	Ratings					
Maturity	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**
1	0,51%	0,52%	1,09%	1,57%	3,32%	5,45%
2	0,52%	0,56%	1,16%	1,67%	3,32%	5,45%
3	0,54%	0,61%	1,23%	1,76%	3,32%	5,45%
4	0,55%	0,65%	1,30%	1,85%	3,32%	5,45%
5	0,56%	0,69%	1,38%	1,94%	3,32%	5,45%

Almeida and Philippon (2007)

The one-year AAA spread in this table is 0,51%, which seems to be an adjustment for taxes and liquidity (Chen et al, 2005). Therefore to calculate the default components for rating *i* and maturity *t*, the next equation can be used:

$$Default - component_i^t = spread_i^t - 0{,}51\% \qquad \textbf{(22)}$$

Based on this equation and table 2, the default spread is increasing rapidly beyond the AA rating corporate bonds.

Only AAA and AA listed firms with short term debt have in most cases a near zero debt beta and can be regarded as risk free debt. In all other cases a debt beta is involved. For non-listed firms the rating starts mostly with a single A. The majority of the non-listed firms have a triple B rating or a double B rating.

Zero Growth Rate

Shannon Pratt and Aswath Damodaran use the M&M/Hamada approach to un-lever the equity beta. In implicit assumption is that they assume that no free cash flow growth is involved. This implicit assumption is based on the comparison between equation (7) and (10). The difference between those two equations is that equation (10) deals with cash flow growth and equation (7) without a growth component. A possible explanation is that Damodaran calculates historic equity betas based on a 5 year period, which is relatively short. He assumes, probably, that in such a short period a zero growth rate is justified. I doubt that due to the fact that the minimum growth rate should be equal to the inflation rate. In the last decade we had only for a very short period an historical inflation rate of 0% (or even deflation), but in general an inflation rate which is higher than zero is common in the western world.

A greater problem is to determine the realized growth rate, which is difficult to extract in a consistent way from the dataset with a certain degree of confidence and stability. In absence of sufficient historical information about the growth rates, it is difficult to quantify what the level of growth rate for a firm, or a peer group of firms, should be. But, this is no reason to discard the growth rate from the historical asset beta analysis.

Financing policy assumptions

If the firm uses debt in a more flexible manner, such as in the dynamic trade off financing policy, the M&M/Hamada or the Myers approach for leveraging and un-leveraging equity betas, represented in equations (7), (10), (18) and (19) are not correct. If firms issues debt as the value of the un-levered assets rises and retires debt as the value of the un-levered assets fall (as in the trade-off theory), then equity betas will move in response to leverage changes than equation (18) suggest they should. In the case where the issuance and retirement of debt is more or less positively correlated with the value of the un-levered assets, the correct formula for leveraging and un-leveraging equity betas is the same as the Harris & Pringle equations (4 and 17). In contrast, in the case where the issuance and retirement of debt is not or less positively correlated, the correct formula for leveraging and un-leveraging equity betas is the same as the M&M or, if growth is involved, the Myers approach.

In practice, practitioners such as Damodaran but also those from Duff & Phelps are very consequent in the way they use the formula for leveraging and un-leveraging equity betas for all kind of firms (with and without a positive correlation). Meaning that Damodaran always use the same formula (equation (7) and (18)) in spite of the fact that a firm could be financed based on the trade-off principle.

In my opinion the practitioner should first be aware what the firm's financing policy assumption is before selecting the correct formula for leveraging and un-leveraging the equity beta. Besides that he or she has to judge whether growth and a debt beta is involved.

9. Using historic betas

An historic beta for a firm is generally used as a proxy for the firm's beta in the future. This is what in practice is called a pragmatic approach. It is important that the practitioner should be aware that the past value of betas for a business may not be a good estimate to its future value, and in most situations it is the future value that is needed. If there is any reason to believe that the nature of the business in the future will be different from that in the past, then the historic calculations may be unreliable. It is important that practitioners should understand the drivers of beta, only by understanding these can judgement be applied in interpreting historic data as a guide to the future. Simply accepting the historic data provided by data providers such as Damodaran, Bloomberg and Datastream at face value can, as I already mentioned in the previous section, lead to errors in forward looking valuation and investment valuation applications.

Historic beta estimation presents challenges. These include:

- The choice of period over which to measure betas;
- The frequency and number of observations used;
- Whether Bayesian adjustment techniques are beneficial or not;
- The choice of data provider;
- Whether there is value in comparator or sector analysis;
- The consideration of other models of estimating betas.

10. Some examples

Assumptions:

Equity Beta	**1,10**
Debt Beta	**0,20**
Risk free rate	**3,75%**
RPm	**6,0%**
Tax	**25,0%**
Debt	**40,0%**
Equity	**60,0%**

$$K_d = R_f + \beta_d (RP_m)$$

Kd =	**4,95%**

(10.1) Harris & Pringle

$$K_e = R_f + \beta_e (RP_m) = 3{,}75\% + 1{,}1 * 6\% = 10{,}35\%$$

$$\beta_u = \beta_e * \frac{E}{TV} + \beta_d * \frac{D}{TV} = 1{,}1 * 0{,}6 + 0{,}2 * 0{,}4 = 0{,}74$$ =equation (17)

$$K_u = R_f + \beta_u (RP_m) = 3{,}75\% + 0{,}74 * 6\% = 8{,}19\%$$

$$\frac{fcf_t}{TV} = wacc = K_u - \frac{D * K_d * T_c}{TV} = 8{,}19\% - \frac{0.4}{1} * 4{,}95\% * 25\% = 7{,}70\%$$ =equation (2)

$$K_e = K_u + (K_u - K_d) * \frac{D}{E} = 8{,}19\% + (8{,}19\% - 4{,}95\%) * \frac{4}{6} = 10{,}35\%$$ =equation (4)

(10.2) M&M / Hamada

$$K_e = R_f + \beta_e (RP_m) = 3{,}75\% + 1{,}1 * 6\% = 10{,}35\%$$

$$\beta_u = \frac{\beta_e}{\left(1 + (1 - T_c) * \frac{D}{E}\right)} + \beta_d * \frac{(1 - T_c) * \frac{D}{E}}{\left(1 + (1 - T_c) * \frac{D}{E}\right)} = \frac{1{,}1}{\left(1 + (1 - 25\%) * \frac{4}{6}\right)} + 0{,}2 * \frac{(1 - 25\%) * \frac{4}{6}}{\left(1 + (1 - 25\%) * \frac{4}{6}\right)} = 0{,}80$$

=equation (18)

$$K_u = R_f + \beta_u (RP_m) = 3{,}75\% + 0{,}8 * 6\% = 8{,}55\%$$

$$K_e = K_u + (K_u - K_d) * (1 - T_c) * \frac{D}{E} = 8{,}55\% + (8{,}55\% - 4{,}95\%) * (1 - 25\%) * \frac{4}{6} = 10{,}35\%$$

=equation (7)

(10.3) Myers (growth rate = 2%)

$$K_e = R_f + \beta_e (RP_m) = 3{,}75\% + 1{,}1 * 6\% = 10{,}35\%$$

$$\beta_u = \frac{\beta_e + \beta_d * \frac{D}{E} * \left(1 - \frac{Kd * T_c}{K_d - g}\right)}{\left(1 + \frac{D}{E} * \left(1 - \frac{K_d * T_c}{K_d - g}\right)\right)} = 0{,}8489$$ =equation (19)

$$K_u = R_f + \beta_u (RP_m) = 3{,}75\% + 0{,}8489 * 6\% = 8{,}8434\%$$

$$K_e = K_u + \left(K_u - K_d * \left(1 + T_c * \left(\frac{K_u - g}{K_d - g} - 1\right)\right)\right) * \frac{D}{E} = 10{,}35\%$$ =equation (10)

11. <u>Discussion</u>

……..

APPENDIX 1. MYERS (growth > 0%)

$$K_u = R_f + \beta_u * RP_m$$

$$K_d = R_f + \beta_d * RP_m$$

$$K_e = R_f + \beta_e * RP_m$$

$$\boxed{A = \beta_e * RP_m \qquad B = \beta_u * RP_m \qquad C = \beta_d * RP_m}$$

$$R_f + \beta_e * RP_m = \left(R_f + \beta_u * RP_m\right) + \left[\left(R_f + \beta_u * RP_m\right) - \left(R_f + \beta_d * RP_m\right) * \left(1 + T_c * \left(\frac{K_u - g}{K_d - g} - 1\right)\right)\right] * \frac{D}{E}$$

$$R_f + A = R_f + B + \left[R_f + B - K_d * \left(1 + T_c * \left(\frac{K_u - g}{K_d - g} - 1\right)\right)\right] * \frac{D}{E}$$

$$A = B + \left[R_f + B - K_d * \left(1 + T_c * \left(\frac{K_u - g}{K_d - g} - 1\right)\right)\right] * \frac{D}{E}$$

$$A = B + \left[R_f + B - K_d - T_c * \left(\frac{K_u - g}{K_d - g} - 1\right) * K_d\right] \frac{D}{E}$$

$$A = B + R_f * \frac{D}{E} + B * \frac{D}{E} - K_d * \frac{D}{E} - K_d * T_c * \frac{D}{E} * \left(\frac{K_u - g}{K_d - g} - 1\right)$$

$$A = B + R_f * \frac{D}{E} + B * \frac{D}{E} - R_f * \frac{D}{E} - C * \frac{D}{E} - K_d * T_c * \frac{D}{E} * \left(\frac{K_u - g}{K_d - g} - 1\right)$$

$$A = B + B * \frac{D}{E} - C * \frac{D}{E} - K_d * T_c * \frac{D}{E} * \left(\frac{K_u - g}{K_d - g} - 1\right)$$

$$B + B * \frac{D}{E} = A + C * \frac{D}{E} + K_d * T_c * \frac{D}{E} * \left(\frac{K_u - g}{K_d - g}\right) - K_d * T_c * \frac{D}{E}$$

$$B + B * \frac{D}{E} - K_d * T_c * \frac{D}{E} * \left(\frac{K_u - g}{K_d - g}\right) = A + \frac{D}{E} * \left(C - K_d * T_c\right)$$

$$B + B * \frac{D}{E} - \frac{K_d * T_c * \frac{D}{E} * K_u}{(K_d - g)} = A + \frac{D}{E} * (C - K_d * T_c) - \frac{K_d * T_c * \frac{D}{E} * g}{(K_d - g)}$$

$$B + B * \frac{D}{E} - \frac{R_f * T_c * \frac{D}{E} * K_d}{(K_d - g)} - B * \frac{K_d * T_c * \frac{D}{E}}{(K_d - g)} = A + \frac{D}{E} * (C - K_d * T_c) - \frac{K_d * T_c * \frac{D}{E} * g}{(K_d - g)}$$

$$B * (1 + \frac{D}{E} - \frac{D}{E} * \left(\frac{K_d * T_c}{K_d - g} \right) = A + \frac{D}{E} * (C - K_d * T_c) + \frac{K_d * T_c * \frac{D}{E}}{(K_d - g)} * (R_f - g)$$

$$B = \frac{A + \frac{D}{E} * (C - K_d * T_c) + \frac{K_d * T_c * \frac{D}{E}}{(K_d - g)} * (R_f - g)}{\left(1 + \frac{D}{E} * \left(1 - \frac{K_d * T_c}{K_d - g} \right) \right)}$$

Dividing both sides by ***RP***

$$\beta_u = \frac{\beta_e + \beta_d * \frac{D}{E} * \left(1 - \frac{Kd * T_c}{K_d - g} \right)}{\left(1 + \frac{D}{E} * \left(1 - \frac{K_d * T_c}{K_d - g} \right) \right)}$$