332.501 Final Project

Fall 2021

Due date is December 19. No late work will be accepted.

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1. (Verify the convolution relationship)

   Let us first generate a row vector $\mathtt{a} = [\mathtt{1\ 1\ 1\ 1\ 0\ 0\ 0\ 0}]$ and make another vector $\mathtt{b} = \mathtt{a}$. Then

   1.) Please plot the vector `a` in Matlab.

   2.) Use the definition of the convolution in the textbook to compute the output $\mathtt{c} = \mathtt{a} \star \mathtt{b}$ and plot the vector `c` in Matlab. That is, please derive the expression of the output $\mathtt{c} = \mathtt{a} \star \mathtt{b}$ in your project document and plot your derived results.

   3.) Use the Matlab command $\mathtt{d} = \mathtt{conv}(\mathtt{a}, \mathtt{b})$; to get the convolution output `d` and also plot `d` in Matlab.

   4.) Compare with `c` and `d` in Matlab plots and make your comments.

2. (Use convolution concept to estimate the relative time delay)

   - a)

     1.) Let vectors $\mathtt{a} = [\mathtt{1\ 1\ 1\ 1\ 0\ 0\ 0\ 0}]$ and $\mathtt{b} = [\mathtt{0\ 0\ 0\ 1\ 1\ 1\ 1\ 0}]$. Use `subplot` to plot them in one figure. What is the relationship between `a` and `b` with your observation?

     2.) Let vector $\mathtt{c} = \mathtt{conv}(\mathtt{a}, \mathtt{b})$ and plot it in Matlab.

     3.) Use Matlab command $[\mathtt{max\_val}, \mathtt{ind}] = \mathtt{max}(\mathtt{c})$ to find the maximum output. What does `ind` mean? (i.e., is there any relationship between `ind` and the relative delay of `a` and `b`?)

     4.) Change `b` to $\mathtt{b} = [\mathtt{0\ 0\ 1\ 1\ 1\ 1\ 0\ 0}\ ]$ and repeat the steps 2.) and 3.). What is the new `ind` value?

     5.) Change `b` to $\mathtt{b} = [\mathtt{1\ 1\ 0\ 0\ 0\ 0\ 1\ 1}\ ]$ and repeat the steps 2.) and 3.). What is the new `ind` value?

     6.) From the observation of 4.) and 5.), what is your conclusion of the meaning for `ind`?

- b)

  1.) Let vectors $\texttt{a} = [\texttt{1; 1; 1; 1; 0; 0; 0; 0;}]$ (Note this $\texttt{a}$ is a column vector but not a row vector) and $\texttt{b} = \texttt{circshift}(\texttt{a}, 3) + 0.1 * \texttt{randn}(8, 1)$. (Note herein it is the function `randn` but not `rand`) Use `subplot` to plot them in one figure. Here `b` is essentially a shifted version of `a` with some noise distortion.

  2.) Repeat steps 2.) and 3.) in the previous section a) multiple times (say, 10 times). What is your observation of the output value `ind`? Does `ind` change during your 10 trials?

  3.) Keep `a` the same and change $\texttt{b} = \texttt{circshift}(\texttt{a}, 3) + 10 * \texttt{randn}(8, 1)$.

  4.) Repeat steps 2.) and 3.) in the previous section a) multiple times (say, 20 times). What is your observation of the output value `ind`? Does `ind` change during your 20 trials? What are the values you get in your test?

  5.) Keep `a` the same and change $\texttt{b} = \texttt{circshift}(\texttt{a}, -2) + 10 * \texttt{randn}(8, 1)$.

  6.) Repeat steps 2.) and 3.) in the previous section a) multiple times (say, 20 times). What is your observation of the output value `ind`? Does `ind` change during your 20 trials? What are the values you get in your test?

  Remarks: *In 2.), 4.) and 6.), even with multiple trials, you only need to print out one plot in each step in your final project document but have to keep all the records of the outputs value* `ind` *from the command* `max(c)`.

3. (Time-delay impacts on frequency domain)

- a)

  1.) Let vectors $\texttt{a} = [\texttt{1 1 1 1 0 0 0 0}]$ and $\texttt{b} = [\texttt{0 0 1 2 3 0 0 0}]$. Plot them in Matlab.

  2.) Let vector $\texttt{c} = \texttt{conv}(\texttt{a}, \texttt{b})$ and plot it in Matlab.

  3.) Let discrete Fourier transforms $\texttt{af} = \texttt{fft}(\texttt{a})$ and $\texttt{bf} = \texttt{fft}(\texttt{b})$. Plot the absolute values of `af` and `bf`, i.e., `abs(af)` and `abs(bf)` in Matlab.

  4.) Let $\texttt{cc} = \texttt{ifft}(\texttt{af}.*\texttt{bf})$ and plot `cc`.

  5.) Compare with `cc` and `c` and make comments of your results.

- b)

  1.) Let vectors $\texttt{a} = [\texttt{1 1 1 1 0 0 0 0}]$ and $\texttt{af1} = \texttt{fft}(\texttt{a}, 8)$. Plot `abs(af1)` in Matlab.

2.) Let $\texttt{af2} = \texttt{fft}(\texttt{a}, 16)$. Plot $\texttt{abs}(\texttt{af2})$ in Matlab.

3.) Compare with `af1` and `af2`, what is your observation?

- c)

1.) Let vector $\texttt{a} = \texttt{randn}(8, 1)$ and $\texttt{af} = \texttt{fft}(\texttt{a})$.

2.) Let vector $\texttt{b} = \texttt{circshift}(\texttt{a}, 3)$ and $\texttt{bf} = \texttt{fft}(\texttt{b})$. Are `af` and `bf` the same?

3.) Let index sequence $\texttt{nn} = [0 : 7]'$ (column vector).

4.) Let $\texttt{delay} = \texttt{exp}(\texttt{j} * 2 * \texttt{pi} * \texttt{nn} * 3/8)$.

5.) Let $\texttt{afa} = \texttt{delay}. * \texttt{bf}$ and compare `afa` with `af`. What is difference between them?

6.) Change vector $\texttt{b} = \texttt{circshift}(\texttt{a}, -2)$ and $\texttt{bf} = \texttt{fft}(\texttt{b})$. Are `af` and `bf` the same?

7.) Let $\texttt{delay} = \texttt{exp}(\texttt{j} * 2 * \texttt{pi} * \texttt{nn} * (-2)/8)$.

8.) Let $\texttt{afa} = \texttt{delay}. * \texttt{bf}$ and compare `afa` with `af`. What is difference between them? Why? What is your conclusion now?

Remarks: *In 8.), if the differences between two vectors are in the order* 10^{-13} *or even smaller, i.e., smaller than* $\texttt{1.0e} - \texttt{14}$*, we believe they are essentially the same.*

4. From time domain to the frequency domain is just to change the view of the signal.

   For example, it is very hard for one to tell what signals containing in a time series from the time domain observations. But it is much easier to distinguish the different frequency components in the frequency domain.

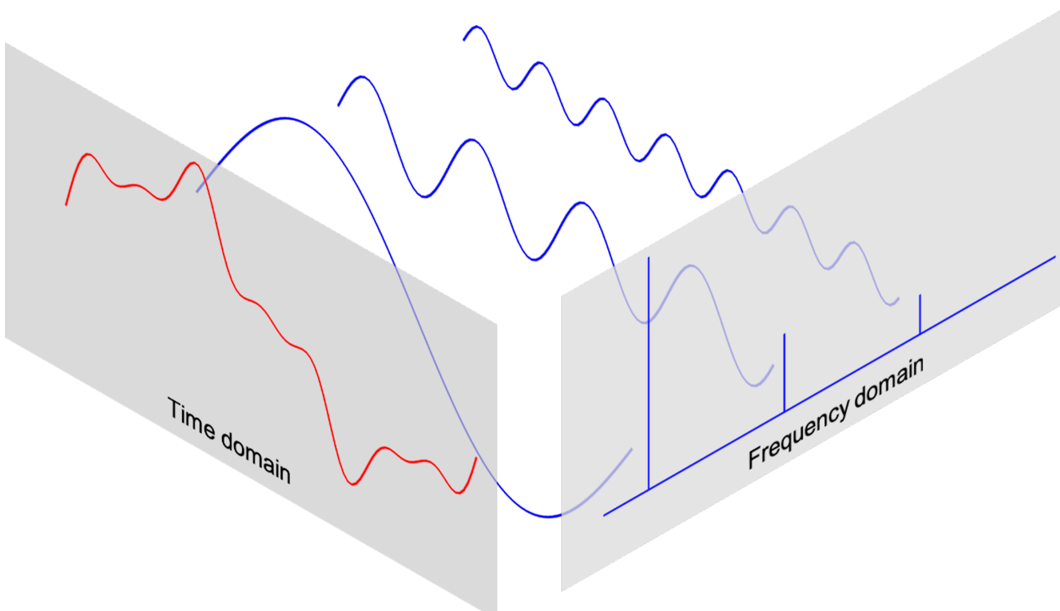
   Filtering a signal using FFT.

   Use the below reference URL to implement how you can manually filter a signal components.

   For example, let us assume the frequency 7 is the signal we like to keep and the rest are interference or noise.

   Use FFT and IFFT to filter out the single sinusoid with frequency 7 and eliminate the rest.

   `https://pythonnumericalmethods.berkeley.edu/notebooks/chapter24.04-FFT-in-Python.html`


Time domain
Frequency domain