Gate 2021 Assignment

EE:1205 Signals and Systems

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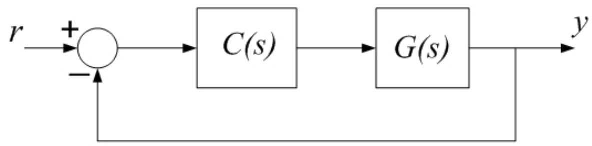
I. Question IN 02

Consider a unity feedback configuration with a plant and a PID controller as shown in the figure. $G(s) = \frac{1}{(s+1)(s+3)}$ and $C(s) = \frac{K(s+3+j)(s+3-j)}{s}$ with K being scalar . The closed loop is :



A only stable for $K < 0$
B stable for all value of K
C only stable for $K > 0$
D only stable for K between -1 and $+1$

II. Solution

TABLE 0
Input Parameters

Parameter	Used to denote	Values
n	Number of forward paths	1
Δ_k	The value of Δ which is not touching the k^{th} forward path	$\Delta = 1$
Δ	1 - sum of the loop gains	$1 - G(s)C(s)$
P	k^{th} forward path gain	$P = G(s)C(s)$

According to Mason's gain formula , transfer function can be given as :

$$\text{TF} = \frac{\sum_{k-1}^{n} P_k \Delta_k}{\Delta} = \frac{P\Delta_1}{\Delta} \tag{1}$$

$$= \frac{G(s)C(s)}{1 + G(s)C(s)} \tag{2}$$

Substituting values of G(s) and C(s) :

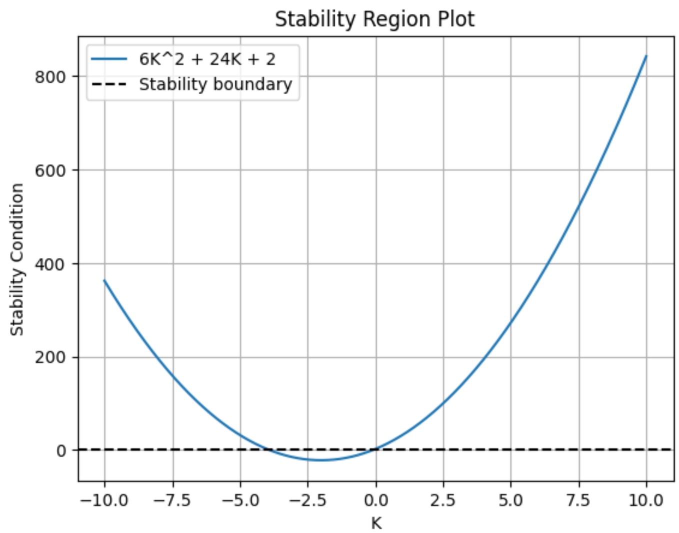
$$\text{TF} = \frac{k(s+3+j)(s+3-j)}{(s+1)(s+3) + k(s+3+j)(s+3-j)} \tag{3}$$

For the system to be stable , the real part of the pole should be negative.

$$s(s+1)(s+3)+K((s+3)^2-j^2)=0 \tag{4}$$
$$s^3+s^2(K+4)+s(3+6K)+10=0 \tag{5}$$

For stability we need :

$$(K+4)(3+6K)>10 \tag{6}$$
$$[\because as^3+bs^2+cs+d=0 \text{ for stability } bc>ad] \tag{7}$$
$$3K+6K^2+12+24K>10 \tag{8}$$
$$6K^2+24K+2>0 \tag{9}$$



If k =1 then above equation is valid, hence option A is wrong.
If k = −1 then above equation is invalid, hence option B is wrong.
If k = 2 then also above equation is valid, hence option D is wrong.
If $k>0$ then always above equation is valid, hence option C is correct.