

NCERT Discrete-10.5.3-7

EE22BTECH11004 - Allu Lohith

1. Find the sum of the first 22 terms of an AP in which $d = 7$ and the 22nd term is 149.

Ans: Let the series be

$$x(0), x(1), x(2), x(3), \dots, x(n)$$

Parameter	Description	Formulae/Value
$x(0)$	First term of A.P	-
d	Common difference	7
n	Count of terms starting from '0'	-
$x(n)$	$(n+1)^{th}$ term of the A.P	$x(0) + nd$
$x(21)$	Value of 22 nd term	149
$y(n)$	Sum of $(n+1)$ terms in A.P	$\left(\frac{n+1}{2}\right)(2x(0) + nd)$

TABLE 0
PARAMETERS

Now, the 22nd term means $a(21)$, so

$$x(21) = x(0) + nd \quad (1)$$

$$149 = x(0) + 21(7) \quad (2)$$

$$x(0) = 2 \quad (3)$$

The general term is $x(n) = 2 + 7d$ The z transform of the general term is

$$x(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (4)$$

$$= \frac{2}{1 - z^{-1}} + \frac{7z^{-1}}{(1 - z^{-1})^2} \quad (5)$$

$$= \frac{2 + 5z^{-1}}{(1 - z^{-1})^2} \quad (6)$$

On convolution for finding the sum

$$y(n) = x(n) * u(n) \quad (7)$$

$$\Rightarrow y(z) = x(z) \cdot u(z) \quad (8)$$

$$\Rightarrow y(z) = \left[\frac{2 + 5z^{-1}}{(1 - z^{-1})^2} \right] \cdot \frac{1}{1 - z^{-1}} \quad (9)$$

$$\Rightarrow y(z) = \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \quad (10)$$

Using Contour integration to find the inverse z-transform,

$$y(n) = \oint_c y(z) \cdot z^{n-1} dz \quad (11)$$

$$y(21) = \oint_c \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot z^{20} dz \quad (12)$$

We can observe there are three poles and thus $m = 3$,

$$R = \frac{1}{(n-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (13)$$

$$= \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \cdot \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot (z^{20}) \right) \quad (14)$$

$$= \frac{1}{2} (1012 + 2310) \quad (15)$$

$$\Rightarrow R = 1661 \quad (16)$$

Python code for finding the sum of terms of the AP:

```
x=2
n=21
d=7
y=((n+1)/2)*(2*x+n*d)
print("The sum of 22 terms is ",y)
```

Parameter	Description	Value
$x(0)$	First term of A.P	2
$y(21)$	Sum of 22 terms in A.P	1661

TABLE 0
RESULTS