

NCERT Discrete-10.5.3-7

EE22BTECH11004 - Allu Lohith

1. Find the sum of the first 22 terms of an AP in which $d = 7$ and the 22nd term is 149.

Parameter	Description	Formulae/Value
$X(0)$	First term of A.P	-
d	Common difference	7
n	Count of terms starting from '0'	-
$X(n)$	$(n+1)^{th}$ term of the A.P	$X(0) + nd$
$X(21)$	Value of 22 nd term	149

TABLE 0
PARAMETERS

Ans: Now, the 22nd term means $x(21)$, so

$$X(21) = X(0) + nd \quad (1)$$

$$149 = X(0) + 21(7) \quad (2)$$

$$X(0) = 2 \quad (3)$$

The general term is $X(n) = 2 + 7n$ The z transform of the general term is

$$X(z) = \frac{X(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (4)$$

$$= \frac{2}{1 - z^{-1}} + \frac{7z^{-1}}{(1 - z^{-1})^2} \quad (5)$$

$$= \frac{2 + 5z^{-1}}{(1 - z^{-1})^2} \quad (6)$$

On convolution for finding the sum

$$Y(n) = X(n) * U(n) \quad (7)$$

On z-transform,

$$Y(z) = X(z) \cdot U(z) \quad (8)$$

$$= \left(\frac{2 + 5z^{-1}}{(1 - z^{-1})^2} \right) \cdot \frac{1}{1 - z^{-1}} \quad (9)$$

$$\Rightarrow Y(z) = \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \quad (10)$$

Using Contour integration to find the inverse z-transform,

$$Y(n) = \oint_c y(z) \cdot z^{n-1} dz \quad (11)$$

$$Y(21) = \oint_c \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot z^{20} dz \quad (12)$$

We can observe there are three poles and thus $m = 3$,

$$R = \frac{1}{(n-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (13)$$

$$= \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \cdot \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot (z^{20}) \right) \quad (14)$$

$$= \frac{1}{2} (1012 + 2310) \quad (15)$$

$$\Rightarrow R = 1661 \quad (16)$$

Parameter	Description	Value
$X(0)$	First term of A.P	2
$Y(21)$	Sum of 22 terms in A.P	1661

TABLE 0
RESULTS

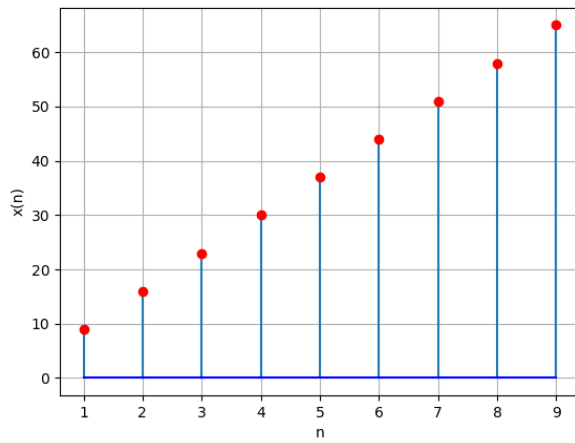


Fig. 0. Prgression