

NCERT Discrete-11.9.4-5

EE22BTECH11004 - Allu Lohith

Find the sum of n terms of this sequence:

$$5^2 + 6^2 + 7^2 \dots + 20^2$$

Solution:

Parameter	Description	Formulae/Value
n	Iteration number starting from zero till 15	-
$x(n)$	General term of the sequence from $n = 0$ to $n = 15$	$(n + 5)^2 u(n)$
$x(0)$	First term of the sequence	5

TABLE I
PARAMETERS

From appendix B.3.1

$$u(n) \xleftrightarrow{z} \frac{1}{1 - z^{-1}}, |z| > 1 \quad (1)$$

$$nu(n) \xleftrightarrow{z} \frac{z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \quad (2)$$

$$n^2 u(n) \xleftrightarrow{z} \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}, |z| > 1 \quad (3)$$

As

$$x(n) = (n^2 + 10n + 25)u(n) \quad (4)$$

On z transform of general term can be written as ,

$$X(z) = \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} + 10 \frac{z^{-1}}{(1 - z^{-1})^2} + \frac{25}{1 - z^{-1}} \quad (5)$$

$$X(z) = \frac{16z^{-2} - 39z^{-1} + 25}{(1 - z^{-1})^3}; |z| > 1 \quad (6)$$

On convolution for finding the sum

$$y(n) = x(n) * u(n) \quad (7)$$

On z transform,

$$Y(z) = X(z) \cdot U(z) \quad (8)$$

$$= \left(\frac{16z^{-2} - 39z^{-1} + 25}{(1 - z^{-1})^3} \right) \cdot \frac{1}{1 - z^{-1}} \quad (9)$$

$$\Rightarrow Y(z) = \frac{16z^{-2} - 39z^{-1} + 25}{(1 - z^{-1})^4}; |z| > 1 \quad (10)$$

Using the contour integration to find the inverse z transform,

$$Y(z) = \oint_c y(z) \cdot z^{n-1} dz \quad (11)$$

$$Y(21) = \oint_c \frac{16z^{-2} - 39z^{-1} + 25}{(1 - z^{-1})^4} z^{14} dz \quad (12)$$

As there are four poles from observation, so $m = 4$

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (13)$$

$$= \frac{1}{3!} \lim_{z \rightarrow 1} \frac{d^3}{dz^3} \left((z-1)^4 \frac{(16z^{-2} - 39z^{-1} + 25)}{(1 - z^{-1})^4} z^{14} \right) \quad (14)$$

$$= \frac{1}{6} \lim_{z \rightarrow 1} \frac{d^3}{dz^3} ((16z^{-2} - 39z^{-1} + 25) z^{18}) \quad (15)$$

$$= \frac{1}{6} \lim_{z \rightarrow 1} \frac{d^3}{dz^3} (16z^{16} - 39z^{17} + 25z^{18}) \quad (16)$$

$$= \frac{1}{6} (16 \times 18 \times 17 \times 16 + 14 \times 17 \times 16 \times 15) \quad (17)$$

$$\Rightarrow R = 2840 \quad (18)$$

Hence the sum of the terms of the sequence is 2840.

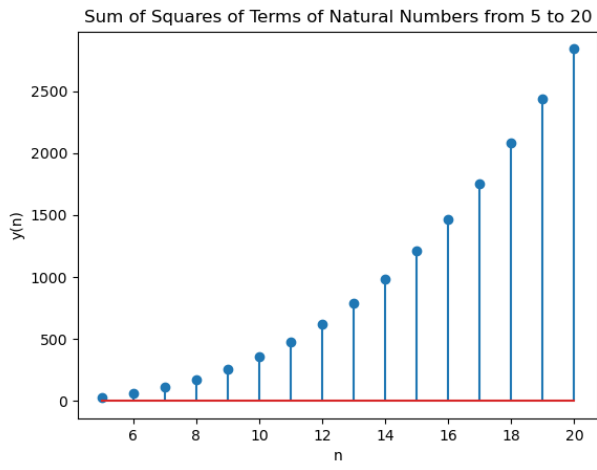


Fig. 1. Plot of $y(n) = (n + 1)^2$