

NCERT Discrete-10.5.3-7

EE22BTECH11004 - Allu Lohith

- 1) Find the sum of the first 22 terms of an AP in which $d = 7$ and the 22nd term is 149.

Solution:

Parameter	Description	Formulae/Value
$u(n)$	Unit step function	$\begin{cases} 0, & \text{if } n < 0, \\ 1, & \text{if } n \geq 0. \end{cases}$
$x(0)$	First term of A.P	-
d	Common difference	7
n	Count of terms starting from '0'	-
$x(n)$	$(n+1)^{th}$ term of the A.P	$(x(0) + nd)u(n)$
$x(21)$	Value of 22 nd term	149

TABLE 1
PARAMETERS

Now, the 22nd term means $x(21)$, so

$$x(21) = (x(0) + nd)u(1) \quad (1)$$

$$149 = (x(0) + 21(7))(1) \quad (2)$$

$$x(0) = 2 \quad (3)$$

The general term is $x(n) = (2 + 7n)u(n)$ The z transform of the general term is

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (4)$$

$$= \frac{2}{1 - z^{-1}} + \frac{7z^{-1}}{(1 - z^{-1})^2} \quad (5)$$

$$= \frac{2 + 5z^{-1}}{(1 - z^{-1})^2}; \quad |z| > 1 \quad (6)$$

$$(7)$$

On convolution for finding the sum

$$y(n) = x(n) * u(n) \quad (8)$$

On z -transform,

$$Y(z) = X(z) \cdot U(z) \quad (9)$$

$$= \left(\frac{2 + 5z^{-1}}{(1 - z^{-1})^2} \right) \cdot \frac{1}{1 - z^{-1}} \quad (10)$$

$$\Rightarrow Y(z) = \frac{2 + 5z^{-1}}{(1 - z^{-1})^3}; \quad |z| > 1 \quad (11)$$

Using Contour integration to find the inverse z -transform,

$$Y(z) = \oint_c y(z) \cdot z^{n-1} dz \quad (12)$$

$$Y(21) = \oint_c \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot z^{20} dz \quad (13)$$

We can observe there are three poles and thus $m = 3$,

$$R = \frac{1}{(n-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (14)$$

$$= \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \cdot \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot (z^{20}) \right) \quad (15)$$

$$= \frac{1}{2} (1012 + 2310) \quad (16)$$

$$\Rightarrow R = 1661 \quad (17)$$

