1

NCERT Discrete-10.5.3-7

EE22BTECH11004 - Allu Lohith

1) Find the sum of the first 22 terms of an AP in which d = 7 and the 22nd term is 149. **Solution:**

Parameter	Description	Formulae/Value
<i>u</i> (<i>n</i>)	Unit step function	$\begin{cases} 0, & \text{if } n < 0, \\ 1, & \text{if } n \ge 0. \end{cases}$
x(0)	First term of A.P	-
d	Commom difference	7
n	Count of terms starting from '0'	-
x(n)	$(n+1)^{th}$ term of the A.P	(x(0) + nd) u(n)
x (21)	Value of 22 nd term	149

TABLE 1 PARAMETERS

Now, the 22^{nd} term means x(21), so

$$x(21) = (x(0) + nd) u(1)$$
 (1)

$$149 = (x(0) + 21(7))(1) \tag{2}$$

$$x(0) = 2 \tag{3}$$

The general term is x(n) = (2 + 7n) u(n) The z transform of the general term is

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}$$
 (4)

$$= \frac{2}{1 - z^{-1}} + \frac{7z^{-1}}{(1 - z^{-1})^2}$$
 (5)

$$=\frac{2+5z^{-1}}{\left(1-z^{-1}\right)^{2}}; \quad |z|>1 \tag{6}$$

(7)

On convolution for finding the sum

$$y(n) = x(n) * u(n)$$
 (8)

On z-transform,

$$Y(z) = X(z) \cdot U(z) \tag{9}$$

$$= \left(\frac{2+5z^{-1}}{(1-z^{-1})^2}\right) \cdot \frac{1}{1-z^{-1}} \tag{10}$$

$$\implies Y(z) = \frac{2 + 5z^{-1}}{(1 - z^{-1})^3}; \qquad |z| > 1 \qquad (11)$$

Using Contour integration to find the inverse *z*-transform,

$$Y(z) = \oint_C y(z) \cdot z^{n-1} dz \tag{12}$$

$$Y(21) = \oint_c \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot z^{20} dz$$
 (13)

We can observe there are three poles and thus m = 3,

$$R = \frac{1}{(n-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z) \right)$$

(14)

$$= \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 \cdot \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot (z^{20}) \right)$$
(15)

$$=\frac{1}{2}(1012+2310)\tag{16}$$

$$\implies R = 1661 \tag{17}$$

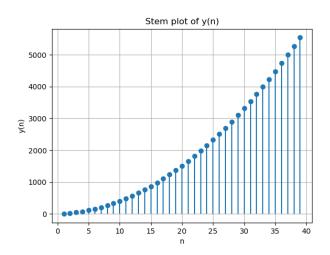


Fig. 1. Sum of terms