GATE 2023-BM.54

EE22BTECH11004 - Allu Lohith

A system is described by the following differential equation

$$(0.01)\frac{d^2y(t)}{dt^2} + (0.2)\frac{dy(t)}{dt} + y(t) = 6x(t)$$

where time t is in seconds. If x(t) is the unit step input applied at t = 0 s to this system, the magnitude of the output at t = 1s is _____. (Round off the answer to two decimal places.)

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Solution: Given,

Parameter	Description	Formulae/Value
x(t)	The unit step input applied at $t = 0$ s to this system	$\begin{cases} 0, & \text{if } t < 0, \\ 1, & \text{if } t \ge 0. \end{cases}$
y (t)	A function of $x(t)$	-
y(1)	Value of y at $t = 1$	-

TABLE I PARAMETERS

$$(0.01)\frac{d^2y(t)}{dt^2} + (0.2)\frac{dy(t)}{dt} + y(t) = 6x(t)$$
 (1)

property:

$$\mathcal{L}\left\{\frac{d^{n}y(t)}{dt^{n}}\right\} = s^{n}Y(s) - s^{n-1}y(0) - \dots - y^{(n-1)}(0)$$
(2)

Taking the Laplace transform of both sides (assuming zero initial conditions):

$$0.01s^{2}Y(s) + 0.2sY(s) + Y(s) = \frac{6}{s}$$
 (3)

$$\implies Y(s) = \frac{6}{s(0.01s^2 + 0.2s + 1)}$$
 (4)

$$\implies Y(s) = \frac{6}{0.01(s)(s+10)^2}$$
 (5)

Using partial fraction decomposition:

$$Y(s) = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{(s+10)^2}$$
 (6)

On solving, we get A = 6, B = -6, C = -60.

$$Y(s) = \frac{6}{s} - \frac{6}{s+10} - \frac{60}{(s+10)^2}$$
 (7)

From standard inverse laplace transforms:

$$\frac{1}{s+a} \longleftrightarrow e^{-at} \tag{8}$$

$$\frac{1}{(s+a)^2} \longleftrightarrow te^{-at} \tag{9}$$

Taking inverse Laplace transform of Y(s),

$$y(t) = u(t) \left(6 - 6e^{-10t} - 60te^{-10t} \right)$$
 (10)

At t = 1s

$$y(1) = u(1) \left(6 - 66e^{-10} \right) \tag{11}$$

$$y(1) = 6 - 66e^{-10} (12)$$

approximately,

$$\implies y(1) = 5.99 \tag{13}$$

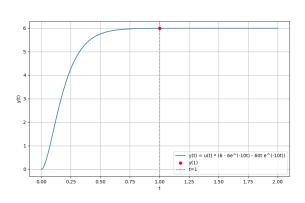


Fig. 1. Plot of $y(t) = u(t) \left(6 - 6e^{-10t} - 60te^{-10t}\right)$