GATE 2023-BM.54

EE22BTECH11004 - Allu Lohith

A system is described by the following differential equation

$$(0.01)\frac{d^2y(t)}{dt^2} + (0.2)\frac{dy(t)}{dt} + y(t) = 6x(t)$$

where time t is in seconds. If $x(t)$ is the unit step input applied at $t = 0$ s to this system, the magnitude of the output at $t = 1$s is __________. (Round off the answer to two decimal places.)

Solution: Given,

Parameter	Description	Formulae/Value
$x(t)$	The unit step input applied at $t = 0$ s to this system	$\begin{cases} 0, & \text{if } t < 0, \\ 1, & \text{if } t \geq 0. \end{cases}$
$y(t)$	A function of $x(t)$	-
$y(1)$	Value of y at $t = 1$	-

TABLE I
PARAMETERS

$$(0.01)\frac{d^2y(t)}{dt^2} + (0.2)\frac{dy(t)}{dt} + y(t) = 6x(t) \tag{1}$$

property:

$$\mathcal{L}\left\{\frac{d^ny(t)}{dt^n}\right\} = s^nY(s) - s^{n-1}y(0) - \cdots - y^{(n-1)}(0) \tag{2}$$

Taking the Laplace transform of both sides (assuming zero initial conditions):

$$0.01s^2Y(s) + 0.2sY(s) + Y(s) = \frac{6}{s} \tag{3}$$

$$\implies Y(s) = \frac{6}{s(0.01s^2 + 0.2s + 1)} \tag{4}$$

$$\implies Y(s) = \frac{6}{0.01(s)(s+10)^2} \tag{5}$$

Using partial fraction decomposition:

$$Y(s) = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{(s+10)^2} \tag{6}$$

On solving, we get $A = 6, B = -6, C = -60$. So,

$$Y(s) = \frac{6}{s} - \frac{6}{s+10} - \frac{60}{(s+10)^2} \tag{7}$$

From standard onverse laplace transforms:

$$\frac{1}{s+a} \longleftrightarrow e^{-at} \tag{8}$$

$$\frac{1}{(s+a)^2} \longleftrightarrow te^{-at} \tag{9}$$

Taking inverse Laplace transform of $Y(s)$,

$$y(t) = u(t)\left(6 - 6e^{-10t} - 60te^{-10t}\right) \tag{10}$$

At $t = 1s$

$$y(1) = u(1)\left(6 - 66e^{-10}\right) \tag{11}$$

$$y(1) = 6 - 66e^{-10} \tag{12}$$

approximately,

$$\implies y(1) = 5.99 \tag{13}$$

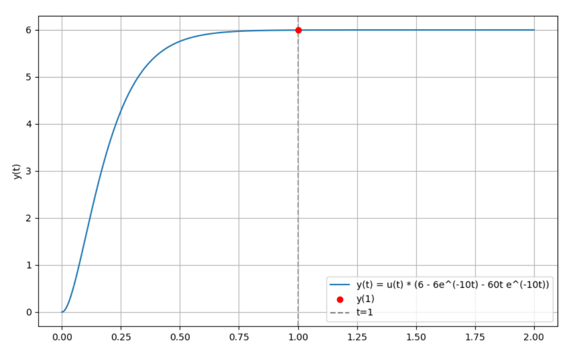

Fig. 1. Plot of $y(t) = u(t)\left(6 - 6e^{-10t} - 60te^{-10t}\right)$