NCERT Discrete-10.5.3-7

EE22BTECH11004 - Allu Lohith

1. Find the sum of the first 22 terms of an AP in which $d = 7$ and the 22nd term is 149.

Ans: Let the series be

$$a(0), a(1), a(2), a(3), \ldots, a(n)$$

Parameter	Description	Formulae/Value
$a(0)$	First term of A.P	-
d	Commom difference	-
n	Count of terms starting from '0'	-
$a(n)$	$(n+1)^{th}$ term of the A.P	$a(0) + nd$
$a(21)$	Value of 22^{nd} term	149
$S(n)$	Sum of (n+1) terms in A.P	$\left(\frac{n+1}{2}\right)(2a(0) + nd)$

TABLE 0
PARAMETERS

Now, the 22^{nd} term means $a(21)$, so

$$a(21) = a(0) + nd \tag{1}$$
$$149 = a(0) + 21(7) \tag{2}$$
$$a(0) = 149 - 147 \tag{3}$$
$$a(0) = 2 \tag{4}$$

As

$$S(n) = \left(\frac{n+1}{2}\right)(a(0) + nd) \tag{5}$$

So,

$$S(21) = \left(\frac{21+1}{2}\right)(2 \times 2 + 21 \times 7) \tag{6}$$
$$S(21) = 11 \times 151 \tag{7}$$
$$S(21) = 1661 \tag{8}$$

Python code for finding the sum of terms of the AP:

```
a=2
n=21
d=7
s=((n+1)/2)*(2*a+n*d)
print("The sum of 22 terms is ",s)
```

Parameter	Description	Value
$a(0)$	First term of A.P	2
$S(21)$	Sum of 22 terms in A.P	1661

TABLE 0
RESULTS

By differentiation property,

$$n(n) \overset{z}{\Longleftrightarrow} (-z)\frac{dX(z)}{dz} \tag{9}$$
$$\implies nu(n) \overset{z}{\Longleftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2}, |z| > 1 \tag{10}$$
$$\implies n^2u(n) \overset{z}{\Longleftrightarrow} \frac{z^{-1}\left(z^{-1}+1\right)}{\left(1-z^{-1}\right)^3}, |z| > 1 \tag{11}$$

For the sequence $x(n)$, the sum of the first $n+1$ terms can be expressed as

$$y(z) = \sum_{n=-\infty}^{n=\infty} a(n) \cdot z^{-n} \tag{12}$$
$$X(z) = \sum_{n=0}^{n=\infty} \left(\frac{n+1}{2}\right)(2(0) + nd) \cdot n \cdot z^{-n} \tag{13}$$

$$X(z) = 2x(0)U(z) - 2x(0)z\frac{dU(z)}{dz} + dz^2\frac{d^2U(z)}{dz^2} \tag{14}$$

$$\therefore X(z) = \frac{z^{-1}(d - x(0)) + x(0)}{2\left(\left(1-z^{-1}\right)^3\right)}, z \in \mathbb{C} : |z| > 1 \tag{15}$$

For the $(n+1)^{th}$ term in the progression,

$$y(n) = y(0) + nd \tag{16}$$

$$n \cdot u(n) \overset{z}{\Longleftrightarrow} \frac{z^{-1}}{\left(1 - z^{-1}\right)^{-2}} \tag{17}$$

$$Y(z) = \sum_{n=-\infty}^{n=\infty} y(n) \times z^{-n} \tag{18}$$

$$Y(z) = \sum_{n=-\infty}^{n=\infty} \left(y(0) + nd\right) u(n) \cdot z^{-n} \tag{19}$$

$$Y(z) = y(0)U(z) - zd\frac{dU(z)}{dz} \tag{20}$$

$$\therefore Y(z) = \frac{y(0) + \left(d - y(0)\right) z^{-1}}{\left(1 - z^{-1}\right)^{2}}, z \in \mathbb{C} : |z| > 1 \tag{21}$$