NCERT Discrete-10.5.3-7

EE22BTECH11004 - Allu Lohith

1) Find the sum of the first 22 terms of an AP in which $d = 7$ and the 22nd term is 149.
Solution:

Parameter	Description	Formulae/Value
$x(0)$	First term of A.P	-
d	Commom difference	7
n	Count of terms starting from '0'	-
$x(n)$	$(n+1)^{th}$ term of the A.P	$x(0) + nd$
$x(21)$	Value of 22nd term	149

TABLE 1
PARAMETERS

Now, the 22nd term means $x(21)$, so

$$x(21) = x(0) + nd \tag{1}$$
$$149 = x(0) + 21(7) \tag{2}$$
$$x(0) = 2 \tag{3}$$

The general term is $x(n) = 2 + 7n$ The z transform of the general term is

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{4}$$
$$= \frac{2}{1 - z^{-1}} + \frac{7z^{-1}}{(1 - z^{-1})^2} \tag{5}$$
$$= \frac{2 + 5z^{-1}}{(1 - z^{-1})^2} \tag{6}$$

Region of convergence: $\left(z^{-1}\right) \neq 1$
On convolution for finding the sum

$$y(n) = x(n) * u(n) \tag{7}$$

On z-transform,

$$Y(z) = X(z) \cdot U(z) \tag{8}$$
$$= \left(\frac{2 + 5z^{-1}}{(1 - z^{-1})^2}\right) \cdot \frac{1}{1 - z^{-1}} \tag{9}$$
$$\implies Y(z) = \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \tag{10}$$

Region of convergence: $\left(z^{-1}\right) \neq 1$
Using Contour integration to find the inverse z-transform,

$$Y(z) = \oint_c y(z) \cdot z^{n-1} dz \tag{11}$$
$$Y(21) = \oint_c \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot z^{20} dz \tag{12}$$

We can observe there are three poles and thus m = 3,

$$R = \frac{1}{(n-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \tag{13}$$
$$= \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \cdot \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot \left(z^{20}\right)\right) \tag{14}$$
$$= \frac{1}{2}(1012 + 2310) \tag{15}$$
$$\implies R = 1661 \tag{16}$$
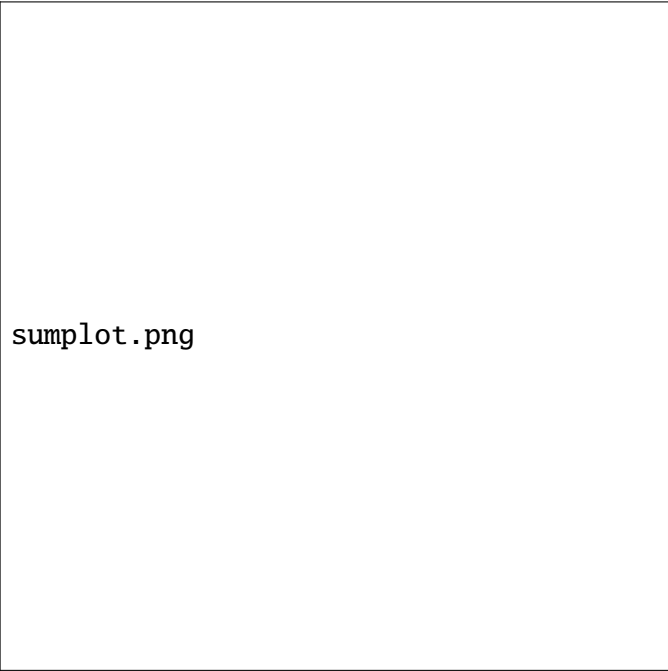
Parameter	Description	Value
$x(0)$	First term of A.P	2
$y(21)$	Sum of 22 terms in A.P	1661

TABLE 1
RESULTS

Fig. 1. Sequence

Fig. 1. Sum of terms