NCERT Discrete-10.5.3-7

EE22BTECH11004 - Allu Lohith

Find the sum of the first 22 terms of an AP in which $d = 7$ and the 22nd term is 149.

Solution:

Parameter	Description	Formulae/Value
$u(n)$	Unit step function	$\begin{cases} 0, & \text{if } n < 0, \\ 1, & \text{if } n \geq 0. \end{cases}$
$x(0)$	First term of A.P	-
d	Commom difference	7
n	Count of terms starting from '0'	-
$x(n)$	$(n+1)^{th}$ term of the A.P	$(x(0) + nd)\, u(n)$
$x(21)$	Value of 22^{nd} term	149

TABLE 0
PARAMETERS

Now, the 22^{nd} term means $x(21)$, so

$$x(21) = (x(0) + nd)\, u(21) \tag{1}$$
$$149 = (x(0) + 21(7))(1) \tag{2}$$
$$x(0) = 2 \tag{3}$$

The standard z transforms,

$$u(n) \overset{z}{\longleftrightarrow} \frac{1}{1 - z^{-1}}, |z| > 1 \tag{4}$$
$$nu(n) \overset{z}{\longleftrightarrow} \frac{z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \tag{5}$$

The general term is $x(n) = (2 + 7n)\, u(n)$, The z transform of the general term is

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{6}$$
$$= \frac{2}{1 - z^{-1}} + \frac{7z^{-1}}{(1 - z^{-1})^2} \tag{7}$$
$$= \frac{2 + 5z^{-1}}{(1 - z^{-1})^2}; \quad |z| > 1 \tag{8}$$

On convolution for finding the sum

$$y(n) = x(n) * u(n) \tag{9}$$

On z-transform,

$$Y(z) = X(z) \cdot U(z) \tag{10}$$
$$= \left(\frac{2 + 5z^{-1}}{(1 - z^{-1})^2}\right) \cdot \frac{1}{1 - z^{-1}} \tag{11}$$
$$\implies Y(z) = \frac{2 + 5z^{-1}}{(1 - z^{-1})^3}; \qquad |z| > 1 \tag{12}$$

Using Contour integration to find the inverse z-transform,

$$y(n) = \oint_c Y(z) \cdot z^{n-1} dz \tag{13}$$
$$y(21) = \oint_c \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot z^{20} dz \tag{14}$$

We can observe there are three poles and thus m = 3,

$$y(21) = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z - a)^m f(z)\right) \tag{15}$$
$$= \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 \cdot \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot \left(z^{20}\right)\right) \tag{16}$$
$$= \frac{1}{2}(1012 + 2310) \tag{17}$$
$$\implies y(21) = 1661 \tag{18}$$

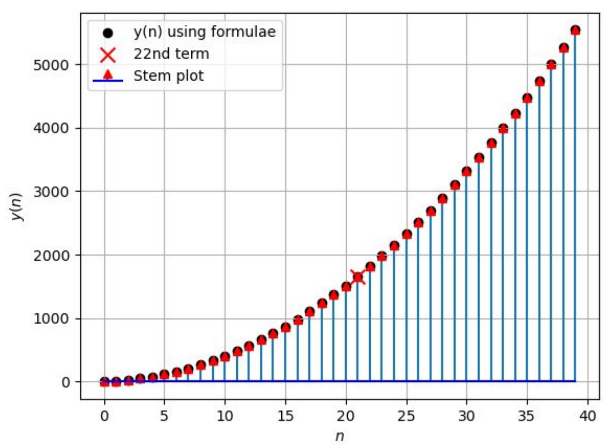

Fig. 0. Simulation v/s theoretical