NCERT-Analog-11.15-6

EE22BTECH11004 - Allu lohith

1) A bat emits ultrasonic sound of frequency $1000kHz$ in air. If the sound meets a water surface, what is the wavelength of
(t)he reflected sound
(b) the transmitted sound?
Speed of sound in air is $340ms^{-1}$ and in water is $1486ms^{-1}$.

Soln: As we know that the frequency of sound does not change with medium, So the frequency in water is equal to in air.
As,

$$wavelength\,(\lambda)\cdot frequency\,(\mathrm{f}) = speed\,(\mathrm{v}) \tag{1}$$

Parameter	Description	Value	Formulae
v	Frequency of sound	$1000KHz$	
v_a	Speed of sound in air	$340ms^{-1}$	
v_w	Speed of sound in water	$1486ms^{-1}$	
λ_a	Wavelength of sound in air		v_a/f
λ_w	Wavelength of sound in water		v_w/f

TABLE 1
Parameters

So,

$$\lambda_w = v_w/\mathrm{f} \tag{2}$$
$$\lambda_w = 1486/1000KHz \tag{3}$$
$$\lambda_w = 1.486mm \tag{4}$$

And similarly,

$$\lambda_a = v_a/\mathrm{f} \tag{5}$$
$$\lambda_a = 340/1000KHz \tag{6}$$
$$\lambda_a = 0.34mm \tag{7}$$

The general equation of a sound wave is

$$y\,(t) = A\sin(\omega t - kx) \tag{8}$$

Parameter	Description	Formula	value
λ_a	Wave length of the reflected sound	v_a/f	$0.34mm$
λ_w	Wave length of the reflected sound	v_w/f	$1.486mm$

TABLE 1
Results

where

$$\omega = 2\pi\mathrm{f} \tag{9}$$
$$A = \text{Amplitude} \tag{10}$$
$$k = \frac{\lambda}{2\pi} \tag{11}$$

So angular frequency $= 2\pi$f

$$\omega = 2\pi\mathrm{f} \tag{12}$$
$$\omega = 2\cdot 3.14\cdot 10^6 \tag{13}$$
$$\omega = 6.28\cdot 10^6 \tag{14}$$

When $y\,(t)$ incident sound wave hits the water, it undergoes transmission into the medium while concurrently experiencing reflection at the air-water interface,leading to a combination of transmitted and reflected waves.
The value of wave number in air (K_a)= $\frac{\lambda_a}{2\pi}$

$$K_a = \left(\frac{0.34\times 10^{-3}}{2\times 3.14}\right) \tag{15}$$
$$K_a = 54\times 10^{-6}\,m^{-1} \tag{16}$$

The value of wave number in water (K_w)= $\frac{\lambda_w}{2\pi}$

$$K_w = \left(\frac{1.486\times 10^{-3}}{2\times 3.14}\right) \tag{17}$$
$$K_w = 236\times 10^{-6}\,m^{-1} \tag{18}$$

Equation of sound wave in air is

$$y\,(t) = A\sin(6.28\times 10^6 t - 54\times 10^{-6}x) \tag{19}$$

Equation of sound wave in water is

$$y(t) = A\sin(6.28^{6}t - 236 \times 10^{-6}x) \qquad (20)$$