

GATE 2023-BM.54

EE22BTECH11004 - Allu Lohith

A system is described by the following differential equation

$$(0.01) \frac{d^2 y(t)}{dt^2} + (0.2) \frac{dy(t)}{dt} + y(t) = 6x(t)$$

where time t is in seconds. If $x(t)$ is the unit step input applied at $t = 0$ s to this system, the magnitude of the output at $t = 1$ s is _____. (Round off the answer to two decimal places.)

Solution: Given,

Parameter	Description	Formulae/Value
$x(t)$	The unit step input applied at $t = 0$ s to this system	$\begin{cases} 0, & \text{if } t < 0, \\ 1, & \text{if } t \geq 0. \end{cases}$
$y(t)$	A function of $x(t)$	-
$y(1)$	Value of y at $t = 1$	-

TABLE I
PARAMETERS

$$(0.01) \frac{d^2 y(t)}{dt^2} + (0.2) \frac{dy(t)}{dt} + y(t) = 6x(t) \quad (1)$$

property:

$$\mathcal{L} \left\{ \frac{d^n y(t)}{dt^n} \right\} = s^n Y(s) - s^{n-1} y(0) - \dots - y^{(n-1)}(0) \quad (2)$$

Taking the Laplace transform of both sides (assuming zero initial conditions):

$$0.01s^2 Y(s) + 0.2sY(s) + Y(s) = \mathcal{L}\{6x(t)\} \quad (3)$$

The Laplace transform of $x(t)$ is $X(s) = \frac{1}{s}$, so

$$0.01s^2 Y(s) + 0.2sY(s) + Y(s) = \frac{6}{s} \quad (4)$$

$$\Rightarrow Y(s) = \frac{6}{s(0.01s^2 + 0.2s + 1)} \quad (5)$$

$$\Rightarrow Y(s) = \frac{6}{0.01(s)(s+10)^2} \quad (6)$$

Using partial fraction decomposition:

$$Y(s) = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{(s+10)^2} \quad (7)$$

On solving, we get $A = 6, B = -6, C = 60$. So,

$$Y(s) = \frac{6}{s} - \frac{6}{s+10} + \frac{60}{(s+10)^2} \quad (8)$$

Taking inverse Laplace transform of $Y(s)$,

$$y(t) = u(t) (6 - 6e^{-10t} - 60te^{-10t}) \quad (9)$$

At $t = 1$ s

$$y(1) = u(1) (6 - 66e^{-10}) \quad (10)$$

$$y(1) = 6 - 66e^{-10} \quad (11)$$

approximately,

$$y(1) = 5.99 \quad (12)$$

$$\therefore y(1) = 5.99$$

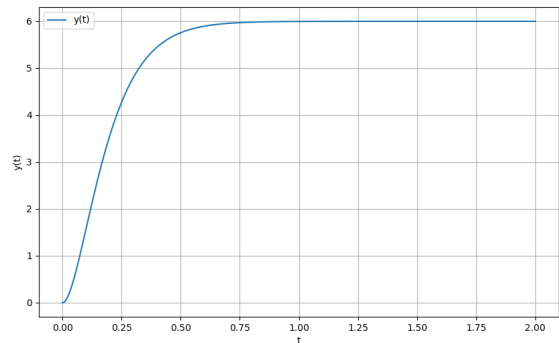


Fig. 1. Plot of $y(t) = u(t)(6 - 6e^{-10t} - 60te^{-10t})$