# NCERT Discrete-10.5.3-7

EE22BTECH11004 - Allu Lohith

1) Find the sum of the first 22 terms of an AP in which $d = 7$ and the 22nd term is 149.
**Solution:**

| Parameter | Description | Formulae/Value |
|---|---|---|
| $u(n)$ | Unit step function | $\begin{cases}0, & \text{if } n < 0,\\ 1, & \text{if } n \geq 0.\end{cases}$ |
| $x(0)$ | First term of A.P | - |
| $d$ | Commom difference | 7 |
| n | Count of terms starting from '0' | - |
| $x(n)$ | $(n+1)^{th}$ term of the A.P | $(x(0)+nd)\,u(n)$ |
| $x(21)$ | Value of $22^{nd}$ term | 149 |

TABLE 1
PARAMETERS

Now, the $22^{nd}$ term means $x(21)$, so

$$x(21) = (x(0) + nd)\,u(1) \tag{1}$$
$$149 = (x(0) + 21(7))\,(1) \tag{2}$$
$$x(0) = 2 \tag{3}$$

The general term is $x(n) = (2 + 7n)\,u(n)$ The $z$ transform of the general term is

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{4}$$
$$= \frac{2}{1 - z^{-1}} + \frac{7z^{-1}}{(1 - z^{-1})^2} \tag{5}$$
$$= \frac{2 + 5z^{-1}}{(1 - z^{-1})^2}; \quad |z| > 1 \tag{6}$$
$$\tag{7}$$

On convolution for finding the sum

$$y(n) = x(n) * u(n) \tag{8}$$

On z-transform,

$$Y(z) = X(z) \cdot U(z) \tag{9}$$
$$= \left(\frac{2 + 5z^{-1}}{(1 - z^{-1})^2}\right) \cdot \frac{1}{1 - z^{-1}} \tag{10}$$
$$\implies Y(z) = \frac{2 + 5z^{-1}}{(1 - z^{-1})^3}; \qquad |z| > 1 \tag{11}$$

Using Contour integration to find the inverse z-transform,

$$Y(z) = \oint_c y(z) \cdot z^{n-1} dz \tag{12}$$
$$Y(21) = \oint_c \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot z^{20} dz \tag{13}$$

We can observe there are three poles and thus m = 3,

$$R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z - a)^m f(z)\right) \tag{14}$$
$$= \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \cdot \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot \left(z^{20}\right)\right) \tag{15}$$
$$= \frac{1}{2}(1012 + 2310) \tag{16}$$
$$\implies R = 1661 \tag{17}$$

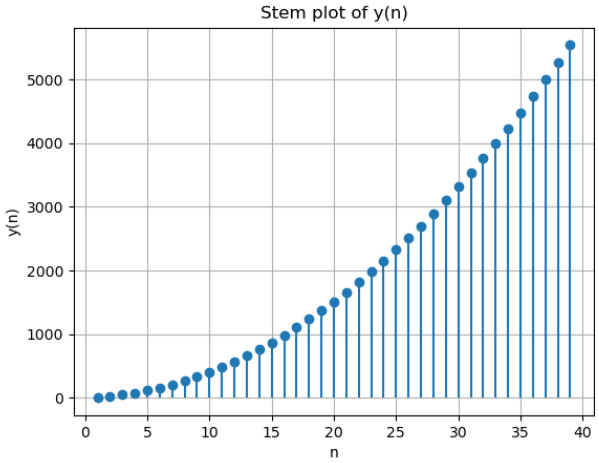


Fig. 1. Sum of terms