NCERT Discrete-10.5.3-7

EE22BTECH11004 - Allu Lohith

1) Find the sum of the first 22 terms of an AP in which $d = 7$ and the 22nd term is 149.
Solution:

Parameter	Description	Value	Formulae
f	Frequency of sound	$1000KHz$	
v_a	Speed of sound in air	$340ms^{-1}$	
v_w	Speed of sound in water	$1486ms^{-1}$	
λ_a	Wavelength of sound wave in air	-	v_a/f
λ_w	Wavelength of sound wave in water	-	v_w/f
K_a	Wavenumber of sound wave in air	-	$\lambda_a/2\pi$
K_w	Wavenumber of sound wave in water	-	$\lambda_w/2\pi$

TABLE 1
PARAMETERS

Now, the 22^{nd} term means $x(21)$, so

$$x(21) = x(0) + nd \tag{1}$$
$$149 = x(0) + 21(7) \tag{2}$$
$$x(0) = 2 \tag{3}$$

The general term is $x(n) = 2 + 7n$ The z transform of the general term is

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{4}$$
$$= \frac{2}{1 - z^{-1}} + \frac{7z^{-1}}{(1 - z^{-1})^2} \tag{5}$$
$$= \frac{2 + 5z^{-1}}{(1 - z^{-1})^2}; \quad \left(z^{-1}\right) \neq 1 \tag{6}$$
$$\tag{7}$$

On convolution for finding the sum

$$y(n) = x(n) * u(n) \tag{8}$$

On z-transform,

$$Y(z) = X(z) \cdot U(z) \tag{9}$$
$$= \left(\frac{2 + 5z^{-1}}{(1 - z^{-1})^2}\right) \cdot \frac{1}{1 - z^{-1}} \tag{10}$$
$$\implies Y(z) = \frac{2 + 5z^{-1}}{(1 - z^{-1})^3}; \quad \left(z^{-1}\right) \neq 1 \tag{11}$$
$$\tag{12}$$

Using Contour integration to find the inverse z-transform,

$$Y(z) = \oint_c y(z) \cdot z^{n-1} dz \tag{13}$$
$$Y(21) = \oint_c \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot z^{20} dz \tag{14}$$

We can observe there are three poles and thus m = 3,

$$R = \frac{1}{(n-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z - a)^m f(z)\right) \tag{15}$$
$$= \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 \cdot \frac{2 + 5z^{-1}}{(1 - z^{-1})^3} \cdot \left(z^{20}\right)\right) \tag{16}$$
$$= \frac{1}{2} (1012 + 2310) \tag{17}$$
$$\implies R = 1661 \tag{18}$$

Parameter	Description	Formula	value
λ_a	Wave length of the reflected sound	v_a/f	$0.34mm$
λ_w	Wave length of the reflected sound	v_w/f	$1.486mm$
K_w	Wavenumber of sound wave in air	$\lambda_a/2\pi$	$54 \times 10^{-6} m^{-1}$
K_a	Wavenumber of sound wave in water	$\lambda_w/2\pi$	$236 \times 10^{-6} m^{-1}$

TABLE 1
RESULTS

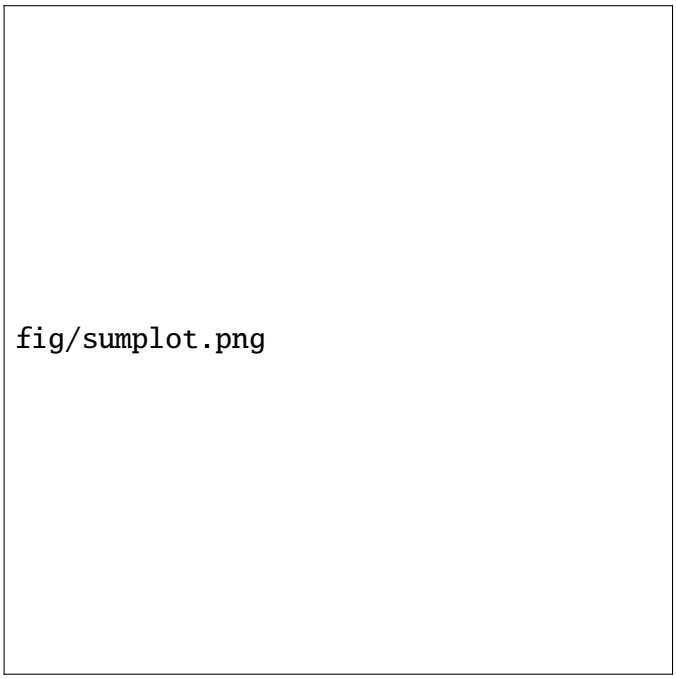

Fig. 1. Sum of terms