GATE 2023-BM.54

EE22BTECH11004 - Allu Lohith

A system is described by the following differential equation

$$(0.01)\frac{d^2y(t)}{dt^2} + (0.2)\frac{dy(t)}{dt} + y(t) = 6x(t)$$

where time t is in seconds. If $x(t)$ is the unit step input applied at $t = 0$ s to this system, the magnitude of the output at $t = 1$s is __________. (Round off the answer to two decimal places.)

Solution: Given,

$$(0.01)\frac{d^2y(t)}{dt^2} + (0.2)\frac{dy(t)}{dt} + y(t) = 6x(t) \tag{1}$$

property:

$$\mathcal{L}\left\{\frac{d^n y(t)}{dt^n}\right\} = s^n Y(s) - s^{n-1}y(0) - \cdots - y^{(n-1)}(0) \tag{2}$$

Taking the Laplace transform of both sides (assuming zero initial conditions):

$$0.01s^2Y(s) + 0.2sY(s) + Y(s) = \mathcal{L}\{6x(t)\} \tag{3}$$

The Laplace transform of $x(t)$ is $X(s) = \frac{1}{s}$, so

$$0.01s^2Y(s) + 0.2sY(s) + Y(s) = \frac{6}{s} \tag{4}$$

$$\implies Y(s) = \frac{6}{s(0.01s^2 + 0.2s + 1)} \tag{5}$$

$$\implies Y(s) = \frac{6}{0.01(s)(s+10)^2} \tag{6}$$

Using partial fraction decomposition:

$$Y(s) = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{(s+10)^2} \tag{7}$$

On solving, we get $A = 6, B = -6, C = -6$.
So,

$$Y(s) = \frac{6}{s} - \frac{6}{s+10} - \frac{6}{(s+10)^2} \tag{8}$$

Taking inverse Laplace transform of $Y(s)$,

$$y(t) = u(t)\left(6 - 6e^{-10t} - 60e^{-10t}\right) \tag{9}$$

$$\implies y(t) = u(t)\left(6 - 66e^{-10t}\right) \tag{10}$$

At $t = 1s$

$$y(1) = u(1)\left(6 - 66e^{-10}\right) \tag{11}$$

$$y(1) = 6 - 66e^{-10} \tag{12}$$

$$\tag{13}$$

approximately,

$$y(1) = 5.99 \tag{14}$$

$$\therefore y(1) = 5.99$$