Problems

Q.1) If $x(n) = \begin{cases} 1+\frac{n}{3} & -3 \le n \le -1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$ then ⓐ determine its values & sketch the signal $x(n)$

ⓑ Sketch the signals –

i) first fold $x(n)$ and then delay the resulting signal by 4 samples.

ii) first delay $x(n)$ by four samples and then fold the resulting signal.

ⓒ Sketch the signal $x(-n+4)$

ⓓ Compare the results in part (b) & (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$

ⓔ Express $x(n)$ in terms of $\delta(n)$ & $u(n)$.

Sol. ⓐ

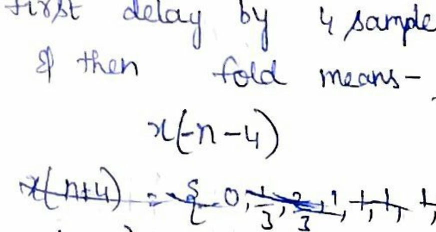
$n=-3 \rightarrow x(n) = 1+\frac{-3}{3} = 0$

$n=-2 \rightarrow x(n) = 1+\frac{-2}{3} = \frac{1}{3}$

$n=-1 \rightarrow x(n) = 1+\frac{-1}{3} = \frac{2}{3}$

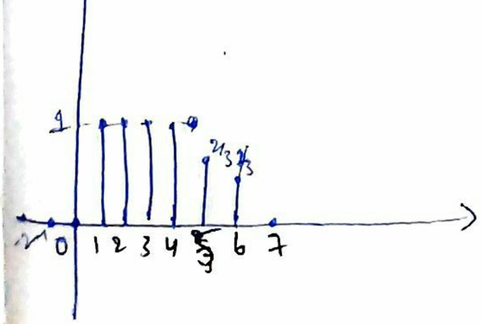
thus we've –

$x(n) = \{\ldots, 0, \frac{1}{3}, \frac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\}$



ⓑ i) fold $x(n)$ & delay by 4 samples means → $x(-n+4)$

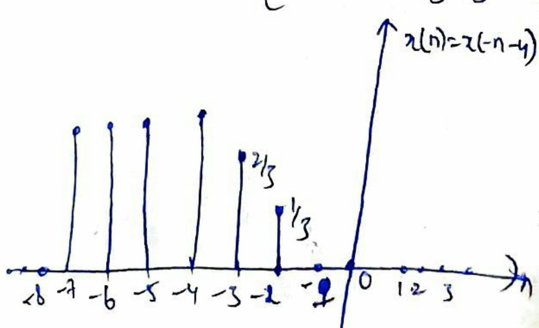
$x(n) = \{\ldots 0, \underset{\uparrow}{0}, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0 \ldots\}$



ii) first delay by 4 samples & then fold means – $x(-n-4)$

~~$x(n+4) = \{0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 1, 0, 0\}$~~

$\rightarrow x(-n-4) = \{\ldots 0, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \underset{\uparrow}{0}\}$



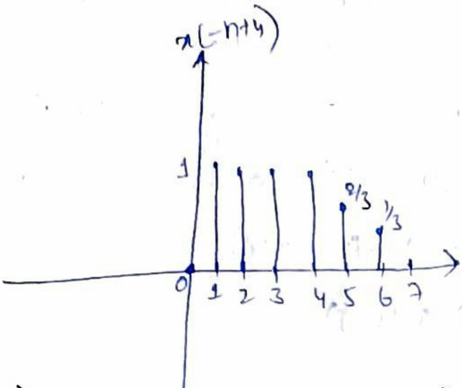
ⓒ $x(-n+4)$:-

i) $x(-n)$ & then $x(-n+4)$

(or)

ii) $x(n-4)$ & then $x(-(n+4))$
$= x(-n+4)$

$x(-n+4) = \{..0, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0...\}$ (↑ at 0)



ⓓ To get $x(-n+k)$ follow these steps -

1st, fold $x(n)$ to get $x(-n)$

2nd, shift $x(-n)$ by k samples to the right if $k>0$

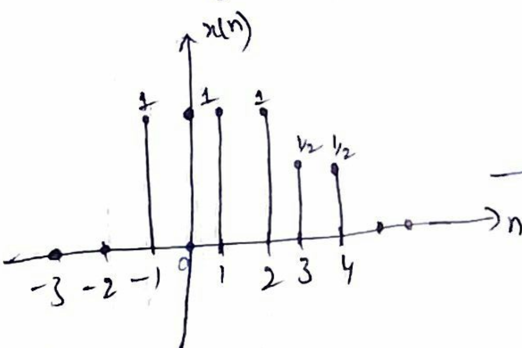
(or)

to the left if $k<0$.

ⓔ Express $x(n)$ in terms of $\delta(n)$ & $u(n)$

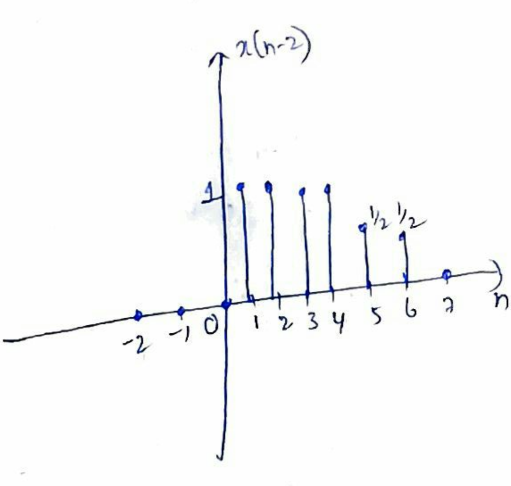
$$x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n+1) + u(n) - u(n-4)$$

2.2) For the following $x(n)$ draw - a) $x(n-2)$ b) $x(4-n)$ c) $x(n+2)$ d) $x(n)u(2-n)$ e) $x(n-1)\delta(n-3)$ f) even part of $x(n)$ g) odd part of $x(n)$ h) $x(n^2)$.



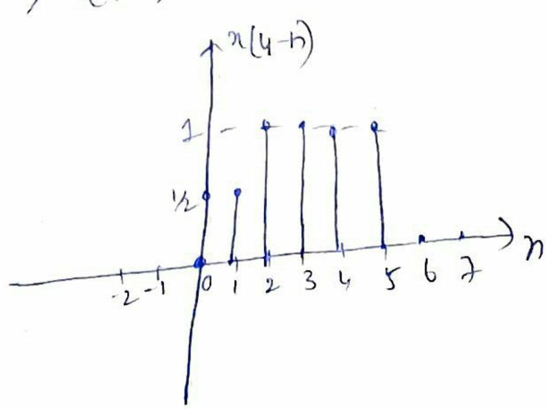
→ $x(n) = \{..0, 0, 0, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 0...\}$ (↑ at 0)

Sol. a) $x(n-2)$



$x(n-2) = \{...0, 0, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 0..\}$ (↑ at 0)

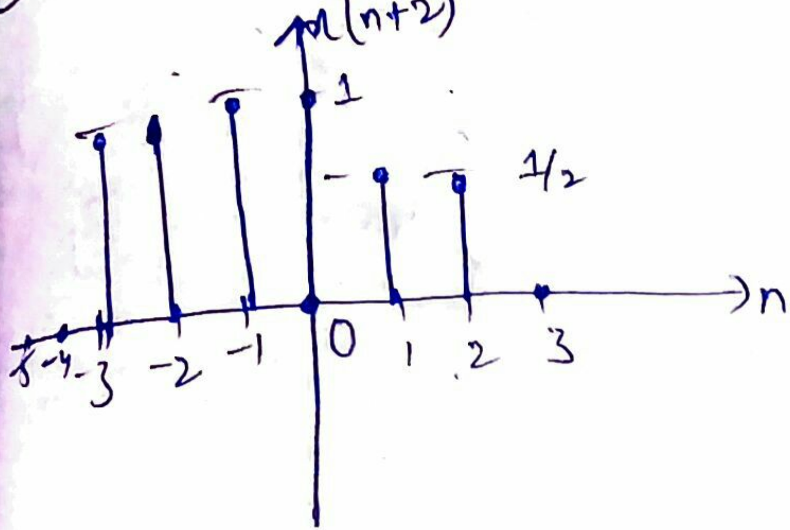
b) $x(4-n)$



$x(-n+4) = x(4-n)$

$= \{0, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, 1, 0, 0..\}$ (↑ at 0)

(c) $x(n+2)$

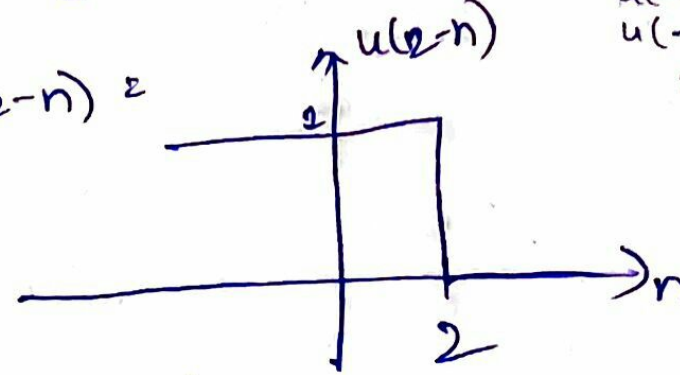


$$x(n+2) = \{\ldots,0,1,1,1,\underset{\uparrow}{1},\tfrac{1}{2},\tfrac{1}{2},0\ldots\}$$

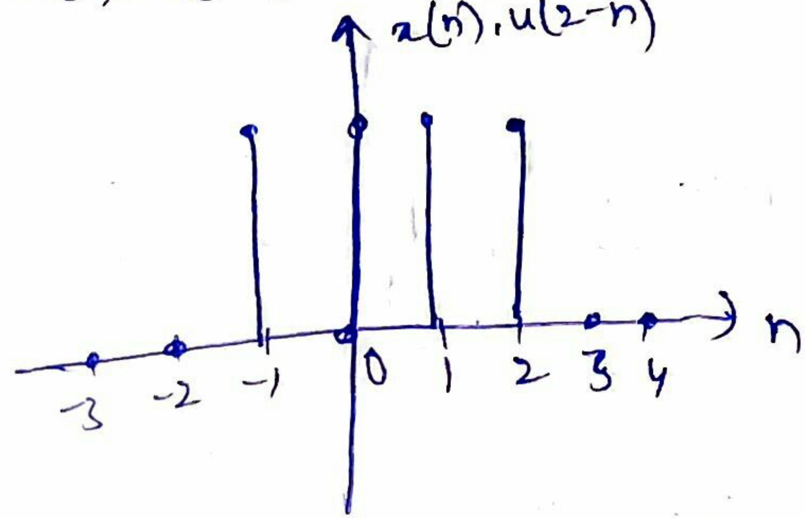
(d) $x(n)\,u(2-n)$

$u(n)$
$u(-n)$
$u(-n+2)$
$2-n \geq 0$
$-n \geq -2$
$n \leq 2$

$u(2-n) =$



$x(n) \cdot u(2-n)$



$$x(n)\cdot u(2-n) = \{\ldots 0,1,\underset{\uparrow}{1},1,1,0,0,\ldots\}$$

(e) $x(n-1)\,\delta(n-3)$

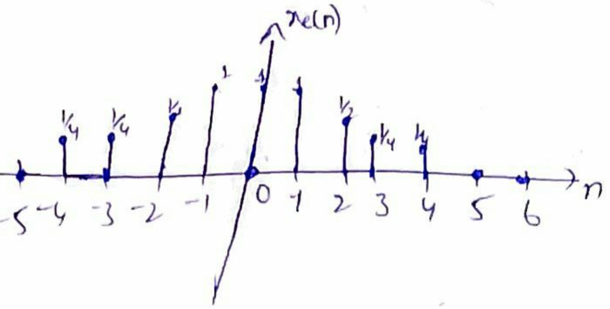
(f) $x_e(n) = \dfrac{x(n)+x(-n)}{2}$

where

$x(n) = \{..0, 1, \underset{\uparrow}{1}, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0..\}$

$x(-n) = \{0, 0, \tfrac{1}{2}, \tfrac{1}{2}, 1, 1, \underset{\uparrow}{1}, 1, 0...\}$

$x_e(n) = \{0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2}, 1, \underset{\uparrow}{1}, 1, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}, 0, 0..\}$

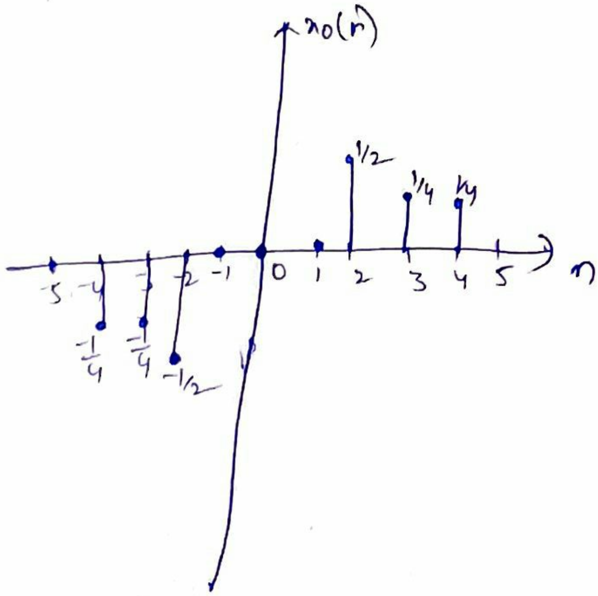


(g) $x_o(n) = \dfrac{x(n)-x(-n)}{2}$

$x(n) = \{..0, 1, \underset{\uparrow}{1}, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0..\}$

$x(-n) = \{0, \tfrac{1}{2}, \tfrac{1}{2}, 1, 1, \underset{\uparrow}{1}, 1, 0, ... \}$

$x_o(n) = \{0, -\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{2}, 0, \underset{\uparrow}{0}, 0, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}, 0..\}$



(h) $x(n^2)$

$x(n) = \{..0, 1, \underset{\uparrow}{1}, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0..\}$

$x(n^2) = y(n)$

$y(0) = x(0)$ $\quad y(3) = x(9)$

$y(1) = x(1)$ $\quad y(-1) = x(1)$

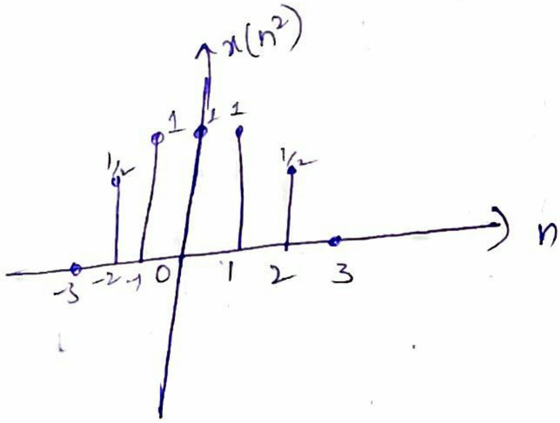
$y(2) = x(4)$ $\quad y(-2) = x(4)$

$\quad y(-3) = x(9)$

thus

$x(n^2) = \{..x(9), x(4), x(1), x(0), x(1), x(4), x(9)..\}$

$= \{..0, \tfrac{1}{2}, 1, \underset{\uparrow}{1}, 1, \tfrac{1}{2}, 0, ..\}$



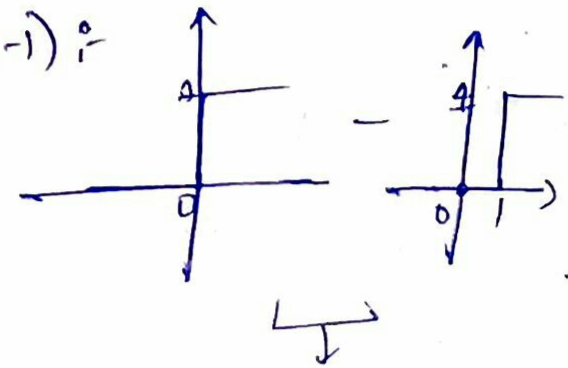
③ Show that a) $\delta(n) = u(n) - u(n-1)$ b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

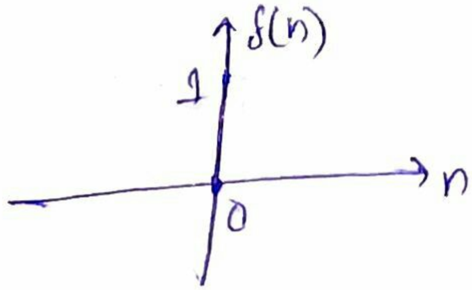
Sol.

ⓐ $u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

$u(n-1) \begin{cases} 1, & n \geq 1 \\ 0, & n < 0 \end{cases}$

$u(n) - u(n-1)$:-



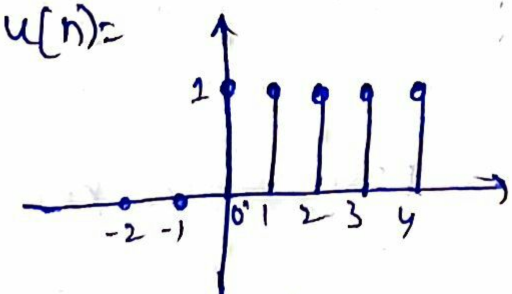


thus,

$u(n) - u(n-1) = \delta(n) = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ 0, & n > 0 \end{cases}$

b) $u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$

$u(n) =$



↓

In terms of $\delta(n)$

$\ldots 0 \cdot \delta(0+2) + 0 \cdot \delta(0+1) + \delta(0) + \delta(0-1) + \delta(0-2) + \ldots$

$u(n) = \sum_{k=-\infty}^{n} \delta(k) =$

$u(n) = \sum_{k=0}^{\infty} \delta(n-k)$

thus,

$u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$

~~2.4~~ ~~Ans~~

2.4 Show that the signal can be decomposed into even and odd component. Is the decomposition unique? Illustrate your arguments using the signal → $x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

Sol. Any signal can be decomposed into Even & odd component.

i.e $x(n) = x_e(n) + x_o(n)$

proof: We know that,

$$x_e(n) = \frac{1}{2}(x(n) + x(-n)) \quad \text{— ①}$$

$$x_o(n) = \frac{1}{2}(x(n) - x(-n)) \quad \text{— ②}$$

Adding ① & ②, we've –

$$x_e(n) + x_o(n) = \frac{1}{2}(x(n)) + \frac{1}{2}(\cancel{x(-n)}) + \frac{1}{2}x(n) - \frac{1}{2}(\cancel{x(-n)})$$

$$\Rightarrow x_e(n) + x_o(n) = \frac{2}{2}x(n)$$

$$\Rightarrow \underline{x_e(n) + x_o(n) = x(n)} \rightarrow \text{thus proved.}$$

Note) here $x_e(n) = x(-n)$

$x_o(n) = -x_o(-n)$.

Decomposition is unique.

Ex) $x(n) = \{2, 3, 4, 5, 6\} \rightarrow x_e(n) = ?$

Sol $\frac{x(n) + x(-n)}{2} = x_e(n) = \{4, 4, \underset{\uparrow}{4}, 4, 4\}$

$\rightarrow x_o(n) = \frac{x(+n) - x(-n)}{2} = x_o(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

thus $x(n)$ is decomposed into $x_e(n)$ & $x_o(n)$.

2.5 Show that Energy(power) of a real-valued energy(power) signal is equal to the sum of the energies (powers) of it's even & odd components.

Sol. → Let Energy = E

→ we know that,

$$\text{Energy of signal } x(n) = \sum_{n=-\infty}^{\infty} x^2(n).$$

&

$\sum_{n=-\infty}^{\infty} x_e(n)x_o(n)$ = Sum of product all even & odd components of a signal is equal to zero thus, $\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} |x_e(n) + x_o(n)|^2$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + \sum_{n=-\infty}^{\infty} 2x_e(n)x_o(n)$$

$$\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + 0.$$

$$E = E_e + E_O$$

thus $\boxed{E = E_e + E_O}$ → saying that energy of $x(n)$ is equal to sum of Energy of even part of $x(n)$ & odd part of $x(n)$.

2.6 Consider the system $y(n) = T[x(n)] = x(n^2)$

a) Determine if the system is time invariant.

b) prove ⓐ part by considering signal $x(n) = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$ is applied to system.

Also find - draw
① $x(n)$ signal
② $y(n) = T[x(n)]$ signal.
③ $y_2(n) = y(n-2)$
④ $x_2(n) = x(n-2)$
⑤ $y_2(n) = T(x_2(n))$
⑥ ~~$y_2(n)$~~ Conclude about $y_2(n)$ & $y(n-2)$

c) Repeat part ⓑ for $y(n) = x(n) - x(n-1)$ & conclude over time invariance of System.

d) Repeat part ⓒ & ⓑ for $y(n) = T[x(n)] = nx(n)$

Sol. ⓐ

$$y(n) = x(n^2)$$

TIME invariance checking!

$$x(n) \rightarrow y(n) = x(n^2)$$

$$x(n-k) \rightarrow y_1(n) = x[(n-k)^2] = y(n-k)$$

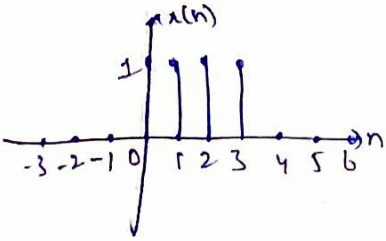
$$= x(n^2 + k^2 - 2nk) \neq y(n-k)$$

thus $x(n-k) \neq y(n-k)$

thus $x(n^2) = y(n)$ is TIME VARIANT system.

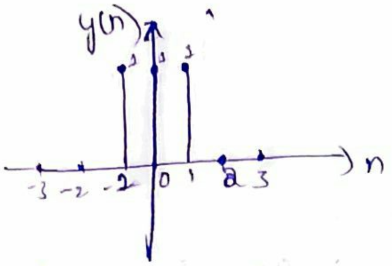
(b)

1) $x(n) = \{\ldots 0, 1, 1, 1, 1, 0 \ldots\}$

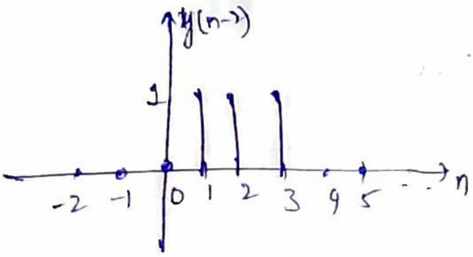


2) $y(n) = \{x(9), x(4), x(1), x(0), x(1), x(4), x(9) \ldots\}$

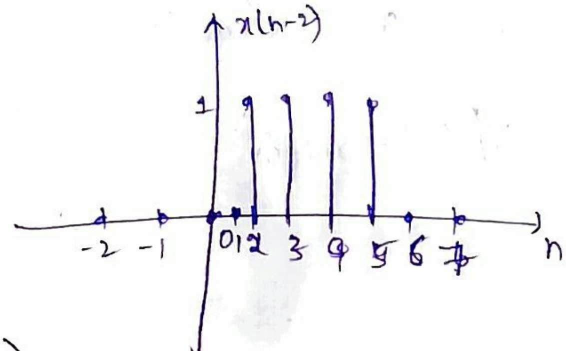
$= \{\ldots 0, 0, 1, 1, 1, 0, 0, \ldots\}$



3) $y(n-2) = \{0, 0, 0, 1, 1, 1, 0, 0 \ldots\}$

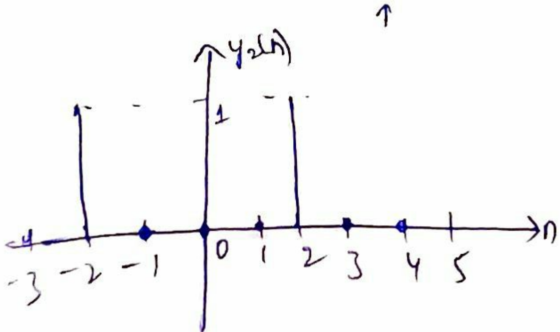


4) $x(n-2) = x_2(n) = \{0, 0, 0, 1, 1, 1, 1, 0 \ldots\}$



5) $y_2(n) = \{x(9), x(4), x(1), x(0), x(1), x(4), x(9) \ldots\}$

$= \{\ldots 0, 1, 0, 0, 0, 1, 0, \ldots\}$



6) $y_2(n)$ & $y(n-2)$ both are having different Values

thus,

$y_2(n) \neq y(n-2)$ ⇒ says that System is time variant

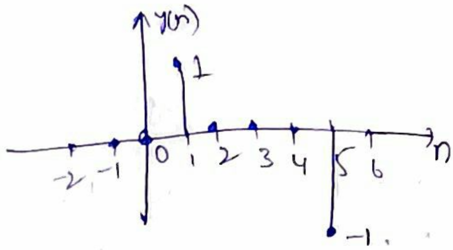
(c)

1) $x(n) = \{0, 1, 1, 1, 1, 0\}$

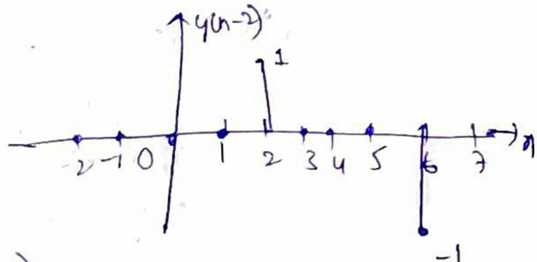
$x(n-1) = \{0, 0, 1, 1, 1, 1, 0\}$

2) $y(n) = x(n) - x(n-1)$

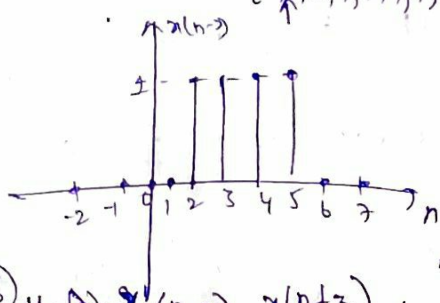
$= \{0, 1, 0, 0, 0, -1, 0 \ldots\}$



3) $y(n-2) = \{\ldots 0, 0, 0, 1, 0, 0, 0, -1, 0 \ldots\}$



4) $x(n-2) = x_2(n) = \{0, 0, 0, 1, 1, 1, 1, 0, \ldots\}$



5) $\cancel{y_2(n) = x(n-2) - x(n+3)}$

$\cancel{\{0, 0, 1, 0, 0, -1\} - \{0, 0, 0, 1, 0, 0, -1\}}$

$\cancel{= \{0, 0, 1, \ldots}$

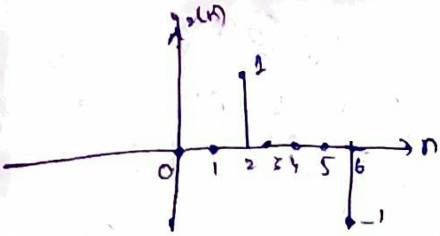
$n-2-1$

$n-3$

$y_2(n) = x_2(n) - x_2(n-1)$

$= \{0, 0, 1, 1, 1, 1, 0\} - \{0, 0, 0, 1, 1, 1, 1\}$

$y_2(n) = \{0, 0, 1, 0, 0, 0, -1\}$

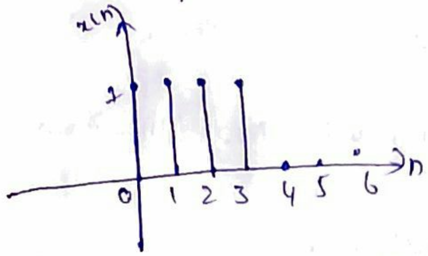


(c) here $y_2(n) = y(n-2)$

$\Rightarrow$ System is time invariant

(d) $y(n) = T[x(n)] = n\,x(n)$

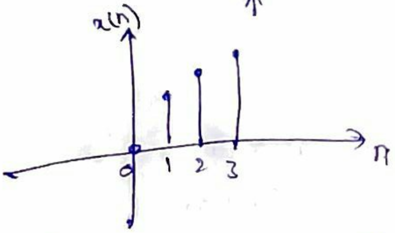
1) $x(n) = \{\ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0 \ldots\}$



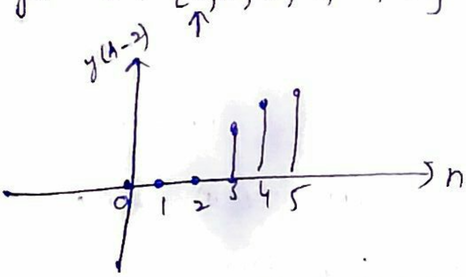
2) $y(n) = n\,x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$

$1\times0 \quad 1\times1 \quad 1\times2 \quad 1\times3$

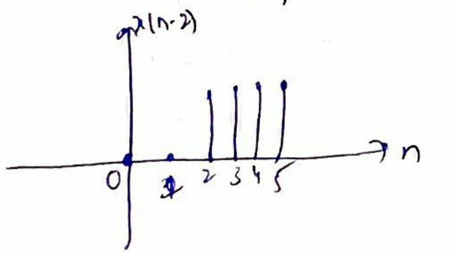
$= \{\underset{\uparrow}{0}, 1, 2, 3\}$



3) $y(n-2) = \{\underset{\uparrow}{0}, 0, 0, 1, 2, 3\}$



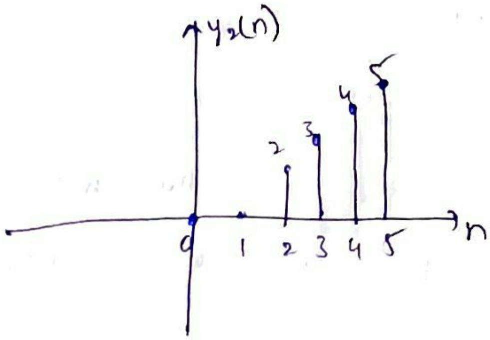
4) $x(n-2) = x_2(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$



5) $y_2(n) = T(x_2(n))$

0	0	1	1	1	1
×	×	×	×	×	×
0	1	2	3	4	5

$y_2(n) = \{\underset{\uparrow}{0}, 0, 2, 3, 4, 5\}$



6) $y_2(n)$ & $y(n-2)$ both are having different values.

thus,

$y_2(n) \neq y(n-2)$

$\Rightarrow$ says that system is time invariant.

2.7) A DTS system can be

1) Static (or) dynamic
2) Linear (or) non-linear
3) Time invariant (or) time varying
4) Causal (or) non-causal
5) stable (or) unstable.

Examine the following systems with respect to the properties above.

a) $y(n) = \cos[x(n)]$ → (a) Static, Nonlinear, Time invariant & Causal & Stable

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$ → (b) Dynamic, linear, time invariant, noncausal & unstable

c) $y(n) = x(n)\cos(\omega_0 n)$ → (c) Static, Linear, time variant, causal, stable

d) $y(n) = x(-n+2)$ → (d) Dynamic, linear, time invariant, non causal, ~~static~~ & stable

e) $y(n) = Tr[x(n)]$, where $Tr[x(n)]$ denotes the integer part of $x(n)$ obtained by Truncation. → (e) Static, Non-linear, time-invariant, Causal, stable.

f) $y(n) = Round[x(n)]$, where $Round[x(n)]$ denotes the integer part of $x(n)$ obtained by Rounding. → (f) Static, non-linear, time-invariant, Causal, stable.

g) $y(n) = |x(n)|$ → (g) Static, non-linear, time invariant, causal, stable.

h) $y(n) = x(n)u(n)$ → (h) static, linear, time-invariant, causal, stable

i) $y(n) = x(n) + n x(n+1)$ → (i) Dynamic, linear, time variant, noncausal, unstable.

Note: Bounded i/p $x(n)$ gives $u(n)$ which is a unbounded o/p.

j) $y(n) = x(2n)$ → (j) Dynamic, linear, time variant, non-causal, stable

k) $y(n) = \begin{cases} x(n) & \text{if } x(n) \geq 0 \\ 0 & \text{if } x(n) < 0 \end{cases}$ → (k) Static, non-linear, time invariant, Causal, stable.

l) $y(n) = x(-n)$ → (l) Dynamic, linear, time invariant, non-causal, Stable.

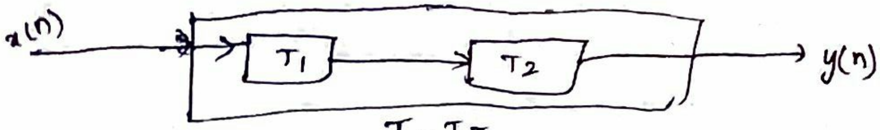
m) $y(n) = sign[x(n)]$

n) The ideal sampling system with i/p $x_0(t)$ & o/p $x(n) = x_0(nT)$, $-\infty < n < \infty$.

Sol. (m) static, non-linear, time invariant, Causal, stable.

(n) static, linear, time invariant, causal, stable

2.8 2 DTS T_1 & T_2 are connected in cascade to form a new system T as shown in fig below. Prove (or) disprove following statements.



$T = T_1 T_2$

a) If T_1 & T_2 are linear, then T is linear [i.e cascade connection of two linear systems is linear]

Sol. True.

If $v_1(n) = T_1(x_1(n))$ & $v_2(n) = T_1(x_2(n))$ then $a_1 x_1(n) + a_2 x_2(n)$ gives –

$a_1 v_1(n) + a_2 v_2(n)$ → from linearity property of T_1

Similarly, If $y_1(n) = T_2[v_1(n)]$ & $y_2(n) = T_2[v_2(n)]$ then

$\beta_1 v_1(n) + \beta_2 v_2(n) \longrightarrow y(n) = \beta_1 y_1(n) + \beta_2 y_2(n)$ [from linearity property of T_2]

Since $v_1(n) = T_1(x_1(n))$ & $v_2(n) = T_2[x_2(n)]$, we've –

$A_1 x_1(n) + A_2 x_2(n) \longrightarrow A_1 T(x_1(n)) + A_2 T(x_2(n))$

where $T = T_1 T_2$ & T is linear.

b) If T_1 & T_2 are time invariant, then T is time invariant.

Sol True.

For T_1, If $x(n) \longrightarrow v(n)$

&

$x(n-k) \longrightarrow v(n-k)$

For T_2, if $v(n) \longrightarrow y(n)$

&

$v(n-k) \longrightarrow y(n-k)$

For $T_1 T_2$, if $x(n) \longrightarrow y(n)$

&

$x(n-k) \longrightarrow y(n-k)$

Therefore $T = T_1 T_2$ is time invariant.

c) If T_1 & T_2 are causal, then T is causal.

Sol. True. T_1 is causal $\Rightarrow$ $v(n)$ depends only on $x(k)$ for $k \le n$

T_2 is causal $\Rightarrow$ $y(n)$ depends only on $v(k)$ for $k \le n$

There $y(n)$ depends only on $x(k)$ for $k \le n$ thus T is causal.

d) If T_1 & T_2 are linear and time invariant, the same holds for T

Sol True proof from both (a) & (b).

e) If T_1 & T_2 are linear and time invariant, then interchanging their order doesn't change the system T.

Sol. True $\rightarrow$ from the reason as $h_1(n) * h_2(n) = h_2(n) * h_1(n)$

f) As in part (e) except that T_1, T_2 are now time varying.

Sol. False.

For Ex:

$T_1 : y(n) = n x(n)$

$T_2 : y(n) = n x(n+1)$

then

$T_2[T_1(\delta(n))] = T_2(0) = 0$

$T_1[T_2(\delta(n))] = T_1[\delta(n+1)]$

$= -\delta(n+1)$

$T_1[T_2(\delta(n))] \neq 0$

thus both are different.

g) If T_1 & T_2 are non-linear, then T is nonlinear.

Sol. False.

For Ex: $T_1 : y(n) = x(n) + b$ & $T_2 : y_2(n) = x(n) - b$ where $b \neq 0$

then

$$T[x(n)] = T_2[T_1[x(n)]]$$

$$= T_2[x(n) + b]$$

$$T[x(n)] = x(n)$$

$\Rightarrow$ says that T is linear.

h) If T_1 & T_2 are stable, then T is stable.

Sol. True.

T_1 is stable $\xRightarrow{\text{when}}$ $v(n)$ is bounded-possible if $x(n)$ is bounded.

T_2 is stable $\xrightarrow{\text{when}}$ $y(n)$ is bounded-possible if $v(n)$ is bounded.

Hence $y(n)$ is bounded if $x(n)$ is bounded $\Rightarrow$ $T = T_1 T_2$ is stable

i) Show by an example that the inverse of parts (c) & (h) don't hold in general.

Sol. Inverse of (c)

If T_1 & T_2 are non causal, then T is noncausal.

Ex: $T_1 : y(n) = x(n+1)$

& $T_2 : y(n) = x(n-2)$

$\Rightarrow T : y(n) = x(n-1)$

↳ causal.

$\Rightarrow$ Inverse of (c) is false

Inverse of (h)

If T_1 and/or T_2 is unstable, then T is unstable.

Ex: $T_1 : y(n) = e^{x(n)} \Rightarrow$ Stable

$T_2 : y(n) = \ln[x(n)] \Rightarrow$ Unstable

$\Rightarrow T : y(n) = x(n) \Rightarrow$ Stable

$\Rightarrow$ Inverse of (h) is false.

2.9 Let T be an LTI, relaxed & BIBO stable system with i/p $x(n)$ & o/p $y(n)$. Then show that -

a) If $x(n)$ is periodic with period N [i.e $x(n) = x(n+N)$ for all $n \geq 0$] then the o/p $y(n)$ tends to a periodic signal with the same period.

Sol. we know that,

$$y(n) = \sum_{k=-\infty}^{n} h(k)\, x(n-k) \qquad \text{for } x(n) = 0,\ n<0$$

↓ periodic with N

$$\Rightarrow y(n+N) = \sum_{k=-\infty}^{n+N} h(k)\, x(n+N-k)$$

$$= \sum_{k=-\infty}^{n+N} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{n} h(k)\, x(n-k) + \sum_{k=n+1}^{n+N} h(k)\, x(n-k)$$

$$y(n+N) = y(n) + \sum_{k=n+1}^{n+N} h(k)\, x(n-k)$$

for a BIBO system, $\lim_{n\to\infty} |h(n)| = 0$

$$\Rightarrow \lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)\, x(n-k) = 0.$$

thus

$$\lim_{n\to\infty} y(n+N) = \lim_{n\to\infty} y(n) + \underbrace{\sum_{k=n+1}^{n+N} h(k)\, x(n-k)}_{0}$$

$$\Rightarrow \boxed{\lim_{n\to\infty} y(n+N) = y(N)}$$ proved.

b) If $x(n)$ is bounded and tends to a constant, the o/p will also tend to a constant.

Sol Let $y(n) = x_0(n) + a u(n)$ where a is a constant & $x_0(n)$ is a bounded signal with $\lim_{n\to\infty} x_0(n) = 0$.

ie

$$y(n) = \underbrace{x_0(n) + a u(n)}$$

convolution gives us

$$y(n) = \sum_{k=0}^{\infty} h(k) x_0(n-k) + a \sum_{k=0}^{\infty} h(k) u(n-k)$$

$$y(n) = a y_0(n) + a \sum_{k=0}^{n} h(k)$$

thus

$$y(n) = y_0(n) + a \sum_{k=0}^{n} h(k) \quad \text{— (1)}$$

Clearly says that when $\sum_n x_0^2(n) < \infty$

$$\Rightarrow \sum_n y_0^2(n) < \infty$$

Hence $\lim_{n\to\infty} |y_0(n)| = 0$

thus (1) eq becomes -

$$y(n) = 0 + a \sum_{k=0}^{n} h(k)$$

$$\Rightarrow \boxed{y(n) = a \sum_{k=0}^{n} h(k)} \Rightarrow \text{constant.}$$

c) If $x(n)$ is an energy signal, the o/p $y(n)$ will also be an energy signal.

Sol, we know that,

$$y(n) = \sum_k h(k) x(n-k)$$

Summing from $-\infty$ to ∞ & squaring on both sides gives us-

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty} \left[\sum_k h(k) x(n-k) \right]^2$$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_k \sum_l h(k) h(l) \sum_n x(n-k) x(n-l)$$

But $\sum_n x(n-k) x(n-l) \le \sum_n x^2(n) = E_x$

then

$$\sum_n y^2(n) \le E_x \sum_k |h(k)| \sum_l |h(l)| \quad \text{(1)}$$

for a BIBO system,

$$\sum_k |h(k)| < M$$

then we've-

$\sum_n y^2(n) \to$ has Energy: E_y

$\sum_k h(k) \to$ has energy: M

$\sum_l h(l) \to$ has Energy: M.

(1) Eq. becomes-

$$\sum_n y^2(n) \le E_x \sum_k h(k) \sum_l h(l)$$

$$E_y \le M \cdot M \cdot E_x$$

$$\boxed{E_y \le M^2 E_x}$$ such that $E_y < 0$ if E_x

Q12 The only available information about a system consists of N i/p-o/p pairs, of signals $y_i(n) = T[x_i(n)]$ $i = 1, 2, \ldots N$

Then find -

a) What is the class of i/p signals for which we can determine the o/p, using the information above, if the system is known to be linear?

b) The same as above, if the system is known to be time invariant.

Sol. To get the o/p as mentioned in the problem, where -

a) System is linear, then our i/p should be -

any weighted linear combination of the signals $x_i(n)$, $i = 1, 2, \ldots N$

b) System is time invariant, then our i/p should be -

any weighted linear combination of the signals as of the form - $x_i(n-k)$ where k = any integer. $i = 1, 2, \ldots N$

2.13 Show that the necessary and sufficient condition for a relaxed LTI system to be BIBO stable is -

$$\sum_{n=-\infty}^{\infty} |h(n)| \leq M_h < \infty \quad \text{for some constant } M_h.$$

Sol. Deriving BIBO stability from convolution formula,

we know that -

Convolution formula :- $y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

Taking absolute value on both sides, for above eq.

$$\Rightarrow |y(n)| = \left|\sum_{k=-\infty}^{\infty} h(k)\, x(n-k)\right|$$

$$\Rightarrow |y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)|\, |x(n-k)|$$

If i/p is bounded with M_x

then we've -

$$\Rightarrow |y(n)| \leq M_x \cdot \sum_{k=-\infty}^{\infty} |h(k)| \quad (\because |x(n)| \leq M_x,\ |x(n-k)| \leq M_x)$$

thus, we've - $\boxed{|y(n)| \leq M_x \cdot \sum_{k=-\infty}^{\infty} |h(k)|}$

here o/p $y(n)$ is bounded if the impulse response satisfies the condition $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$.

Q.14 Show that :

a) A relaxed LTI is causal if and only if for any i/p $x(n)$ such that $x(n)=0$ for $n \le n_0$ $\Rightarrow$ $y(n)=0$ for $n < n_0$.

b) A relaxed LTI system is causal if and only if $h(n)=0$ for $n \lessgtr 0$

Sol. a) From convolution formula, we've -

$$y(n) = \sum_{k=0}^{n} x(k)\,h(n-k) = \sum_{k=0}^{n} h(k)\,x(n-k)$$

* If i/p $x(n)$ is causal i.e $x(n)=0$ for $n < n_0$

then $\underline{y(n)=0}$ for $n < n_0$.

b) From convolution formula, we've -

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\,x(n_0-k)$$

$$= \sum_{k=0}^{\infty} h(k)\,x(n_0+k) + \sum_{k=-\infty}^{-1} h(k)\,x(n_0-k)$$

$$y(n) = \underbrace{\left[h(0)x(n_0)+h(1)x(n_0-1)+h(2)x(n_0-2)+\dots\right]}_{\text{past and present values}} + \underbrace{\left[h(-1)x(n_0+1)+h(-2)x(n_0+2)+\dots\right]}_{\text{future values}}$$

→ To make the system causal, o/p should depend on only past and present values, this is possible if $\underline{h(n)=0}$ for $\underline{n<0}$

Q.15 a) Show that for any real (or) complex constant a, and any finite integer no's M and N, we have -

$$\sum_{n=M}^{N} a^n = \begin{cases} \dfrac{a^M - a^{N+1}}{1-a} & \text{if } a \ne 1 \\ N-M+1 & \text{if } a = 1 \end{cases}$$

a)

Sol. We know that sum of no's in a Range from L to R.

$R-(L-1) = \underline{R-L+1}$.

Similarly, $\sum_{n=M}^{N} a^n = N-M+1$ for $a=1$

for $a \neq 1$ $\sum_{n=M}^{N} a^n$ is -

$$\sum_{n=M}^{N} a^n = a^M + a^{M+1} + \dots + a^N$$

$$(1-a)\sum_{n=M}^{N} a^n = (a^M + a^{M+1} + \dots + a^N)(1-a)$$

$$= a^M + \cancel{a^{M+1}} - \cancel{a^{M+1}} + \dots + \cancel{a^N} - \cancel{a^N} - a^{N+1}$$

$$(1-a)\sum_{n=M}^{N} a^n = a^M - a^{N+1}$$

$$\sum_{n=M}^{N} a^n = \frac{a^M - a^{N+1}}{1-a}$$

thus we've -

$$\sum_{n=M}^{N} a^n = \begin{cases} \frac{a^M - a^{N+1}}{1-a} & ; a \neq 1 \\ N-M+1 & ; a = 1 \end{cases}$$

b) Show that if $|a| < 1$, then $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$

Sol. We know that,

$$\sum_{k=0}^{\infty} a\gamma^k = \frac{a}{1-\gamma}, \quad |\gamma| < 1.$$

Similarily

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad |a| < 1$$

this can be obtained if we substitute $M = 0$, $N = \infty$ in $\sum_{n=M}^{N} a^n = \frac{a^M - a^{N+1}}{1-a}$; $a \neq 1$.

2.16 a) If $y(n) = x(n) * h(n)$, show that $\mathcal{E}_y = \mathcal{E}_x \mathcal{E}_h$ where $\mathcal{E}_x = \sum_{n=-\infty}^{\infty} x(n)$.

b) Compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the correctness of the results by using the test in (a)

1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

4) $x(n) = \{1,2,3,4,5\}$, $h(n) = \{1\}$

5) $x(n) = \{-1, \underset{\uparrow}{1}, 2, 3\}$, $h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1\}$

6) $x(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$, $h(n) = \{1, \underset{\uparrow}{-2}, 3\}$

7) $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$, $h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$

8) $x(n) = \{\underset{\uparrow}{1}, 1, 2\}$, $h(n) = u(n)$

9) $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$, $h(n) = \{1, -2, -3, \underset{\uparrow}{4}\}$

10) $x(n) = \{1, 2, \underset{\uparrow}{0}, 2, 1\}$, $h(n) = x(n)$

11) $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

Sol. a) From convolution formula, we've-

$$y(n) = \sum_k h(k)x(n-k)$$

Summation on both sides w.r.t. n

$$\sum_n y(n) = \sum_n \sum_k h(k)\, x(n-k)$$

$$= \sum_k h(k) \cdot \sum_{n=-\infty}^{\infty} x(n-k)$$

✓ from given $\sum_{n=-\infty}^{\infty} x(n-k) = \sum_{n=-\infty}^{\infty} x(n)$

$$\sum_n y(n) = \sum_k h(k) \cdot \sum_n x(n)$$

thus

$$\sum_n y(n) = \left[\sum_k h(k)\right] \cdot \left[\sum_n x(n)\right]$$

$$\boxed{\sum y = \sum h \cdot \sum x} \Rightarrow \text{is proved.}$$

b)

① $x(n) = [1,2,4]$, $h(n) = \{1,1,1,1,1\}$ proof

$x(n) * h(n)$

$x(n)$ \ $h(n)$	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

$\Rightarrow x(n) * h(n) = y(n)$

$y(n) = \{1, 3, 7, 7, 7, 6, 4\}$

$\sum y(n) = 35$. — ⓐ

$\sum x(n) = [1,2,4] = 7$

$\sum h(n) = [1,1,1,1,1] = 5$

$x(n) * h(n) = 35$

— ⓑ .

thus ⓐ & ⓑ are true.

2) $x(n) = [1, 2, -1]$, $h(n) = x(n)$

a)

$x(n) \backslash h(n)$	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$y(n) = x(n) * h(n)$

$= \{1, 4, 2, -4, 1\}$

$y(n) = 4.$

b) $x(n) = [1, 2, -1] = 2$

$h(n) = x(n) = [1, 2, -1] = 2$

$x(n) * h(n) = y(n) = 4.$

thus a & b are true

3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

a)

$x(n) \backslash h(n)$	0	1	-2	3	-4
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$\frac{3}{2}$	-2
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$\frac{3}{2}$	-2
1	0	1	-2	3	-4
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$\frac{3}{2}$	-2

$x(n) * h(n) = y(n)$

$y(n) = \{0, \frac{1}{2}, \frac{-1}{2}, \frac{3}{2}, -2, 0, \frac{-5}{2}, -2\}$

$y(n) = -5$ $\frac{-10}{2}$ $-1+1+3-4-5-4$ -6 $\frac{10}{2}$

b) $x(n) = \{0, 1, -2, 3, -4\} = -2$

$h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\} = 2.5$

then $x(n) * h(n) = y(n)$

$y(n) = -2 * 2.5$

$y(n) = -5$

thus a & b are true.

4) a) $x(n) = \{1, 2, 3, 4, 5\}$ $h(n) = 1$

$x(n) * h(n) = y(n)$

$y(n) = \{1, 2, 3, 4, 5\} = 15.$

b) $x[n] = \{1, 2, 3, 4, 5\} = 15$

$h[n] = \{1\} = 1$

$x[n] * h[n] = 15 = y(n)$

thus a & b are true.

5) a) $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$

$h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

$x(n) \backslash h(n)$	1	-2	3
→ 0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$x(n) * h(n) = y(n)$

$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$y(n) = 8.$

b) $x(n) = \{1, -2, 3\} = 2$

$h(n) = \{0, 0, 1, 1, 1, 1\} = 4$

$x(n) * h(n) = 4 \times 2 = 8 = y(n)$

thus (a) & (b) are true

(6)

(a) $x(n) = \{0,0,1,1,1,1\}$, $h(n) = \{1,-2,3\}$

$x(n) \backslash h(n)$	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

$x(n) * h(n) = y(n)$

$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\Sigma y(n) = 8.$

(b) $x(n) = \{0,0,1,1,1,1\} = 4$

$h(n) = \{1,-2,+3\} = 2.$

$x(n) * h(n) = 4 \times 2 = 8 = y(n)$

thus (a) & (b) are true.

(7)

(a) $x(n) = (0, 1, 4, -3)$, $h(n) = (1, 0, -1, -1)$

$x(n) \backslash h(n)$	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

$x(n) * h(n) = y(n)$

$y(n) = \{0, 1, 4, -4, -5, -1, 3\}$

$y(n) = -2$

(b) $x(n) = (0,1,4,-3) = 2$

$h(n) = (1,0,-1,-1) = -1$

$x(n) * h(n) = 2 \times -1 = -2 = y(n)$

thus (a) & (b) are true.

(8)

a) $x(n) = (1,1,2)$, $h(n) = u(n)$

$\Rightarrow h(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

$x(n) * h(n) = y(n)$

$1,1,2 * u(n) = y(n)$

$\Rightarrow y(n) = u(n) + u(n-1) + 2u(n-2)$

$\sum_n y(n) = \infty.$

(b) $x(n) = \{1,1,2\} = 3$

$h(n) = u(n) = \infty$

$x(n) * h(n) = y(n) = \infty$

thus (a) & (b) are true.

(9)

(a) $x(n) = \{1,1,0,1,1\}$, $h(n) = \{1,-2,-3,4\}$

$x(n) \backslash h(n)$	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
4	4	4	0	4	4

$x(n) * h(n) = y(n)$

$y(n) = \{1, -1, -5, 2, 3, -5, -1, 4\}$

$\Sigma y(n) = 0$

(b) $x(n) = \{1,1,0,1,1\} = 4$

$h(n) = \{1,-2,-3,4\} = 0$

$x(n) * h(n) = 4 \times 0 = 0$

$\Rightarrow y(n) = 0$

thus (a) & (b) are true

(10)

(a) $x(n) = \{1, 2, 0, 2, 1\}$, $h(n) = x(n)$

$x(n) \backslash h(n)$	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	1
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

$x(n) * h(n) = y(n)$

$y(n) = \{1, 4, 4, 4, 10, 4, 4, 4, 1\}$

$\Sigma y(n) = 36$

(b) $x(n) = \{1, 2, 0, 2, 1\} = 6$

$h(n) = x(n) = 6$.

$x(n) * h(n) = 6 \times 6 = 36 = y(n)$

thus (a) & (b) are true

(11)

a) $x(n) = \left(\frac{1}{2}\right)^n u(n)$

$h(n) = \left(\frac{1}{4}\right)^n u(n)$

$x(n) * h(n) = y(n)$

$y(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right] u(n)$

$\Sigma y(n) = \frac{8}{3}$

(b) $x(n) = \left(\frac{1}{2}\right)^n u(n) = 2$

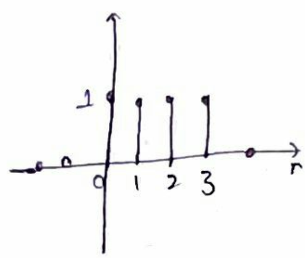
$h(n) = \left(\frac{1}{4}\right)^n u(n) = \frac{4}{3}$

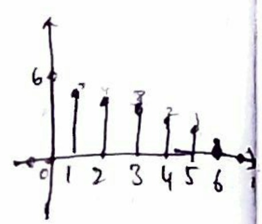
$x(n) * h(n) = 2 \times \frac{4}{3} = \frac{8}{3} = y(n)$

thus (a) & (b) are true.

Q2.17 Compute and plot the convolutions $x(n) * h(n)$ and $h(n) * x(n)$ for the following pairs -

a) $x(n)$ $h(n)$

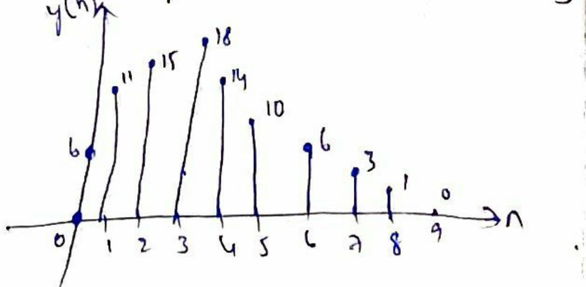




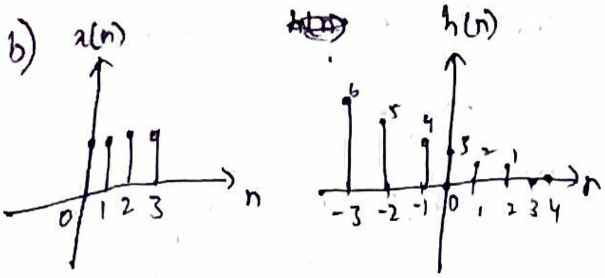
Sol. $x(n) * h(n) = \Sigma x(k) h(n-k) = y(n)$

$x(n) \backslash h(n)$	1	1	1	1
6	6	6	6	6
5	5	5	5	5
4	4	4	4	4
3	3	3	3	3
2	2	2	2	2
1	1	1	1	1

$y(n) = \{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1\}$



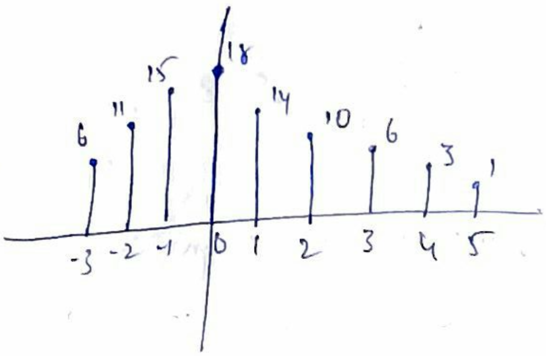
b)



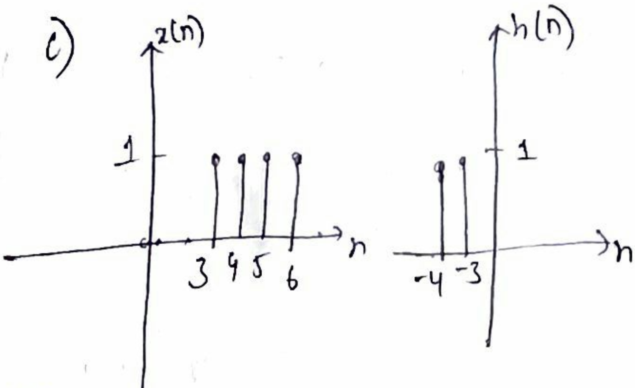
Sol. $x(n) * h(n) = y(n)$

x(n) \ h(n)	1	1	1	1
6	6	6	6	6
5	5	5	5	5
4	4	4	4	4
→ 3	3	3	3	3
2	2	2	2	2
1	1	1	1	1

$y(n) = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$



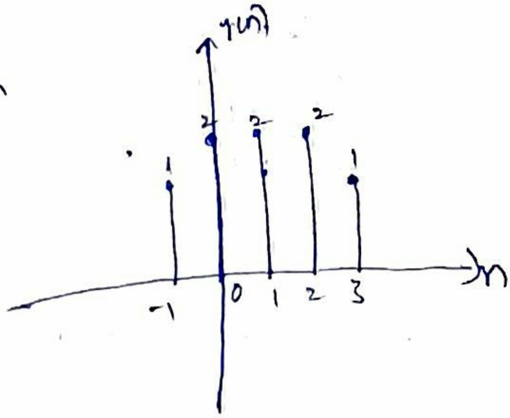
c)



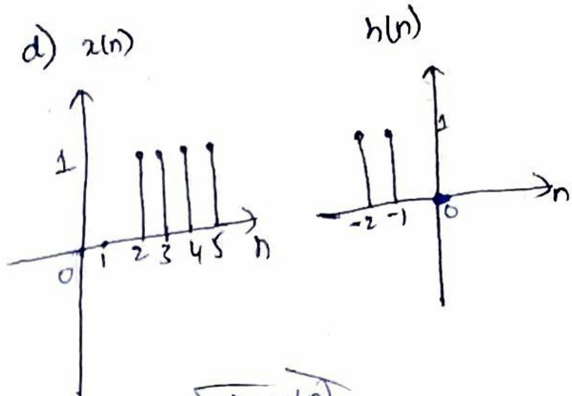
Sol. $x(n) * h(n) = y(n)$

x(n) \ h(n)	1	1	0	0	0
→ 0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0

$y(n) = \{0, 0, 0, 1, \underset{\uparrow}{2}, 2, 2, 1, 0, 0, 0\}$



d)



Sol. $\cancel{x(n) * h(n)} = y(n)$

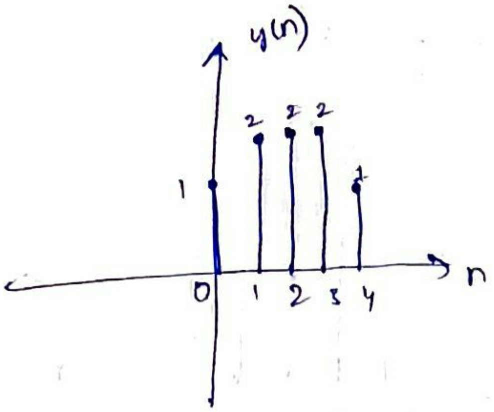
x(n) \ h(n)	0	0	1	1	1	1
0	0	0	1	1	1	1
1	0	0	1	1	1	1
→ 0	0	0	1	1	1	1

$\cancel{y(n) = \{0, 0, \underset{\uparrow}{1}, 2, 3, 3, 2, 1\}}$

$x(n) * h(n) = y(n)$

x(n) \ h(n)	1	1	0
→ 0	0	0	0
0	0	0	0
1	1	1	0
1	1	1	0
1	1	1	0
1	1	1	0

$y(n) = \{0, 0, \underset{\uparrow}{1}, 2, 2, 2, 1, 0\}$



2.19 Compute the convolution $y(n)$ of the signals-

$$x(n) = \begin{cases} \alpha^n, & -3 \le n \le 5 \\ 0, & \text{Elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{Elsewhere} \end{cases}$$

Sol. $y(n) = \sum_{k=0}^{4} h(k) \cdot x(n-k)$

$\Rightarrow x(n) = \{ \alpha^{-3}, \alpha^{-2}, \alpha^{-1}, 1, \alpha, \alpha^2, \ldots \alpha^5 \}$

$h(n) = \{1, 1, 1, 1\}$

$$\Rightarrow y(n) = \begin{cases} \sum_{k=0}^{4} x(n-k) & -3 \le n \le 9 \\ 0 & \text{Otherwise} \end{cases}$$

2.20 Consider the following operations

a) Multiply the integer numbers: 131 and 122

Sol. $131 \times 122 = 15982$

b) Compute the convolution of signals: $\{1, 3, 1\} * \{1, 2, 2\}$

Sol.

$x(n)$ \ $h(n)$	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

$x(n) * h(n)$

$y(n) = \{ \underset{\uparrow}{1}, 5, 9, 8, 2 \}$

c) Multiply the polynomials: $1+3z+z^2$ and $1+2z+2z^2$

Sol. $(1+3z+z^2)(1+2z+2z^2)$

$= 1 + 3z + z^2 + 2z + 6z^2 + 2z^3 + 2z^2 + 6z^3 + 2z^4$

$= 2z^4 + 8z^3 + 9z^2 + 5z + 1$

Thus,

$(1+3z+z^2)(1+2z+2z^2)$

$= 2z^4 + 8z^3 + 9z^2 + 5z + 1$

d) Multiply 1.31 and 12.2

Sol. $1.31 \times 12.2 = 15.982$

Q.18 Determine and sketch the convolution $y(n)$ of the signals -

$$x(n) = \begin{cases} \frac{1}{3}n & 0 \le n \le 6 \\ 0 & \text{Elsewhere} \end{cases} \qquad h(n) = \begin{cases} 1 & -2 \le n \le 2 \\ 0 & \text{Elsewhere} \end{cases}$$

a) Graphically b) - Analytically.

Sol.

$$x(n) = \begin{cases} \frac{1}{3}n & 0 \le n \le 6 \\ 0 & \text{Elsewhere} \end{cases} \qquad h(n) = \begin{cases} 1 & -2 \le n \le 2 \\ 0 & \text{Elsewhere} \end{cases}$$

$n=0 \rightarrow \frac{1}{3} \times 0 = 0$

$n=1 \rightarrow \frac{1}{3} \times 1 = \frac{1}{3}$

$n=2 \rightarrow \frac{2}{3}$

$n=3 \rightarrow 1$

$n=4 \rightarrow \frac{4}{3}$

$n=5 \rightarrow \frac{5}{3}$

$n=6 \rightarrow 2$

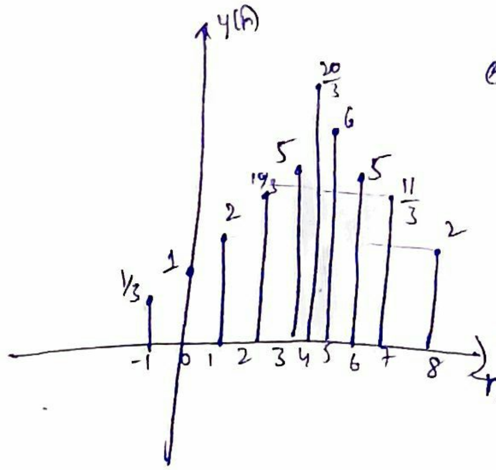
$\Rightarrow h(n) = \{0, 1, 1, \underset{\uparrow}{1}, 1, 1, 0\}$

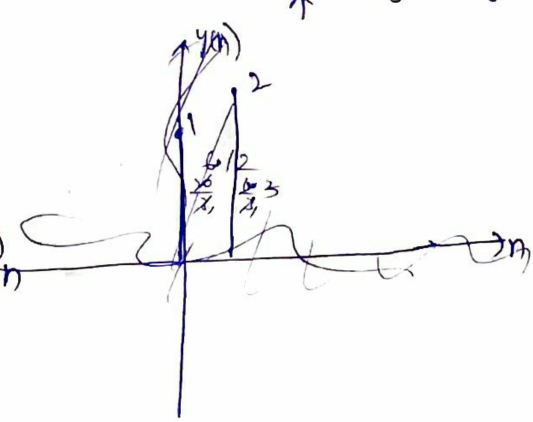
thus $x(n) = \{\underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$

a) Graphically:

$x(n) \backslash h(n)$	0	1	1	1 ↓	1	1	0
→ 0	0	0	0	0	0	0	0
$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0
$\frac{2}{3}$	0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
1	0	1	1	1	1	1	0
$\frac{4}{3}$	0	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	0
$\frac{5}{3}$	0	$\frac{5}{3}$	$\frac{5}{3}$	$\frac{5}{3}$	$\frac{5}{3}$	$\frac{5}{3}$	0
2	0	2	2	2	2	2	0

$y(n) = \{0, 0, \frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$





(b) Analytically:

$$x(n) = \begin{cases} \frac{1}{3}n & 0 \le n \le 6 \\ 0 & \text{Elsewhere} \end{cases} = \frac{1}{3}n\left[u(n) - u(n-7)\right]$$

$$h(n) = \begin{cases} 1 & -2 \le n \le 2 \\ 0 & \text{Elsewhere} \end{cases} = u(n+2) - u(n-3)$$

$$y(n) = x(n) * h(n)$$

$$= \frac{1}{3}n\left[u(n) - u(n-7)\right] * \left[u(n+2) - u(n-3)\right]$$

$$= \frac{1}{3}n\,u(n) * u(n+2) - \frac{1}{3}n\,u(n) * u(n-3) - \frac{1}{3}n\,u(n-7) * u(n+2)$$
$$+ \frac{1}{3}n\,u(n-7) * u(n-3)$$

$$=$$

$$\Rightarrow y(n) = \frac{1}{3}\delta(n+1) + \delta(n) + 2\delta(n-1) + \frac{10}{3}\delta(n-2) + 5\delta(n-3) + \frac{2}{3}\delta(n-4) +$$
$$6\delta(n-5) + 5\delta(n-6) + 5\delta(n-6) + \frac{11}{3}\delta(n-7) + \delta(n-8)$$

Q.Q1 Compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

a) $x(n) = a^n u(n)$, $h(n) = b^n u(n)$ when $a \neq b$ & when $a = b$.

Sol. $y(n) = \sum_k h(k)\, x(n-k) \approx \sum_k x(k)\, h(n-k)$

$$y(n) = \sum_{k=0}^{n} a^k u(k) \cdot b^{n-k} u(n-k)$$

$$= \sum_{k=0}^{n} a^k \cdot b^n \cdot b^{-k}\ (1)$$

$$y(n) = b^n \sum_{k=0}^{} (ab^{-1})^k \Rightarrow y(n) = \begin{cases} \dfrac{b^{n+1} - a^{n+1}}{b-a} u(n), & a \neq b \\ b^n (n+1)\, u(n), & a = b \end{cases}$$

b) $x(n) = \begin{cases} 1 & n = -2, 0, 1 \\ 2 & n = -1 \\ 0 & \text{Elsewhere} \end{cases}$, $h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

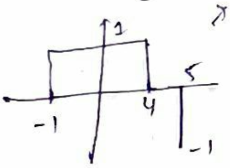
Sol. $x(n) = \{0, 1, 2, \underset{\uparrow}{1}, 1, 0\}$, $h(n) = \{\underset{\uparrow}{1}, -1, 0, 0, 1, 1\}$

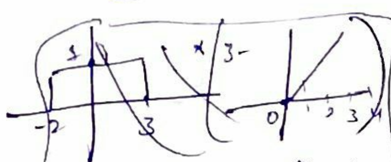
x(n)\h(n)	1	-1	0	0	1	1
0	0	0	0	0	0	0
1	1	-1	0	0	1	1
2	2	-2	0	0	2	2
→ 1	1	-1	0	0	1	1
1	1	-1	0	0	1	1
0	0	0	0	0	0	0

$y(n) = \{0, 1, +1, \underset{\uparrow}{-1}, 0, 0, 3, 3, 2, 1, 0\}$

c) $x(n) = u(n+1) - u(n-4) - \delta(n-5)$, $h(n) = [u(n+2) - u(n-3)] \cdot (3 - |n|)$

Sol. $x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$

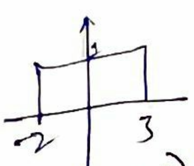




$(1, 1, \underset{\uparrow}{1}, 1, 1, 0) \times (3, 2, 1, \underset{\uparrow}{0}, -1,$

~~1, 1, 0, 2, 1, 0~~

$h(n) = (1, 2, \underset{\uparrow}{3}, 2, 1)$



$\times\ 3 - |n|$

$(1, 1, \underset{\uparrow}{1}, 1, 1, 0) \times \begin{pmatrix} n = -2 \to 1 \\ n = -1 \to 2 \\ n = 0 \to 3 \\ n = 1 \to 2 \\ n = 2 \to 1 \\ n = 3 \to 0 \end{pmatrix} = (1, 2, \underset{\uparrow}{3}, 2, 1)$

↓

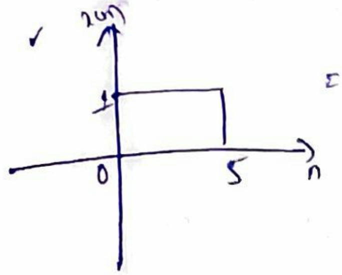
Thus

$x(n) \backslash h(n)$	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
0	0	0	0	0	0
-1	-1	-2	-3	-2	-1

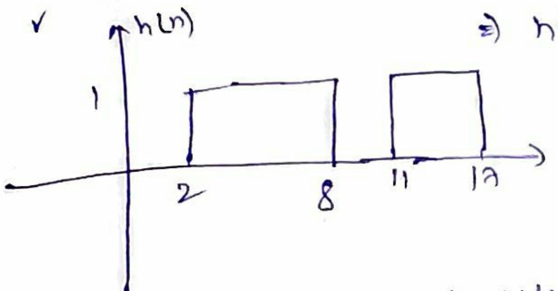
$$y(n) = \{1, 3, 6, \underset{\uparrow}{8}, 9, 8, 5, 1, -2, -3, -1\}$$

d) $x(n) = u(n) - u(n-5)$, $h(n) = u(n-2) - u(n-8) + u(n-11) - u(n-17)$

Sol.



$$= x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1, 0\}$$



$$\Rightarrow h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1, 0, \cancel{0, 0}$$
$$0, 0, 1, 1, 1, 1, 1, 1, 0\}$$

Let $h'(n) = \{0, 0, 1, 1, 1, 1, 1, 1\}$

$$h(n) = h'(n) + h'(n-9)$$

$$\Rightarrow y(n) = y'(n) + y'(n-9)$$

where

$$y'(n) = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$$

2.22 Let $x(n)$ be the i/p signal to a Discrete-time filter with impulse response $h_i(n)$ & let $y_i(n)$ be the corresponding o/p.

a) Compute & sketch $x(n)$ & $y_i(n)$ for the following cases, using the same scale in all figures. $x(n) = \{1,4,2,3,5,3,3,4,5,7,6,9\}$

$h_1(n) = (1,1)$; $h_2(n) = (1,2,1)$; $h_3(n) = (\frac{1}{2}, \frac{1}{2})$, $h_4(n) = (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$,

$h_5(n) = (\frac{1}{4}, \frac{-1}{2}, \frac{1}{4})$

Sketch $x(n)$, $y_1(n)$, $y_2(n)$ on one graph & $x(n)$, $y_3(n)$, $y_4(n)$, $y_5(n)$ on another graph.

Sol. (i)

$x(n)$ \ $h(n)$	1	1
1	1	1
4	4	4
2	2	2
3	3	3
5	5	5
3	3	3
3	3	3
4	4	4
5	5	5
7	7	7
6	6	6
9	9	9

$\Rightarrow y_1(n) = \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 12, 15, 9\}$

↳ similar to $x(n) + x(n-1)$

(ii)

$x(n)$ \ $h_2(n)$	1	2	1
1	1	2	1
4	4	8	4
2	2	4	2
3	3	6	3
5	5	10	5
3	3	6	3
3	3	6	3
4	4	8	4
5	5	10	5
7	7	14	7
6	6	12	6
9	9	18	9

$y_2(n) = \{1, 6, 11, 11, 13, 16, 14, 13, 15, 21, 25, 28, 24, 9\}$

↳ similar to

(iii)

$x(n)$ \ $h_3(n)$	$\frac{1}{2}$	$\frac{1}{2}$
1	$\frac{1}{2}$	$\frac{1}{2}$
4	2	2
2	1	1
3	$\frac{3}{2}$	$\frac{3}{2}$
5	$\frac{5}{2}$	$\frac{5}{2}$
3	$\frac{3}{2}$	$\frac{3}{2}$
3	$\frac{3}{2}$	$\frac{3}{2}$
4	2	2
5	$\frac{5}{2}$	$\frac{5}{2}$
7	$\frac{7}{2}$	$\frac{7}{2}$
6	3	3
9	$\frac{9}{2}$	$\frac{9}{2}$

$y_3(n) = \{0.5, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, 4.5\}$

(ii)

$x(n)$ \ $h_4(n)$	$\frac{1}{4}$	$+\frac{1}{2}$	$\frac{1}{4}$
1	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
4	1	2	1
2	$\frac{1}{2}$	1	$\frac{1}{2}$
3	$\frac{3}{4}$	$\frac{3}{2}$	$\frac{3}{4}$
5	$\frac{5}{4}$	$\frac{5}{2}$	$\frac{5}{4}$
3	$\frac{3}{4}$	$\frac{3}{2}$	$\frac{3}{4}$
3	$\frac{3}{4}$	$\frac{3}{2}$	$\frac{3}{4}$
4	1	2	1
5	$\frac{5}{4}$	$\frac{5}{2}$	$\frac{5}{4}$
7	$\frac{7}{4}$	$\frac{7}{2}$	$\frac{7}{4}$
6	$\frac{3}{2}$	3	$\frac{3}{2}$
9	$\frac{9}{4}$	$\frac{9}{2}$	$\frac{9}{4}$

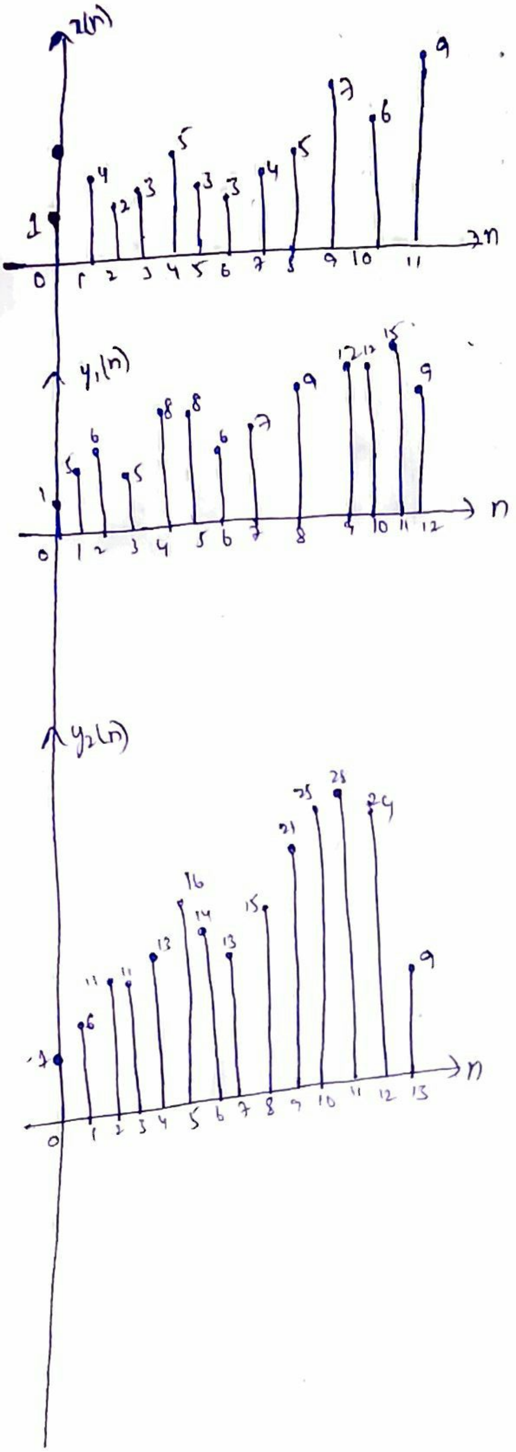
$y_4(n) = \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 3.75, 5.25, 6.25, 7, 6, 2.25\}$

v)

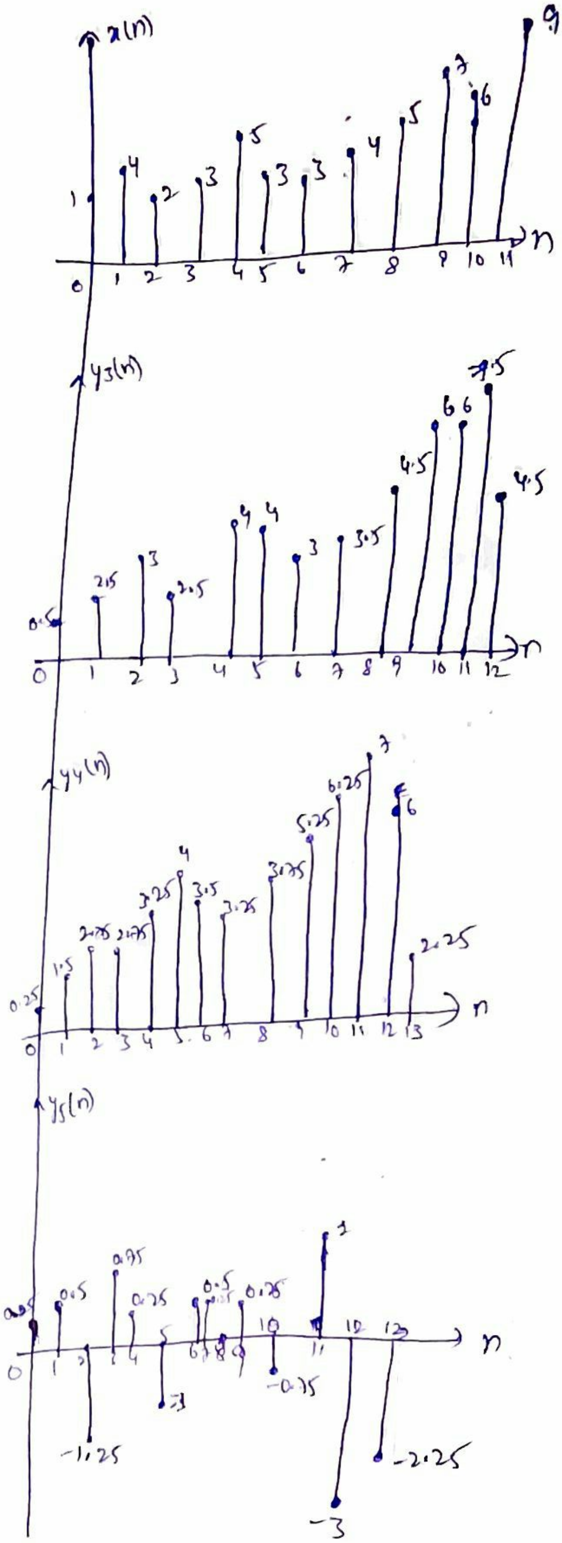
$x(n)$ \ $h_5(n)$	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$
1	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$
4	1	-2	1
2	$\frac{1}{2}$	-1	$\frac{1}{2}$
3	$\frac{3}{4}$	$-\frac{3}{2}$	$\frac{3}{4}$
5	$\frac{5}{4}$	$-\frac{5}{2}$	$\frac{5}{4}$
3	$\frac{3}{4}$	$-\frac{3}{2}$	$\frac{3}{4}$
3	$\frac{3}{4}$	$-\frac{3}{2}$	$\frac{3}{4}$
4	1	-2	1
5	$\frac{5}{4}$	$-\frac{5}{2}$	$\frac{5}{4}$
7	$\frac{7}{4}$	$-\frac{7}{2}$	$\frac{7}{4}$
6	$\frac{3}{2}$	-3	$\frac{3}{2}$
9	$\frac{9}{4}$	$-\frac{9}{2}$	$\frac{9}{4}$

$y_5(n) = \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0, 0.25, -0.75, 1, -3, +2.25\}$

Graph-1:



Graph-2:



b) what is the difference between $y_1(n)$ & $y_2(n)$ & between $y_3(n)$ & $y_4(n)$?

Sol.

→ $y_3(n) = \frac{1}{2} y_1(n)$ because $h_3(n) = \frac{1}{2} h_1(n)$

→ $y_4(n) = \frac{1}{4} y_2(n)$ because $h_4(n) = \frac{1}{4} h_2(n)$

c) comment on smoothness of $y_2(n)$ & $y_4(n)$, which factors affect the smoothness?

Sol. $y_2(n)$ & $y_4(n)$ are smoother than $y_1(n)$, but $y_4(n)$ will appear even smoother because of the smaller scale factor.

d) compare $y_4(n)$ with $y_5(n)$, what is the difference? Can you explain it?

Sol. System 4 results in a smoother o/p. The negative value of $h_5(0)$ is responsible for non-smooth characteristics of $y_5(n)$

e) Let $h_6(n) = \{\frac{1}{2}, -\frac{1}{2}\}$. Compute $y_6(n)$. Sketch $x(n)$, $y_2(n)$ & $y_6(n)$ on the same figure & comment on the results.

Sol.
$$y_6(n) = \{\frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, \frac{-1}{2}, \frac{3}{2}, \frac{-9}{2}\}$$

$y_2(n)$ is smoother than $y_6(n)$

Q3) The DTS having $y(n) = ny(n-1) + x(n)$; $n \geq 0$ is at rest. ($y(-1)=0$)

Check if the system is linear time invariant & BIBO stable.

Sol. * If $y_1(n) = ny_1(n-1) + x_1(n)$ & $y_2(n) = ny_2(n-1) + x_2(n)$

then $x(n) = a x_1(n) + b x_2(n)$
$\downarrow$
produces o/p
$\downarrow$
$y(n) = a y_1(n) + b y_2(n)$

Similarly $x(n) = a x_1(n) + b x_2(n)$
$\downarrow$
gives
$\downarrow$
$y(n) = ny(n-1) + x(n)$
$\downarrow$
thus system is linear.

* If i/p is delayed $x(n-1)$ we've $\rightarrow$ $y(n-1) = (n-1) y(n-2) + x(n-1)$ — (a)

If System is delay $\longrightarrow$ $y(n-1) = ny(n-2) + x(n-1)$ — (b)

from (a) & (b), we've System is time variant.

* If $x(n) = u(n)$, then $|x(n)| \leq 1$.

But for this bounded i/p,

the o/p is -
$$\left.\begin{aligned} y(0) &= 1 \\ y(1) &= 1+1 = 2 \\ y(2) &= 2\times2+1 = 5 \\ &\vdots \end{aligned}\right\} \Rightarrow \text{which is unbounded}$$

thus,

System is unstable.

2.24 Consider the signal $\gamma(n) = a^n u(n)$, $0 < a < 1$

a) Show that any sequence $x(n)$ can be decomposed as
$$x(n) = \sum_{n=-\infty}^{\infty} c_k \gamma(n-k)$$ & express c_k in terms of $x(n)$

b) Use the properties of linearity & time invariance to express the o/p $y(n) = T(x(n))$ in terms of i/p $x(n)$ & the signal $g(n) = T(\gamma(n))$, where $T(.)$ is an LTI system.

c) Express the impulse response $h(n) = T[\delta(n)]$ in terms of $g(n)$.

Sol. a) From given $\gamma(n)$, $\delta(n)$ is expressed as -
$$\delta(n) = \gamma(n) - a\gamma(n-1)$$
$$\delta(n-k) = \gamma(n-k) - a\gamma(n-k-1)$$

Then,
$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$
$$= \sum_{k=-\infty}^{\infty} x(k)\left[\gamma(n-k) - a\gamma(n-k-1)\right]$$
$$= \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\gamma(n-k-1)$$

similar to

$$= \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k-1)\gamma(n-k)$$
$$x(n) = \sum_{k=-\infty}^{\infty}\left[x(k) - a x(k-1)\right]\gamma(n-k)$$
$$x(n) = \sum_{k=-\infty}^{\infty} c_k\,\gamma(n-k)$$

where
$$c_k = x(k) - a x(k-1)$$

(b) $y(n) = T(x(n))$

$\quad$ from given

$$= T\left[\sum_{k=-\infty}^{\infty} C_k\, \delta(n-k)\right]$$

$\quad$ from linear property

$$= \sum_{k=-\infty}^{\infty} C_k\left(T\left[\delta(n-k)\right]\right)$$

$\quad$ from given

thus $y(n) = \sum_{k=-\infty}^{\infty} C_k\, g(n-k) = T(g(n))$

ie o/p is expressed in terms of $g(n)$

(c) $h(n) = T[\delta(n)]$

$$= T[\delta(n) - a\delta(n-1)]$$

$$= T[\delta(n)] - T[a\delta(n-1)]$$

$h(n) = g(n) - a\,g(n-1)$

proved.

2.25 Determine the zero-i/p response of the system described by second-order difference equation $\rightarrow x(n) - 3y(n-1) - 4y(n-2) = 0$.

Sol. Given,

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

Zero-i/p response $\Rightarrow x(n) = 0$.

$$\Rightarrow x(n) - 3y(n-1) - 4y(n-2) = 0$$

$$0 - 3y(n-1) - 4y(n-2) = 0$$

$$3y(n-1) + 4y(n-2) = 0$$

$$y(n-1) + \frac{4}{3}y(n-2) = 0$$

$$y(n-1) = -\frac{4}{3}y(n-2)$$

when $n = 0$.

$$\cancel{y(0)}\; y(-1) = -\frac{4}{3}y(-2)$$

similarly

$$y(0) = \left(-\frac{4}{3}\right)^2 y(-2)$$

$$y(1) = \left(-\frac{4}{3}\right)^3 y(-2)$$

$$\vdots$$

Zero i/p response $\rightarrow$ $y(k) = \left(-\frac{4}{3}\right)^{k+2} y(-2)$

2.26 Determine the particular solution of the difference equation $y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$ when the forcing function is $x(n) = 2^n u(n)$

Sol

Given,

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$$

$\Rightarrow$ finding ~~particular~~ homogeneous solution by considering $x(n)=0$

$$y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2) = 0.$$

$$\lambda^n - \frac{5}{6}\lambda^{n-1} + \frac{1}{6}\lambda^{n-2} = 0.$$

$$\lambda^{n-2}\left[\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6}\right] = 0$$

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0$$

$$\Rightarrow \lambda = \frac{1}{2}, \frac{1}{3}$$

$$\Rightarrow y_h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{3}\right)^n.$$

$\Rightarrow$ finding particular solution to $x(n)$

$$x(n) = 2^n u(n)$$

for a unit-step sequence, the particular solution of o/p is a Constant K multiplied with i/p $\hookrightarrow u(n)$

i.e

$$y_p(n) = K\, 2^n u(n)$$

Substituting this $y_p(n)$ into D.E equation.

i.e $$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n).$$

$$y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2) = x(n)$$

$$K 2^n u(n) - \frac{5}{6} K\, 2^{n-1} u(n-1) + \frac{1}{6} K\, 2^{n-2} u(n-2) = 2^n u(n)$$

highest order in equation of D.E for $n=2$

$$2^2 K - \frac{5}{6} 2^1 K + \frac{1}{6} K 2^0 = 2^2$$

$$4K - \frac{5K}{3} + \frac{K}{6} = 4$$

$$\underline{K = \frac{8}{5}}$$

thus, $$y_p(n) = \frac{8}{5} 2^n u(n)$$

→ then, the total solution is–

$$y(n) = y_p(n) + y_h(n)$$

$$\Rightarrow y(n) = \frac{8}{5} 2^n u(n) + c_1 \left(\frac{1}{2}\right)^n u(n) + c_2 \left(\frac{1}{3}\right)^n u(n).$$

→ Determining c_1, c_2 :-

* Assuming initial conditions as zero.

i.e $y(-1) = y(-2) = 0$ — ⓐ

* Substitue $n = 0, 1$ in D.E Equation.

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$$

$$n=0 \rightarrow y(0) = \frac{5}{6} y(-1) - \frac{1}{6} y(0-2) + 2^0 u(0)$$

[from ⓐ]

$$= \frac{5}{6}(0) - \frac{1}{6}(0) + 1$$

$$y(0) = 1 \text{ — ⓒ}$$

$$n=1 \rightarrow y(1) = \frac{5}{6} y(1-1) - \frac{1}{6} y(1-2) + 2^1 u(1)$$

$$= \frac{5}{6} y(0) - \frac{1}{6} y(-1) + 2$$

$$= \frac{5}{6}(1) - \frac{1}{6}(0) + 2$$

$$y(1) = \frac{17}{6} \text{ — ⓓ}$$

* Substitue $n = 0, 1$ in total solution, then we've -

$$y(n) = \frac{8}{5} 2^n u(n) + c_1 \left(\frac{1}{2}\right)^n u(n) + c_2 \left(\frac{1}{3}\right)^n u(n)$$

$$n=0 \longrightarrow y(0) = \frac{8}{5} 2^0 u(0) + c_1 \left(\frac{1}{2}\right)^0 u(0) + c_2 \left(\frac{1}{3}\right)^0 u(0)$$

↓ from ⓒ

$$1 = \frac{8}{5} + c_1 + c_2$$

$$\Rightarrow \boxed{c_1 + c_2 = \frac{-3}{5}} \text{ — ①}$$

$$n=1 \longrightarrow y(1) = \frac{8}{5} 2^1 u(1) + c_1 \left(\frac{1}{2}\right)^1 u(1) + c_2 \left(\frac{1}{3}\right)^1 u(1)$$

↓ from ⓓ

$$\frac{17}{6} = \frac{16}{5} + \frac{c_1}{2} + \frac{c_2}{3}$$

$$\Rightarrow \boxed{3c_1 + 2c_2 = \frac{-11}{5}} \text{ — ②}$$

Solving ① & ② gives us-

$C_1 = -1, \quad C_2 = \frac{2}{5}$

$\Rightarrow$ thus total solution is:-

$$y(n) = \left[\frac{8}{5}(2^n) - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n \right] u(n)$$

2.27 Determine the response $y(n), n \geq 0$ of the system described by the 2nd order difference equation:

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1) \text{ to the i/p } x(n) = 4^n u(n)$$

Sol. Given, $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

$\Rightarrow$ C.E Equation:- $y(n) - 3y(n-1) - 4y(n-2) = 0$

$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$

$\Rightarrow \lambda^{n-2}[\lambda^2 - 3\lambda - 4] = 0$

$\Rightarrow \boxed{\lambda^2 - 3\lambda - 4 = 0}$

$\Rightarrow \lambda = 4, -1$

$\Rightarrow y_h(n) = C_1 4^n + C_2(-1)^n$

$\Rightarrow$ Particular solution:-

we've- $x(n) = 4^n u(n)$

Since 4 is a characteristic root & Excitation value

we've-

$y_p(n) = Kn\, 4^n u(n)$

Substitute this $y_p(n)$ in D.E equation.

$Kn4^n u(n) - 3K(n-1)4^{n-1}u(n-1) - 4K(n-2)4^{n-2}u(n-2) = 4^n u(n) + 2(4)^{n-1}u(n-1)$

for $n = 2$, highest order in given D.E equation

$\Rightarrow K(2)4^2 - 3K(2-1)4^1 - 4K(2-2)4^0 = 4^2 + 2(4)^1$

$\Rightarrow 32K - 12K - 0 = 16 + 8$

$20K = 24$

$K = \frac{24}{20} = \frac{6}{5}$

$\Rightarrow K = \frac{6}{5} \quad \rightarrow y_p(n) = \frac{6}{5} n 4^n u(n)$

⇒ Total solution is:

$$y(n) = y_p(n) + y_h(n)$$

$$y(n) = \left[\frac{6}{5} n 4^n + C_1 4^n + C_2 (-1)^n\right] u(n)$$

⇒ Determining C_1 & C_2:

* Assuming initial conditions as zero.

⇒ $y(-1) = y(-2) = 0$ — ⓐ

* Substitute $n=0, 1$ in D.E Equation.

$n=0$ ⟶ $y(0) - 3y(0-1) - 4y(0-2) = x(0) + 2x(0-1)$

$y(0) - 3y(-1) - 4y(-2) = 4^0 u(0) + 2 4^{-1} u(-1)$

$y(0) - 0 - 0 = 1 + 0.$

⇒ $y(0) = 1.$

$n=1$ ⟶ $y(1) - 3y(1-1) - 4y(1-2) = x(1) + 2x(1-1)$

$y(1) - 3y(0) - 4y(-1) = 4^1 u(1) + 2 4^0 u(0)$

$y(1) - 3 - 0 = 4 + 2$

$y(1) = 6 + 3$

$y(1) = 9.$

* Substitute $n=0, 1$ in Total solution eq

$n=0$ ⟶ $y(0) = \left[\frac{6}{5}(0) 4^0 + C_1 4^0 + C_2 (-1)^0\right] u(0)$

↓

$1 = C_1 + C_2$ — ⓑ

$n=1$ ⟶ $y(1) = \left[\frac{6}{5}(1) 4^1 + C_1 4^1 + C_2 (-1)^1\right] u(1)$

↓

$9 = \frac{24}{5} + 4C_1 - C_2$

$4C_1 - C_2 = \frac{21}{5}$ — ⓒ

Solving ⓑ & ⓒ we've — $C_1 = \frac{26}{25}$, $C_2 = \frac{-1}{25}$

then the total solution is —

$$y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n\right] u(n)$$

2

Q.28 Determine Impulse response of the following Causal system:

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

Sol. $\Rightarrow y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

$\hookrightarrow \lambda = 4, -1 \Rightarrow y_h(n) = c_1 4^n + c_2(-1)^n \Big\} \Rightarrow y(n) = c_1 4^n + c_2(-1)^n$

$\Rightarrow$ when $x(n) = \delta(N) \Rightarrow y_p(n) = 0$

then C_1 & C_2 are?

* Assuming Initial Conditions as zero $\Rightarrow y(-1) = y(-2) = 0$

* substitute $n = 0, 1$ in D.E. Equation

$n = 0 \Rightarrow y(0) - 3y(0-1) - 4y(0-2) = x(0) + 2x(0-1)$

$\Rightarrow y(0) - 3y(-1) - 4y(-2) = \delta(0) + 2\delta(-1)$

$\Rightarrow y(0) - 0 - 0 = 1 + 0$

$\Rightarrow y(0) = 1$

$n = 1 \rightarrow y(1) - 3y(1-1) - 4y(1-2) = x(1) + 2x(1-1)$

$\Rightarrow y(1) - 3y(0) - 4y(-1) = \delta(1) + 2\delta(0)$

$y(1) - 3(1) - 0 = 0 + 2$

$\Rightarrow y(1) = 2 + 3$

$\Rightarrow y(1) = 5$

$\Rightarrow \delta(1) = 1.$

$\Rightarrow$ ~~Similar to last problem,~~

~~we've $C_1 + C_2 = y(0)$ & $4C_1 - C_2 =$~~

* Substitue $n = 0, 1$ in total solution

then we've -

$y(0) = C_1 + C_2 \Rightarrow C_1 + C_2 = 1$ — (B)

$y(1) = 4C_1 - C_2 \Rightarrow 4C_1 - C_2 = 5$ — (C)

Solving (B) & (C) we've -

$C_1 = \frac{6}{5}, \quad C_2 = \frac{-1}{5}$

then the total solution is -

Impulse response $= h(n) = y(n) = \left[\frac{6}{5} 4^n - \frac{1}{5}(-1)^n\right] u(n)$

2.29 Let $x(n) \to N_1 \le n \le N_2$ & $h(n) \to M_1 \le n \le M_2$ be 2 finite-duration signals

a) Determine the range $L_1 \le n \le L_2$ of their convulution in terms of N_1, N_2, M_1 & M_2

b) Determine the limits of the cases of partial overlap from left, full overlap & partial overlap from the right. for convenience, assume that h(n) has shorter duration than x(n).

c) Illustrate the validity of your results by computing the convulution of the signals $x(n) = \begin{cases} 1 & -2 \le n \le 4 \\ 0 & \text{Elsewhere} \end{cases}$ & $h(n) = \begin{cases} 2 & -1 \le n \le 2 \\ 0 & \text{elsewhere} \end{cases}$

Sol: ~~a) $L_1 \le n \le L_2$ convulution~~

Sol a) Convulution of x(n) & h(n) gives the range $L_1 \le n \le L_2$

where $L_1 : N_1 + M_1$ & $L_2 : N_2 + M_2$

b) L_1 & L_2 for different ~~ranges~~ cases:

partial overlap from left: $L_1 = N_1 + M_1$, $L_2 = N_1 + M_2 - 1$

Full overlap: $L_1 = N_1 + M_2$, $L_2 = N_2 + M_1$

Partial overlap from right: $L_1 = N_2 + M_1 + 1$, $L_2 = N_2 + M_2$

c) finding the above values of L_1 & L_2 for the different mentioned above cases for $x(n) = \{1, 1, 1, 1, 1, 1, 1\}$ & $h(n) = \{2, 2, 2, 2\}$

here $N_1 = -2$, $N_2 = 4$ $\Rightarrow$ $N_1 \le n \le N_2$

$M_1 = -1$, $M_2 = 2$ $\Rightarrow$ $M_1 \le n \le M_2$

then L_1, L_2 for below cases are:-

partial overlap from left: $L_1 = -3$, $L_2 = -1$

full overlap:- $L_1 = 0$, $L_2 = 3$

partial overlap from right: $L_1 = 4$, $L_2 = 6$.

Q.30 Determine the impulse response and the unit step response of the systems described by the difference equation.

a) $y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

b) $y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$

Sol. Impulse response = total solution.

a) (i) Impulse response

$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

$\rightarrow y_h(n)$:-

$x(n) = 0$

$\Rightarrow y(n) = 0.6y(n-1) - 0.08y(n-2)$

$\lambda^n = 0.6\lambda^{n-1} - 0.08\lambda^{n-2}$

$\Rightarrow \lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$

$\lambda^{n-2}\left[\lambda^2 - 0.6\lambda + 0.08\right] = 0$

$\Rightarrow \lambda^2 - 0.6\lambda + 0.08 = 0$

$\Rightarrow \lambda = 0.2, 0.4$

then

$y_h(n) = C_1\left(\frac{1}{5}\right)^n + C_2\left(\frac{2}{5}\right)^n$

$\rightarrow y_p(n)$:-

$x(n) = \delta(n)$.

$\Rightarrow y_p(n) = 0$

$\rightarrow y(n) = y_p(n) + y_h(n)$

$\Rightarrow y(n) = C_1\left(\frac{1}{5}\right)^n + C_2\left(\frac{2}{5}\right)^n$

$\Rightarrow$ Determining C_1, C_2:-

Assu & $y(-1) = y(-2) = 0$

& $n = 0, 1$ in D.E equation

$\xrightarrow{n=0} y(0) = 0.6y(-1) - 0.08y(-2) + \delta(0)$

$\Rightarrow y(0) = 0.6(0) - 0.08(0) + 1$

$\Rightarrow y(0) = 1$

$\xrightarrow{n=1} y(1) = 0.6y(0) - 0.8y(-1) + \delta(1)$

$= 0.6(1) - 0 + 0$

$\Rightarrow y(1) = 0.6$

& $n = 0, 1$ in total solution gives us-

$C_1 + C_2 = 1$ ← (B)

$\frac{1}{5}C_1 + \frac{2}{5}C_2 = 0.6$ — (C)

Solving (B) & (C) gives us-

$C_1 = -1, \quad C_2 = 3$

then

$h(n) = y(n) =$ Impulse response is

$h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right]u(n)$

(ii) step response

$s(n) = \sum_{k=0}^{n} h(n-k), \quad n \geq 0$

$= \sum_{k=0}^{n}\left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$

$s(n) = \left\{\frac{1}{0.12}\left[\left(\frac{2}{5}\right)^{n+1} - 1\right] - \frac{1}{0.16}\left[\left(\frac{1}{5}\right)^{n+1} - 1\right]\right\}$

(b) i) Impulse response

$y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$

$\to y_h(n)$:-

$x(n) = 0$ & $x(n-2) = 0$.

$\Rightarrow y(n) = 0.7y(n-1) - 0.1y(n-2)$

$\lambda^n = 0.7\lambda^{n-1} - 0.1\lambda^{n-2}$

$\lambda^n - 0.7\lambda^{n-1} + 0.1\lambda^{n-2} = 0$

$\lambda^{n-2}[\lambda^2 - 0.7\lambda + 0.1] = 0$

$\Rightarrow \lambda^2 - 0.7\lambda + 0.1 = 0$

$\Rightarrow \lambda = \frac{1}{2}, \frac{1}{5}$

then

$y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{5}\right)^n$

$\to y_p(n)$:

$x(n) = \delta(n)$

$\Rightarrow y_p(n) = 0$

$y(n) = y_p(n) + y_h(n)$.

$\Rightarrow y(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{5}\right)^n$

determining c_1, c_2 :-

* $y(-1) = y(-2) = 0$

* $n = 0, 1$ in D.E equation

$\xrightarrow{n=0} y(0) = 0.7y(-1) - 0.1y(-2) + 2\delta(0) - \delta(-2)$

$y(0) = 0.7(0) - 0.1(0) + 2 - 0$

$\Rightarrow y(0) = 2$

$\xrightarrow{n=1} y(1) = 0.7y(0) - 0.1y(-1) + 2\delta(1) - \delta(-1)$

$= 0.7y(0) - 0.1(0) + 2(0) - 0$

$y(1) = 0.7 \times 2 = 1.4 = y(1)$

* $n = 0, 1$ in total solution gives us-

$\to c_1 + c_2 = 2$ — (b).

$\to \frac{1}{2}c_1 + \frac{1}{5}c_2 = 1.4$ — (c)

Solving (b) & (c) gives us-

$c_1 = \frac{10}{3}, \quad c_2 = -\frac{4}{3}$

then

$h(n) = y(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$

(ii) Step response -

$s(n) = \sum_{k=0}^{n} h(n-k)$

$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$

$= \frac{10}{3}\left(\frac{1}{2}\right)^n\sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n\sum_{k=0}^{n} 5^k$

$s(n) = \left[\frac{10}{3}\left[\frac{1}{2}^n\left(2^{n+1} - 1\right)\right] - \frac{1}{3}\left[\frac{1}{5}^n\left(5^{n+1} - 1\right)\right]\right] \times u(n)$

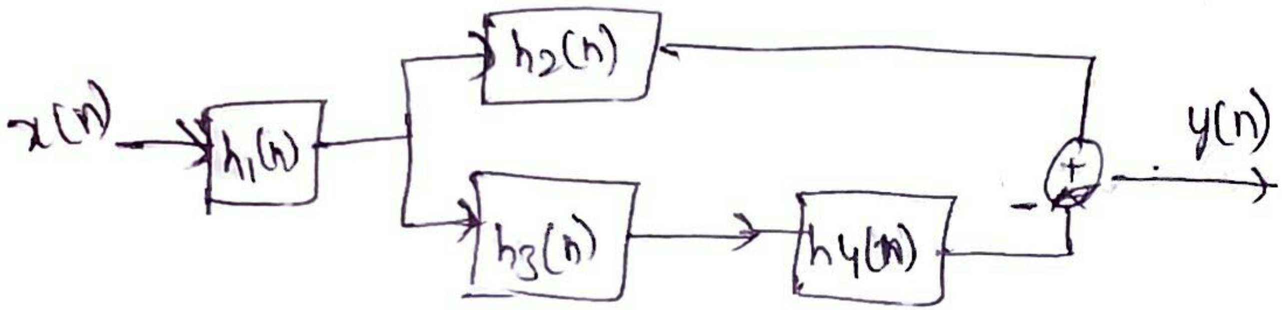
Q.32 Consider the interconnection of LTI systems as shown in fig below

a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$ & $h_4(n)$

b) Determine $h(n)$ when $h_1(n) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{2}\right)$

$$h_2(n) = h_3(n) = (n+1)u(n)$$

$$h_4(n) = \delta(n-2)$$



c) Determine the response of the system in part (b) if

$$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$$

Sol. a) $h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$

b) $\rightarrow h_3(n) * h_4(n) = (n+1)\,u(n) * \delta(n-2) \rightarrow (\because x(n) * \delta(n-n_0) = x(n-n_0))$

$= (n+1-2)\,u(n-2)$

$h_3(n) * h_4(n) = (n-1)\,u(n-2)$

$\rightarrow h_2(n) - h_3(n) * h_4(n) = (n+1)u(n) - (n-1)u(n-2)$

$= nu(n) + u(n) - nu(n-2) + u(n-2)$

$h_2(n) - h_3(n) * h_4(n) = 2u(n) - \delta(n)$

$\rightarrow h_1(n) * [h_2(n) - h_3(n) * h_4(n)] = \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right] * [2u(n) - \delta(n)]$

where
$h_1(n) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{2}\right)$
$= \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

$= \frac{1}{2}\delta(n) * 2u(n) + \frac{1}{4}\delta(n-1) * 2u(n)$

$+ \frac{1}{2}\delta(n-2) * 2u(n) - \frac{1}{2}\delta(n) * \delta(n)$

$+ \frac{1}{4}\delta(n-1) * \delta(n) - \frac{1}{2}\delta(n-2) * \delta(n)$

$=$

$\Rightarrow h(n) = \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

c) $x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$

$\Rightarrow x(n) = \{+1, 0, \underset{\uparrow}{0}, 3, 0, -4\}$

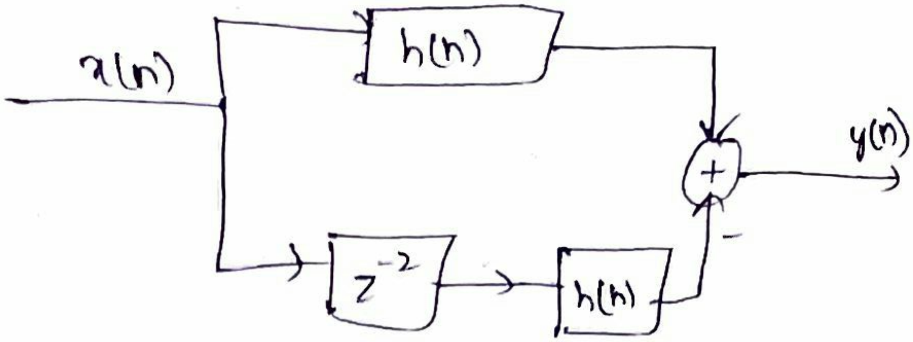
$h(n) = \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

$\Rightarrow h(n) = \left\{\frac{1}{2}, \frac{5}{4}, 2, \frac{5}{2}, \frac{5}{2}, \ldots\right\}$

then $y(n) = x(n) * h(n)$ is

$\Rightarrow y(n) = \left\{ \frac{1}{2}, \frac{5}{4}, 2, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, \dots \right\}$

2.33 Consider the system in below figure with $h(n) = a^n u(n)$, $-1 < a < 1$. Determine the response $y(n)$ of the system to the excitation: $x(n) = u(n+5) - u(n-10)$



Sol

$z^{-2} \Rightarrow$ Delay by 2 units.

here

$$y(n) = x(n) * h(n)$$

$$y(n) = x(n) * (h(n) - h(n-2))$$

$$y(n) = x(n) * h(n) - x(n) * h(n-2)$$

~~We know that~~, Let us find out - $s(n) = x(n) * h(n)$

$$s(n) = u(n) * h(n) \qquad \text{Let } x(n) = u(n)$$

$$= \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k)$$

Let $h(n) = a^n u(n)$

$$= \sum_{k=0}^{n} a^{n-k} u(n-k)$$

$$s(n) = \frac{a^{n+1} - 1}{a - 1} u(n), \quad n \geq 0$$

thus we've -

- for $x(n) = u(n)$ & $h(n)$ $\rightarrow$ o/p $s(n) = \frac{a^{n+1} - 1}{a-1} u(n)$, $n \geq 0$

- then

for

$x(n) = u(n+5) - u(n-10)$ & $h(n)$ $\rightarrow$ o/p: $s(n+5) - s(n-10) = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10)$

then back to our problem

$y(n) = x(n) * h(n) - x(n) * h(n-2)$ gives ↓

$$= \left(\frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10) \right) -$$

$$\left(\frac{a^{n+4} - 1}{a-1} u(n+3) - \frac{a^{n-11} - 1}{a-1} u(n-12) \right)$$

take n values as $n-2$. i.e if we've $n+6$ then $n+6-2 = n+4$

$$\Rightarrow \underline{y(n)} = \frac{a^{n+6} - 1}{a-1} u(n+5) - \frac{a^{n-9}}{a-1} u(n-10) - \frac{a^{n+4} - 1}{a-1} u(n+3) + \frac{a^{n-11} - 1}{a-1} u(n-12)$$

2.35 Determine the range of values of the parameter a for which the linear time-invariant system with impulse response

$$h(n) = \begin{cases} a^n & n \geq 0, \text{ n even} \\ 0 & \text{otherwise} \end{cases}$$ is stable.

Sol. Given $h(n) = \begin{cases} a^n & n \geq 0, \; n = \text{even} \\ 0 & \text{otherwise} \end{cases}$

$$\Rightarrow \sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0, n=\text{even}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n}$$ (2 is there for the sake of even)

$$\sum_{n=-\infty}^{\infty} |h(n)| = \frac{1}{1-|a|^2}$$ $\rightarrow$ stable $\underline{\text{if}}$ $|a| < 1$.

2.34 compute and sketch the step response of the system

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

Sol. $h(n) = \frac{1}{M}[u(n) - u(n-M)]$

$$s(n) = \sum_{k=-\infty}^{\infty} u(k) h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k)$$

$$s(n) = \begin{cases} \frac{n+1}{M}, & n < M \\ 1, & n \geq M \end{cases}$$

2.35 Determine the response of the system with impulse response $h(n) = a^n u(n)$ to the i/p signal $x(n) = u(n) - u(n-10)$

Sol. we've to find $\rightarrow y(n) = ?$

$$y_1(n) = x(n) * h(n)$$

$$= \sum_{k=0}^{\infty} x(k) h(n-k)$$ Let $x(n) = u(n)$.

$$= \sum_{k=0}^{\infty} u(k) h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k}$$

$$= a^n \sum_{k=0}^{n} a^{-k}$$

$$= a^n \cdot \frac{1-a}{1-a} u(n)$$

$$y_1(n) = \frac{1-a^{n+1}}{1-a} u(n)$$

Similiarily

$y_1(n-10)$ is the o/p when $x(n) = u(n-10)$

$$y_1(n-10) = \frac{1-a^{n-9}}{1-a} u(n-10)$$

then $y(n) = y_1(n) - y_1(n-10)$

$$y(n) = \frac{1}{1-a}\left[(1-a^{n+1})u(n) - (1-a^{n-9})u(n-10)\right]$$

2.37 Determine the response of the system characterized by the impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$ to i/p signal

$$x(n) = \begin{cases} 1 & 0 \le n < 10 \\ 0 & \text{otherwise} \end{cases}$$

Sol. we know that

$$y(n) = \frac{1}{1-a}\left[(1-a^{n+1})u(n) - (1-a^{n-9})u(n-10)\right],$$ when $h(n) = a^n u(n)$ & $x(n) = \begin{cases} 1 & 0 \le n < 10 \\ 0 & \text{otherwise} \end{cases}$ ↳ similar to $u(n) - u(n-10)$

now ~~what~~ when $h(n) = \left(\frac{1}{2}\right)^n u(n)$ $y(n) = ?$

It is clear that -

$$a = \frac{1}{2}$$

then

$$y(n) = \frac{1}{1-\frac{1}{2}}\left[\left(1-\left(\frac{1}{2}\right)^{n+1}\right)u(n) - \left(1-\left(\frac{1}{2}\right)^{n-9}\right)u(n-10)\right]$$

$$y(n) = 2\left[\left(1-\left(\frac{1}{2}\right)^{n+1}\right)u(n) - \left(1-\left(\frac{1}{2}\right)^{n-9}\right)u(n-10)\right]$$

$\frac{1}{1-\frac{1}{2}} = \frac{2}{2-1} = 2$

2.38 Determine the response of the (relaxed) system characterised by impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$ to i/p signals-

a) $x(n) = 2^n u(n)$ b) $x(n) = u(-n)$

Sol.

$$y(n) = ?$$

a) $$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u(k) \cdot 2^{n-k} u(n-k)$$

$$= \sum_{k=-\infty}^{n} \left(\frac{1}{2}\right)^k \cdot 2^{n-k}$$

$$= \sum_{k=-\infty}^{n} \left(\frac{1}{2}\right)^k \cdot 2^n \cdot \left(\frac{1}{2}\right)^k$$

$$= 2^n \sum_{k=-\infty}^{n} \left(\frac{1}{4}\right)^k$$

$$= 2^n \left[1 - \left(\frac{1}{4}\right)^{n+1}\right] \cdot \frac{4}{3}$$

$$= 2^{n+1} \left[1 - \left(\frac{1}{4}\right)^{n+1}\right] \cdot \frac{2}{3}$$

$$= \left[2^{n+1} - \left(\frac{2}{4_2}\right)^{n+1}\right] \cdot \frac{2}{3}$$

$$\underline{y(n)} = \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] u(n)$$

b) $y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u(k) \cdot u(-n-k)$

$$= \sum_{k=-\infty}^{\infty} h(k) \cancel{u(-n+k)}$$

$$= \sum_{k=0}^{\infty} h(k)$$

$$y(n) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$$

$$\text{@} \underline{y(n) = 2}, \quad n < 0$$

(b) $y(n) = \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k$$

$$= 2 - \left(\frac{1 - \left(\frac{1}{2}\right)^n}{1/2}\right)$$

$$=$$

$$y(n) = 2\left(\frac{1}{2}\right)^n, \quad n \geq 0$$

thus

$$y(n) = \begin{cases} 2, & n < 0 \\ 2\left(\frac{1}{2}\right)^n, & n \geq 0 \end{cases}$$

2.39) Three systems with impulse response $h_1(n) = \delta(n) - \delta(n-1)$; $h_2(n) = h(n)$ and $h_3(n) = u(n)$ are connected in cascade.

a) what is the impulse response of overall system $h_e(n)$

b) Does the order of the interconnection affect the overall system?

Sol. a) $h_e(n) = h_1(n) * h_2(n) * h_3(n)$

$$= [\delta(n) - \delta(n-1)] * h(n) * u(n)$$

$$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$$

$$= [u(n) - u(n-1)] * h(n)$$

$$= u(n) * h(n)$$

$$\underline{h_e(n) = h(n)}$$

b) No, it doesn't affect. Because convulutions satisfies commutative law.

$$\underline{a * b = b * a}$$

2.40) a) Prove and explain graphically the difference between the relations $x(n)\delta(n-n_0) = x(n_0)\delta(n-n_0)$ & $x(n) * \delta(n-n_0) = x(n-n_0)$

b) Show that a discrete-time system, which is described by a convulution summation is LTI & relaxed.

Sol a) (i) $x(n)\delta(n-n_0) = x(n_0)\delta(n-n_0)$

This is true, Because, only at $n = n_0$ the value of $x[n]$ is existed with an impulse.

(ii) $x(n) * \delta(n-n_0) = x(n-n_0)$

This can be obtained from the shifted version of $x(n)$

b) $y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) = h(n) * x(n)$

Linearity:

$$x_1(n) \longrightarrow y_1(n) = h(n) * x_1(n)$$

$$x_2(n) \longrightarrow y_2(n) = h(n) * x_2(n)$$

$$x(n) = ax_1(n) + bx_2(n) \longrightarrow y(n) = h(n) * x(n)$$

$$y(n) = h(n) * [ax_1(n) + bx_2(n)]$$

$$= ah(n) * x_1(n) + bh(n) * x_2(n)$$

thus, $\boxed{y(n) = a\,y_1(n) + b\,y_2(n)}$

Time invariance:

$x(n) \rightarrow y(n) = h(n) * x(n) \rightarrow$ delay ↓

$h(n) * x(n-n_0) = y(n-n_0)$ — (b)

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$

$$= \sum_{k=0}^{\infty} h(k)\, x(n-n_0-k)$$

$$y_1(n) = y(n-n_0) \quad -(a)$$

thus (a) & (b) shows system is time invariant.

c) what is the impulse response of the system described by $y(n) = x(n-n_0)$

Sol. $y(n) = h(n) * x(n) = x(n-n_0)$

To get o/p $x(n-n_0)$, when let $h(n)$ be -

$\Rightarrow \delta(n-n_0) * x(n) = x(n-n_0)$

therefore, Impulse response, $h(n) = \delta(n-n_0)$

2.41) Two signals $s(n)$ and $v(n)$ are related through the following difference equations.

$$s(n) + a_1 s(n-1) + \dots + a_N s(n-N) = b_0 v(n)$$

i) Design the system that generates $s(n)$ when excited by $v(n)$

ii) The system that generates $v(n)$ when excited by $s(n)$.

iii) what is the impulse response of the cascade interconnection of the systems in parts of (i) & (ii)

Sol. i) $s(n) = -a_1 s(n-1) - a_2 s(n-2) - \dots - a_N s(n-N) + b_0 v(n)$

ii) $v(n) = \frac{1}{b_0}\left[s(n) + a_1 s(n-1) + \dots + a_N s(n-N)\right]$

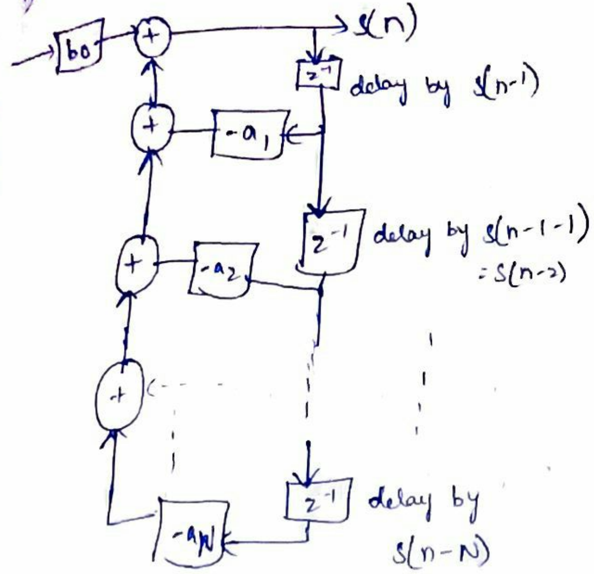
Block diagram for (i) & (ii)

↓

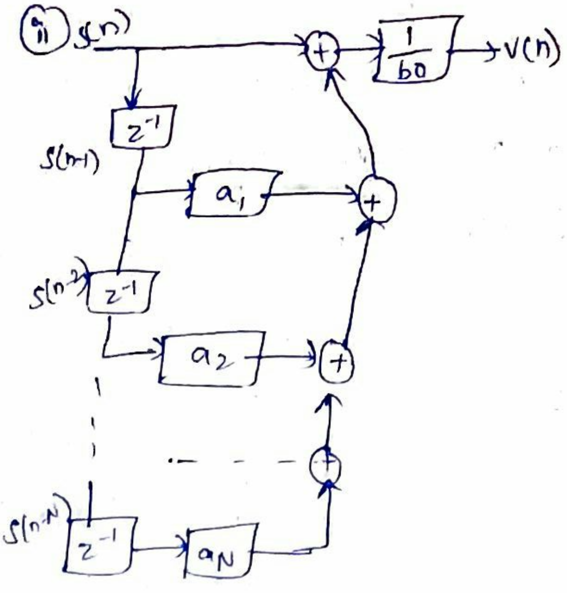
10)

(b)

i)



(ii)



(iii) In (i) → Impulse response is:- $-a_1, -a_2, -a_3 \cdots -a_N = -a_n$

In (ii) → Impulse response is:- $a_1, a_2, a_3 \cdots a_N = +a_n$

Q.42) Compute the zero-state response of the system described by the difference equation → $y(n) + \frac{1}{2}y(n-1) = x(n) + 2x(n-2)$ to the i/p $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 2, 1\}$ by solving the difference equation recursively.

Sol.

$$y(n) + \frac{1}{2}y(n-1) = x(n) + 2x(n-2)$$

zero state response → $y(n) = -\frac{1}{2}y(n-1) + x(n) + 2x(n-2)$

considering unknown as 0

At

At $n=-2$, $x(n)=1$ then $y(-2) = -\frac{1}{2}y(-3) + x(-2) + 2x(-4) = 1$ $x(0) = 3$

$n=-1$, $x(n)=2$ then $y(-1) = -\frac{1}{2}y(-2) + x(-1) + 2x(-3) = 3/2$

$n=0$, $x(n)=3$ then $y(0) = -\frac{1}{2}y(-1) + x(0) + 2x(-2) = -\frac{3}{4} + 3 + 2 = \frac{17}{4}$

$n=1$, $x(n)=4$ then $y(1) = -\frac{1}{2}y(0) + x(1) + 2x(-1) = -\frac{17}{8} + 4 + 4 = \frac{47}{8}$

$n=2$, $x(n)=2$ then $y(2) = -\frac{1}{2}y(1) + x(2) + 2x(0) = -\frac{47}{16} + 2 + 6 = \frac{81}{16}$

$n=3$, $x(n)=1$ then $y(3) = -\frac{1}{2}y(2) + x(3) + 2x(1) = -\frac{81}{32} + 1 + 8 = \frac{207}{32}$

thus,

$$y(n) = \{1, \frac{3}{2}, \underset{\uparrow}{\frac{17}{4}}, \frac{47}{8}, \frac{81}{16}, \frac{207}{32}\}.$$

6

2.43) Determine the direct form II realization for each of the following LTI systems.

a) $2y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-5)$

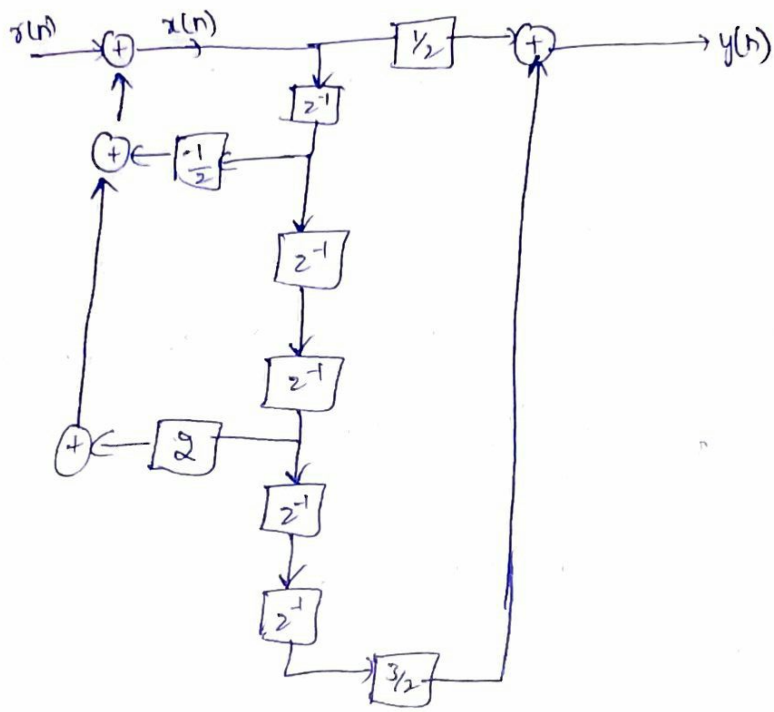
b) $y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$

Sol. a) $2y(n) + y(n-1) + 4y(n-3) = x(n) + 3x(n-5)$

$$y(n) = \frac{1}{2}\left[-y(n-1) + 4y(n-3) + x(n) + 3x(n-5)\right]$$

$$y(n) = -\frac{1}{2}y(n-1) + 2y(n-3) + \frac{1}{2}x(n) + \frac{3}{2}x(n-5)$$

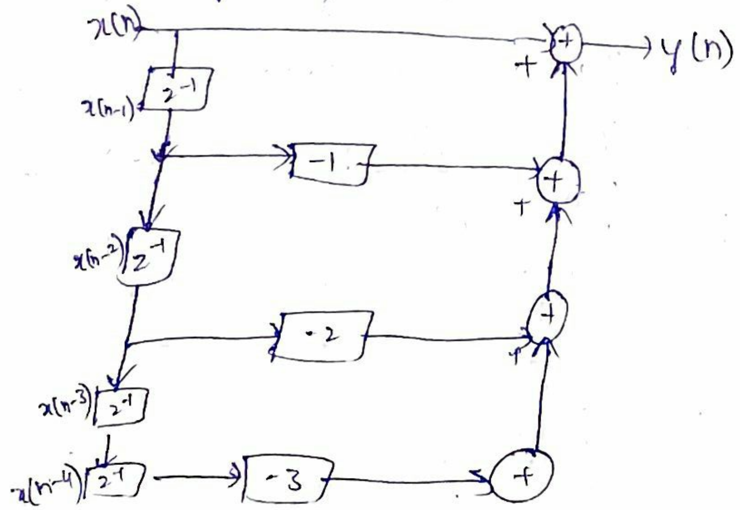
gives Block diagram



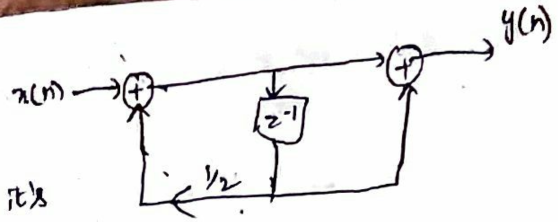
b)

$$y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$$

Block diagram



Q.4) consider the discrete system



a) compute the 1st 10 samples of it's impulse response.

b) Find the i/p-o/p relation

c) Apply i/p $x(n) = \{1, 1, \dots\}$ & compute the 1st 10 samples of o/p

d) compute the 1st 10 samples of o/p for the i/p given in part (c) by using convolution.

e) Is the system causal & stable.

Sol. a) here $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

Let $x(n) = \{0, \overset{\downarrow}{1}, 0, 0, 0, \dots\} = \delta(n)$

At

$n=0 \rightarrow x(n)=1 \rightarrow$ then $y(0) = \frac{1}{2} y(-1) + x(0) + x(-1) = 1$

$n=1 \rightarrow x(n)=0 \rightarrow$ then $y(1) = \frac{1}{2} y(0) + x(1) + x(0) = \frac{1}{2} + 0 + 1 = 3/2$

$n=2 \rightarrow x(n)=0 \rightarrow$ then $y(2) = \frac{1}{2} y(1) + x(2) + x(1) = \frac{1}{2}\left(\frac{3}{2}\right) + 0 + 0 = 3/4$

$n=3 \rightarrow x(n)=0 \rightarrow$ then $y(3) = \frac{1}{2} y(2) + x(3) + x(2) = \frac{1}{2}\left(\frac{3}{4}\right) + 0 + 0 = 3/8$

$\vdots$

$n=n \rightarrow x(n)=0 \rightarrow$ then $y(n) = \left\{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \frac{3}{64}, \frac{3}{128}, \frac{3}{256} \dots\right\}$

b) $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

c) $x(n) = \{\underset{\uparrow}{1}, 1, 1, \dots\}$

At

$n=0 \rightarrow x(n)=1 \rightarrow$ then $y(0) = \frac{1}{2} y(-1) + x(0) + x(-1) = 1$

$n=1 \rightarrow x(n)=1 \rightarrow$ then $y(1) = \frac{1}{2} y(0) + x(1) + x(0) = \frac{1}{2} + 1 + 1 = 5/2$

$n=2 \rightarrow x(n)=1 \rightarrow$ then $y(2) = \frac{1}{2} y(1) + x(2) + x(1) = \frac{5}{4} + 1 + 1 = \frac{13}{4}$

$n=3 \rightarrow x(n)=1 \rightarrow$ then $y(3) = \frac{1}{2} y(2) + x(3) + x(2) = \frac{13}{8} + 2 = \frac{29}{8}$

$\vdots$

$n=n \rightarrow x(n)=1 \rightarrow$ then $y(n) = \left\{1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \dots\right\}$

d) $y(n) = u(n) * h(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$

$y(n) = \sum_{k=0}^{n} h(n-k)$

At $n=0$, $y(0) = h(0) = 1$

At $n=1$, $y(1) = h(0) + h(1) = 5/2$

At $n=2 \rightarrow y(2) = h(0) + h(1) + h(2) = \frac{13}{4}$

e) yes the system is causal & stable

$\downarrow$ from the ~~dif~~ zerostate response equation

$\downarrow$ from $\sum_{n=0}^{\infty} |h(n)| = 1 + \frac{3}{2}\left(1 + \frac{1}{2} + \frac{1}{4} \cdots\right) = 4$

stable o/p

2.45) Consider the system described ~~the~~ by the equation.

$y(n) = a y(n-1) + b x(n)$

then a) Determine b in terms of a. So that $\sum_{n=-\infty}^{\infty} h(n) = 1$

b) compute zero-state step response $s(n)$ of the system. & choose b so that $s(\infty) = 1$

c) compare the values of b obtained in parts (a) & (b)

Sol. a) $y(n) = a y(n-1) + b x(n)$

$h(n) = b a^n u(n)$

$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$

$\Rightarrow \boxed{b = 1-a}$

b) $s(n) = \sum_{k=0}^{n} h(n-k)$

$s(n) = b\left[\frac{1-a^{n+1}}{1-a}\right] u(n)$

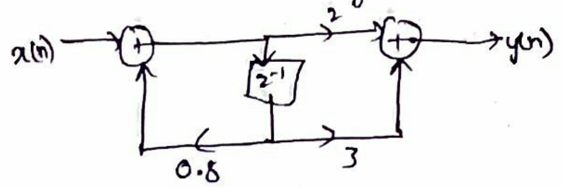
At $n = \infty$

$\Rightarrow s(\infty) = \frac{b}{1-a} = 1$

$\Rightarrow \boxed{b = 1-a}$

(c) In both cases, b is same & equal.

2.4b) A discrete time system is realized by the structure.



then a) determine impulse response

b) determine a realization for it's inverse of the system. That is the system which produces $x(n)$ as an o/p when $y(n)$ is used as an i/p.

Sol. from Block diagram,

we've -
$$y(n) = 2x(n) + 3x(n-1) + 0.8y(n-1)$$
$$\Rightarrow y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$$

$y_h(n)$:-
$$y(n) - 0.8y(n-1) = 0$$
$$\lambda^n - 0.8\lambda^{n-1} = 0$$
$$\lambda^{n-1}[\lambda - 0.8] = 0$$
$$\lambda - 0.8 = 0.$$
$$\Rightarrow \lambda = 0.8$$
$$\Rightarrow y_h(n) = c(0.8)^n$$

$y_p(n)$: Let $x(n) = \delta(N)$ $\Rightarrow y_p(n) = 0$

$y(n) = y_p(n) + y_h(n)$:-
$$\Rightarrow y(n) = c(0.8)^n$$

Substitute $n=0$ in d.E equation & initial conditions ~~$x(n-1)=0$~~, $y(-1)=0$.

$$\Rightarrow y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$$

At $n=0$ we've -
$$y(0) - 0.8y(-1) = 2x(0) + 3x(-1)$$
$$y(+0) - 0.8(0) = 2\delta(0) + 3\delta(-1)$$
$$y(0) = 2$$

Let us consider the response of the system is
$$y(n) - 0.8y(n-1) = x(n)$$

~~At n=0 we~~
→ Substitute $n=0$ in above equation at initial conditions $y(-1)=0$

then we've -

$$y(0) - 0.8\,y(-1) = x(0)$$

$$\Rightarrow y(0) - 0.8(0) = \delta(0)$$

$$\underline{y(0) = 1} \quad \text{— (a)}$$

similiarly substitue $n=0$ in $y(n)$

then we've - $y(0) = c\,(0.8)^0$

$$\Rightarrow y(0) = c \quad \text{— (b)}$$

Equating (a) & (b) gives us -

$$\boxed{c = 1}$$

- then impulse response, $h(n)$ is -

$$h(n) = y(n) = 2\,(0.8)^n u(n) + 3\,(0.8)^{n-1} u(n-1)$$

$$\Rightarrow h(n) = 2\delta(n) + 4.6\,(0.8)^{n-1} u(n-1).$$

b) Inverse of the system is characterized by the difference equation - $y(n) = 2x(n) + 3x(n-1) + 0.8y(n-1)$

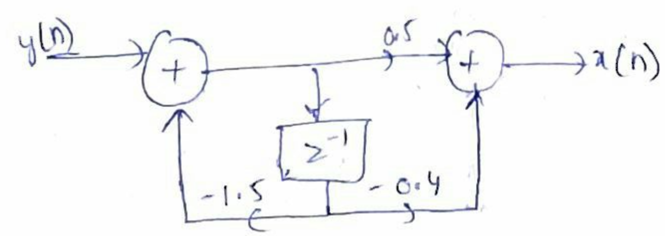
$$\frac{y(n) - 0.8y(n-1) - 3x(n-1)}{2} = x(n)$$

$$\Rightarrow \underline{x(n)} = -1.5\,x(n-1) + \frac{1}{2}y(n) - 0.4\,y(n-1)$$

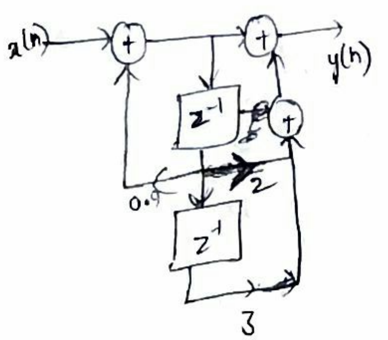
↓

gives block diagram.

↓



Q.47 Consider the discrete time system below.



a) Compute the 1st six values of the impulse response of the system.

b) Compute the first six values of the zero-state step response of the system.

c) Determine an analytical expression for the impulse response of the system.

Sol. From Block diagram,

$$y(n) = x(n) + 0.9y(n-1) + 2x(n-1) + 3x(n-2)$$

a) $x(n) = \{\underset{\uparrow}{1}, 0, 0, \dots\}$

At $n=0 \to x(n)=1$: then $y(0) = 0.9y(-1) + x(0) + 2x(-1) + 3x(-2) = 1$.

At $n=1 \to x(n)=0$: then $y(1) = 0.9y(0) + x(1) + 2x(0) + 3x(-1) = 2.9$

At $n=2 \to x(n)=0$: then $y(2) = 0.9y(1) + x(2) + 2x(1) + 3x(0) = 5.61$

thus

$$y(n) = \{1, 2.9, 5.61, 5.049, 4.544, 4.090 \dots\}$$

b) $x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1, \dots\}$

At $n=0 \to x(n)=1$; then $y(0) = 0.9y(-1) + x(0) + 2x(-1) + 3x(-2) = 1$

At $n=1 \to x(n)=1$; then $y(1) = 0.9y(0) + x(1) + 2x(0) + 3x(-1) = 3.9$

At $n=2 \to x(n)=1$; then $y(2) = 0.9y(1) + x(2) + 2x(1) + 3x(0) = 9.51$

then

$$y(n) = \{1, 3.91, 9.51, 14.56, 19.10, 23.19 \dots\}.$$

$\to y(4) + y(3) + y(2) + y(1) + y(0)$

$y(3) + y(2) + y(1) + y(0)$.

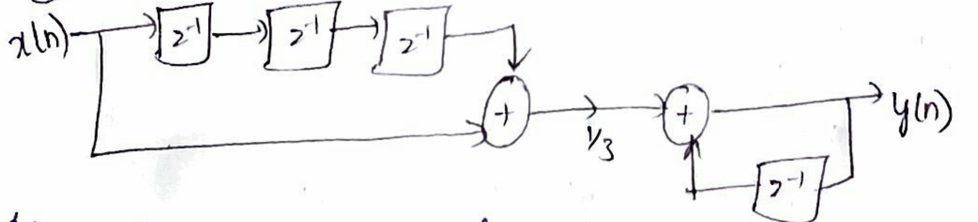
c) $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$

$h(n) = \delta(n) + 2.9\delta(n-1) + 5.61(0.9)^{n-2} u(n-2)$

$2(0.9)(0.9)^{n-1}$

2.48 Determine & sketch the impulse response of the following systems for $n \geq 0$.

(a)



Sol. from the above figure, we've

$$y(n) = \frac{1}{3}[x(n) + x(n-3)] + y(n-1)$$

At $x(n) = \delta(n) = \{1, 0, 0, \dots\}$

At $n=0 \to x(n)=1 \Rightarrow y(n) = \frac{1}{3}x(0) + \frac{1}{3}x(-3) + y(-1) = 1/3$

At $n=1$, $x(n)=0$: $y(1) = \frac{1}{3}x(1) + \frac{1}{3}x(-2) + y(0) = 1/3$

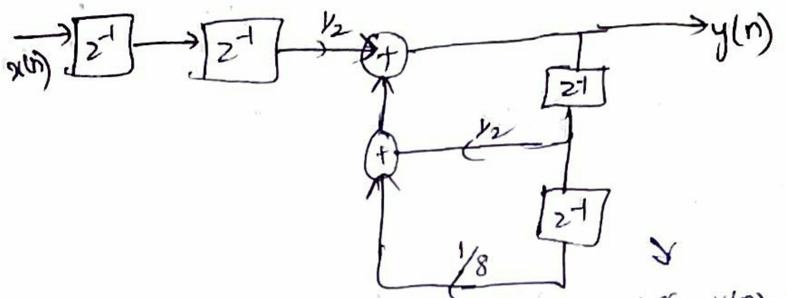
At $n=2$, $x(n)=0$: $y(2) = \frac{1}{3}x(2) + \frac{1}{3}x(-1) + y(1) = 1/3$

At $n=3$, $x(n)=0$: $y(3) = \frac{1}{3}x(3) + \frac{1}{3}x(0) + y(2) = 2/3$

thus

$h(n) = y(n) = \{ \underset{\uparrow}{\frac{1}{3}}, \frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3} 3,$

(b)



here $y(n)$ is ↓

$y(n) = \frac{1}{2}y(n-1) + \frac{1}{8}y(n-2) + \frac{1}{2}x(n-2)$

Sol. $x(n) = \delta(n) = \{\underset{\uparrow}{1}, 0, 0, \ldots\}$

At $n=0$, $x(n)=1$: then $y(0) = \frac{1}{2}y(-1) + \frac{1}{8}y(-2) + \frac{1}{2}x(-2) = 0$

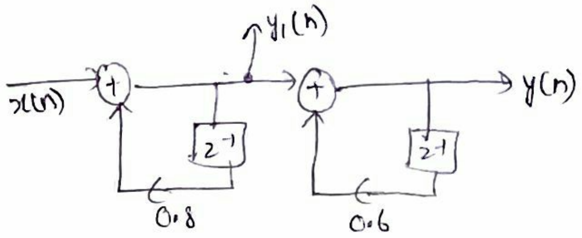
At $n=1$, $x(n)=0$: then $y(1) = \frac{1}{2}y(0) + \frac{1}{8}y(-1) + \frac{1}{2}x(-1) = 0$

At $n=2$, $x(n)=0$; then $y(2) = \frac{1}{2}y(1) + \frac{1}{8}y(0) + \frac{1}{2}x(0) = 1/2$

Similiarly we've -

$h(n) = y(n) = \{\underset{\uparrow}{0}, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{11}{128}, \ldots\}$

(c)



Sol. $y(n) = 1.4\,y(n-1) - 0.48\,y(n-2) + x(n)$

$x(n) = \delta(n) = \{1, 0, 0, \ldots\}$

At $n=0$, $x(n)=1$: then $y(0) = 1.4\,y(-1) - 0.48\,y(-2) + x(0) = 1$

At $n=1$, $x(n)=0$; then $y(1) = 1.4\,y(0) - 0.48\,y(-1) + x(1) = 1.4$

At $n=2$, $x(n)=0$; then $y(2) = 1.4\,y(1) - 0.48\,y(0) + x(2) = 1.48$

thus we've -

$h(n) = y(n) = \{1, 1.4, 1.48, 1.4, 1.2496, \ldots\}$

d) classify the systems above as FIR (or) IIR.

Sol. All three systems are IIR.

e) find an explicit expression for impulse response of the system in part c.

Sol.

$$y(n) = 1.4\,y(n-1) - 0.48\,y(n-2) + x(n)$$

then $\underline{y_h(n)}$: $x(n) = 0$.

$$\Rightarrow y(n) = 1.4y(n-1) - 0.48\,y(n-2)$$

$$\Rightarrow y(n) - 1.4y(n-1) + 0.48y(n-2) = 0.$$

$$\Rightarrow \lambda^n - 1.4\lambda^{n-1} + 0.48\,\lambda^{n-2} = 0$$

$$\lambda^{n-2}\left[\lambda^2 - 1.4\lambda + 0.48\right] = 0$$

$$\lambda^2 - 1.4\lambda + 0.48 = 0$$

then

$$\lambda = 0.8,\ 0.6$$

$$y_h(n) = c_1(0.8)^n + c_2(0.6)^n$$

$\underline{y_p(n)}$:

for $x(n) = \delta(n)$ $\quad y_p(n) = 0$

$$y(n) = y_h(n) + y_p(n) = \underline{c_1(0.8)^n + c_2(0.6)^n}$$

Assuming conditions $y(-1) = y(-2) = 0$

Substitue $n = 0, 1$ in D.E equation

$$\rightarrow y(0) = 1.4y(-1) - 0.48y(-2) + x(0)$$

$$y(0) = 1.4(0) - 0.48(0) + \delta(0)$$

$$\boxed{y(0) = 1}$$

$$\rightarrow y(1) = 1.4\,y(0) - 0.48\,y(-1) + x(1)$$

$$y(1) = 1.4 - 0 + 0 \Rightarrow \boxed{y(1) = 1.4}$$

Substitue $n = 0, 1$ in $y(n)$ we've

$\rightarrow y(0) = c_1 + c_2$ $\qquad \rightarrow y(1) = c_1(0.8) + c_2(0.6)$

$\Rightarrow c_1 + c_2 = 1$ — (b) $\qquad \Rightarrow 0.8c_1 + 0.6c_2 = 1.4$ — (c)

Solving (b) & (c) we've -

$$c_1 = 4,\quad c_2 = -3$$

then

$$h(n) = y(n) = \left[4(0.8)^n - 3(0.6)^n\right]u(n)$$

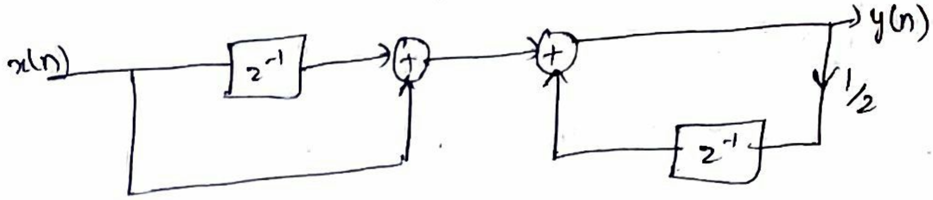
2.50 Consider the systems shown below.

a) Determine it's impulse response h(n)

b) Show that h(n) is equal to the convolution of the following signal

$h_1(n) = \delta(n) + \delta(n-1)$

$h_2(n) = \left(\frac{1}{2}\right)^n u(n)$



Sol. a) $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

Impulse response

$\Rightarrow y(n) - \frac{1}{2} y(n-1) = x(n) + x(n-1)$

$y_h(n)$:-

$x(n) \& x(n-1) = 0$

$\Rightarrow y(n) - \frac{1}{2} y(n-1) = 0$

$\lambda - \frac{1}{2} = 0$

$\lambda = 1/2$

$\Rightarrow y_h(n) = C\left(\frac{1}{2}\right)^n u(n)$

$y_p(n)$:-

$x(n) = \delta(n)$

$\Rightarrow y_p(n) = 0$

$\Rightarrow y(n) = y_p(n) + y_h(n)$

$y(n) = C\left(\frac{1}{2}\right)^n u(n)$

Assuming $y(-1) = 0$

Substitue n=0 in D.E Equation

$n=0 \rightarrow y(0) - \frac{1}{2} y(-1) = x(0) + x(-1)$

$y(0) - \frac{1}{2}(0) = \delta(0) + \delta(-1)$

$y(0) = 1$

Substitue n=0 in y(n)

$\Rightarrow y(0) = C\left(\frac{1}{2}\right)^0 u(0)$

$\Rightarrow y(0) = C$

$\Rightarrow C = 1$

then Impulse response

$h(n) = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$

b) $h_1(n) * h_2(n) = \left[\delta(n) + \delta(n-1)\right] * \left(\frac{1}{2}\right)^n u(n)$

$h_1(n) * h_2(n) = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$

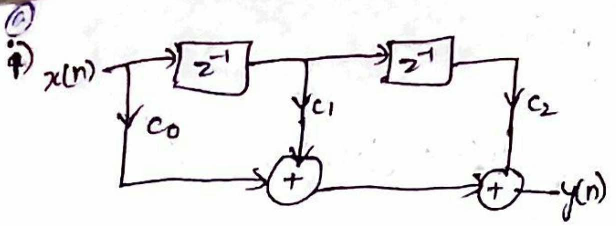
2.49 Consider the systems below.

a) Determine & sketch their impulse reponses $h_1(n)$, $h_2(n)$ & $h_3(n)$

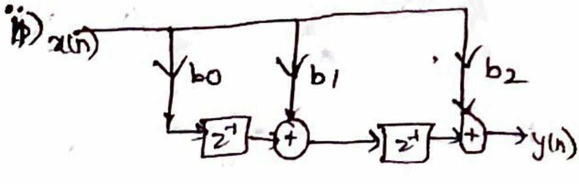
b) Is it possible to choose the coefficients of these systems in such a way that $h_1(n) = h_2(n) = h_3(n)$

(a)

i)



ii)



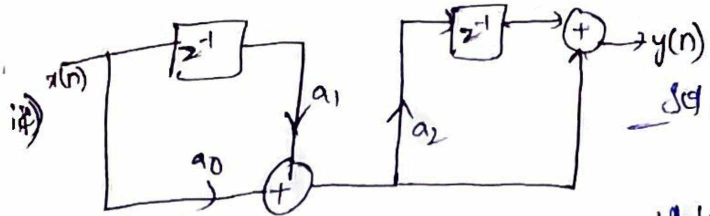
Sol. $h_1(n) = c_0\delta(n) + c_1\delta(n-1) + c_2\delta(n-2)$

$y_1(n) = c_0 x(n) + c_1 x(n-1) + c_2 x(n-2)$

Sol $h_2(n) = b_2\delta(n) + b_1\delta(n-1) + b_0\delta(n-2)$

$y_2(n) = b_2 x(n) + b_1 x(n-1) + b_0 x(n-2)$

iii)



Sol. $h_3(n) = a_0\delta(n) + (a_1 + a_0 a_2)\delta(n-1) + a_1 a_2\delta(n-2)$

$y_3(n) = a_0 x(n) + a_1 x(n-1) + a_0 a_2 x(n-1) + a_1 a_2 x(n-2)$.

(b) To make $h_3(n) = h_2(n) = h_1(n)$

Let $a_0 = c_0$, $a_1 + a_2 c_0 = c_1$, $a_2 a_1 = c_2$

Thus we've -

$$\frac{c_2}{a_2} + a_2 c_0 - c_1 = 0$$

$$\Rightarrow c_2 a_2^2 - c_1 a_2 + c_2 = 0$$

for $c_0 \neq 0$, the Q.E has a real solution iff & only if $c_1^2 - 4c_0c_2 \geq 0$

2.51 Compute the sketch the convolution $y_i(n)$ & correlation $r_i(n)$ sequences for the following pair of signals & comment on the results obtained.

a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 4\}$ $\quad h_1(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1\}$

b) $x_2(n) = \{0, \underset{\uparrow}{1}, -2, 3, -4\}$ $\quad h_2(n) = \{\frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}\}$

c) $x_3(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $\quad h_3(n) = \{4, \underset{\uparrow}{3}, 2, 1\}$

d) $x_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $\quad h_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Note: ① convolution is done as $h(n) * x(n)$

② Correlation means writing h(n) in reverse order. $h(n)\downarrow$ $x(n)\rightarrow$

Sol.

a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 4\}$, $h_1(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

$h_1(n)$ \ $x_1(n)$	↓ 1	2	4
→ 1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

Convolution: $y_1(n) = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

Correlation: $y_1(n) = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

(when $h_1(n)$ is considered u'll get 7 at $y_1(n) = 7$

b) $x_2(n) = (\underset{\uparrow}{0}, 1, -2, 3, -4)$, $h_2(n) = (\frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2})$

$h_2(n)$ \ $x_2(n)$	↓ 0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
→ 2	0	2	-4	6	-8
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

Convolution: $y_2(n) = \{0, \frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, \ldots$

Correlation: $y_2(n) = \{0, \frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, \ldots -2\}$

c) $x_3(n) = (\underset{\uparrow}{1}, 2, 3, 4)$, $h_3(n) = (\underset{\uparrow}{4}, 3, 2, 1)$

$h_3(n)$ \ $x_3(n)$	↓ 1	2	3	4
→ 4	4	8	12	16
3	3	6	9	12
2	2	4	6	8
1	1	2	3	4

$h_3(n)$ \ $x_3(n)$	↓ 1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
→ 4	4	8	12	16

Convolution:
$y_3(n) = \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$

Correlation:
$y_3(n) = \{1, 4, 10, 20, \underset{\uparrow}{25}, 24, 16\}$

d) $x_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$; $h_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

$h_4(n)$ \ $x_4(n)$	↓ 1	2	3	4
→ 1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

$h_4(n)$ \ $x_4(n)$	↓ 1	2	3	4
4	4	8	12	16
3	3	6	9	12
2	2	4	6	8
→ 1	1	2	3	4

Convolution: $y_4(n) = \{\underset{\uparrow}{1}, 4, 10, 20, 25, 24, 16\}$

Correlation: $y_4(n) = \{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\}$

2.52 The zero state response of a causal LTI system to the i/p $x(n) = \{1, 3, 3, 1\}$ is $y(n) = \{1, 4, 6, 4, 1\}$. Determine it's impulse response.

Sol.

$h(n)$ \ $x(n)$	1	3	3	1
h_0	$h(0)x(0)$	$h(0)x(1)$	$h(0)x(2)$	$h(0)x(3)$
h_1	$h(1)x(0)$	$h(1)x(1)$	$h(1)x(2)$	$h(1)x(3)$

$\rightarrow h(0)x(0) = 1$

$h(0) = 1$

$\rightarrow h(1)\, x(0) + h(0)x(1) = 4$

$h(1)\cancel{1} + 1\cdot(3) = 4$

$h(1) = 4 - 3$

$h(1) = 1.$

thus ~~h(0)~~ $h(0) = 1,\ h(1) = 1$

$\Rightarrow h(n) = \{h_0, h_1\} = \{1, 1\}.$

2.55 Determine the impulse response $h(n)$ for the system described by the 2nd order difference equation.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

Sol.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

$y_h(n)$:

$x(n) = x(n-1) = 0$

$\Rightarrow y(n) - 4y(n-1) + 4y(n-2) = 0$

$\lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2} = 0$

$\lambda^{n-2}(\lambda^2 - 4\lambda + 4) = 0$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$

$\Rightarrow y_h(n) = [C_1 2^n + C_2 n 2^n] u(n)$

$y_p(n)$:

$x(n) = \delta(n)$.

$\Rightarrow y_p(n) = 0$

$y(n)$:

$y(n) = y_p(n) + y_h(n)$

$\Rightarrow y(n) = (C_1 2^n + C_2 n 2^n) u(n)$

* Assuming $y(-1) = y(-2) = 0$
* Substitue $n=0, n=1$ in D.E Equation we've -

$\rightarrow y(0) - 4y(-1) + 4y(-2) = x(0) - x(-1)$

$y(0) - 4(0) + 4(0) = \delta(0) - \delta(-1)$

$\Rightarrow \underline{y(0) = 1}$

$\rightarrow y(1) - 4y(0) + 4y(-1) = x(1) - x(0)$

$y(1) - 4(1) + 4(0) = \delta(1) - \delta(0)$

$y(1) - 4 + 0 = 0 - 1$

$y(1) = 4 - 1$

$\underline{y(1) = 3}$

* Substitue $n=0, n=1$ in $y(n)$

$\rightarrow y(0) = C_1 + C_2(0)$

$\Rightarrow C_1 = 1$

$\rightarrow y(1) = C_1(2) + C_2(2)$

$\Rightarrow y(1) = 2 + 2C_2$

$\Rightarrow 3 - 2 = 2C_2$

$\frac{1}{2} = C_2$

$$\Rightarrow h(n) = y(n) = \left(2^n + \frac{1}{2} n 2^n\right) u(n)$$

2.54 Determine the response $y(n)$, $n \geq 0$ of the system described by the 2nd order difference equation -

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ when the i/p is $x(n) = (-1)^n u(n)$

Sol

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

$y_h(n) = (C_1 2^n + C_2 n 2^n) u(n)$

$y_p(n)$: $x(n) = (-1)^n u(n)$ $n \geq 0$

$u(n) \rightarrow$ has a constant K.

thus $x(n)$ has $\underline{y_p(n) = K \cdot (-1)^n u(n)}$

~~then~~ ~~$y(n) = y_h(n) + y_p(n)$~~ Substituting this $y_p(n)$ into D.E equation.

~~$\Rightarrow y(n) =$~~

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

highest $\rightarrow$ ~~for~~ $n = 2$

$$k(-1)^2 u(2) - 4k(-1)^1 u(1) + 4k(-1)^0 u(0) = (-1)^2 u(2) - (-1)^1 u(1)$$

$$k + 4k + 4k = 1 + 1$$

$$9k = 2$$

$$k = 2/9.$$

then

$$y_p(n) = \frac{2}{9}(-1)^n u(n)$$

then $y(n) = y_h(n) + y_p(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$

* Assuming initial conditions, $y(-1) = y(-2) = 0$
* Substitue $n=0$, $n=1$ in D.E equation.

we've -

$\rightarrow y(0) - 4y(-1) + 4y(-2) = x(0) - x(-1) \Rightarrow y(0) = 1$

$\rightarrow y(1) - 4y(0) + 4y(-1) = x(1) - x(0)$ ~~$\Rightarrow y(1) = 3$~~

~~Substi~~

$$y(1) - 4(1) + 4(0) = (-1)^1 u(1) - (-1)^0 u(0)$$

$$y(1) - 4 + 0 = -1 - 1$$

$$y(1) = 4 - 2$$

$$y(1) = 2$$

* Substitue $n=0$, $n=1$ in $y(n)$

$\rightarrow y(0) = \left(c_1 + \frac{2}{9}\right) u(n)$

$\Rightarrow c_1 + \frac{2}{9} = 1 \Rightarrow c_1 = 7/9$

$\rightarrow y(1) = 2c_1 + 2c_2 - \frac{2}{9}$

$\Rightarrow 2c_1 + 2c_2 - \frac{2}{9} = 2$

$\Rightarrow c_1 + c_2 = \frac{10}{9}$

$\Rightarrow c_2 = \frac{10}{9} - \frac{7}{9} = \frac{3}{9} = 1/3 \Rightarrow c_2 = 1/3$

then $h(n) = y(n) = \left[\frac{7}{9} 2^n + \frac{1}{3} n 2^n + \frac{2}{9}(-1)^n\right] u(n)$

2.56 Show that any DT signal $x(n)$ can be expressed as $x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u[n-k]$ where $u(n-k)$ is a unit step delayed by k units in time, that is $u(n-k) = \begin{cases} 1 & n \geq k \\ 0 & \text{otherwise} \end{cases}$

Sol.

$$x(n) = x(n) * \delta(n).$$

$$= x(n) * [u(n) - u(n-1)]$$

$$= [x(n) - x(n-1)] * u(n)$$

$$= x(n) * u(n) - x(n-1) * u(n)$$

$$= \sum_k x(k) u(n-k) - \sum_k x(k-1) u(n-k)$$

$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$$

thus proved.

2.57 Show that the o/p of an LTI system can be expressed in terms of it's unit-step response $s(n)$ as follows.

$$y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k)$$

$$y(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] s(n-k)$$

Sol. Let $h(k) = s(k) - s(k-1)$ where $s(k)$ = unit step response.

$$s(k) = \sum_{m=-\infty}^{\infty} h(m)$$

then $y(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k)$$

thus proved.

Similarly other can be proved.

2.58 Compute the correlation sequences $r_{xx}(l)$ & $r_{xy}(l)$ for the following signal sequences.

$$x(n) = \begin{cases} 1 & n_0 - N \leq n \leq n_0 + N \\ 0 & \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} 1 & -N \leq n \leq N \\ 0 & \text{otherwise} \end{cases}$$

Sol. (i) We know that -

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) x(n-l)$$

Range of $r_{xx}(l)$ is -
i) $n_0 - N \leq n \leq n_0 + N$: $x(n)$
ii) $n_0 - N \leq n - l \leq n_0 + N$.
$n_0 - N + l \leq n \leq n_0 + N + l$: $x(n-l)$

we know that $x(n)\, x(n-l)$ summation values gives us $r_{xx}(l)$

similarly summing those range values gives us $r_{xx}(l)$ range.

ie adding/summing ⓐ & ⓑ gives us $-\,2N+1-|l|$

$r_{xx}(l) \rightarrow$ range: $\underline{\underline{-2N \le l \le 2N}}$.

(iv) thus we've -

$$r_{xx}(l) = \begin{cases} 2N+1-|l| & -2N \le l \le 2N \\ 0 & \text{otherwise} \end{cases}$$

$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0| & n_0-2N \le l \le n_0+2N \\ 0 & \text{otherwise.} \end{cases}$$

2.59 Determine the autocorrelation sequences of the following signals & give the conclusion

a) $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$ b) $y(n) = \{1, \underset{\uparrow}{1}, 2, 1\}$

Sol.

$x(n)$ \ $x(n)$	$\overset{0}{1}$	2	1	1
1	1	2	1	1
1	1	2	1	1
2	2	4	2	2
→ 1	1	2	1	1

$y(n)$ \ $y(n)$	1	1	2	1
1	1	1	2	1
2	2	2	4	2
1	1	1	2	1
→ 1	1	1	2	1

$r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$

$r_{yy}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$

we observe that $y(n) = x(-n+3)$ which is equivalent to reversing the sequence $x(n)$. this has not changed the auto correlation sequence

2.60. What is the normalized autocorrelation sequence of the signal, $x(n)$ given by $x(n) = \begin{cases} 1 & -N \le n \le N \\ 0 & \text{otherwise} \end{cases}$

Sol. we know that,

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$\downarrow$

$$\text{range} = \begin{cases} 2N+1-|l| & ; -2N \le l \le 2N \\ 0 & ; \text{otherwise.} \end{cases}$$

here $r_{xx}(0) = 2N+1$

$\downarrow$

then normalized auto correlation is:-

$$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|) ; & -2N \le l \le 2N \\ 0 ; & \text{otherwise} \end{cases}$$

Solved by

R161503.