A preliminary study of the para-model control of HIV

- Controling without knowing -

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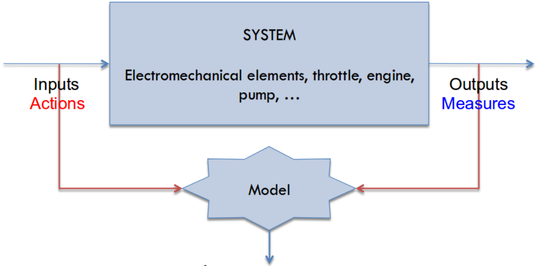
OUTLINE

- Presentation of the model-free control methodology
- Toward Para-Model control
- A preliminary study of the para-model of the HIV-1

Main idea of model-free control

Control today

- Most existing works: need precise mathematical (often difficult) modelling



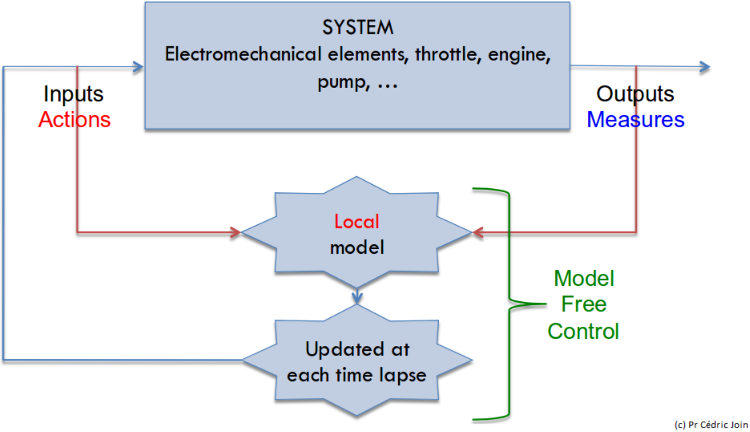
$$\begin{cases} \dot{x} = f(x,u,t) \\ y = g(x,u,t) \end{cases}$$

- PID (Proportional Integral Derivative) most common control law in practice (>95 %)

Main idea of model-free control

Model-free based control architecture

- From a complex system... to a simple local approximation



Main idea of model-free control

Model-free based control overview

Estimated and updated at each time lapse

~~Complex Model~~

$$\cancel{\begin{cases} \dot{x} = f(x, u, t) \\ y = g(x, u, t) \end{cases}}$$

Remplaced by ultra local model

$$y^{(\nu)} = \alpha u + F$$

Ultra local model control based

$$u = \frac{1}{\alpha}\left(-F + y_*^{(\nu)}\right) + Correction(e)$$

(c) Pr Cédric Join

- Same conceptual simplicity as PID controllers
- No need of precise modelling

Model-free control

Model-free based control discrete formal definition

(Fliess, Join - 2008)

Consider an unknown system $E : u \mapsto y$,

$$E(t, y, \dot{y}, \ldots, y^{(\iota)}, u, \dot{u}, \ldots, u^{(\kappa)}) = 0$$

MFC definition

To control y relating to a reference y^*, one considers the intelligent PI (i-PI) controller that reads:

$$u_k = u_{k-1} - \frac{1}{\beta}\left(\left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{k-1} - \left.\frac{\mathrm{d}y^*}{\mathrm{d}t}\right|_{k} \right) + \mathcal{C}(y^*|_k - y|_{k-1})$$

where $\mathcal{C}$ is a PI controller and β is a real constant; the tracking error is $\varepsilon = y^*|_k - y|_{k-1}$

- Numerical derivation of y necessary

Model-free control

In general, performances increase:

- development costs decrease
- maintenance costs decrease
- energetic costs decrease

Towards recent first applications in biology...

- Acute inflammation
- Wastewater denitrification
- Dynamic compensation for homeostasis
- Glycemia regulation

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Para-model control

Based on the original Model-free control...

- Simplified version of the orignal model-free control

Para-model control definition

To control y relating to a reference y^*, one considers the $\mathcal{C}_\pi$ controller that reads:

$$u_k = \int_0^t K_i \varepsilon_{k-1} d\,\tau \Big|_{k-1} \left\{ u_{k-1} + K_p (k_\alpha e^{-k_\beta k} - y_{k-1}) \right\}$$

where $K_i, K_p, k_\alpha, k_\beta$ are real coefficients to adjust

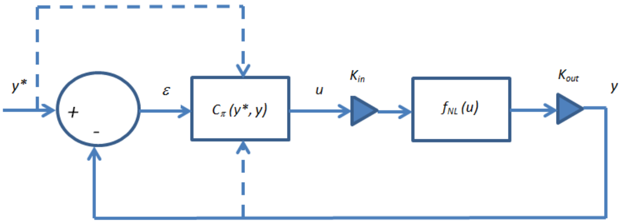
- Derivative-free algorithm

Para-model control

Structural properties

Given an output reference y^* and a nonlinear system $y = f_{nl}(u)$, it is *a priori* possible:

- to *control* dynamical or static f_{nl} systems
- to *optimize* (look for extremum) dynamical or static f_{nl} systems



Para-model control

Matlab code

```
y_int(i) = M_alpha*exp(-M_beta*tt(i));
para_exp_err = y_int(i-1) - y(i-1);
para_stand_err(i) = y_ref(i) - y(i-1);
para_u(i) = para_u(i-1) + Kp*para_exp_err;
para_G(i) = Kint*para_stand_err(i);
para_tr(i) = para_tr(i-1) + h*(para_G(i) + para_G(i-1))/2;
para_u_final = para_u(i)*para_tr(i);
```

BFO-tuning of the Para-model control

Tuning of the $\mathcal{C}_\pi$ coefficients in simulation

(Porcelli, Toint - 2015)

- DFO-based Brute Force Optimization (BFO) Matlab package
- For a given closed-loop, consider a performance index $\mathbb{P}$ (IAE, ISE, ITAE or ITSE) to minimize

PMA Tuning Optimization Procedure

Consider $\mathcal{C}_\pi$ that controls an unknown system E over $[0, t]$. We expect to minimize a performance index $\mathbb{P}$ of the closed-loop:

$$\min_{K_i, K_p, k_\alpha, k_\beta} \mathbb{P} \quad \text{e.g.} \quad \min_{K_i, K_p, k_\alpha, k_\beta} \int_0^t (y - y^*)^2 \mathrm{d}t \qquad \text{(ISE)}$$

BFO-tuning of the Para-model control

An example

- Consider a first order E dynamical system
- We expect to minimize $\mathbb{P} = IAE + ISE + ITAE + ITSE$ over $[0, t]$

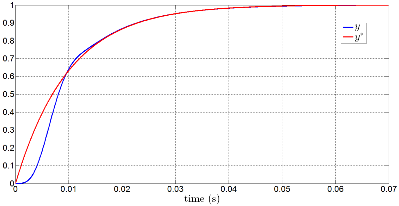


Figure: $\mathcal{C}_\pi$ control of E without optimization

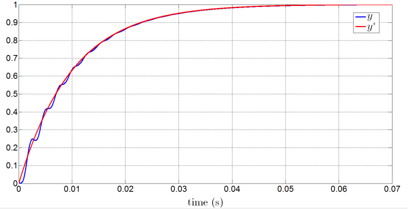


Figure: $\mathcal{C}_\pi$ control of E with optimization

First applications in simulation

Présentation

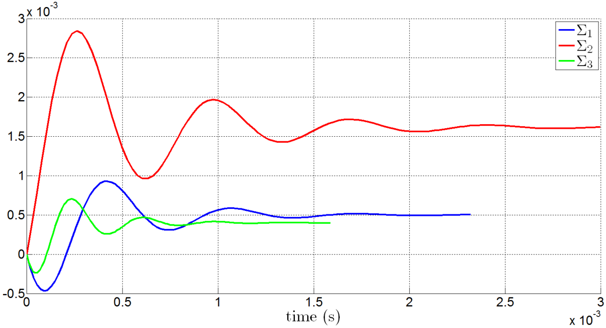
Application example:

- Power electronics : control of nonlinear converters
- Mechanics : balistic fire and robot trajectory
- Magnetism : characterization of magnetic materials (experimentally validated)
- Optimization : "extremum seeking-control"
- Nonlinear swiched systems

A basic Example

An example of controlled switched systems

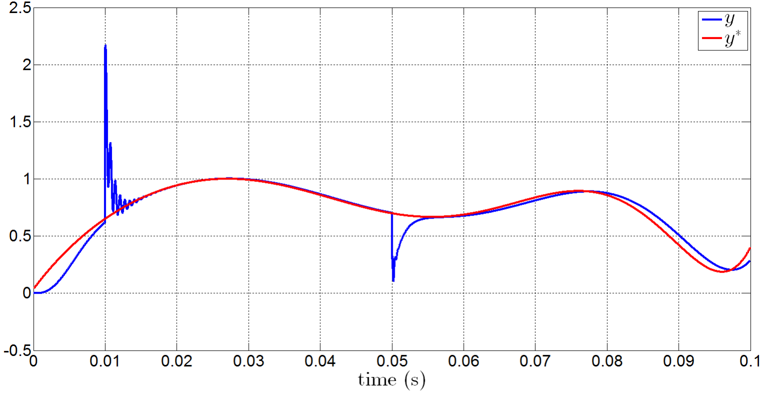
- Consider controlling three linear "unknown" systems that switch



$$\Sigma_p(u \longmapsto y) := \begin{cases} \dot{x}(t) = A_p x(t) + B_p u(t) \\ y = C_p x(t) \end{cases}$$

A basic Example

An example of controlled switched systems



- Switch of the model at $t_1 = 0.01$ s and $t_2 = 0.05$ s

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Para-model control of the HIV-1 model

Presentation

(Craig, Xia, Venter - 2004)

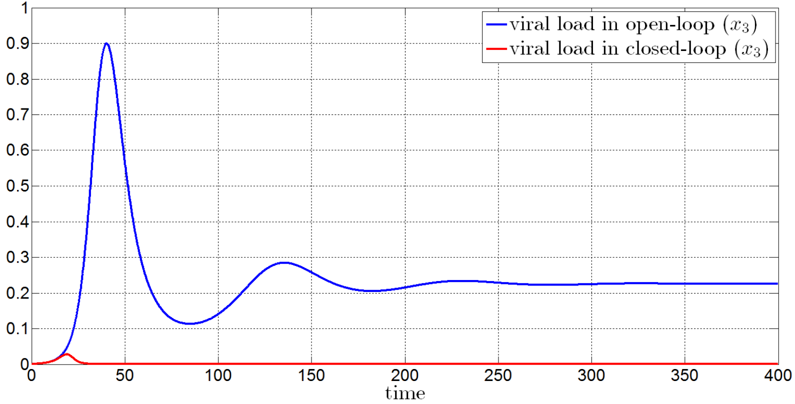
- Control of the predator-prey like model that describes the evolution of the HIV-1 dynamics when subjected to an external "medical agent"
- We do not take into account the constraints that are medically imposed

$$\begin{cases} \dot{x}_1 = s - dx_1 - (1-u_1)\beta x_1 x_3 \\ \dot{x}_2 = (1-u_1)\beta x_1 x_3 - \mu x_2 \\ \dot{x}_3 = (1-u_2)k x_2 - c x_3 \\ y = \begin{pmatrix} 0 & 0 & \gamma \end{pmatrix} x \end{cases}$$

Para-model control of the HIV-1 model

Control of the viral load

- Open-loop vs closed-loop



Para-model control of the HIV-1 model

Control of the viral load

- the associated u variable