Permanent Magnet Synchronous Machine

Implicit Euler SMC Controller

E. Koçak, A. Genidy, M.A. Hamida and L. Michel

Preprint. Work in progress

Ecole Centrale de Nantes - LS2N UMR 6004
06 February 2020

Contents

1 Introduction

Permanent Magnet Synchronous Machine (PMSM) drives present several advantages over other drives due to their high efficiency and high power density capabilities. The control of these drives can be performed through the vector control which was developed as a way to get better torque responses through the decoupling of the system in two different control actions (one for the torque and one for the flux).

In this work, a nonlinear control strategy based on advanced sliding mode algorithms is developed and implemented to improve the speed efficiency of PM synchronous machines. Sliding mode control (SMC) has very attractive features like robustness and simplicity of implementation, with few gains to tune. However, its main drawback is the existence of the so-called chattering phenomenon, which may be due to actuators' limitations, or unmodelled dynamics, and this motivates the proposed solution in this research. That solution involves the use of a simpler discretization method, which is the Euler implicit method in order to design the SMC control law in the digital setting.

1.1 Implicit Euler Summary

The implicit method is implemented as follows:

Consider a nominal unperturbed system with state x_k, from which the implicit control input is computed as shown in the following equations:

$$x_{k+1} = x_{k+1} + hKU_{k+1}$$

$$U_{k+1} = -sgn(x_{k+1})$$

This is a so-called "generalized equation" with unknown x_{k+1}. Its solution yields after few manipulations illustrated in [1]:

$$U_{k+1} = -\mathbf{proj}([-1,1], \frac{x_k}{\mathbf{Kh}})$$

That is the projection of $\frac{x_k}{kh}$ on the interval $[-1, 1]$, Thus, U_{k+1} develops into a causal input (not depending on any future values of the state), where:

$$proj([-1,1]; \frac{x_k}{kh}) = \left\{ \begin{array}{ll} sgn(x_k), & \text{for } |x_k| \geq kh \\ \frac{x_k}{hk}, & \text{otherwise} \end{array} \right\}$$

The Implicit controller which is also called as "projected sliding-mode controller" is limited the value of the sampling period h where h is greater than 0 [4].

1.2 Numerical Simulations of the simplest system and understanding Implicit Euler Method

Before conducting the motor model, the main parameters - h: sampling time and the gain K- of the Implicit Euler Method in discrete time are studied in various cases illustrated in the table below. Also, the presence of disturbance -P- is included to observe how it will affect the system. For simulations MATLAB software is used.

Table 1: Numerical simulation parameters for the simplest system

Parameter 1	Value	Parameter 2	Value
Sampling time 1 (ha)	0.1	Gain 1 (K1)	1
Sampling time 2 (hb)	0.01	Gain 2 (K2)	0.1
Sampling time 3 (hc)	0.0001	Gain 3 (K3)	10

Case 1: No Disturbance (P=0) when Initial Condition (IC) for X_K is $X_0 = 1$

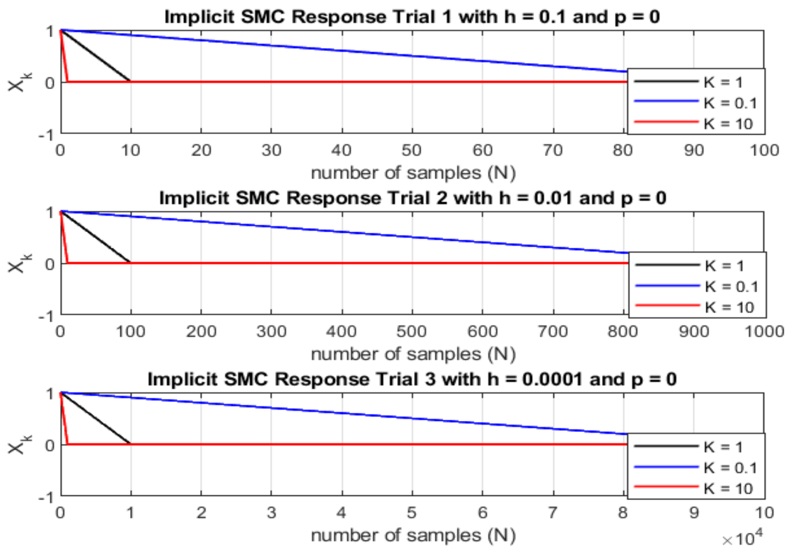


Figure 1: Simulation results without disturbance

Case 2: Presence of a disturbance (P=0.01) when IC for X_K is $X_0 = 1$

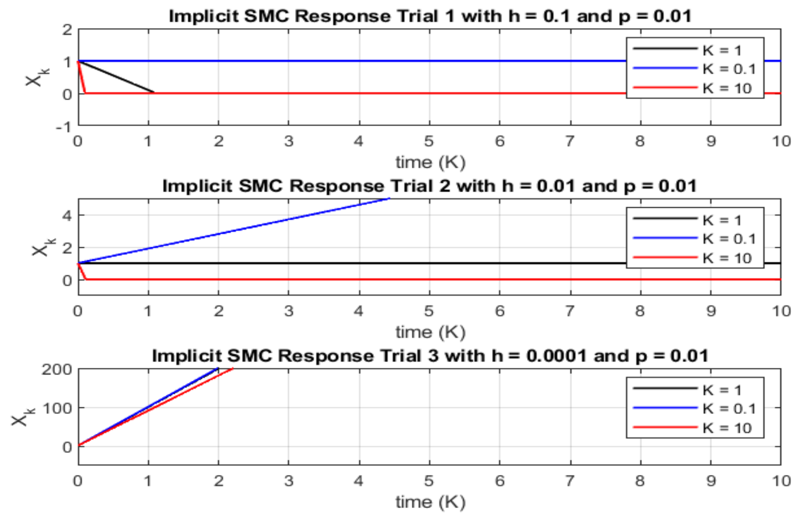


Figure 2: Simulation results with a disturbance of P=0.01

Case 3: Presence of a disturbance (P=0.1) when IC for X_K is $X_0 = 1$

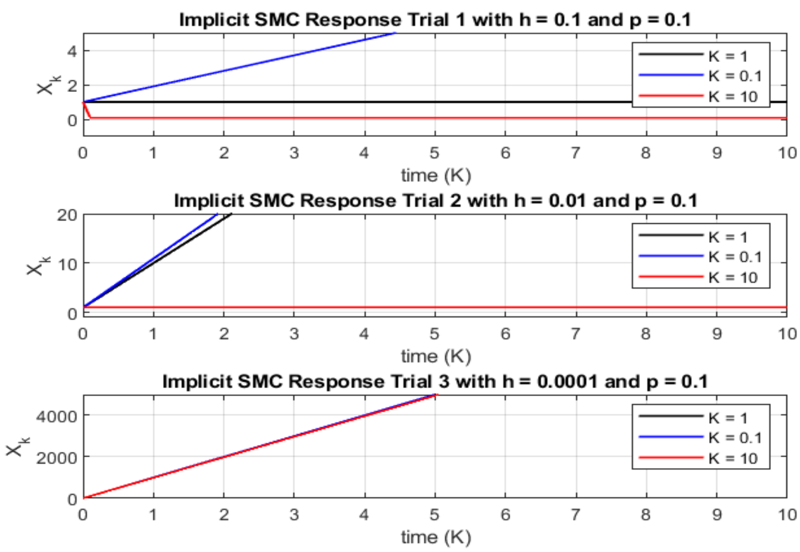


Figure 3: Simulation results with a disturbance of P=0.1

From the initial work of the project, the following conclusions are drawn:

1. h – sampling time- and K –the gain- has the same inverse proportional effect on the time needed for the IC of the state to reach zero.

2. For no disturbance case, the stability of the system does not depend on h.

3. If $hKsgn(X_k) = P$, disturbance and the control input terms cancel out.Therefore, X_K is always constant and equal to its IC (X_0) i.e. never converges to zero.

4. If $hKsgn(X_k) = X_K$, X_K never reaches to zero,it converges only on the disturbance value.

5. If, $hKsgn(X_k) < (X_K + P)$, disturbance dominates and system becomes unstable.

Shortly, and while considering real-life applications, to avoid the domination of disturbance over the system, the gain has to be tuned sufficiently large. If the controller gain is set too low, it will not respond adequately to disturbances. Also, taking a very precise sampling time is not a healthy choice for systems under the effect of considerable disturbances, as seen in case 3. (h=10e-4 and P=0.1). To illustrate, to stabilize the system in case 3 one must choose a gain of 10e8 that is extremely large.

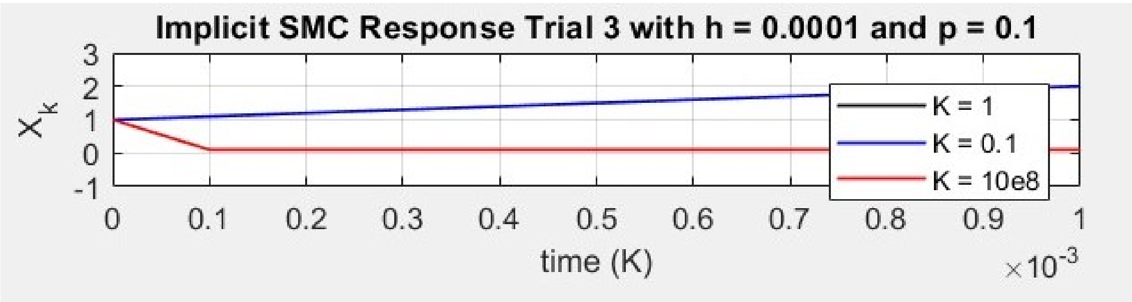


Figure 4: Detail view to stabilizing the system under disturbance with a large gain is set

2 Permanent Magnet Synchronous Machine

2.1 Permanent Magnet Synchronous Machine Model in d-q Frame

$$\frac{di_d}{dt} = -\frac{R_s}{L_d}i_d + p\frac{L_q}{L_d}i_q\Omega + \frac{1}{L_d}v_d \tag{1}$$

$$\frac{di_q}{dt} = -\frac{R_s}{L_q}i_q - p\frac{L_d}{L_q}i_d\Omega - p\frac{1}{L_q}\phi_f\Omega + \frac{1}{L_q}v_q \tag{2}$$

$$\frac{d\Omega}{dt} = -\frac{f_v}{J}\Omega + \frac{3}{2}(\frac{p}{J})(L_d - L_q)i_d i_q + \frac{3}{2}(\frac{p}{J})\phi_f i_q - \frac{1}{J}T_L \tag{3}$$

$$\frac{d\theta}{dt} = \Omega \tag{4}$$

Where i_d and i_q are the measured outputs and v_d and v_q are inputs [3] [5], with

- R_s stator resistance
- L_d, L_q dq-axis inductances
- ϕ_f permanent-magnet flux linkage
- i_d, i_q stator currents
- v_d, v_q stator voltages
- Ω rotor mechanical speed
- p number of pole pairs
- θ rotor angular position
- J moment of inertia
- f_v viscous friction coefficient
- T_l load torque

2.2 Implicit Discrete-Time SMC

$$i_{d,k+1} = i_{d,k}(1 - \mathbf{h}\frac{R_s}{L_d}) + \mathbf{h}p\frac{L_q}{L_d}i_{q,k}\Omega_k + \mathbf{h}\frac{1}{L_d}v_{d,k} \tag{5}$$

$$i_{q,k+1} = i_{q,k}(1 - \mathbf{h}\frac{R_s}{L_q}) - \mathbf{h}p\frac{L_d}{L_q}i_{d,k}\Omega_k - \mathbf{h}p\frac{1}{L_q}\phi_f\Omega_k + \mathbf{h}\frac{1}{L_q}v_{q,k} \tag{6}$$

$$\Omega_{k+1} = (1 - \mathbf{h}\frac{f_v}{J})\Omega_k + \mathbf{h}\frac{3}{2}(\frac{p}{J})(L_d - L_q)i_{d,k}i_{q,k} + \mathbf{h}\frac{3}{2}(\frac{p}{J})\phi_f i_{q,k} - \mathbf{h}\frac{1}{J}T_L \tag{7}$$

2.2.1 Sliding Variable for i_d

$$s_{1,k} = i_{d,k} - i_{dref,k} \tag{8}$$

$$s_{1,k+1} = i_{d,k+1} - i_{dref,k+1} \tag{9}$$

Recalling equation (5), where:

$$i_{d,k+1} = i_{d,k}(1 - \mathbf{h}\frac{R_s}{L_d}) + \mathbf{h}p\frac{L_q}{L_d}i_{q,k}\Omega_k + \mathbf{h}\frac{1}{L_d}v_{d,k}$$

Substituting equation (5) into (9) we get:

$$s_{1,k+1} = i_{d,k}(1 - \mathbf{h}\frac{R_s}{L_d}) + \mathbf{h}p\frac{L_q}{L_d}i_{q,k}\Omega_k + \mathbf{h}\frac{1}{L_d}v_{d,k} - i_{dref,k+1} \tag{10}$$

Forcing sliding variable to reach zero by designing the control input $v_{d,k}$:

$$v_{d,k} = -pL_q i_{q,k}\Omega_k + \frac{L_d}{\mathbf{h}}(-i_{d,k}(1 - \mathbf{h}\frac{R_s}{L_d}) + i_{dref,k+1} + s_{1,k}) - L_d\mathbf{K}_1\mathbf{U}_{1,k+1} \tag{11}$$

Substituting equation (11) into (10) to design SMC control input $\mathbf{U}_{1,k+1}$:

$$s_{1,k+1} = s_{1,k} + \mathbf{h}\mathbf{K}_1\mathbf{U}_{1,k+1} \tag{12}$$

$$\mathbf{U}_{1,k+1} = -sgn(s_{1,k+1}) \tag{13}$$

Solving the two equations (12) and (13) using the projection:

$$\mathbf{U}_{1,k+1} = -\mathbf{proj}([-1,1], \frac{s_{1,k}}{\mathbf{K}_1\mathbf{h}}) \tag{14}$$

Taking into consideration equation (15) that is driven in [3]:

$$i_{dref,k} = \frac{-\phi_f}{2(L_d - L_q)} - \sqrt{\frac{\phi_f^2}{4(L_d - L_q)^2} + i_{q,k}^2} \tag{15}$$

Taking a step a head

$$i_{dref,k+1} = \frac{-\phi_f}{2(L_d - L_q)} - \sqrt{\frac{\phi_f^2}{4(L_d - L_q)^2} + i_{q,k+1}^2} \tag{16}$$

Recalling equation (6), where:

$$i_{q,k+1} = i_{q,k}(1 - \mathbf{h}\frac{R_s}{L_q}) - \mathbf{h}p\frac{L_d}{L_q}i_{d,k}\Omega_k - \mathbf{h}p\frac{1}{L_q}\phi_f\Omega_k + \ \mathbf{h}\frac{1}{L_q}v_{q,k}$$

Substituting equation (6) into (16) so that $i_{dref,k+1}$ does not depend on any future values of $i_{q,k}$

$$i_{dref,k+1} = \frac{-\phi_f}{2(L_d - L_q)} - \sqrt{\frac{\phi_f^2}{4(L_d - L_q)^2} + (i_{q,k}(1 - \mathbf{h}\frac{R_s}{L_q}) - \mathbf{h}p\frac{L_d}{L_q}i_{d,k}\Omega_k - \mathbf{h}p\frac{1}{L_q}\phi_f\Omega_k + \ \mathbf{h}\frac{1}{L_q}v_{q,k})^2} \tag{17}$$

2.2.2 Sliding Variable for i_q

$$s_{2,k} = i_{q,k} - i_{qref,k} \tag{18}$$

$$s_{2,k+1} = i_{q,k+1} - i_{qref,k+1} \tag{19}$$

Recalling equation (6), where:

$$i_{q,k+1} = i_{q,k}(1 - \mathbf{h}\frac{R_s}{L_q}) - \mathbf{h}p\frac{L_d}{L_q}i_{d,k}\Omega_k - \mathbf{h}p\frac{1}{L_q}\phi_f\Omega_k + \ \mathbf{h}\frac{1}{L_q}v_{q,k}$$

Substituting equation (6) into (19) we get:

$$s_{2,k+1} = i_{q,k}(1 - \mathbf{h}\frac{R_s}{L_q}) - \mathbf{h}p\frac{L_d}{L_q}i_{d,k}\Omega_k - \mathbf{h}p\frac{1}{L_q}\phi_f\Omega_k + \ \mathbf{h}\frac{1}{L_q}v_{q,k} - i_{qref,k+1} \tag{20}$$

Forcing sliding variable to reach zero by designing the control input $v_{q,k}$:

$$v_{q,k} = pL_d i_{d,k}\Omega_k + \frac{L_q}{\mathbf{h}}(-i_{q,k}(1-\mathbf{h}\frac{R_s}{L_d}) + \mathbf{h}p\frac{1}{L_q}\phi_f\Omega_k + i_{qref,k+1} + s_{2,k}) - L_q\mathbf{K}_2\mathbf{U}_{2,k+1} \tag{21}$$

Substituting equations (21) into (20) to design SMC control input $\mathbf{U}_{2,k+1}$:

$$s_{2,k+1} = s_{2,k} + \mathbf{h}\mathbf{K}_2\mathbf{U}_{2,k+1} \tag{22}$$

$$\mathbf{U}_{2,k+1} = -sgn(s_{2,k+1}) \tag{23}$$

Solving the two equations (22) and (23) using the projection:

$$\mathbf{U}_{2,k+1} = -\mathbf{proj}([-1,1], \frac{s_{2,k}}{\mathbf{K}_2\mathbf{h}}) \tag{24}$$

2.2.3 Sliding Variable for Ω

$$s_3 = \dot{\Omega_{ref}} - \dot{\Omega} + \lambda(\Omega_{ref} - \Omega) \tag{25}$$

Recalling equation (3), where:

$$\frac{d\Omega}{dt} = -\frac{f_v}{J}\Omega + \frac{3}{2}(\frac{p}{J})(L_d - L_q)i_d i_q + \frac{3}{2}(\frac{p}{J})\phi_f i_q - \frac{1}{J}T_L$$

Substituting equation (3) into (25), under the assumption that Ω_{ref} is constant:

$$s_3 = \frac{f_v}{J}\Omega - \frac{3}{2}(\frac{p}{J})(L_d - L_q)i_d i_q - \frac{3}{2}(\frac{p}{J})\phi_f i_q + \frac{1}{J}T_L + \lambda(\Omega_{ref} - \Omega) \tag{26}$$

Considering s_3 in discrete time domain :

$$s_{3,k} = \frac{f_v}{J}\Omega_k - \frac{3}{2}(\frac{p}{J})(L_d - L_q)i_{d,k}i_{q,k} - \frac{3}{2}(\frac{p}{J})\phi_f i_{q,k} + \frac{1}{J}TL + \lambda(\Omega_{ref,k} - \Omega_k) \tag{27}$$

$$s_{3,k+1} = \frac{f_v}{J}\Omega_{k+1} - \frac{3}{2}(\frac{p}{J})(L_d - L_q)i_{d,k+1}i_{q,k+1} - \frac{3}{2}(\frac{p}{J})\phi_f i_{q,k+1} + \frac{1}{J}TL + \lambda(\Omega_{ref,k+1} - \Omega_{k+1}) \tag{28}$$

Recalling equations (5) and (7), where:

$$i_{d,k+1} = i_{d,k}(1 - \mathbf{h}\frac{R_s}{L_d}) + \mathbf{h}p\frac{L_q}{L_d}i_{q,k}\Omega_k + \mathbf{h}\frac{1}{L_d}v_{d,k}$$

$$\Omega_{k+1} = (1 - \mathbf{h}\frac{f_v}{J})\Omega_k + \mathbf{h}\frac{3}{2}(\frac{p}{J})(L_d - L_q)i_{d,k}i_{q,k} + \mathbf{h}\frac{3}{2}(\frac{p}{J})\phi_f i_{q,k} - \mathbf{h}\frac{1}{J}T_L$$

Substituting equations (5) and (7) into (28):

$$\begin{aligned} s_{3,k+1} = {} & i_{q,k+1}(-\frac{3}{2}\frac{p}{J}(L_d - L_q)(i_{d,k}(1 - \mathbf{h}\frac{R_s}{L_d}) + \mathbf{h}p\frac{L_q}{L_d}i_{q,k}\Omega_k + \mathbf{h}\frac{1}{L_d}v_{d,k}) - \frac{3}{2}\frac{p}{J}\phi_f) \\ & + (\frac{f_v}{J} - \lambda)((1 - \mathbf{h}\frac{f_v}{J})\Omega_k + \mathbf{h}\frac{3}{2}(\frac{p}{J})(L_d - L_q)i_{d,k}i_{q,k} + \mathbf{h}\frac{3}{2}(\frac{p}{J})\phi_f i_{q,k} - \mathbf{h}\frac{1}{J}T_L) \\ & + \frac{1}{J}T_L + \lambda\Omega_{ref,k+1} \end{aligned} \tag{29}$$

Forcing sliding variable to reach zero by designing $i_{qref,k+1}$; $i_{qref,k+1} = i_{q,k+1}$:

$$i_{qref,k+1} = \frac{((\lambda - \frac{f_v}{J})((1-\mathbf{h}\frac{f_v}{J})\Omega_k + \mathbf{h}\frac{3}{2}(\frac{p}{J})(L_d - L_q)i_{d,k}i_{q,k} + \mathbf{h}\frac{3}{2}(\frac{p}{J})\phi_f i_{q,k} - \mathbf{h}\frac{1}{J}T_L) - \frac{1}{J}T_L - \lambda\Omega_{ref,k+1} + s_{3,k} + \mathbf{h}\mathbf{K}_3\mathbf{U}_{k+1})}{(-\frac{3}{2}\frac{p}{J}(L_d - L_q)(i_{d,k}(1-\mathbf{h}\frac{R_s}{L_d}) + \mathbf{h}p\frac{L_q}{L_d}i_{q,k}\Omega_k + \mathbf{h}\frac{1}{L_d}v_{d,k}) - \frac{3}{2}\frac{p}{J}\phi_f)} \quad (30)$$

Recalling equation (6), where:

$$i_{q,k+1} = i_{q,k}(1 - \mathbf{h}\frac{R_s}{L_q}) - \mathbf{h}p\frac{L_d}{L_q}i_{d,k}\Omega_k - \mathbf{h}p\frac{1}{L_q}\phi_f\Omega_k + \ \mathbf{h}\frac{1}{L_q}v_{q,k}$$

Substituting equation (30) into (6) to get $i_{qref,k}$ to be used in calculating the sliding variable $s_{2,k}$ as shown in equation (18):

$$i_{qref,k} = \frac{i_{qref,k+1} + \mathbf{h}p\frac{L_d}{L_q}i_{d,k}\Omega_k + \mathbf{h}p\frac{1}{L_q}\phi_f\Omega_k - \ \mathbf{h}\frac{1}{L_q}v_{q,k}}{(1 - \mathbf{h}\frac{R_s}{L_q})} \quad (31)$$

As well, $i_{qref,k+1}$ is used to calculate the control input $v_{q,k}$ shown in equation (21).

Substituting equation (30) into (29) to design SMC control input $\mathbf{U}_{3,k+1}$:

$$s_{3,k+1} = s_{3,k} + \mathbf{h}\mathbf{K}_3\mathbf{U}_{3,k+1} \quad (32)$$

$$\mathbf{U}_{3,k+1} = -sgn(s_{3,k+1}) \quad (33)$$

Solving the two equations (32) and (33) using the projection:

$$\mathbf{U}_{3,k+1} = -\mathbf{proj}([-1,1], \frac{s_{3,k}}{\mathbf{K}_3\mathbf{h}}) \quad (34)$$

2.3 Continuous-Time SMC

In order to evaluate the performance of the implicit SMC controller, it has to be compared to performance of the continuous-time SMC controller in controlling the speed of the PMSM. So, a continuous-time SMC controller is designed in this section as follows [6]:

2.3.1 Sliding Variable for i_d

$$s_1 = i_{dref} - i_d \tag{35}$$

$$\dot{s_1} = \dot{i_{dref}} - \dot{i_d} \tag{36}$$

Recalling equation (1), where:

$$\frac{di_d}{dt} = -\frac{R_s}{L_d} i_d + p\frac{L_q}{L_d} i_q \Omega + \frac{1}{L_d} v_d$$

Substituting equation (1) into (36):

$$\dot{s_1} = \dot{i_{dref}} - (-\frac{R_s}{L_d} i_d + p\frac{L_q}{L_d} i_q \Omega) - \frac{1}{L_d} v_d \tag{37}$$

Forcing sliding variable to reach zero by designing the control input v_d:

$$v_d = -L_d(-\frac{R_s}{L_d} i_d + p\frac{L_q}{L_d} i_q \Omega - \dot{i_{dref}} + \mathbf{K}_1\mathbf{U}_1) \tag{38}$$

Substituting equation (38) into (37) to design SMC control input $\mathbf{U}_1$:

$$\dot{s_1} = \mathbf{K}_1\mathbf{U}_1 \tag{39}$$

$$\mathbf{U}_1 = -sgn(s_1) \tag{40}$$

Taking into consideration that [3]:

$$i_{dref} = \frac{-\phi_f}{2(L_d - L_q)} - \sqrt{\frac{\phi_f^2}{4(L_d - L_q)^2} + i_q^2} \tag{41}$$

$$\dot{i_{dref}} = -\frac{i_q}{\sqrt{\frac{\phi_f^2}{4(L_d - L_q)^2} + i_q^2}} \tag{42}$$

2.3.2 Sliding Variable for i_q

$$s_2 = i_{qref} - i_q \tag{43}$$

$$\dot{s_2} = \dot{i_{qref}} - \dot{i_q} \tag{44}$$

Recalling equation (2), where:

$$\frac{di_q}{dt} = -\frac{R_s}{L_q}i_q - p\frac{L_d}{L_q}i_d\Omega - p\frac{1}{L_q}\phi_f\Omega + \frac{1}{L_q}v_q$$

Substituting equation (2) into (44):

$$\dot{s_2} = \dot{i_{qref}} - (-\frac{R_s}{L_q}i_q - p\frac{L_d}{L_q}i_d\Omega - p\frac{1}{L_q}\phi\Omega) - \frac{1}{L_q}v_q \tag{45}$$

Forcing sliding variable to reach zero by designing the control input v_q:

$$v_q = -L_q(-\dot{i_{qref}} - \frac{R_s}{L_q}i_q - p\frac{L_d}{L_q}i_d\Omega - p\frac{1}{L_q}\phi\Omega + \mathbf{k}_2\mathbf{U}_2) \tag{46}$$

Substituting equation (46) into (45) to design SMC control input $\mathbf{U}_2$:

$$\dot{s_2} = \mathbf{K}_2\mathbf{U}_2 \tag{47}$$

$$\mathbf{U}_2 = -sgn(s_2) \tag{48}$$

2.3.3 Sliding Variable for Ω

$$s_3 = \dot{\Omega_{ref}} - \dot{\Omega} + \lambda(\Omega_{ref} - \Omega) \tag{49}$$

Recalling equation (3), where:

$$\frac{d\Omega}{dt} = -\frac{f_v}{J}\Omega + \frac{3}{2}(\frac{p}{J})(L_d - L_q)i_d i_q + \frac{3}{2}(\frac{p}{J})\phi_f i_q - \frac{1}{J}T_L$$

Substituting equation (3) into (49):

$$s_3 = \dot{\Omega_{ref}} + \frac{f_v}{J}\Omega - \frac{3}{2}(\frac{p}{J})(L_d - L_q)i_d i_q - \frac{3}{2}(\frac{p}{J})\phi_f i_q + \frac{1}{J}T_L + \lambda(\Omega_{ref} - \Omega) \tag{50}$$

$$\begin{aligned}\dot{s_3} = {} & \ddot{\Omega_{ref}} - \frac{3}{2}\frac{p}{J}(L_d - L_q)(\frac{-R_s}{L_d}i_d + p\frac{L_q}{L_d}i_q\Omega + \frac{1}{L_d}v_d)i_q \\ & + \frac{f_v}{J}(\frac{3}{2}\frac{p}{J}(L_d - L_q)i_d i_q - \frac{f_v}{J}\Omega + \frac{3}{2}\frac{p}{J}\phi_f i_q - \frac{T_l}{J}) - (\frac{3}{2}\frac{p}{J}(L_d - L_q)i_d + \frac{3}{2}\frac{p}{J}\phi_f)\dot{i_q} \\ & + \frac{\dot{T_l}}{J} + \lambda\dot{\Omega_{ref}} - \lambda(\frac{3}{2}\frac{p}{J}(L_d - L_q)i_d i_q - \frac{f_v}{J}\Omega + \frac{3}{2}\frac{p}{J}\phi_f i_q - \frac{T_l}{J})\end{aligned} \tag{51}$$

Forcing sliding variable to reach zero by designing $\dot{i_{qref}}$; $\dot{i_{qref}} = \dot{i_q}$:

$$\begin{aligned}\dot{i_{qref}} = {} & \frac{-1}{\frac{3}{2}\frac{p}{J}(L_d - L_q)i_d + \frac{3}{2}\frac{p}{J}\phi_f}(-\ddot{\Omega_{ref}} + \frac{3}{2}\frac{p}{J}(L_d - L_q)(\frac{-R_s}{L_d}i_d + p\frac{L_q}{L_d}i_q\Omega + \frac{1}{L_d}v_d)i_q \\ & - \frac{f_v}{J}(\frac{3}{2}\frac{p}{J}(L_d - L_q)i_d i_q - \frac{f_v}{J}\Omega + \frac{3}{2}\frac{p}{J}\phi_f i_q - \frac{T_l}{J}) - \frac{\dot{T_l}}{J} - \lambda\dot{\Omega_{ref}} \\ & + \lambda(\frac{3}{2}\frac{p}{J}(L_d - L_q)i_d i_q - \frac{f_v}{J}\Omega + \frac{3}{2}\frac{p}{J}\phi_f i_q - \frac{T_l}{J}) + \mathbf{k}_3\mathbf{U}_3)\end{aligned} \tag{52}$$

Integrating equation (52)to get i_{qref} to be used in calculating the sliding variable s_2 as shown in equation (43):

$$i_{qref} = \int \dot{i_{qref}} dt \tag{53}$$

As well, $\dot{i_{qref}}$ is used to calculate the control input $v_{q,}$ shown in equation (46). Substituting equation (52) into (51) to design SMC control input $\mathbf{U}_3$:

$$\dot{s_3} = \mathbf{K}_3\mathbf{U}_3 \tag{54}$$

$$\mathbf{U}_3 = -sgn(s_3) \tag{55}$$

3 Simulation Results

To illustrate the mathematical analysis and, hence, to investigate the performance of the proposed Implicit SMC control of PMSM, simulations are carried out in comparison to the continuous-time SMC corresponding to distinct scenarios as shown in the consecutive sections. Furthermore, simulations are executed twice taking into account various motors' parameters to demonstrate the robustness of the proposed controller.

3.1 Simulation with Projected Sliding Mode Algorithm

The following results are obtained after simulating the previously designed sliding mode control inputs on simulink under the assumption that no disturbance acts on the system. As well, ω_{ref} is constant.

The following tables show the parameters used to implement the simulation.

Table 2: Implicit SMC Parameters

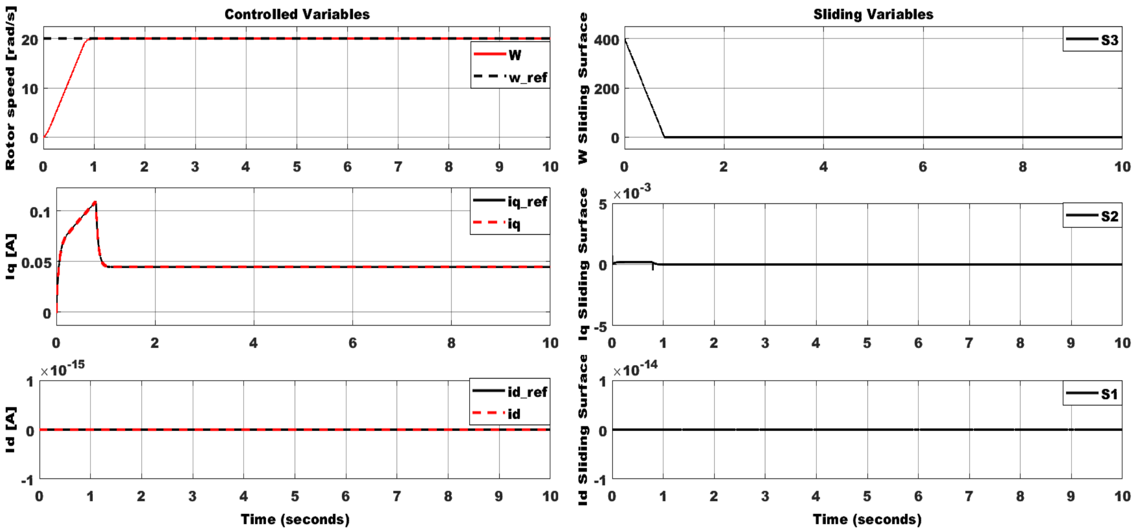
Parameters	Value
$\mathbf{k}_1$	100
$\mathbf{k}_2$	150
$\mathbf{k}_3$	500
λ	20
$\mathbf{h}$	0.5 ms
Ω_{ref}	20 rad/s

Table 3: Motor parameters [3]

Electrical Parameters	Value	Mechanical Parameters	Value	Magnetic Parameter	Value
R_s	3.25 Ω	T_L	5.3 $N.m$	ϕ_f	0.341 Wb
L_d	18 mH	J	0.00417 $kg\ m^2$		
L_q	34 mH	f_v	0.0034 $Kg\ m^2\ s^{-1}$		
p	3				

3.1.1 First Test Scenario with I_d equals zero

Firstly, the simulation is carried out, with no load torque and zero I_d giving the following results for the three implicitly designed SMC controllers in terms of sliding variables, control inputs and targeted variables to control.



(a) (b)

Figure 5: Simulation results with no load torque and zero I_d

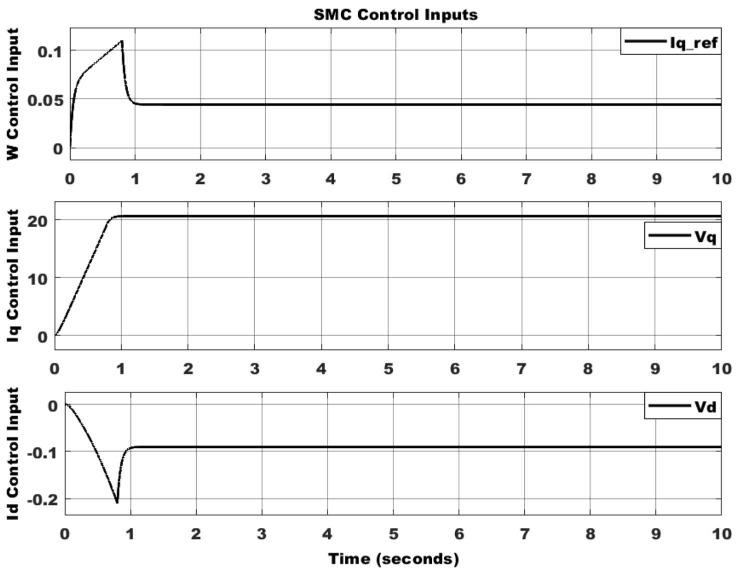
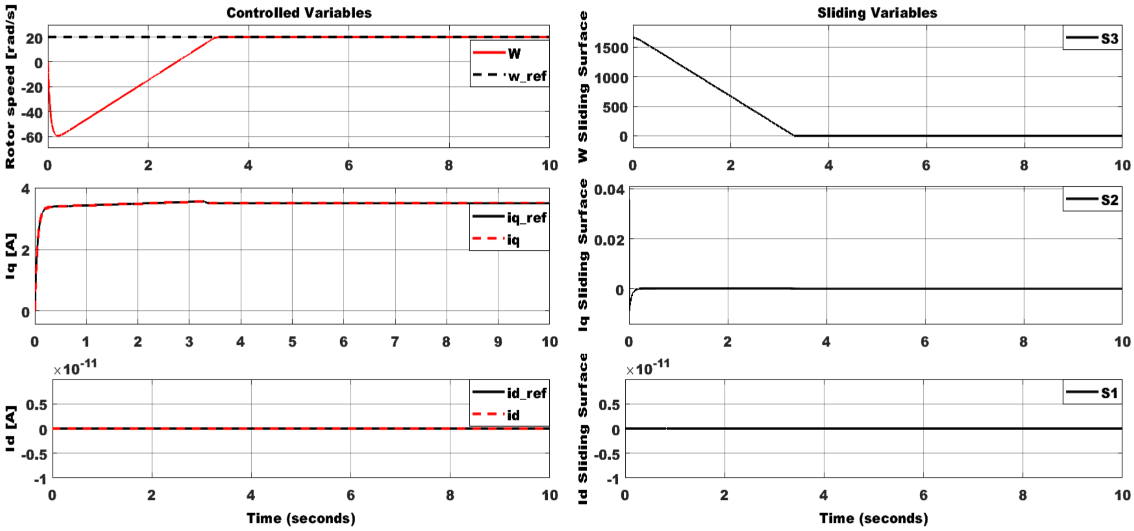


Figure 6: No chattering SMC Control Inputs

In Figures 5 and 6, the three Implicit SMCs succeeded in controlling the speed of the PMSM to track its reference speed, in addition to eliminating the chattering from the control inputs.

The same simulation is repeated, however, load torque is taken is applied on the motor besides the zero I_d giving the following results for the three implicit SMC controllers.



(a) (b)

Figure 7: Simulation results with load torque of 5.3 NM and zero I_d

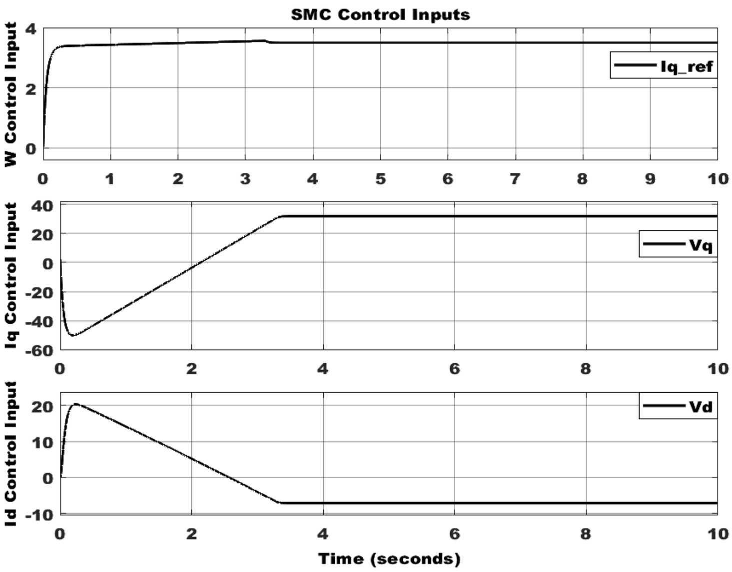


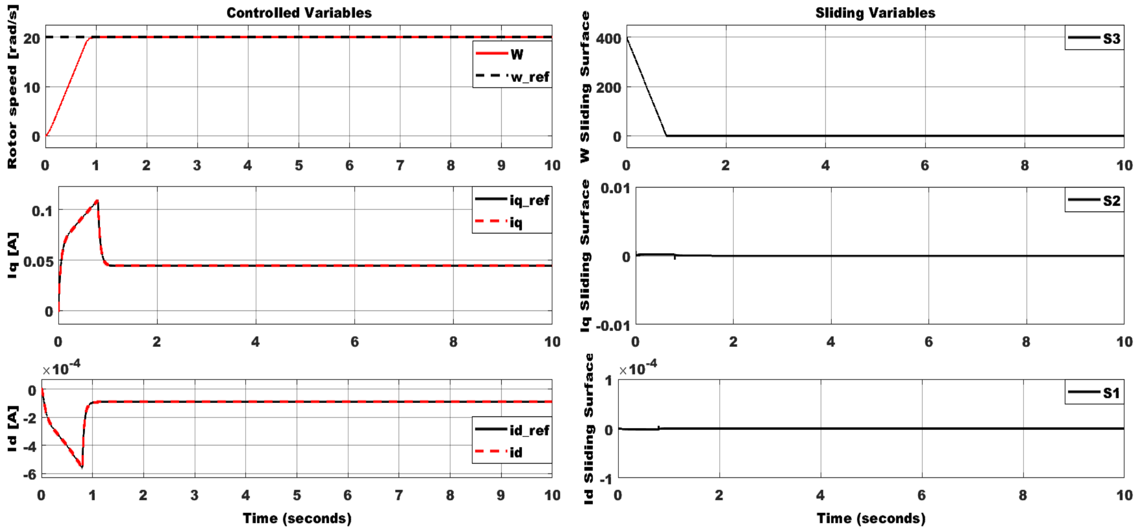
Figure 8: No chattering SMC Control Inputs

Comparing figures 5 and 6 to figures 7 and 8, the presence of the load torque affected the time taken by the actual rotational speed to track the desired value. When there is a load torque, the rotational speed became negative therefore , speed SMC needed more time to overcome its effect and reach the positive value of the reference given as 20 rad/s.

3.1.2 Second Test Scenario with varying I_d

This testing scheme is precisely a duplicate of the previous test, except that it considers I_d varying depending on I_q as explained previously to allow the motor to overwhelm heavier load torque.

Firstly, the simulation is carried out, with no load torque and fluctuating I_d giving the following results for the three implicitly designed SMC controllers



(a) (b)

Figure 9: Simulation results with no load torque and varying I_d

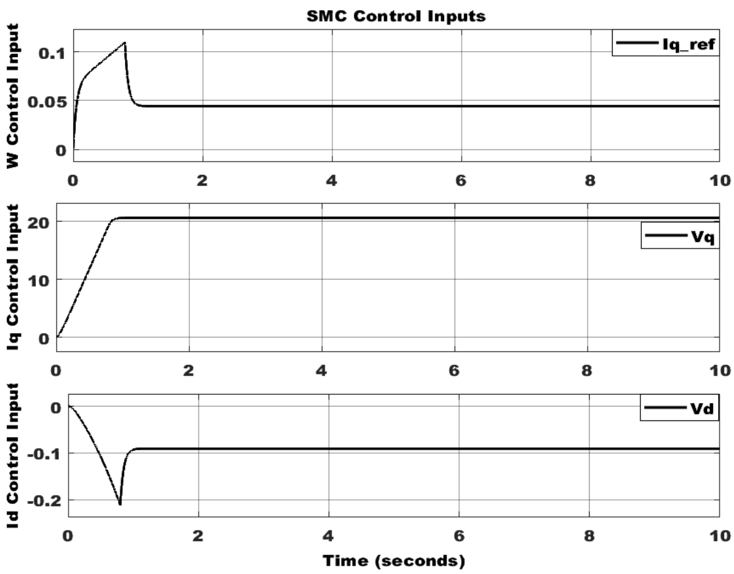
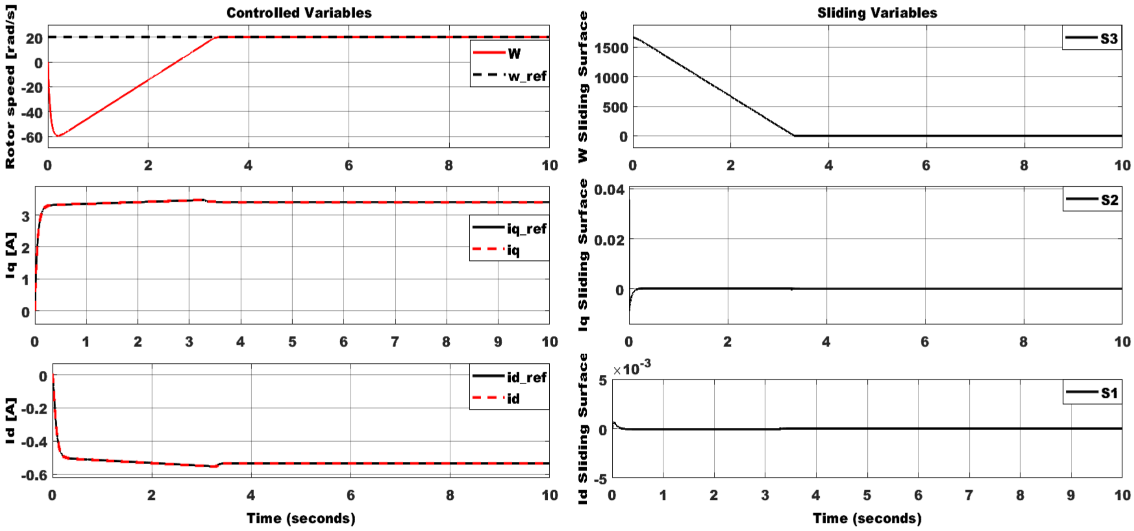


Figure 10: No chattering SMC Control Inputs

The same simulation is repeated, however, load torque is taken is applied on the motor besides the varying I_d giving the following results for the three implicit SMC controllers.



(a) (b)

Figure 11: Simulation results with load torque of 5.3 NM and varying I_d

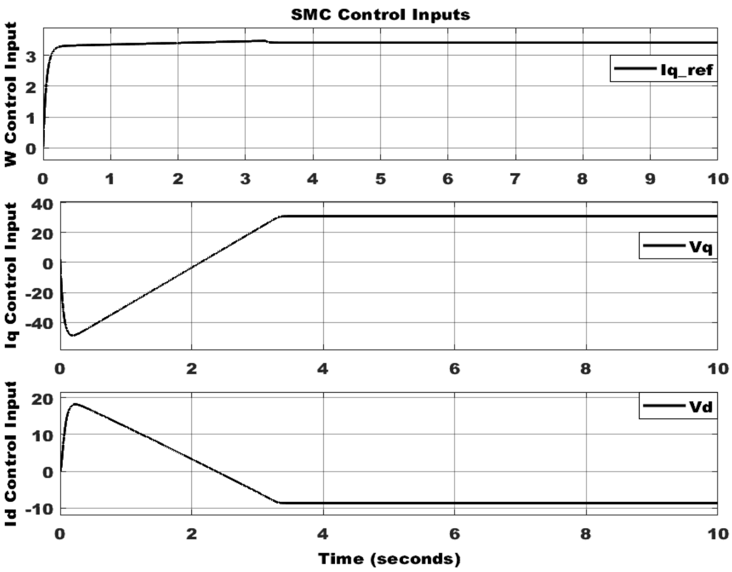


Figure 12: No chattering SMC Control Inputs

3.2 The effect of increasing the gains and the sample time independently

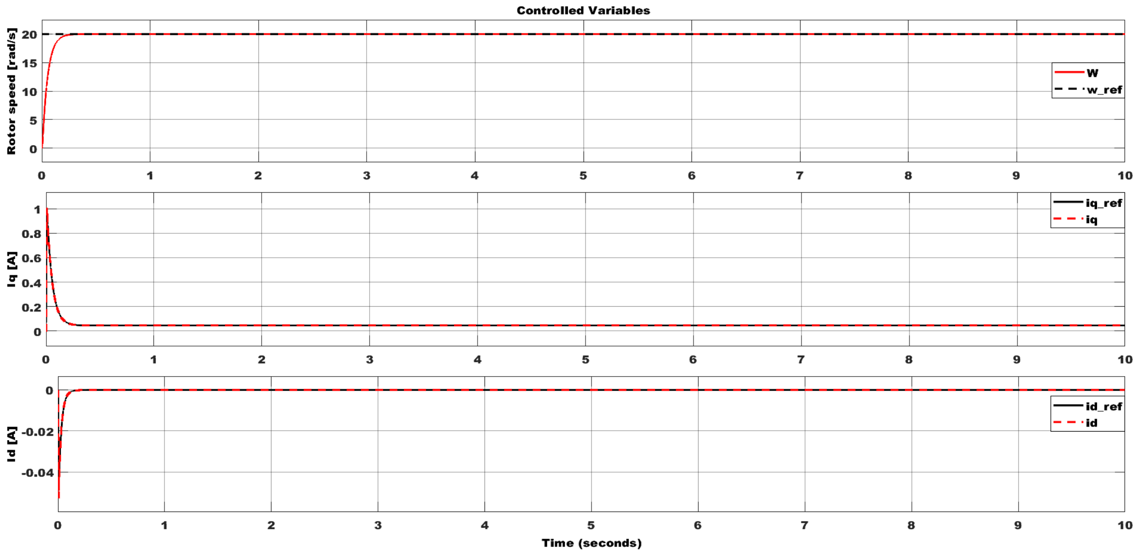


Figure 13: Simulation results with zero load torque and varying I_d for gains multiplied by 100

Without changing the sample time, increasing the gains by 100 fold lets each variable stabilize 100 times quicker. The results shown in Figure 13 meet the previously attained conclusions of part 1 for the simplest simplest system.

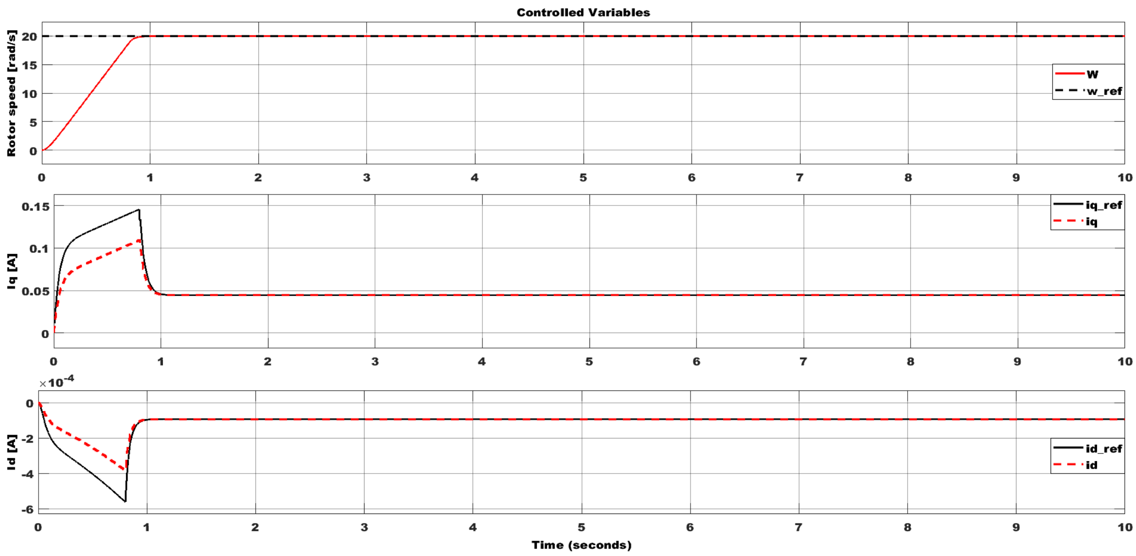


Figure 14: Simulation results with a zero load torque and varying I_d for 10 times less precise h

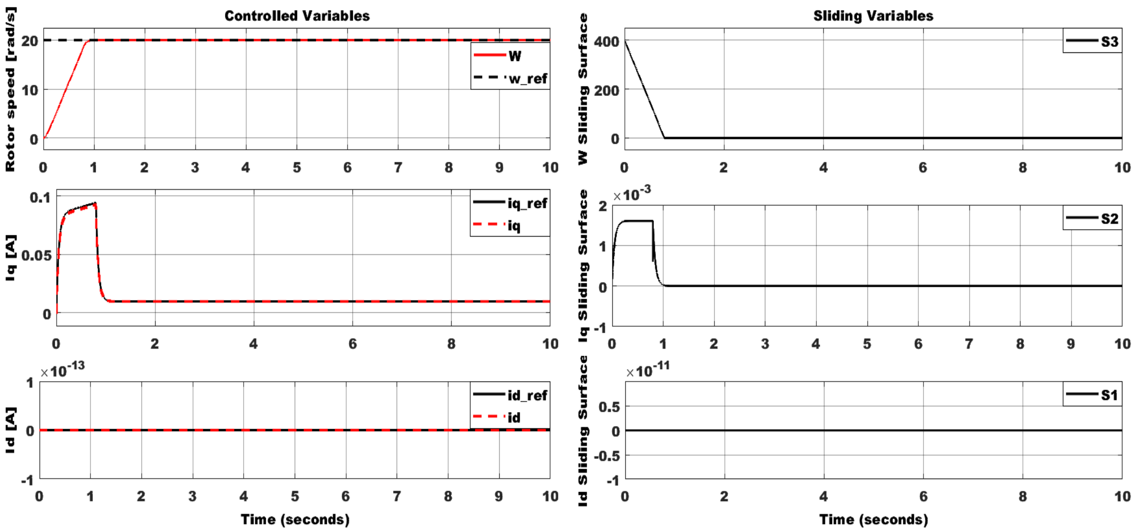
On the other hand, without changing the gains, 10 times larger sample time increased the tracking errors and also time to reach the stability.

3.3 Robustness Test

In this section, the implicit control Simulink model of the PMSM is tested using the parameters on the work of Kakosimos & Abu-Rub [7].

Table 4: Motor parameters for robustness test

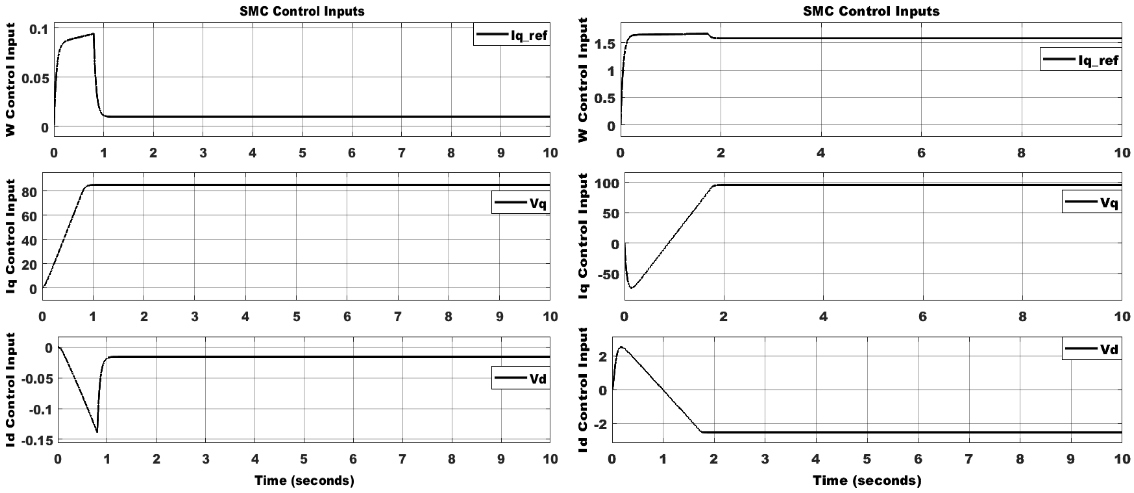
Electrical Parameters	Value	Mechanical Parameters	Value	Magnetic Parameter	Value
R_s	6.98 Ω	T_L	10 $N.m$	ϕ_f	1.06 Wb
L_d	20 mH	J	0.0212 $kg\,m^2$		
L_q	20 mH	f_v	0.0031 $Kg\,m^2\,s^{-1}$		
p	4				



(a) (b)

Figure 15: Simulation results with no load torque and zero I_d for data in Table 4

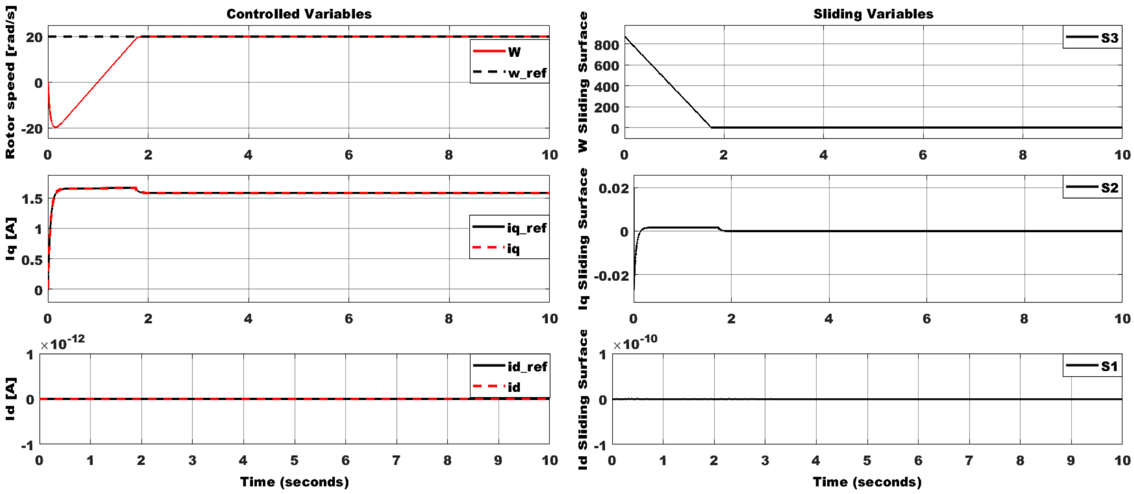
All the given simulation results in this section assure the robustness of the designed SMC controllers because they succeeded in controlling the speed of another PMSM with different parameters as well as phasing out the chattering phenomena.



(a) No chattering SMC Control Inputs with no load torque and zero I_d

(b) No chattering SMC Control Inputs with 10 NM load torque and zero I_d

Figure 16



(a)

(b)

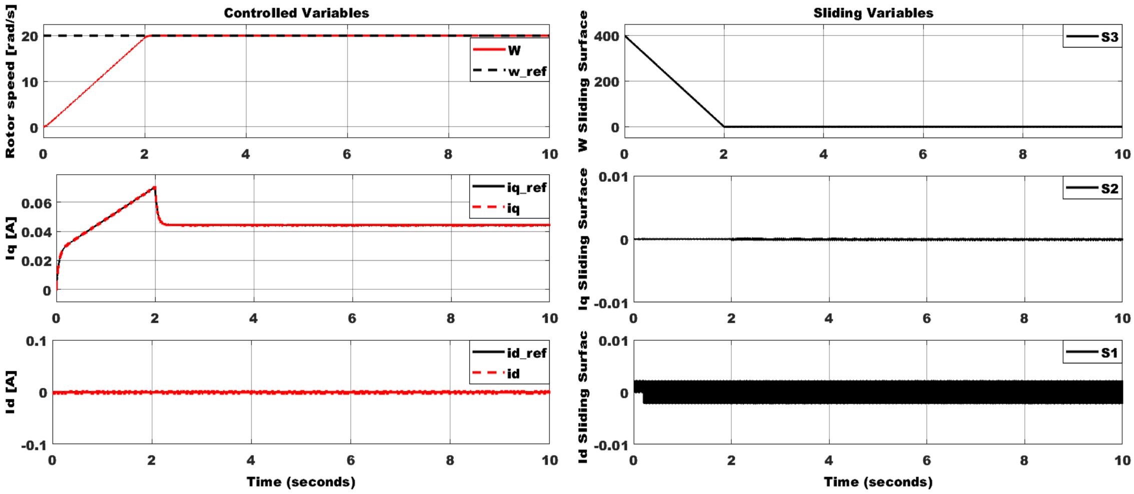
Figure 17: Simulation results with 10 NM load torque and zero I_d for data in Table 4

3.4 Simulation with Classical Sliding Mode Algorithm

The objective of this section is to highlight the performance of the continuous-time sliding mode controller counting its well-known chattering incident in order to compare its performance to the proposed implicit SMC.

In this attempt, the following results are obtained after simulating the previously designed classical sliding mode control inputs on simulink under the assumption that no disturbance or load torque acts on the system.

3.4.1 Constant Reference Speed



(a) (b)

Figure 18: Simulation results with no load torque and zero I_d for Classical SMC

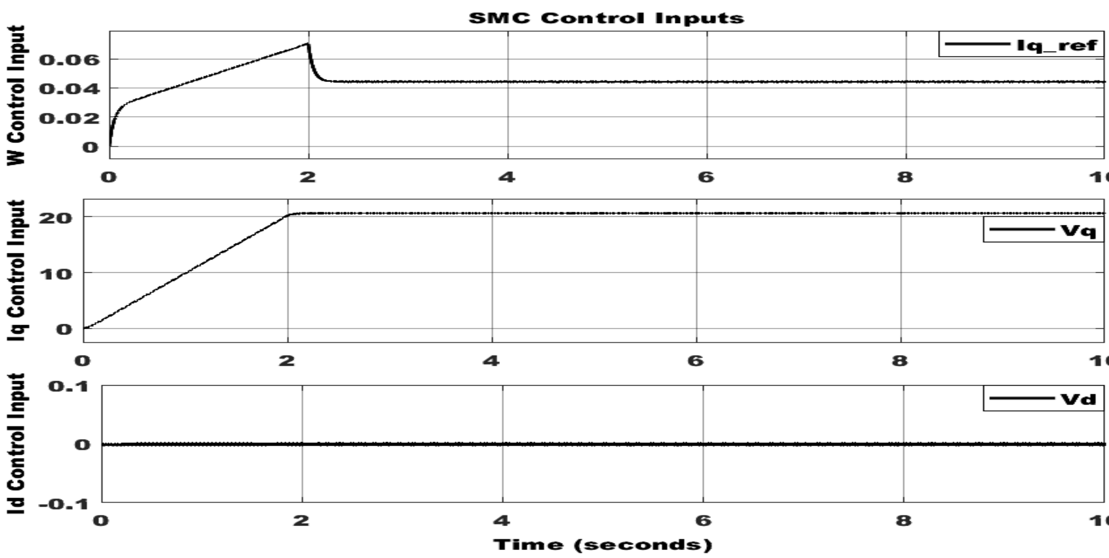
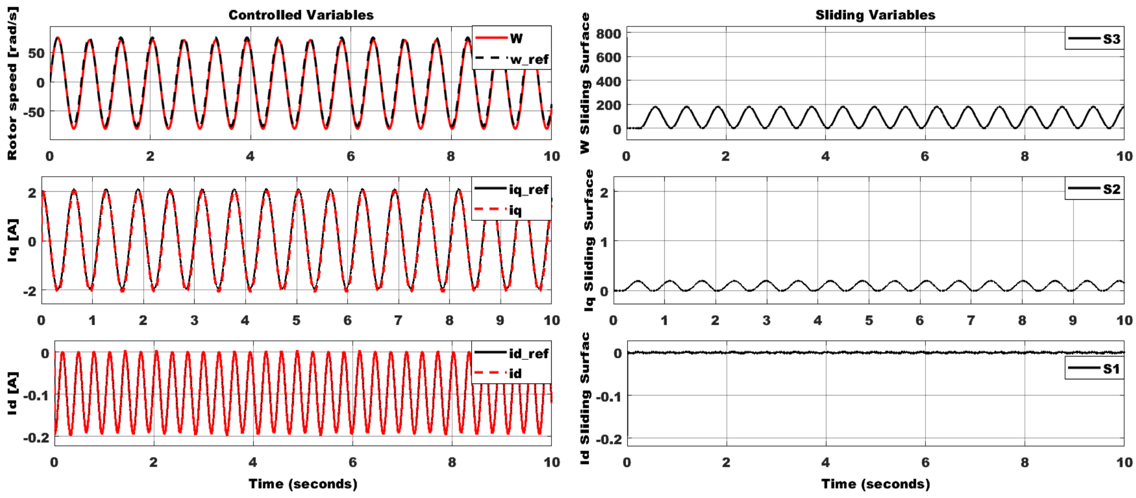


Figure 19: Chattering Control Input

3.4.2 Varying Reference Speed



(a) (b)

Figure 20: Simulation results with no load torque and zero I_d for Classical SMC

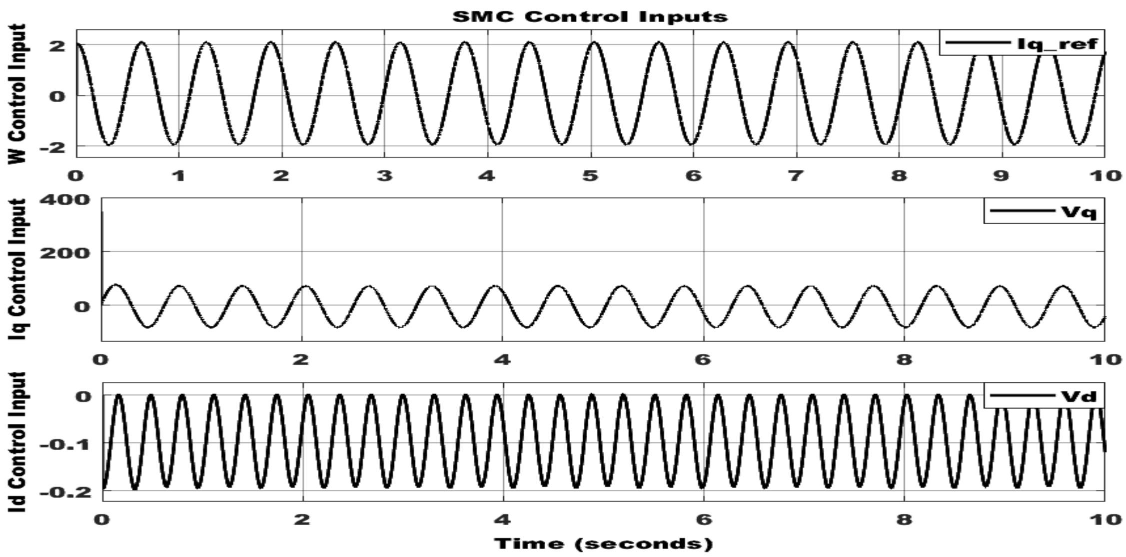


Figure 21: Chattering Control Input

4 Conclusion

In E-PiCo Project, Nonlinear control of permanent magnet synchronous machine (PMSM) is performed with the use of projected sliding mode control [4]. The main objective was to develop the Implicit - or also called as projected- SMC was to obtain an efficient tracking behavior for the rotor speed and rotor currents of their reference values. In other words, the aim was to eliminate the chattering phenomenon of classical sliding mode which may cause poor control accuracy, wearing of moving mechanical parts, and high heat losses in power circuits.

For the controller implementation, initially, the continuous-time PMSM model and classical SMC is studied. Next, the system is discretized and three sliding surfaces: one for the rotor speed, two for the rotor currents are defined referring to the implicit equations. Thirdly, the control inputs evaluated using the projector and mathematical model of the implicit controller is completed. To examine the tracking behavior, Matlab/Simulink software is used and the results for zero loads torque/ non-zero load torque; zero d-frame currents, variable d-frame current are simulated. As presented in the results section, with the use of projected SMC algorithms, all variables are followed the desired values without chattering. As a final step, the robustness of the system is tested changing the motor parameters.

References

[1] Vincent Acary, Bernard Brogliato, "Implicit Euler numerical scheme and chattering-free implementation of sliding mode systems", Systems & Control Letters, Volume 59, Issue 5, 2010, Pages 284-293, ISSN 0167-6911, https://doi.org/10.1016/j.sysconle.2010.03.002.

[2] A. Navarrete, J. Rivera, J. J. Raygoza and S. Ortega, "Discrete-Time Modeling and Control of PMSM," 2011 IEEE Electronics, Robotics and Automotive Mechanics Conference, Cuernavaca, Morelos, 2011, pp. 258-263, doi: 10.1109/CERMA.2011.48.

[3] M.A. Hamida, J. De Leon & A. Glumineau (2014) "High-order sliding mode observers and integral backstepping sensorless control of IPMS motor", International Journal of Control, 87:10, 2176-2193, DOI: 10.1080/00207179.2014.904523

[4] Olivier Huber, Bernard Brogliato, Vincent Acary, Ahcene Boubakir, Franck Plestan, et al., "Experimental results on implicit and explicit time-discretization of equivalent-control-based sliding-mode control". L. Fridman; J.P. Barbot; F. Plestan. Recent Trends in Sliding Mode Control, 102, IET, pp.207-235, 2016, IET Control, Robotics and Sensors Series, 978-1-78561-076-9.

[5] P. Pillay and R. Krishnan, "Modeling of permanent magnet motor drives", in IEEE Transactions on Industrial Electronics, vol. 35, no. 4, pp. 537-541, Nov. 1988, doi: 10.1109/41.9176.

[6] M.S. Merzoug, H. Benall, L. Louze, "Sliding Mode Control (SMC) Of Permanent Magnet Synchronous Generators (PMSG)", Energy Procedia, Volume 18, 2012, Pages 43-52, ISSN 1876-6102.

[7] P. Kakosimos and H. Abu-Rub, "Predictive Speed Control With Short Prediction Horizon for Permanent Magnet Synchronous Motor Drives," in IEEE Transactions on Power Electronics, vol. 33, no. 3, pp. 2740-2750, March 2018, doi: 10.1109/TPEL.2017.2697971.