Experimental validation of two semi-implicit homogeneous discretized differentiators on a cable-driven parallel robot

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Interest of the discretized differentiators with projectors

The objectives

- Real-time discrete signal differentiation from a measured signal
- An alternative solution to the usual backward difference scheme to reduce
  - Chattering effect
  - Noise
  - Disturbances

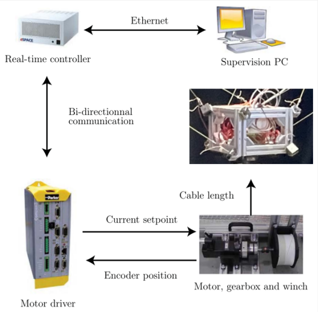
Some application domains in engineering

- Robotics
- Automatic control
- Signal processing
- and so on...

The CRAFT parallel robot 1/2

A cable-driven parallel robot

a)



b)

- Base frame of *CRAFT*: 4 m long, 3.5 m wide, and 2.7 m high
- Suspended moving-platform (*MP*): 3-DoF translational, 3-DoF rotational motions, and 8 cables; 0.28 m long, 0.28 m wide and 0.2 m high; 5 kg mass

The CRAFT parallel robot 2/2

Dynamic model

$$\mathbb{I}_p \ddot{\mathrm{p}} + \mathrm{C}\dot{\mathrm{p}} - \mathrm{w}_g = \mathrm{W}\boldsymbol{\tau} + \mathrm{w}_e \tag{1}$$

where:

- $$\dot{\mathrm{p}} = \begin{bmatrix} \dot{\mathrm{t}} \\ \boldsymbol{\omega} \end{bmatrix} \quad \ddot{\mathrm{p}} = \begin{bmatrix} \ddot{\mathrm{t}} \\ \boldsymbol{\alpha} \end{bmatrix}, \tag{2}$$

  $\dot{\mathrm{t}} = [\dot{t}_x, \dot{t}_y, \dot{t}_z]^\top$ and $\ddot{\mathrm{t}} = [\ddot{t}_x, \ddot{t}_y, \ddot{t}_z]^\top$ *MP* linear velocity and acceleration, respectively; $\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^\top$ and $\boldsymbol{\alpha} = [\alpha_x, \alpha_y, \alpha_z]^\top$ *MP* angular velocity and acceleration
- W wrench matrix that maps the cable tension vector $\boldsymbol{\tau}$ exerted by the cables onto *MP*
- External wrench w_e, a 6-dimensional vector / Wrench w_g due to gravity
- Matrix $\mathbb{I}_p$ is the spatial inertia of the platform
- C is the matrix of the centrifugal and Coriolis wrenches

Problem statement 1/4

Continuous-time state model systems

- $p(t)$ is a bounded perturbation, unknown such as:

$$p_M > 0 \;\text{ such that }\; |p(t)| < p_M \;\text{ for all }\; t > 0. \tag{3}$$

- The continuous model under consideration:

$$\Sigma : \begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= p(t) \\ y &= x_1 \end{cases} \tag{4}$$

x_1 and x_2 are respectively the angular variable and velocity; y is the measure of x_1 with additional noise η

Problem statement 2/4

Discretized state model systems

- Following notation for the discretized variable:

$$
\begin{array}{rcl}
\bullet(t=(k+1)h) & = & \bullet^{+} \\
\bullet(t=kh) & = & \bullet.
\end{array}
\tag{5}
$$

the implicit Euler discretization of the continuous-time model can be written

$$
\left\{
\begin{array}{l}
x_1^{+} = x_1 + h\,x_2^{+} = x_1 + h(x_2 + hp^{+}) \\
\\
x_2^{+} = x_2 + hp^{+} \\
\\
y = x_1
\end{array}
\right. .
\tag{6}
$$

where $p(t)$ is assumed to be a constant parameter or a slowly variable

Problem statement 3/4

Homogeneous continuous-time differentiator

- Homogeneity approach, interesting: Due to the dilatation, this local property to global settings $\Rightarrow$ continuous-time homogeneous differentiator is therefore chosen under the assumption $|p(t)| < p_M$

$$\begin{cases} \dot{z}_1 = z_2 + \lambda_1 \mu \lceil \epsilon_1 \rfloor^{\alpha} \\ \dot{z}_2 = \lambda_2 \mu^2 \lceil \epsilon_1 \rfloor^{2\alpha-1} \\ \hat{y} = z_1 \end{cases} \tag{7}$$

where

- $\alpha \in]0.5 \quad 1[$, $\epsilon_1 = y - z_1$, and the notation $\lceil \bullet \rfloor^{\alpha} = |\bullet|^{\alpha} \mathrm{sgn}(\bullet)$
- If $\alpha = 0.5$, it becomes the classical super-twisting algorithm
- If $\alpha \searrow$ then accuracy $\nearrow$ but noise rejection $\nearrow$

Problem statement 4/4

Existing Euler discretization schemes

- Explicit method: z_i and $\dot{z}_i$, known at $t = kh$, z_i^+ *deduced* from $z_i^+ = z_i + h\dot{z}_i$

$$\begin{cases} z_1^+ = z_1 + h\left(z_2 + \lambda_1 \mu \lceil \epsilon_1 \rfloor^{\alpha}\right) \\ z_2^+ = z_2 + h\lambda_2 \mu^2 \lceil \epsilon_1 \rfloor^{2\,\alpha - 1} \end{cases} \tag{8}$$

  Chattering effect remains $\Rightarrow$ the numerical solution is not attractive

- Implicit method: z_i is known, $\dot{z}_i^+$ is *calculated* such as z^+ is equal to $z_i^+ = z_i + h\dot{z}_i^+$.

$$\begin{cases} z_1^+ = z_1 + h\left(z_2^+ + \lambda_1 \mu \lceil \epsilon_1^+ \rfloor^{\alpha}\right) \\ z_2^+ = z_2 + h\lambda_2 \mu^2 \lceil \epsilon_1^+ \rfloor^{2\,\alpha - 1} \end{cases} \tag{9}$$

If $\epsilon_1^+ = 0$, $z_2^+ = 0 \Longrightarrow z_2 = 0$ Hence, the two correction terms $\lambda_1 \lceil \epsilon_1^+ \rfloor^{\alpha}$ and $\lambda_2 \lceil \epsilon_1^+ \rfloor^{2\alpha - 1}$ with $\epsilon_1^+ = 0$ become inoperative

Semi-implicit Homogeneous Euler differentiators 1/2

Semi-implicit Euler homogeneous differentiator (SIHD-1 version)

The first scheme SIHD1 allows to overcome the drawbacks of these two previous numerical schemes

$$\begin{cases} z_1^+ = z_1 + h\left(z_2^+ + \lambda_1 \mu |\epsilon_1|^{\alpha} \mathcal{N}_1\right) \\ z_2^+ = z_2 + E_1^+ h \lambda_2 \mu^2 |\epsilon_1|^{2\alpha - 1} \mathcal{N}_1 \end{cases} \tag{10}$$

The def. of the single projector $\mathcal{N}_1$ (associated to the enabling flag E_1^+) reads:

$$\mathcal{N}_1(\epsilon_1) := \begin{cases} \epsilon_1 \in SD \quad \to \mathcal{N}_1 = \dfrac{\lceil \epsilon_1 \rfloor^{1-\alpha}}{\lambda_1 \mu h}, \quad E_1^+ = 1 \\ \epsilon_1 \notin SD \quad \to \mathcal{N}_1 = \operatorname{sign}(\epsilon_1), \quad E_1^+ = 0 \end{cases} \tag{11}$$

Semi-implicit Homogeneous Euler differentiators 2/2

Semi-implicit homogeneous Euler discretization (SIHD-2 version)

$$\begin{cases} z_1^+ = z_1 + h\left(z_2^+ + \lambda_1 \mu |\epsilon_1|^{\alpha} \mathcal{N}_1\right) \\ z_2^+ = z_2 + E_1^+ h \lambda_2 \mu^2 |\epsilon_1|^{2\,\alpha - 1} \mathcal{N}_2 \end{cases} \quad (12)$$

with the projector $\mathcal{N}_1$ and the flag E_1^+ defined in (11) and when $\epsilon_1 \in SD$ $\epsilon_1 = h\,\epsilon_2$ holds, $\mathcal{N}_2$ reads as:

$$\mathcal{N}_2 := \begin{cases} \epsilon_1 \in SD' \to \mathcal{N}_2 = \dfrac{\lceil \epsilon_1 \rfloor^{2(1-\alpha)}}{\lambda_2 h^2 \mu^2} \\ \epsilon_1 \notin SD' \to \mathcal{N}_2 = \mathrm{sign}(\epsilon_1) \end{cases} \quad (13)$$

$SD' = \{\epsilon_1 \in SD / \, |\epsilon_1| \leq (\lambda_1 \mu^2 h^2)^{\frac{1}{2(1-\alpha)}} \equiv |\epsilon_2| \leq (\lambda_1 \mu^2)^{\frac{1}{2(1-\alpha)}} h^{\frac{\alpha}{1-\alpha}}\}$.

Experimental validation 1/4

Condition of data capture

- For each of the eight electrical motors an encoder sensor measures the angular variable of its shaft
- The eight motors with a gearbox reducer of ratio $n = 8$
- The measured value of the angular position at the output shaft of the gearbox reducer
- The sampling period of the acquisition data is equal to 1 ms
- The recording data in position are processed off-line in order to apply the semi-implicit homogeneous Euler discretized differentiators SIHD-1 and SIHD-2

Experimental validation 2/4

Attenuation noise projectors (SIHD$_\theta$-1)

Measured angular positions noisy $y \Rightarrow y_m = x_1 + \eta$ where η is a measurement noise (the output corrective term e_1 becomes $e_{1m} = y_m - z_1$)
$\Rightarrow$ a modified projector including a new parameter θ *to extend* SIHD-1 and SIHD-2 in order to mitigate the influence of noise

$$\begin{cases} z_1^+ = z_1 + h\left(z_2^+ + \lambda_1 \mu |\epsilon_{1m}|^{\alpha} \mathcal{N}_{\theta_1}\right) \\ z_2^+ = z_2 + E_{\theta_1}^+ h \lambda_2 \mu^2 |\epsilon_{1m}|^{2\alpha - 1} \mathcal{N}_{\theta_1} \end{cases} \tag{14}$$

$$\mathcal{N}_{\theta_1} := \begin{cases} (1-\theta)|\epsilon_{1m}|^{1-\alpha} < \lambda_1 \mu h & \to \mathcal{N}_{\theta_1} = \dfrac{(1-\theta)\lceil \epsilon_{1m} \rfloor^{1-\alpha}}{\lambda_1 h \mu} \\ (1-\theta)|\epsilon_{1m}|^{1-\alpha} \geq \lambda_1 \mu h & \to \mathcal{N}_{\theta_1} = \mathrm{sign}(\epsilon_{1\mathrm{m}}) \end{cases}$$

Experimental validation 3/4

Attenuation noise projectors (SIHD$_\theta$-2)

$$
\begin{cases}
z_1^+ = z_1 + h\left(z_2^+ + \lambda_1 \mu |\epsilon_{1m}|^{\alpha} \mathcal{N}_{\theta_1}\right) \\
z_2^+ = z_2 + E_{\theta_1}^+ h \lambda_2 \mu^2 |\epsilon_{1m}|^{2\alpha - 1} \mathcal{N}_{\theta_2}
\end{cases} \tag{15}
$$

$$
\mathcal{N}_{\theta_2} := \begin{cases}
(1-\theta)\,|\epsilon_{1m}|^{2(1-\alpha)} < \lambda_2 \mu^2 h^2 \rightarrow \mathcal{N}_{\theta_2} = \dfrac{(1-\theta)\lceil \epsilon_{1m} \rfloor^{2(1-\alpha)}}{\lambda_2 h^2 \mu^2} \\
(1-\theta)\,|\epsilon_{1m}|^{2(1-\alpha)} \geq \lambda_2 \mu^2 h^2 \rightarrow \mathcal{N}_{\theta_2} = \mathrm{sign}(\epsilon_{1\mathrm{m}})
\end{cases}
$$

Experimental validation 4/4

Parameter setting

- λ_i, $i = 1, 2$ parameters chosen such as the linear part stable
- Value of homogeneous exponent α is chosen between the coefficient of Levant's differentiator ($\alpha = 0.5$) and the linear solution of the discretized differentiators SIHD-1 and SIHD-2 ($\alpha = 1$)
- The parameter θ is chosen by numerical test trial and error allowing a good filtering of the noise *i.e.* $0.5 < \theta < 1$
- Numerical values:

$$\lambda_1 = 2\ 10^4, \quad \lambda_2 = 1\ 10^4, \quad \alpha = 0.81, \quad \theta = 0.9, \quad \mu = 1 \tag{16}$$

Experimental Results 1/5

Comparison between back. difference method, SIHD-1 and SIHD-2

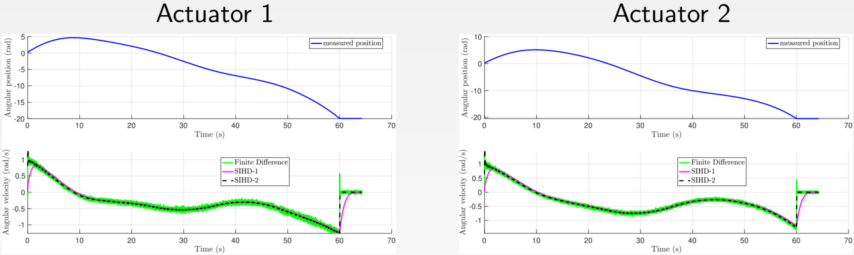
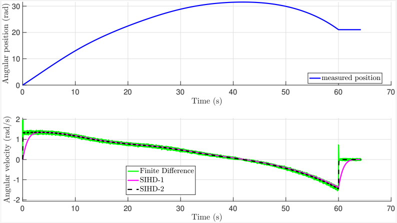


Figure: Recording angular positions (blue color), reference velocity calculated by backward difference (green color), estimated velocities with SIHD-1 (solid line red color) and estimated velocities with SIHD-2 (dashed line black color).

Experimental Results 2/5

Comparison between back. difference method, SIHD-1 and SIHD-2

Actuator 3



Actuator 4

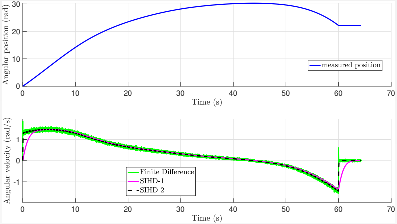
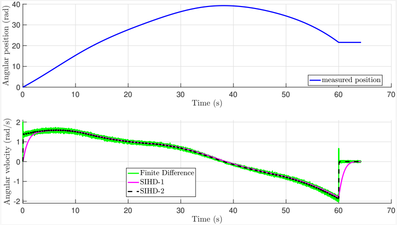


Figure: Recording angular positions (blue color), reference velocity calculated by backward difference (green color), estimated velocities with SIHD-1 (solid line red color) and estimated velocities with SIHD-2 (dashed line black color).

Experimental Results 3/5

Comparison between back difference method, SIHD-1 and SIHD-2

Actuator 5



Actuator 6

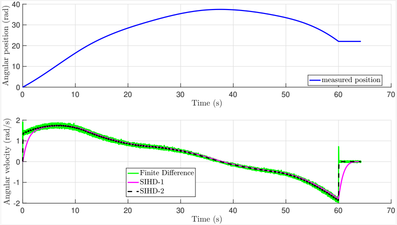
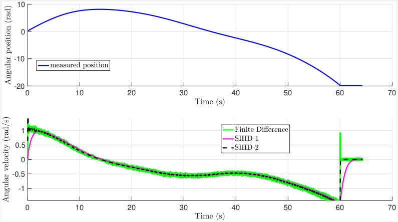


Figure: Recording angular positions (blue color), reference velocity calculated by backward difference (green color), estimated velocities with SIHD-1 (solid line red color) and estimated velocities with SIHD-2 (dashed line black color).

Experimental Results 4/5

Comparison between back difference method, SIHD-1 and SIHD-2

Actuator 7



Actuator 8

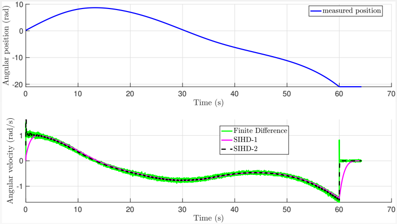


Figure: Recording angular positions (blue color), reference velocity calculated by backward difference (green color), estimated velocities with SIHD-1 (solid line red color) and estimated velocities with SIHD-2 (dashed line black color).

Experimental Results 5/5

Discussion

- Performances of these differentiators are almost uniform whatever the motor
- Angular velocities are smoother *i.e.* less noisy than the reference velocities obtained by backward difference
- Dynamics of the three signals are similar / Transient behavior of the velocity with SIHD-2 is better than with Euler differentiator SIHD-1
- No tachymeter sensor on the motor shaft $\Rightarrow$ difficult to consider the backward difference signal as the reference velocity
- Evaluation of the sensitivity to noise, for example with motor 4

	angular velocity (rad/s)		
	σ, BD	σ, SIHD-1	σ SIHD2
motor 4	0.032	0.017	0.017

Conclusion, perspectives

Conclusion

- Cable-driven parallel robot *CRAFT* : a complex mechanical system
- *CRAFT* promising for handling, rescue, or personal assistance
- Two new semi-implicit homogeneous differentiators applied with success to estimate the angular velocity of the output shaft of the eight motors of *CRAFT*
- Good experimental results: less noisy that the one calculated with backward difference

Perspectives.

- *Cascaded* utilization to estimate the acceleration ;-)
- Semi-implicit homogeneous differentiators for identification tasks of model parameters
- Co-manipulation between its effector and human thanks to a force sensor