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Calcul Numeric
- BONUS -

$$\dot{I} = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2\sigma^2}} dx$$

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Schimbare de variabilă: $y = \frac{x}{\sigma\sqrt{2}}$

$$\Rightarrow \begin{cases} dy = \frac{1}{\sigma\sqrt{2}} dx \\ +\infty \longrightarrow +\infty \\ -\infty \longrightarrow -\infty \end{cases} \qquad \Rightarrow y^2 = \frac{x^2}{2\sigma^2}$$

Ecuația devine:

$$\dot{I} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-y^2} \cdot dy$$

Demonstrez că integrala Gaussiană $\int_{-\infty}^{+\infty} e^{-y^2} dy = \sqrt{\pi}$

Fie $f : \mathbb{R} \to \mathbb{R}$, $f(x) = e^{-x^2}$. $f(-x) = e^{-(-x)^2} = e^{-x^2} = f(x)$

$\Rightarrow f$ funcție pară

$$\dot{I} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} f(y)\, dy \quad \Rightarrow \dot{I} = \frac{1}{\sqrt{\pi}} \cdot 2 \cdot \int_{0}^{\infty} f(y)\, dy$$

Schimbare de variabilă: $y = \sqrt{t}$

$$\Rightarrow \begin{cases} dy = \frac{1}{2\sqrt{t}} dt \\ +\infty \rightarrow +\infty \\ 0 \rightarrow 0 \end{cases} \qquad \Rightarrow y^2 = t$$

Ecuația devine:

$$\dot{I} = \frac{1}{\sqrt{\pi}} \cdot 2 \cdot \int_{0}^{+\infty} e^{-t} \cdot \frac{1}{2\sqrt{t}} dt$$

$$\overset{\circ}{I} = \frac{1}{\sqrt{\pi}} \cdot \cancel{2} \cdot \frac{1}{\cancel{2}} \int_0^{+\infty} t^{\frac{1}{2}-1} \cdot e^{-t} \, dt$$

$$\overset{\circ}{I} = \frac{1}{\sqrt{\pi}} \cdot \Gamma\left(\frac{1}{2}\right)$$

$$\overset{\circ}{I} = \frac{1}{\sqrt{\pi}} \cdot \sqrt{\pi}$$

$$\overset{\circ}{I} = 1$$

Așadar:

$$\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2\sigma^2}} \, dx = 1$$