# Homework 1

PHYS209

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# (1) Problem One

## (1.1)

The higher order function *sqr2* can be defined as:

$$sqr :: (\mathbb{C} \to \mathbb{C}) \to (\mathbb{C} \to \mathbb{C}) \tag{1.1}$$

$$sqr2 = (x \to f(x)) \to (x \to 2f(x)) \tag{1.2}$$

## (1.2)

Since $\cos(x)' = -\sin(x)$ the type and definition of high order function $(\frac{d}{dx} + I)$:

$$\left(\frac{d}{dx} + \mathcal{I}\right) \cdot \cos(x) :: (\mathbb{C} \to \mathbb{C}) \to (\mathbb{C} \to \mathbb{C}) \tag{1.3}$$

$$\left(\frac{d}{dx} + \mathcal{I}\right) \cdot \cos(x) = (x \to \cos(x)) \to (x \to \cos(x) - \sin(x)) \tag{1.4}$$

## (1.3)

Type of operator $C$ when acting on real variables and real functions is:

$$C :: [(\mathbb{R} \to \mathbb{R}) \to (\mathbb{R} \to \mathbb{R})] \to [(\mathbb{R} \to \mathbb{R}) \to (\mathbb{R} \to \mathbb{R})] \tag{1.5}$$

## (1.4)

Action of $\exp\left(\frac{d}{dx}\right) x^4$ can be calculated as:

$$\exp\left(\frac{d}{dx}\right) x^4 = (x^4)^{(0)} + \frac{1}{1!}(x^4)^{(1)} + \frac{1}{2!}(x^4)^{(2)} + \frac{1}{3!}(x^4)^{(3)} + \frac{1}{4!}(x^4)^{(4)} + ... = \\ x^4 + 4x^3 + 6x^2 + 4x + 1 \tag{1.6}$$

Since $(x^4)^{(n)}$ is equal to 0 for any $n > 4$ it is easy to calculate analytically

However, if we simplify (6) even further it can be seen that

$$4x^3 + 6x^2 + 4x + 1 = (x + 1)^4 \tag{1.7}$$

Which amazingly coincides with its geometrical meaning of shifting argument