Homework 10

PHYS209

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(1) Problem 1

(1.1)

If we declare $z = e^{i\pi\theta}$, then $z^* = e^{-i\pi\theta}$. Therefore $f(z^*)$:

$$\begin{aligned} \sin(1) + \cos(1) &= 1.38177329068 \\ \sin(e^{-i\pi/4}) + \cos(e^{-i\pi/4}) &= 0.76536686473 \\ \sin(e^{-i\pi/2}) + \cos(e^{-i\pi/2}) &= 0.38177329068 \\ \sin(e^{-3i\pi/4}) + \cos(e^{-3i\pi/4}) &= 0.76536686473 \end{aligned} \tag{1.1}$$

(1.2)

From Euler's formula we know that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$. From which we can derive:

$$\begin{aligned} \cos(\theta) &= \frac{e^{i\theta} + e^{-i\theta}}{2} \\ \sin(\theta) &= \frac{e^{i\theta} - e^{-i\theta}}{2i} \end{aligned} \tag{1.2}$$

Putting these into $f(z)$ we get:

$$f(z) = \frac{e^{iz} - e^{-iz}}{2i} + \frac{e^{iz} + e^{-iz}}{2} \tag{1.3}$$

Getting complex conjugate of $f(z)$:

$$\begin{aligned} (f(z))^* &= \left(\frac{e^{iz} - e^{-iz}}{2i} + \frac{e^{iz} + e^{-iz}}{2} \right)^* \\ &= \left(\frac{e^{iz} - e^{-iz}}{2i} \right)^* + \left(\frac{e^{iz} + e^{-iz}}{2} \right)^* \\ &= \frac{e^{-iz^*} - e^{iz^*}}{-2i} + \frac{e^{-iz^*} + e^{iz^*}}{2} \\ &= \frac{e^{iz^*} - e^{-iz^*}}{2i} + \frac{e^{-iz^*} + e^{iz^*}}{2} \\ &= f(z^*) \end{aligned} \tag{1.4}$$

(1.3)

If we define $g(z)$ to be equal $\cos(iz)$ then using 1.2 we get:

$$g(z) = \cos(iz) = \frac{e^{-z} + e^{z}}{2} \tag{1.5}$$

Then to find complex conjugate of $g(z)$ we get:

$$\begin{aligned}(g(z))^* &= \left(\frac{e^{-z} + e^{z}}{2}\right)^* \\ &= \frac{e^{-z^*} + e^{z^*}}{2} \\ &= g(z^*)\end{aligned} \tag{1.6}$$

Similarly if we define $h(z)$ to be equal $\sin(iz)$ then using 1.2 we get:

$$h(z) = \sin(iz) = \frac{e^{-z} - e^{z}}{2i} \tag{1.7}$$

Then to find complex conjugate of $h(z)$ we get:

$$\begin{aligned}(h(z))^* &= \left(\frac{e^{-z} - e^{z}}{2i}\right)^* \\ &= \frac{e^{-z^*} - e^{z^*}}{-2i} \\ &= -h(z^*) \\ &\neq h(z^*)\end{aligned} \tag{1.8}$$

(2) Problem 2

(2.1)

To derive most general $3x3$ Hermitian matrix, we can start with a general $3x3$ matrix:

$$\begin{pmatrix} a + ia' & b + ib' & c + ic' \\ d + id' & e + ie' & f + if' \\ g + ig' & h + ih' & k + ki' \end{pmatrix} \tag{2.9}$$

Where z is real part z_{ij} and z' is imaginary part. Now finding complex conjugate of the matrix:

$$\begin{pmatrix} a-ia' & d-id' & g-ig' \\ b-ib' & e-ie' & h-ih' \\ c-ic' & f-if' & k-ki' \end{pmatrix} \tag{2.10}$$

Since matrix is Hermitian, it must be equal to its complex conjugate. Therefore:

$$\begin{gathered} a+ia' = a-ia' \Rightarrow a' = 0 \\ b+ib' = d-id' \Rightarrow b' = -d', b = d \\ c+ic' = g-ig' \Rightarrow c' = -g', c = d \\ e+ie' = e-ie' \Rightarrow e' = 0 \\ f+if' = h-ih' \Rightarrow f' = -h', f = h \\ k+ki' = k-ki' \Rightarrow k' = 0 \end{gathered} \tag{2.11}$$

Therefore most general $3x3$ Hermitian matrix is:

$$\begin{pmatrix} a & b-ib' & c-ic' \\ b+ib' & e & f-if' \\ c+ic' & f+if' & k \end{pmatrix} \tag{2.12}$$