Homework 12

PHYS209

Hikmat Gulaliyev

28.12.2023

(1) Problem 1

(1.1)

Since $\mathcal{D} \cdot f(x)$ is equal to:

$$\left(\frac{\mathrm{d}^3}{\mathrm{d}x^3} + \cos(x) \frac{\mathrm{d}^2}{\mathrm{d}x^2} + x^3 \frac{\mathrm{d}}{\mathrm{d}x} + 1 \right) f(x) = 0 \tag{1.1}$$

And since $\mathcal{A}$ is equal to 1.1, we can rewrite it as matrix as:

$$\frac{\mathrm{d}}{\mathrm{d}x} \begin{pmatrix} f(x) \\ f'(x) \\ f''(x) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -\cos(x) & -x^3 \end{pmatrix} \begin{pmatrix} f(x) \\ f'(x) \\ f''(x) \end{pmatrix} \tag{1.2}$$

(2) Problem 2

(2.1)

Starting with matrix form of equation:

$$\frac{\mathrm{d}}{\mathrm{d}x} \begin{pmatrix} f(x) \\ f'(x) \\ \vdots \\ f^{(n-2)}(x) \\ f^{(n-1)}(x) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & a_1 \end{pmatrix} \begin{pmatrix} f(x) \\ f'(x) \\ \vdots \\ f^{(n-2)}(x) \\ f^{(n-1)}(x) \end{pmatrix} \tag{2.3}$$

We can apply differential operator, and multiply matrices to get:

$$\begin{pmatrix} f'(x) \\ f''(x) \\ \vdots \\ f^{(n-1)}(x) \\ f^{(n)}(x) \end{pmatrix} = \begin{pmatrix} f'(x) \\ f''(x) \\ \vdots \\ f^{(n-1)}(x) \\ -a_n f(x) - a_{n-1} f'(x) - \cdots - a_1 f^{(n-1)}(x) \end{pmatrix} \tag{2.4}$$

And last line gives us:

$$f^{(n)}(x) = -a_n f(x) - a_{n-1} f'(x) - \cdots - a_1 f^{(n-1)}(x) \tag{2.5}$$

Which is the same as:

$$f^{(n)}(x) + a_1 f^{(n-1)}(x) + \cdots + a_n f(x) = 0 \tag{2.6}$$

(2.2)

We are given that:

$$\det(\mathcal{M}) = \sum_{i_1,\ldots,i_n} \epsilon_{i1,\ldots,in} \mathcal{M}_{1i1} \mathcal{M}_{2i2} \cdots \mathcal{M}_{nin} \tag{2.7}$$

asasa