This document is based on Amir's 2711h tutorial 2 Question 3, whose solution I disagree with. The following is written to find the general solution to determine which player must win.

Given two stacks of blocks. Name the one with fewer blocks as A and another as B. Let the number of blocks that A and B have be a and b respectively.

Let player M be the player first moves and another player N.

Hypothesis: if there exists $d \in \mathbb{N}_{\geq 1}$ such that a and b can be expressed in form of $1 + d$ and $1 + 2d$, then the play must lose, else must win.

Lemma 1: player facing $a = b \geq 1$ must win.

Proof: The player can reduce (a, b) to $(0, 0)$ easily.

Lemma 2: The player facing $a = 0, b \geq 1$ must win.

Proof: The player can reduce $(0, b)$ to $(0, 0)$ easily.

Base case: when $d = 1$:

When player M faces $(1, 2)$, he can have the following moves only:

$$\begin{cases} (1,2) \to (1,1) & \text{loses according to Lemma 1} \\ (1,2) \to (1,0) & \text{loses according to Lemma 2} \\ (1,2) \to (2,0) & \text{loses according to Lemma 2} \end{cases}$$

Note that if $b < 2$, the situation becomes $(0, 1)$ or $(1, 1)$, then player M must win

if $b > 2$, then player M can $(1, b) \to (1, 2)$, which means player N must lose.

Induction steps: Assume there exists $k \in \mathbb{N}$ such that the statement is true for all $i \in \mathbb{N}_{\leq k}$, ie player M must lose if he faces $(1 + i, 1 + 2i)$. Consider $k + 1$:

If player M faces $(1 + k + 1, 1 + 2k + 2)$, he can have the following move:

1. Removing $j \leq k$ blocks from A:

$$(1 + k + 1, 1 + 2k + 2) \to (1 + k + 1 - j, 1 + 2k + 2)$$

Then player N can remove $2j$ blocks from B:

$$(1 + k + 1 - j, 1 + 2k + 2) \to (1 + k + 1 - j, 1 + 2(k + 1 - j))$$

where player M must lose by assumption.

2. Removing $j \leq k$ blocks from B:

$$(1 + k + 1, 1 + 2k + 2) \to (1 + k + 1, 1 + 2k + 2 - j)$$

Then player N can remove j blocks from both A and B:

$$(1 + k + 1, 1 + 2k + 2 - j) \to (1 + k + 1 - j, 1 + 2(k + 1 - j))$$

where player M must lose by assumption.

3. Removing $j \leq k$ blocks from both A and B:

$$(1+k+1, 1+2k+2) \to (1+k+1-j, 1+2k+2-j)$$

Then player N can remove j blocks from B:

$$(1+k+1-j, 1+2k+2-j) \to (1+k+1-j, 1+2(k+1-j))$$

where player M must lose by assumption.

4. Removing $j : k+1 < j < 2k+2$ blocks from B:

$$(1+k+1, 1+2k+2) \to (1+k+1, 1+2k+2-j)$$

Then player N can remove $3k+3-j$ blocks from both A and B:

$$(1+k+1, 1+2k+2-j) \to (1-2k-2+2j, 1-k-1+j) = (1+j-k-1, 1+2(j-k-1))$$

where player M must lose by assumption.

5. Removing $k+1, k+2$ blocks from A, $2k+2, 2k+3$ blocks from B,
   or $k+1, k+2$ blocks from both A and B:
   These will result in $(0, n), (1, n)$, which must lose by Lemma 1 & 2.

If player M faces $(1+k+1, 1+2k+2-j)$ or $(1+k+1, 1+2k+2+j)$, where $1 \leq j \leq k$, he can do the following to make sure he can win:

$$\begin{cases} \text{Remove } j \text{ blocks from both A and B} & (1+k+1, 1+2k+2-j) \to (1+k+1-j, 1+2(k+1-j)) \\ \text{Remove } j \text{ blocks from and B} & (1+k+1, 1+2k+2+j) \to (1+k+1, 1+2(k+1)) \end{cases}$$

Therefore, by strong induction, we have successfully prove that if there exists $d \in \mathbb{N}_{\geq 1}$ such that a and b can be expressed in form of $1+d$ and $1+2d$, then the play must lose, else must win.