Computer Aided Control Design

Project-Lab 3
Automatic Control, General Course, EL1000/EL1010

Revised September 3, 2024

> NB! The preparation required for this project lab are the previous computer exercises 2.6, 2.7, 3.4, 3.21, 3.22, 3.23, 4.5, 5.13, 6.10, 9.14. Check the solutions to these exercises for **useful Matlab code**.

1 Introduction

In this computer lab we are going to see how the control laws that we have studied in this course can be used for controlling the position of a robot arm shown in Figure 1. The project should be completed in groups of maximum two students.

1.1 Background

Control of robot arms is a common problem in industry today. Imagine a robot that picks up a part from a crate and puts it on a conveyor belt. If this robot is a bottleneck in a factory, a speed increase of the robot will directly influence the total production of the factory. Even a small improvement of a few percent in the performance can give a big gain in the total number of produced goods.
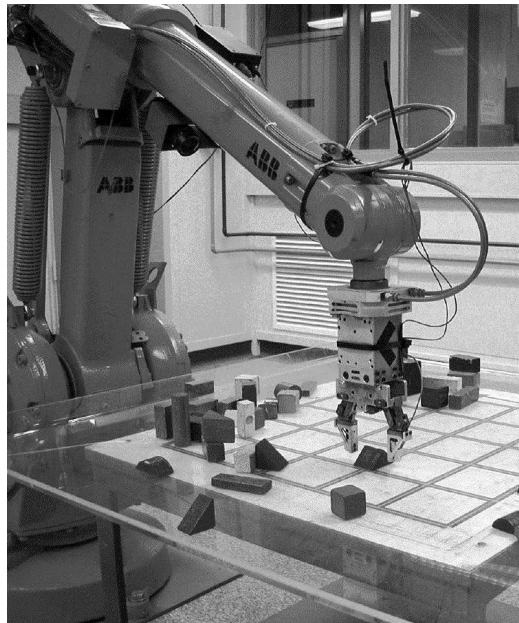

Figure 1: Robot

1.2   Goal

The goal of this lab is to design a lead-lag compensator and a state feedback controller that can be used to control the robot in order to meet certain specifications. The following concepts should have been understood by the student by the end of the project:

- The relationship between time and frequency domain properties (for both open and closed loop systems).

- The process of designing a lead-lag compensator to meet certain specifications.

- The concept of robustness and sensitivity.

- The principle behind a state feedback controller.

- The concept of pole placement design.

1.3   Examination

Examination is performed in a two step procedure.

The first step is when the whole project is completed. A MATLAB program will be used to check that the designed controllers satisfy all the specifications. See Section 4 for more information.

The second step is a 15-minute oral examination. Before this examination the following should have been prepared:

- Answers to all assignments on a separate paper. (Not to be handed in.)

- MATLAB program that can instantaneously reproduce the results, and with as many constants as possible analytically defined.

- Relevant plots, transfer functions and state space models should be easily retrievable in your MATLAB program. (*Hint: Look into dividing the program into cells. See MATLAB help on cell dividers.*)

- The students should be logged into a computer with the MATLAB program ready to run *before* the examination starts.

- The code generated by correct input to the `lab3robot` program (see below).

Note that the assignments should be solved numerically in MATLAB unless they explicitly request an analytical answer.

2   Preparation

To be able to complete the project more easily the answers to the following questions need to be understood.

- What is the cross-over frequency and how can it be found in a Bode plot?

- What is the phase-margin and how can it be found in a Bode plot?

- What is the amplitude-margin and how can it be found in a Bode plot? *(1 - amplitude) at phase: -180 deg*

- How to derive the errors e_0 and e_1? *e0 = s->0 S(s) e1 = s->0 S(s)/s*

- How is the cross-over frequency, bandwidth, and rise time related?

- How is the overshoot, phase margin, amplitude margin and stability related?

- Concerning a lead-lag controller:

  - How does the design parameter β affect the phase margin?
  - How does the design of the lag-part affect the design of the lead-controller?
  - Which frequency should the lead-lag controller mainly be designed for? *Td = 1/(wc * sqrt(B))*
  - How are τ_D and τ_I usually defined? *Ti = 10/wc*
  - What effect does the parameter γ have and what is the effect of setting γ to zero? *Low freq gain when w = 0 gain is 1/gamma   Gamma = 0 => PI-regulator*

- What more information is contained in a state space model than in a transfer function? *The "initial values" of the differential equations*

- What is the main difference between input-output feedback and state feedback? *You have more parameters to work with*

- Why can it be interesting to plot the sensitivity function? What information can be found in the sensitivity plot? *To know wich frequency disturbances affect the system*

- What is a model error? What problems can a model error cause?

- What is the *robustness criterion* and what does it guarantee? *Its the error between the real world and the model. Can come from linearization. Can cause instability*

|T(iw)| < |1/delta_G(iw)|

3 Exercises

3.1 Modeling

The block diagram for the robot arm is given in Figure 2.
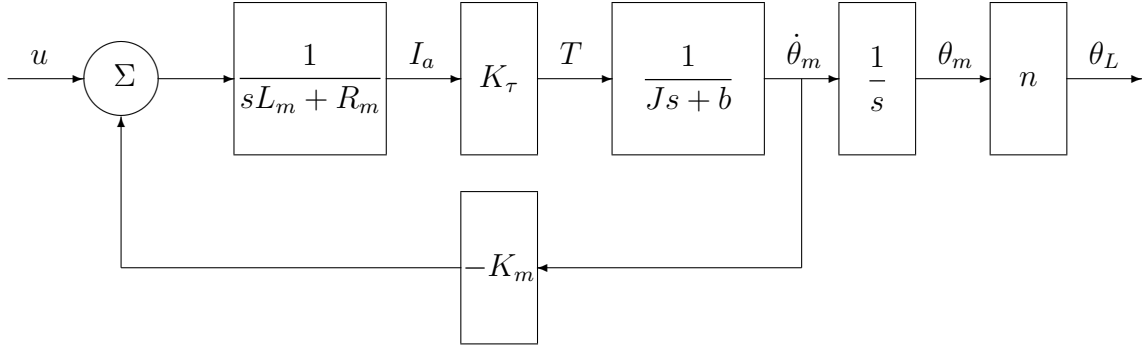
Figure 2: Block diagram of robot arm.

where the variables are defined as

u - voltage
θ_L - robot arm angle
θ_m - motor axle angle
I_a - motor current
T - torque
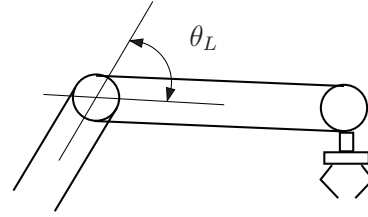
Table 1: Desciptions and definitions of constants in Figure 2

Constant	Description	Value
L_m	Induction	2
R_m	Resistance	21
b	Friction coefficient	1
J	Moment of inertia	see below
K_τ	Material constant	38
K_m	Material constant	0.5
n	Gearing factor	1/20

Moment of inertia J

The moment of inertia and a maximum control signal will be provided by the function `lab3robot`. The file `lab3robot.p` (which can be downloaded from the course homepage) contains a MATLAB function which can be used as

```
[J,umax] = lab3robot(PersonalNumber)
```

where `PersonalNumber` denotes the birth date of one of the team members which should be in the format YYMMDD (year, month and day). Since only one value of J is needed, only use data from one of the team members (which data it is does not matter). The same number is to be used when correcting the solutions. The maximum value of the control signal, u_{max}, which will be used in the control design, is given by `umax`.

Assignment 1. Based on the block diagram in Figure 2, and the parameter values in Table 1, calculate the transfer function of the robot arm first analytically and then print it out with numerical values.

$$G(s) = \underline{\hspace{2cm}} G = \frac{nK_\tau}{s[(sL_m+R_m)(Js+b)+K_\tau K_m]} =$$

$$= \underline{\hspace{6cm}} \tag{1}$$

Check $G(s)$ by writing `lab3robot(G,PersonalNumber)` in MATLAB. Be sure that the current directory is where `lab3robot.p` has been placed after being downloaded from the course homepage.

3.2 Proportional control

First we want to control the system using only a proportional controller. This is shown in Figure 3 where $F(s) = K$. The signal v is a disturbance that will be considered later, for now disregard it, i.e., $v = 0$.

Now assume that we have the following requirements on the unit step response of the closed loop system:

1. The overshoot should be less than 5%.

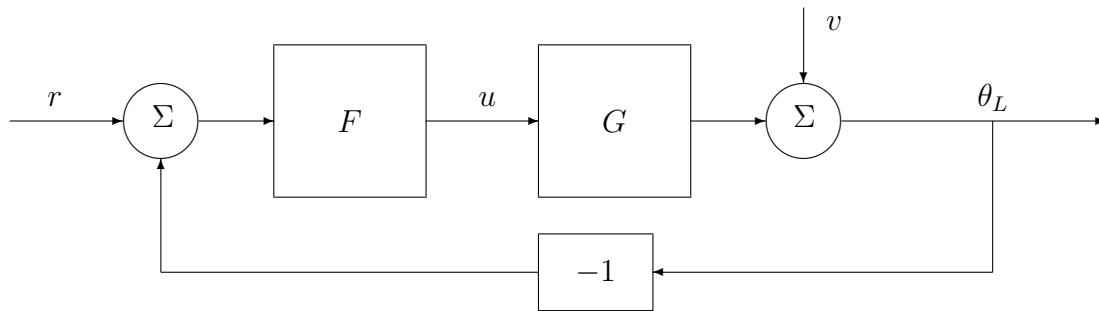2. The rise time should be as short as possible.

Figure 3: Control system.

Assignment 2. Design a proportional controller which satisfies the above requirements. Make a note of the resulting gain, K, and the corresponding rise time.

3,75   8,4 seconds

. . . .

Assignment 3. Calculate the cross-over frequency and phase-margin for the open loop system (including the proportional controller) and the bandwidth of the closed loop system.

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Assignment 4. Answer the following questions (Assume P-control in the discussion):

- What happens to the magnitude and phase plots when a P-controller is implemented?

- What is limiting the speed of the system if ignoring the criteria of max 5% overshoot?

- What happens to the robot arm when the closed loop system becomes unstable?

- At what gain, K, does the system become unstable? What happens to the phase margin using this gain?

- What part of a Bode diagram/Nyqvist curve indicates how much the gain can be increased before instability occurs?

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3.3 Improved control

In this part we want to improve the performance of the control system by determining a loop-shaping (lead-lag) compensator which satisfies the following specifications.

1. The overshoot should be less than 5% when the reference signal is a unit step (same as before).

2. The closed loop system should be four times faster than what was possible with proportional control.

3. The control signal should satisfy $|u(t)| < u_{max}$ for all t when the reference signal is a unit step, where u_{max} is given by the function `lab3robot` (see Section 3.1).

4. The stationary control error should be smaller than 0.05 when the reference signal is a unit ramp (hint: a ramp is an integrated step),

$$r(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}. \tag{2}$$

Since we eventually want to apply the control law in practice we also have the following requirements.

5. We do not want to amplify measurement disturbances too much, i.e. the high-frequency amplification of the controller should not be made unnecessarily large.

6. In order to decrease possible problems with nonlinearities we do not want the controller gain for low frequencies to be too large.

Assignment 5. Answer the following questions:

- Which frequency domain property of the open-loop system is most closely related to the rise time of the system? How should this be changed in the case with a lead-lag compensator to make it four times faster than the P-controller?

- What frequency domain property of the open-loop system should be kept approximately the same to make sure the overshoot is under 5%?

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Assignment 6. Determine the transfer function $F(s)$ of a lead-lag compensator that satisfies the new specifications. Motivate the choice of parameters τ_D, β, τ_I, and γ.

Hint: Start with designing a controller that satisfies specifications 1, 2 and 3, and then augment this controller in order to satisfy specification 4. To check specification 3, simulate $u(t)$ when $r(t)$ is a unit step.

To solve the assignment, the τ_I parameter in the lead-lag compensator may need to deviate from the rule of thumb (see p. 109 in Glad & Ljung ver. 4). If doing so, remember to compensate the modified phase lag by choosing another β.

$$F(s) = F = K\frac{\tau_D s+1}{\beta\tau_D s+1}\frac{\tau_I s+1}{\tau_I s+\gamma} \; \text{-} \tag{3}$$

Assignment 7. Why is the control error for a ramp input not reduced to zero?

Because we only have one integrating factor in G_closed

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3.4   Sensitivity

Assignment 8. Calculate and plot the Bode diagram of the sensitivity functions $S(s)$ with the proportional controller **and** with the controller $F(s)$ from Assignment 6 in the same plot (*hint:* `Bodemag(S1,S2)`). The sensitivity function is the transfer function from v to θ_L in Figure 3. For which frequencies do the controllers attenuate the effect of disturbances on the output? For which frequencies are the effect of disturbances amplified?

0 - 0,17 rad/s

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3.5   Robustness

The axis that turns the robot arm is not completely stiff and thus the true transfer function is given by

$$G^0(s) = G(s)(1 + \Delta G(s)) \tag{4}$$

where $\Delta G(s)$ is the relative model error.

Two alternative descriptions of the model error have been proposed:

$$\Delta G_1(s) = \frac{s + 10}{40} \quad \text{and} \quad \Delta G_2(s) = \frac{s + 10}{4(s + 0.01)}$$

Assignment 9. Will the robustness criterion guarantee robust stability for both of these model error descriptions (see p. 125 in Glad & Ljung ver. 4) with the controller $F(s)$ from Assignment 6? Motivate the answer with frequency plots of the complementary sensitivity function $T(s)$ and $\frac{1}{\Delta G}$.

No. Only G1

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3.6 State space model

Here we will work under the assumption that we are able to measure the motor current, I_a, the motor shaft speed, $\dot{\theta}_m$, in addition to the robot arm angle, θ_L.

Assignment 10. Again study the block diagram of Figure 2. Find the state space description of the system with the states $x_1 = \theta_L$, $x_2 = \dot{\theta}_m$ and $x_3 = I_a$. Compute the matrices A, B and C analytically (*do **not** use MATLAB, this should be done by hand calculations*)

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t)$$

$$A = \begin{pmatrix} 0 & n & 0 \\ 0 & -\frac{b}{J} & \frac{K_T}{J} \\ 0 & -\frac{K_m}{L_m} & -\frac{R_m}{L_m} \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_m} \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \qquad \overline{X} = \begin{pmatrix} \theta_L \\ \dot{\theta}_m \\ I_m \end{pmatrix} \begin{matrix} - \text{ Armens vinkel} \\ - \text{ Motorns acceleration} \\ - \text{ Motorns ström} \end{matrix}$$

Assignment 11. Is the system controllable and observable? Motivate the answer.

Yes. See rank of matrices in matlab

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3.7 Pole placement

Having measurements of all states available allow for state feedback control. The state feedback can be designed to meet different performance criteria. Here the task is to design the feedback controller so as to obtain desired poles in the closed-loop system, i.e., pole placement design.
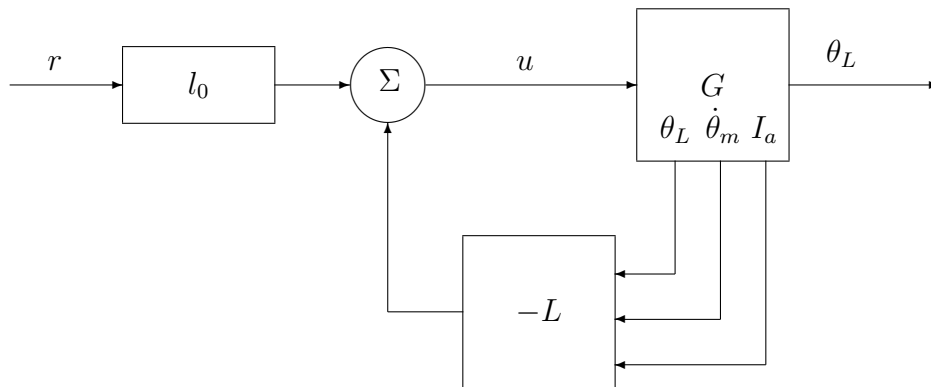

Figure 4: Control system with state feedback.

The pole placement specifications are:

1. The system should be as fast as possible.

2. The overshoot should be less than 5% when the reference signal is a unit step.

3. When the reference signal is a unit step the control signal must not exceed u_{max} (given by the function lab3robot).

4. The static gain of the closed loop system should be equal to 1.

Hint: see chapter 9 in Glad&Ljung ver. 4.

Assignment 12. Use state feedback and pole-placement to find a control law which gives a closed loop system that satisfies the above requirements.

See matlab

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Assignment 13. Answer the following questions:

- What is the effect of having a static gain of 1 in terms of reference tracking $(e = r - y)$? e -> 0 when t -> inf

- Motivate the choice of the poles/feedback constants of the system? Same distance from origo

- Which rise time is achieved and how does this compare to the lead-lag design in assignment 6? See matlab

- What is the biggest difference between a lead-lag compensator and a state feedback controller in terms of the physical implementation?

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We need to have neccesary sensors or observers for a state feedbakc controller.
For lead-lag we only need one output measurement.

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4 Examination

The examination is done by a two step procedure.

4.1 Validation of controllers

The validation program `lab3robot.p` should be downloaded from the course homepage and put in the same folder as the newly written MATLAB program. To validate the derived controllers and models run the command `lab3robot(G,K,F,A,B,C,L,L0,PersonalNumber)` where:

G	is the transfer function for the robot system (Assignment 1)
K	is the gain of P-controller (Assignment 2)
F	is the lead-lag compensator (Assignment 6)
A, B, C	is the state space model (Assignment 10)
L, L0	is the state feedback controller (Assignment 12)
PersonalNumber	The birthdate of one group member in the format YYMMDD

If approved, a code will be generated. Bring this to the oral examination.

4.2 Oral Examination

The second step is a 15-minute oral examination. Before this examination the following should have been prepared:

- Answers to all assignments on seperate paper. (Not to be handed in.)

- MATLAB program which can instantaneously reproduce the results with as many constants as possible analytically defined.

- Relevant plots, transfer functions and state space models should be easily retrievable in your MATLAB program. (*Hint: Look into dividing the program into cells. See MATLAB help on cell dividers.*)

- The students should be logged into a computer with the MATLAB program ready and open in time *before* the examination starts.

- The code generated by correct input to the `lab3robot` program.

A successful oral examination can only be performed when the first step is complete and all values in the program have been validated as correct and a code has been generated