

To solve Berger's equation using the fourth-order Runge-Kutta (RK4) method with central differences for approximating derivatives, we need to discretize the space and time domains. Let's denote x_i as the spatial grid points and t_n as the temporal grid points. Then, we can use the following discretization:

$$x_i = i \cdot \Delta x, \quad i = 0, 1, 2, \dots, N_x$$

$$t_n = n \cdot \Delta t, \quad n = 0, 1, 2, \dots, N_t$$

where Δx and Δt are the spatial and temporal step sizes, respectively. Now, let's apply the RK4 method to solve the equation.

1. Define initial and boundary conditions. 2. Update the solution using RK4 in each time step.

Let's denote u_i^n as the numerical solution at spatial point x_i and temporal point t_n .

The RK4 update formula for the given equation becomes:

$$u_i^{n+1} = u_i^n + \frac{\Delta t}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$\begin{aligned} k_1 &= -\frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} u_i^n + \nu \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2} \\ k_2 &= -\frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} \left(u_i^n + \frac{\Delta t}{2} k_1 \right) + \nu \frac{u_{i+1}^n - 2(u_i^n + \frac{\Delta t}{2} k_1) + u_{i-1}^n}{\Delta x^2} \\ k_3 &= -\frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} \left(u_i^n + \frac{\Delta t}{2} k_2 \right) + \nu \frac{u_{i+1}^n - 2(u_i^n + \frac{\Delta t}{2} k_2) + u_{i-1}^n}{\Delta x^2} \\ k_4 &= -\frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} (u_i^n + \Delta t k_3) + \nu \frac{u_{i+1}^n - 2(u_i^n + \Delta t k_3) + u_{i-1}^n}{\Delta x^2} \end{aligned}$$