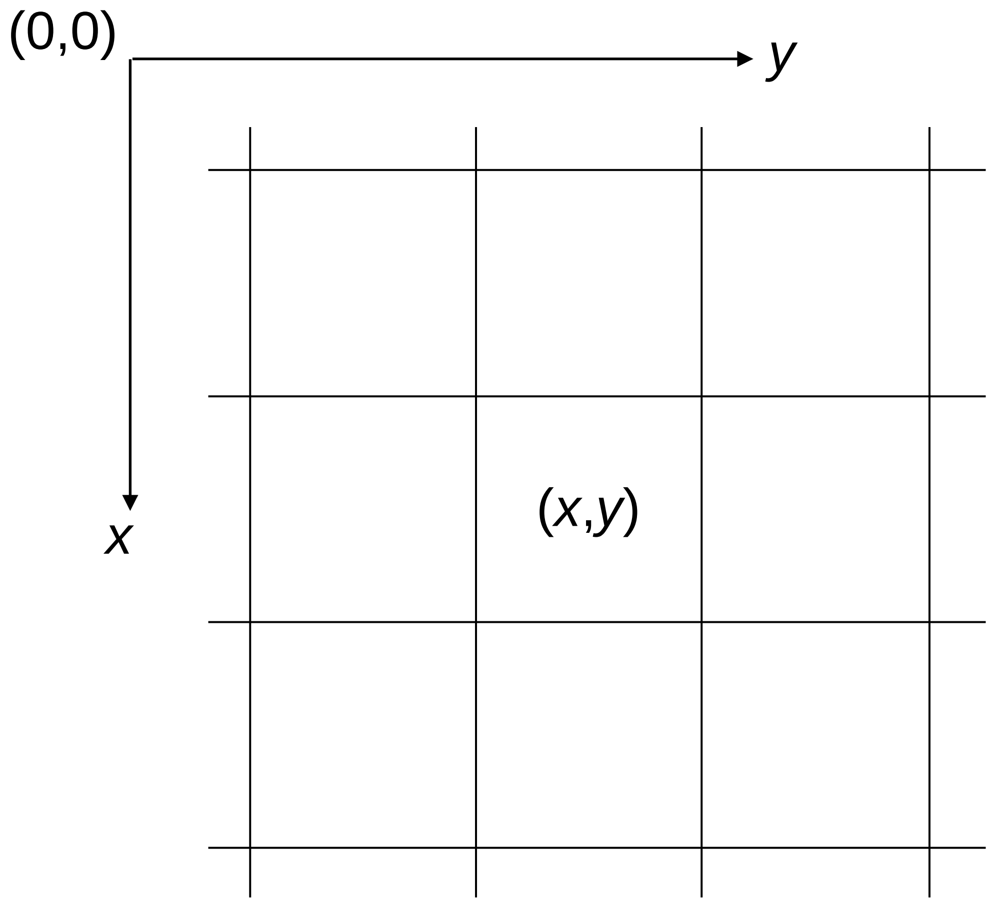
Objectives

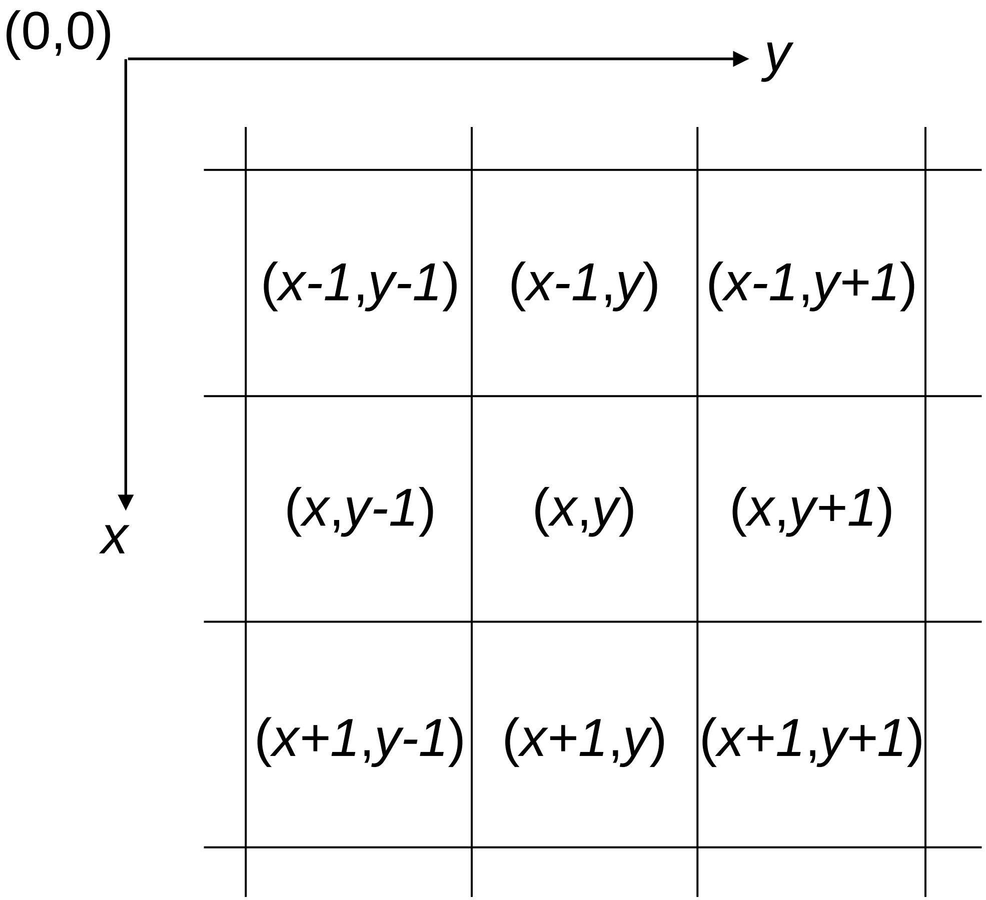
- We will learn the followings:
  - Pixel neighborhood
  - Adjacency
  - Connectivity
  - Distance measure

Basic Relationship of Pixels



Conventional indexing method

Basic Relationship of Pixels



Conventional indexing method

Neighbors of a Pixel

Neighborhood relation is used to find adjacent pixels. It is useful for analyzing regions.

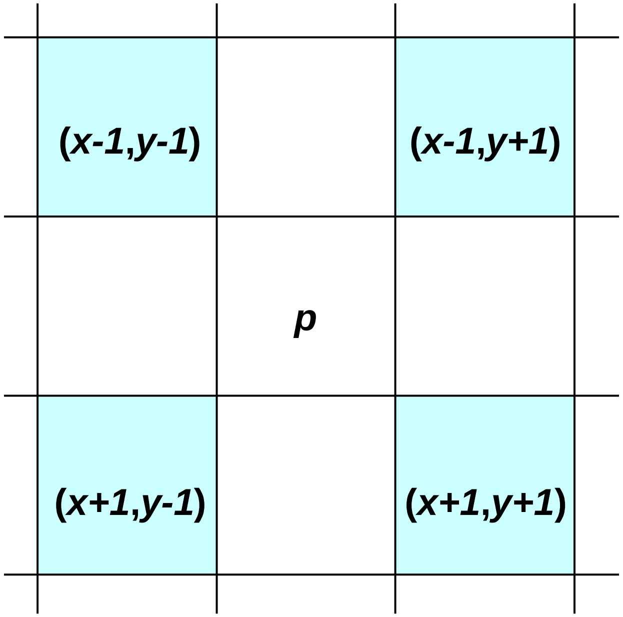
	(*x-1*,*y*)	
(*x*,*y-1*)	***p***	**(*x*,*y+1*)**
	(*x+1*,*y*)	

4-neighbors of *p*:

$$N_4(p) = \left\{ \begin{array}{c} (x-1,y) \\ (x+1,y) \\ (x,y-1) \\ (x,y+1) \end{array} \right\}$$

4-neighborhood relation considers only vertical and horizontal neighbors.

Neighbors of a Pixel



Diagonal neighbors of *p*:

$$N_D(p) = \left\{ \begin{array}{c} (x-1,y-1) \\ (x+1,y-1) \\ (x-1,y+1) \\ (x+1,y+1) \end{array} \right\}$$

Diagonal -neighborhood relation considers only diagonal neighbor pixels.

Neighbors of a Pixel

(*x-1*,*y-1*)	**(*x-1*,*y*)**	**(*x-1*,*y+1*)**
(*x*,*y-1*)	***p***	**(*x*,*y+1*)**
(*x+1*,*y-1*)	**(*x+1*,*y*)**	**(*x+1*,*y+1*)**

8-neighbors of *p*:

$$N_8(p) = \left\{ \begin{array}{c} (x-1,y-1) \\ (x,y-1) \\ (x+1,y-1) \\ (x-1,y) \\ (x+1,y) \\ (x-1,y+1) \\ (x,y+1) \\ (x+1,y+1) \end{array} \right\}$$

8-neighborhood relation considers all neighbor pixels.

Adjacency

Let V be the set of intensity values used to define adjacency

- **4-adjacency.** Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.
- **8-adjacency.** Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.
- **m-adjacency** (also called mixed adjacency). Two pixels p and q with values from V are m-adjacent if
  a) q is in $N_4(p)$. or
  b) q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$. has no pixels whose values are from V

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

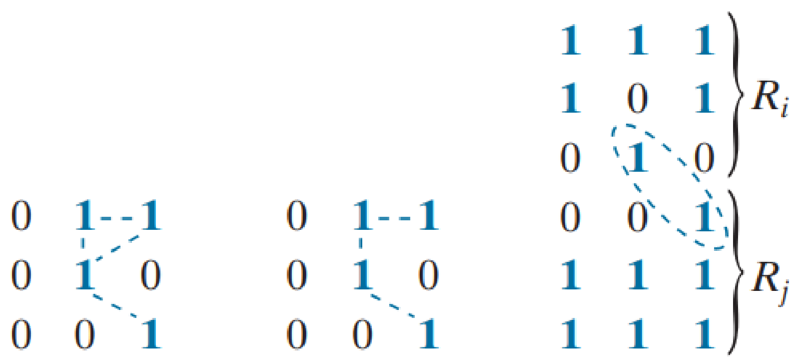
In a binary image, V = {1} if we are referring to adjacency of pixels with value 1.

Path

A digital path (or curve) from pixel p with coordinates (x_0,y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

$(x_0,y_0), (x_1,y_1), \ldots\ldots\ldots\ldots\ldots\ldots, (x_n,y_n)$

where points (x_0,y_0) and (x_{i-1},y_{i-1}) are adjacent for $1 \le i \le n$. In this case, n is the length of the path. If $(x_0,y_0) = (x_n,y_n)$ the path is a closed path.



Connectivity

- Let S represent a subset of pixels in an image.
- Two pixels p and q are said to be connected in S if there exists a path between them consisting entirely of pixels in S.
- For any pixel p in S, the set of pixels that are connected to it in S is called a connected component of S.
- If it only has one component, and that component is connected, then S is called a connected set.

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

Region

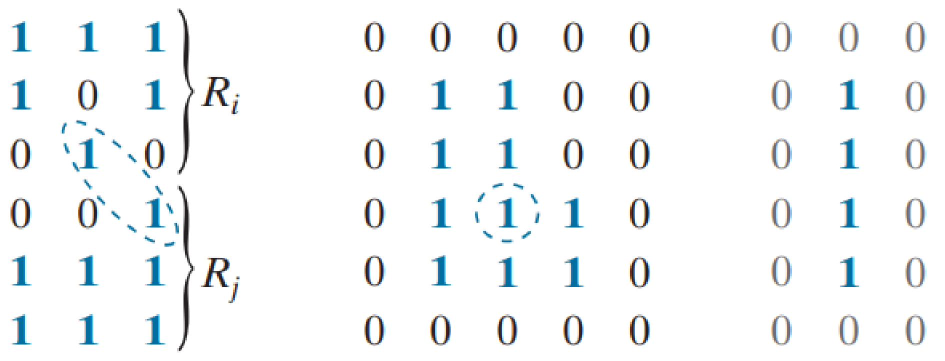
- Let R represent a subset of pixels in an image.
- We call R a region of the image if R is a connected set.
- Two regions, R_i and R_j are said to be adjacent if their union forms a connected set.
- Regions that are not adjacent are said to be disjoint.
- We consider 4- and 8-adjacency when referring to regions.
- The type of adjacency used must be specified.

$$
\begin{array}{l}
\left.\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}\right\} R_i \\
\left.\begin{matrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}\right\} R_j
\end{array}
\qquad
\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}
\qquad
\begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}
$$

Boundary

- The boundary of a region R is the set of pixels in R that are adjacent to pixels in the complement of R.

  or
- The border of a region is the set of pixels in the region that have at least one background neighbor.
- we must specify the connectivity being used to define adjacency



Distance

For pixel p, q, and z with coordinates (x,y), (s,t) and (u,v), D is a ***distance function*** or ***metric*** if

- $D(p,q) \geq 0$ ($D(p,q) = 0$ if and only if $p = q$)
- $D(p,q) = D(q,p)$
- $D(p,z) \leq D(p,q) + D(q,z)$

Example: Euclidean distance

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

Distance

D_4-distance (*city-block distance*) is defined as

$$D_4(p,q) = |x - s| + |y - t|$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Pixels with $D_4(p) = 1$ is 4-neighbors of p.

Distance

D_8-distance (*chessboard distance*) is defined as

$$D_8(p,q) = \max(|x-s|,|y-t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Pixels with $D_8(p) = 1$ are 8-neighbors of p.