

Deflection of Beam

1. BACKGROUND

The Euler-Bernoulli beam theory primarily addresses vertical deflection due to bending moments and shear forces. Also, one of the key assumptions it makes is that plane sections of the beam normal to the axis before bending remain plane and normal to the axis after bending. This implies that this theory does not take into account the axial stresses involved.

Also, in the case of smaller loads for which this equation is valid, the curvature of the beam is much less. Thus, horizontal displacements, which are a function of the sine of curvature, are negligible compared to vertical displacements, which are a function of the cosine of curvature. Therefore, the horizontal displacement of a point is usually ignored for simplicity.

No, the beam does not stretch in real life. Though the top of the beam (in tension) and the bottom of the beam (in compression) undergo slight longitudinal strains, the combined effect results in no stretching.

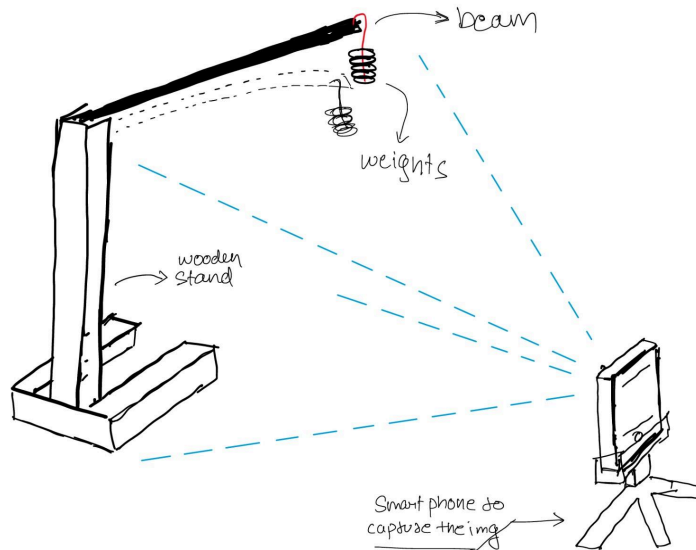
As mentioned above, in cases of small loads, the horizontal displacements are negligible, which causes it to appear that the deflections of the free end of the beam fell on a straight vertical line for increasing loads. However, for large loads, a significant horizontal displacement can be observed.

2. SETUP DESIGN

The setup included:

- **Cantilever Beam:** This is the primary subject of the experiment. It's fixed at one end to the stand, and the free end is subjected to loads. The beam's material and dimensions would be chosen based on the specific experimental requirements.
- **Weights:** These are applied to the beam's free end to induce deflection. The weight magnitude can be varied to study deflection under different loads.
- **Wooden Stand:** The stand serves as the rigid support for the beam. It's made of wood, which is strong enough to hold the beam securely while also being relatively easy to work with and modify if necessary. The wooden material also minimises the risk of introducing additional metal-on-metal effects, such as corrosion and prevents the introduction of strains within the setup.
- **Extended Base:** The base of the wooden stand extends outward, which provides stability and prevents the apparatus from toppling over when the beam deflects. This is important when dealing with large deflections, as the system's centre of gravity can shift.
- **Smartphone to Capture the Image:** A smartphone is positioned at a suitable angle to capture images of the deflected beam. The images taken by the smartphone can be processed using the image processing script. Using a smartphone is advantageous

because it's a readily available device with a high-resolution camera suitable for capturing detailed images, but any image-capturing device can work.



The working of the image processing script can be explained as follows:

- Loads and resizes images to fit the screen resolution.
- A graphical user interface allows the manual selection of reference and measurement points on the beam.
- Calculates the angle needed to rotate the image so that the beam appears horizontal, aiding in accurate deflection measurement.
- Applies the calculated rotation to align the beam correctly in the image.
- Transforms selected pixel coordinates into real-world measurements using a scaling factor derived from known dimensions in the two directions.
- Outputs the deflection data in a structured format for analysis.

3. PROCEDURE

1. Choose the appropriate beam specimen based on the material and dimensions required for the experiment.
2. Set up the support apparatus to ensure proper and stable support for the beam.
3. Use levelling equipment to ensure the beam is level; mark the points at which you want to measure the deflection.

4. Place a high-resolution camera (your smartphone) in front of the apparatus at a fixed location to capture clear images of the unloaded beam during the experiment.
5. Capture an image of the unloaded beam as a reference. This image will serve as a baseline for deflection measurements.
6. Apply the load gradually to the beam using weights or the chosen load application system.
7. Record the applied load at each increment.
8. Once the beam is stable under load, capture an image of the deflected beam. This image will be used for image processing to measure deflections.
9. To ensure an accurate pixel-to-mm conversion ratio, calibrate the image processing system by integrating a known measurement in both the x and y directions.
10. Utilise image processing software to analyse the captured images and measure the deflection of the marked points on the beam at each load increment.
11. Repeat the above procedure by measuring and recording the deflection at each load increment using the image processing system.
12. Plot a load-deflection curve using the collected data from both the traditional measuring devices and the image processing system.

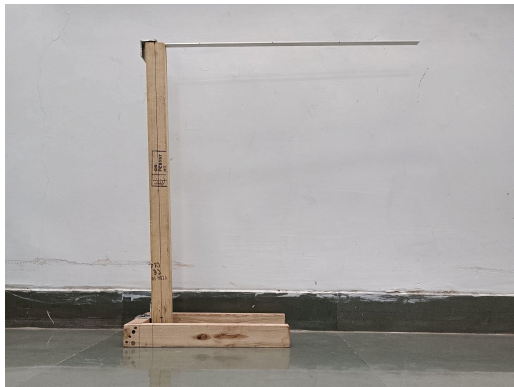


Fig. 1: Setup with beam without loading.

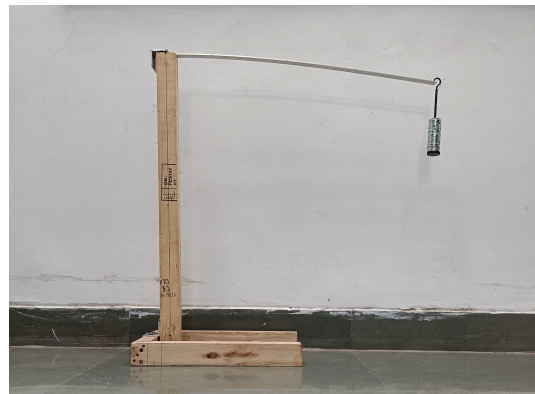


Fig. 2: Setup with beam under loading.

4. OBSERVATIONS

● Data and Graphs

1. Deflection of the Aluminium beam at various points.

$$L = 614 \text{ mm}; E = 70,000$$

Load(in gm)	dl(dx, dy) at L=614(in mm)	dl(dx, dy) at L=435(in mm)	dl(dx, dy) at L=278.8(in mm)	dl(dx, dy) at L=108.7(in mm)
260	(1.4, -57.8)	(0.8, -25.9)	(0.7,-8.1)	(0, -1.9)
320	(4, -65.7)	(1.6, -33.8)	(0.7,-10.9)	(0, -2.3)
350	(6, -74.9)	(2.4, -40.6)	(0.9, -14.9)	(0.1, -2.6)
400	(9, -83.5)	(4, -44.5)	(1.4, -20.7)	(0.1, -3.01)

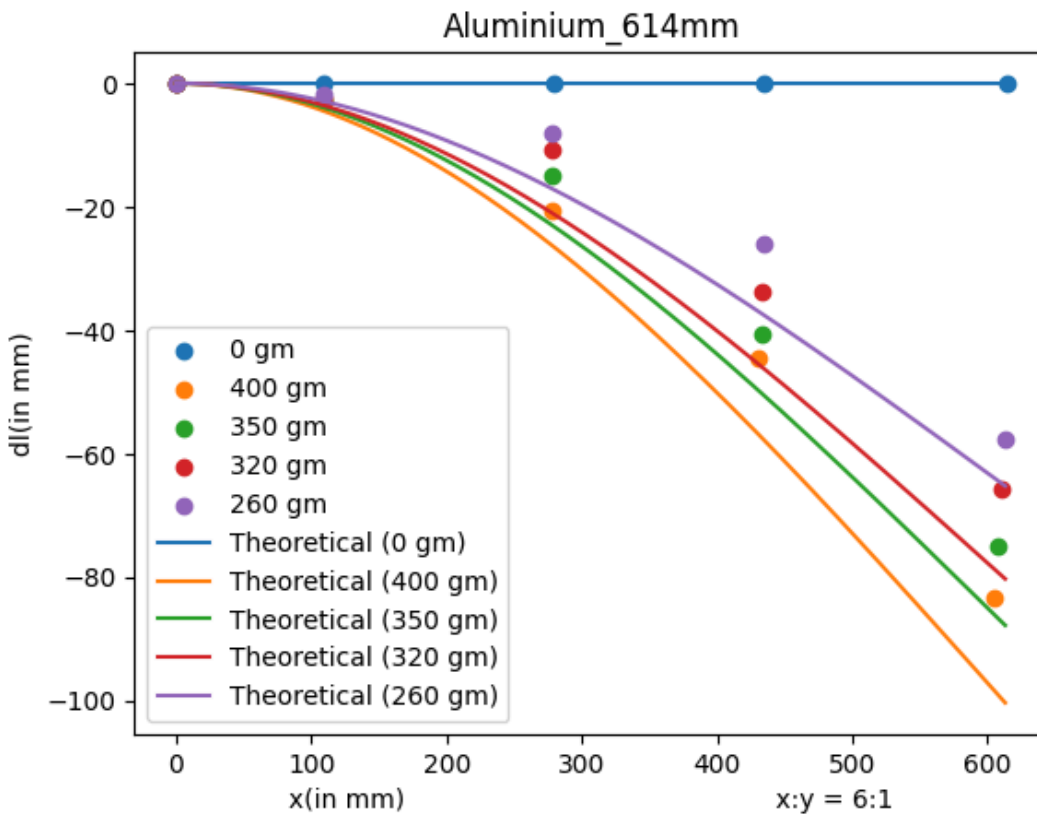


Fig. 3: Graph showing variation in theoretical and experimental values of deflection of Aluminium beam at various loads.

Average %error w.r.t experimental values in y-direction at L = 614 mm at various loads = 18.8%

2. Deflection of the Brass beam at various points.

L = 625 mm; E = 100,000

Load(in gm)	dl(dx, dy) at L=625(in mm)	dl(dx, dy) at L=482(in mm)	dl(dx, dy) at L=333(in mm)	dl(dx, dy) at L=182(in mm)
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200	(0.9, -42.2)	(0.9, -25.9)	(0.6, -11.5)	(0.4, -3.9)
250	(2.1, -51.2)	(1.8, -32.2)	(0.5, -15.6)	(0.4, -6.2)
300	(3.7, -60.9)	(2.1, -36.2)	(1.1, -19.6)	(0.5, -8.1)
350	(5.3, -70.1)	(3.4, -39.5)	(1.4, -22.7)	(0.9, -10.2)

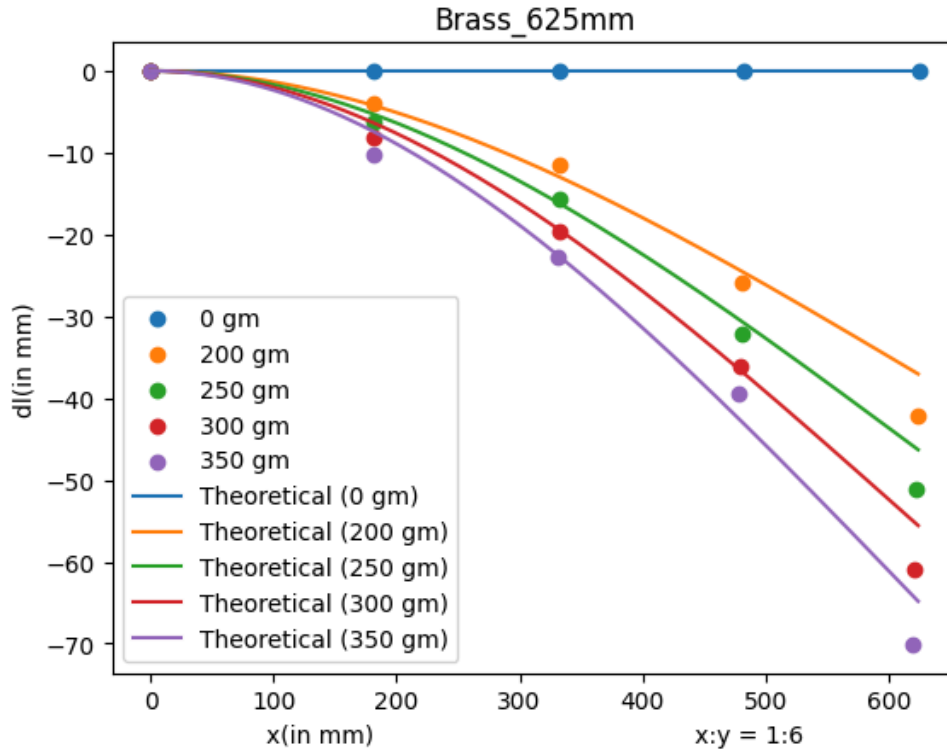


Fig. 3: Graph showing variation in theoretical and experimental values of deflection of Brass beam at various loads.

Average %error w.r.t experimental values in y-direction at $L = 625$ mm at various loads = 11.68%

5. INFERENCES

- For smaller loads, horizontal displacements appear negligible, as predicted by theory. However, there is evidence of horizontal displacement for larger loads, which the Euler-Bernoulli beam theory does not account for, suggesting the need for a more advanced theory for large deflections.
- Comparing the experimental data with theoretical values from the Euler-Bernoulli beam theory reveals discrepancies quantified by the average percentage error. We find that these errors are

18.8% and 11.68% for aluminium and brass beams, respectively, suggesting that while the theory provides reasonable predictions for the overall pattern, it cannot be used for precise calculations.

- The experiment demonstrates that the deflection of beams under various loads can be successfully measured using the described setup (the use of an extended base to prevent toppling and wooden materials to maintain setup integrity proved effective) and image processing techniques.
- The image processing method provides a non-intrusive method to measure deflections, which might be preferable over traditional contact methods, especially in sensitive or small-scale applications, as it shields the setup from possible errors caused due to disturbances.
- The percentage errors also hint towards the limitations of our experimental setup, which may include the resolution of the image capture device, the precision of load application, the calculation of pixel-to-mm and potential parallax errors in image analysis.

6. REFERENCES

- T. BeleÂndez, C. Neipp and A. BeleÂndez, Large and small deflections of a cantilever beam, Eur. J.Phys., 23, 2002
- <https://stackoverflow.com/questions/69083799/find-distance-between-points-in-opencv> (reference for code)