LaTeX 9.10.6.7

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Class 9, Chapter, 10, Exercse 6.7

Q. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is a rectangle.

Solution: Let, we have a unit circle with center at origin, i.e. $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, and radius $r = 1$. Let's consider points A, B, C and D on the circle such that AC and BD are diameter of the circle. The points on circle that we consider are available in Table (I).

A	$\begin{pmatrix} \cos 0 \\ \sin 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
B	$\begin{pmatrix} \cos \frac{\pi}{2} \\ \sin \frac{\pi}{2} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
C	$\begin{pmatrix} \cos \pi \\ \sin \pi \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
D	$\begin{pmatrix} \cos \frac{3}{2}\pi \\ \sin \frac{3}{2}\pi \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

TABLE I

1) AC and BD are diameters of the circle. Let's check if they bisect each other,

$$\mathbf{A} + \mathbf{C} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{1}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{B} + \mathbf{D} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{3}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{4}$$

$$\tag{5}$$

From equation (2) and (4) AC and BD bisect each other. Hence, we can say that if two chords bisect each other then they are diameters.

2) Let's check if $ABCD$ is a rectangle. The sides of a rectangle are parallel to each other. Let's check if AB and BC are parallel to each other.

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{7}$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{8}$$

$$= \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{9}$$

$$\tag{10}$$

From equation (7) and (9), AB and CD are anti-parallel to each other. $\implies ABCD$ is a parallelogram.

Now let's check if its a rectangle. Let's check the angle between adjacent sides of this quadrilateral, i.e. AB and BC.

$$\mathbf{A}-\mathbf{B} = \begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix} \tag{11}$$

$$= \begin{pmatrix}1\\-1\end{pmatrix} \tag{12}$$

$$\mathbf{B}-\mathbf{C} = \begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}-1\\0\end{pmatrix} \tag{13}$$

$$= \begin{pmatrix}1\\1\end{pmatrix} \tag{14}$$

$$(\mathbf{A}-\mathbf{B})^\top(\mathbf{B}-\mathbf{C}) = \begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} \tag{15}$$

$$= 0 \tag{16}$$

From equation (15), we can say that the angle between AB and BC is $90°$. Hence, the quadrilateral $ABCD$ is a rectangle.

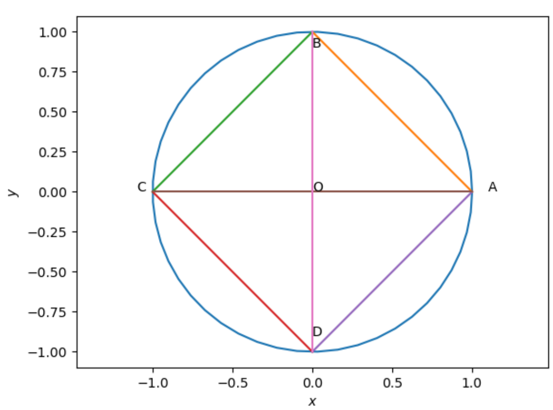

Fig. 1: circle