11.10.4.21

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Class 11, Chapter 10, Exercise 4.21

Q.21. Find equation of the line which is equidistant from parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.

Solution: Let's first rewrite the given equation of line in the form $x + by + c = 0$ where b, c are constants. Equation of lines are

$$L_1 : 9x + 6y - 7 = 0 \tag{1}$$
$$\implies L_1 : x + \frac{2}{3}y - \frac{7}{9} = 0 \tag{2}$$
$$\implies \mathbf{n}_1 = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} \text{ and } c_1 = \frac{7}{9} \tag{3}$$
$$L_2 : 3x + 2y + 6 = 0 \tag{4}$$
$$\implies L_2 : x + \frac{2}{3}y + 2 = 0 \tag{5}$$
$$\implies \mathbf{n}_2 = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} \text{ and } c_2 = -2 \tag{6}$$

Using the given equidistant condition for desired line

$$\frac{\left|\mathbf{n}_1^\top \mathbf{x} - c_1\right|}{\|\mathbf{n}_1\|} = \frac{\left|\mathbf{n}_2^\top \mathbf{x} - c_2\right|}{\|\mathbf{n}_2\|} \tag{7}$$
$$\frac{\left|\begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix}\mathbf{x} - \frac{7}{9}\right|}{\sqrt{\frac{13}{9}}} = \frac{\left|\begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix}\mathbf{x} - (-2)\right|}{\sqrt{\frac{13}{9}}} \tag{8}$$
$$\left|\begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix}\mathbf{x} - \frac{7}{9}\right| = \left|\begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix}\mathbf{x} + 2\right| \tag{9}$$

Case 1.

$$\begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix}\mathbf{x} - \frac{7}{9} = \begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix}\mathbf{x} + 2 \tag{10}$$
$$\implies -\frac{7}{9} = 2 \tag{11}$$
$$\text{(not possible)} \tag{12}$$

Case 2.

$$\begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix}\mathbf{x} - \frac{7}{9} = -\begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix}\mathbf{x} - 2 \tag{13}$$
$$\implies \begin{pmatrix} 2 & \frac{4}{3} \end{pmatrix}\mathbf{x} = \frac{7}{9} - 2 \tag{14}$$
$$\implies \begin{pmatrix} 2 & \frac{4}{3} \end{pmatrix}\mathbf{x} = -\frac{11}{9} \tag{15}$$

The equation of line is

$$L : 2x + \frac{4}{3}y + \frac{11}{9} = 0 \tag{16}$$
$$\implies L : x + \frac{2}{3}y + \frac{11}{18} = 0 \tag{17}$$
$$\implies L : 18x + 12y + 11 = 0 \tag{18}$$