# 12.10.5.10

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CLASS 12, CHAPTER 10, EXERCISE 5.10

Q.10. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $1\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

**Solution:** The sides of the parallelogram are given as $\mathbf{a} = \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$. The diagonals of the parallelogram are given by

$$\mathbf{D_1} = \mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} \tag{1}$$

$$\mathbf{D_2} = \mathbf{a} - \mathbf{b} = \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} \tag{2}$$

The unit vectors parallel to the diagonals are given by

$$\hat{D}_1 = \frac{\mathbf{D_1}}{\|\mathbf{D_1}\|} = \frac{\begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix}}{\sqrt{3^2+6^2+2^2}} = \begin{pmatrix} \frac{3}{\sqrt{45}} \\ -\frac{6}{\sqrt{45}} \\ \frac{2}{\sqrt{45}} \end{pmatrix} \tag{3}$$

$$\hat{D}_2 = \frac{\mathbf{D_2}}{\|\mathbf{D_2}\|} = \frac{\begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix}}{\sqrt{1^2+2^2+8^2}} = \begin{pmatrix} \frac{1}{\sqrt{69}} \\ -\frac{2}{\sqrt{69}} \\ \frac{8}{\sqrt{69}} \end{pmatrix} \tag{4}$$

The area of the parallelogram is given by the cross product or vector product of $\mathbf{A}, \mathbf{B}$ is defined as

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} \begin{vmatrix} \mathbf{a}_{23} & \mathbf{b}_{23} \end{vmatrix} \\ \begin{vmatrix} \mathbf{a}_{31} & \mathbf{b}_{31} \end{vmatrix} \\ \begin{vmatrix} \mathbf{a}_{12} & \mathbf{b}_{12} \end{vmatrix} \end{pmatrix} \tag{5}$$

Hence

$$\begin{vmatrix} \mathbf{a}_{23} & \mathbf{b}_{23} \end{vmatrix} = \begin{vmatrix} -4 & -2 \\ 5 & 3 \end{vmatrix} = 22 \tag{6}$$

$$\begin{vmatrix} \mathbf{a}_{31} & \mathbf{b}_{31} \end{vmatrix} = \begin{vmatrix} 5 & -3 \\ 2 & 1 \end{vmatrix} = -11 \tag{7}$$

$$\begin{vmatrix} \mathbf{a}_{12} & \mathbf{b}_{12} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -4 & -2 \end{vmatrix} = 0 \tag{8}$$

Substituting the values

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 22 \\ -11 \\ 0 \end{pmatrix} \tag{9}$$

The area of the parallelogram is given by

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{22^2 + 11^2 + 0^2} = \sqrt{605} \tag{10}$$