11.11.5.3

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Class 11, Chapter 11, Exercise 5.3

Q. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

Solution: Uniformly loaded suspension bridge cable hangs in the form of a parabola facing upwards.

O	Lowest point of cable	$\begin{pmatrix}0\\0\end{pmatrix}$
d	Length of the cable	100 m
d_1	Length of longest wire	30 m
d_2	Length of shortest wire	6 m
A	End point of cable	$\begin{pmatrix}\frac{d}{2}\\d_1-d_2\end{pmatrix}$
B	End point of cable	$\begin{pmatrix}-\frac{d}{2}\\d_1-d_2\end{pmatrix}$

TABLE I: points

This will give us a setup similar to figure 1,

Here A and B are the points on the parabola where the cable is attached to the roadway, i.e. longest wire is attached at this points. And vertex of parabola O is point where shortest wire is attached, which is 6m from the ground. With the assumption of point O being $\begin{pmatrix}0\\0\end{pmatrix}$, we'll get

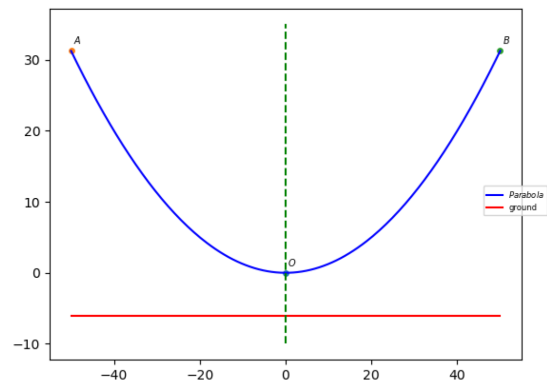

Fig. 1: Representation of parabola with vertex at origin.

$$\mathbf{A} = \begin{pmatrix}\frac{d}{2}\\d_1-d_2\end{pmatrix} = \begin{pmatrix}50\\24\end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix}-\frac{d}{2}\\d_1-d_2\end{pmatrix} = \begin{pmatrix}-50\\24\end{pmatrix} \tag{2}$$

The generic equation of conic is

$$g(\mathbf{x}) = \mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{3}$$

Point $\begin{pmatrix}0\\0\end{pmatrix}$ is on conic, so

$$\implies f = 0 \tag{4}$$

As conic is upward facing parabola,

$$\mathbf{V} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}. \tag{5}$$

As points A and B are on parabola

$$\implies \begin{pmatrix} 50 & 24 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 50 \\ 24 \end{pmatrix} + 2\mathbf{u}^\top \begin{pmatrix} 50 \\ 24 \end{pmatrix} = 0 \quad (6)$$

$$\implies \mathbf{u}^\top \begin{pmatrix} 50 \\ 24 \end{pmatrix} = -1250 \quad (7)$$

$$\implies \begin{pmatrix} 50 & 24 \end{pmatrix} \mathbf{u} = -1250 \quad (8)$$

and

$$\implies \begin{pmatrix} -50 & 24 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -50 \\ 24 \end{pmatrix} + 2\mathbf{u}^\top \begin{pmatrix} -50 \\ 24 \end{pmatrix} = 0 \quad (9)$$

$$\implies \mathbf{u}^\top \begin{pmatrix} -50 \\ 24 \end{pmatrix} = -12 \quad (10)$$

$$\implies \begin{pmatrix} -50 & 24 \end{pmatrix} \mathbf{u} = -12 \quad (11)$$

From (8) and (11), we get

$$\begin{pmatrix} 50 & 24 \\ -50 & 24 \end{pmatrix} \mathbf{u} = = \begin{pmatrix} -1250 \\ -1250 \end{pmatrix} \quad (12)$$

$$\implies \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{625}{12} \end{pmatrix} \quad (13)$$

we get parabola

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 & -\frac{625}{12} \end{pmatrix} \mathbf{x} = 0 \quad (14)$$

At a point λ_1 m from middle, i.e.

$$\lambda_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (15)$$

From equation (14), we get

$$\begin{pmatrix} \lambda_1 & \lambda_2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} + 2 \begin{pmatrix} 0 & -\frac{625}{12} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0 \quad (16)$$

$$\implies \lambda_2 = \frac{625}{12} \lambda_1 \quad (17)$$

Now we have given the point $18m$ from middle, so $\lambda_1 = 18$ and we have to find λ_2.

Using equation (17), we get

$$\lambda_2 = \frac{625}{12} \lambda_1 = \frac{1944}{625} \quad (18)$$

$\implies$ Length of a supporting wire attached to the roadway $18m$ from the middle is

$$= \lambda_2 + 6 = \frac{1944}{625} + 6 = \frac{5694}{625} m \quad (19)$$

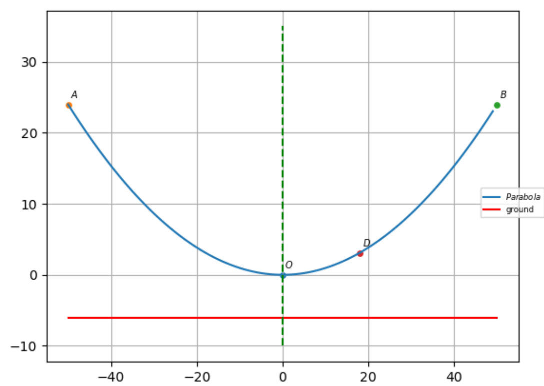

Fig. 2: Parabola