

12.11.3.9

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CLASS 12, CHAPTER 11, EXERCISE 4.19

Q. Find the vector equation of the line passing through $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and parallel to the planes $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}^T \mathbf{r} = 5$ and $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}^T \mathbf{r} = 6$.

Solution: The line equations are given as

$$\mathbf{r} = \mathbf{A} + \lambda \mathbf{m} \quad (1)$$

where \mathbf{m} is the direction vector of the line and \mathbf{A} is any point on the line.

The planes are given as

$$P_1 : \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}^T \mathbf{r} = 5 \quad (2)$$

$$\Rightarrow \mathbf{n}_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad (3)$$

$$P_2 : \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}^T \mathbf{r} = 6 \quad (4)$$

$$\Rightarrow \mathbf{n}_2 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \quad (5)$$

The expected line is parallel to both the planes, then the direction vector of the line must be perpendicular to both the normal vectors. This means that

$$\mathbf{n}_1^T \mathbf{m} = 0 \quad (6)$$

$$\mathbf{n}_2^T \mathbf{m} = 0 \quad (7)$$

$$\Rightarrow \begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \end{pmatrix} \mathbf{m} = 0 \quad (8)$$

echelon form:

$$\begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \end{pmatrix} \xrightarrow{R_2 \rightarrow -\frac{3}{4}R_1 + \frac{1}{4}R_2} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -\frac{5}{4} \end{pmatrix} \quad (9)$$

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -\frac{5}{4} \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 + R_2} \begin{pmatrix} 1 & 0 & \frac{3}{4} \\ 0 & 1 & -\frac{5}{4} \end{pmatrix} \quad (10)$$

Using (8), (9) and (10), we get:

$$\Rightarrow \begin{pmatrix} 1 & 0 & \frac{3}{4} \\ 0 & 1 & -\frac{5}{4} \end{pmatrix} \mathbf{m} = 0 \quad (11)$$

$$\Rightarrow \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} -\frac{3}{4}m_3 \\ \frac{5}{4}m_3 \\ m_3 \end{pmatrix} \quad (12)$$

$$\Rightarrow \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = m_3 \begin{pmatrix} -\frac{3}{4} \\ \frac{5}{4} \\ 1 \end{pmatrix} \quad (13)$$

$$\Rightarrow \mathbf{m} = \begin{pmatrix} -3 \\ 5 \\ 4 \end{pmatrix} \quad (14)$$

It is given that line passes through point $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, so the final equation of line implies

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 5 \\ 4 \end{pmatrix} \quad (15)$$

$$(16)$$

Let's reduce the matrix from equation (8) to row-