12.10.3.13

1

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Class 12, Chapter 10, Exercise 3.13

13) If are \mathbf{a} , \mathbf{b} , \mathbf{c} are unit vectors such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=$ 0, find the value of $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$. Solution: The inner product of given unit vectors with sum of all unit vectors (Which is given to be 0) is zero

$$\mathbf{a}^{\mathsf{T}}(\mathbf{a} + \mathbf{b} + \mathbf{c}) = 0 \tag{1}$$

$$\Rightarrow (\mathbf{a}^{\mathsf{T}}\mathbf{b} + \mathbf{a}^{\mathsf{T}}\mathbf{c}) = -1 \tag{2}$$

$$\mathbf{b}^{\mathsf{T}}(\mathbf{a} + \mathbf{b} + \mathbf{c}) = 0 \tag{3}$$

$$\Rightarrow (\mathbf{b}^{\mathsf{T}}\mathbf{c} + \mathbf{b}^{\mathsf{T}}\mathbf{a}) = -1 \tag{4}$$

$$\mathbf{c}^{\mathsf{T}}(\mathbf{a} + \mathbf{b} + \mathbf{c}) = 0 \tag{5}$$

$$\Rightarrow (\mathbf{c}^{\mathsf{T}}\mathbf{a} + \mathbf{c}^{\mathsf{T}}\mathbf{b}) = -1 \tag{6}$$

$$\Rightarrow (\mathbf{a}^{\mathsf{T}}\mathbf{b} + \mathbf{b}^{\mathsf{T}}\mathbf{c} + \mathbf{c}^{\mathsf{T}}\mathbf{a}) = \frac{-3}{2}$$
 (7)

$$\Rightarrow (\mathbf{a}^{\mathsf{T}}\mathbf{b} + \mathbf{b}^{\mathsf{T}}\mathbf{c} + \mathbf{c}^{\mathsf{T}}\mathbf{a}) = \frac{-3}{2}$$
(7)
$$\Rightarrow \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a} = \frac{-3}{2}$$
(8)