LaTeX 9.10.5.3

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Class 9, Chapter 10, Exercse 5.3

Q7. $\angle PQR = 100°$, where $\mathbf{P}, \mathbf{Q}$ and $\mathbf{R}$ are points on a circle with centre $\mathbf{O}$. Find $\angle OPR$

Solution: Let, we have a unit circle with center at origin, i.e. $\mathbf{O}$, and radius $r = 1$. Then let following points be on the circle

O	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}0\\0\end{pmatrix}$
R	$\begin{pmatrix}\cos 0\\\sin 0\end{pmatrix}$	$\begin{pmatrix}1\\0\end{pmatrix}$
Q	$\begin{pmatrix}\cos\frac{\pi}{6}\\\sin\frac{\pi}{6}\end{pmatrix}$	$\begin{pmatrix}\frac{\sqrt{3}}{2}\\\frac{1}{2}\end{pmatrix}$
P	$\begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}$	$\begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}$

TABLE I: points

As per given condition, we have

$$\angle PQR = 100° \tag{1}$$

$$\implies \cos 100° = \frac{(\mathbf{P}-\mathbf{Q})^\top(\mathbf{R}-\mathbf{Q})}{\|\mathbf{P}-\mathbf{Q}\|\|\mathbf{R}-\mathbf{Q}\|} \tag{2}$$

$$\|\mathbf{P}-\mathbf{Q}\| = \sqrt{\left(\cos\theta - \frac{\sqrt{3}}{2}\right)^2 + \left(\sin\theta - \frac{1}{2}\right)^2} \tag{3}$$

$$\|\mathbf{R}-\mathbf{Q}\| = \sqrt{\left(1 - \frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} \tag{4}$$

$$= \sqrt{2-\sqrt{3}} \tag{5}$$

$$(\mathbf{P}-\mathbf{Q})^\top(\mathbf{R}-\mathbf{Q}) = \left(\cos\theta - \tfrac{\sqrt{3}}{2}\right)\cdot\left(1 - \tfrac{\sqrt{3}}{2}\right) + \left(\sin\theta - \tfrac{1}{2}\right)\cdot\left(-\tfrac{1}{2}\right) \tag{6}$$

$$= 1 - \frac{\sqrt{3}}{2} + \cos\theta - \cos\left(\theta - \frac{\pi}{6}\right) \tag{7}$$

Using (2), (3), (5) and (7), we get

$$\cos 100° = \frac{1 - \frac{\sqrt{3}}{2} + \cos\theta - \cos\left(\theta - \frac{\pi}{6}\right)}{\left(\sqrt{2-\sqrt{3}}\right)\left(\sqrt{\left(\cos\theta - \frac{\sqrt{3}}{2}\right)^2 + \left(\sin\theta - \frac{1}{2}\right)^2}\right)} \tag{8}$$

$$\theta = \tag{9}$$

$$\mathbf{P} = \begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix} \tag{10}$$

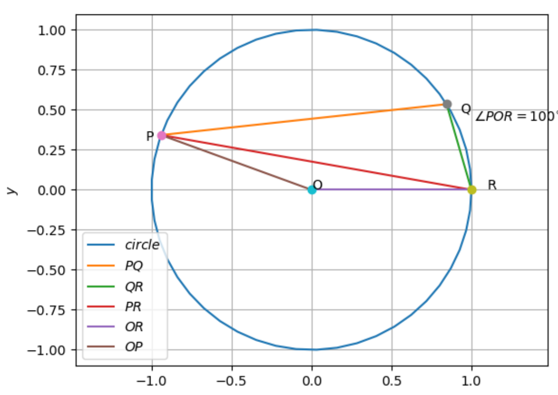

Fig. 1: circle

Now, we'll find the $\angle OPR$,

$$\mathbf{OP} = \mathbf{P} - \mathbf{O} = \begin{pmatrix} \cos 160^\circ - 0 \\ \sin 160^\circ - 0 \end{pmatrix} = \begin{pmatrix} \cos 160^\circ \\ \sin 160^\circ \end{pmatrix} \tag{11}$$

$$\mathbf{OR} = \mathbf{R} - \mathbf{O} = \begin{pmatrix} 1 - 0 \\ 0 - 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{12}$$

$$\implies \angle OPR = \arccos\left(\frac{\mathbf{OP}^\top \mathbf{OR}}{\|\mathbf{OP}\|\|\mathbf{OR}\|}\right) \tag{13}$$

$$= \arccos\left(\frac{\cos 160^\circ + 1}{\sqrt{2}}\right) \tag{14}$$

$$= 20.87^\circ \tag{15}$$