$\LaTeX$ 11.10.1.7

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Class 11, Exercise 10.1

Q7. $\angle PQR = 100°$, where $\mathbf{P}, \mathbf{Q}$ and $\mathbf{R}$ are points on a circle with centre $\mathbf{O}$. Find $\angle OPR$

Solution: Let, we have a unit circle with center at origin, i.e. $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, and radius $r = 1$. The points $\mathbf{P}, \mathbf{Q}$ and $\mathbf{R}$ are on the circle with center $\mathbf{O}$ and radius $r = 1$, such as

$$\mathbf{P} = \begin{pmatrix} \cos 160° \\ \sin 160° \end{pmatrix} \tag{1}$$

$$\mathbf{Q} = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \tag{2}$$

$$\mathbf{R} = \begin{pmatrix} \cos 0° \\ \sin 0° \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3}$$

$$\implies \mathbf{PQ} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} \cos \theta - \cos 160° \\ \sin \theta - \sin 160° \end{pmatrix} \tag{4}$$

$$\implies \mathbf{QR} = \mathbf{R} - \mathbf{Q} = \begin{pmatrix} 1 - \cos \theta \\ 0 - \sin \theta \end{pmatrix} \tag{5}$$

As per given condition, we have

$$\begin{aligned} \angle PQR = 100° \implies \cos 100° &= \frac{\mathbf{PQ}^\top \mathbf{QR}}{\|\mathbf{PQ}\|\|\mathbf{QR}\|} && (6) \\ & && (7) \end{aligned}$$

$$\|\mathbf{PQ}\| = \sqrt{(\cos \theta - \cos 160°)^2 + (\sin \theta - \sin 160°)^2} \tag{8}$$

$$\|\mathbf{QR}\| = \sqrt{(1 - \cos \theta)^2 + (0 - \sin \theta)^2} \tag{9}$$

$$\mathbf{PQ}^\top \mathbf{QR} = -1 + \cos \theta + \cos 160° - \theta \tag{10}$$

Using (6), (8), (9) and (10), we get

$$\theta = 32.31° \tag{11}$$

$$\mathbf{Q} = \begin{pmatrix} \cos 32.31° \\ \sin 32.31° \end{pmatrix} = \begin{pmatrix} 0.999 \\ 0.032 \end{pmatrix} \tag{12}$$

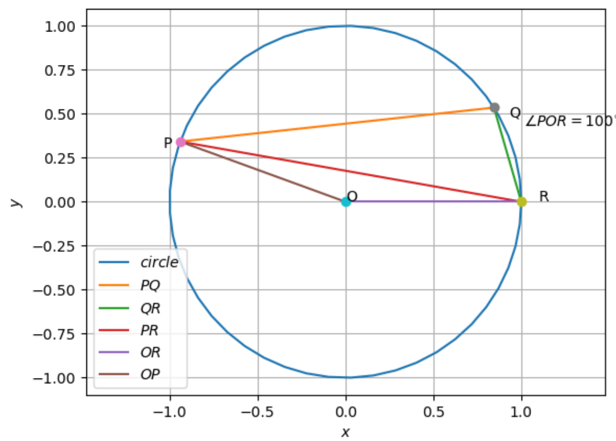

Fig. 1: circle

Now, we'll find the $\angle OPR$,

$$\mathbf{OP} = \mathbf{P} - \mathbf{O} = \begin{pmatrix} \cos 160° - 0 \\ \sin 160° - 0 \end{pmatrix} = \begin{pmatrix} \cos 160° \\ \sin 160° \end{pmatrix} \tag{13}$$

$$\mathbf{OR} = \mathbf{R} - \mathbf{O} = \begin{pmatrix} 1 - 0 \\ 0 - 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{14}$$

$$\begin{aligned} \implies \angle OPR &= \arccos\left(\frac{\mathbf{OP}^\top \mathbf{OR}}{\|\mathbf{OP}\|\|\mathbf{OR}\|}\right) && (15) \\ &= \arccos\left(\frac{\cos 160° + 1}{\sqrt{2}}\right) && (16) \\ &= 20.87° && (17) \end{aligned}$$