

# L<sup>A</sup>T<sub>E</sub>X 9.10.6.7

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CLASS 9, CHAPTER, 10, EXERCISE 6.7

Q. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is a rectangle.

**Solution:** Let point  $O = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$  be the center of circle, and

$$\mathbf{A} = \begin{pmatrix} \cos^\circ 0 \\ \sin^\circ 0 \end{pmatrix} \quad (1)$$

$$\Rightarrow (A) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2)$$

$$\mathbf{B} = \begin{pmatrix} \cos^\circ 90 \\ \sin^\circ 90 \end{pmatrix} \quad (3)$$

$$\Rightarrow (B) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (4)$$

$$\mathbf{C} = \begin{pmatrix} \cos^\circ 180 \\ \sin^\circ 180 \end{pmatrix} \quad (5)$$

$$\Rightarrow (C) = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad (6)$$

$$\mathbf{D} = \begin{pmatrix} \cos^\circ 270 \\ \sin^\circ 270 \end{pmatrix} \quad (7)$$

$$\Rightarrow (D) = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad (8)$$

- 1) As it is given that chords AC and BD bisect each other, we have

$$\mathbf{A} + \mathbf{C} = \mathbf{B} + \mathbf{D} \quad (9)$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad (10)$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (11)$$

$\Rightarrow$  both chords AC and BD pass through the center of the circle. Hence, AC and BD are diameters of the circle.

- 2) Here the AC and BD i.e. the diagonals of quadrilateral bisect each other. Hence, the quadrilateral ABCD is either a rectangle or a parallelogram. Let's check the angle between

adjacent sides of this quadrilateral, i.e. AB and BC.

$$\mathbf{AB} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (12)$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (13)$$

$$\mathbf{BC} = \mathbf{B} - \mathbf{C} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad (14)$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (15)$$

$$(16)$$

Let  $\theta$  be the angle between AB and BC, then as per cosine rule

$$\cos \theta = \frac{\mathbf{AB} \cdot \mathbf{BC}}{\|\mathbf{AB}\| \|\mathbf{BC}\|} \quad (17)$$

$$= 0 \quad (18)$$

$$\Rightarrow \theta = 90^\circ \quad (19)$$

Hence, the quadrilateral ABCD is a rectangle.