LATEX 9.10.5.3

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CLASS 9, CHAPTER 10, EXERCSE 5.3

Q. $\angle PQR = 100°$, where $\mathbf{P}, \mathbf{Q}$ and $\mathbf{R}$ are points on a circle with centre $\mathbf{O}$. Find $\angle OPR$
Solution: Let, we have a unit circle with center at origin, i.e. $\mathbf{O}$, and radius $r = 1$. Then let following points be on the circle

$\mathbf{O}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\mathbf{R}$	$\begin{pmatrix} \cos 0 \\ \sin 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\mathbf{Q}$	$\begin{pmatrix} \cos\left(-\frac{\pi}{6}\right) \\ \sin\left(-\frac{\pi}{6}\right) \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{pmatrix}$
$\mathbf{P}$	$\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$	$\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

TABLE I: points

Using the theorem from appendix of matrix analysis book, Let

$$\mathbf{A} = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \quad (1)$$

be points on a unit circle. Then

$$\cos ACB = \frac{(\mathbf{C} - \mathbf{A})^\top (\mathbf{C} - \mathbf{B})}{\|\mathbf{C} - \mathbf{A}\|\|\mathbf{C} - \mathbf{B}\|} \tag{2}$$

$$= \cos\left(\frac{\theta_1 - \theta_2}{2}\right) \tag{3}$$

For our question we have 3 points

$$\mathbf{P} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} \cos\left(-\frac{\pi}{6}\right) \\ \sin\left(-\frac{\pi}{6}\right) \end{pmatrix}, \mathbf{R} = \begin{pmatrix} \cos 0 \\ \sin 0 \end{pmatrix}, \tag{4}$$

on a unit circle. Then using this theorem, we get

$$\cos PQR = \frac{(\mathbf{Q} - \mathbf{P})^\top (\mathbf{Q} - \mathbf{R})}{\|\mathbf{Q} - \mathbf{P}\|\|\mathbf{Q} - \mathbf{R}\|} \tag{5}$$

$$= \cos\left(\frac{\theta - 0}{2}\right) \tag{6}$$

As per given condition, we have

$$\angle PQR = 100° \tag{7}$$

$$\implies \cos 100° = \cos\left(\frac{\theta - 0}{2}\right) \tag{8}$$

$$\theta = 200° \tag{9}$$

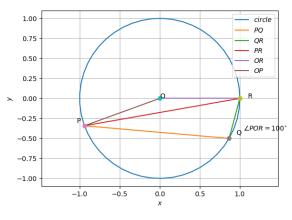$$\mathbf{P} = \begin{pmatrix} \cos 200° \\ \sin 200° \end{pmatrix} \tag{10}$$

Fig. 1: circle