11.11.5.3

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CLASS 11, CHAPTER 11, EXERCISE 5.3

Q. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

Solution: Uniformly loaded suspension bridge cable hangs in the form of a parabola facing upwards.

O	Lowest point of cable	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
AB	Length of the cable	100 m
OC	Length of shortest wire	6 m
C_1A	Length of longest wire	30 m

TABLE I: points

This will give us a setup similar to figure 1,

Here A and B are the points on the parabola where the cable is attached to the roadway, i.e. longest wire is attached at this points. And vertex of parabola O is point where shortest wire is attached, which is 6m from the ground. With the assumption of point O being $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, we'll get Point $A = \begin{pmatrix} 50 \\ 24 \end{pmatrix}$ and Point $B = \begin{pmatrix} -50 \\ 24 \end{pmatrix}$.

The generic equation of conic is

$$g(\mathbf{x}) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{1}$$

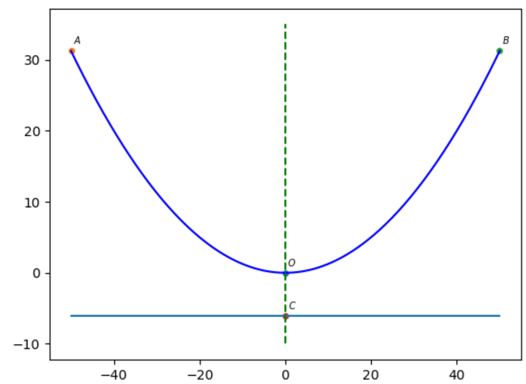

Fig. 1: Representation of parabola with vertex at origin.

Point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is on conic, so

$$\implies f = 0 \tag{2}$$

As conic is upward facing parabola,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{3}$$

As points A and B are on parabola

$$\implies \begin{pmatrix} 50 & 24 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 50 \\ 24 \end{pmatrix} + 2\mathbf{u}^\top \begin{pmatrix} 50 \\ 24 \end{pmatrix} = 0 \tag{4}$$

$$\implies \mathbf{u}^\top \begin{pmatrix} 50 \\ 24 \end{pmatrix} = -1250 \tag{5}$$

and

$$\implies \begin{pmatrix}-50 & 24\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}-50\\24\end{pmatrix} + 2\mathbf{u}^\top\begin{pmatrix}-50\\24\end{pmatrix} = 0 \tag{6}$$

$$\implies \mathbf{u}^\top\begin{pmatrix}-50\\24\end{pmatrix} = -1250 \tag{7}$$

From (5) and (7), we get

$$\mathbf{u}^\top\begin{pmatrix}50 & -50\\24 & 24\end{pmatrix} = \begin{pmatrix}1250 & -1250\end{pmatrix} \tag{8}$$

$$\implies \mathbf{u} = \begin{pmatrix}0\\-\frac{625}{12}\end{pmatrix} \tag{9}$$

we get parabola

$$\mathbf{x}^\top\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}0\\-\frac{625}{12}\mathbf{x}\end{pmatrix} = 0 \tag{10}$$

At a point $18m$ from middle, let's call it $D = \begin{pmatrix}18\\x_2\end{pmatrix}$.

$$\begin{pmatrix}18 & x_2\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}18\\x_2\end{pmatrix} + 2\begin{pmatrix}0\\-\frac{625}{12}\end{pmatrix}\begin{pmatrix}18\\x_2\end{pmatrix} = 0 \tag{11}$$

$$\implies x_2 = 3.3 \tag{12}$$

$\implies$ Length of a supporting wire attached to the roadway $18m$ from the middle is

$$= x_2 + 6 = 3.3 + 6 = 9.3m \tag{13}$$

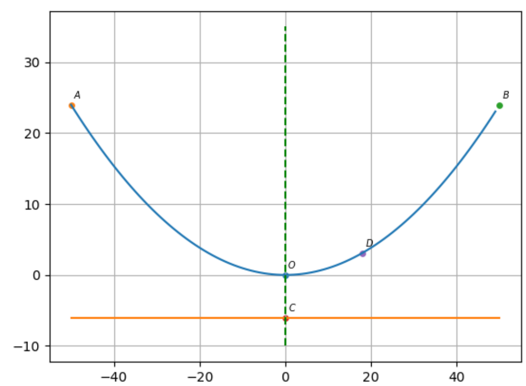

Fig. 2: Parabola