

12.10.3.13

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CLASS 12, CHAPTER 10, EXERCISE 3.13

- 13) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are unit vectors such that $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$, find the value of $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$.

Solution: The inner product of given unit vectors with sum of all unit vectors (Which is given to be 0) will be zero.

$$\mathbf{a}^T(\mathbf{a} + \mathbf{b} + \mathbf{c}) = 0 \quad (1)$$

$$\Rightarrow (\mathbf{a}^T \mathbf{b} + \mathbf{a}^T \mathbf{c}) = -1 \quad (2)$$

$$\mathbf{b}^T(\mathbf{a} + \mathbf{b} + \mathbf{c}) = 0 \quad (3)$$

$$\Rightarrow (\mathbf{b}^T \mathbf{c} + \mathbf{b}^T \mathbf{a}) = -1 \quad (4)$$

$$\mathbf{c}^T(\mathbf{a} + \mathbf{b} + \mathbf{c}) = 0 \quad (5)$$

$$\Rightarrow (\mathbf{c}^T \mathbf{a} + \mathbf{c}^T \mathbf{b}) = -1 \quad (6)$$

Adding equations (2), (4) and (6) we get

$$\Rightarrow 2(\mathbf{a}^T \mathbf{b} + \mathbf{b}^T \mathbf{c} + \mathbf{c}^T \mathbf{a}) = -3 \quad (7)$$

$$\Rightarrow (\mathbf{a}^T \mathbf{b} + \mathbf{b}^T \mathbf{c} + \mathbf{c}^T \mathbf{a}) = \frac{-3}{2} \quad (8)$$

$$\Rightarrow \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a} = \frac{-3}{2} \quad (9)$$