

11.10.3.16

Lokesh Surana

CLASS 11, CHAPTER 10, EXERCISE 3.16

Q16. If p and q are the lengths of perpendiculars from the origin to the lines $x \cos \theta - y \sin \theta = k \cos 2\theta$ and $x \sec \theta + y \csc \theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$

Solution: Equation of lines are as follows:

$$L_1 : x \cos \theta - y \sin \theta = k \cos 2\theta \quad (1)$$

$$\Rightarrow \mathbf{n}_1 = \begin{pmatrix} \cos \theta \\ -\sin \theta \end{pmatrix} \text{ and } c_1 = k \cos 2\theta \quad (2)$$

$$L_2 : x \sec \theta + y \csc \theta = k \quad (3)$$

$$L_2 : x \sin \theta + y \cos \theta = k \cos \theta \sin \theta \quad (4)$$

$$L_2 : x \sin \theta + y \cos \theta = \frac{1}{2}k \sin 2\theta \quad (5)$$

$$\Rightarrow \mathbf{n}_2 = \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix} \text{ and } c_2 = \frac{1}{2}k \sin 2\theta \quad (6)$$

The lengths of perpendiculars from origin can be found by using the following formula:

$$p = \frac{|\mathbf{n}_1^\top \mathbf{x} - c_1|}{\|\mathbf{n}_1\|} \quad (7)$$

$$p = \frac{\left| \begin{pmatrix} \cos \theta & -\sin \theta \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - k \cos 2\theta \right|}{\sqrt{\cos^2 \theta + \sin^2 \theta}} \quad (8)$$

$$p = |k \cos 2\theta| \quad (9)$$

$$\Rightarrow p^2 = k^2 \cos^2 2\theta \quad (10)$$

$$q = \frac{|\mathbf{n}_2^\top \mathbf{x} - c_2|}{\|\mathbf{n}_2\|} \quad (11)$$

$$q = \frac{\left| \begin{pmatrix} \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \frac{1}{2}k \sin 2\theta \right|}{\sqrt{\sin^2 \theta + \cos^2 \theta}} \quad (12)$$

$$q = \left| \frac{1}{2}k \sin 2\theta \right| \quad (13)$$

$$(14)$$

Therefore,

$$p^2 + 4q^2 = k^2 \cos^2 2\theta + 4\left(\frac{1}{4}\right)k^2 \sin^2 2\theta = k^2$$