12.11.3.9

Lokesh Surana

Class 12, Chapter 11, Exercise 3.9

Q.9. Find the equation of the plane through the intersection of the planes $3x–y+2z–4=0$ and $x+y+z–2=0$ and the point $\begin{pmatrix}2\\2\\1\end{pmatrix}$.

Solution: The equation of given are given by

$$P_1 : \mathbf{n}_1^\top \mathbf{x} = c_1 \tag{1}$$
$$P_1 : \mathbf{n}_1^\top \mathbf{x} = c_2 \tag{2}$$

The intersection of the planes is given by the solution of the system of equations

$$P : P_1 + \lambda P_2 = 0 \tag{3}$$
$$P : \mathbf{n}_1^\top \mathbf{x} - c_1 + \lambda\left(\mathbf{n}_2^\top \mathbf{x} - c_2\right) = = 0 \tag{4}$$

If this plane is passing through a point $\mathbf{P}$, then following will be satisfied

$$P : \left(\mathbf{n}_1^\top - \lambda \mathbf{n}_2^\top\right)\mathbf{P} - (c_1 + \lambda c_2) = 0 \tag{5}$$

Solving above equation would give us value of λ which can be substituted to get final plane equation.

The equation of given planes are given by

$$P_1 : \begin{pmatrix}3 & -1 & 2\end{pmatrix}\mathbf{x} = 4 \tag{6}$$
$$P_2 : \begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{x} = 2 \tag{7}$$

The intersection of the planes is given by the solution of the system of equations

$$P : P_1 + \lambda P_2 = 0 \tag{8}$$
$$P : \begin{pmatrix}3+\lambda & -1+\lambda & 2+\lambda\end{pmatrix}\mathbf{x} - (4+2\lambda) = 0 \tag{9}$$

These plane shall pass through point $\begin{pmatrix}2\\2\\1\end{pmatrix}$, which means that

$$\begin{pmatrix}3+\lambda & -1+\lambda & 2+\lambda\end{pmatrix}\begin{pmatrix}2\\2\\1\end{pmatrix} - (4+2\lambda) = 0 \tag{10}$$

$$\lambda = -\frac{2}{3} \tag{11}$$

The equation of plane is as follows:

$$\frac{1}{3}\begin{pmatrix}7 & -5 & 4\end{pmatrix}\mathbf{x} = \frac{8}{3} \tag{12}$$
$$\implies P : \begin{pmatrix}7 & -5 & 4\end{pmatrix}\mathbf{x} = 8 \tag{13}$$