$\LaTeX$ 9.10.5.3

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CLASS 9, CHAPTER 10, EXERCSE 5.3

Q7. $\angle PQR = 100°$, where $\mathbf{P}, \mathbf{Q}$ and $\mathbf{R}$ are points on a circle with centre $\mathbf{O}$. Find $\angle OPR$

Solution: Let, we have a unit circle with center at origin, i.e. $\mathbf{O}$, and radius $r = 1$. Then let following points be on the circle

$\mathbf{O}$	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}0\\0\end{pmatrix}$
$\mathbf{R}$	$\begin{pmatrix}\cos 0\\\sin 0\end{pmatrix}$	$\begin{pmatrix}1\\0\end{pmatrix}$
$\mathbf{Q}$	$\begin{pmatrix}\cos\left(-\frac{\pi}{6}\right)\\\sin\left(-\frac{\pi}{6}\right)\end{pmatrix}$	$\begin{pmatrix}-\frac{\sqrt{3}}{2}\\-\frac{1}{2}\end{pmatrix}$
$\mathbf{P}$	$\begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}$	$\begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}$

TABLE I: points

Using the theorem from appendix of matrix analysis book, Let

$$\mathbf{A} = \begin{pmatrix}\cos\theta_1\\\sin\theta_1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}\cos\theta_2\\\sin\theta_2\end{pmatrix}, \mathbf{C} = \begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}, \tag{1}$$

be points on a unit circle. Then

$$\cos ACB = \frac{(\mathbf{C}-\mathbf{A})^\top(\mathbf{C}-\mathbf{B})}{\|\mathbf{C}-\mathbf{A}\|\|\mathbf{C}-\mathbf{B}\|} \tag{2}$$

$$= \cos\left(\frac{\theta_1-\theta_2}{2}\right) \tag{3}$$

For our question we have 3 points

$$\mathbf{P} = \begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}, \mathbf{Q} = \begin{pmatrix}\cos\left(-\frac{\pi}{6}\right)\\\sin\left(-\frac{\pi}{6}\right)\end{pmatrix}, \mathbf{R} = \begin{pmatrix}\cos 0\\\sin 0\end{pmatrix}, \tag{4}$$

on a unit circle. Then using this theorem, we get

$$\cos PQR = \frac{(\mathbf{Q}-\mathbf{P})^\top(\mathbf{Q}-\mathbf{R})}{\|\mathbf{Q}-\mathbf{P}\|\|\mathbf{Q}-\mathbf{R}\|} \tag{5}$$

$$= \cos\left(\frac{\theta-0}{2}\right) \tag{6}$$

As per given condition, we have

$$\angle PQR = 100° \tag{7}$$

$$\implies \cos 100° = \cos\left(\frac{\theta-0}{2}\right) \tag{8}$$

$$\theta = 200° \tag{9}$$

$$\mathbf{P} = \begin{pmatrix}\cos 200°\\\sin 200°\end{pmatrix} \tag{10}$$

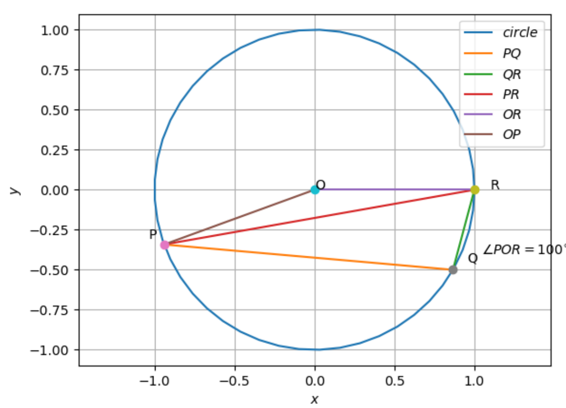

Fig. 1: circle