

12.6.5.19

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CLASS 12, CHAPTER 6, EXERCISE 5.19

Q. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

Solution: The rectangle inscribed in circle is shown below in figure (1)

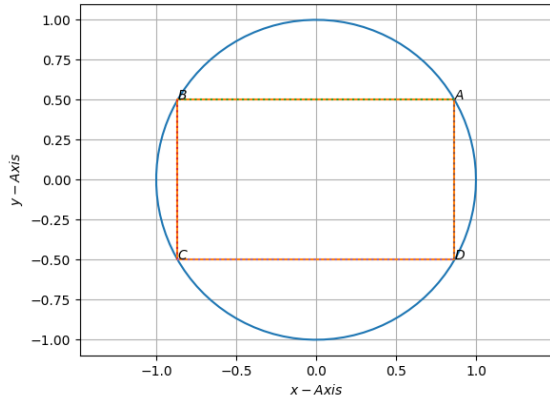


Fig. 1: Circle

Point	Coordinates
A	$\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$
B	$\begin{pmatrix} -\cos\theta \\ \sin\theta \end{pmatrix}$
C	$\begin{pmatrix} -\cos\theta \\ -\sin\theta \end{pmatrix}$
D	$\begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}$

TABLE I: points

For $0 < \theta < \frac{\pi}{2}$

The area of rectangle inscribed in circle is given by

$$\text{Area} = AB \times BC \quad (1)$$

$$= 2 \cos \theta \times 2 \sin \theta \quad (2)$$

$$= 4 \cos \theta \sqrt{1 - \cos^2 \theta} \quad (3)$$

$$(4)$$

Let $x = \cos \theta$, area function is given by

$$A(x) = 4x \sqrt{1 - x^2} \quad (5)$$

$$(6)$$

For $0 < \theta < \frac{\pi}{2}$, i.e. $0 < x < 1$

$$A(x) > 0 \quad (7)$$

So

$$\min_{\lambda} A^2(\lambda) = \min_{\lambda} A(\lambda) \quad (8)$$

$$A^2(x) = 16x^2(1 - x^2) \quad (9)$$

$$= -16x^4 + 16x^2 \quad (10)$$

This is concave function, so using cvxpy, we get

$$x = 1.4142 \approx \frac{1}{\sqrt{2}} \quad (11)$$

$$x = \cos \frac{\pi}{4} \quad (12)$$

So area of square is maximum in all rectangles inscribed in a circle.