LaTeX 9.10.6.7

Lokesh Surana

Class 9, Chapter, 10, Exercse 6.7

Q. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is a rectangle.

Solution: Let, we have a unit circle with center at origin, i.e. $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, and radius $r = 1$. The points on circe that we consider are available in Table (**??**).

$$\mathbf{A} = \begin{pmatrix} \cos^\circ 0 \\ \sin^\circ 0 \end{pmatrix} \tag{1}$$

$$\implies \mathbf{A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{B} = \begin{pmatrix} \cos^\circ 90 \\ \sin^\circ 90 \end{pmatrix} \tag{3}$$

$$\implies \mathbf{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

$$\mathbf{C} = \begin{pmatrix} \cos^\circ 180 \\ \sin^\circ 180 \end{pmatrix} \tag{5}$$

$$\implies \mathbf{C} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{6}$$

$$\mathbf{D} = \begin{pmatrix} \cos^\circ 270 \\ \sin^\circ 270 \end{pmatrix} \tag{7}$$

$$\implies \mathbf{D} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{8}$$

1) As it is given that chords AC and BD bisect each other, we have

$$\mathbf{A} + \mathbf{C} = \mathbf{B} + \mathbf{D} \tag{9}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{10}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{11}$$

$\implies$ both chords AC and BD pass through the center of the circle. Hence, AC and BD are diameters of the circle.

2) Here the AC and BD i.e. the diagonals of quadrilateral bisect each other. Hence, the quadrilateral $ABCD$ is either a rectangle or a parallelogram. Let's check the angle between adjacent sides of this quadrilateral, i.e. AB and BC.

$$\mathbf{AB} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{12}$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{13}$$

$$\mathbf{BC} = \mathbf{B} - \mathbf{C} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{14}$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{15}$$

$$\tag{16}$$

Let θ be the angle between $\mathbf{AB}$ and $\mathbf{BC}$, then as per cosine rule

$$\cos\theta = \frac{\mathbf{AB} \cdot \mathbf{BC}}{\|\mathbf{AB}\|\|\mathbf{BC}\|} \tag{17}$$

$$= 0 \tag{18}$$

$$\implies \theta =^\circ 90 \tag{19}$$

Hence, the quadrilateral $ABCD$ is a rectangle.

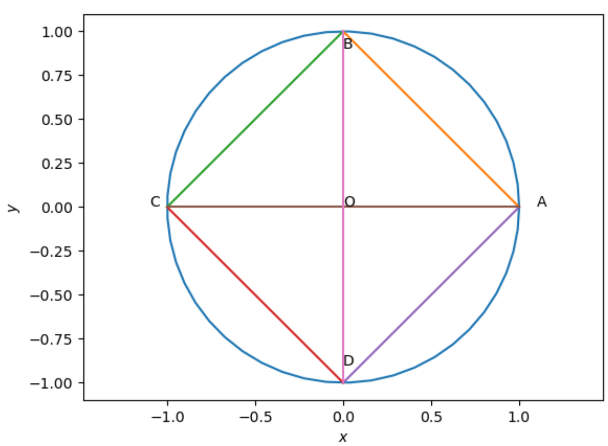

Fig. 1: circle

[12pt]report
setspace gensymb [latin1]inputenc fullpage color array longtable calc multirow hhline ifthen
xcolor caption

siunitx [cmex10]amsmath mathtools hyperref amsthm mathrsfs txfonts stfloats cite cases subfig longtable multirow enumitem mathtools listings tikz circuitikz *Res Res
Theorem[section] Problem Proposition[section] Lemma[section] [theorem]Corollary Example[section] Definition[section]
Remark
english
¿

A	$\begin{pmatrix}1\\0\end{pmatrix}$	$\begin{pmatrix}\cos^\circ 0\\ \sin^\circ 0\end{pmatrix}$
B	$\begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}\cos^\circ 90\\ \sin^\circ 90\end{pmatrix}$
C	$\begin{pmatrix}-1\\0\end{pmatrix}$	$\begin{pmatrix}\cos^\circ 180\\ \sin^\circ 180\end{pmatrix}$
D	$\begin{pmatrix}0\\-1\end{pmatrix}$	$\begin{pmatrix}\cos^\circ 270\\ \sin^\circ 270\end{pmatrix}$

english

TABLE II