

12.10.3.17

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CLASS 12, CHAPTER 10, EXERCISE 3.17

17) Show that the vectors form the vertices $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$,

$\begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix}$ of a right angled triangle.

Solution: Let's name the triangle ABC , where

$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix}$. So sides of triangle are:

$$\mathbf{a} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \quad (1)$$

$$\mathbf{b} = \mathbf{A} - \mathbf{C} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad (2)$$

$$\mathbf{c} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ -2 \\ -6 \end{pmatrix} \quad (3)$$

If inner product of two vectors is zero, then they are perpendicular. So, we have:

$$\mathbf{a}^\top \mathbf{b} = 2 \times (-1) + (-1) \times 3 + 1 \times 5 = 0 \quad (4)$$

$$\mathbf{b}^\top \mathbf{c} = (-1) \times (-1) + 3 \times (-2) + 5 \times (-6) = -35 \quad (5)$$

$$\mathbf{c}^\top \mathbf{a} = (-1) \times 2 + (-2) \times (-1) + (-6) \times 1 = -6 \quad (6)$$

So, \mathbf{a} and \mathbf{b} are perpendicular and therefore triangle ABC is right angled triangle.