

11.11.5.3

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CLASS 11, CHAPTER 11, EXERCISE 5.3

Q. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

Solution: Uniformly loaded suspension bridge cable hangs in the form of a parabola facing upwards.

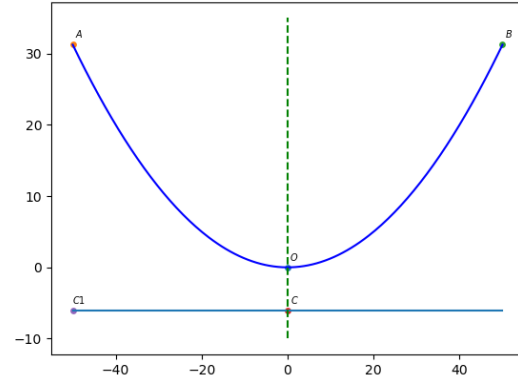


Fig. 1: Representation of parabola with vertex at origin.

O	Lowest point of cable	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
AB	Length of the cable	100 m
OC	Length of shortest wire	6 m
C_1A	Length of longest wire	30 m

TABLE I: points

This will give us a setup similar to figure 1,

Here A and B are the points on the parabola where the cable is attached to the roadway, i.e. longest wire is attached at this points. And vertex of parabola O is point where shortest wire is attached, which is 6m from the ground.

With the assumption of point O being $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, we'll get Point A = $\begin{pmatrix} 50 \\ 24 \end{pmatrix}$ and Point B = $\begin{pmatrix} -50 \\ 24 \end{pmatrix}$.

The generic equation of conic is

$$g(\mathbf{x}) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1)$$

Point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is on conic, so

$$\Rightarrow f = 0 \quad (2)$$

As conic is upward facing parabola,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \quad (3)$$

As points A and B are on parabola

$$\Rightarrow \begin{pmatrix} 50 & 24 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 50 \\ 24 \end{pmatrix} + 2\mathbf{u}^T \begin{pmatrix} 50 \\ 24 \end{pmatrix} = 0 \quad (4)$$

$$\Rightarrow \mathbf{u}^T \begin{pmatrix} 50 \\ 24 \end{pmatrix} = -1250 \quad (5)$$

and

$$\Rightarrow (-50 \ 24) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -50 \\ 24 \end{pmatrix} + 2\mathbf{u}^\top \begin{pmatrix} -50 \\ 24 \end{pmatrix} = 0 \quad (6)$$

$$\Rightarrow \mathbf{u}^\top \begin{pmatrix} -50 \\ 24 \end{pmatrix} = -1250 \quad (7)$$

From (5) and (7), we get

$$\mathbf{u}^\top \begin{pmatrix} 50 & -50 \\ 24 & 24 \end{pmatrix} = \begin{pmatrix} 1250 & -1250 \end{pmatrix} \quad (8)$$

$$\Rightarrow \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{625}{12} \end{pmatrix} \quad (9)$$

we get parabola

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 \\ -\frac{625}{12} \end{pmatrix} \mathbf{x} = 0 \quad (10)$$

At a point 18m from middle, let's call it $D = \begin{pmatrix} 18 \\ x_2 \end{pmatrix}$.

$$\begin{pmatrix} 18 & x_2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 18 \\ x_2 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ -\frac{625}{12} \end{pmatrix} \begin{pmatrix} 18 \\ x_2 \end{pmatrix} = 0 \quad (11)$$

$$\Rightarrow x_2 = 3.3 \quad (12)$$

\Rightarrow Length of a supporting wire attached to the roadway 18m from the middle is

$$= x_2 + 6 = 3.3 + 6 = 9.3m \quad (13)$$

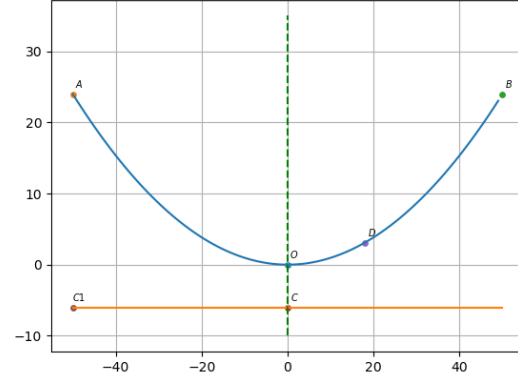


Fig. 2: Parabola