

L^AT_EX 9.10.5.3

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CLASS 9, CHAPTER 10, EXERCISE 5.3

Q7. $\angle PQR = 100^\circ$, where \mathbf{P} , \mathbf{Q} and \mathbf{R} are points on a circle with centre \mathbf{O} . Find $\angle OPR$

Solution: Let, we have a unit circle with center at origin, i.e. \mathbf{O} , and radius $r = 1$. Then let following points be on the circle

\mathbf{O}	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
\mathbf{R}	$\begin{pmatrix} \cos 0 \\ \sin 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
\mathbf{Q}	$\begin{pmatrix} \cos \frac{\pi}{6} \\ \sin \frac{\pi}{6} \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$
\mathbf{P}	$\begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$	$\begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$

TABLE I: points

As per given condition, we have

$$\angle PQR = 100^\circ \quad (1)$$

$$\Rightarrow \cos 100^\circ = \frac{(\mathbf{P} - \mathbf{Q})^\top (\mathbf{R} - \mathbf{Q})}{\|\mathbf{P} - \mathbf{Q}\| \|\mathbf{R} - \mathbf{Q}\|} \quad (2)$$

$$\|\mathbf{P} - \mathbf{Q}\| = \sqrt{\left(\cos \theta - \frac{\sqrt{3}}{2}\right)^2 + \left(\sin \theta - \frac{1}{2}\right)^2} \quad (3)$$

$$\|\mathbf{R} - \mathbf{Q}\| = \sqrt{\left(1 - \frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} \quad (4)$$

$$= \sqrt{2 - \sqrt{3}} \quad (5)$$

$$(\mathbf{P} - \mathbf{Q})^\top (\mathbf{R} - \mathbf{Q}) = \left(\cos \theta - \frac{\sqrt{3}}{2}\right) \cdot \left(1 - \frac{\sqrt{3}}{2}\right) + \left(\sin \theta - \frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) \quad (6)$$

$$= 1 - \frac{\sqrt{3}}{2} + \cos \theta - \cos\left(\theta - \frac{\pi}{6}\right) \quad (7)$$

Using (2), (3), (5) and (7), we get

$$\theta = \quad (8)$$

$$\mathbf{P} = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (9)$$

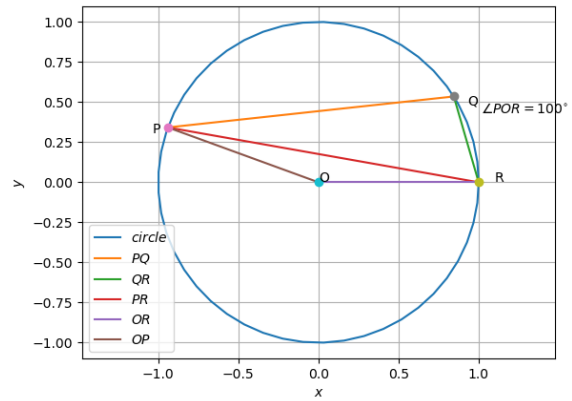


Fig. 1: circle

Now, we'll find the $\angle OPR$,

$$\mathbf{OP} = \mathbf{P} - \mathbf{O} = \begin{pmatrix} \cos 160^\circ - 0 \\ \sin 160^\circ - 0 \end{pmatrix} = \begin{pmatrix} \cos 160^\circ \\ \sin 160^\circ \end{pmatrix} \quad (10)$$

$$\mathbf{OR} = \mathbf{R} - \mathbf{O} = \begin{pmatrix} 1 - 0 \\ 0 - 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (11)$$

$$\Rightarrow \angle OPR = \arccos\left(\frac{\mathbf{OP}^\top \mathbf{OR}}{\|\mathbf{OP}\| \|\mathbf{OR}\|}\right) \quad (12)$$

$$= \arccos\left(\frac{\cos 160^\circ + 1}{\sqrt{2}}\right) \quad (13)$$

$$= 20.87^\circ \quad (14)$$