12.10.3.17

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Class 12, Chapter 10, Exercise 3.17

17) Show that the vectors form the vertices $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix}$ of a right angled triangle.

Solution: Let's name the triangle ABC, where $\mathbf{A} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix}$. So sides of triangle are:

$$\mathbf{a} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\mathbf{b} = \mathbf{A} - \mathbf{C} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$$

$$\mathbf{c} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ 2 \\ -6 \end{pmatrix}$$

If inner product of two vectors is zero, then they are perpendicular. So, we have:

$$\mathbf{a}^\top \mathbf{b} = 2 \cdot (-1) + (-1) \cdot 3 + 1 \cdot 5 = 0$$
$$\mathbf{b}^\top \mathbf{c} = (-1) \cdot (-1) + 3 \cdot 2 + 5 \cdot (-6) = -23$$
$$\mathbf{c}^\top \mathbf{a} = (-1) \cdot 2 + 2 \cdot (-1) + (-6) \cdot 1 = -10$$

So, $\mathbf{a}$ and $\mathbf{b}$ are perpendicular and therefore triangle ABC is right angled triangle.