11.10.4.21

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Class 11, Chapter 10, Exercise 4.21

Q.21. Find equation of the line which is equidistant from parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.

Solution: Equation of lines are

$$L_1 : \begin{pmatrix} 9 & 6 \end{pmatrix} \mathbf{x} - 7 = 0 \tag{1}$$

$$\implies L_1 : \begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix} \mathbf{x} - \frac{7}{9} = 0 \tag{2}$$

$$\implies \mathbf{n}_1 = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} \text{ and } c_1 = \frac{7}{9} \tag{3}$$

$$L_2 : \begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} + 6 = 0 \tag{4}$$

$$\implies L_2 : \begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix} \mathbf{x} + 2 = 0 \tag{5}$$

$$\implies \mathbf{n}_2 = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} \text{ and } c_2 = -2 \tag{6}$$

The equation of desired line will be $\mathbf{n}^\top \mathbf{x} = c$. As it is given that lines are parallel, $\mathbf{n}$ must be same as $\mathbf{n}_1$ and $\mathbf{n}_2$.

$$L : \begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix} \mathbf{x} - c = 0 \tag{7}$$

The distance between two parallel lines $\mathbf{n}^\top \mathbf{x} = c_1$ and $\mathbf{n}^\top \mathbf{x} = c_2$ is given as

$$d = \frac{|c_1 - c_2|}{||n||} \tag{8}$$

So for given lines, we need to solve

$$\frac{|c_1 - c|}{||n||} = \frac{|c - c_2|}{||n||} \tag{9}$$

$$\implies \frac{\left|\frac{7}{9} - c\right|}{\sqrt{1^2 + \left(\frac{2}{3}\right)^2}} = \frac{|c + 2|}{\sqrt{1^2 + \left(\frac{2}{3}\right)^2}} \tag{10}$$

Case 1.

$$\frac{7}{9} - c = -c - 2 \tag{11}$$

$$\implies \frac{7}{9} = -2 \tag{12}$$

$$\text{(not possible)} \tag{13}$$

Case 2.

$$\frac{7}{9} - c = c + 2 \tag{14}$$

$$\implies 2c = -\frac{11}{9} \tag{15}$$

$$\implies c = -\frac{11}{18} \tag{16}$$

The equation of line is

$$L : \begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix} \mathbf{x} - c = 0 \tag{17}$$

$$L : \begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix} \mathbf{x} + \frac{11}{18} = 0 \tag{18}$$

$$\implies L : \begin{pmatrix} 18 & 12 \end{pmatrix} \mathbf{x} + 11 = 0 \tag{19}$$

$$\tag{20}$$

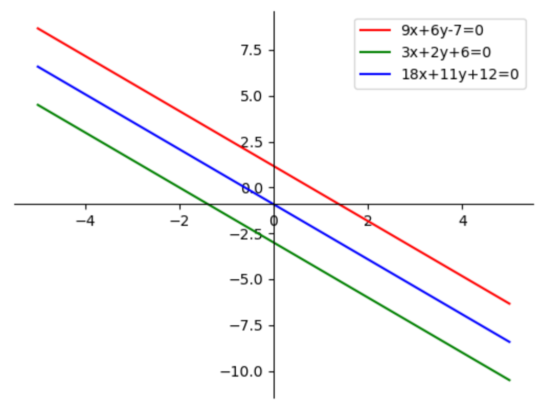

Fig. 1: Given lines and equidistant line