

12.11.3.9

Lokesh Surana

CLASS 12, CHAPTER 11, EXERCISE 3.9

Q.9. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$.

Solution: The equation of given are given by

$$P_1 : \mathbf{n}_1^\top \mathbf{x} = c_1 \quad (1)$$

$$P_2 : \mathbf{n}_2^\top \mathbf{x} = c_2 \quad (2)$$

The intersection of the planes is given by the solution of the system of equations

$$P : P_1 + \lambda P_2 = 0 \quad (3)$$

$$P : \mathbf{n}_1^\top \mathbf{x} - c_1 + \lambda (\mathbf{n}_2^\top \mathbf{x} - c_2) = 0 \quad (4)$$

If this plane is passing through a point \mathbf{P} , then following will be satisfied

$$P : (\mathbf{n}_1^\top - \lambda \mathbf{n}_2^\top) \mathbf{P} - (c_1 + \lambda c_2) = 0 \quad (5)$$

Solving above equation would give us value of λ which can be substituted to get final plane equation.

The equation of given planes are given by

$$P_1 : \begin{pmatrix} 3 & -1 & 2 \end{pmatrix} \mathbf{x} = 4 \quad (6)$$

$$P_2 : \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = 2 \quad (7)$$

The intersection of the planes is given by the solution of the system of equations

$$P : P_1 + \lambda P_2 = 0 \quad (8)$$

$$P : \begin{pmatrix} 3 + \lambda & -1 + \lambda & 2 + \lambda \end{pmatrix} \mathbf{x} - (4 + 2\lambda) = 0 \quad (9)$$

These plane shall pass through point $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$, which means that

$$\begin{pmatrix} 3 + \lambda & -1 + \lambda & 2 + \lambda \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - (4 + 2\lambda) = 0 \quad (10)$$

$$\lambda = -\frac{2}{3} \quad (11)$$

The equation of plane is as follows:

$$\frac{1}{3} \begin{pmatrix} 7 & -5 & 4 \end{pmatrix} \mathbf{x} = \frac{8}{3} \quad (12)$$

$$\Rightarrow P : \begin{pmatrix} 7 & -5 & 4 \end{pmatrix} \mathbf{x} = 8 \quad (13)$$