12.11.3.9

Lokesh Surana

Class 12, Chapter 11, Exercise 3.9

Q.9. Find the equation of the plane through the intersection of the planes $3x–y + 2z–4 = 0$ and $x + y + z–2 = 0$ and the point $\begin{pmatrix}2\\2\\1\end{pmatrix}$.

Solution: The equation of given planes are given by

$$P_1 : \begin{pmatrix}3 & -1 & 2\end{pmatrix}\mathbf{x} = 4 \tag{1}$$

$$P_2 : \begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{x} = 2 \tag{2}$$

The intersection of the planes is given by the solution of the system of equations

$$P : P_1 + \lambda P_2 = 0 \tag{3}$$

$$P : \begin{pmatrix}3 & -1 & 2\end{pmatrix}\mathbf{x} - 4 + \lambda\begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{x} - 2 = 0 \tag{4}$$

$$P : \begin{pmatrix}3+\lambda & -1+\lambda & 2+\lambda\end{pmatrix}\mathbf{x} - (4+2\lambda) = 0 \tag{5}$$

These plane shall pass through point $\begin{pmatrix}2\\2\\1\end{pmatrix}$, which means that

$$\begin{pmatrix}3+\lambda & -1+\lambda & 2+\lambda\end{pmatrix}\begin{pmatrix}2\\2\\1\end{pmatrix} - (4+2\lambda) = 0 \tag{6}$$

$$\lambda = -\frac{2}{3} \tag{7}$$

The equation of plane is as follows:

$$\frac{1}{3}\begin{pmatrix}7 & -5 & 4\end{pmatrix}\mathbf{x} = \frac{8}{3} \tag{8}$$

$$\implies P : \begin{pmatrix}7 & -5 & 4\end{pmatrix}\mathbf{x} = 8 \tag{9}$$