# 11.10.4.21

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CLASS 11, CHAPTER 10, EXERCISE 4.21

Q.21. Find equation of the line which is equidistant from parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.

**Solution:** Let's first rewrite the given equation of line in the form $x + by + c = 0$ where $b, c$ are constants. Equation of lines are

$$L_1 : 9x + 6y - 7 = 0 \tag{1}$$
$$\implies L_1 : x + \frac{2}{3}y - \frac{7}{9} = 0 \tag{2}$$
$$\implies \mathbf{n}_1 = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} \text{ and } c_1 = \frac{7}{9} \tag{3}$$
$$L_2 : 3x + 2y + 6 = 0 \tag{4}$$
$$\implies L_2 : x + \frac{2}{3}y + 2 = 0 \tag{5}$$
$$\implies \mathbf{n}_2 = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} \text{ and } c_2 = -2 \tag{6}$$

The equation of desired line will be $\mathbf{n}^\top \mathbf{x} = c$. As it is given that lines are parallel, $\mathbf{n}$ must be same as $\mathbf{n}_1$ and $\mathbf{n}_2$.

$$L : \begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix} \mathbf{x} - c = 0 \tag{7}$$

The distance between two parallel lines $\mathbf{n}^\top \mathbf{x} = c_1$ and $\mathbf{n}^\top \mathbf{x} = c_2$ is given as

$$d = \frac{|c_1 - c_2|}{||n||} \tag{8}$$

So for given lines, we need to solve

$$\frac{|c_1 - c|}{||n||} = \frac{|c - c_2|}{||n||} \tag{9}$$
$$\implies \frac{\left|\frac{7}{9} - c\right|}{||\begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix}||} = \frac{|c + 2|}{||\begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix}||} \tag{10}$$

Case 1.

$$\frac{7}{9} - c = -c + 2 \tag{11}$$
$$\implies -\frac{7}{9} = 2 \tag{12}$$
$$\text{(not possible)} \tag{13}$$

Case 2.

$$\frac{7}{9} - c = c + 2 \tag{14}$$
$$\implies 2c = -\frac{11}{9} \tag{15}$$
$$\implies c = -\frac{11}{18} \tag{16}$$

The equation of line is

$$L : \begin{pmatrix} 1 & \frac{2}{3} \end{pmatrix} \mathbf{x} - c = 0 \tag{17}$$
$$L : x + \frac{2}{3}y + \frac{11}{18} = 0 \tag{18}$$
$$\implies L : 18x + 12y + 11 = 0 \tag{19}$$

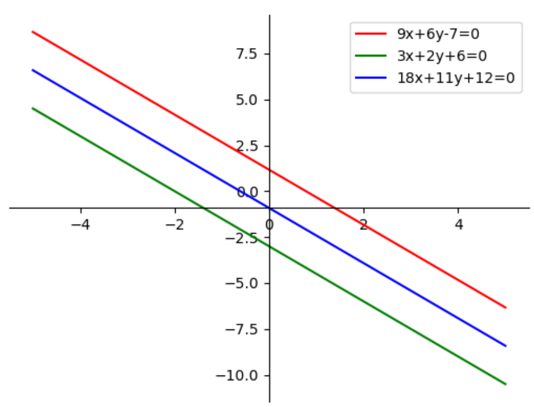


Fig. 1: Given lines and equidistant line