

# 11.11.5.3

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CLASS 11, CHAPTER 11, EXERCISE 5.3

Q. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

**Solution:** Uniformly loaded suspension bridge cable hangs in the form of a parabola facing upwards.

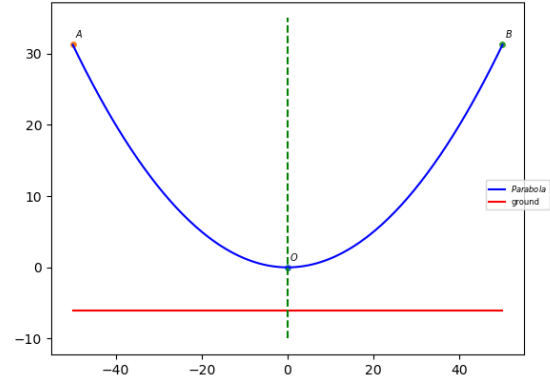


Fig. 1: Representation of parabola with vertex at origin.

<b>O</b>	Lowest point of cable	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$d$	Length of the cable	100 m
$d_1$	Length of longest wire	30 m
$d_2$	Length of shortest wire	6 m
<b>A</b>	End point of cable	$\begin{pmatrix} \frac{d}{2} \\ d_1 - d_2 \end{pmatrix}$
<b>B</b>	End point of cable	$\begin{pmatrix} -\frac{d}{2} \\ d_1 - d_2 \end{pmatrix}$

TABLE I: points

This will give us a setup similar to figure 1,

Here A and B are the points on the parabola where the cable is attached to the roadway, i.e. longest wire is attached at this points. And vertex of parabola O is point where shortest wire is attached, which is 6m from the ground.

With the assumption of point O being  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ , we'll get

$$\mathbf{A} = \begin{pmatrix} \frac{d}{2} \\ d_1 - d_2 \end{pmatrix} = \begin{pmatrix} 50 \\ 24 \end{pmatrix} \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} -\frac{d}{2} \\ d_1 - d_2 \end{pmatrix} = \begin{pmatrix} -50 \\ 24 \end{pmatrix} \quad (2)$$

The generic equation of conic is

$$g(\mathbf{x}) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (3)$$

As conic is upward facing parabola,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \quad (4)$$

Points **O**, **A**, and **B** are on conic, so we have

$$\mathbf{O}^T \mathbf{V} \mathbf{O} + 2\mathbf{u}^T \mathbf{O} + f = 0 \quad (5)$$

$$\mathbf{A}^T \mathbf{V} \mathbf{A} + 2\mathbf{u}^T \mathbf{A} + f = 0 \quad (6)$$

$$\mathbf{B}^T \mathbf{V} \mathbf{B} + 2\mathbf{u}^T \mathbf{B} + f = 0 \quad (7)$$

Rewrite the equations as

$$2\mathbf{O}^\top \mathbf{u} + f = -\mathbf{O}^\top \mathbf{V}\mathbf{O} \quad (8)$$

$$2\mathbf{A}^\top \mathbf{u} + f = -\mathbf{A}^\top \mathbf{V}\mathbf{A} \quad (9)$$

$$2\mathbf{B}^\top \mathbf{u} + f = -\mathbf{B}^\top \mathbf{V}\mathbf{B} \quad (10)$$

This can be formulated as matrix, as follows:

$$\begin{pmatrix} 2\mathbf{O}^\top & 1 \\ 2\mathbf{A}^\top & 1 \\ 2\mathbf{B}^\top & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = - \begin{pmatrix} \mathbf{O}^\top \mathbf{V}\mathbf{O} \\ \mathbf{A}^\top \mathbf{V}\mathbf{A} \\ \mathbf{B}^\top \mathbf{V}\mathbf{B} \end{pmatrix} \quad (11)$$

Substituting the values of  $\mathbf{O}$ ,  $\mathbf{A}$ , and  $\mathbf{B}$  in the above equation, we get

$$\begin{pmatrix} \begin{pmatrix} 0 & 0 \end{pmatrix} & 1 \\ \begin{pmatrix} 100 & 48 \end{pmatrix} & 1 \\ \begin{pmatrix} -100 & 48 \end{pmatrix} & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = - \begin{pmatrix} 0 \\ -1250 \\ -1250 \end{pmatrix} \quad (12)$$

$$\Rightarrow f = 0 \text{ and } \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{625}{12} \end{pmatrix} \quad (13)$$

So, equation of parabola is

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 & -\frac{625}{12} \end{pmatrix} \mathbf{x} = 0 \quad (14)$$

At a point  $\lambda_1$  m from middle, i.e.

$$\mathbf{x} = \lambda_1 \mathbf{e}_1 + \lambda_2 \mathbf{e}_2 \quad (15)$$

Substitute this in parabola equation, we get

$$(\lambda_1 \mathbf{e}_1 + \lambda_2 \mathbf{e}_2)^\top \mathbf{V} (\lambda_1 \mathbf{e}_1 + \lambda_2 \mathbf{e}_2) + 2\mathbf{u}^\top (\lambda_1 \mathbf{e}_1 + \lambda_2 \mathbf{e}_2) + f = 0 \quad (16)$$

$$\begin{pmatrix} \lambda_1 & \lambda_2 \end{pmatrix} \mathbf{V} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} + 2\mathbf{u}^\top \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} + f = 0 \quad (17)$$

$$2\mathbf{u}^\top \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = - \left( f + \begin{pmatrix} \lambda_1 & \lambda_2 \end{pmatrix} \mathbf{V} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \right) \quad (18)$$

Substituting  $\mathbf{u}$ ,  $\lambda_1$ ,  $f$ ,  $\mathbf{V}$ , we get

$$2 \begin{pmatrix} 0 & -\frac{625}{12} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = - \begin{pmatrix} \lambda_1 & \lambda_2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \quad (19)$$

$$\Rightarrow \lambda_2 = \frac{6}{625} (\lambda_1)^2 \quad (20)$$

It is given that  $\lambda_1 = 18$

$$\Rightarrow \lambda_2 = \frac{6}{625} (18)^2 = \frac{1944}{625} \quad (21)$$

$\Rightarrow$  Length of a supporting wire attached to the roadway 18m from the middle is

$$= \lambda_2 + d_2 = \frac{1944}{625} + 6 = \frac{5694}{625} \text{ m} \quad (22)$$

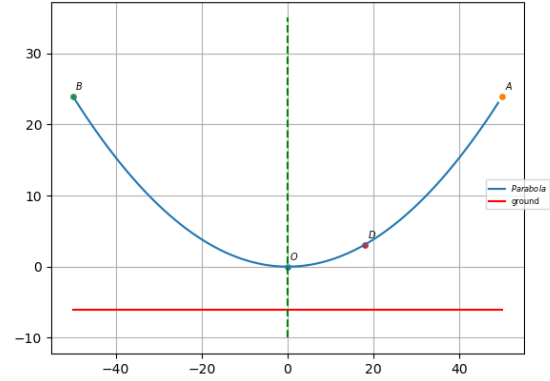


Fig. 2: Parabola