11.10.4.21

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Class 11, Chapter 10, Exercise 4.21

Q.21. Find equation of the line which is equidistant from parallel lines 9x + 6y - 7 = 0 and 3x + 2y + 6 = 0.

Solution: Let's first rewrite the given equation of line in the form x + by + c = 0 where b, c are constants. Equation of lines are

$$L_1: 9x + 6y - 7 = 0 \tag{1}$$

$$\implies L_1: x + \frac{2}{3}y - \frac{7}{9} = 0 \tag{2}$$

$$\implies$$
 $\mathbf{n}_1 = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix}$ and $c_1 = \frac{7}{9}$ (3)

$$L_2: 3x + 2y + 6 = 0 \tag{4}$$

$$\implies L_2: x + \frac{2}{3}y + 2 = 0 \tag{5}$$

$$\implies$$
 $\mathbf{n}_2 = \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix}$ and $c_2 = -2$ (6)

The equation of desired line will be $\mathbf{n}^{\mathsf{T}}\mathbf{x} + c = 0$. As it is given that lines are parallel, \mathbf{n} must be same as \mathbf{n}_1 and \mathbf{n}_2 .

$$L: \left(1 \quad \frac{2}{3}\right)\mathbf{x} - c = 0 \tag{7}$$

The distance between two parallel lines $\mathbf{n}^{\mathsf{T}}\mathbf{x}+c_1=0$ and $\mathbf{n}^{\mathsf{T}}\mathbf{x}+c_2=0$ is given as

$$d = \frac{|c_1 - c_2|}{\|n\|} \tag{8}$$

So for given lines, we need to solve

$$\frac{|c_1 - c|}{\|n\|} = \frac{|c - c_2|}{\|n\|} \tag{9}$$

$$\implies \frac{\left|\frac{7}{9} - c\right|}{\left\|\begin{pmatrix}1\\\frac{2}{2}\end{pmatrix}\right\|} = \frac{\left|c + 2\right|}{\left\|\begin{pmatrix}1\\\frac{2}{2}\end{pmatrix}\right\|} \tag{10}$$

Case 1.

$$\frac{7}{9} - c = -c + 2 \tag{11}$$

$$\implies -\frac{7}{9} = 2 \tag{12}$$

(not possible) (13)

Case 2.

$$\frac{7}{9} - c = c + 2 \tag{14}$$

$$\implies 2c = -\frac{11}{9} \tag{15}$$

$$\implies c = -\frac{11}{18} \tag{16}$$

The equation of line is

$$L: \left(1 \quad \frac{2}{3}\right)\mathbf{x} - c = 0 \tag{17}$$

$$L: x + \frac{2}{3}y + \frac{11}{18} = 0 \tag{18}$$

$$\implies L: 18x + 12y + 11 = 0 \tag{19}$$

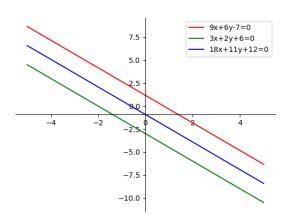


Fig. 1: Given lines and equidistant line