

# 11.11.2.5

Lokesh Surana

CLASS 11, CHAPTER 11, EXERCISE 2.5

we get

$$c = \frac{5^2 - 1(0)}{-2(5 \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix}} = -\frac{5}{2} \quad (12)$$

Q. Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum  $y^2 = 10x$

**Solution:** The given equation of the parabola can be rearranged as

$$y^2 - 10x = 0 \quad (1)$$

The above equation can be equated to the generic equation of conic sections

$$g(\mathbf{x}) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2)$$

Comparing coefficients of (1) and (2),

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (3)$$

$$\mathbf{u} = -\begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (4)$$

$$f = 0 \quad (5)$$

1) From (3), since  $\mathbf{V}$  is already diagonalized, the Eigen values  $\lambda_1$  and  $\lambda_2$  are given as

$$\lambda_1 = 0 \quad (6)$$

$$\lambda_2 = 1 \quad (7)$$

and the eigenvector matrix

$$\mathbf{P} = \mathbf{I}. \quad (8)$$

$$\therefore \mathbf{n} = \sqrt{\lambda_2} \mathbf{p}_1 \quad (9)$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (10)$$

Since

$$c = \frac{\|\mathbf{u}\|^2 - \lambda_2 f}{2\mathbf{u}^T \mathbf{n}}, \quad (11)$$

Substituting values of  $\mathbf{u}, \mathbf{n}, \lambda_2$  and  $f$  in (11),

The focus  $\mathbf{F}$  of parabola is expressed as

$$\mathbf{F} = \frac{ce^2 \mathbf{n} - \mathbf{u}}{\lambda_2} \quad (14)$$

$$= \frac{-\frac{5}{2}(1)^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix}}{1} \quad (15)$$

$$= \begin{pmatrix} \frac{5}{2} \\ 0 \end{pmatrix} \quad (16)$$

2) The directrix is given by

$$\mathbf{n}^T \mathbf{x} = c \quad (17)$$

$$\Rightarrow (1 \ 0) \mathbf{x} = -\frac{5}{2} \quad (18)$$

$$(19)$$

3) The equation for the axis of parabola passing through  $\mathbf{F}$  and orthogonal to the directrix is given as

$$\mathbf{m}^T (\mathbf{x} - \mathbf{F}) = 0 \quad (20)$$

where  $\mathbf{m}$  is the normal vector to the axis and also the slope of the directrix.

$$\therefore \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (21)$$

$$\Rightarrow (0 \ 1) \left( \mathbf{x} - \begin{pmatrix} \frac{5}{2} \\ 0 \end{pmatrix} \right) = 0 \quad (22)$$

$$\text{or, } (0 \ 1) \mathbf{x} = 0 \quad (23)$$

4) The latus rectum of a parabola is given by

$$l = \frac{\eta}{\lambda_2} = -\frac{2\mathbf{u}^\top \mathbf{p}_1}{\lambda_2} \quad (24)$$

$$= -\frac{2 \begin{pmatrix} -5 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{1} \quad (25)$$

$$= 10 \text{ units} \quad (26)$$