$\LaTeX$ 9.10.6.7

Lokesh Surana

CLASS 9, CHAPTER, 10, EXERCSE 6.7

Q. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is a rectangle.

Solution: Let, we have a unit circle with center at origin, i.e. $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, and radius $r = 1$. The points on circle that we consider are available in Table (I).

A	$\begin{pmatrix} \cos 0 \\ \sin 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
B	$\begin{pmatrix} \cos \frac{\pi}{2} \\ \sin \frac{\pi}{2} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
C	$\begin{pmatrix} \cos \pi \\ \sin \pi \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
D	$\begin{pmatrix} \cos \frac{3}{2}\pi \\ \sin \frac{3}{2}\pi \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

TABLE I

1) As it is given that chords AC and BD bisect each other, we have

$$\mathbf{A} + \mathbf{C} = \mathbf{B} + \mathbf{D} \tag{1}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

$\implies$ both chords AC and BD pass through the center of the circle. Hence, AC and BD are diameters of the circle.

2) Here the AC and BD i.e. the diagonals of quadrilateral bisect each other. Hence, the quadrilateral $ABCD$ is either a rectangle or a parallelogram. Let's check the angle between adjacent sides of this quadrilateral, i.e. AB and BC.

$$\mathbf{AB} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{5}$$

$$\mathbf{BC} = \mathbf{B} - \mathbf{C} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{7}$$

$$\tag{8}$$

Let θ be the angle between $\mathbf{AB}$ and $\mathbf{BC}$, then as per cosine rule

$$\cos\theta = \frac{\mathbf{AB} \cdot \mathbf{BC}}{\|\mathbf{AB}\|\|\mathbf{BC}\|} \tag{9}$$

$$= 0 \tag{10}$$

$$\implies \theta =^\circ 90 \tag{11}$$

Hence, the quadrilateral $ABCD$ is a rectangle.

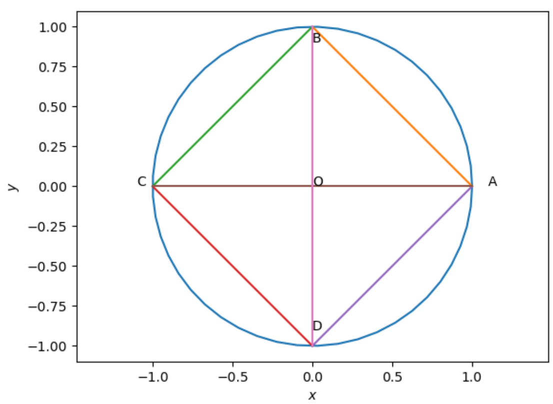

Fig. 1: circle