12.6.5.19

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Class 12, Chapter 6, Exercise 5.19

Q. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

Solution: The rectangle inscribed in circle is shown below in figure (1)

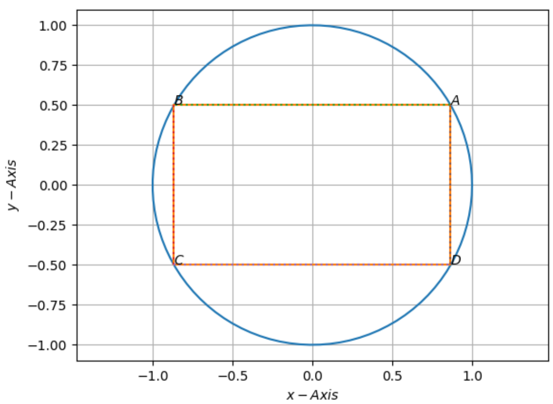

Fig. 1: Circle

Point	Coordinates
A	$\begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix}$
B	$\begin{pmatrix} -cos\theta \\ sin\theta \end{pmatrix}$
C	$\begin{pmatrix} -cos\theta \\ -sin\theta \end{pmatrix}$
D	$\begin{pmatrix} cos\theta \\ -sin\theta \end{pmatrix}$

TABLE I: points

For $0 < \theta < \frac{\pi}{2}$

The area of rectangle inscribed in circle is given by

$$\begin{aligned} \text{Area} &= \text{AB} \times \text{BC} && (1) \\ &= 2\cos\theta \times 2\sin\theta && (2) \\ &= 4\cos\theta\sqrt{1-\cos^2\theta} && (3) \\ & && (4) \end{aligned}$$

Let x = $\cos\theta$, area function is given by

$$\begin{aligned} A(x) &= 4x\sqrt{1-x^2} && (5) \\ & && (6) \end{aligned}$$

For $0 < \theta < \frac{\pi}{2}$, i.e. $0 < x < 1$

$$A(x) > 0 \tag{7}$$

So

$$\min_{\lambda} A^2(\lambda) = \min_{\lambda} A(\lambda) \tag{8}$$

$$\begin{aligned} A^2(x) &= 16x^2\left(1-x^2\right) && (9) \\ &= -16x^4 + 16x^2 && (10) \end{aligned}$$

Let $y = x^2$, as $0 < x < 1$, $\implies$ $0 < y < 1$

$$A^2(x) = -16y^2 + 16y \tag{11}$$

Using 2.3.1 from Optimization book, we get This is concave function, so using cvxpy, we get

$$\begin{aligned} x &= 1.4142 \approx \frac{1}{\sqrt{2}} && (12) \\ x &= \cos\frac{\pi}{4} && (13) \end{aligned}$$

So area of square is maximum in all rectangles inscribed in a circle.