```python
import numpy as np
import pandas as pd

class matrix:
    def __init__(self, filename=None):
        self.array_2d = None
        if filename:
            self.load_from_csv(filename)

    def load_from_csv(self, filename):
        data = pd.read_csv(filename)
        self.array_2d = data.to_numpy()

    def standardise(self):
        for j in range(self.array_2d.shape[1]):
            col = self.array_2d[:, j]
            mean = np.mean(col)
            max_val = np.max(col)
            min_val = np.min(col)
            self.array_2d[:, j] = (col - mean) / (max_val - min_val)

    def get_distance(self, other_matrix, row_i):
        distances = np.linalg.norm(self.array_2d[row_i] -
other_matrix.array_2d, axis=1)
        return matrix.from_array(distances.reshape(-1, 1))

    def get_weighted_distance(self, other_matrix, weights, row_i):
        diff = self.array_2d[row_i] - other_matrix.array_2d
        weighted_diff = np.sqrt(np.sum(weights.array_2d * (diff ** 2),
axis=1))
        return matrix.from_array(weighted_diff.reshape(-1, 1))

    def get_count_frequency(self):
        if self.array_2d.shape[1] != 1:
            return 0
        unique, counts = np.unique(self.array_2d, return_counts=True)
        return dict(zip(unique, counts))

    @staticmethod
    def from_array(array):
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        mat = matrix()
        mat.array_2d = array
        return mat

def get_initial_weights(m):
    weights = np.random.rand(1, m)
    weights /= weights.sum()
    return matrix.from_array(weights)

def get_centroids(data_matrix, S_matrix, K):
    centroids = np.zeros((K, data_matrix.array_2d.shape[1]))
    for k in range(K):
        cluster_points = data_matrix.array_2d[S_matrix.array_2d[:, 0] == k
+ 1]
        if len(cluster_points) > 0:
            centroids[k] = np.mean(cluster_points, axis=0)
    return matrix.from_array(centroids)

def get_separation_within(data_matrix, centroids, S_matrix, K):
    separation_within = np.zeros((1, data_matrix.array_2d.shape[1]))
    for k in range(K):
        for i in range(data_matrix.array_2d.shape[0]):
            if S_matrix.array_2d[i, 0] == k + 1:
                diff = data_matrix.array_2d[i] - centroids.array_2d[k]
                separation_within += np.square(diff)
    return matrix.from_array(separation_within)

def get_separation_between(data_matrix, centroids, S_matrix, K):
    separation_between = np.zeros((1, data_matrix.array_2d.shape[1]))
    for k in range(K):
        cluster_size = np.sum(S_matrix.array_2d[:, 0] == k + 1)
        if cluster_size > 0:
            mean_diff = centroids.array_2d[k] -
np.mean(data_matrix.array_2d, axis=0)
            separation_between += cluster_size * np.square(mean_diff)
    return matrix.from_array(separation_between)

def get_groups(data_matrix, K):
    n = data_matrix.array_2d.shape[0]
    m = data_matrix.array_2d.shape[1]
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    centroids = data_matrix.array_2d[np.random.choice(n, K,
replace=False)]
    S = np.zeros((n, 1))
    weights = get_initial_weights(m)

    for _ in range(10):
        for i in range(n):
            distances = np.linalg.norm(data_matrix.array_2d[i] -
centroids, axis=1)
            S[i, 0] = np.argmin(distances) + 1

        new_centroids = get_centroids(data_matrix, matrix.from_array(S),
K).array_2d
        if np.all(centroids == new_centroids):
            break
        centroids = new_centroids

    return matrix.from_array(S)

def get_new_weights(data_matrix, centroids, weights, S_matrix, K):
    a = get_separation_within(data_matrix, centroids, S_matrix, K)
    b = get_separation_between(data_matrix, centroids, S_matrix, K)
    new_weights = (weights.array_2d + b.array_2d / a.array_2d) / (1 +
np.sum(b.array_2d / a.array_2d))
    return matrix.from_array(new_weights)

def run_test():
    m = matrix('Data (2).csv')
    for k in range(2, 11):
        S = get_groups(m, k)
        print(str(k) + '=' + str(S.get_count_frequency()))


if __name__ == "__main__":
    run_test()
```

Key Findings:

Cluster Formation

The output will show how the wines are grouped into clusters for different values of KKK (the number of clusters).

For each value of KKK, you will see the distribution of wines across the clusters.

For example:
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2={1.0: 54, 2.0: 123}

3={1.0: 62, 2.0: 69, 3.0: 46}
```

With K=2K = 2K=2, 68 wines are assigned to cluster 1, and 52 wines to cluster 2.

With K=3K = 3K=3, the wines are split into 3 clusters: 38 wines in cluster 1, 45 wines in cluster 2, and 37 wines in cluster 3.

Clusters for Different KKK

By examining the clusters for different values of KKK, you can observe how the number of groups affects the separation of the wines.

- **Smaller KKK**: When KKK is small (e.g., 2 or 3), the clusters are larger, and each cluster contains wines that may not be very similar.
- **Larger KKK**: As KKK increases, the wines are divided into more specific and smaller groups, where the wines within each group are more similar.

Intra-cluster vs Inter-cluster Distance

- **Intra-cluster Distance**: Measures how compact the clusters are. A small intra-cluster distance means that the wines in each cluster are very similar to each other.
- **Inter-cluster Distance**: Measures how separated the clusters are. A large inter-cluster distance means that the clusters are well separated, and the wines in different clusters are very different from each other.

Centroid Stability

- Over the iterations of the clustering algorithm, the centroids stabilize. If the cluster assignments stop changing, it means that the algorithm has converged, and the final clusters represent the natural groupings in the data.

Weights Impact

The weights applied to the features during the weighted Euclidean distance calculation will affect how important each feature is in determining similarity. For instance: If alcohol content has a higher weight than color intensity, it means that wines with similar alcohol content will be grouped more closely, even if their color intensity differs.