Assignment-2

EE3900: Linear Systems and Signal Processing

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***Abstract*—This document contains solution to Assignment-2 [Question 2.4 from Discrete-Time Signal Processing by Alan V. Oppenheim and Ronald W. Schafer]**

1. Difference Equation

1.1 **[Question 2.4 from Discrete-Time Signal Processing by Alan V. Oppenheim] :** Consider the linear constant-coefficient difference equation:

$$y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=2x[n-1] \tag{1.1}$$

Determine $y[n]$ for $n \geq 0$ when $x(n)=\delta[n]$ and $y(n)=0, n<0$.

Solution: By applying Z-transform,

$$\mathcal{Z}\{y[n]\}-\frac{3}{4}\mathcal{Z}\{y[n-1]\}+\frac{1}{8}\mathcal{Z}\{y[n-2]\}=2\mathcal{Z}\{x[n-1]\} \tag{1.2}$$

As,

$$\mathcal{Z}\{x(n-k)\}=z^{-k}X(z) \tag{1.3}$$

$$Y(z)-\frac{3}{4}z^{-1}Y(z)+\frac{1}{8}z^{-2}Y(z)=2z^{-1}X(z) \tag{1.4}$$

As,

$$x(n)=\delta[n] \tag{1.5}$$

$$\mathcal{Z}\{x(n)\}=\mathcal{Z}\{\delta(n)\}=1 \tag{1.6}$$

$$Y(z)-\frac{3}{4}z^{-1}Y(z)+\frac{1}{8}z^{-2}Y(z)=2z^{-1} \tag{1.7}$$

$$Y(z)(1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2})=2z^{-1} \tag{1.8}$$

$$Y(z)=\frac{2z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} \tag{1.9}$$

$$=\frac{-8}{4z-1}+\frac{8}{2z-1} \tag{1.10}$$

$$=\frac{4z^{-1}}{1-\frac{1}{2}z^{-1}}-\frac{2z^{-1}}{1-\frac{1}{4}z^{-1}} \tag{1.11}$$

Case-1: When ROC is $|z|>\frac{1}{2}$, the signal x(n) is causal. Both terms in (1.11) are causal. As,

$$a^{n-1}u(n-1) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{z^{-1}}{1-az^{-1}} \quad |z|>|a| \tag{1.12}$$

$$y(n)=4\left(\frac{1}{2}\right)^{n-1}u(n-1)-2\left(\frac{1}{4}\right)^{n-1}u(n-1) \tag{1.13}$$

$$y(n)=\left(\frac{1}{2}\right)^{n-3}u(n-1)-\left(\frac{1}{4}\right)^{n-\frac{3}{2}}u(n-1) \tag{1.14}$$

Case-2: When ROC is $\frac{1}{4}<|z|<\frac{1}{2}$, ROC is a ring. The signal is two-sided. Thus the pole $p_1=\frac{1}{2}$ provides anti-causal part and $p_2=\frac{1}{4}$ provides causal part.

$$-a^{n-1}u(-n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{z^{-1}}{1-az^{-1}} \quad |z|<|a| \tag{1.15}$$

$$y(n)=-4\left(\frac{1}{2}\right)^{n-1}u(-n)-2\left(\frac{1}{4}\right)^{n-1}u(n-1) \tag{1.16}$$

$$=\left(\frac{1}{2}\right)^{n-3}u(-n)-\left(\frac{1}{4}\right)^{n-\frac{3}{2}}u(n-1) \tag{1.17}$$

Case-3: When ROC is $|z|<\frac{1}{4}$, the signal x(n) is anti-causal. Both terms in (1.11) are anti-causal.
Using (1.12),

$$y(n)=-4\left(\frac{1}{2}\right)^{n-1}u(-n)+2\left(\frac{1}{4}\right)^{n-1}u(-n) \tag{1.18}$$

$$y(n)=\left(\frac{1}{2}\right)^{n-3}u(-n)+\left(\frac{1}{4}\right)^{n-\frac{3}{2}}u(-n) \tag{1.19}$$