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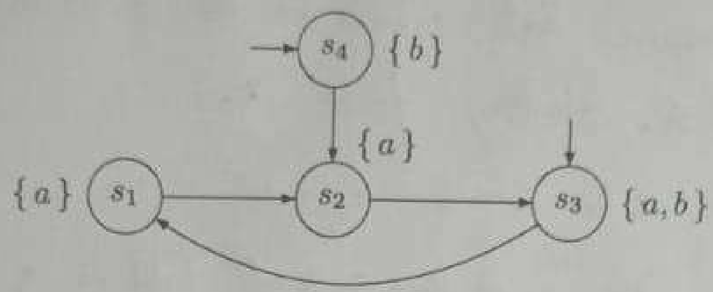
Indian Institute of Technology Kanpur
CS637 Embedded and Cyber-Physical Systems
Homework Assignment 3
Deadline: November 8, 2024

Total: **40 marks**

Problem 1. (10 points)
Consider the following state machine over the set of atomic propositions $\{a, b\}$:



Decide for each of the following LTL specifications whether the model satisfies it. For the positive outcome, provide a proof. For the negative outcome, provide a counterexample trace.
Note that the symbols $\bigcirc$, $\Box$, $\Diamond$, and $\mathtt{U}$ represent the "next", "always", "eventually", and "until" temporal operators respectively.

(a) $\bigcirc\bigcirc\bigcirc a$
(b) $\Box b$
(c) $\Box\Diamond a$
(d) $\Box(b \,\mathtt{U}\, a)$
(e) $\Diamond(a \,\mathtt{U}\, b)$

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Ans 1 a) ○ ○ ○ a

This formula requires that a holds in the third state following the current state. Starting from s_1;

→ The path from s_1 is $s_1 \to s_2 \to s_3 \to s_1$.

→ Following this path, a ~~do~~ does indeed hold in s_1 (~~the~~ the third state after the starting state)

Therefore, the model satisfies ○○○a

b) □b

The specification requires that b holds in all states in every possible path.

→ In the model, state s_2 does not satisfy b (only a holds in s_2)

→ This means that there is at least one state in the model where b does not hold.

Thus, the model ~~does~~ does not ~~at~~ satisfy □b. ~~Ano~~

→ A counterexample is the path $s_1 \to s_2$, where b is not true in s_2

c) □ ~~◇~~ ◇a

This formula means that there is some point in the future where a will hold in all subsequent states.

→ However, there is a cycle $s_1 \to s_2 \to s_3 \to s_1$ that does not guarantee a holds always. (for instance, s_3 has both a and b)

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→ Thus, the model does not satisfy $\square\Diamond a$.

→ A counterexample is the path $s_1 \to s_2 \to s_3 \to s_1$, where b appears periodically.

d) $\square(b \cup a)$

This specification means that in all paths, b must hold until a eventually holds.

→ In the path $s_4 \to s_2$, b holds in s_4, and a holds in ~~s~~ s_2, which satisfies $b \cup a$.

→ In the ~~cy~~ cycle $s_1 \to s_2 \to s_3 \to s_1$, a is present in each state, which also satisfies $b \cup a$.

Therefore, the model satisfies $\square(b \cup a)$

e) $\Diamond(a \cup b)$

This formula specifies that there exists a path where a holds ~~unt~~ until b eventually holds.

→ In the path $s_1 \to s_3$, a holds in s_1, and b holds in s_3.

→ This satisfies the condition that a holds until b is encountered.

Thus, the model satisfies $\Diamond(a \cup b)$

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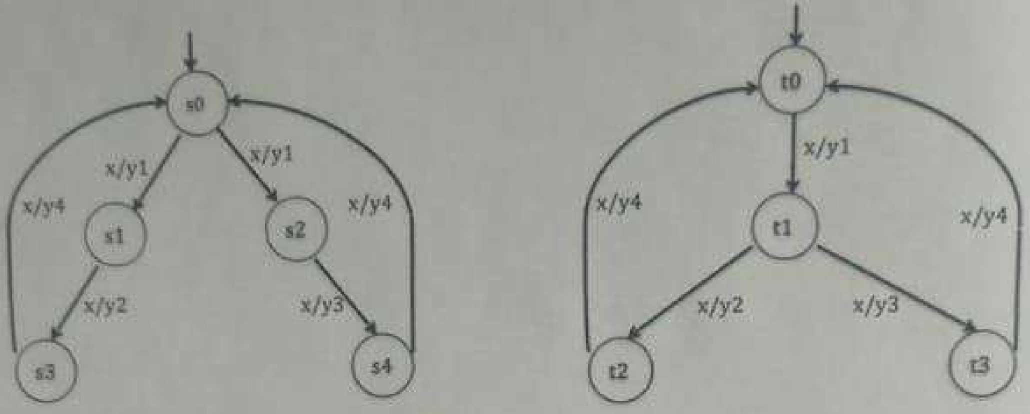
Problem 2. (10 points)
Consider the two state machines in Figure 1 and answer the following questions:



(a) M_1 (b) M_2

Figure 1

(a) Does the state machine M_1 simulate the state machine M_2? If yes, provide the simulation relation. If no, provide a transition of M_2 that M_1 cannot match.

(b) Does the state machine M_2 simulate the state machine M_1? If yes, provide the simulation relation. If no, provide a transition of M_1 that M_2 cannot match.

(c) Are the two state machines bisimilar? If yes, provide the bisimulation relation. If no, provide one reason.

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a) To verify if M_1 simulates M_2, we need to check if there ~~is~~ exists a simulation relation $R \subseteq M_1 \times M_2$ such that for every pair of states $(s, t) \in R$, every transition from t in M_2 can be matched by a corresponding transition from s in M_1.

- From t_0 in M_2, we have three transitions: $(t_0, x/y_1, t_1)$, $(t_0, x/y_4, t_2)$, and $(t_0, x/y_4, t_3)$
- From s_0 in M_1, there are also three transitions: $(s_0, x/y_1, s_1)$, $(s_0, x/y_4, s_3)$ and $(s_0, x/y_4, s_4)$.

By mapping s_1 to t_1, s_3 to t_2, and s_4 to t_3, we can ~~create~~ create a relation $R = \{(s_0, t_0), (s_1, t_1), (s_3, t_2), (s_4, t_3)\}$ such that every transition from a state in M_2 can be matched by a transition from the corresponding state in M_1.

Therefore, M_1 simulates M_2.

b) To verify if M_2 simulates M_1, we again need to check if there exists a simulation relation $R' \subseteq M_2 \times M_1$ such that every transition from a state in M_1 can be matched by a corresponding transition in M_2.

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Let's examine the states:

- From s_0 in M_1, there is a transition $(s_0, x/y2, s_3)$
- In M_2, t_0 does not have a corresponding $x/y2$ transition

Since M_2 cannot match the transition $(s_0, x/y2, s_3)$ from M_1, there is no simulation relation from M_2 to M_1. Therefore, M_2 does not simulate M_1.

(c) Bisimilarity requires both M_1 simulates M_2 and M_2 simulates M_1. As we know for (a & b),

M_1 simulates M_2 but M_2 does not simulate M_1,

Since mutual simulation does not hold, M_1 and M_2 are not bisimilar

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Problem 3. (10 points)
Consider the processes P_1 and P_2 with the shared variables b_1, b_2, and x. Variables b_1 and b_2 are Boolean variables, while variable x can take either the value 1 or 2. Initially, each process P_i is in the non-critical section (i.e., P_i is in location $noncrit_i$). The scheduling strategy for giving the processes access to the critical section is realized using x as follows. If both processes want to enter the critical section (i.e., P_i is in location $wait_i$), the value of variable x decides which of the two processes may enter its critical section: if $x = i$, then P_i may enter its critical section $crit_i$ (for $i = 1, 2$). On entering location $wait_1$, process P_1 performs $x := 2$, thus giving privilege to process P_2 to enter the critical section. The value of x thus indicates which process has its turn to enter the critical section. Symmetrically, P_2 sets x to 1 when starting to wait. The variables b_i provide information about the current location of P_i. More precisely, b_i is set when P_i starts to wait, and is reset when the process exits the critical section. In pseudocode, P_1 performs as follows (the code for process P_2 is similar):

P_1 **loop** forever

⋮ (*noncritical actions*)

$b_1 := true; x := 2$

wait until $(x = 1 \vee \neg b_2)$ (*request*)

do critical section **od**

$b_1 := false$ (*release*)

⋮ (*noncritical actions*)

end loop

(a) Draw the state machines for P_1 and P_2.

(b) Show the state machine that is obtained by asynchronous composition of P_1 and P_2.

(c) How many total states are there in the composed state machine? How many of them are reachable?

(d) Provide an LTL formula that captures the requirement that the process P_1 and P_2 will not enter the critical section simultaneously. Using the composed state machine, determine whether the two systems satisfy the formula (property).

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Ans The pseudocode for P_1 and P_2 indicates that each process has five main states:

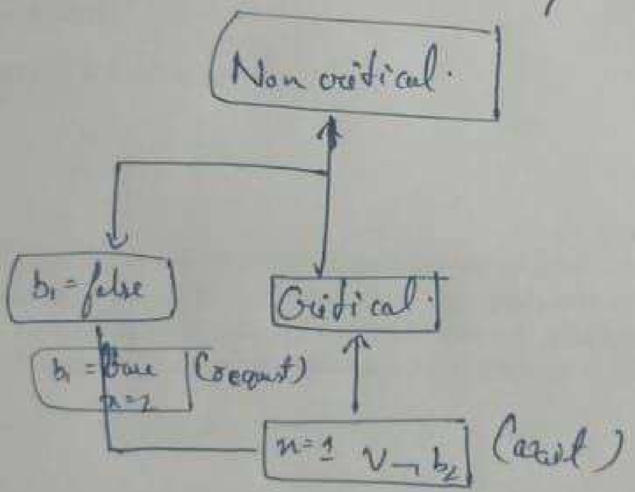
N → Noncritical Actions R → Request (setting b_i := true and $x = i$)

W → Wait (waiting for the condition to enter the critical section)

C → Critical section (executing critical section code)

L → Release (resetting b_i := false upon exiting the critical section)

The state machines for P_1 and P_2 are as follows.



b) To compose the state machines asynchronously we consider the Cartesian product of the states of P_1 and P_2. Each combined state represents a pair of states, one from P_1's state machine and one from P_2's state machine.

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(N, N), (N, R), (N, W), (N, C), (N, L),
(R, N), (R, R), (R, W), (R, C), (R, L),
(W, N), (W, R), (W, W), (W, C), (W, L),
(C, N), (C, R), (C, W), (C, C), (C, L),
(L, N), (L, R), (L, W), (L, C), (L, L)

Thus, there are a total of $5 \times 5 = 25$ states.

(c) Total states in the Composed State machine and Reachability
We need to analyse which state pairs can be reached based on the ~~rules~~ rules set by x, b_1, b_2. For example:- (C, C) is unreachable because it violates mutual exclusion (both processes cannot be in the critical section at the same time).
- Other unreachable states might include cases where both are waiting indefinitely due to incompatible conditions in x and b_i.

d) To specify that P_1 and P_2 cannot both be in the critical section simultaneously, we use the following Linear Temporal Logic (LTL) formula:-

$$\Box \neg (C_1 \wedge C_2)$$

where :- C_1 denotes P_1 in the critical section. - C_2 denotes P_2 in the critical section.
This formula asserts that at no point in time ($\Box$, meaning "always") will both P_1 and P_2 be in critical section simultaneously

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Problem 4. (10 points)
Consider the following program:

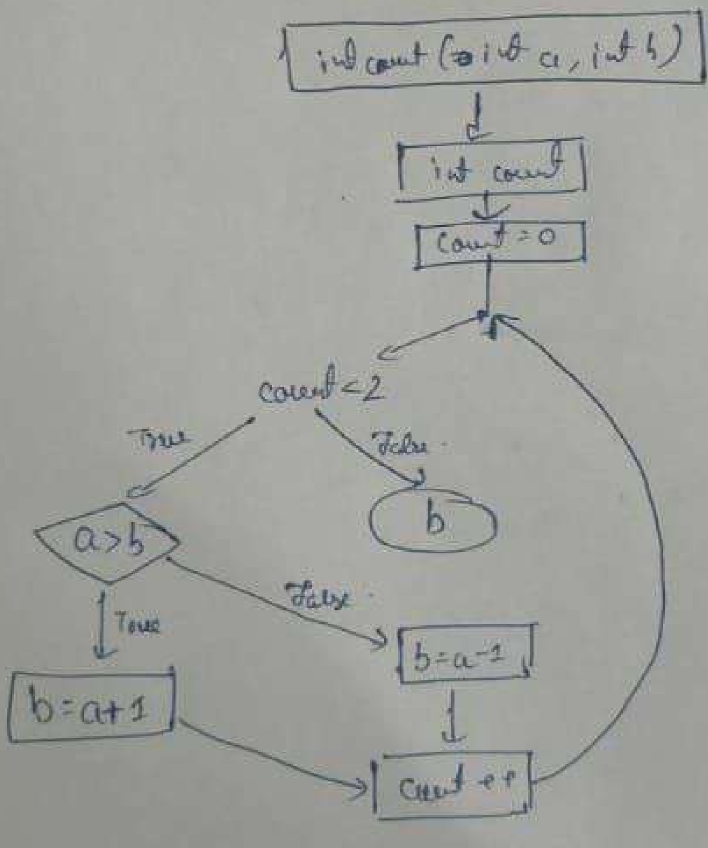
```
int count (int a, int b)
{
    int count;
    for (count = 0; count < 2; count++)
    {
        if (a > b)
            b = a + 1;
        else
            b = a - 1;
    }
    return b;
}
```

(a) Draw a control flow graph for the program.
(b) How many paths are there in the program? How many paths are feasible?
(c) Assume the following:

- An assignment statement (for example, count = 0) requires 2 unit time for execution.
- A statement involving an arithmetic operation followed by an assignment (for example, count ++ b = a + 1) requires 6 unit time for execution.
- A comparison statement (for example, count < 2) requires 4 unit time for execution.

Compute a tight bound on the worst-case execution time for the program.

Ans a)



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b) Total no. of paths = 4

i) ~~(a>b)~~ Path 1 : (a > b) is true in both iterations.

ii) Path 2 :- (a > b) is true in the first iteration and false in the second.

iii) Path 3 :- (a > b) is false in ~~the~~ the first iteration and true in the second.

iv) Path 4 : (a > b) is false in both iterations.

→ Since b is modified in each iteration, the outcome of a > b can change between iterations.

→ All four paths are feasible depending on the initial values of a and b.

c) Worst Case Execution Time (WCET) Calculation :-

Given Execution times :-

- Initialization
  - count = 0 ; → 2 units.
- First Loop condition Check :
  - count < 2 ; → 4 units
- First iteration
  - count ++ ; → 6 units
  - a > b ; → 4 units
  - b = a ± 1 ; → 6 units
  - Total for first iteration : 6 + 4 + 6 = 16 units

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- Second Loop Condition Check:
  - count < 2; → 4 units
- Second Iteration:
  - count ++; → 6 units
  - a > b; → 4 units
  - b = a + 1; → 6 units
  - Total for second iteration: 6 + ~~4~~ + 6 = 16 units
- Third Loop Condition Check:
  - count < 2; → 4 units (condition fails, loop ~~exit~~ exits)
- Return Statement:
  - return b; → Execution time negligible or not specified.

Total WCET Calculation:-

Total time = Initialization + First Loop Check + First Iteration + Second Loop Check + Second Iteration + Third Loop Check

= 2 + 4 + 16 + 4 + 16 + 4

= 46 units