Assignment 3

Ans: $V_{DD} = 5V$, $\left(\frac{W}{L}\right)_p = \left(\frac{W}{L}\right)_n$ $\quad k_n' = 81\mu A\, V^{-2}$

$k_p' = 27\mu A\, V^{-2}$

$V_{tn} = |V_{tp}| = 0.75\,V$

Assuming saturation mode for both transistors to calculate V_{in}

$$\frac{K_n}{2}(V_{GSn} - V_{tn})^2 = \frac{K_p}{2}(|V_{GSp}| - |V_{tp}|)^2$$

$$\frac{1}{\left(\frac{K_p}{K_n}\right)} = \frac{(|V_{GSp}| - |V_{tp}|)^2}{(V_{GSn} - V_{tn})^2}$$

Let $r = \frac{1}{\sqrt{\frac{K_p}{K_n}}} \Rightarrow r = \frac{|V_{GSp}| - |V_{tp}|}{V_{GSn} - V_{tn}}$

$$rV_{GSn} - rV_{tn} = V_{DD} - V_{GSn} - |V_{tp}|$$

$$(1+r)(V_{GSn}) = V_{DD} + rV_{Tn} - |V_{tp}|$$

$$V_{GSn} = \frac{V_{DD} + rV_{tn} - |V_{Tp}|}{1+r}$$

$$V_{GSn} = \frac{V_{DD} + \sqrt{\frac{K_n}{K_p}}\,V_{tn} - |V_{Tp}|}{1 + \sqrt{\frac{K_n}{K_p}}}$$

or $$V_{GSn} = \frac{V_{DD}\left(\sqrt{\frac{K_p}{K_n}}\right) + V_{tn} - \left(\sqrt{\frac{K_p}{K_n}}\right)|V_{tp}|}{1 + \sqrt{\frac{K_p}{K_n}}}$$

Verification

for NMOS, $V_{in} = V_{out} = V_M \Rightarrow V_{GS} = V_M = V_{DS}$

$\therefore (V_{DS} > V_{GS} - V_{Tn})$

↳ saturation

for PMOS; $V_{out} = V_M = V_{in} \Rightarrow V_{SG} = V_{DD} - V_M$

$V_{SD} = V_{DD} - V_M$

$\Rightarrow V_{SD} > |V_{GS}| - |V_{TP}|$

- sat^n.

Hence, we can ~~satisfy~~ safely take transistors to be in sat.

$$V_M = V_{GSn} = \frac{V_{DD}\sqrt{\frac{K_p}{K_n}} + V_{Tn} - \sqrt{\frac{K_p}{K_n}}\,|V_{Tp}|}{1 + \sqrt{\frac{K_p}{K_n}}}$$

$$V_M = \frac{5\sqrt{\frac{27}{81}} + 0.75 - 0.75\sqrt{\frac{27}{81}}}{1 + \sqrt{\frac{27}{81}}}$$

$$= \frac{\frac{5}{\sqrt{3}} + 0.75 - \frac{0.75}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$= 2.031V$

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Ans2 1) t_{pLH} , At $t=0$, $V_{out}=0$ $V_{in}=0$

$\Rightarrow$ NMOS is in cutoff, PMOS is in sat $\Rightarrow V_{SG}=V_{DD}$

$(\because V_{SD}=V_{DD}>V_{SG}-|V_{Tp}|)$

Calculating $\Rightarrow$

$$I_{D_p}=\frac{K_p}{2}(|V_{GS}|-|V_{Tp}|)^2=C_{ox}\mu_p\left(\frac{W}{L}\right)_p(5-1)^2$$

$$=\frac{\cancel{4}2\times10^{-5}}{\cancel{2}}(16)=32\times10^{-5}A$$

$$R_{V_{out}=0}=\frac{V_{DD}-V_{out}}{I_{Dp}}$$

$$=\frac{5-0}{32\times10^{-5}}=\frac{10^5\times5}{32}=1.5625\times10^4\,\Omega$$

Calculating R @ $V_o=\frac{V_{DD}}{2}$ $\left\{\text{Intermediate value of } V_o=\frac{V_{OH}+V_{OL}}{2}=\frac{V_{DD}+0}{2}=\frac{V_{DD}}{2}\right\}$

$V_{in}=0$

$\Rightarrow$ NMOS is in cutoff.

$V_{SG}=V_{DD}-V_{in}=V_{DD}=5V$

$\Rightarrow$ PMOS in linear mode $(5-|V_{GS}|-|V_{Tp}|=4)$

$$I_{Dp}=K_p\left((V_{SG}-|V_{Tp}|)V_{SD_p}-\frac{(V_{SD_p})^2}{2}\right)$$

$$=4\times10^{-5}\left(4\times\frac{5}{2}-\frac{(5/2)^2}{2}\right)$$

$$=4\times10^{-5}\left(10-\frac{25}{8}\right)$$

$$=27.5\times10^{-5}A$$

$R_{V_{out} = \frac{V_{DD}}{2}} = \frac{V_{DD} - V_{out}}{I_{DP}} = \frac{2.5}{27.5 \times 10^{-5}}$

$= 0.0909 \times 10^{5}\ \Omega \approx 9091\ \Omega$

$R_{eq} = (15625 + 9091)/2 = 12358\ \Omega$

$\Rightarrow t_{PLH} = (0.69) \times R_{eq} \times C_L$

$= 0.69 \times 12358 \times 10^{-15}$

$= 8.527 \times 10^{-10}$ sec

$t_{PHL} \Rightarrow V_{in} = V_{DD}$

$V_{SG} = V_{DD} - V_{in} = 0$

$\Rightarrow V_{SG} < |V_{Tp}| \Rightarrow$ Pmos cutoff.

R at $V_o = V_{DD} \Rightarrow$

$V_{GS} = V_{DD}$

$V_{DS} = V_{DD}$

$V_{DS} > V_{GS} - V_{Tn}$

$\Rightarrow$ Nmos in sat^n mode

$I_{Dn} = \frac{k_n}{2}(V_{DD} - V_{Tn})^2 = 2 \times 10^{-5} \times (5-1)^2$

$= \frac{64}{2} \times 10^{-5}\ A = 32 \times 10^{-5}\ A$

$R(V_o = V_{DD}) \Rightarrow \frac{V_{out}}{I_{Dn}} = \frac{5}{32 \times 10^{-5}} = 15625\ \Omega$

$R_{(V_O = \frac{V_{DD}}{2})}$ $\Rightarrow$ $V_{in} = V_{DD}$
$\Rightarrow V_{SG} = 0 < |V_{Tp}| \rightarrow$ PMOS in cutoff mode

$V_{GS} = V_{DD} = 5V$, $V_{DS} = \frac{V_{DD}}{2} = 2.5V \Rightarrow V_{GS} - V_{Tn} = 4 > 2.5$

$\Rightarrow$ NMOS is in linear mode

$$I_{Dn} = K_n \left((V_{GS} - V_{Tn}) V_{DS} - \frac{V_{DS}^2}{2} \right)$$

$$= 4 \times 10^{-5} \left(4 \times 2.5 - \frac{2.5^2}{2} \right)$$

$$= 27.5 \times 10^{-5} A$$

$$R_{(V_O = \frac{V_{DD}}{2})} = \frac{V_{DD}/2}{I_{Dn}} = \frac{2.5}{27.5 \times 10^{-5}}$$

$$= 0.0909 \times 10^5 \Omega = 9091 \Omega.$$

$$R_{eq} \simeq \left(\frac{15625 + 9091}{2} \right) \simeq 12358 \Omega$$

$$t_{PHL} = 0.69 \times R_{eq} \times C_L$$

$$= 8.527 \times 10^{-10} \text{ sec}$$

$$t_p = \frac{t_{PHL} + t_{PLH}}{2}$$

$$= 8.527 \times 10^{-10} \text{ sec}$$

ii) t_{PHL} sol^n.

V_{out} goes from high to low
V_{in} goes from low to high

$$\Rightarrow V_{in}(t = 0^+) = V_{DD}$$

$$V_{GS} - V_{Tn} = 4\ V.$$

So NMOS is in the sat^n mode $(V_{DS} > V_{GS} - V_{Tn})$

For V_{out} from 5 to 4 V and PMOS in cutoff $(V_{SG} < V_{SG} - V_{Tp})$

$$I_{Dn} = \frac{k_n}{2}(V_{GS} - V_{Tn})^2$$

$$= 3.2 \times 10^{-4}\ A$$

time from 5 to 4 V $\Rightarrow \frac{C\Delta V}{\Delta t} = I_{Dn} \Rightarrow \Delta t = 3.12 \times 10^{-7}$ s

for V_{out} from 4 to 0 V.

At $V_{out} = \frac{V_{DD}}{2}$ $\Rightarrow$ NMOS in linear mode

$$(V_{DS} < V_{GS} - V_{Tn})$$
$$2.5_V < 4\ V$$

$$I_{Dn} @ \frac{V_{DD}}{2} = k_n\left((V_{GS} - V_{Tn})V_{DS} - \frac{V_{DS}^2}{2}\right)$$

$$= k_n\left(4 \times 2.5 - \frac{2.5^2}{2}\right) = 2.75 \times 10^{-4}\ A$$

$$I_{av} = 2.975 \times 10^{-4}\ A = \left(\frac{3.2 + 2.75}{2}\right) \times 10^{-4}$$

time taken from 4 V to 2.5 V.

$$\Rightarrow \int C\,dV = \int dt\, I_{av} = \frac{C(4-2.5)}{2.975 \times 10^{-4}} = \Delta t$$

$\Delta t \approx 5.04 \times 10^{-16}$ sec

$\Rightarrow t_{pHL} = 8.165 \times 10^{-10}$ sec

t_{pLH} $sol^n \Rightarrow$ $V_{in}(0^+) = 0V \Rightarrow$ Nmos in cutoff
($V_{GS} < V_{Tn}$)

V_{out} from 0V to 1V gives
$\Rightarrow$ pmos in sat^n.

$V_{SD} > V_{SG} - |V_{Tp}| = 4V$

$I_{Dp} = K_p (V_{SG} - |V_{Tp}|)^2 = 3.2 \times 10^{-4} A$

Δt from 0 to 1V $\Rightarrow \frac{C\Delta V}{\Delta t} = I_{Dp} \Rightarrow \Delta t = 3.125 \times 10^{-10}$ sec

from 1V to $\frac{V_{DD}}{2} \Rightarrow$ pmos in linear region
($V_{SD} < 4V = V_{SG} - V_{Tp}$)

$\Rightarrow I_{Dp} = K_p \left((V_{SG} - |V_{Tp}|) V_{SDp} - \frac{V_{SDp}^2}{2} \right)$

$I_{Dp} = 2.75 \times 10^{-4} A$

Δt from 1V to 2.5V $\Rightarrow \frac{C\Delta V}{\Delta t} = I_{av}$

$\frac{C\Delta V}{\Delta t} = \frac{2.75 + 3.2}{2 \times 10^{-4}}$

$\Rightarrow \Delta t = 5.04 \times 10^{-40}$ sec

$\Rightarrow t_{pLH} \approx 8.165 \times 10^{-10}$ sec

$\Rightarrow t_p = \frac{t_{pLH} - t_{pHL}}{2} \approx 8.165 \times 10^{-10}$ sec

$\Rightarrow \boxed{t_p = t_{pLH} = t_{pHL}}$

t_{pLH} = time for V_{out} goes from low (0 V) to $\left(\frac{high+low}{2}\right)$ i.e. $\frac{V_{DD}}{2}$

At $t = 0^+$ $V_{in} = 0V$

$V_{out} = 0V$

$V_{SG} = V_{DD}$, $V_{SD} = V_{DD}$

$V_{SD} > V_{SG} - |V_{Tp}|$

$\Rightarrow$ sat^n mode for pMOS.

Magnitude of current I_D through pMOS

$= \frac{k_p}{2}(V_{SG} - |V_{Tp}|)^2$

$= 2 \times 10^{-5}(5-1)^2$

$= 0.32$ mA

At $V_{out} = 2.5V$
$V_{in} = 0V$ $\Rightarrow \begin{cases} V_{SG} = 5V \\ V_{SD} = 2.5V \end{cases}$

$V_{SG} - |V_{Tp}| > V_{SD}$

$\Rightarrow$ pMOS is linear

(Nmos in cutoff
$V_{GS} < V_{Tn}$)

Mag of I_D flowing $= k_p\left((V_{SG} - |V_{Tp}|)V_{SD} - \frac{V_{SD}^2}{2}\right)$

$= 10^{-5} \times 4\left(4 \times \frac{5}{2} - \frac{25}{8}\right)$

$= 2.75 \times 10^{-4} = 0.275$ mA

$I_{avg} = \frac{0.275 + 0.32}{2}$ mA

$= 0.2975$ mA

We know $C_L \frac{dV_o}{dt} = I_{av}$

$\Rightarrow C_L\left(\frac{V_{DD}}{2} - V_{OL}\right) = t_{PLH} I_{av}$

$\Rightarrow \frac{10^{-13}(2.5)}{0.2975\times10^{-3}} = t_{PLH}$

$t_{PLH} = 8.4034\times10^{-10}$ sec

for t_{PHL} $\Rightarrow$ At $t = 0$, $V_{in} = V_{DD} = 5V$

$V_{out} = 5V$

$\Rightarrow V_{SG} < |V_{Tp}|$ $\Rightarrow$ PMOS cutoff

for NMOS, $V_{GS} = V_{DD}$, $V_{DS} = V_{DD} \Rightarrow V_{DS} > V_{GS} - V_{Tn}$

$\Rightarrow$ NMOS sat.

$I_D = \frac{K_n}{2}(V_{GS} - V_{Tn})^2 = 0.32$ mA

At $V_{out} = \frac{V_{DD}}{2}$ $V_{in} = V_{DD} = 5V$
$V_{out} = 2.5V$

$\Rightarrow V_{SG} < |V_{Tp}| \Rightarrow$ PMOS in cutoff

for NMOS, $V_{GS} = V_{DD}$, $V_{DS} = \frac{V_{DD}}{2} = 2.5$ V.

$\Rightarrow$ NMOS in linear $(V_{GS} - V_{Tn} > V_{DS})^2$

$I_D = K_n\left((V_{GS} - V_{Tn})V_{DS} - \frac{V_{DS}^2}{2}\right)$

$= K_n\left(4\times2.5 - \frac{25}{8}\right) = 0.275$ mA

$I_{avg} = \frac{0.275 + 0.32}{2}$ mA $= 0.2975$ mA

$C_L \frac{dV_o}{dt} = I_{avg} \Rightarrow C_L\frac{\times 2.5}{I_{avg}} = t_{PHL} \Rightarrow t_{PHL} = 8.4034\times10^{-10}$ sec

$t_P = t_{PHL} + t_{PLH} = 8.4034\times10^{-10}$ sec

~~Value differ in (ii) and (iii) are due although value~~ is

	t_{PLH}	t_{PHL}	t_p
i)	8.527×10^{-10} sec	8.52×10^{-10} s	8.527×10^{-10} sec
ii)	8.165×10^{-10} s	8.165×10^{-10} sec	8.165×10^{-10} s.
iii)	8.4034×10^{-10} s.	807.3 ps	8.4034×10^{-10} sec

Ans 3 Assuming V_{in} as step input from high to low.

$\Rightarrow V_{in} = 0\,V$ $\qquad (5V \to 0V)$

We know, $t_{PLH} = (\ln 9)\, C_L R_{eq}$

$= 2.2\, C_L R_{eq}$

Here, $R_{eq} \approx \dfrac{R_{(10\%)} + R_{(90\%)}}{2}$

@ 10% $\Rightarrow V_{out} = 0.1 V_{DD} = 0.5\,V$

$V_{in} = 0 \Rightarrow$ nmos in cutoff

$\Rightarrow (\because V_{in} = V_{GS} = 0 < V_{Tn} = 1V$

$V_{SG} = V_{DD} - V_{in} = 5V$

$V_{SD} = V_{DD} - V_{out} = 4.5\,V$

$V_{SD} = 4.5 > V_{SG} - |V_{Tp}| = 4V$

$\Rightarrow$ pmos is in sat^n

$I_{Dp} = \dfrac{k_p}{2} (V_{SG} - |V_{Tp}|)^2 = 32 \times 10^{-5}$ A

$R_{(t=10\%)} = \frac{V_{DD} - V_{out}}{I_{Dp}} = \frac{4.5}{32 \times 10^{-5}} = 14062.5\ \Omega$

@ 90% $\Rightarrow V_{out} = 4.5\ V$, $V_{in} = 0$

NMOS in cutoff $(\because V_{GS} = 0 < V_{Tn} = 1V)$

for PMOS, $V_{SG} = V_{DD} - V_{in} = 5\ V$

$V_{SG} - |V_{Tp}| = 4V > V_{SD} = 0.5\ V$

PMOS in linear mode.

$$R(t = 90\%) = \frac{V_{DD} - V_{out}}{I_{Dp}} = \frac{0.5}{k_p\left((V_{SG} + V_{Tp})V_{SD} - \frac{V_{SD}^2}{2}\right)}$$

$$R(t=90\%) = \frac{0.5}{4 \times 10^{-5}\left((5-1)\frac{1}{2} - \frac{1}{8}\right)}$$

$= 6.6666\ k\Omega$

$$R_{eq} = \frac{14062.5 + 6.6666k}{2}$$

$\simeq$ ~~10364~~ $10364.583\ \Omega$

$t_{TLH} = 2.2 \times 0.1 \times 10^{-12} \times (10364.583)$

$= 2.28 \times 10^{-9}$ seconds

$= 2.28\ ns$

$t_{THL} \Rightarrow$ Assuming V_{in} as step i/p from low to high

$\Rightarrow V_{in} = 5V \quad (0V \to 5V)$

$t_{THL} = 2.2 C_L R_{eq}$

$R_{eq} = \frac{R_{(90\%)} + R_{(10\%)}}{2}$

@ 90% $V_{out} = 4.5\ V$
$V_{in} = 5V$

for PMOS $\Rightarrow V_{SG} = 0 < |V_{Tp}| \rightarrow$ Cutoff

for NMOS , $V_{GS} = 5V$

$V_{GT} = V_{GS} - V_{Tn} = 4V < V_{DS} = 4.5\ V$

$\rightarrow$ Sat^n mode

$$I_{Dn} = \frac{k_n}{2} V_{GT}^2 = \frac{20 \times 2}{2} \times 10^{-6} \times 16$$
$$= 32 \times 10^{-5}\ A$$

$$R_{90\%} = \frac{V_{out}}{I_{Dn}} = \frac{4.5}{32 \times 10^{-5}} = 14062.5\ \Omega$$

@ 10% $\Rightarrow V_{out} = 0.5V$
$V_{in} = 5V$

for PMOS $\Rightarrow V_{SG} = 0 < |V_{Tp}| \rightarrow$ Cutoff

NMOS $\rightarrow$ $V_{GS} = 5V$

$V_{GT} = 4V > 0.5V = V_{DS}$

$\rightarrow$ Linear mode.

$$I_{Dn} = k_n \left(V_{GT} V_{DS} - \frac{V_{DS}^2}{2} \right)$$
$$= k_n \left(4 \times 1/2 - 1/8 \right) = 7.5 \times 10^{-5}$$

$$R_{10\%} = \frac{V_{out}}{I_{Dn}} = \frac{0.5}{I_{Dn}} \simeq 6.6666\ k\Omega$$

$R_{eq} =$ ~~1020~~ $10364.583\ \Omega$

$t_{PHL} = 2.2 \times R_{eq} \times C_L = ?$

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$t_{THL} = t_{TLH}$ as expected from symmetry of given CMOS

Ans 4 $I_{sc\,max}$ occurs at $V_{in} = V_m$ (when both mosfets are in sat^n).

for NMOS, $V_{in} = V_{out} = V_m = V_{GS}$

$V_{DS} = V_m > V_{GS} - V_{Tn} \Rightarrow$ sat^n

for PMOS, $V_{in} = V_{out} = V_m$

$\Rightarrow V_{SG} = V_{DD} - V_m$

$V_{SD} = V_{DD} - V_m > V_{SG} - |V_{Tp}| \rightarrow$ saturation

$$\frac{K_n}{2}(V_{GS} - V_{Tn})^2 = \frac{K_p}{2}(V_{SG} - |V_{Tp}|)^2$$

$$(V_{in} - 1)^2 = (V_{DD} - V_m - 1)^2$$

$$(V_m - 1)^2 = (V_{DD} - V_m - 1)^2$$

$$\Rightarrow V_m = \frac{V_{DD}}{2} = 2.5\ V$$

$$I_{sc\,max} = \frac{K_n}{2}(V_{in} - 1)^2 = \frac{K_n}{2}(2.5 - 1)^2$$

$$= 4.5 \times 10^{-5}\ A.$$

(P_{sc}) Power dissipated $= \alpha_{0\to 1}\, f\, E_{sc}$

also $E_{sc} = V_{DD}\, Q_{V_{DD}\to gnd} = \frac{1}{2} V_{DD}\, I_{sc\,max}\left(\frac{t_r + t_f}{0.8}\right)\left(\frac{V_{DD} - V_{Tn} - |V_{Tp}|}{V_{DD}}\right)$

$$= \frac{1}{2} \times 4.5 \times 10^{-5}\,(t_r + t_f)(3)$$

$P_{dynamic} = \alpha_{0\to 1}\, f\, C_L V_{DD}^2 \quad | \quad P_{sc} = \alpha_{0\to 1}\, E_{sc}\, f$

Given condition: $P_{sc} < 0.1\, P_{dynamic}$

$\alpha_{0\to 1} f P_{SC} < \alpha_{0\to 1} \times f \times C_L \times V_{DD}^2 \times 0.1$

Taking $t_r + t_f = 2t_s$ gives $\Rightarrow$

$\frac{9.5}{2} \times 10^{-5} (2t_s) \times 3 < C_L V_{DD}^2 \times 0.1$

$t_s < 1.84\ ns$

Ans $V_{Tn} = |V_{Tp}| = 0.6\,V$

$K_n' = 400\,\mu A$, $K_p' = 200\,\mu A V^{-2}$

i) For $V_{out} = V_{OH} = V_{DD} = 2.5\,V$.

PMOS is in cutoff $(V_{GS} < |V_{Tp}|)$

$\Rightarrow I_{Dp} = 0.$

Static Power dissipation $\Rightarrow V_{DD} \times I_{DD} = 0$

In $V_{out} = V_{OL} = 0\,V$

NMOS is in cutoff $(V_{GS} < V_{Tn})$

$\Rightarrow I_{Dn} = 0$

$\Rightarrow V_{DD} \times I_{Dn} = 0$

$\Rightarrow$ Static Power Dissipation $= 0$

$\Rightarrow$ Avg Static Power dissipation $= 0$.

ii) for transition of Vout from low to high.

$\Rightarrow$ NMOS is in cutoff

$\Rightarrow$ PMOS is in operating regime

Avg Dyn Power dissipation $= \frac{\alpha_{0\to 1}}{T} \times C_L \times V_{supply} \times V_{swing}$

$= 0.02 \times 235 \times 10^6 \times (5\times 3 (\times f)) \times 2.2 \times 2.5$

$= 0.01 \times 235 \times 10^6 \times 10^{-15} \times 3 \times 6.25$

$= 44.06\,nW$

We already know that when Vout goes from high to low there is no current I_{Dp} ∵ PMOS is in cutoff

no dynamic power dissipation for this transition

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Q.17) $t_r = 1ns$ $t_f = 1.5ns$

$$P_{sc} = \alpha_{0\to1} \times f_c\, V_{DD} \times \frac{1}{2}\left(\frac{t_r + t_f}{0.8}\right) \times \left(\frac{V_{DD} - V_{Tn} - |V_{Tp}|}{V_{DD}}\right)$$

We know, I_{sc} max occurs when
$V_{in} = V_m$

$\Rightarrow$ both mosfets are in sat^n

$\because$ for PMOS $\Rightarrow$ $V_{SD} = V_{DD} - V_m > V_{SG} - |V_{Tp}|$
$\Rightarrow$ sat^n.

for NMOS $\Rightarrow$ $V_{DS} = V_m > V_{GS} - V_{Tn}$
$\hookrightarrow$ sat^n.

$$\frac{K_n}{2}(V_{GS} - V_{Tn})^2 = \frac{K_p}{2}(V_{SG} - |V_{Tp}|)^2$$

$$\frac{K_n'}{2}\left(\frac{W}{L}\right)_N (V_m - 0.6)^2 = \frac{K_p'}{2}\left(\frac{W}{L}\right)_P (2.5 - V_m - 0.6)^2$$

for NMOS we take min values of W&L

$$K_n'\left(\frac{W}{L}\right)_N = 400\frac{\mu A}{V^2} \times \left(\frac{W}{L}\right)_N$$

Also $\left(\frac{W}{L}\right)_P = \beta\left(\frac{W}{L}\right)_n = 2\left(\frac{W}{L}\right)_N$

$$K_p'\left(\frac{W}{L}\right)_P = 200\frac{\mu A}{V^2}\left(\frac{W}{L}\right)_P$$

$$= 400\frac{\mu A}{V^2}\left(\frac{W}{L}\right)_N$$

$\Rightarrow$ $K_p = K_N$

$$(V_m - 0.6)^2 = (V_m - 1.9)^2$$

$$\Rightarrow \boxed{V_m = 1.25\,V}$$

$$I_{sc\,max} = \frac{k_n'}{2} \times \left(\frac{W}{L}\right)_N \times (1.25 - 0.6)^2$$

$$= \frac{400 \times 10^{-6}}{2} (0.4225) \frac{W_{min}}{L_{min}}$$

$$= 8.45 \times 10^{-5} \times \frac{W_{min}}{L_{min}}$$

for $W_{min} = L_{min}$) $\Rightarrow I_{sc} = 8.45 \times 10^{-5}\,A$

$$P_{sc} = 0.01 \times 235 \times 10^{6} \times 2.5 \times \frac{1}{2}\left(\frac{1+1.5}{2.8}\right) \times \frac{1.3}{2.5} \times I_{sc\,max}$$

$$= 0.01 \times 2.5 \times 235 \times \frac{10^{-6}}{2}\left(\frac{2.5}{8}\right)\left(\frac{1.5}{2.5}\right) \times 8.45 \times 10^{-5} \times \frac{W_{min}}{L_{min}}$$

$$= 4.0325 \times 10^{-7} \frac{W_{min}}{L_{min}} \text{ watt}$$

for $W_{min} = L_{min}$. $\boxed{P_{sc} = 0.4\,\mu W}$.

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Ans $C_{gid} = \gamma(1+\beta)C_{go}$

$= 3C_{go} = 3\times10^{-15}$ F

$C_L = 1\,pF$

$f = e^{(1+\gamma/f)}$

$\ln f = 1 + \frac{\gamma}{f} = 1 + \frac{1}{f} \quad (\gamma \approx 1)$

Consider f^n $\quad L(f) = \ln f - 1 - \frac{1}{f}$

$L(f) = \ln f_0 - \frac{1}{f_0} - 1 = 0$

Take initial value of f_0 as 3.5

$\ln f_0 = 1 + \frac{1}{f_0}$

$\ln f_0 = 1 + \frac{1}{3.5} \approx 1.28571$

$\Rightarrow f_0 = 3.61725$

Now for II iteration $\Rightarrow \ln f_0 = 1 + \frac{1}{3.61725} \approx 1.27645$

$f_0 = 3.5839$

Now for III iteration $\Rightarrow \ln f_0 = 1 + \frac{1}{3.5839}$

$\Rightarrow f_0 = 3.5931$

IV iteration $\Rightarrow f_0 = 3.59057$

V iteration $\Rightarrow f_0 = 3.59127$

VI iteration $\Rightarrow f_0 = 3.591026$

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VI Iteration $\Rightarrow f_0 = 3.5911$

VII Iteration $\Rightarrow f_0 = 3.5911$

$f_0 = 3.5911$

$\Rightarrow f = 3.5911$.

$$N = \frac{\ln(F)}{\ln f} \Rightarrow N = \frac{\ln(100/3)}{\ln(f)} = \frac{\ln(1000/3)}{\ln(3.5911)}$$

$$\left[\text{where } F = \frac{C_1}{C_{g1}} = \frac{C_L}{C_g(1+\beta)\gamma}\right] \quad N = 4.54$$

$N = 4$ Ans

$$t_p = N t_{p0}\left(1 + \frac{f}{\gamma}\right) = 4 t_{p0}\left(1 + \frac{4.2728}{\gamma}\right) = 21.09 t_{p0}$$

(ii) $N = 5$, $F = \frac{C_L}{C_g} = \frac{10^{-12}}{10^{-15}\gamma(1+\beta)} = \frac{1000}{3}$

$$f = F^{1/N} = \left(\frac{1000}{3}\right)^{1/5} = 3.196$$

$$t_p = N t_{p0}\left(1 + \frac{f}{\gamma}\right)$$

$$= t_{p0}(5)\left(1 + \frac{3.196}{2}\right)$$

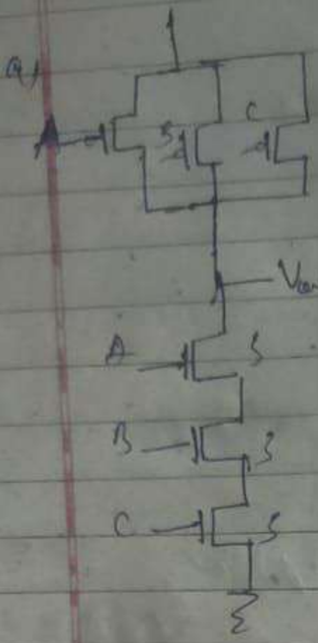
$$= \cancel{2}\ 20.98\ t_{p0}$$

Case (i) ~~but has~~ has a bit smaller delay than (ii)

Ans 7.) NAND (for 3 i/p) $o/p = \overline{(A \cdot B \cdot C)}$

$= \bar{A} + \bar{B} + \bar{C}$

a)



$size = 3(\beta A_0) + 3 \times 3 A_0$

$= 3\beta A_0 + 9 A_0$

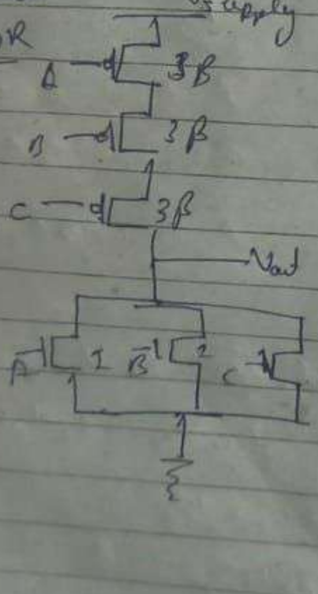
$= 9A_0 + 3\beta A_0$

$Or = 3(3+\beta) A_0$

$= 16.5\ A_0$

each PMOS is β scaled and each NMOS is 3 scaled

NOR



$o/p = \overline{A+B+C}$

$= \bar{A} \cdot \bar{B} \cdot \bar{C}$

each PMOS is 3β scaled

each NMOS is unity scaled

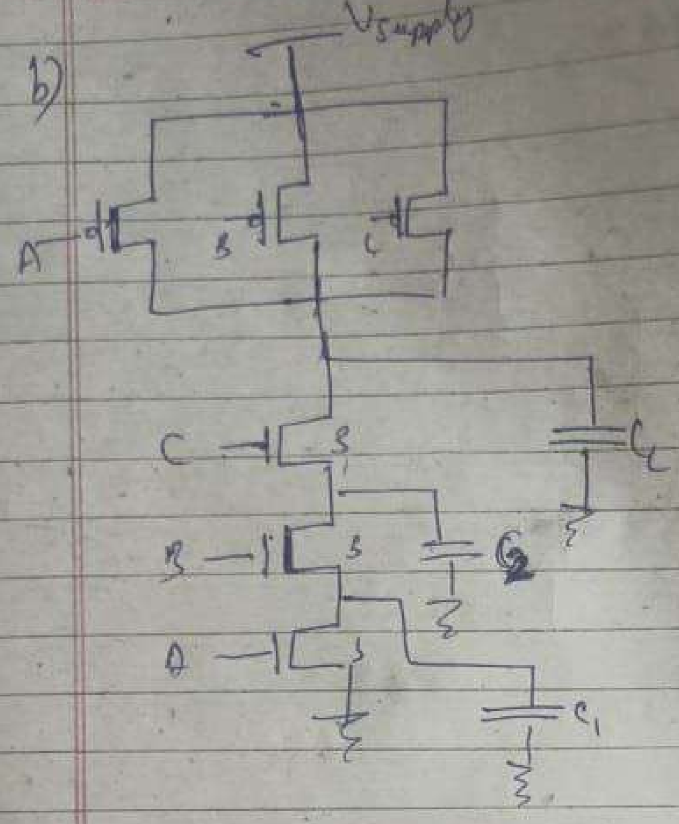
$size = 3 \times 3\beta A_0 + 3 \times A_0$

$= 9\beta A_0 + 3A_0$

$= 3A_0(3\beta + 1) = 25.5\ A$

NAND

b)



Note → To get C_L, C_2, C_1, we focus on neighbouring transistor

$\Rightarrow C_L = 3\gamma C_{g0} + 3\times\beta\gamma C_{g0}$

$= 3\gamma(1+\beta)C_{g0}$

$= 3\times1.05\,(3.5)\times10^{-15}$

$= 11.025\,fF$

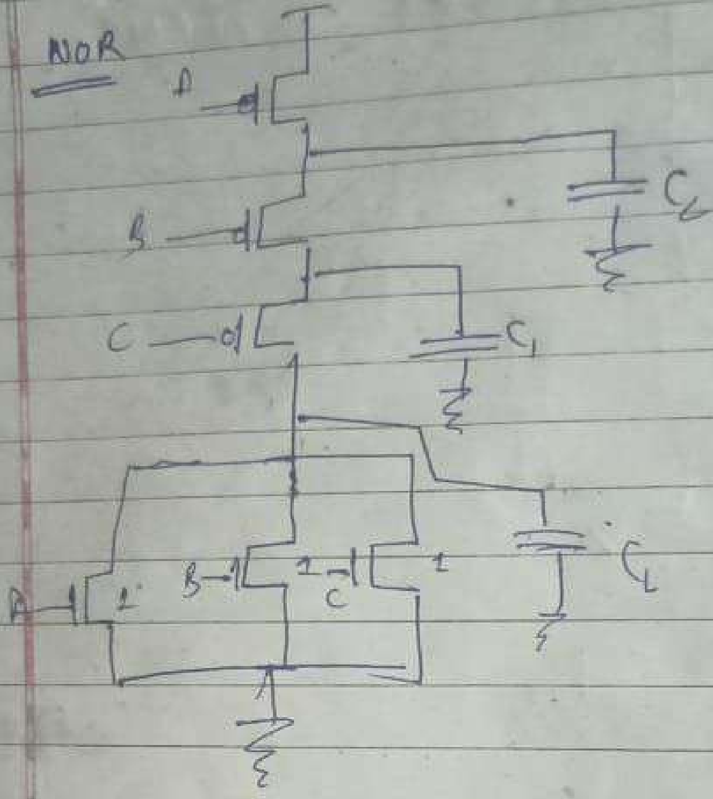
$C_1 = 3\gamma C_{g0} + 3\gamma C_{g0}$

$= 6\gamma C_{g0}$

$= 6.3\,fF$

$C_2 = 3\gamma C_{g0} + 3\gamma C_{g0} = 6\gamma C_{g0} = 6.3\,fF$

NOR



$C_L = 3 \times \gamma C_{g0} + 3\beta\gamma C_{g0}$

$= 3\gamma C_{g0}(1+\beta)$

$= 11.025\ fF$

$C_1 = 3\beta\gamma C_{g0} + 3\beta\gamma C_{g0}$

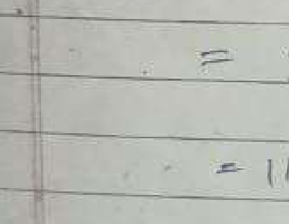
$= 6\beta\gamma C_{g0}$

$= 15.75\ fF$

$C_2 = 3\beta\gamma C_{g0} + 3\beta\gamma C_{g0} = 6\beta\gamma C_{g0}$

$= 15.75\ fF$

c)



Inverter

Size $= \beta A_0 + A_0$

$= (\beta+1)A_0 = 3.5A_0$

Worst case delay for inverter ⇒.

$t_{pLH} = (\ln 2)(R_{eq})(C_L) = 0.69\,\gamma(1+\beta)(C_{g0})\,R_{eq}$

$= 2.53575\, R_{eq}\, f\,sec$

$t_{pHL} = 0.69 \times R_{eq} C_L = 0.69 \times R_{eq}\,\gamma(1+\beta) C_{g0}$

$= 2.53575\, R_{eq}\, f\,sec$

Worst case power dissipation $= \alpha_{0\to1} \times f \times C_L V_{DD}^2$

$= \alpha_{0\to1} \times f \times (\gamma C_{g0} + \beta\gamma C_{g0}) \times V_{DD}^2$

For $P_A = \frac{1}{4} \Rightarrow P_{worst} = \frac{1}{4} \times f \times (10^{-15})(3.675)(V_{DD}^2)$

$= 8.26875 \times f \times 10^{-15}\, W$

$t_p = \frac{t_{pLH} + t_{pHL}}{2} = 2.53575\, R_{eq}\, f\,sec$

NAND $A_{seq} = 3A_0(\beta+3)$

Worst case delay,

Using Elmore delay,

$t_{pHL} = (0.69)\left(\frac{R_{eq}}{3} \times C_1 \times \left(\frac{R_{eq}}{3} + \frac{R_{eq}}{3}\right) C_2\right.$

$\left. + \left(\frac{R_{eq}}{3} + \frac{R_{eq}}{3} + \frac{R_{eq}}{3}\right) C_L\right)$

$= 11.954\, R_{eq}\, f\,sec$

$t_{pLH} = 0.69\, R_{eq} C_L$

$= 0.69\, R_{eq} \times 11.025\, f \times 10^{-15}$

$= 7.6072 \times R_{eq}$ fsec

$t_p = 9.78\, R_{eq}$ fsec

Worst case Power dissipation $\Rightarrow P_{stat} = 0$, so we need $P_{dynamic}$ for worst case dynamic power dissipation when $A = B = 1$ and C goes from 1 to 0.

A	B	C	F
1	1	1	0
1	1	0	1
1	0	1	1
0	1	1	1
0	0	1	1
1	0	0	1
0	1	0	1
0	0	0	1

$P_{worst} = P_{dyn\,worst}$

$= f\, P_A P_B P_C \left(P_A P_B (1-P_C)\right)$
$\times \left(C_L V_{DD}^2 + C_2 (V_{DD} - V_{th}) V_{DD} + C_1 (V_{DD} - V_{th}) V_{DD}\right)$

$= f\, P_A^2 P_B^2 P_C (1-P_C)$
$\times \left(99.225 + 2(1 - 3V_{th})\right)$

$P_{worst} = f P_A^2 P_B^2 P_C (1-P_C)(99.225 + 12.6(9 - 3V_{th}))$
(NAND)

To get worst value, assuming $P_A = P_B = 1$, $P_C = \frac{1}{2}$

$P_{worst} = f(53.15621 - 9.45 V_{th}) \times 10^{-1}$

NBR $Area = A_0 (3 + 9\beta)$

Worst case $t_{pLH} = 0.69 \left(\frac{R_{eq}}{3} \times C_2 + \left(\frac{R_{eq}}{3} + \frac{R_{eq}}{3}\right) C_1 + \left(\frac{R_{eq}}{3} + \frac{R_{eq}}{3} + \frac{R_{eq}}{3}\right) C_L\right)$

$= 18.475\, R_{eq}$ fsec

Worst case

$t_{pHL} \Rightarrow t_{pHL} = 0.69\, R_{eq} \times C_L$

$= 7.6072\ 9's\ R_{eq}\ t_{sec}$

$t_p = 13\ R_{eq}\ t_{sec}$

Worst case Power dissipation $\Rightarrow P_{stat} = 0$, so we need to get $P_{dynamic}$

Refer NOR diagram

$P_{worst} = P_{dynamic} \Rightarrow$ which occur when $A =$ goes to $A = 0$ and $B = C = 0$

$P_{worst} = f\, P_A P_B P_C\, ((1-P_A) P_B P_C)$
$\times (C_L V_{DD}^2 + C_1 V_{DD}(V_{DD} - |V_{Tp}|) + C_2 V_{DD}(V_{DD} - |V_{Tp}|))$

$= f\, P_C^2 P_B^2 P_A (1-P_A)$
$\times (99.225 + 94.5(3 - |V_{Tp}|)) \times 10^{-15}$ Watts

A	B	C	F
1	1	1	0
1	1	0	0
1	0	1	0
0	1	1	0
0	0	1	0
1	0	0	0
0	1	0	0
0	0	0	1

for $P_A = \frac{1}{2}$, $P_B = P_C = 1$

$\Rightarrow P_{worst} = f \times 10^{-15} (205.68125 - 23.625\, |V_{Tp}|)$ Watts

	NAND	NOR	Inverter
Area	$16.5\,A_0$	$23.5\,A_0$	$3.5\,A_0$

$t_p = 9.78\,R_{eq}$ / $t_p = 13\,R_{eq}$ fsec

Worst case delay $t_{pHL} = 11.95\,R_{eq}$ fsec $\quad t_{pHL} = 7.60725\,R_{eq}$

$t_{pLH} = 7.60725\,R_{eq}$ $\quad t_{pLH} = 18.44\,\beta R_{eq}$

$t_p = t_{pLH} = t_{pHL} = 2.53575\,R_{eq}$ fsec

~~Ex (5.3 1546 = 9.4 V...)~~

Largest area for NOR

smallest for Inverter

Power dissipation largest for NOR

and lowest for Inverter

assuming $|V_{Tp}| = V_{Tn} \sim 0.5V$

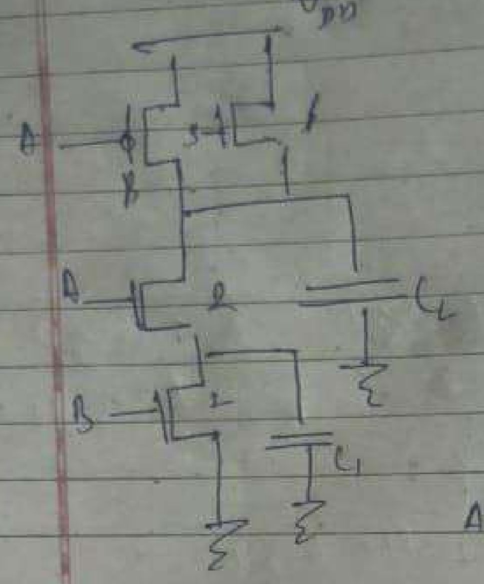
t_{pLH} & t_{pHL} delay min for inverter

t_{pLH} max for NOR

t_{pHL} max for NAND

t_p smallest for inverter.

Ans 8 NAND



$Size = 4A + 2\gamma A_0$

$C_L = (2\beta + 2)\gamma C_{g_0}$

$C_1 = (2+2)\gamma C_{g_0}$

$= 4\gamma C_{g_0}$

A	B	F	Probability
0	0	1	$\bar{P}_A \bar{P}_B$
0	1	1	$\bar{P}_A P_B$
1	0	1	$P_A \cdot \bar{P}_B$
1	1	1	$P_A \cdot P_B$

3 cases
$A = B = 1 \rightarrow A = B = 0$
$A = B = 1 \rightarrow A = 0,\ B = 1$
$A = B = 1 \rightarrow A = 1,\ B = 0$

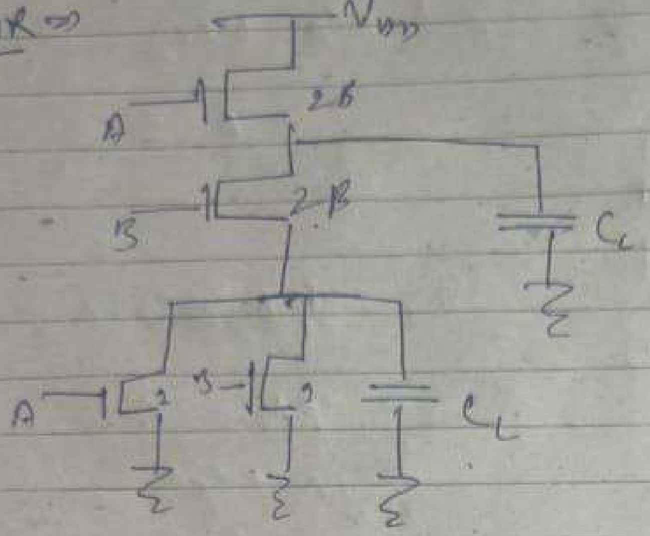
Total $P_{dynamic} = f \times (P_A P_B)\big(\bar{P}_A \bar{P}_B C_L V_{DD}^2$
$+ \bar{P}_A P_B C_L V_{DD}^2$
$+ P_A \bar{P}_B (C_L V_{DD}^2 + C_1 V_{DD} (V_{DD}$

$= f P_A P_B (\bar{P}_A \bar{P}_B (2\beta + 2)\gamma C_{g_0} V_{DD}^2$
$+ V_{DD}^2 \bar{P}_A P_B (2\beta + 2)\gamma C_{g_0}$
$+ P_A \bar{P}_B (2\beta + 2)\gamma C_{g_0} V_{DD}^2$
$+ P_A \bar{P}_B V_{DD}(V_{DD} - V_{Tn})(4\gamma C_{g_0})$

$P_{dyn} = f(1 - P_A P_B)(P_A P_B) V_{DD}^2 (2\gamma C_{g_0})(1+\beta)$
$+ f(P_A P_B)(P_A(1 - P_B))(V_{DD} - V_{Tn}) V_{DD}$

Date : Page :

NOR ⇒



$$C_L = 2\beta\gamma C_{g_0} + 2\gamma C_{g_0} = 2(1+\beta)\gamma C_{g_0}$$

$$C_1 = 2\beta \times \gamma C_{g_0} \times 2 = 4\beta\gamma C_{g_0}$$

A	B	R	Prob
0	0	1	$\bar{P}_A \bar{P}_B$
1	0	0	$P_A \bar{P}_B$
0	1	0	$\bar{P}_A P_B$
1	1	0	$P_A P_B$

Again 3 cases

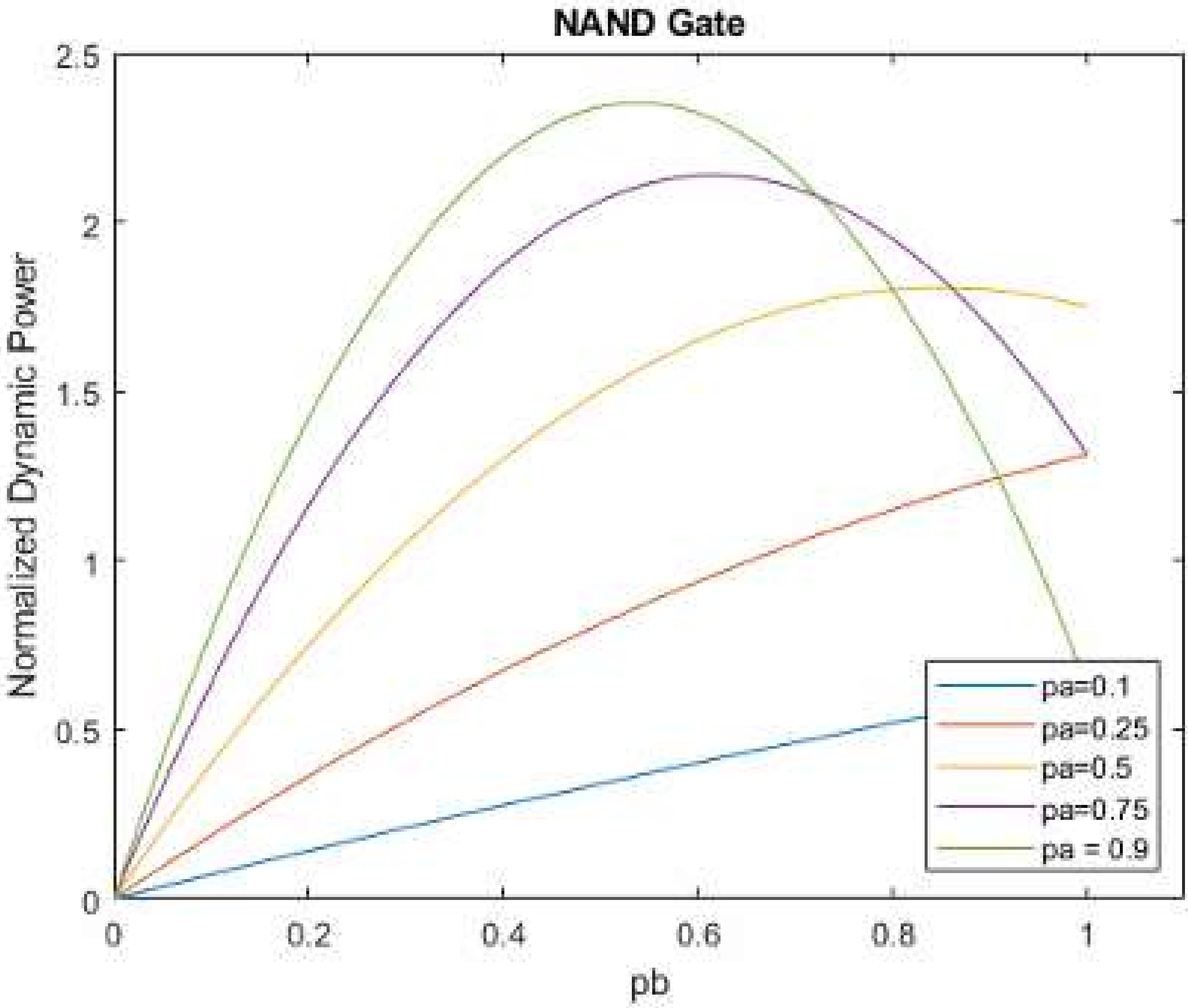
$A = B = 1$, $A = B = 0$

$A = 1$, $B = 0$, $A = 0$, $B = 0$

$A = 0$, $B = 1$, $A = B = 0$

$$P_{dynamic} = f(P_A P_B)\bar{P}_A \bar{P}_B (C_L V_{DD}^2 + C_1 V_{DD}(V_{DD} - |V_{Tp}|))$$
$$+ f P_A \bar{P}_B (\bar{P}_A)(\bar{P}_B)(C_L V_{DD}^2 + C_1 V_{DD}(V_{DD} - |V_{Tp}|))$$
$$+ f \bar{P}_A P_B \bar{P}_A \bar{P}_B \times C_L \times V_{DD}^2$$
$$= f \bar{P}_A \bar{P}_B \bar{P}_A \bar{P}_B \times C_L \times V_{DD}^2$$
$$= f \bar{P}_A \bar{P}_B (1 - \bar{P}_A \bar{P}_B)(C_L V_{DD}^2)$$
$$+ f P_A \bar{P}_A \bar{P}_B C_1 V_{DD}((V_{DD}) - |V_{Tp}|)$$
$$= f \bar{P}_A \bar{P}_B (1 - \bar{P}_A \bar{P}_B)(2 + 2\beta)\gamma C_{g_0} V_{DD}^2$$
$$+ f P_A \bar{P}_A \bar{P}_B (4\beta\gamma C_{g_0}) V_{DD}(V_{DD} - |V_{Tp}|)$$

```
%NAND
% Normalizing and assuming certain values where V_th : Vsupply as 1:4
pa = [0.1,0.25,0.5,0.75,0.9];
beta=2.5;
for i = 1:5
    pb = 0:0.01:1;
    dp = pa(i).*pb.*((2*beta+2).*(1-pa(i)).*(1-pb)+pb.*(1-
pa(i)).*(2*beta+2)+pa(i).*(1-pb).*(2*beta+2)+4*pa(i).*(1-
pb).*(1-0.25));
    plot(pb,dp);
    hold on;
end
legend('pa=0.1','pa=0.25','pa=0.5', 'pa=0.75','pa =
 0.9','Location','southeast')
title('NAND Gate')
xlim([0 1.1])
xlabel('pb')
ylabel('Normalized Dynamic Power')
```



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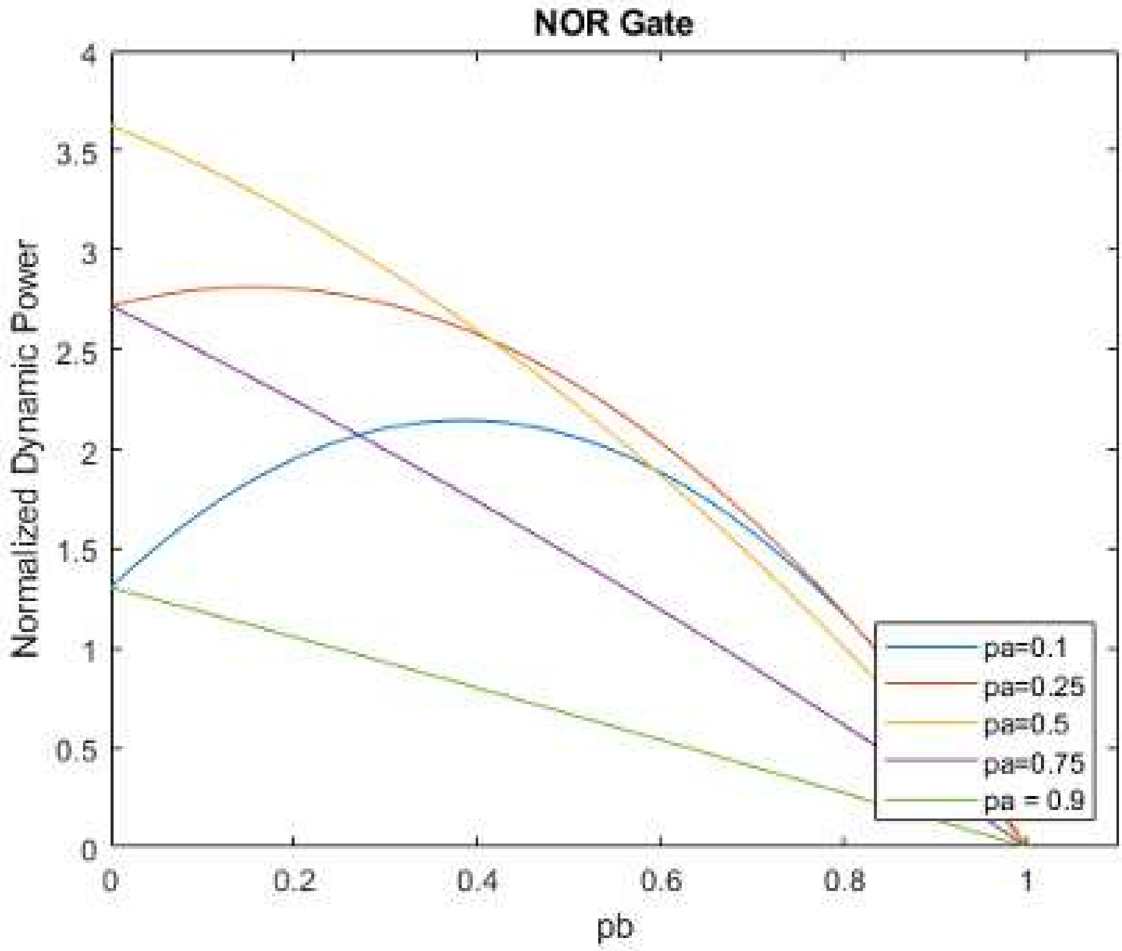
```
%NOR
% Normalizing and assuming certain values where V_th : Vsupply as 1:4
pa = [0.1,0.25,0.5,0.75,0.9];
beta=2.5;
for i = 1:5
    pb = 0:0.01:1;
    dp = (1-pa(i)).*(1-pb).*(1-(1-pa(i)).*(1-
pb)).*(2+2*beta)+pa(i).*(1-pa(i)).*(1-pb).*4.*beta*(1-0.25);
    plot(pb,dp);
    hold on;
end
legend('pa=0.1','pa=0.25','pa=0.5', 'pa=0.75','pa =
 0.9','Location','southeast')
title('NOR Gate')
xlim([0 1.1])
xlabel('pb')
ylabel('Normalized Dynamic Power')
```



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