

Lecture 23

2D Array or Matrix

- ① Creation of 2D Array
- 1) Inside memory 2D Array is stored in form of linear array only.
- 2)

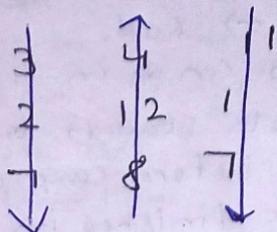
```
int arr[3][3]; // make a matrix of 3x3
```
- 3) which can be accessed using a formula like $c \times i + j$
 $c \rightarrow$ no. of col
 $i \rightarrow$ row, $j \rightarrow$ col index
- 4) Input in 2D Array
`cin >> arr[i][j];`
- 5) Output

```
for (i = 0 → i < arr[i].size())
{
    for (j = 0 → arr[i].size())
    {
        cout << arr[i][j];
    }
}
```

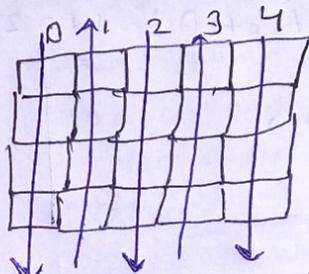
- 6) While passing 2D Array as a parameter, we need to give the column size mandatorily.

So that compiler can pre-calculate memory addresses of individual elements

7) Wave Print



O/p: {3, 7, 8, 12, 4,
1, 1, 7}

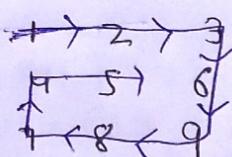


For every odd index we go up (bottom to top)

For every even index we go Top to bottom

So we can apply Top to bottom & bott to top using for loop

8) Spiral Print



O/p = {1, 2, 3, 6, 9, 8, 7, 4, 5}

→ Print starting Row

→ Print ending col

→ Print ending Row

→ Starting Col Print

(After printing each step move row & col)

row = matrix.size(); //size
col = matrix[0].size(); //size

9) Rotate Matrix by 90°.

$$\begin{matrix} 1 & 2 & 3 & 7 & 4 & 1 \\ 4 & 5 & 6 & \Rightarrow & 8 & 5 & 3 \\ 7 & 8 & 9 & & 9 & 6 & 3 \end{matrix}$$

$$\begin{matrix} (0,0) & (0,1) & (0,2) \\ 1 & 2 & 3 \\ (1,0) & (1,1) & (1,2) \\ 4 & 5 & 6 \\ (2,0) & (2,1) & (2,2) \\ 7 & 8 & 9 \end{matrix}$$

$$\begin{matrix} & \downarrow & \\ (0,0) & (0,1) & (0,2) \\ 7 & 4 & 1 \\ (1,0) & (1,1) & (1,2) \\ 8 & 5 & 2 \\ (2,0) & (2,1) & (2,2) \\ 9 & 6 & 3 \end{matrix}$$

$$(2,0) \rightarrow (0,0)$$

$$(1,0) \rightarrow (0,1)$$

$$(0,0) \rightarrow (0,2)$$

$$(2,1) \rightarrow (1,0)$$

Swap (mat[i][j], mat[j][i])

$$1 \quad 4 \quad 7$$

$$2 \quad 5 \quad 8$$

$$3 \quad 6 \quad 9$$

reverse (mat[i].begin(),
mat[i].end()) ↓ Reverse only
row

$$7 \quad 4 \quad 1$$

$$8 \quad 5 \quad 2$$

$$9 \quad 6 \quad 3$$

10) Binary Search in 2D Array

$$\begin{matrix} 1 & 3 & 5 & 7 \\ 10 & 11 & 16 & 20 \\ 28 & 30 & 34 & 60 \end{matrix}$$

cond'n given: whole matrix is in sorted orders we need to find the target.

it looks like a linear sorted array only, so we can apply binary search in 1D array only

start = 0;

end = row * col - 1;

mid = start + $\frac{(end - start)}{2}$

case 1: arr[mid] == target
return 1

case 2: arr[mid] < target
start = mid + 1

case 3: arr[mid] > target
end = mid - 1

How to find mid element in 2D array?

We use column index for it

mid row = mid / col;

mid col = mid % col;

so element = matrix [mid / col]
[mid % col];

$$TC = O(\log \text{row} * \text{col})$$

11) Search in a 2D Matrix-II

Cond'n given: now each element is sorted in Row and Col manner both

1	4	7	11	15
2	5	8	13	19
3	6	9	16	22
4	10	13	14	17
5	18	21	23	26

target = 5

sol = true

1) Let say if our target > 9 then we can say all values before 9 are worthless then we do row++

2) if our target < 9, we can say all values after 9 are worthless, so do col--

3) if we start our soln from last Col and 0th row, then we can move accordingly.

```
int row = mat.size()
int col = mat[0].size()
int start = 0
int end = col - 1;
while (start < row && end >= 0)
    { int element = mat[start][end];
```

if (ele == target)

return 1;

if (ele < target)

start++;

if else

{ end--;

}

return 0;

— X — X — X —

Lecture 24

Basic Maths for DSA

1) Sieve of Eratosthenes

Find prime no. in optimised way.

let say. n = 40

find all prime no. less than 40

X	2	3	X	5	X	7	X	9	11	X	13	X	X	X	17	X	19	X	23	X	29	X	X	31	X	37	X	39	X
---	---	---	---	---	---	---	---	---	----	---	----	---	---	---	----	---	----	---	----	---	----	---	---	----	---	----	---	----	---

① mark every no. as prime no. initially

② we know prime no. does not come in table of other numbers.

③ we reach i=2, we mark all no. which comes in table of 2 as non-prime, and keep 2 as prime.

④ we come at 3, we keep 3 as prime but mark all in table of 3 as non-prime

- ⑤ Some we go to 7 and do some
 ⑥ we do this till < 40
 ⑦ we mark $\text{prime}[0] = \text{prime}[1] = \text{false}$ initially.
 Q find prime no. till n in optimise way
 {
 vector<bool> prime(
 int cnt = 0;
 prime[0] = prime[1] = false;
 for (i = 2; i < n; i++)
 { if (prime[i])
 { cnt++;
 for (j = 2 * i; j < n; j += i)
 prime[j] = false;
 }
 }
 }
 return cnt;
 }

$$\begin{aligned}
 \text{TC} : & \left(\frac{n}{2} + \frac{n}{3} + \frac{n}{5} + \dots + \frac{n}{11} + \dots \right) \\
 & n \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{11} + \dots \right) \\
 & \quad \quad \quad \text{--- HP of prime no.}
 \end{aligned}$$

$$\text{TC} = O(n^{\log(\log n)})$$

2) Segmented Sieve of Eratosthenes
 It is a space optimised version of simple sieve:
 1) n is very large
 2) we have a range [low, high]
 as input and high is very large.
 Let say low = 6, High = 10
 1) Generate all prime no. from 0 to $\text{floor}(\sqrt{H})$.
 $\text{floor}(\sqrt{10}) = 3$
 $\text{prime}[] = \{2, 3\}$
 we do this using normal sieve algorithm, $\text{TC} = O(n \log(\log n))$
 $\text{SC} = O(\sqrt{H})$

2) Create an array of size (bool)
 $(H-1+1)$, i.e. $(10-6+1) = 5$
 (initially all true)
 $\text{TC} = O(H-1+1)$
 we overall ~~mark~~ index like
 for 0 to 10, we will fill values in array of size 5 only

1	T	X	F	T	X	F
	0	1	2	3	4	
	5	6	7	8	9	

3) we mark multiple of 2, 3 as False
 in above array.

F	T	F	F	F
---	---	---	---	---

$$\text{TC} = O(H \log \log H)$$

$$\text{SC} = O(\sqrt{H}) + O(H-1+1)$$

(TC remains same but SC reduces)

3) GCD / HCF

Factor which is maximum and can divide both the nos.

$$\boxed{\gcd(a, b) = \gcd(a - b, b)}$$

↓
 $\gcd(a - b, b)$

$$\begin{aligned} \gcd(72, 24) &= \gcd(48, 24) \\ &\quad \downarrow \\ &\quad \gcd(24, 24) \\ &\quad \downarrow \\ &\quad \underline{\gcd(0, 24)} \end{aligned}$$

24 is the answer as [when one becomes 0, other becomes our answer]

```

3 if (a == 0)
    return b;
if (b == 0)
    return a;
while (a != b)
    {
        if (a > b)
            { a = a - b; }
        else
            { b = b - a; }
    }
return a;
}

```

4) Relation b/w LCM and GCD

$$\boxed{\text{LCM}(a, b) * \gcd(a, b) = a * b}$$

5) Modulo Arithmetics

$$a \cdot l \cdot m \rightarrow [0 \rightarrow cn-1]$$

Print ans modulo $10^9 + 7$?
It is done to bring ans within the range.

Properties of mod

- 1) $(a+b) \cdot l \cdot m = a \cdot l \cdot m + b \cdot l \cdot m$
- 2) $a \cdot l \cdot m - b \cdot l \cdot m = (a-b) \cdot l \cdot m$
- 3) $((a \cdot l \cdot m) \cdot l \cdot m) \cdot l \cdot m = a \cdot l \cdot m$
- 4) $a \cdot l \cdot m * b \cdot l \cdot m = (a * b) \cdot l \cdot m$

6) Fast Exponentiation

$$a^b \rightarrow (a^{b/2})^2 \text{ if } b \text{ is even}$$

$$\rightarrow (a^{b/2})^2 \times a \rightarrow \text{if } b \text{ is odd}$$

$$\text{example: } 2^{10} \rightarrow (2^5)^2$$

$$2^{11} \rightarrow (2^5)^2 \times 2$$

ITC of Fast expo: $O(\log n)$

Q Given an $a \geq n \geq m$
find $(a^n) \cdot l \cdot m$

```
{ int res = 1  
    while (m > 0)  
    { if (m & 1)      means  
        { if odd    typecast  
            res = (1LL * (res) *  
                    (aL) * l * m) * l * m  
        }  
        x = (1LL * (x) * l * m)  
             * (a) * l * m;  
        m = m >> 1;  
    }  
    return res;  
}
```