Assignment 3

Q1)

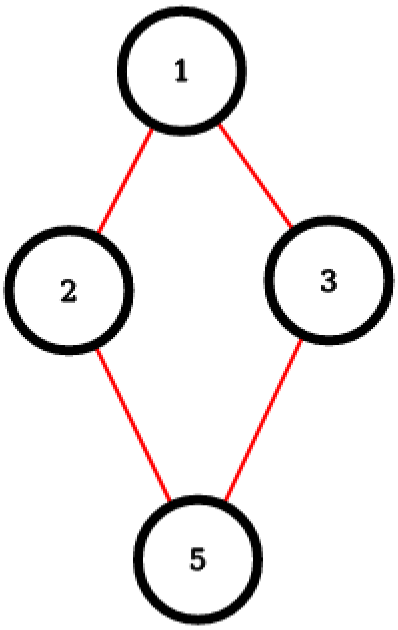
Jen-Schmidt Algorithm for Ear Decomposition

1. Find a rooted DFS tree.
2. Give DFS number to each vertex.
3. Direct Edges in the DFS tree towards the `root` vertex
4. Direct Non Tree edges away from the `root` vertex.
5. Have {visited, not-visted} flag for each vertex.
6. Take the Non tree edge with minimum DFS number`(min(dfsnum(u), dfsnum(v)))`
7. Trace the simple cycle traced by that edge and mark all vertices in the cycle as `visited`
8. All edges in this cycle belong to a EAR component
9. Loop:

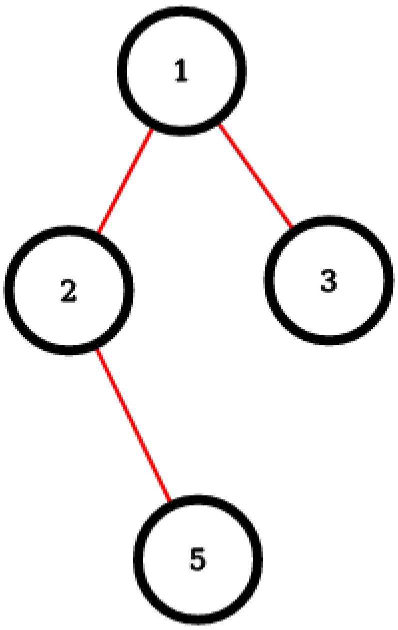
- i. Take the Non tree edge with minimum DFS number`(min(dfsnum(u), dfsnum(v)))`
- ii. Trace the simple cycle traced by that edge until we reach a vertex that is visted.
- iii. Mark all vertices in the cycle as `visited`
- Iv. All edges traversed till finding the visited vertex belong to a EAR component

Our Question

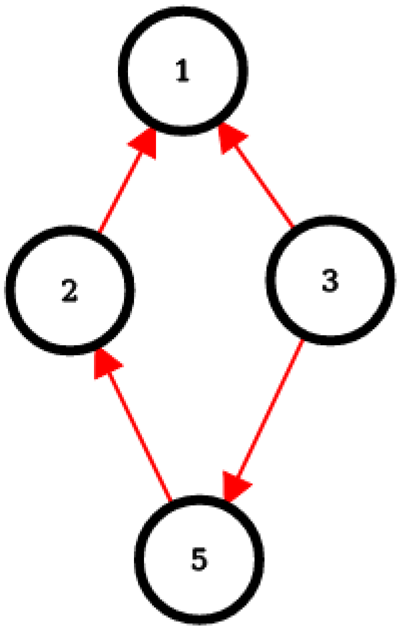
- We are asked used a Non-DFS tree instead of DFS tree.
- One direct observation is in a Non DFS tree there might be `cross edges`, where as in DFS tree only `back edges` exist.[graph is un-directional]
- So what essentially happens assume there is `cross edge`,then it will form a cycle with LCA of the two vertices, which need not be one of u,v.
- The problem is when X->(lca(u,v)) is not one of u,v.
- In which case path to both u and v from X will contain only edges going towards the root,only edge `(u,v)` away from the root.
- In which case, simple cycle does(need) not exist for all these nodes in the path so edge `(u,v)` has possibilty of not ending up in any ear.
- We can see from this directed graph for jen algorithm below(which the directed graph with BFS tree rooted at 1), that there is simple cycle induced by non tree edge `(3,5)`
- So Any spanning tree will not work , DFS trees works becuase there are no cross edges, cross edges as non tree edges need not induce a simple cycle , when directions are given according to our algorithm.
- Back edges dont cause these problem
- Graph:



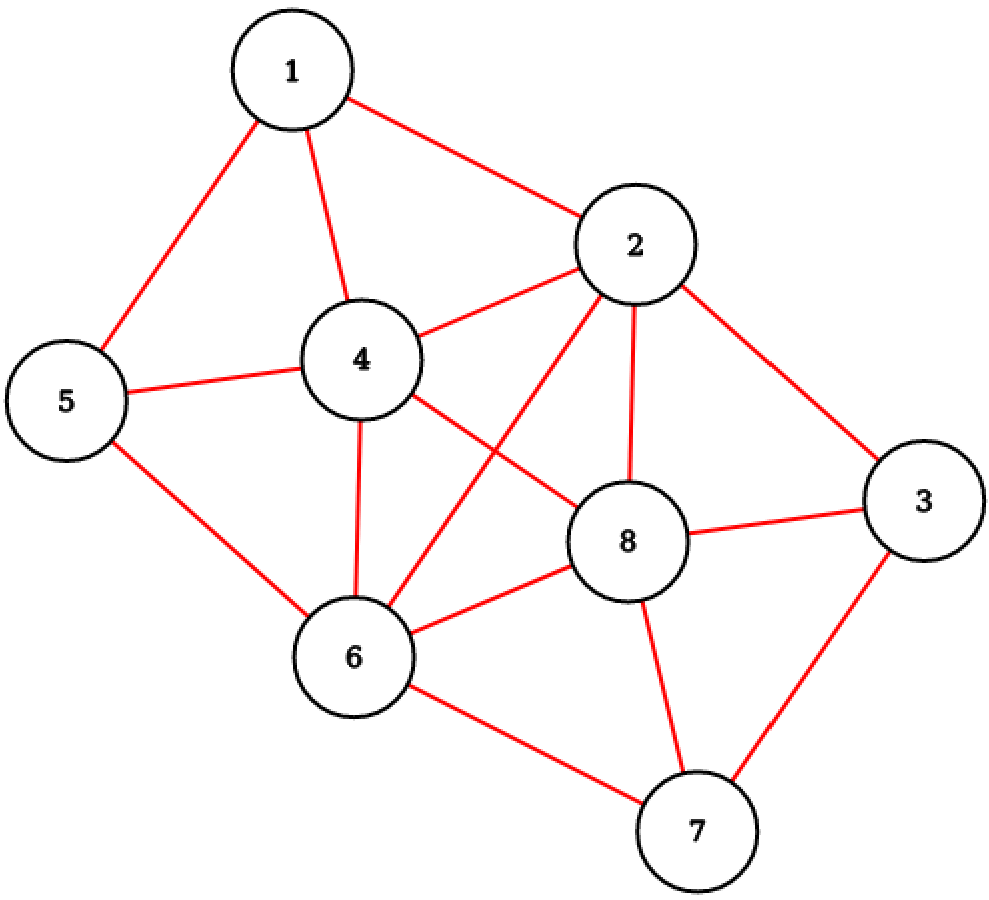
- BFS tree:



- Directed graph for jen schmidt



Question 2



Jen schmidt algorithm

- Non tree edges:

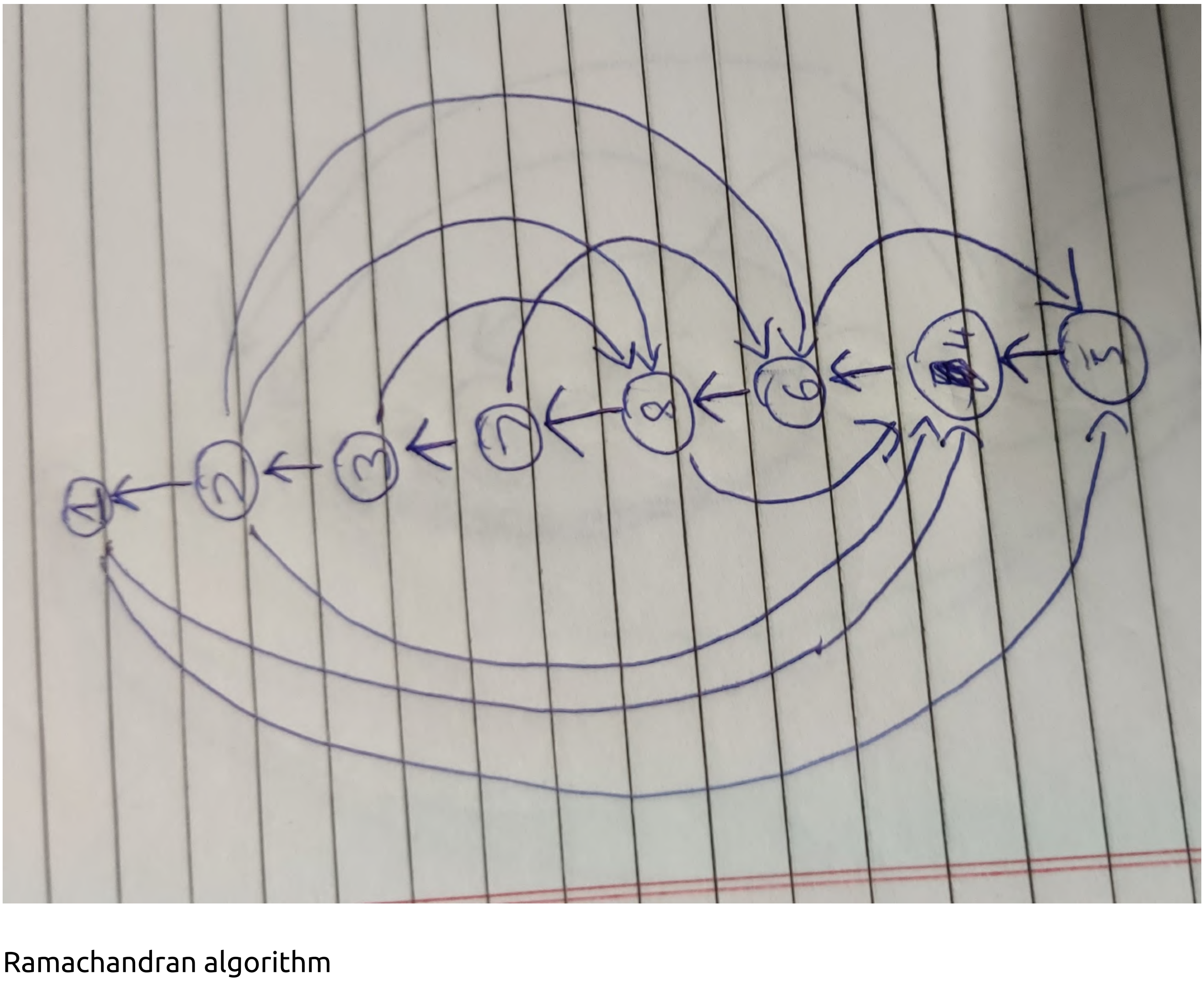
edge(u,v) -> DFS number

1. (1,5) -> 1

2. (1,4) -> 1
3. (2,4) -> 2
4. (2,6) -> 2
5. (2,8) -> 2
6. (3,8) -> 3
7. (7,6) -> 4
8. (8,4) -> 5
9. (6,3) -> 6

- Now the Ear each non tree edge induces:

1. (1,5) -> [(1,5),(5,4),(4,6),(6,8),(8,7),(7,3),(3,2),(2,1)] , vis = [1,2,3,4,5,6,7,8]
2. (1,4) -> [(1,4)], vis = [1,2,3,4,5,6,7,8]
3. (2,4) -> [(2,4)], vis = [1,2,3,4,5,6,7,8]
4. (2,6) -> [(2,6)], vis = [1,2,3,4,5,6,7,8]
5. (2,8) -> [(2,8)], vis = [1,2,3,4,5,6,7,8]
6. (3,8) -> [(3,8)], vis = [1,2,3,4,5,6,7,8]
7. (7,6) -> [(7,6)], vis = [1,2,3,4,5,6,7,8]
8. (8,4) -> [(8,4)], vis = [1,2,3,4,5,6,7,8]
9. (6,3) -> [(6,3)], vis = [1,2,3,4,5,6,7,8]



Ramachandran algorithm

- Tree rooted at vertex 1

vertex	Preoder number
1	1
2	6
3	8
4	5
5	2
6	3
7	4
8	7

- Non tree edges:

index	edge(u,v)	LCA	Preder Number
1.	(1,4)	1	1
2.	(4,2)	1	1
3.	(6,2)	1	1
4.	(4,6)	5	2
5.	(4,8)	1	1
6.	(3,8)	2	6
7.	(6,8)	1	1
8.	(7,8)	1	1
9.	(7,3)	1	1

- Sort non tree edges by their preorder number LCA

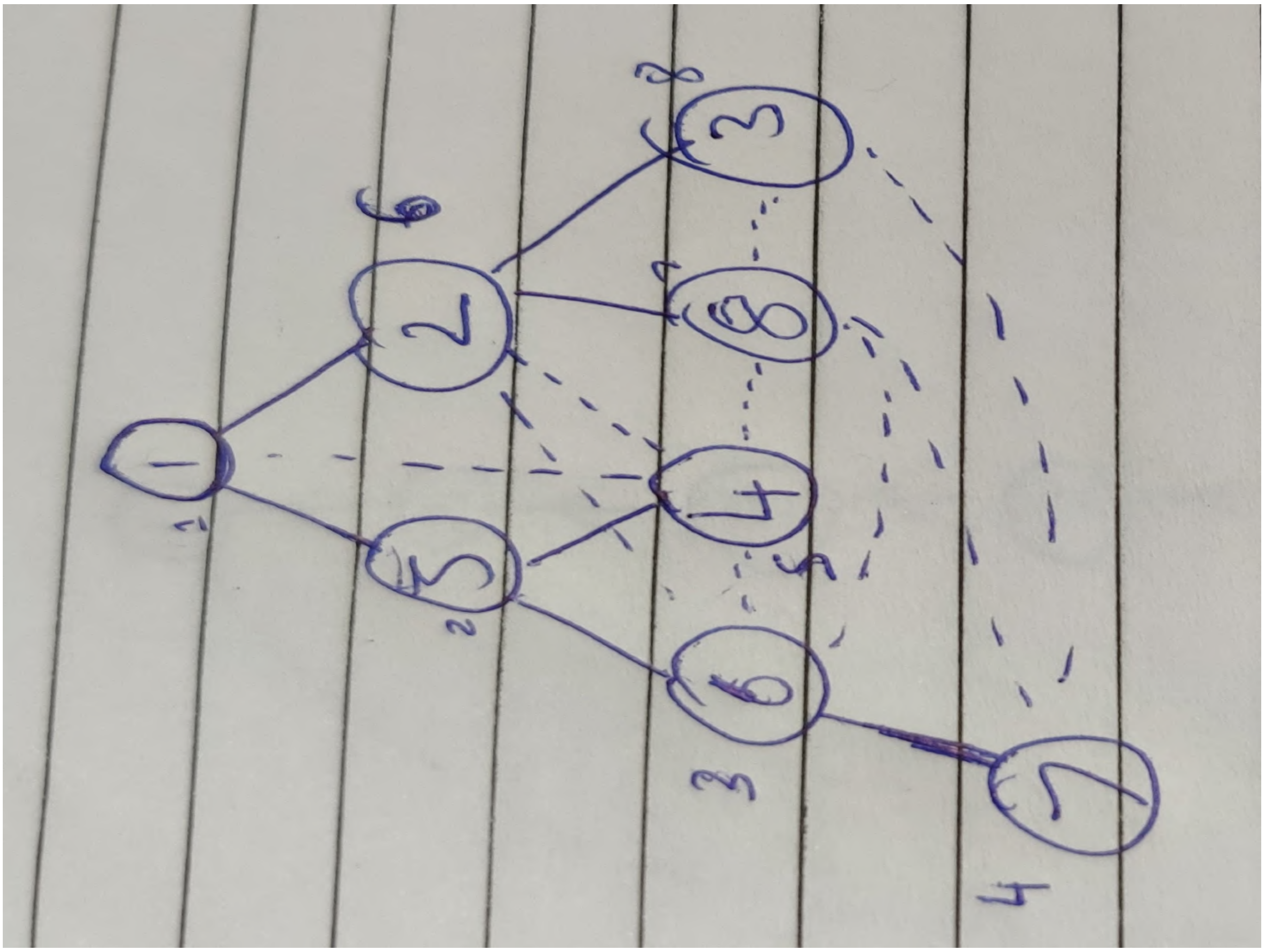
Sort Number	edge(u,v)	LCA	Preder Number
1.	(1,4)	1	1
2.	(4,2)	1	1
3.	(6,2)	1	1
4.	(6,8)	1	1
5.	(7,8)	1	1
6.	(7,3)	1	1
7.	(4,8)	1	1

Sort Number	edge(u,v)	LCA	Preder Number
8.	(4,6)	5	2
9.	(3,8)	2	6

- Labeling Tree edges:

Index	edge(u,v)	lables of cycle in which present	label of edge
1.	(1,2)	2,3,4,5,6,7	2
2.	(1,5)	1,2,3,4,5,6,7	1
3.	(5,4)	1,2,7,8	1
4.	(5,6)	3,4,5,6,8	3
5.	(6,7)	5,6	5
6.	(2,8)	4,5,7,9	4
7.	(2,3)	6,9	6

- Final Ear decomisition:
    - Label 1: [(1,4),(1,5),(5,4)]
    - Label 2: [(4,2),(1,2)]
    - Label 3: [(6,2),(5,6)]
    - Label 4: [(6,8),(2,8)]
    - Label 5: [(7,8),(6,7)]
    - Label 6: [(7,3),(2,3)]
    - Label 7: [(4,8)]
    - Label 8: [(4,6)]
    - Label 9: [(3,8)]
- We can move intial non tree edge to Label 0(here the above in itself ear decompsed):
    - Label 0 [(1,4)]
    - Label 1: [(1,5),(5,4)]
    - Label 2: [(4,2),(1,2)]
    - Label 3: [(6,2),(5,6)]
    - Label 4: [(6,8),(2,8)]
    - Label 5: [(7,8),(6,7)]
    - Label 6: [(7,3),(2,3)]
    - Label 7: [(4,8)]
    - Label 8: [(4,6)]
    - Label 9: [(3,8)]



Question 3

Algorithm

- We have seen in class that if we multiply Adjacency matrix with itself it gives no of paths of length `2` between(i,j).
- Now for it form a triangle there should path on length `1` ans `2` between given vertices `(i,j)`
- And we also known that AxA [i,j] gives no of paths of length 2 between `(i,j)`
- So total number of triangles = sum of all{ (AxA[i,j] * A[i,j])}/3
- division by 3 comes becuase of each edge in traingle will given one value to the sum.
- Pseudo Code:

1. Sum ← 0
2. compute A2
3. for i = 1 . . . n do
4. for j = 1 . . . n do
5. Sum ← Sum + A[i,j] · (A*A) [i,j]
6. return Sum/3

Timecomplexity

- Basically finally triangle couting after matrix multplication cause `O(N^2)`
- Matrix Multuplication - fastest known is `O(N^2.37)`

- So overall time complexity is - `O(N^2)+O(N^2.37)` = `O(N^2.37)`