Unit - III

RANDOM VARIABLES AND MATHEMATICAL EXPECTATIONS

Random variable :- Random variable is a real valued functions defined on the sample space of a random experiment.

Ex :- when two coins are tossed, then the sample space is, $S = \{HH, HT, TH, TT\}$.

Let x be a variable can be defined as the number of tails in the experiment. That is Here x takes the values $0, 1, 2$.

$x = 0, 1, 2$.

$\therefore$ x is a Random variable (R.V). Since it is defined on the sample space of a random experiment.

Types of Random Variables:-

Random variables are two types.

They are (i) Discrete Random Variable
(ii) Continuous Random Variable.

(i) **Discrete Random Variable:-** A random variable X which can take only a finite number of discrete values in an interval of domain is called a "discrete random variable".

In other words, if the random variable takes the values only on the set {0, 1, 2, ---} is called a "Discrete random variable".

Ex:- Tossing of a coin, throwing of a die, the number of defectives in a sample of electric bulbs, the number of printing mistakes in each page of book are examples of discrete random variable.

(ii) <u>Continuous Random Variable</u>:- A random variable x which can take values continuously, i.e., which takes all possible values in a given interval is called a "continuous random variable".

<u>For example</u>:- The height, age and weight of individuals are examples of continuous random variable. Also temperature and time are continuous random variables.

<u>Distribution Function</u>:- Let x be a random variable then the function $F(x)$ [i.e., $F_x(x)$] defined by

$$F_x(x) = F(x) = P(X \leq x), \quad -\infty < x < \infty$$

is called "the distribution function of the random variable x".

<u>Properties of Distribution Function</u>:-

1) If $F(x)$ is the distribution function of the random variable x. Then

$$0 \leq F(x) \leq 1$$

ii) $F(x)$ is right continuous and non decreasing function.

iii) If $F(x)$ is the distribution of the random variable x. then

$$F(-\infty) = \underset{x \to -\infty}{Lt} F(x) = 0$$

$$F(+\infty) = \underset{x \to +\infty}{Lt} F(x) = 1$$

iv) If $a \le x \le b$, then $P(a \le x \le b) = F(b) - F(a)$

<u>Probability mass function</u>:- suppose x is a discrete random variable with possible outcomes $x_1, x_2, \ldots$, then the probability of each possible outcome "x_i" is

$$P_i = P(x = x_i) = ~~P_i~~\ P(x_i), \text{ for } i = 1, 2, 3, \ldots$$

If the number $P(x_i)$, $i = 1, 2, 3, \ldots$ satisfy the two conditions

(i) $P(x_i) > 0$ for all values of i, $0 < P_i \le 1$

(ii) $\Sigma P(x_i) = 1$, $i = 1, 2, 3, \ldots$

then the function $P(x)$ is called the Probability mass function of the random variable X and the set $\{P(x_i)\}$, $i=1,2,\ldots$ is called the discrete Probability distribution of the discrete random variable X.

P) A R.V X has the following Probability distribution

$X=x$	0	1	2	3	4	5	6	7	8
$P(X=x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) Determine the value of a.

(ii) Find $P(X<3)$, $P(X\geq 3)$, $P(0<X<5)$

(iii) Find the distribution function $F(x)$.

Sol^n:- we know that

$$\Sigma P(x_i) = 1$$

$$a+3a+5a+7a+9a+11a+13a+15a+17a=1$$

$$81a = 1$$

$$a = \frac{1}{81}$$

(ii) $P(x<3) = P(X=0,1,2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= a + 3a + 5a$

$= 9a$

$= 9\left(\frac{1}{81}\right)$

$= \frac{9}{81} = \frac{1}{9}$.

(iii) $P(X \geq 3) = P(X=3,4,5,6,7,8)$

$= P(X=3) + P(X=4) + P(X=5) + P(X=6) +$

$P(X=7) + P(X=8)$

$= 7a + 9a + 11a + 13a + 15a + 17a$

$= 72a$

$= 72\left(\frac{1}{81}\right) = \frac{\cancel{72}^{8}}{\cancel{81}_{9}} = 8/9$

(or)

we know that

$P(X \geq 3) + P(X<3) = 1 \quad (\because P(X \geq a) + P(X<a) = 1)$

$P(X \geq 3) = 1 - P(X<3)$

$= 1 - \frac{1}{9} \quad (\text{since } P(X<3) = \frac{1}{9})$

$= 8/9$

(iv) $P(0 < X < 5) = P(X = 1,2,3,4)$

$= P(x=1)+P(x=2)+P(x=3)+P(x=4)$

$= 3a+5a+7a+9a$

$= 24a$

$= 24\left(\frac{1}{81}\right) = \frac{24}{81} = 8/27.$

$\therefore$ The distribution function of the R.V X is given by

$X=x$	0	1	2	3	4	5	6	7	8
$P(X=x)$	$\frac{1}{81}$	$3/81$	$5/81$	$7/81$	$9/81$	$11/81$	$13/81$	$15/81$	$17/81$
$F(x) = P(X \le x)$	$1/81$	$4/81$	$9/81$	$\frac{16}{81}$	$25/81$	$36/81$	$49/81$	$64/81$	$\frac{81}{81}=1$

P) A R.V X has the following Probability distribution

$X=x$	1	2	3	4	5	6	7	8
$P(X=x)$	K	2K	3K	4K	5K	6K	7K	8K

(i) Find the value of K.

(ii) Find the $P(X \le 2)$

(iii) Find $P(2 \le X \le 5)$

(iv) Find the distribution function of Random variable "X".

Sol^n:- (i) we know

$$\Sigma P(x_i) = 1.$$

$$K + 2K + 3K + 4K + 5K + 6K + 7K + 8K = 1$$

$$36K = 1$$

$$\Rightarrow K = \frac{1}{36}.$$

(ii) $P(X \leq 2) = P(X = 1, 2)$

$$= P(X=1) + P(X=2)$$

$$= 1 + 2K = 3K = 3\left(\frac{1}{36}\right) = \frac{1}{12}.$$

(iii) $P(2 \leq X \leq 5) = P(X = 2, 3, 4, 5)$

$$= P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$= 2K + 3K + 4K + 5K$$

$$= 14K = 14\left(\frac{1}{36}\right) = \frac{7}{18}.$$

(iv) The distribution function of the Random Variable X is given by

$X = x$	1	2	3	4	5	6	7	8
$P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{7}{36}$	$8/36$
$F(x) = P(X \leq x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{15}{36}$	$\frac{21}{36}$	$28/36$	$\frac{36}{36} = 1$

P) A R.V "x" has the following Probability function.

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

(i) Determine K.

(ii) Evaluate $P(x<6)$, $P(x \geq 6)$, $P(0<x<5)$ and $P(0<x\leq 4)$.

(iii) If $P(X \leq K) > \frac{1}{2}$, Then find the minimum Value of K

(iv) Determine the distribution function of x.

(v) Find Mean and Variance

Soln:- (i) We know that

$$\sum_{i=1}^{n} P(x_i) = 1$$

$$0+K+2K+2K+3K+K^2+2K^2+7K^2+K=1$$

$$\Rightarrow 10K^2+9K-1=0$$

$$\Rightarrow 10K^2+10K-K-1=0$$

$$10K(K+1)-1(K+1)=0$$

$$(K+1)(10K-1)=0$$

$\Rightarrow K+1 = 0$ (or) $10K-1 = 0$

$10K = 1$

$K = -1$ $K = 1/10$.

(k does not take the value -1. because Probability does not take negative value)

$\therefore K = \frac{1}{10}$

(ii) $P(X<6) = P(X=0,1,2,3,4,5)$

$= P(X=0)+P(X=1)+P(X=2)+P(X=3)+$
$P(X=4)+P(X=5)$

$= 0+K+2K+2K+3K+K^2$

$= 8K+K^2$

$= 8\left(\frac{1}{10}\right)+\left(\frac{1}{10}\right)^2$

$= \frac{8}{10}+\frac{1}{100}$

$= \frac{80+1}{100} = \frac{81}{100}$.

$P(X\geq 6) = 1-P(X<6)$

$= 1-\frac{81}{100}$

$= \frac{100-81}{100}$

$= \frac{19}{100}$.

[we know that
$P(X<a)+P(X\geq a)=1$
$P(X\geq a)=1-P(X<a)$]

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$$P(0 < x < 5) = P(x = 1, 2, 3, 4)$$
$$= P(x=1) + P(x=2) + P(x=3) + P(x=4)$$
$$= k + 2k + 2k + 3k$$
$$= 8k$$
$$= 8\left(\frac{1}{10}\right) = \frac{8}{10}.$$

$$P(0 < x \leq 4) = P(x = 1, 2, 3, 4)$$
$$= P(x=1) + P(x=2) + P(x=3) + P(x=4)$$
$$= k + 2k + 2k + 3k$$
$$= 8k = 8\left(\frac{1}{10}\right) = \frac{8}{10}$$

(iii) $P(x \leq k) > \frac{1}{2}$

$$P(x \leq 0) = 0$$
$$P(x \leq 1) = P(x=0) + P(x=1)$$
$$= 0 + k = k = \frac{1}{10}.$$
$$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$$
$$= 0 + k + 2k = 3k = \frac{3}{10}.$$
$$P(x \leq 3) = P(x = 0, 1, 2, 3)$$
$$= P(x=0) + P(x=1) + P(x=2) + P(x=3)$$
$$= 0 + k + 2k + 2k = 5k = 5\left(\frac{1}{10}\right) = \frac{5}{10} = \frac{1}{2}$$
$$P(x \leq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$$
$$= 0 + k + 2k + 2k + 3k$$
$$= 8k = 8\left(\frac{1}{10}\right) = 0.8$$

$P(X \le 4) = P(X \le 3) + P(X = 4)$

$= \frac{1}{2} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5} > \frac{1}{2}.$

$\therefore$ The minimum value of K for which

$P(X \le K) > \frac{1}{2}$ is $K = 4$.

(iv) The distribution function of the R.V X is given by.

$X = x$	0	1	2	3	4	5	6	7
$P(X=x)$	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{7}{100}+\frac{1}{10}=\frac{17}{100}$
$F(x) = P(X \le x)$	0	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{5}{10}$	$\frac{8}{10}$	$\frac{81}{100}$	$\frac{83}{100}$	$\frac{100}{100}=1$

(v) Mean $= E(X) = \Sigma x_i P(x_i)$

$= (0)(0) + (1)(K) + 2(2K) + 3(2K) +$
$4(3K) + 5(K^2) + 6(2K^2) + 7(7K^2 + K)$

$= K + 4K + 6K + 12K + 5K^2 + 12K^2 + 49K^2 + 7K$

$= 30K + 66K^2$

$= 30\left(\frac{1}{10}\right) + 66\left(\frac{1}{100}\right) = \frac{366}{100} = 3.66.$

$E(x^2) = \sum_{i=1}^{n} x_i^2 p_i$

$= (0)^2(0) + (1)^2 k + (2)^2(2k) + 3^2(2k) +$

$4^2(3k) + 5^2(k^2) + 6^2(2k^2) + 7^2(7k^2+k)$

$= k + 8k + 18k + 48k + 25k^2 + 72k^2 +$

$343k^2 + 49k$

$= 440k^2 + 124k = 440\left(\frac{1}{100}\right) + 124\left(\frac{1}{10}\right)$

$= 16.8$

Variance $= E(x^2) - (E(x))^2$

$= 16.8 - (3.66)^2$

$= 3.4044$

Standard Deviation (S.D) $= \sqrt{\text{Variance}}$

$= \sqrt{3.4044}$

$= 1.845$

P) Calculate expectation (mean) and Variance of "x". The Probability distribution of the Random Variable "x" is given by

x	-1	0	1	2	3
$P(x)$	0.3	0.1	0.1	0.3	0.2

$\underline{\underline{sol^n}}$:-

Mean $= E(x_i) = \sum_{i=1}^{n} x_i P(x_i)$

$= (-1)(0.3) + 0(0.1) + 1(0.1) + 2(0.3)$
$+ 3(0.2)$

$= -0.3 + 0.1 + 0.6 + 0.6$

$= 1.2 - 0.2$

$= 1.$

$E(x^2) = \sum_{i=1}^{n} x_i^2 P_i = (-1)^2(0.3) + 0^2(0.1) + 1^2(0.1)$
$+ 2^2(0.3 + 3^2(0.2)$

$= 0.3 + 1.2 + 1.8 + 0.1$

$= 3.4$

Variance $= E(x^2) - (E(x))^2$

$= 3.4 - (1)^2$

$= 2.4$

P) If the Probability mass function of a variable X is

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K.

Find (i) $P(X<4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$.

(ii) what will be the minimum Value of K such that $P(X \leq 2) > 0.3$?

Soln:- (i) we know that

$$\sum_{i=1}^{n} P(x_i) = 1.$$

$$K + 3K + 5K + 7K + 9K + 11K + 13K = 1.$$

$$\Rightarrow 49K = 1$$

$$K = \frac{1}{49}.$$

$$P(X<4) = P(X = 0, 1, 2, 3)$$

$$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= K + 3K + 5K + 7K$$

$$= 16K$$

$$= 16\left(\frac{1}{49}\right) = \frac{16}{49}.$$

$$P(X \geq 5) = P(X = 5, 6)$$

$$= P(X=5) + P(X=6)$$

$$= 11K + 13K$$

$$= 24K = 24\left(\frac{1}{49}\right) = \frac{24}{49}.$$

$$P(3 < X \leq 6) = P(X = 4, 5, 6)$$

$$= P(X=4) + P(X=5) + P(X=6)$$

$$= 9K + 11K + 13K$$

$$= 33K = \frac{33}{49}.$$

(ii) Given

$$P(X \leq 2) > 0.3.$$

$$P(X=0) + P(X=1) + P(X=2) > 0.3$$

$$K + 3K + 5K > 0.3$$

$$9K > 0.3$$

$$K > \frac{0.3}{9}$$

$$\Rightarrow K > \frac{1}{30}.$$

$\therefore$ The minimum value of K so that $P(X \leq 2) > 0.3$ is $\frac{1}{30}$.

P) Find the mean and variance of the uniform probability distribution, which is given by

$f(x) = \frac{1}{n}$, for $x = 1, 2, 3, \dots n$.

Solⁿ:- The given probability distribution is

x	1	2	3	4	. . . . . .	n
$f(x)$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	. . . . . .	$\frac{1}{n}$

(i) Mean $= E(X) = \sum_{i=1}^{n} x_i \, p(x_i)$

$= 1\left(\frac{1}{n}\right) + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right) + \dots + n\left(\frac{1}{n}\right)$

$= \frac{1}{n}\left[1 + 2 + 3 + \dots + n\right]$

$= \frac{1}{\cancel{n}}\left[\frac{\cancel{n}(n+1)}{2}\right]$ $\left[\text{since } 1+2+3+\dots+n = \frac{n(n+1)}{2}\right]$

$= \frac{n+1}{2}$

(ii) Variance $= E(x^2) - [E(x)]^2$

$E(x^2) = \sum x_i^2 \, p(x_i)$

$$= (1)^2\left(\frac{1}{n}\right) + 2^2\left(\frac{1}{n}\right) + 3^2\left(\frac{1}{n}\right) + \cdots + n^2\left(\frac{1}{n}\right)$$

$$= \frac{1}{n}\left[1^2 + 2^2 + 3^2 + \cdots + n^2\right]$$

$$= \frac{1}{\cancel{n}}\left[\frac{\cancel{n}(n+1)(2n+1)}{6}\right]$$

$$\left(\because 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}\right)$$

$$= \frac{(n+1)(2n+1)}{6}$$

$$\therefore \text{Variance} = V(x) = E(x^2) - (E(x))^2$$

$$= \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$

$$= \frac{n+1}{2}\left[\frac{2(2n+1) - 3(n+1)}{6}\right]$$

$$= \frac{n+1}{2}\left[\frac{4n - 3n + 2 - 3}{6}\right]$$

$$= \left(\frac{n+1}{2}\right)\left(\frac{n-1}{6}\right) = \frac{n^2-1}{12}$$

$$\therefore V(x) = \frac{n^2-1}{12}$$

P) The Probability density function of a random Variable "x" is given by

$$f(x) = Ax^2, \quad 0 \leq x \leq 1.$$

Determine A and

find the probability that

(i) x lies between 0.2 and 0.5

(II) $x < 0.3$

(III) $\frac{1}{4} < x < \frac{1}{2}$

Sol^n:- we know that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

$$\int_0^1 A x^2\,dx = 1$$

$$\Rightarrow A \cdot \left[\frac{x^3}{3}\right]_0^1 = 1$$

$$\Rightarrow A\left[\frac{1}{3} - 0\right] = 1$$

$$\Rightarrow \frac{A}{3} = 1$$

$$\Rightarrow A = 3.$$

∴ The given Probability density function becomes.

$$f(x) = 3x^2, \quad 0 \le x \le 1.$$

(i) $P(0.2 < X < 0.5) = \int_{0.2}^{0.5} 3x^2\,dx.$

$$= 3\left[\frac{x^3}{3}\right]_{0.2}^{0.5}$$

$$= (0.5)^3 - (0.2)^3.$$

$$= 0.117.$$

(II) $P(x < 0.3) = \int_0^{0.3} 3x^2 dx$

$= 3 \int_0^{0.3} x^2 dx$

$= \cancel{3} \cdot \frac{(x^3)_0^{0.3}}{\cancel{3}}$

$= 3((0.3)^3 - (0)^3)$

$= 0.027$

(III) $P\left(\frac{1}{4} < x < \frac{1}{2}\right) = \int_{0.25}^{0.5} 3x^2 dx$

$= 3 \cdot \frac{(x^3)_{0.25}^{0.5}}{3}$

$= [(0.5)^3 - (0.25)^3]$

$= 0.1.$

P) A Continuous Random Variable "x" has the following Probability function.

$$f(x) = \begin{cases} 6x(1-x), & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

(i) check whether the above function is a Probability density function

(ii) Find $P(x>0.3)$, $P(x<0.1)$, $P(\frac{1}{2}<x<\frac{3}{4})$, $P(\frac{1}{3}<x<2)$.

Soln:- If the given function is a Probability density function, then it must satisfy the following condition

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 6x(1-x)\,dx = 1.$$

$$\Rightarrow \int_0^1 6x(1-x)\,dx = \int_0^1 6x\,dx - \int_0^1 6x^2\,dx$$

$$= 6.\frac{(x^2)_0^1}{2} - 6\frac{(x^3)_0^1}{3}$$

$$= 3(1^2-0^2) - 2(1^3-0^3)$$

$$= 3-2$$

$$= 1.$$

$$\therefore \int_0^1 6x(1-x)\,dx = 1.$$

$\therefore$ The given function is a Probability density function

(ii) $P(X>0.3) = \int_{0.3}^{1} f(x)\,dx$

$= \int_{0.3}^{1} 6x(1-x)\,dx$

$= \int_{0.3}^{1} 6x\,dx - \int_{0.3}^{1} 6x^2\,dx$

$= 6\left(\frac{x^2}{2}\right)_{0.3}^{1} - 6\left(\frac{x^3}{3}\right)_{0.3}^{1}$

$= 3\left((1)^2 - (0.3)^2\right) - 2\left((1)^3 - (0.3)^3\right)$

$= 2.73 - 1.946$

$= 0.784$

$P(X<0.1) = \int_{0}^{0.1} f(x)\,dx$

$= \int_{0}^{0.1} 6x(1-x)\,dx$

$= \int_{0}^{1} 6x\,dx - \int_{0}^{0.1} 6x^2\,dx$

$= \frac{\cancel{6}^{3}\,(x^2)_{0}^{0.1}}{\cancel{2}} - \frac{\cancel{6}^{2}\,(x^3)_{0}^{0.1}}{\cancel{3}}$

$= 3\left((0.1)^2 - (0)^2\right) - 2\left((0.1)^3 - (0)^3\right)$

$= 0.0281$

$$P\left(\frac{1}{2} < x < \frac{3}{4}\right) = \int_{1/2}^{3/4} f(x)\,dx$$

$$= \int_{1/2}^{3/4} 6x(1-x)\,dx$$

$$= \int_{1/2}^{3/4} 6x\,dx - \int_{1/2}^{3/4} 6x^2\,dx$$

$$= \cancel{6}^{3} \frac{(x^2)_{1/2}^{3/4}}{\cancel{2}} - 6\left(\frac{x^3}{3}\right)_{1/2}^{3/4}$$

$$= 3\left[\left(\frac{3}{4}\right)^2 - \left(\frac{1}{2}\right)^2\right] - 2\left[\left(\frac{3}{4}\right)^3 - \left(\frac{1}{2}\right)^3\right]$$

$$= 0.34$$

$$P(1/3 < x < 2) = P\left(\frac{1}{3} < x \leq 1\right) + P(1 < x < 2)$$

$$= \int_{1/3}^{1} 6x(1-x)\,dx + 0$$

$$= \int_{1/3}^{1} 6x\,dx - \int_{1/3}^{1} 6x^2\,dx$$

$$= \frac{6}{2}(x^2)_{1/3}^{1} - \frac{6}{3}(x^3)_{1/3}^{1}$$

$$= 3\left[(1)^2 - \left(\frac{1}{3}\right)^2\right] - 2\left((1)^3 - \left(\frac{1}{3}\right)^3\right)$$

$$= \underline{0.74}$$

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P) A Continuous Random Variable has the following Probability density function

$$f(x) = \begin{cases} kx\,e^{-\lambda x}, & \text{for } x \geq 0, \lambda > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Determine (i) K (ii) Mean (iii) Variance

Sol^n:- we know that,

the total Probability is always equal to "1".

$\therefore$ we have

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{0}^{\infty} kx\,e^{-\lambda x}\,dx = 1$$

$$\Rightarrow K\int_{0}^{\infty} x\,e^{-\lambda x}\,dx = 1$$

$$\Rightarrow K\left[x\frac{e^{-\lambda x}}{-\lambda} - (1)\frac{e^{-\lambda x}}{(-\lambda)(-\lambda)}\right]_{0}^{\infty}$$

$$\Rightarrow K\left[(0-0) - \left(0 - \frac{e^{-0}}{\lambda^2}\right)\right]$$

$$\Rightarrow K\left(\frac{1}{\lambda^2}\right) = 1$$

$$K = \lambda^2.$$

$\therefore$ The above Probability density function becomes.

$$f(x) = \begin{cases} \lambda^2 x e^{-\lambda x}, & \text{for } x \geq 0, \lambda > 0. \\ 0, & \text{otherwise.} \end{cases}$$

(ii) Mean $= E(X) = \int_{-\infty}^{\infty} x f(x)\, dx.$

$$= \int_0^{\infty} x \, \lambda^2 x e^{-\lambda x} dx.$$

$$= \lambda^2 \int_0^{\infty} x^2 e^{-\lambda x} dx$$

$$= \lambda^2 \left[(x^2)\left(\frac{e^{-\lambda x}}{-\lambda}\right) - (2x)\left(\frac{e^{-\lambda x}}{(-\lambda)(-\lambda)}\right) + (2)\left(\frac{1}{\lambda^2}\left(\frac{e^{-\lambda x}}{-\lambda}\right)\right)\right]_0^{\infty}$$

$$= \lambda^2 \left[(0 - 0 + 0) - \left(0 - 0 - \frac{2 \cdot 1 e^{-0}}{\lambda^2 \lambda}\right)\right].$$

$$= \lambda^2 \cdot \frac{2}{\lambda^3}$$

$$= \frac{2}{\lambda}$$

$\therefore$ Mean $= E(X) = 2/\lambda$

(iii) $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx.$

$$= \int_0^{\infty} x^2 (\lambda^2 x\, e^{-\lambda x})\, dx$$

$$= \lambda^2 \int_0^{\infty} x^3\, e^{-\lambda x}\, dx$$

$$= \lambda^2 \left[x^3 \cdot \frac{e^{-\lambda x}}{-\lambda} - (3x^2)\left(\frac{e^{-\lambda x}}{(-\lambda)(-\lambda)}\right) + (6x)\left(\frac{1}{\lambda^2}\left(\frac{e^{-\lambda x}}{-\lambda}\right)\right) - (6)\frac{1}{\lambda^2}\left(\frac{e^{-\lambda x}}{-\lambda}\right)\right]_0^{\infty},$$

$$= \lambda^2 \left[(0-0+0-0) - \left(0-0+0-\frac{6}{\lambda^4}\right)\right]$$

$$= \lambda^2 \cdot \frac{6}{\lambda^4}$$

$$= \frac{6}{\lambda^2}.$$

Variance $= V(X) = E(x^2) - (E(x))^2$

$$= \frac{6}{\lambda^2} - \left(\frac{2}{\lambda}\right)^2$$

$$= \frac{6}{\lambda^2} - \frac{4}{\lambda^2}$$

$$V(x) = \frac{2}{\lambda^2}$$

P) For the Continuous Probability function (16)

$f(x) = K x^2 e^{-x}$. when $x \geq 0$. Find

(i) K (ii) Mean (iii) Variance.

Sol:- (i) Since we know that, the total Probability is always unity.

ie., $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\Rightarrow \int_0^{\infty} K x^2 e^{-x}\,dx = 1.$$

$$\Rightarrow K\left[(x^2)(-e^{-x}) - (2x)(-(-e^{-x})) + (2)(-e^{-x})\right]_0^{\infty} = 1$$

$$\Rightarrow K\left[(0-0+0) - (0-0-2e^{-0})\right] = 1$$

$$K(2) = 1$$

$$2K = 1$$

$$K = 1/2.$$

∴ The above Probability density function becomes.

$$f(x) = \frac{1}{2} x^2 e^{-x}, \text{ when } x \geq 0.$$

(ii) Mean $= E(X) = \int_{-\infty}^{\infty} x f(x)\, dx.$

$$= \int_0^{\infty} x \cdot k x^2 e^{-x} dx.$$

$$= k \int_0^{\infty} x^3 e^{-x} dx$$

$$= k \left[x^3 \cdot \frac{e^{-x}}{(-1)} - (3x^2) \frac{e^{-x}}{(-1)^2} + (6x) \frac{e^{-x}}{(-1)^3} - (6) \frac{e^{-x}}{(-1)^4} \right]_0^{\infty}$$

$$= \frac{1}{2} \left[(0 - 0 + 0 - 0) - (-0 + 0 - 0 - 6) \right]$$

$$= \frac{6}{2} = 3.$$

$$E(x^2) = \int_0^{\infty} x^2 f(x)\, dx.$$

$$= \int_0^{\infty} x^2 \left(\frac{1}{2} x^2 e^{-x} \right) dx.$$

$$= k \int_0^\infty x^4 e^{-x} dx$$

$$= \frac{1}{2}\Big[(x^4)\left(\frac{e^{-x}}{-1}\right) - (4x^3)\left(\frac{e^{-x}}{(-1)^2}\right) +$$

$$(12x^2)\left(\frac{e^{-x}}{(-1)^3}\right) - (24x)\left(\frac{e^{-x}}{(-1)^4}\right) +$$

$$(24)\left(\frac{e^{-x}}{(-1)^5}\right)\Big]_0^\infty$$

$$= \frac{1}{2}\left[(0-0+0-0+0) - (0-0+0-0-24)\right]$$

$$= \frac{24}{2} = 12.$$

$$\therefore \text{Variance} = V(X) = E(x^2) - (E(x))^2$$

$$= 12 - (3)^2$$

$$= 12 - 9$$

$$= 3.$$

$\therefore$ Variance $= 3.$

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Unit – IV

Probability Distributions

Binomial Distribution:- Binomial Distribution was discovered by James Bernouli. It was first published in 1713.

Definition of Binomial Distribution:-

A Random Variable X is said to follow Binomial Distribution, if it assumes only non-negative values, then it's Probability mass function is given by

$$P(X=x) = P(x) = \begin{cases} {}^{n}C_{x}\, p^{x} q^{n-x}, & x = 0,1,2,3,\ldots n \\ & p+q=1 \\ 0, & \text{otherwise} \end{cases}$$

Constants of Binomial Distribution (or) Mean and Variance of Binomial Distribution (B.D):-

(i) The mean of B.D is "np"

(ii). The variance of B.D is "npq".

<u>Problems</u>

P) A fair coin is tossed 6 times. Find the Probability of getting four heads ?

Sol^n:- The Probability mass function of B.D is

$$P(X=x) = P(x) = \begin{cases} {}^{n}C_{x}\, p^{x} q^{n-x}, & x = 0,1,2,\dots n \\ & p+q=1 \\ 0, & \text{otherwise.} \end{cases} \rightarrow (1)$$

Here $n = 6$

p = Probability of getting a head = $1/2$

q = Probability of getting a tail = $1/2$

$x = 4$.

From eq (1) =)

$\therefore$ The Probability of getting four heads = $P(X=4)$.

$$P(X=4) = {}^{6}C_{4} \left(\frac{1}{2}\right)^{4} \left(\frac{1}{2}\right)^{6-4}$$

$\therefore$ $= 0.23437$.

P) A discrete Random Variable "x" has the mean "6" and variance "2". If it is assumed that the distribution is Binomial, Find the $P(5 \leq x \leq 7)$?

Sol:- Given

The mean of Binomial Distribution $np = 6 \rightarrow ①$

Variance of Binomial Distribution $npq = 2 \rightarrow ②$.

$$\frac{Eq ②}{Eq ①} \Rightarrow \frac{\cancel{n}pq}{\cancel{n}p} = \frac{2}{6} = 1/3$$

$$\Rightarrow q = 1/3.$$

we know that

$$p + q = 1 \quad \Rightarrow \quad p = 1 - q$$

$$p = 1 - 1/3 = 2/3.$$

$$p = 2/3.$$

From eq ① $\Rightarrow$ $np = 6$

$$n \times \frac{\cancel{2}}{3} = \cancel{6}^{3}$$

$$n = 3 \times 3 = 9.$$

$$\therefore n = 9.$$

$\therefore$ The fitted Binomial Distribution is

$$P(X = x) = n_{C_x} \, p^x q^{n-x}, \quad x = 0, 1, 2 \cdots n$$

$$p + q = 1$$

$$P(X = x) = 9_{C_x} \, (2/3)^x \, (1/3)^{9-x}, \quad x = 0, 1, 2 \cdots n \rightarrow ③$$

$$P(5 \le x \le 7) = P(x=5,6,7)$$

$$= P(x=5) + P(x=6) + P(x=7)$$

$$= {}^9C_5 (2/3)^5 \left(\tfrac{1}{3}\right)^{9-5} + {}^9C_6 (2/3)^6 \left(\tfrac{1}{3}\right)^{9-6}$$

$$+ {}^9C_7 (2/3)^7 \left(\tfrac{1}{3}\right)^{9-7} \quad \text{(From eq (3))}$$

$$P(5 \le x \le 7) = 0.711$$

P). The mean and Variance of a Binomial Variable X with Parameter n and p are 16 and 8. Find $P(X \ge 1)$ and $P(X > 2)$?

Sol^n:- Given,

the mean of Binomial distribution, $np = 16 \to (1)$

Variance $= npq = 8 \to (2)$

$$\frac{\text{Eq}(2)}{\text{Eq}(1)} \Rightarrow \frac{\cancel{n}pq}{\cancel{n}p} = \frac{8}{16} = \frac{1}{2}$$

$$q = \frac{1}{2}$$

$$p+q=1 \Rightarrow p = 1-q = 1-\frac{1}{2} = \frac{1}{2}$$

$$p = \frac{1}{2}$$

From eq (1) $\Rightarrow n \times \frac{1}{2} = 16$

$$n = 32.$$

$\therefore$ The fitted Binomial Distribution is

$$P(X=x) = P(x) = n_{C_x} p^x q^{n-x}, \quad x = 0,1,2, \ldots n$$

$$P(X=x) = 32_{C_x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{32-x}, \quad x = 0,1,2, \ldots 32.$$

$$P(X \geq 1) = 1 - P(X<1)$$

$\left[\begin{array}{l} \text{since} \\ P(X \geq a) + P(X<a) = 1 \\ P(X \geq a) = 1 - P(X<a) \end{array}\right]$

$$= 1 - P(X=0)$$

$$= 1 - 32_{C_0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{32-0}$$

$$= 0.999.$$

$$P(X>2) = 1 - P(X \leq 2)$$

$$= 1 - P(X = 0,1,2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[32_{C_0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{32-0} + 32_{C_1} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{32-1} + 32_{C_2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{32-2}\right]$$

$= 1$ (nearly)

P) 20% of items Produced from a factory are defective. Find the Probability that in a sample of 5 chosen at random.

(i) none is defective

(ii) one is defective

(iii) $P(1 < x < 4)$

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$\underline{\underline{sol^n}}$:- Probability of defective items $= p = 20\% = \frac{20}{100} = 0.2$.

Probability of non defective items $= q = 1-p = 1-0.2 = 0.8$

Total number of items $(n) = 5$.

x = number of defective items

$\therefore$ The fitted Binomial Distribution is

$P(X=x) = P(x) = {}^nC_x \, p^x q^{n-x}, \quad x = 0,1,2,\ldots n.$

$P(X=x) = {}^5C_x \, (0.2)^x \, (0.8)^{n-x}, \quad x = 0,1,2,\ldots 5 \rightarrow (1)$.

(i) Probability that none is defective = Prob of zero defective items. (That is $x=0$)

$P(X=0) = {}^5C_0 \, (0.2)^0 \, (0.8)^{5-0}$ (From (1))

$= \cancel{0.41}\ 0.33.$

(ii) Probability that one is defective $= P(X=1)$

$= {}^5C_1 \, (0.2)^1 \, (0.8)^{5-1}$

$= 0.41$

(iii) $P(1 < X < 4) = P(X = 2, 3)$

$= P(X=2) + P(X=3)$

$= {}^5C_2 \, (0.2)^2 (0.8)^{5-2} + {}^5C_3 \, (0.2)^3 (0.8)^{5-3}$

$= 0.256$

P) Assume that 50% of all engineering students are good in mathematics. Determine the probability that among 18 engineering students

(i) Exactly 10

(ii) At least 10

(iii) Atmost 8

(iv) At least 2 and Atmost 9 are good in mathematics.

Sol^n:- Let p = The Probability that students who are good in mathematics.

$p = 50\% = \frac{50}{100} = \frac{1}{2} = 0.5$

$q = 1-p = 1-\frac{1}{2} = \frac{1}{2} = 0.5$

n = number of students = 18.

Let x be the no of engineering students who are good in mathematics.

Hence the fitted Binomial distribution is,

$P(X=x) = P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}, \quad x = 0,1,2 \ldots n.$

$P(X=x) = {}^{18}C_{x}\, (0.5)^{x} (0.5)^{18-x}, \quad x = 0,1,2 \ldots 18.$

(i) $P(\text{exactly } 10) = P(X=10) = {}^{18}C_{10}\, (0.5)^{10}\, (0.5)^{18-10}$

$= 0.166.$

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(ii) $P(\text{at least } 10) = P(X \geq 10) = P(X=10) + P(X=11) + P(X=12)$

$+ \cdots \cdots + P(X=18)$

$= {}^{18}C_{10} (0.5)^{10} (0.5)^{18-10} + {}^{18}C_{11} (0.5)^{11}$

$(0.5)^{18-11} + {}^{18}C_{12} (0.5)^{12} (0.5)^{18-12} +$

$\cdots\cdots + {}^{18}C_{18} \cdot (0.5)^{18} (0.5)^{18-18}$

$= {}^{18}C_{10} (0.5)^{18} + {}^{18}C_{11} (0.5)^{18} + {}^{18}C_{12} (0.5)^{18}$

$+ \cdots + {}^{18}C_{10} (0.5)^{18}$

$= (0.5)^{18} \left[{}^{18}C_{10} + {}^{18}C_{11} + {}^{18}C_{12} + \cdots + {}^{18}C_{18} \right]$

~~(iv) P(at least 2 and atmost 9) = $P(2 \leq X \leq 9)$~~

(iii) $P(\text{at most } 8) = P(X \leq 8)$

$= P(X = 1, 2, 3, 4, 5, 6, 7, 8)$

$= P(X=1) + P(X=2) + P(X=3) + \cdots + P(X=8)$

$= {}^{18}C_1 (0.5)^{18} + {}^{18}C_2 (0.5)^{18} + {}^{18}C_3 (0.5)^{18}$

$+ \cdots + {}^{18}C_8 (0.5)^{18}$

$= (0.5)^{18} \left[{}^{18}C_1 + {}^{18}C_2 + {}^{18}C_3 + \cdots + {}^{18}C_8 \right]$

(iv) P(at least 2 and atmost 9) $= P(2 \le X \le 9)$

$= P(X = 2, 3, 4, 5, --- 9)$

$= P(X=2) + P(X=3) + P(X=4) + --- + P(X=9)$

$= {}^{18}C_2 (0.5)^{18} + {}^{18}C_3 (0.5)^{18} + --- + {}^{18}C_9 (0.5)^{18}$

$= (0.5)^{18} \left[{}^{18}C_2 + {}^{18}C_3 + --- + {}^{18}C_9 \right]$

P) out of 800 families with 5 children each, how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys (d) atleast one boy? Assume equal probabilities for boys and girls.

Soln:- (i) Let the number of boys in each family $= x$.

p = The Probability of each boy $= \frac{1}{2}$ (since equal Probability for boys and girls).

Number of children, $n = 5$.

$\therefore$ The Binomial distribution is

$P(X=x) = P(x) = {}^nC_x p^x q^{n-x}$

$= {}^5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x}$

$= {}^5C_x \left(\frac{1}{2}\right)^5$

$= {}^5C_x \frac{1}{2^5}$.

(a) $P(3\text{ boys}) = P(x=3) = P(3) = \frac{1}{2^5} \cdot {}^5C_3 = \frac{10}{32} = \frac{5}{16}$ per family.

Thus for 800 families, the probability of ~~no~~ families having 3 boys $= \frac{5}{16}(800) = 250$ families.

(b) $P(5\text{ girls}) = P(\text{no boys}) = P(x=0) = P(0) = \frac{1}{2^5} \cdot {}^5C_0$

$= \frac{1}{32}$ per family.

Thus for 800 families, the probability of number of families having 5 girls.

$= \frac{1}{32}(800) = 25$ families.

(c) $P(\text{either } 2 \text{ or } 3 \text{ boys}) = P(x=2) + P(x=3)$

$= \frac{1}{2^5} \cdot {}^5C_2 + \frac{1}{2^5} \cdot {}^5C_3 = \frac{1}{2^5}(10+10)$

$= \frac{20}{32} = \frac{5}{8}$ per family.

$\therefore$ Expected number of families with 2 or 3 boys $=$

$\frac{5}{8}(800) = 500$ families

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(ii) P(at least one boy) = $P(x \geq 1)$

$= P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$= \frac{1}{2^5}({}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5)$

$= \frac{1}{2^5}(5+10+10+5+1)$

$= \frac{31}{32}$.

$\therefore$ Expected number of families with at least one boy $= \frac{31}{32}(800) = \underline{\underline{775}}$

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P) Seven coins are tossed and the number of heads are noted. The experiment is repeated 128 times and the following distribution is obtained.

Number of heads	0	1	2	3	4	5	6	7	Total
Frequency	7	6	19	35	30	23	7	1	128

Fit a Binomial Distribution and calculate the expected frequencies by assuming
(i) The coin is unbiased.
(ii) The nature of the coin is not known.

Soln:- (i) The coin is unbiased.

Since the coin is unbiased.

$p = \frac{1}{2} = 0.5$ (Probability of getting head = Probability of getting tail)

$q = \frac{1}{2} = 0.5$

$N = \Sigma f_i = 7+6+19+35+30+23+7 = 128$

$n = 7$.

∴ The fitted Binomial Distribution is given by

$P(X=x) = n_{C_x} \, p^x q^{n-x}$, $x = 0,1,2 --- n$

$= 7_{C_x} (0.5)^x (0.5)^{7-x}$, $x = 0,1,2 --- 7 \rightarrow (1)$

Then, the expected frequencies are calculated by using the following formula.

$$F(x) = N \cdot P(X=x)$$
$$= 128 \cdot \left({}^7C_x (0.5)^x (0.5)^{7-x}\right), \quad x = 0,1,2,3,\dots 7 \quad (2)$$

Put $x=0$ in eq (2) =)

$$F(0) = 128\left({}^7C_0 (0.5)^0 (0.5)^{7-0}\right) = 1.$$

Put $x=1$ in eq (2) =)

$$F(1) = 128\left({}^7C_1 (0.5)^1 (0.5)^{7-1}\right) = 27.$$

Put $x=2$ in eq (2) =)

$$F(2) = 128\left({}^7C_2 (0.5)^2 (0.5)^{7-2}\right) = 21$$

Put $x=3$ in eq (2) =)

$$F(3) = 128\left({}^7C_3 (0.5)^3 (0.5)^{7-3}\right) = 35$$

Put $x=4$ in eq (2) =)

$$F(4) = 128\left({}^7C_4 (0.5)^4 (0.5)^{7-4}\right) = 35$$

Put $x=5$ in eq (2) =)

$$F(5) = 128\left({}^7C_5 (0.5)^5 (0.5)^{7-5}\right) = 21$$

Put $x = 6$ in eq ② ⇒

$$F(6) = 128\left({}^{7}C_{6} (0.5)^{6} (0.5)^{7-6} \right) = 7$$

Put $x = 7$ in eq ② ⇒

$$F(7) = 128\left({}^{7}C_{7} (0.5)^{7} (0.5)^{7-7} \right) = 1.$$

∴ The expected frequencies are 1, 7, 21, 35, 35, 21, 7 and 1 such that their sum is 128.

(ii) when the nature of the coin is not known.

Then $n = 7$, $N = \Sigma f_i = 7+6+19+35+30+23+7+1 = 128$.

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{(0\times7)+(1\times6)+(2\times19)+(3\times35)+(4\times30)+(5\times23)+(6\times7)+(7\times1)}{128}$$

$$= \frac{433}{128} = 3.38 \rightarrow ①$$

But we know that, for Binomial Distribution

Mean $= np \rightarrow$ ②

From eq ① and eq ② ⇒

$$np = 3.38$$

$$7p = 3.38$$

$$p = \frac{3.38}{7} = 0.48.$$

$$q = 1 - p = 1 - 0.48 = 0.52.$$

∴ The fitted Binomial Distribution is

given by

$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}, \quad x = 0, 1, 2, \ldots n$$

$$P(X=x) = {}^{7}C_{x}\, (0.48)^{x} (0.52)^{7-x}, \quad x = 0, 1, 2, 3, \ldots 7, \quad ③$$

∴ Then the expected frequencies are calculated

by using the following formula.

$$F(x) = N\,P(x)$$

$$= 128 \cdot \left[{}^{7}C_{x} (0.48)^{x} (0.52)^{7-x}\right] \quad ④$$

Put $x = 0$ in eq ④ ⇒

$$F(0) = 128 \left[{}^{7}C_{0} (0.48)^{0} (0.52)^{7-0}\right]$$

$$= 1.316 \simeq 1$$

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Put $x = 1$ in eq (4) =)

$$F(1) = 128\left[{}^{7}C_{1}(0.48)^{1}(0.52)^{7-1}\right]$$

$$= 8.\cancel{4}2 \simeq 8$$

Put $x = 2$ in eq (4) =)

$$F(2) = 128\left[{}^{7}C_{2}(0.48)^{2}(0.52)^{7-2}\right]$$

$$= 23.4146 \simeq 23$$

Put $x = 3$ in eq (4) =)

$$F(3) = 128\left[{}^{7}C_{3}(0.48)^{3}(0.52)^{7-3}\right]$$

$$= 36.226 \simeq 36.$$

Put $x = 4$ in eq (4) =)

$$F(4) = 128\left[{}^{7}C_{4}(0.48)^{4}(0.52)^{7-4}\right]$$

$$= 34.439 \simeq 34.$$

Put $x = 5$ in eq (4) =)

$$F(5) = 128\left[{}^{7}C_{4}(0.48)^{5}(0.52)^{7-5}\right]$$

$$= 18.52 \simeq 19.$$

Put $x = 6$ in eq (1) =)

$$F(6) = 128\left[{}^{7}C_{6} (0.48)^{6} (0.52)^{7-6}\right]$$

$$= 5.69 \simeq 6.$$

Put $x = 7$ in eq (1) =)

$$F(7) = 128\left[{}^{7}C_{7} (0.48)^{7} (0.52)^{7-7}\right]$$

$$= 0.75 \simeq 1$$

$\therefore$ The expected frequencies are. 1, 8, 23, 36, 34, 19, 6, and 1. Such that their sum is 128.

P) Fit a Binomial Distribution to the following data and Calculate the expected frequencies

x	0	1	2	3	4	5
f	2	14	20	34	22	8

Solⁿ:-

Here $n = 5$ and $N = \Sigma f_i$

$= 2 + 14 + 20 + 34 + 22 + 8$

$N = 100$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{(0\times2)+(1\times14)+(2\times20)+(3\times34)+(4\times22)+(5\times8)}{100}$$

$$= \frac{14+40+102+88+40}{100}$$

$$= \frac{284}{100}$$

$$= 2.84 \rightarrow (1)$$

For Binomial Distribution,

we know that

$\text{Mean} = np \rightarrow (2)$

From eq (1) and eq (2) $\Rightarrow$

$$np = 2.84$$
$$5p = 2.84$$
$$p = \frac{2.84}{5} = 0.568$$
$$q = 1 - p = 1 - 0.568 = 0.432$$

$\therefore$ The fitted Binomial Distribution is

$$P(X=x) = P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}, \quad x = 0, 1, 2 \ldots n$$
$$= {}^{5}C_{x}\, (0.568)^{x} (0.432)^{5-x}, \quad x = 0,1,2,3,4,5 \longrightarrow (3)$$

Then, the expected frequencies are calculated by using the following formula.

$$F(x) = N \cdot P(X=x)$$
$$= 100 \times \left({}^{5}C_{x}\, (0.568)^{x} (0.432)^{5-x} \right) \longrightarrow (4)$$

Put $x=0$ in eq (4) $\Rightarrow$

$$F(0) = 100 \times \left({}^{5}C_{0}\, (0.568)^{0} (0.432)^{5-0} \right)$$
$$\doteq 1.5 \simeq 2. \text{ (approximately)}$$

Put $x=1$ in eq (4) $\Rightarrow$

$$F(1) = 100 \times \left({}^{5}C_{1}\, (0.568)^{1} (0.432)^{5-1} \right)$$
$$= 9.89 \simeq 10.$$

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Put $x=2$ in eq (4) $\Rightarrow$

$$F(2) = 100\left[{}^5C_2 (0.568)^2 (0.432)^{5-2}\right]$$

$$= 26.01 \simeq 26.$$

Put $x=3$ in eq (4) $\Rightarrow$

$$F(3) = 100\left[{}^5C_3 (0.568)^3 (0.432)^{5-3}\right]$$

$$= 34.199 \simeq 34$$

Put $x=4$ in eq (4) $\Rightarrow$

$$F(4) = 100\left[{}^5C_4 (0.568)^4 (0.432)^{5-4}\right]$$

$$= 22.4 \simeq 22.$$

Put $x=5$ in eq (4) $\Rightarrow$

$$F(5) = 100\left[{}^5C_5 (0.568)^5 (0.432)^{5-5}\right]$$

$$= 5.91 \simeq 6.$$

$\therefore$ The expected frequencies are 2, 10, 26, 34, 22, 6. Such that their sum is 100.

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