UNIT-I
WAVE OPTICS
1. Interference

Syllabus: Introduction - Interference in thin films (reflected light) – Newton's rings - Determination of wavelength.

Introduction:

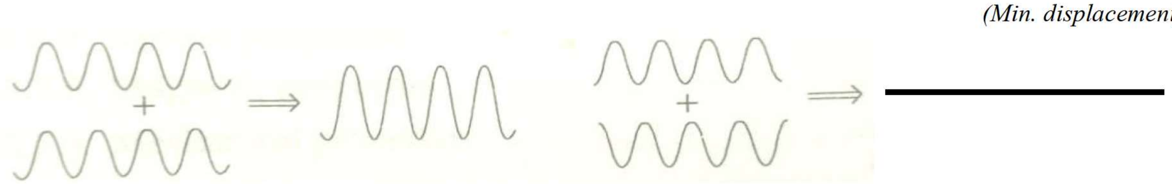
* Light is a form of energy that stimulates our vision.
* According to Huygens's theory, Light is considered to propagate in the form of waves.
* Maxwell showed that light is an electromagnetic wave.
* Refractive index (μ) of an optical medium is defined as the ratio of the velocity of light in a vacuum to the velocity of light in the medium.
* A light source emitting a single color / wavelength is called a *monochromatic source*.
* Two light waves which are having the same wavelength, amplitude, and constant phase difference are said to be *coherent sources*.
* *Phase:* The relative displacement between waves having the same frequency.
* Phase difference (ϕ) between the two waves is $\frac{2\pi}{\lambda}$ (path difference)
* Intensity is directly proportional to the square of the amplitude.

**** Principle of superposition of waves:***

When two or more waves travel simultaneously in a medium, the resultant displacement at any point is the vector sum of the individual displacement of the waves.

Let y_1 and y_2 be displacements of particles, then the resultant displacement due to two waves is $Y = y_1 + y_2$. *(Max. displacement)*

If two displacements are in opposite directions, the instantaneous displacement due to the two waves is $Y = y_1 - y_2$. *(Min. displacement)*



(Waves in phase) $Y = y_1 + y_2$ (Waves in out of phase) $Y = y_1 - y_2$.

Interference:

The physical effect of the superposition of waves from two coherent sources is called interference of light.

Interference is classified into two types: Division of wavefront and division of amplitude.

Division of wavefront: The wavefront originating from a common source is divided into two parts by using a mirror (or) prism and the two new wavefronts travel a long distance and are brought together to produce interference.

Ex. Young's double-slit experiment, Fresnel biprism, Lloyd's single mirror, etc…

Division of Amplitude: The light ray's incident on film/glass, the amplitude of the incident beam is divided into two or more parts either by partial reflection or refraction, these rays exhibit coherent nature. These rays travel different paths and are finally combined together and produce interference.

Ex. Newton's ring methods, Wedge methods, Interference in thin films, etc…..

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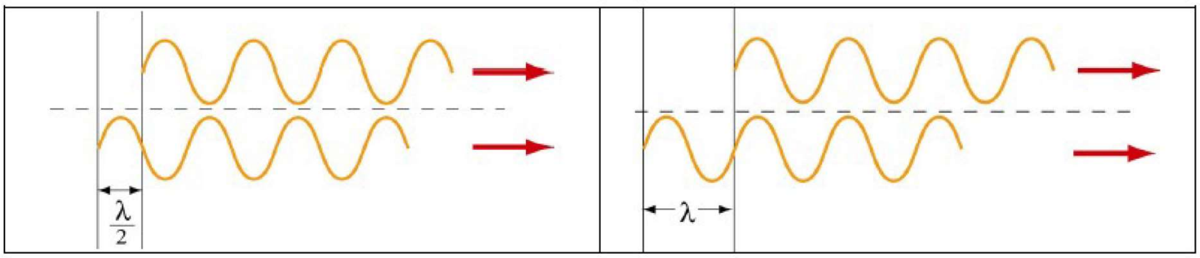
** Interference:

The physical effect of the superposition of waves from two coherent sources is called interference of light.

Interference is classified into two types: constructive interference and destructive interference.

The crests of one wave meet the crests of another wave, the resultant amplitude is maximum and is said to be ***constructive interference***. (Bright fringe)

In ***destructive interference***, the crest of one wave meets the trough of another wave, the resultant amplitude is minimum. (Dark fringe)



Destructive interference   Constructive interference

** Interference in thin films / Interference due to reflected light / Oblique incidence of a plane wave on a thin film / Cosine Law equation:

Let XY and X^lY^l be the two surfaces (top & bottom surfaces) of a transparent thin film. Let 't' be the uniform thickness and 'μ' refractive index of the film. Suppose a light ray AB (of monochromatic light) is incident on its upper surface of the film. This ray is partly reflected along with BC (into the air) and partly refracted along with BD. At point 'D' once again the light undergoes reflection (DE) as well as refraction (oblique downwards). At 'E' the light ray undergoes reflection (oblique downwards) as well as refraction (EF). The ray EF emerges into the air. The rays BC is parallel to EF. These two rays exhibit a coherent nature. <u>*Our aim is to find the path difference between the rays BC and EF.*</u> For this purpose, let's draw normal PE on BC.

The optical path difference (Δ) between the two reflected light rays (BC and EF) is

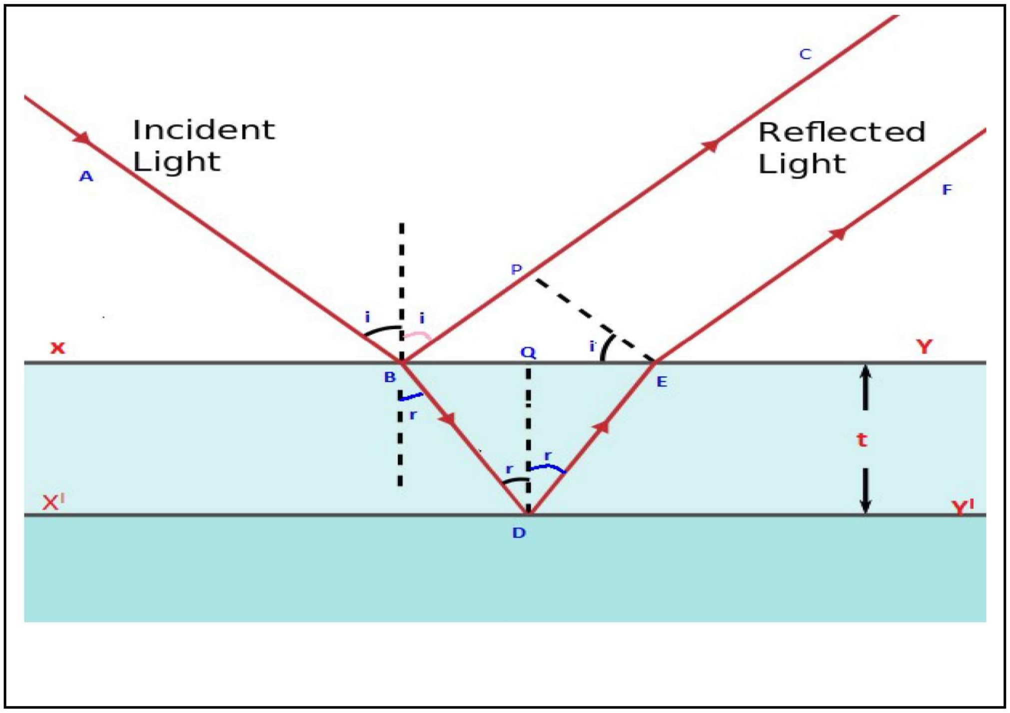
$$\Delta = \text{Path (BD + DE) in film} - \text{Path BP in air}$$

$$\Delta = \mu_{film}\,(BD + DE) - \mu_{air}\,BP$$

$$\Delta = \mu\,(BD + DE) - BP \qquad (\because \mu_{air} = 1) \quad \text{------(1)}$$

From ΔBQD, $\cos r = \dfrac{DQ}{BD}$ → $\cos r = \dfrac{t}{BD}$

$$BD = \frac{t}{\cos r} = DE \qquad \text{------(2)}$$



From ΔBPE, $\sin i = \dfrac{BP}{BE}$ → $BP = BE \sin i$

$$BP = (BQ + QE) \sin i \qquad \text{------(3)}$$

From ΔBQD, $\tan r = \dfrac{BQ}{QD}$ → $BQ = QD \tan r$

$$BQ = t \tan r = QE \qquad \text{------(4)}$$

Substituting Eq. (4) in Eq.(3) → $BP = 2t \tan r \sin i$

$$BP = 2t \frac{\sin r}{\cos r} \sin i$$

We know that $\mu = \dfrac{\text{Sin i}}{\text{Sin r}}$ → $\sin i = \mu \sin r$

$$\therefore BP = 2t \frac{\sin r}{\cos r} \mu \sin r \qquad \text{------(5)}$$

Substituting Eq. (2) & Eq. (5) in Eq.(1)

$$\Delta = \mu \left(\frac{t}{\cos r} + \frac{t}{\cos r} \right) - 2 \mu t \frac{\sin^2 r}{\cos r}$$

$$\Delta = \frac{2 \mu t}{\cos r} - 2 \mu t \frac{\sin^2 r}{\cos r}$$

$$\Delta = 2\mu t\left[\frac{1-\sin^2 r}{\cos r}\right]$$

$$\Delta = 2\mu t\left[\frac{\cos^2 r}{\cos r}\right]$$

$$\boxed{\Delta = 2\mu t \, Cos \, r}$$

This optical path difference is usually called the **Cosine law**.

When a ray is reflected at the boundary of a *rarer to denser medium then the abrupt phase change is π, which is equivalent to a path difference of* $\lambda/2$

∴ **The effective path difference is $2\mu t \, Cos \, r \pm \lambda/2$**

Case:(1)

We know that maxima occur when the effective path difference is $n\lambda$

$\therefore \; \Delta = 2\mu t \, Cos \, r \pm \lambda/2 = n\lambda$

$2\mu t \, Cos \, r = (2n \pm 1)\lambda/2$

If the reflected rays satisfy this condition, we will get bright fringes.

Case:(2)

We know that minima occur when the effective path difference is $(2n \pm 1)\,\lambda/2$

$\therefore \; \Delta = 2\mu t \, Cos \, r \pm \lambda/2 = (2n \pm 1)\,\lambda/2$

$2\mu t \, Cos \, r = n\lambda$

If the reflected rays satisfy this condition, we will get dark fringes.

Note:

Colors of thin films:

When a thin film is exposed to sunlight (or) white light then beautiful colors are observed.
For example ***(i)*** *White light is exposed to a soap bubble of thickness 't'. At different points, the reflected light rays satisfied the equation $2\mu t$ Cos r = $(2n \pm 1)\lambda/2$ and we will observe different colors from the soap bubble. (In a soap bubble 'μ and λ are variables)*

(ii) *When white light is exposed to a thin layer of oil film floating on water, then also we will get the different colors from the films. At different points on the oil film, the reflected rays satisfy the equation $2\mu t$ Cos r = $(2n \pm 1)\lambda/2$ and produce different colors. Here 'r' is constant and the thickness of the film 't' may not be constant throughout the film ('t' is constant at the center region while at the edges it reduces).*

** Newton's Rings:

Let's take a plano-convex lens (P) and its convex surface is placed on a plane glass plate (P_1) as a result air film is formed between the lower surface of the lens and the upper surface of the glass plate (*shown in Fig.1*). At the point of contact (0) where the lens touches the glass plate the thickness of the air film is zero. The thickness of air film increases along its radial direction.

Let 'S' be the monochromatic source (*sodium lamp*), the light coming from the source (S) is incident on the lens (L) and we will get a fine parallel beam of light. This parallel beam of light is incident on a glass plate 'G' held at an angle of 45°. A part of the light beam is reflected from glass plate G and incident on the air film enclosed between the plano-convex lens (P) and glass plate (P_1). These light rays are reflected from the bottom surface of the lens (P) and the top surface of the glass plate (P_1) *(i.e. top and bottom surface of air film)*. These reflected rays exhibit a coherent nature. These rays combine together and produce an interference pattern. This can be seen with a microscope (M). The experimental setup is shown in Fig.1.

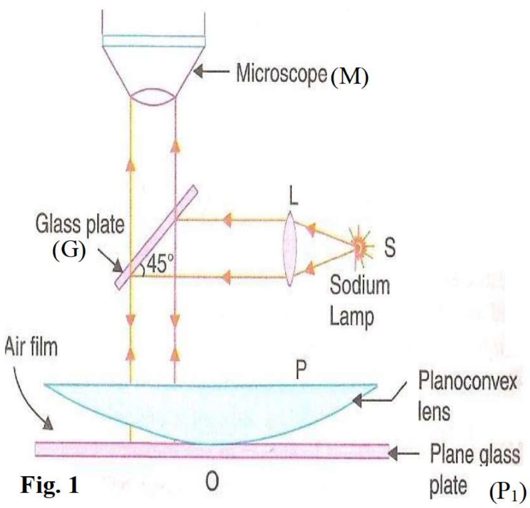


Fig. 1

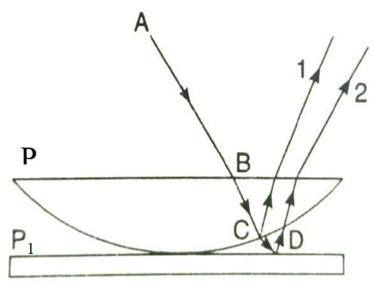


Fig. 2

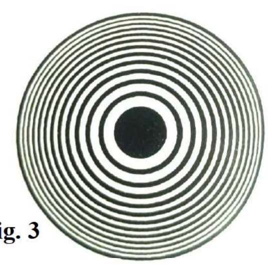


Fig. 3

Formation of Newton's rings:

AB is a monochromatic light ray incident on a plano-convex lens. This ray is partly reflected at C and comes out in the form of ray 1. The other part is refracted along CD *(into the air)*, at 'D' it's again reflected and comes out in the form of ray 2 (*with a phase change π*). The reflected rays 1 & 2 exhibits a coherence nature and these rays combine and produce interference fringes. It is shown in Fig. 2 & 3. Between P & P_1, air film is circularly hence Newton's rings are appearing circularly.

As the rings are observed in the reflected light, then the path difference is $2\mu t \cos r + \lambda/2$

For the air film $\mu = 1$ and form normal incidence $r = 0^\circ$ then the path difference is $2t + \lambda/2$

Calculation of diameter of rings:

Let LOL_1 be the plano-convex lens placed on a glass plate (P_1). The curved surface LOL_1 is the part of the spherical surface (in Fig.4 - shown in dotted points) with center C. Let 'R' be the radius of curvature and 'r' be the radius of Newton's ring corresponding to the thickness of the constant film 't'.

We have the condition for the bright ring is $2t + \lambda/2 = n\lambda$

$$2t = (2n-1)\lambda/2 \quad \text{where } n = 1, 2, 3, \ldots \qquad \text{------(1)}$$

We have the condition for the dark ring is $2t + \lambda/2 = (2n+1)\lambda/2$

$$2t = n\lambda \qquad \text{where } n = 1, 2, 3, \ldots \qquad \text{------(2)}$$

From the property of the circle

$$NP \times NQ = NO \times ND$$

(OR)

From Fig.4 $\quad r \times r = t(2R-t)$

$$r^2 = 2Rt - t^2$$

$$r^2 = 2Rt$$

($\because$ t is very small, then t^2 can be neglected)

$$t = r^2/2R \qquad \text{------(3)}$$

From Fig.4 $\Delta PNC \quad PC^2 = NP^2 + NC^2$

$$R^2 = r^2 + (R-t)^2$$

$$R^2 = r^2 + R^2 + t^2 - 2Rt$$

($\because$ t is very small, then t^2 can be neglected)

$$r^2 = 2Rt$$

$$t = r^2/2R \qquad \text{------(3)}$$

Substituting Eq. (3) in Eq.(1) → Thus, for a bright ring

$$\frac{2r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$r^2 = (2n-1)\frac{\lambda R}{2}$$

Replacing 'r' by '$D/2$' (r is the radius of the ring, D is the diameter of the ring)

then $$\frac{D^2}{4} = (2n-1)\frac{\lambda R}{2}$$

$$D = \sqrt{(2\lambda R)}\sqrt{(2n-1)}$$

$$D \,\alpha\, \sqrt{(2n-1)}$$

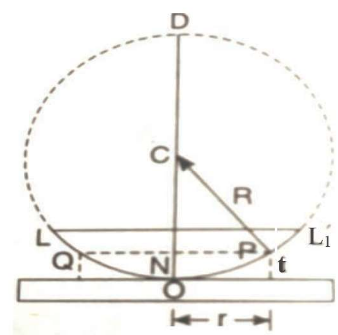


Fig. 4

Thus, the diameters of the bright rings are proportional to the square roots of odd numbers.

Similarly for the dark ring *(substituting Eq. 3 in Eq.2)* $\quad \frac{2r^2}{2R} = n\lambda \quad \rightarrow \quad r^2 = n\lambda R$

Replacing r by $D/2$ (r is the radius of the ring, D is the diameter of the ring)

then $$\frac{D^2}{4} = n\lambda R \quad \rightarrow \quad D = 2\sqrt{n\lambda R} \qquad \text{------ (4)}$$

$$D \,\alpha\, \sqrt{n}$$

Thus, the diameters of the dark rings are proportional to the square roots of natural numbers.

Determination of wavelength of sodium light:

Let 'R' be the radius of curvature of the plano-convex lens (P_1), 'λ' be the wavelength of light used, D_m and D_n be the diameter of m^{th} and n^{th} dark rings respectively. From Eq.(4) we have

$$(D_n)^2 = 4\,n\,\lambda\,R \qquad \text{------ (5)}$$

$$(D_m)^2 = 4\,m\,\lambda\,R \qquad \text{------ (6)}$$

Eq.(6) – Eq.(5) → $(D_m)^2 - (D_n)^2 = 4\,m\,\lambda\,R - 4\,n\,\lambda\,R$

$$(D_m)^2 - (D_n)^2 = 4\,\lambda\,R\,(m - n)$$

$$\lambda = \frac{D_m^2 - D_n^2}{4(m-n)\,R}$$

Note:

i. Radius of curvature of the lens (P_1) can be determined using Boy's method.

ii. The radius of the plano-convex lens also can be determined using a spherometer

$$R = \frac{l^2}{6h} + \frac{h}{2}$$

Where 'l' is the distance between the legs of the spherometer,

'h' is the difference in the reading of the spherometer when it is placed on the lens as well as when placed on the plane surface.

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Unit-I: Wave Optics

2. Diffraction

Syllabus: Introduction - Fraunhofer diffraction - Single slit diffraction (qualitative) - Double slit diffraction (qualitative).

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* When light falls on obstacles / small apertures whose size is comparable with the wavelength of light, then the light bends around the corners of the obstacles. This bending of light waves is called diffraction.

* Diffraction pattern that produces bright and dark fringes are known as diffraction fringes.

* In diffraction fringes, the central portion on the screen is known as the central maximum (it is always bright).

Difference between interference and Diffraction:

	Interference	Diffraction
i.	Superposition is due to two separate wavefronts originating from two coherent sources.	The bending of light waves is called diffraction. (or) Superposition is due to secondary wavelets originating from different parts of the same wavefront.
ii.	Fringes are having uniform width.	Fringes are having non-uniform width.
iii.	The region of minimum intensities is perfectly dark.	The region of minimum intensities is not perfectly dark.
iv.	All bright bands are of the same intensity.	All bright bands are not of the same intensity.

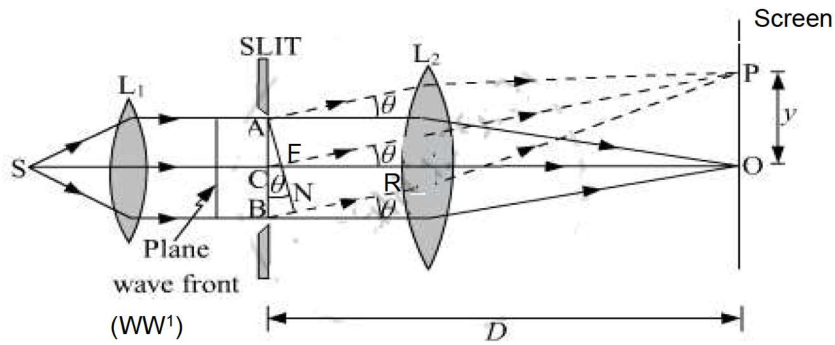
The diffraction phenomenon is classified into two types.

	Fresnel Diffraction	Fraunhofer Diffraction
i.	The source and screen are placed at a finite distance from the aperture.	The source and screen are placed at an infinite distance from the aperture.
ii.	No lenses are used to focus the rays.	A converging lens is used to focus parallel rays.
iii.	The incident wavefronts are either spherical or cylindrical.	The incident wavefront must be plane.

** Fraunhofer Diffraction at single slit:

Fraunhofer diffraction by a single slit is shown in Fig. Let 'S' be the monochromatic source of wavelength λ and L_1 is a collimating lens. Let a plane wavefront WW^1 is incident normally on the slit (AB) of width 'e'. Let the diffracted light be focused using a convex lens (L_2) on a screen placed in the focal plane of the lens. According to Huygens-Fresnel at the slit, every point on the wavefront will act as a source of secondary wavelets, which spread out to the right in all directions. The secondary wavelets traveling normally to the slit *(i.e along direction CO)* are brought to focus at O by the lens. The path difference between the rays AO & BO is zero hence the intensity at O is maximum and is called central maximum.

The secondary wavelets traveling at an angle 'θ' with the normal are focused at a point P on the screen. The intensity at point P may be maximum or minimum depending upon the path difference between the secondary waves originating from the corresponding points of the wavefront (ACB).



In order to find intensity at P, lets us draw a perpendicular AN on BR. After AFN all the waves travel the same distance to reach P.

∴ The path difference between secondary wavelets from ACB in direction 'θ' is BN.

From ΔANB, Sin θ = $^{BN}/_{AB}$

BN = AB Sin θ

Path difference (BN) = e Sin θ

∴ Phase difference = $\frac{2\pi}{\lambda}$ (path difference) = $\frac{2\pi}{\lambda}$ e Sin θ

Let's consider that the width of the slit is divided into 'n' equal parts and the amplitude of the wave from each part is 'a'. The phase difference between any two consecutive waves from these parts is $\frac{1}{n}$ (total Phase) = $\frac{1}{n}\left(\frac{2\pi}{\lambda} \text{ e Sin } \theta\right)$ = d (say) ------(1)

From the vector addition of amplitude, the resultant amplitude (R) = $a \, \frac{\text{Sin}\ ^{nd}/_2}{\text{Sin}\ ^{d}/_2}$ ------(2)

Substituting Eq.(1) in Eq.(2)

$$R = a\frac{sin\left(\frac{\pi\ e\ sin\theta}{\lambda}\right)}{sin\left(\frac{\pi\ e\ sin\theta}{n\lambda}\right)}$$

$$R = a\frac{sin\alpha}{sin\left(\frac{\alpha}{n}\right)} \qquad \text{where } \alpha = \frac{\pi\ e\ sin\theta}{\lambda} \qquad \text{------ (3)}$$

$$R = a\frac{sin\alpha}{\left(\frac{\alpha}{n}\right)}$$ ∵ $(^{\alpha}/_{n})$ is very small, hence Sin $(^{\alpha}/_{n})$ = $(^{\alpha}/_{n})$

$$R = an\frac{sin\alpha}{\alpha}$$

Thus, the resultant amplitude is $R = A\frac{\text{Sin }\alpha}{\alpha}$ ------(4)

We know that intensity is the square of the amplitude

$$I = R^2 = A^2\left(\frac{\text{Sin }\alpha}{\alpha}\right)^2 \quad \text{------(5)}$$

$$I = I_o\left(\frac{\text{Sin }\alpha}{\alpha}\right)^2 \quad \text{------(6)}$$

Principal maximum: Expression for resultant amplitude R can be written in ascending powers of α as

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3\,!} + \frac{\alpha^5}{5\,!} - \frac{\alpha^7}{7\,!} + \ldots\ldots\right]$$

$$R = A\left[1 - \frac{\alpha^2}{3\,!} + \frac{\alpha^4}{5\,!} - \frac{\alpha^6}{7\,!} + \ldots\ldots\right]$$

If $\alpha = 0$, then R will be maximum.

From Eq.(3) $\alpha = \frac{\pi\, e \text{ Sin }\theta}{\lambda} = 0 \rightarrow \text{Sin }\theta = 0$ (or) $\theta = 0$

$\therefore R = A$, Intensity (I) is proportional to A^2

When θ = 0, the intensity at 'O' is maximum and is called principal maximum (or) central maximum.

Minimum intensity positions: Intensity will be minimum when $\text{Sin }\alpha = 0$

i.e $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots\ldots\ldots = \pm m\pi$

Eq.(3) $\alpha = \frac{\pi\, e \text{ Sin }\theta}{\lambda} = \pm m\pi$

$\text{Sin }\theta = \pm m\lambda$ where m = 1,2,3,……..

This is the condition for the minimum intensity on either side of the principal maximum.

(m = 0 is inadmissible, since θ = 0 corresponds to principal maximum)

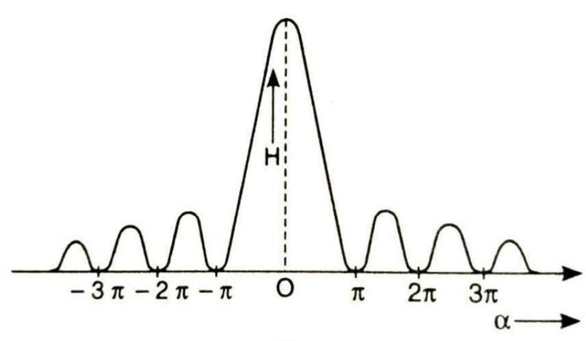
Conditions for Maxima: In addition to a principal maximum at $\alpha = 0$, there are weak secondary

maxima were obtained when $\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots\ldots$

Substituting 'α' in Eq.(5) $I = A^2$

$I_o = A^2\left(\frac{\text{Sin }\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = \frac{A^2}{22}$ (1^{st} subsidiary Maximum)

$I_1 = A^2\left(\frac{\text{Sin }\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2 = \frac{A^2}{62}$ (2^{nd} subsidiary Maximum)



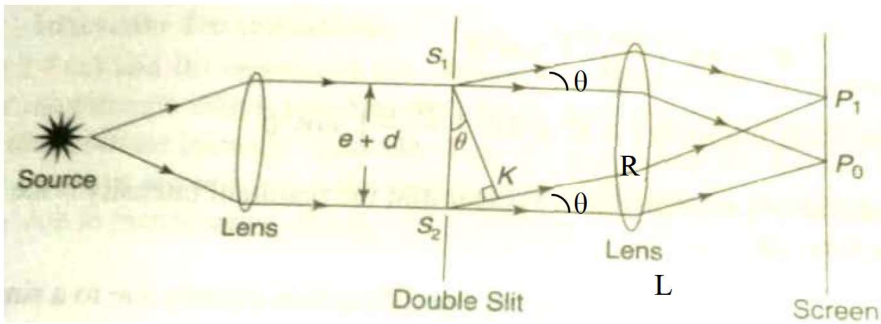
The variation of intensity distribution with α is shown in Fig.2

Fig.2

The subsidiary maxima are decreasing intensity obtained on either side of the central maxima (i.e. at $\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}\ldots$), between these maxima, there are minima at $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots\ldots$

** Fraunhofer Diffraction at a double slit:

Let S be the monochromatic source. Let S_1 and S_2 be the two slits of width 'e' and separated by an opaque length 'd'. The distance between the middle points of the two slits is (e+d) and is called grating element. Let a monochromatic light of wavelength λ be incident normally on two slits. The light is diffracted from these slits and is brought to focus at P_o on the screen using a converging lens L. The distance traveled by these rays is the same, hence the intensity at P_o is maximum and is called central maximum.



When the plane wavefront is incident normally on both slits, all the points within the slits become the source of secondary wavelets which travels in all directions. The secondary waves traveling in a direction are inclined at an angle 'θ' and these waves come to focus at P_1. The intensity at P_1 will be dark or bright depending on the path difference between the secondary waves. *The diffraction at two slits is the combination of diffraction as well as interference. The pattern on the screen is the diffraction pattern due to a single slit on which a system of interference fringes is superposed.*

In order to find intensity at P_1, lets us draw a perpendicular S_1K on S_2R. After S_1K all the waves travel the same distance to reach P_1.

∴ The path difference between the wavelets from S_1 & S_2 in direction 'θ' is S_2K.

From ΔS_1KS_2 $\sin\theta = {}^{S_2K}/_{S_1S_2}$

$$S_2K = S_1S_2 \sin\theta$$

Path difference (S_2K) = (e+d) Sin θ

Where 'e' be the width of each slit, 'd' be the width of opaque part, (e+d) is called grating element.

∴ Phase difference (ϕ) = $\frac{2\pi}{\lambda}$ (path difference)

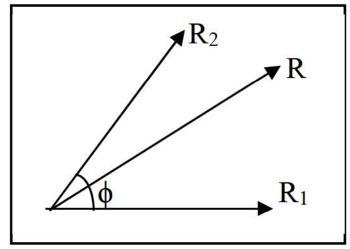
$$\phi = \frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta \quad \text{(say)} \qquad \text{------ (1)}$$

We know that the resultant amplitude due to a single slit is $R = A\frac{\sin\alpha}{\alpha}$

Let R_1 & R_2 be resultant amplitude due to each slit.

The resultant intensity (R) due to two slits can be calculated using the parallelogram law of vectors.

$$R = \sqrt{R_1^2 + R_2^2 + 2R_1R_2\cos\phi}$$



$$= \sqrt{A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 + A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 + 2A\left(\frac{\sin\alpha}{\alpha}\right)A\left(\frac{\sin\alpha}{\alpha}\right)\cos\phi}$$

$$= A\left(\frac{\sin\alpha}{\alpha}\right)\sqrt{1+1+2\cos\phi}$$

$$= A\left(\frac{\sin\alpha}{\alpha}\right)\sqrt{2\,(1+\cos\phi)}$$

$$= A\left(\frac{\sin\alpha}{\alpha}\right)\sqrt{2\,[2\cos^2\left(\frac{\phi}{2}\right)]}$$

$$= A\left(\frac{\sin\alpha}{\alpha}\right)2\cos\left(\frac{\phi}{2}\right)$$

$$R = A\left(\frac{\sin\alpha}{\alpha}\right)2\cos\beta \qquad \text{------ (2)} \qquad (\because \text{Eq. } 1 \rightarrow \phi = 2\beta)$$

Intensity $I = R^2$

$$I = 4A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \cos^2(\beta)$$

The above equation $A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$ represents intensity due to the diffraction pattern of a single slit and $\cos^2\beta$ represents the interference pattern due to wavelets from double slits. The resultant intensity is due to diffraction and interference.

Diffraction Effect:

We get principal maximum at $\theta = 0$

The position of secondary maxima occurs for when $\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$……

Let's check if $\cos^2\beta$ is due to the interference effect or not:

The interference term $\cos^2\beta$ gives the equidistant bright and dark fringes.

The maxima will occur for $\cos^2\beta = 1$

When $\beta = \pm n\pi$ Where n = 1, 2, 3, ……

$$\text{Eq.(1)} \rightarrow \quad \frac{\pi\,(e+d)\sin\theta}{\lambda} = \pm n\pi$$

$$(e+d)\sin\theta = \pm n\lambda$$

i.e. Path difference between two rays = $n\,\lambda$

This represents the condition for constructive interference (maxima).

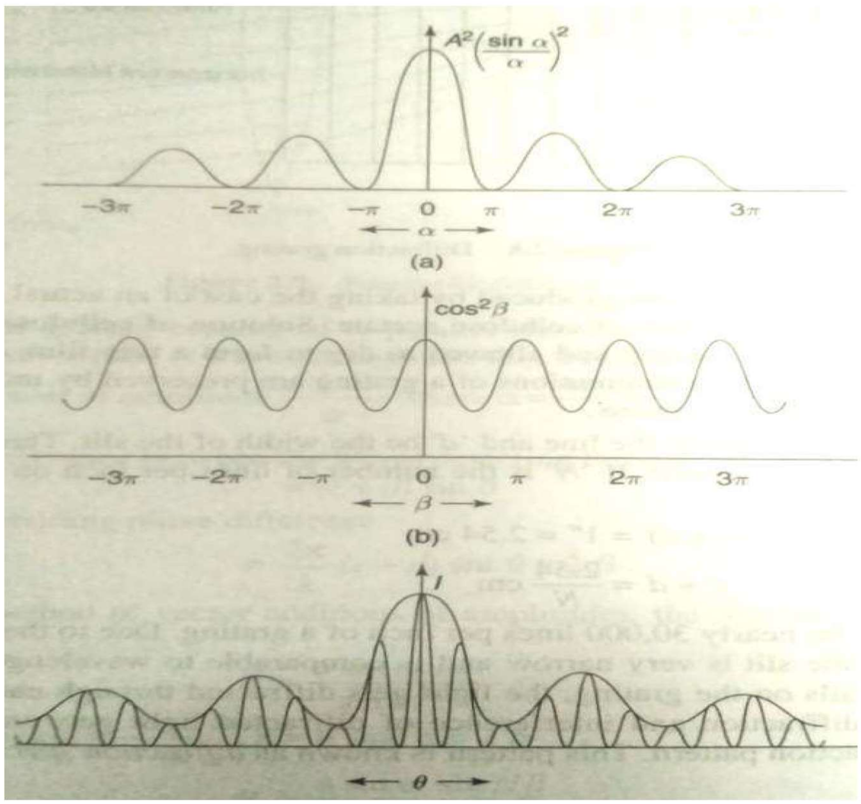
The minima will occur for $\cos^2\beta = 0$, when $\beta = \pm (2n + 1)\frac{\pi}{2}$ Where n = 0, 1, 2, …..

Eq.(1) → $$\frac{\pi\,(e+d)\,\text{Sin}\,\theta}{\lambda} = \pm (2n + 1)\frac{\pi}{2}$$

$$(e+d)\,\text{Sin}\,\theta = \pm (2n + 1)\frac{\lambda}{2}$$

i.e. Path difference between two rays = $\pm (2n + 1)\frac{\lambda}{2}$

This represents the condition for destructive interference (minima).



Intensity distribution: a) Diffraction effect b) Interference effect c) Resultant Intensity

Unit-I: Wave Optics

3. Polarization

Syllabus: Introduction - Polarization by reflection and double refraction - Nicol's prism - Half wave and Quarter wave plate - Engineering applications of interference - diffraction and polarization.

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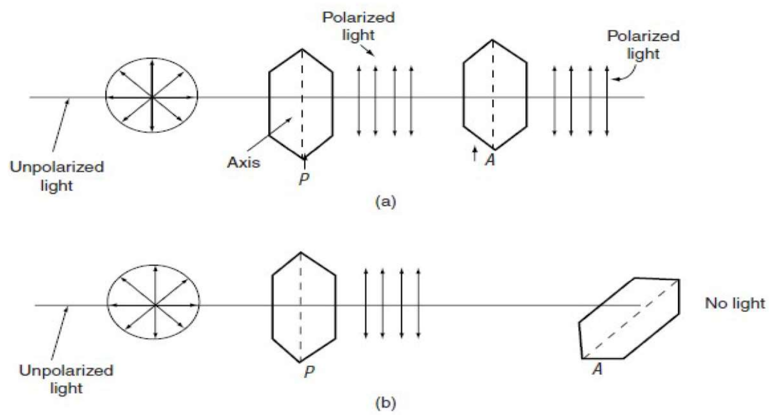
***Introduction**:* Interference and diffraction say that light exhibits a wave nature. But these phenomena didn't reveal the character of wave motion. i.e. Whether it is longitudinal or transverse in nature. The phenomenon of polarization concludes that light exhibits transverse nature. Light is made up of electromagnetic rays, it has electric and magnetic fields. These fields are perpendicular to each other. Electrical vibrations are responsible for vision. The light coming from conventional light sources such as electric bulbs, burning candles, sodium lamps, mercury lights, etc. are unpolarized (equally distributed in all directions).

When unpolarized light is passing through tourmaline (or) calcite crystals, then the transmitted light travels only in one direction, this phenomenon is called polarization.

Let an unpolarized light be passing through a tourmaline crystal (P), the transmitted light contains electric vectors in a direction parallel to the axis of the crystal, as a result, we will get polarized light. This polarized light passes through another tourmaline crystal (A) [called analyzer] which is kept parallel to the polarizer (P), as a result, the polarized light passes through A also (Shown in Fig. a).



If crystal A is rotated about the axis of an incident ray (shown in Fig. b), the emergent beam from A varies. When the crystal axes of P and A are parallel, then the intensity of the emergent ray is maximum, if P & A are perpendicular then the intensity is minimum (no light). *Note: A crystal used to produce polarized light is called a polarizer (P) and the crystal used to analyze the polarized light is called an analyzer (A).*

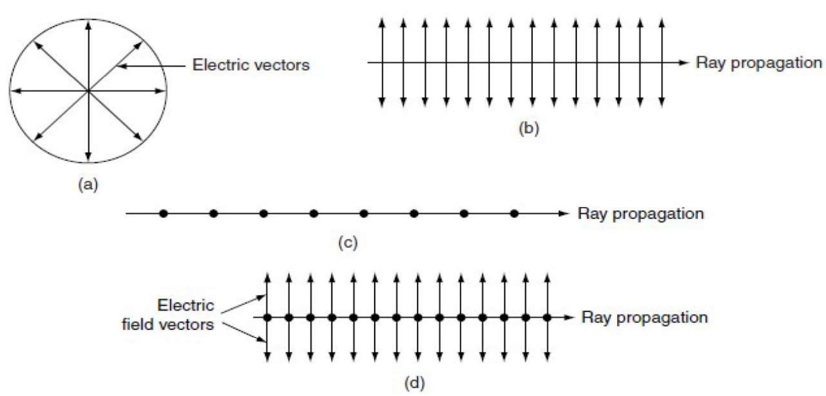
Representation of various types of light:

Unpolarized light is shown in Fig. a.

Linearly polarized light is shown in Fig. b & c.

In Fig. b, the direction of electric field vectors lies in the plane of the paper and it is perpendicular to the plane of the paper in Fig. c.

Fig. d is unpolarized light (or) ordinary light.



The light beam which has to vibrate in no. of plains is called unpolarized light
The light beam which has vibrated in only one plain is called polarized light

** Polarization by reflection:

In 1809, Malus discovered when unpolarized light is incident on a transparent medium (like glass or water), reflected light is partially or completely polarized.

Brewster studied the polarization of light by reflection on different transparent surfaces. He observed that for a particular angle of incidence [θ_i], the reflected light is completely plane polarized while the refracted / transmitted light is partially plane-polarized. This angle of incidence is known as *Brewster's angle* or *angle of polarization* [θ_p].

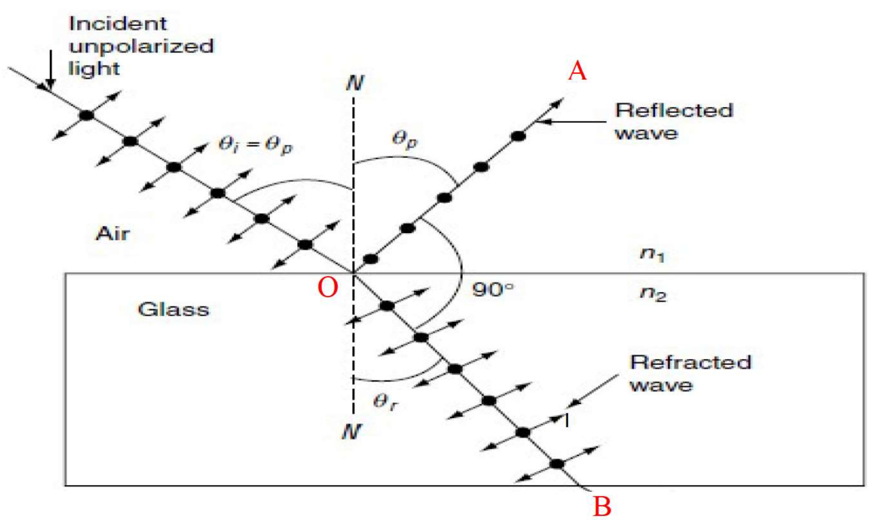
Brewster's Law: The tangent of the angle of polarization [θ_p] is numerically equal to the refractive index of the medium. i.e. $\mu = \tan(\theta_p)$ ------ (1)

When an unpolarized light incident on glass, at O part of the light, undergoes reflection (OA) and part undergoes refraction (OB). Now by changing the incidence angle, at a particular angle of incidence, the reflected light is polarized that incidence angle is called the *angle of polarization* [θ_p].

Note:

For glass θ_p = 57.5°

and water θ_p = 53°.



From Snell's law, $n_1 \sin\theta_i = n_2 \sin\theta_r$

$$n_1 \sin\theta_p = n_2 \sin\theta_r \qquad (\because \theta_i = \theta_p)$$

Where n_1 is the refractive index of air ($n_1 = 1$), n_2 is the refractive index of the glass plate, θ_i is the angle of incidence = angle of reflection (θ_p), θ_r is the angle of refraction

$$\mu = n_2 = \frac{\sin\theta_p}{\sin\theta_r} \qquad \text{------ (2)}$$

Comparing Eq. (1) and (2)

$$\mu = \tan\theta_p = \frac{\sin\theta_p}{\sin\theta_r} \rightarrow \frac{\sin\theta_p}{\cos\theta_p} = \frac{\sin\theta_p}{\sin\theta_r} \rightarrow \cos\theta_p = \sin\theta_r$$

$$\sin(90° - \theta_p) = \sin\theta_r$$

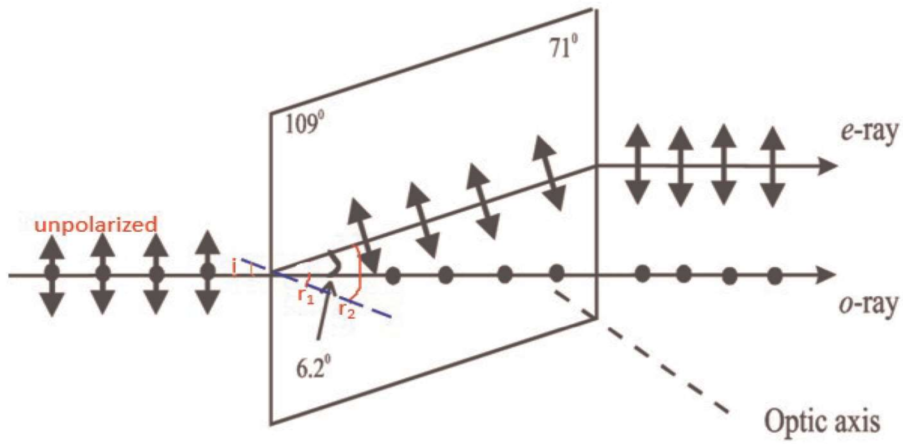
$$\therefore 90° - \theta_p = \theta_r$$

$$\theta_p + \theta_r = 90°$$

Brewster's proved that the reflected and refracted rays are perpendicular to each other.

** Double refraction / Birefringence:

When an unpolarized light beam incidents on anisotropic crystals such as quartz or calcite, the refracted beam is split into two rays, this phenomenon is known as double refraction. Double refraction in calcite crystal is shown in Fig. The refracted beams are plane-polarized. One beam is polarized along the direction of the optic axis and is known as an extraordinary ray (e-ray), while the other refracted beam is polarized along the direction perpendicular to the optic axis and is known as an ordinary ray (o-ray). At the splitting position, the angle between e-ray & o-ray is 6.2°. The emergent rays (e-ray & o-ray) are parallel to each other.



The refractive indices of o-ray and e-ray are

$$\mu_o = \frac{\sin i}{\sin r_1}, \qquad \mu_e = \frac{\sin i}{\sin r_2}$$

For calcite crystal, $\mu_o > \mu_e$ $\quad (\because r_1 < r_2)$

$\therefore$ Inside the crystal, the velocity of the o-ray is less than the velocity of the e-ray.

If the velocity of the e-ray is less than the o-ray, such a type of crystal is called a ***positive crystal***.

Ex. Quartz

If the velocity of the e-ray is greater than the o-ray, such a type of crystal is called a ***negative crystal***.

Ex. Calcite

Note:

Along the optic axis, the refractive indices $\mu_e = \mu_o$. So, the wavefronts of e-ray and o-ray will coincide. The velocities of o-ray and e-ray are the same along the optic axis. It is shown in Fig. In quartz, crystal $\mu_e > \mu_o$ in all other directions, and it is very large in the direction perpendicular to the optic axis. In calcite crystal, $\mu_e < \mu_o$ in all directions except the optic axis. So, the speed of the e-ray is larger than that of the o-ray.

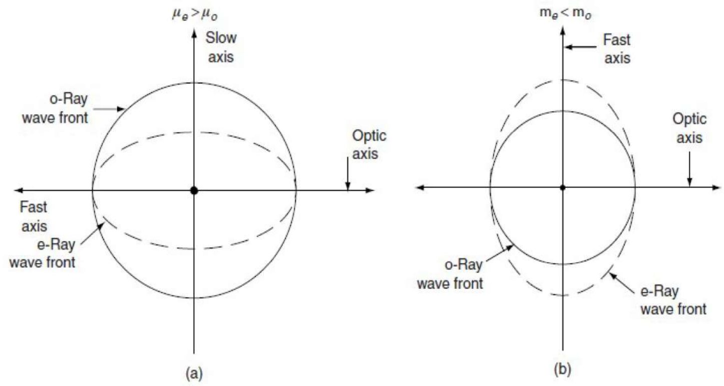


Fig. Wavefronts in a quartz crystal (b) wavefronts in a calcite crystal

Note: Isotropic crystal: Physical & mechanical properties of refraction isotropic medium are the same in all directions, if not is called an anisotropic crystal.

An imaginary line passing from peak to peak is known as the *optic axis*.

If a crystal contains only one optic axis is known as an *un-axial crystal*. Ex. Quartz, Calcite

If a crystal contains two optic axes is known as a *biaxial crystal*. Ex. Mica

Any plane which contains the optic axis & is perpendicular to two opposite faces is called a principal section.

** Nicol's prism:

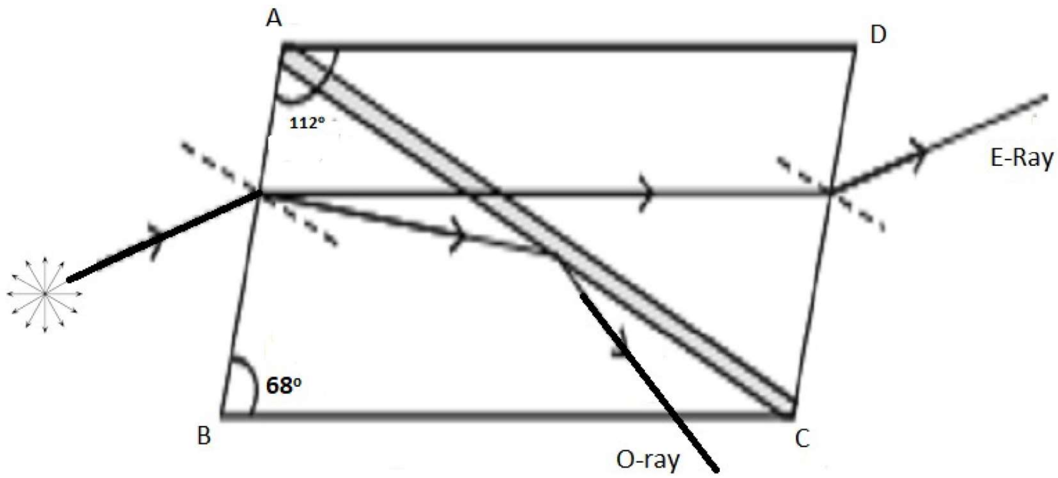
Nicol's prism is an optical device used to produce and analyze plane-polarized light. This was invented by W. Nicol in the year 1828. Nicol's prism is made from a double-refracting calcite crystal.

Construction:

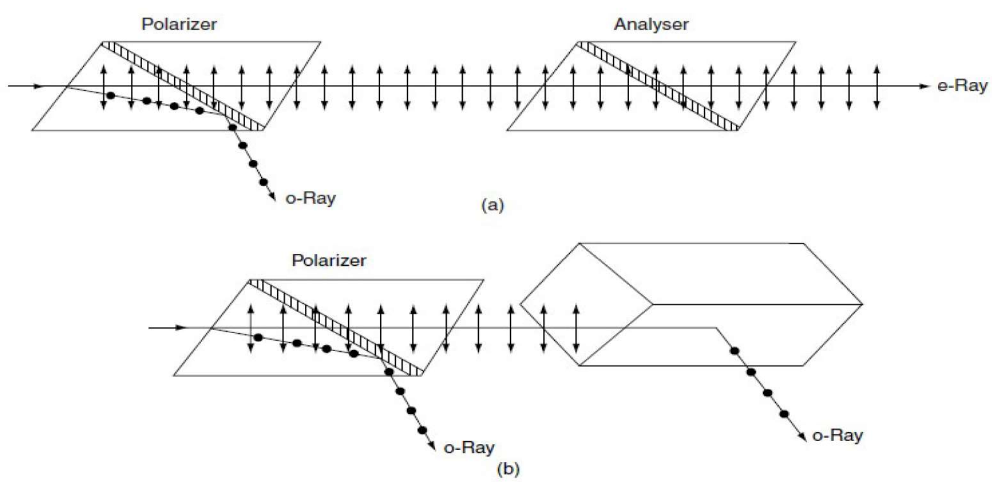
A calcite crystal whose length is three times its width is taken. The end faces of this crystal are grounded in such a way that the angles in the principal section become 68° and 112° instead of 71° and 109°. The crystal is cut into two pieces by a plane perpendicular to the principal section (along with the diagonal AC). The two cut surfaces are grounded, polished optically flat, and then cemented together by the Canada balsam layer (along with AC). (μ of Canada balsam is 1.55). This polished prism is called the Nicol prism.

Action: A beam of unpolarized light incident on Nicol prism *(doubly refracted crystal: calcite)*, the refracted beam is split into two rays. One ray is called an e-ray and the other is an o-ray. The e-ray moves in the direction of the optic axis and the o-ray moves perpendicular to the optic axis.

In calcite crystal, the refractive indices of the ordinary ray are 1.658 and the extraordinary ray is 1.486. From the refractive indices' values, the Canada balsam acts as a rarer medium for the o-ray and it acts as a denser medium for the e-ray. When the angle of incidence for an ordinary ray is greater than the critical angle then total internal reflection takes place (i.e. O-ray is reflecting from Canada balsam), while the extraordinary ray gets transmitted through the calcite-balsam surface *(Since light ray passes from rarer to denser medium).*



Nicol's prism can be used as an analyzer. Two Nicol prisms are placed adjacently as shown in Fig.(a). When an unpolarized light passes through a polarizer, we get polarized light. This polarized light passes through another prism (analyzer). When the second prism (analyzer) is slowly rotated, the intensity of the e-ray gradually decreases. When two prisms are perpendicular to each other, the intensity of light is minimum. Because the e-ray that comes out from the first prism will enter the second prism and act as an o-ray. So, this light is reflected in the second prism (Fig. b). Hence Nicol prism can be used as both a polarizer and an analyzer.



Limitations: To obtain plane-polarized light, the incident beam should be confined with an angle of 14°.

In prism, the angle at B must be 68° since the critical angle is 69° (*for o-ray at the calcite-balsam surface).*

** Quarter-wave plate:

A quarter-wave plate is a thin double-refracting calcite / quartz crystal. Let's take the crystal in the form of a plate with its optic axis along the surface. When unpolarized light of wavelength λ falls normally on the crystal surface, o-wave and e-wave move along the propagating direction with different velocities. The path difference between the two polarized light waves is $\frac{\lambda}{4}$(phase difference is $\frac{\pi}{2}$).

Let 't' be the thickness of plate calcite crystal, the optic axis is parallel to the surface. The optical path covered by the rays in the crystal of thickness 't' is $\mu_o t$ and $\mu_e t$.

$\therefore$ Path difference $(\Delta) = \mu_o t - \mu_e t$

$$\Delta = t\,(\mu_o - \mu_e) \qquad \text{------ (1)}$$

As the crystal is a quarter-wave plate, then the path difference between o-ray and e-ray is

$$\Delta = \frac{\lambda}{4} \qquad \text{------ (2)}$$

From Eq. (1) & (2) → $t = \frac{\lambda}{4(\mu_o - \mu_e)}$

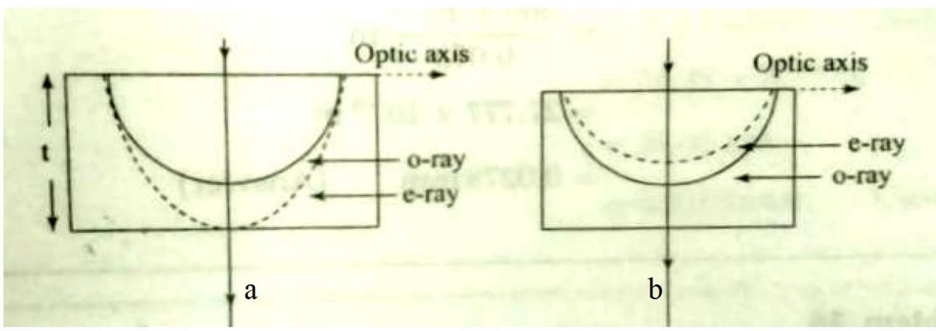


Fig. Waveplates in a) Negative Crystal b) Positive crystal

Note: The crystals like quartz, where $\mu_e > \mu_o$, the thickness of the quarter-wave plate $d = \frac{\lambda}{4(\mu_e - \mu_o)}$

** Half-wave plate:

A half-wave plate is a thin double-refracting calcite / quartz crystal. Let's take the crystal in the form of a plate with its optic axis along the surface. When unpolarized light of wavelength λ falls normally on the crystal surface, o-wave and e-wave move along the propagating direction with different velocities. The path difference between the two polarized light waves is $\frac{\lambda}{2}$ (phase difference is π).

Let 't' be the thickness of plate calcite crystal, the optic axis is parallel to the surface.

The optical path covered by the rays in the crystal of thickness 't' is $\mu_o t$ and $\mu_e t$.

$\therefore$ The path difference $(\Delta) = \mu_o t - \mu_e t$

$$\Delta = t\ (\mu_o - \mu_e) \qquad \text{------ (1)}$$

As the crystal is a half-wave plate, then the path difference between o-ray and e-ray is

$$\Delta = \frac{\lambda}{2} \qquad \text{------ (2)}$$

From Eq. (1) & (2) → $t = \dfrac{\lambda}{2(\mu_o - \mu_e)}$

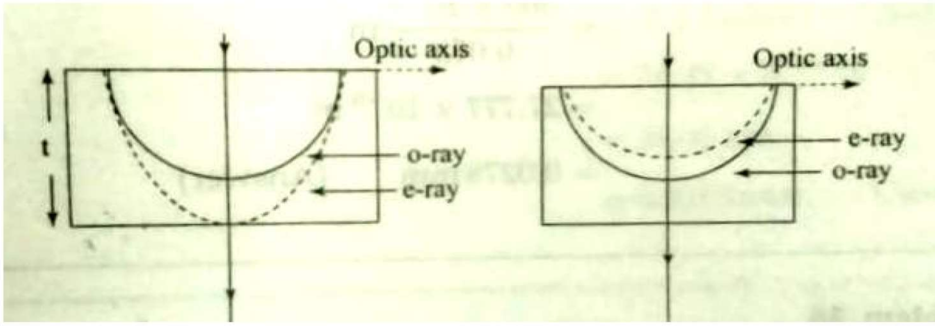


Fig. Waveplates in a) Negative Crystal b) Positive crystal

Note: The crystals like quartz, where $\mu_e > \mu_o$, the thickness of the quarter-wave plate $d = \dfrac{\lambda}{2(\mu_e - \mu_o)}$

** Engineering applications of interference, diffraction, and polarization:

Interference:

i. Interference technique is used in testing surface quality (like: flat surface, spherical surface, the roughness of the surface, etc.)

ii. used in space applications (in retrieving images from the telescopes)

iii. This technique is used in recording holography.

iv. Used in recording and reproducing 3D films.

iv. Destructive interference is used in the antireflection coating on spectacle glasses.

v. Destructive interference is used to reduce outside sound in a room, this technique is called noise cancellation.

vi. it is used in the determination of the thickness of films, the wavelength of the light source, the radius of curvature of the lens, the refractive index of liquid, etc.

vii. Tuning an instrument by singing beats is one application of interference.

viii. rainbow formation on rainy days.

Diffraction:

i. It is used in separate white light

ii. It is used in crystallographic studies (x-ray diffraction), spectroscopic studies

iii. It is used in the determination of the wavelength of the light source, particle size, etc..

Polarization:

i. Polarization filters are used in LCD screens and the Calculator display
ii. Polarization filters are camera lenses.
iii. Used in anti-reflection coating in spectacles.
iv. Polaroid sunglasses are used to reduce the headlight glare of a car.
v. Polaroid sunglasses are used to locate fish underwater
vi. Used in the determination of the concentration of sugar in sugar solution.

1. In a Newton's rings experiment the diameter of 15th ring was found to be 0.59 cm and that of 5th ring is 0.336 cm. if the radius of curvature of the lens is 100 cm, then find the wavelength of the light.

$D_{15} = 0.59$ cm $\quad D_5 = 0.336$ cm

m=15 $\quad$ n= 5 $\quad$ R=100 cm $\quad \lambda = ?$

$$\lambda = \frac{D_m^2 - D_n^2}{4(m-n)R}$$

$$\lambda = \frac{0.59^2 - 0.336^2}{4(15-5)\,100} = 5880 \times 10^{-8}\ \text{cm} = \mathbf{5880\text{Å}}$$

2. Newton's rings are observed in the reflected light 5900 Å. The diameter of 15th ring is 0.5 cm, then find the radius of curvature of the lens used.

$\lambda = 5900$ Å $= 5900 \times 10^{-8}$ cm

$D_{15} = 0.5$ cm $\quad R = ?$

$(D_n)^2 = 4\, n\, \lambda\, R$

$$R = \frac{D_n^2}{4\, n\, \lambda}$$

$$R = \frac{0.5^2}{4 \times 15 \times 5900 \times 10^{-8}} = \mathbf{106\ cm}$$

3. Light of wavelength 500 nm forms interference pattern on a screen at a distance of 2 m from the slit. If 100 fringes are formed within a distance of 5 cm on the screen, then find the distance between the slits.

$\lambda = 500 \text{ nm} = 500 \times 10^{-9}\text{ m}$ $\qquad$ $D = 2\text{ m}$

$2d = ?$ $\qquad$ $\beta = \frac{5\,cm}{100} = 5 \times 10^{-2} \times 10^{-2}\text{ m}$

$$\beta = \frac{\lambda D}{2d} \quad \rightarrow \quad 2d = \frac{\lambda D}{\beta}$$

$$2d = \frac{500 \times 10^{-9} \times 2}{5 \times 10^{-2} \times 10^{-2}} = 2 \times 10^{-3}\text{ m}$$

$2d = 2$ mm

4. A soap film of refractive index 1.33 and thickness 5000 Å is exposed to white light. What wavelength in the visible region are reflected.

$\mu = 1.33$ $\qquad$ $t = 5000\text{ Å} = 5000 \times 10^{-10}\text{ m}$

$\lambda = ?$

Condition for Max. reflection : $2\mu t \cos r = (2n \pm 1)\lambda/2$

Let's assume $\cos r = 1$

$$\lambda = \frac{4\mu t}{(2n+1)}$$

$$\lambda = \frac{4 \times 1.3 \times 5000 \times 10^{-10}}{(2n+1)} = \frac{26600 \times 10^{-10}}{(2n+1)}$$

$n = 0 \rightarrow$ $\qquad$ $\lambda = \frac{26600 \times 10^{-10}}{(1)}$ = infrared region

$n = 1 \rightarrow$ $\qquad$ $\lambda = \frac{26600 \times 10^{-10}}{(3)} = 8867 \times 10^{-10}\text{ m}$ $\qquad$ (IR)

$n = 2 \rightarrow$ $\qquad$ $\lambda = \frac{26600 \times 10^{-10}}{(5)} = 5320 \times 10^{-10}\text{ m}$ $\qquad$ (Visible region)

$n = 3 \rightarrow$ $\qquad$ $\lambda = \frac{26600 \times 10^{-10}}{(7)} = 3800 \times 10^{-10}\text{ m}$ $\qquad$ (Ultraviolet region)

5. A glass plate is to be used as a polarizer. Find the angle of polarization for it. Also, find the angle of refraction. Given that the refractive index of glass is 1.54.

$\theta_p = ?$ $\theta_r = ?$

Brewster's Law → $\mu = \tan(\theta_p)$

$1.54 = \tan(\theta_p)$

$\theta_p = \tan^{-1}(1.54)$

$\theta_p = 57^o$

Brewster's proved: $\theta_p + \theta_r = 90°$

$\theta_r = 90° - 57^o = 33^o$

6. Plane polarized light is incident on a piece of quartz cut parallel to the axis. Find the least thickness for which the ordinary and extra-ordinary rays combine to form a plane-polarized line. Given that $\mu_o = 1.544$, $\mu_e = 1.553$ and $\lambda = 5 \times 10^{-5}$ cm.

$\mu_o = 1.544$ $\mu_e = 1.553$ $\lambda = 5 \times 10^{-5}$ cm.

$$d = \frac{\lambda}{4(\mu_e - \mu_o)}$$

$$= \frac{5 X 10^{-5}}{2(1.553 - 1.544)}$$

The thickness of quartz is 2.73×10^{-3} cm

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