Module 2: ANTENNA ARRAYS

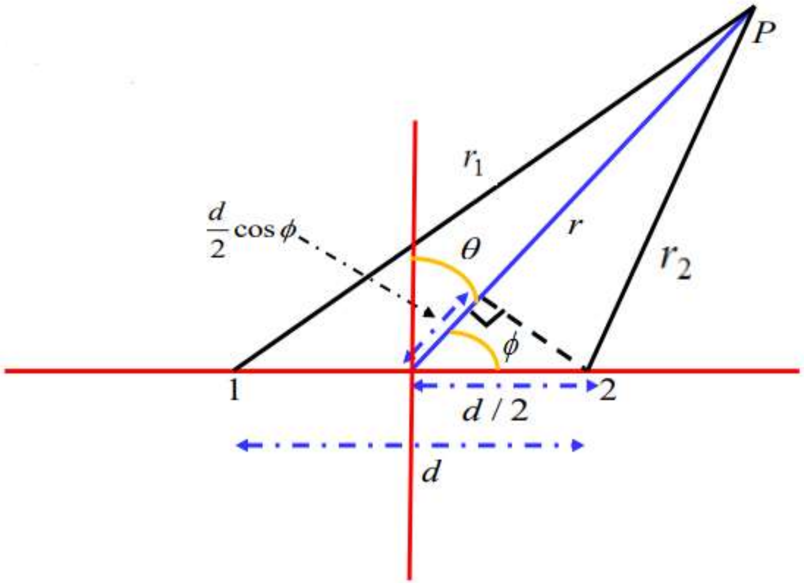
Point sources - Definition, Patterns, arrays of 2 isotropic sources different cases; Principle of pattern multiplication, Uniform linear arrays - Broadside arrays, End fire arrays, EFA with increased directivity, Derivation of their characteristics and comparison, BSA with non-uniform amplitude distribution - General considerations and Binomial arrays, Yagi-Uda arrays, Folded dipoles & their characteristics.

Arrays of Two Isotropic Point Sources
Case I: Same Amplitude and Phase

$$E = E_o e^{-j\beta r_1} + E_o e^{-j\beta r_2}$$

$$E = E_o e^{-j\beta r}\left[e^{-j\beta\frac{d}{2}\cos\phi} + e^{j\beta\frac{d}{2}\cos\phi}\right]$$

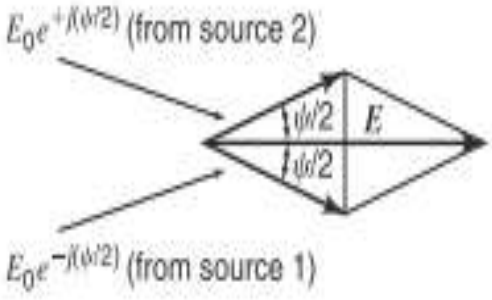
$$= E_o e^{-j\beta r}\left[e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}\right]$$



$$\psi = d_r \cos\phi$$

$$d_r = \frac{2\pi d}{\lambda} = \beta d$$

$$\psi = \beta d\cos\phi = \frac{2\pi d}{\lambda}\cos\phi$$
$$= \beta d\sin\theta = \frac{2\pi d}{\lambda}\sin\theta$$



Arrays of Two Isotropic Point Sources Case I: Same Amplitude and Phase

$$\boxed{E = 2E_o \cos\left(\frac{\psi}{2}\right) = 2E_o \cos\left(\frac{\pi d}{\lambda}\cos\phi\right)}$$

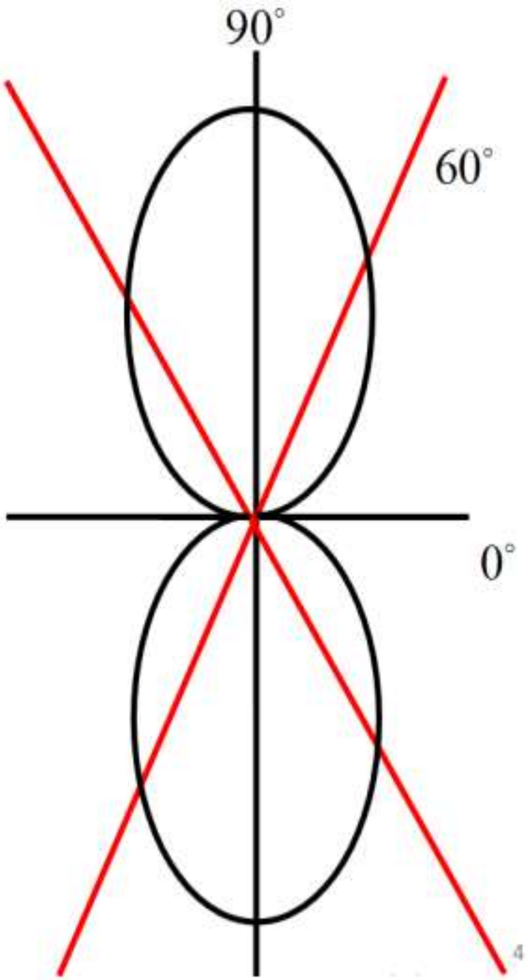
Normalized E: $E = \cos\left(\frac{d_r}{2}\cos\phi\right)$

$$d_r = \frac{2\pi d}{\lambda} = \beta d$$

For $d = \frac{\lambda}{2}$ $\qquad E = \cos\left(\frac{\pi}{2}\cos\phi\right)$

φ	0°	90°	60°
E	0	1	$1/\sqrt{2}$

HPBWs = 60° in one plane and 360° in another pl

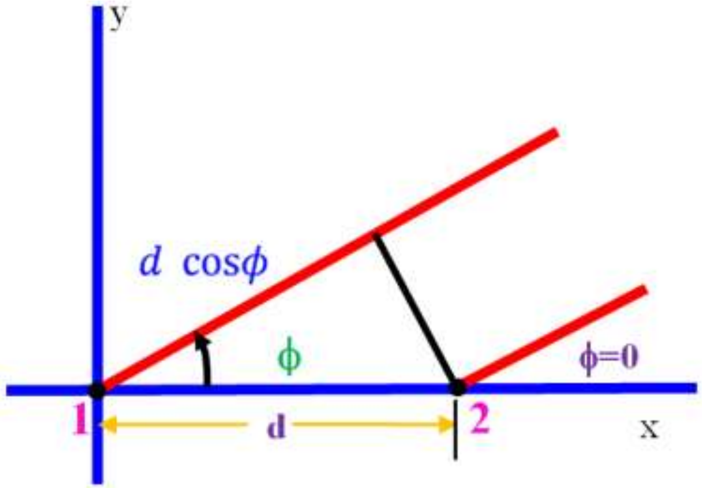


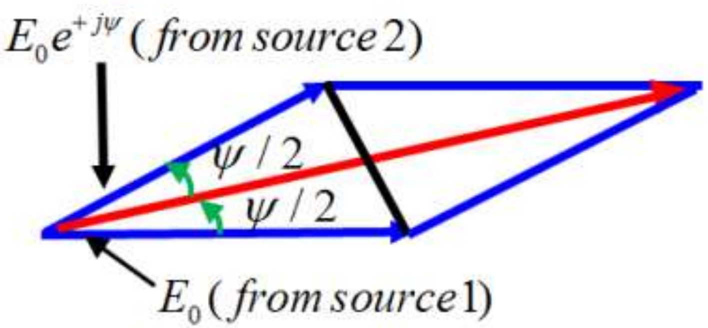
Origin at Element 1

$$E = E_0\left(1 + e^{j\psi}\right)$$
$$= 2E_0 e^{j\psi/2}\left(\frac{e^{j\psi/2} + e^{-j\psi/2}}{2}\right)$$
$$= 2E_0 e^{j\psi/2}\cos\frac{\psi}{2}$$

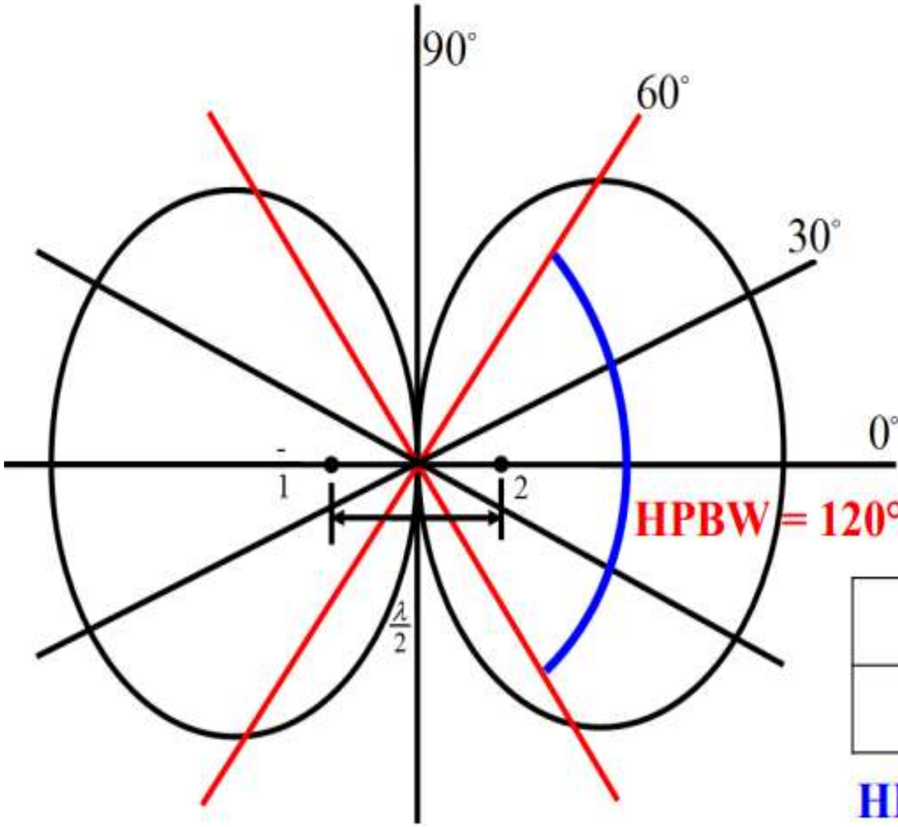
Normalizing by setting $2E_0 = 1$

$$E = e^{j\psi/2}\cos\frac{\psi}{2}$$
$$= \cos\frac{\psi}{2}\underline{\left|\psi/2\right.}$$





Two Isotropic Point Sources of Same Amplitude and Opposite Phase



$$E = E_0 e^{+j\psi/2} - E_0 e^{-j\psi/2}$$

$$E = 2jE_0 \sin\frac{\psi}{2} = 2jE_0 \sin\left(\frac{d_r}{2}\cos\phi\right)$$

For $d = \frac{\lambda}{2}$

$$E = sin\left(\frac{\pi}{2}\cos\phi\right)$$

ϕ	0°	90°	60°
E	1	0	$1/\sqrt{2}$

HPBWs = 120° in both orthogonal planes

Two Isotropic Point Sources of Same Amplitude and Opposite Phase

- The total field component as compared with case I, this is unimportant here. Thus putting $2jE_0 = 1$ and considering the special case of $d = \lambda/2$,

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right)$$

Two Isotropic Point Sources of Same Amplitude and Opposite Phase

The directions ϕ_m of maximum field are obtained by setting the argument of (11) equal to $\pm(2k+1)\pi/2$. Thus,

$$\frac{\pi}{2}\cos\phi_m = \pm(2k+1)\frac{\pi}{2}$$

where $k = 0, 1, 2, 3 \ldots$. For $k = 0$, $\cos\phi_m = \pm 1$ and $\phi_m = 0^\circ$ and 180°.

The null directions ϕ_0 are given by

$$\frac{\pi}{2}\cos\phi_0 = \pm k\pi$$

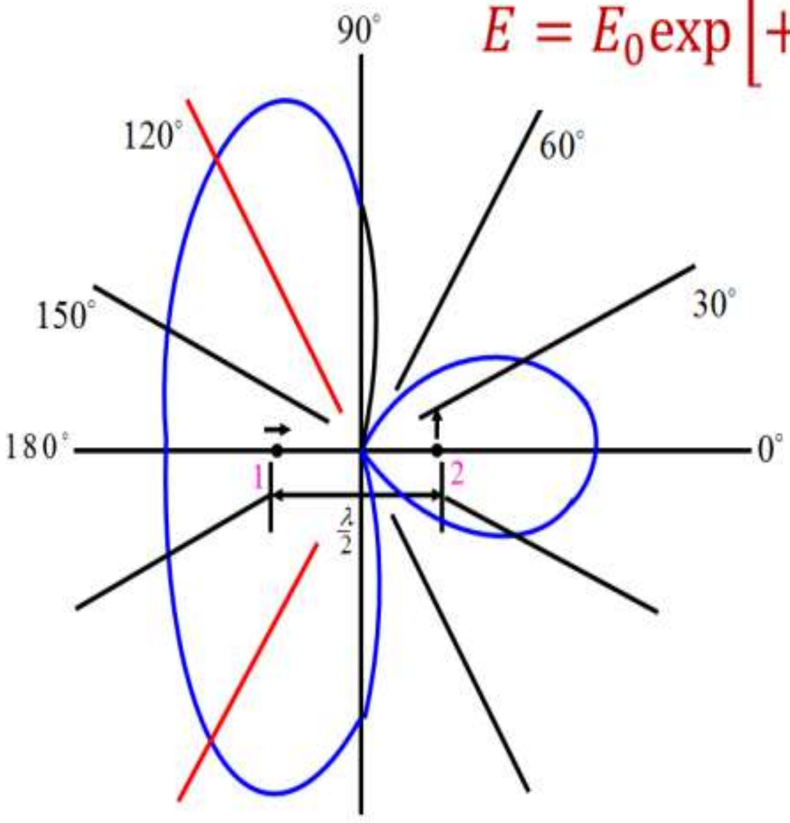
For $k = 0$, $\phi_0 = \pm 90^\circ$.

The half-power directions are given by

$$\frac{\pi}{2}\cos\phi = \pm(2k+1)\frac{\pi}{4}$$

For $k = 0$, $\phi = \pm 60^\circ, \pm 120^\circ$.

Two Isotropic Point Sources of Same Amplitude and In-Phase Quadrature



$$E = E_0 \exp\left[+j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right] + E_0 \exp\left[-j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right]$$

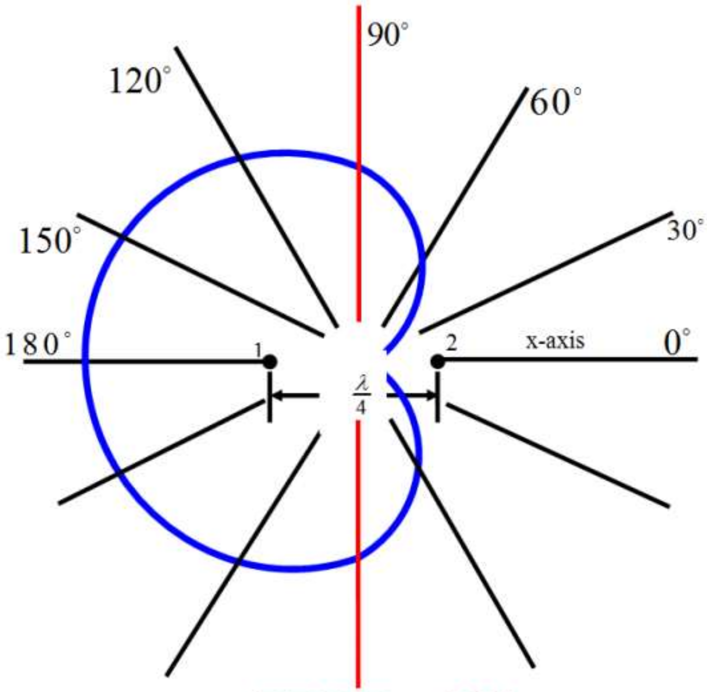
$$E = 2E_0 \cos\left(\frac{\pi}{4} + \frac{d_r}{2}\cos\phi\right)$$

Letting $2E_0 = 1$, and $d = \frac{\lambda}{2}$

$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right)$$

φ	**0°**	**60°**	**90°**	**120°**	**180°**
E	$1/\sqrt{2}$	**0**	$1/\sqrt{2}$	**1**	$1/\sqrt{2}$

Two Isotropic Point Sources of Same Amplitude with 90^0 Phase Difference at λ/4



Spacing between the sources is reduced to λ/4

$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{4}\cos\phi\right)$$

ϕ	**0°**	**90°**	**120°**	**150°**	**180°**
E	**0**	$1/\sqrt{2}$	**0.924**	**0.994**	**1**

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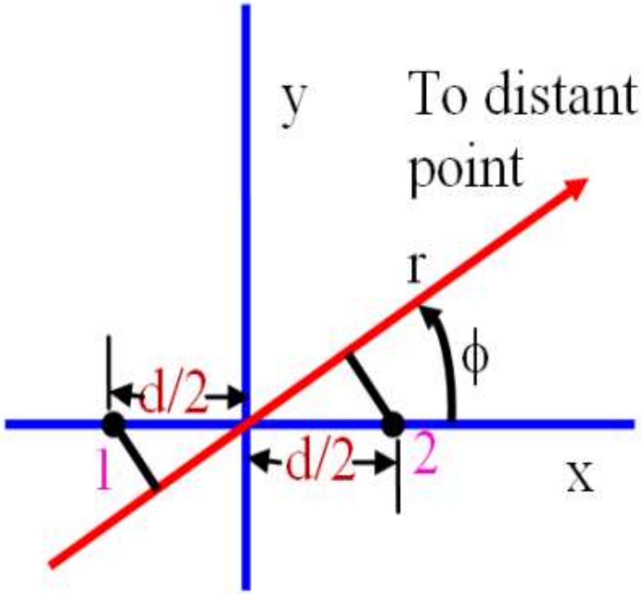
Two Isotropic Point Sources Of Equal Amplitude and any Phase Difference



$$\psi = d_r \cos\phi + \delta$$

$$E = E_0\left(e^{j\psi/2} + e^{-j\psi/2}\right)$$

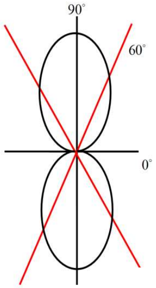
$$= 2E_0 \cos\frac{\psi}{2}$$

Normalizing by setting $2E_0 = 1$

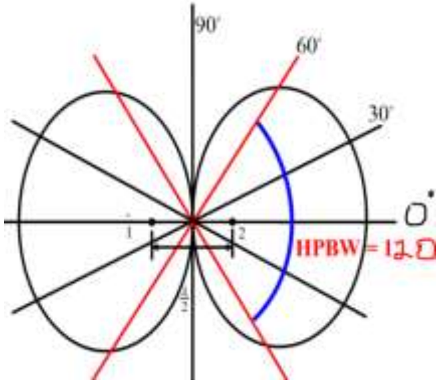
$$E = \cos\frac{\psi}{2}$$

$$E = \cos\left(\frac{dr\cos\emptyset + \delta}{2}\right)$$

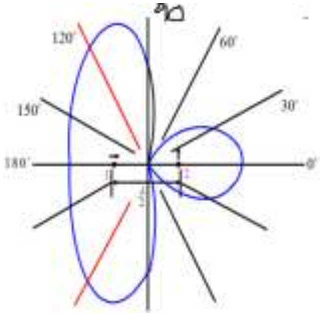
- $\delta = 0^0$ → case:I →



- $\delta = 180^0$ → case:II →



- $\delta = 90^0$ → case:III →

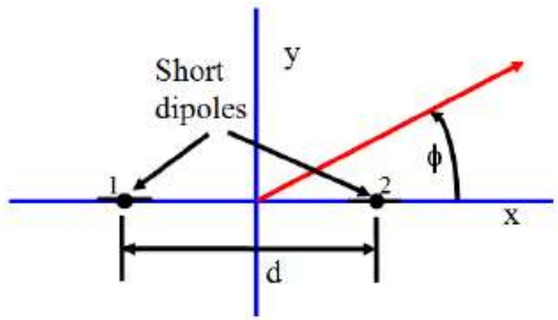


Most General Case of Two Isotropic Point Sources of Unequal Amplitude and any Phase Difference

$$E = E_0\sqrt{(1 + a\cos\psi)^2 + a^2\sin^2\psi}\ \underline{/[a\sin\psi/(1 + a\cos\psi)]}$$

where $\psi = d_r\cos\phi + \delta$ and the phase angle ($\angle$) is referred to source 1. This is the phase angle ξ

Two Same Dipoles and Pattern Multiplication



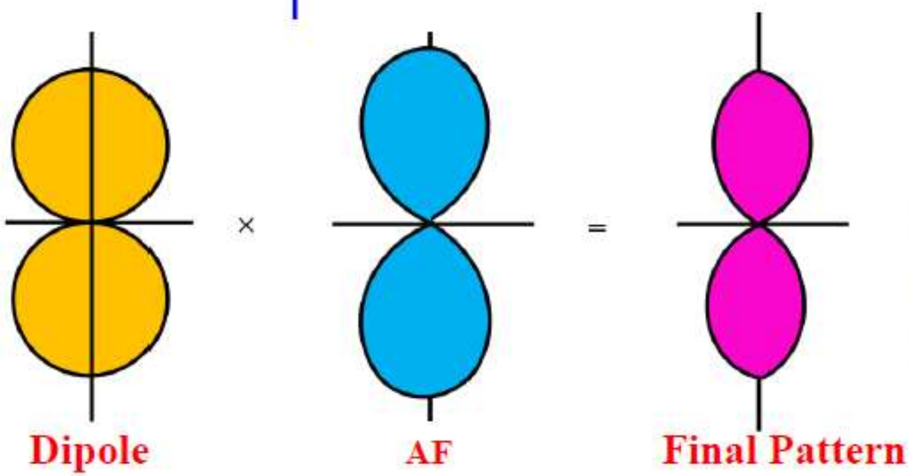
Horizontal Dipole: $E_0 = E'_0 \sin\phi$

$$AF = \cos(\psi/2)$$

$$E = \sin\phi \cos\frac{\psi}{2}$$

where, $\psi = d_r \cos\phi + \delta$

For $\delta = 0$, Array Factor (AF) will give max. radiation in Broadside Direction



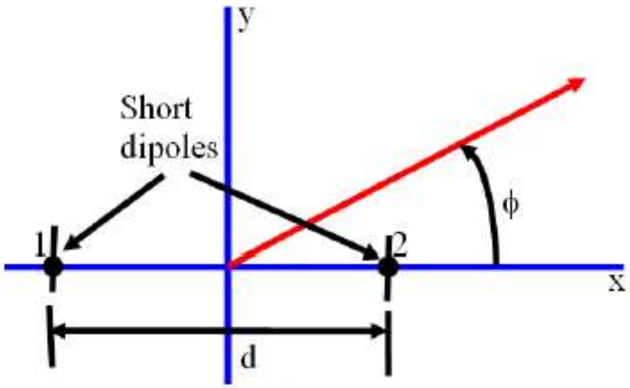
PATTERN MULTIPLICATION

Dipole E-Field for Vertical Orientation:

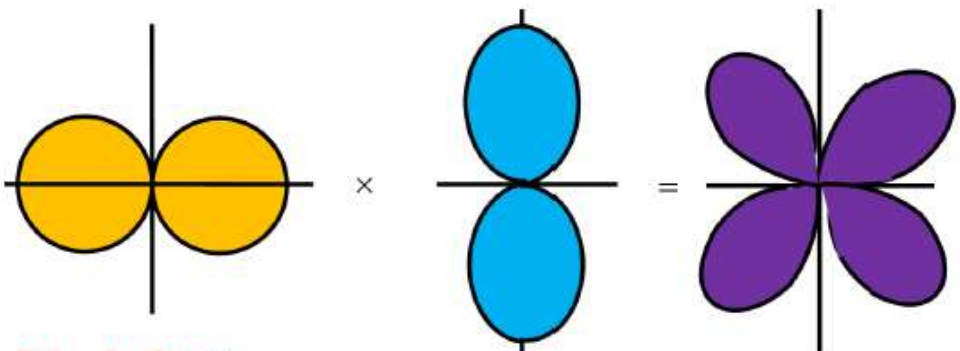
$$E_0 = E'_0 \cos\phi$$

Combined E-Field

$$E = \cos\phi \cos\left(\frac{\pi}{2}\cos\phi\right)$$



Array of two vertical dipole antennas



Dipole Pattern **AF** **Product of Patterns**

Nonisotropic but Similar Point Sources and The Principle of Pattern Multiplication

- The field pattern of an array of nonisotropic but similar point sources is the product of the pattern of the individual source and the pattern of an array of isotropic point sources having the same locations, relative amplitudes, and phase as the nonisotropic point sources.
- The total phase pattern is referred to the phase center of the array. In symbols, the total field E is then

$$E = \underbrace{f(\theta,\phi)F(\theta,\phi)}_{\text{Field pattern}} \underbrace{\big/ f_p(\theta,\phi) + F_p(\theta,\phi)}_{\text{Phase pattern}}$$

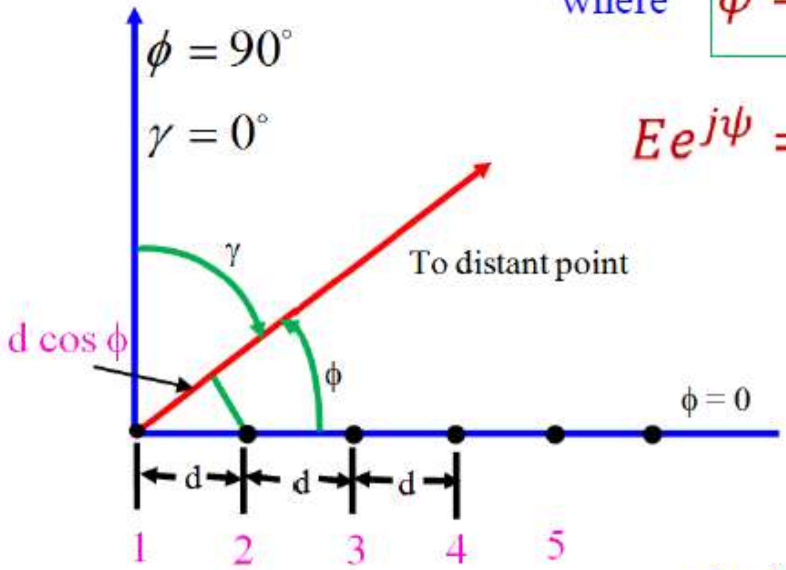
where

$$f(\theta,\phi) = \text{field pattern of individual source}$$
$$f_p(\theta,\phi) = \text{phase pattern of individual source}$$
$$F(\theta,\phi) = \text{field pattern of array of isotropic sources}$$
$$F_p(\theta,\phi) = \text{phase pattern of array of isotropic sources}$$

N Isotropic Point Sources of Equal Amplitude and Spacing

$$E = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{j(n-1)\psi}$$

where $\boxed{\psi = \frac{2\pi d}{\lambda}\cos\phi + \delta = d_r \cos\phi + \delta}$



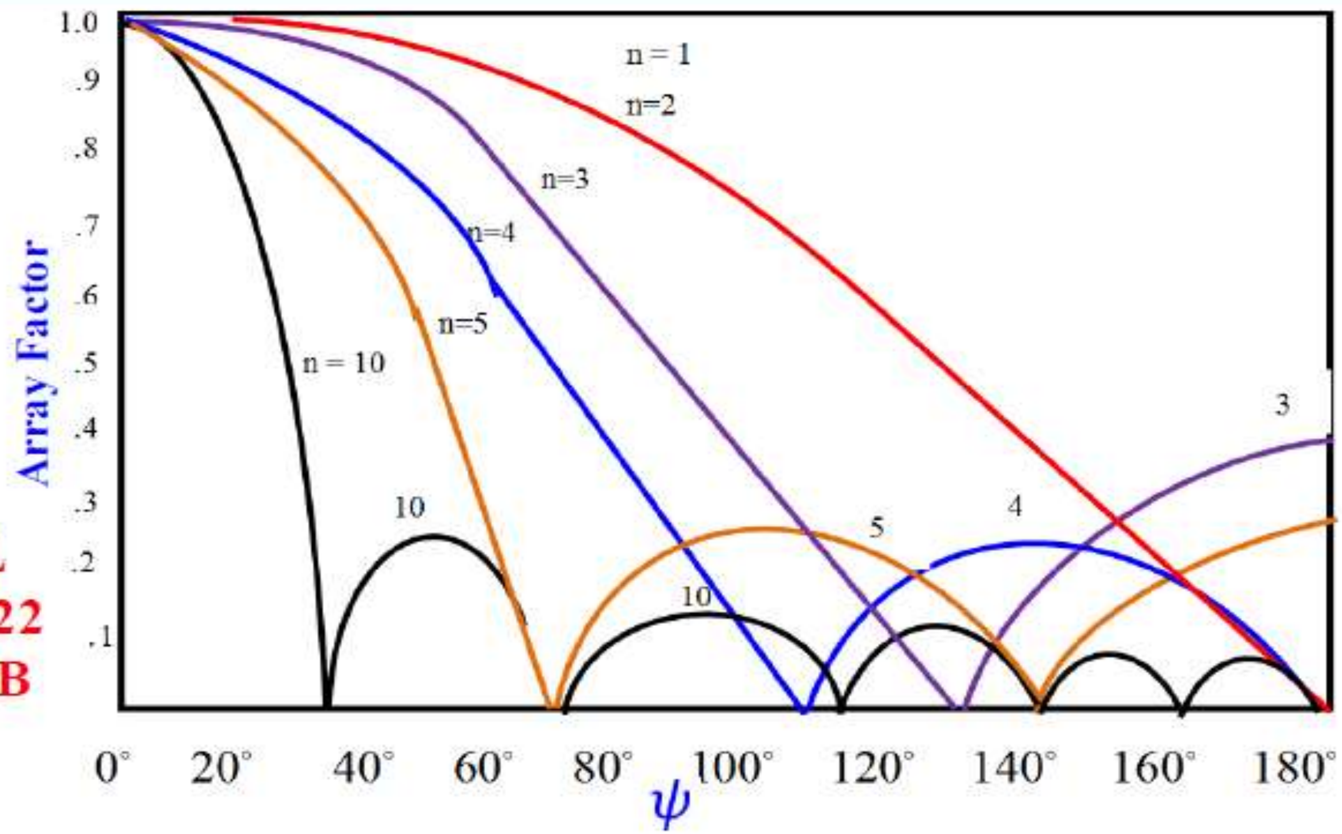
$$E e^{j\psi} = e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{jn\psi}$$

$$E - E e^{j\psi} = 1 - e^{jn\psi}$$

$$\boxed{E = \frac{1 - e^{jn\psi}}{1 - e^{j\psi}} = \frac{\sin(n\,\psi/2)}{\sin(\psi/2)}}$$

As $\Psi \to 0$, $E_{max} = n$, $E_{norm} = \frac{\sin(n\,\psi/2)}{n\sin(\psi/2)}$

Radiation Pattern of N Isotropic Elements Array



First SLL
= 20log0.22
= -13.15dB

Radiation Pattern for array of n isotropic radiators of equal amplitude and spacing.

Broadside Array (Sources In Phase)

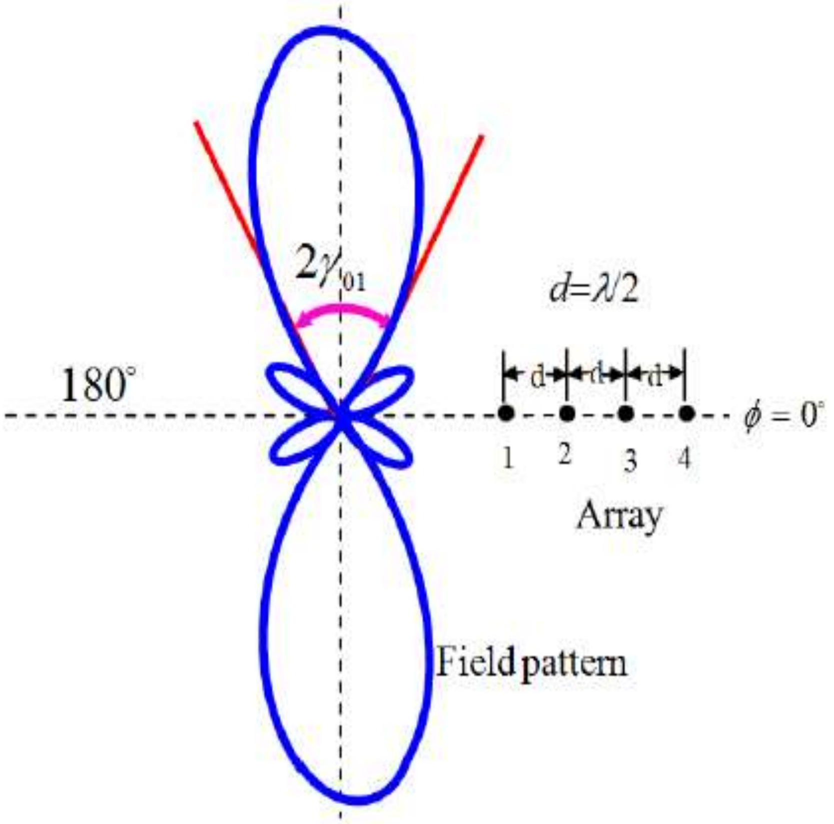
$$\psi = \frac{2\pi d}{\lambda}\cos\phi + \delta$$

$$\delta=0,\ d = \frac{\lambda}{2} \quad \text{and } n = 4$$

$$\psi = \pi\cos\phi \qquad E = \frac{\sin(n\,\psi/2)}{n\sin(\psi/2)}$$

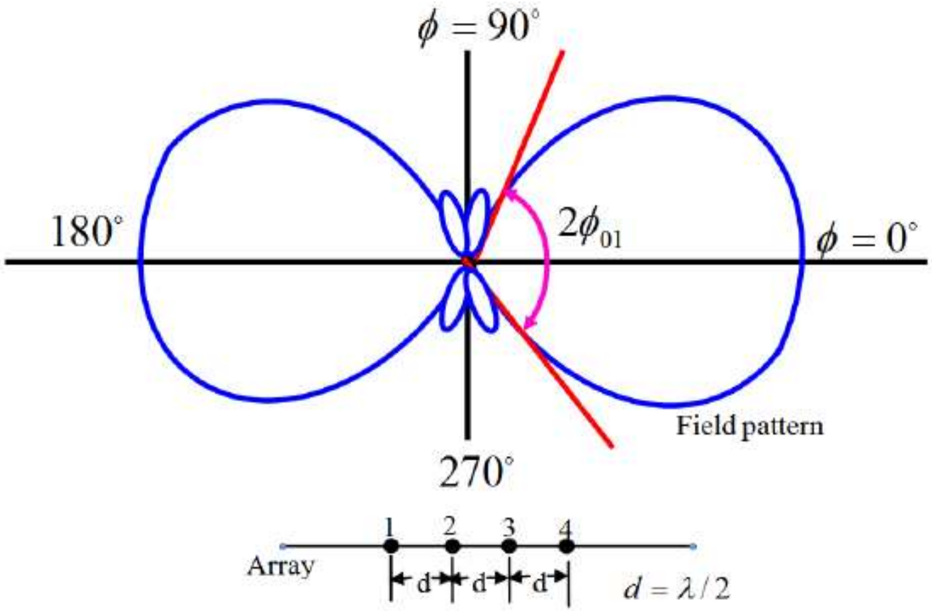
φ	Ψ	E
0°	π	0
60°	π/2	0
90°	0	1

$$BWFN = 2\gamma_{01} = 60°$$



Field pattern of 4 isotropic point sources with the same amplitude and phase. Spacing = λ/2.

Ordinary Endfire Array



$$\psi = \frac{2\pi d}{\lambda}\cos\phi + \delta$$

For $d = \lambda/2$, $\phi = 0^0$ and $\Psi = 0$

$$\delta = -\pi$$

$$\psi = \pi(\cos\phi - 1)$$

BWFN=120°

Field pattern of ordinary end-fire array of 4 isotropic point sources of same amplitude. Spacing is $\lambda/2$ and the phase angle $\delta = -\pi$.

Endfire Array With Increased Directivity

For endfire array

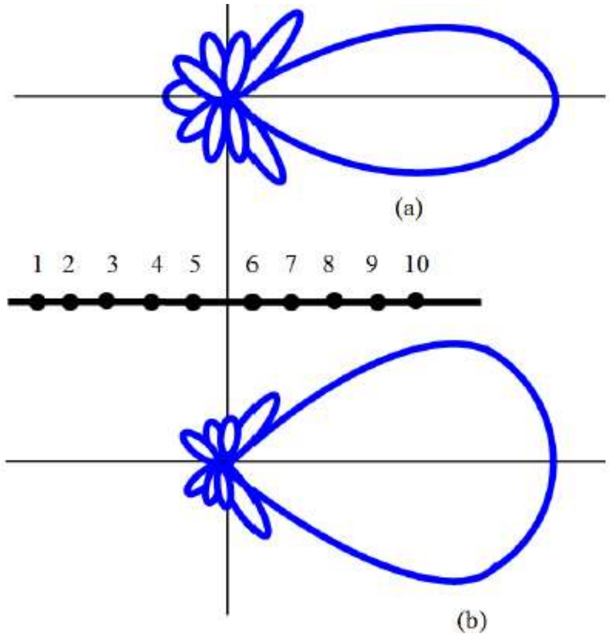
$$\psi = d_r(\cos\phi - 1)$$

For increased directivity endfire array

$$\psi = d_r(\cos\phi - 1) - \frac{\pi}{n}$$

$$E_{norm} = \sin\left(\frac{\pi}{2n}\right)\frac{\sin(n\,\psi/2)}{\sin(\psi/2)}$$

Parameter	Ordinary end fire array	Increased Directivity endfire array
HPBW	69°	38°
FNBW	106°	74°
Directivity	11	19



Field patterns of end-fire arrays of 10 isotropic point sources of equal amplitude spaced λ/4 apart.
(a) Phase for increased directivity (δ = - 0.6π),
(b) Phase of an ordinary end-fire array (δ = - 0.5π). 16

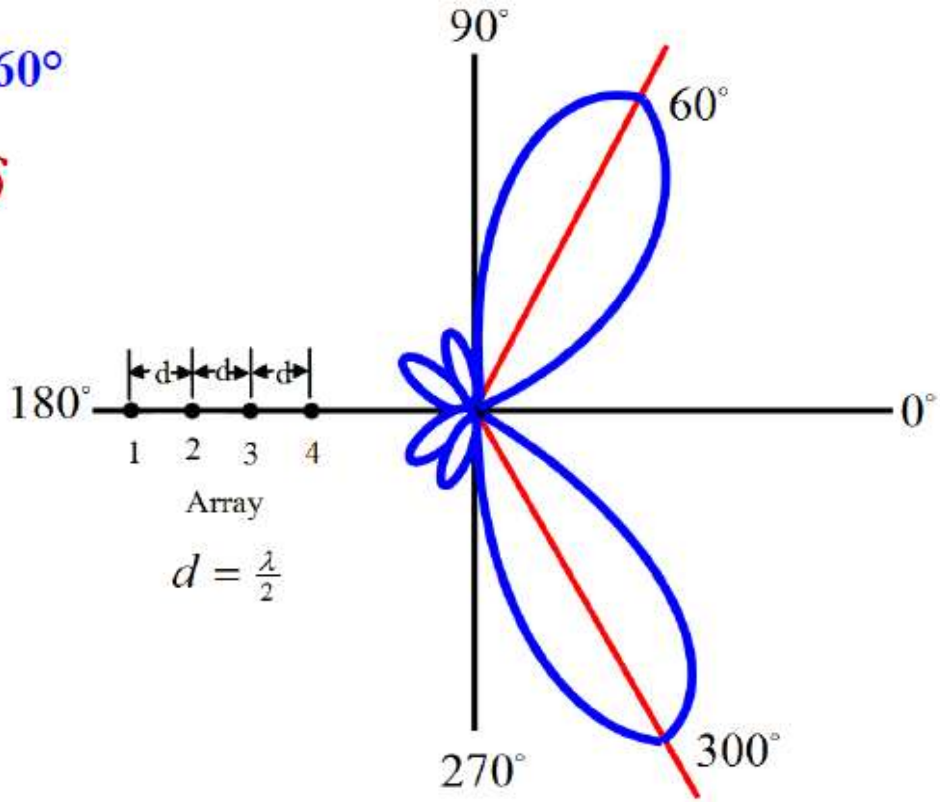
Array with Maximum Field in any Arbitrary Direction

For Beam Maxima at $\phi = 60°$

$$\psi = 0 = d_r \cos 60° + \delta$$

For $d = \lambda / 2$, $d_r = \pi$

$$\delta = -\frac{\pi}{2}$$



Field pattern of array of 4 isotropic point sources of equal amplitude with phase adjusted to give the maximum at $\phi = 60°$ for spacing $d = \lambda/2$

Null Directions for Arrays of N Isotropic Point Sources

$$E_{norm} = \frac{\sin(n\psi/2)}{n \sin(\psi/2)}$$

For Finding Direction of Nulls:

$$\sin\left(\frac{n\psi}{2}\right) = 0 \rightarrow \frac{n\psi}{2} = \pm k\pi \text{ where, } k=1,2,3.\dots$$

$$\psi = \pm \frac{2k\pi}{n}$$

For Broadside Array, δ = 0

$$\frac{2\pi d}{\lambda} cos\phi_0 = \pm \frac{2k\pi}{n} \rightarrow \phi_0 = \pm\cos^{-1}\left(\frac{k\lambda}{nd}\right)$$

Null Direction and First Null Beamwidth

Null directions and beam width between first nulls for linear arrays of n isotropic point sources of equal amplitude and spacing

Type of array	Null directions (array any length)	Null directions (long array)	Beam width between first nulls(long array)
General case	$\phi_0 = \arccos\left[\left(\pm\frac{2K\pi}{n} - \delta\right)\frac{1}{d_r}\right]$		
Broadside	$\gamma_0 \simeq \arcsin\left(\pm\frac{K\lambda}{nd}\right)$	$\gamma_0 \simeq \pm\frac{K\lambda}{nd}$	$2\gamma_{01} \simeq \frac{2\lambda}{nd}$
Ordinary end-fire	$\phi_0 = 2\arcsin\left(\pm\sqrt{\frac{K\lambda}{2nd}}\right)$	$\phi_0 \simeq \pm\sqrt{\frac{2K\lambda}{nd}}$	$2\phi_{01} \simeq 2\sqrt{\frac{2\lambda}{nd}}$
End-fire with increased directivity	$\phi_0 = 2\arcsin\left[\pm\sqrt{\frac{\lambda}{4nd}(2K-1)}\right]$	$\phi_0 \simeq \pm\sqrt{\frac{\lambda}{nd}(2K-1)}$	$2\phi_{01} \simeq 2\sqrt{\frac{\lambda}{nd}}$

An EFA composed of λ/2 radiators with axis at right angle to the line of the array required to have a power again of 20. determine the array length width of a major lobe between the nulls, derive the formula used

- D=4 L/λ

  20= 4 L/λ

  L=5 λ

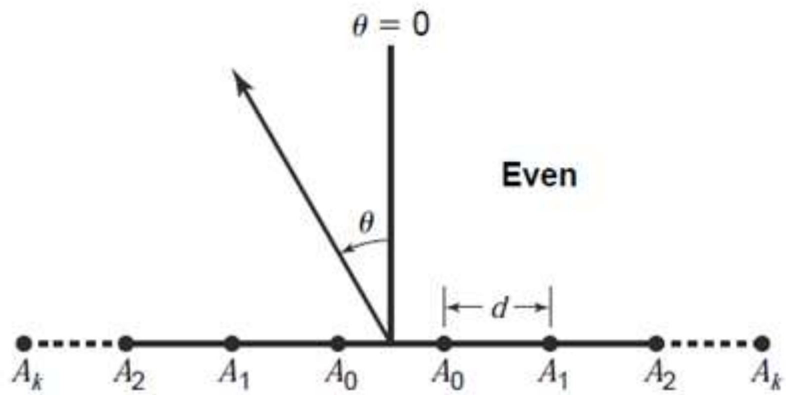
  $$BW= \pm 114.6\sqrt{\frac{2}{L/\lambda}}$$

  =72.5

an array consists of 4 identical isotropic sources located at the corner of a square having diagonal 3/4 and excited with equal current in same phase calculation and plot the field pattern

Linear Arrays with Nonuniform Amplitude Distributions.

- consider a linear array of an even number n_e *of isotropic point sources of uniform spacing d* arranged.



$$E_{n_e} = 2A_0 \cos\frac{\psi}{2} + 2A_1 \cos\frac{3\psi}{2} + \cdots + 2A_k \cos\left(\frac{n_e - 1}{2}\psi\right) \qquad (1)$$

where

$$\psi = \frac{2\pi d}{\lambda}\sin\theta = d_r \sin\theta \qquad (2)$$

Linear Arrays with Nonuniform Amplitude Distributions.

- Each term in (1) represents the field due to a symmetrically disposed pair of the sources.

  Now let

$$2(k+1) = n_e$$

where $k = 0, 1, 2, 3, \ldots$ so that

$$\frac{n_e - 1}{2} = \frac{2k+1}{2}$$

Then 1 becomes

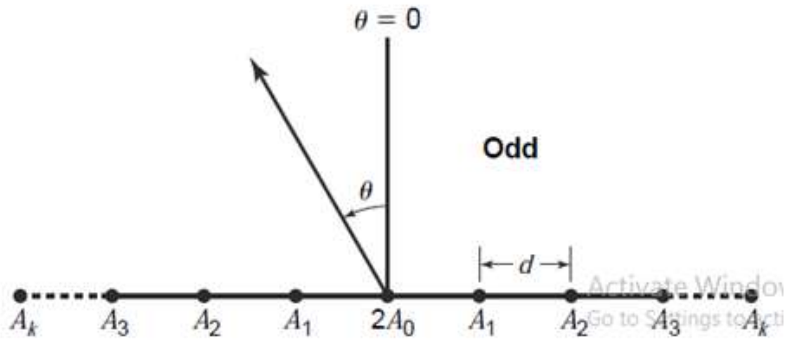
$$\boxed{E_{n_e} = 2 \sum_{k=0}^{k=N-1} A_k \cos\left(\frac{2k+1}{2}\psi\right) \qquad \textit{Even number Fourier series}} \tag{3}$$

Where

$$N = n_e/2$$

Linear Arrays with Nonuniform Amplitude Distributions.

- Next let us consider the case of a linear array of an odd number no of isotropic point sources of uniform spacing.



- The amplitude distribution is symmetrical about the center source.
- The amplitude of the center source is taken as $2A_0$, the next as A_1, the next as A_2, etc. The total field E_{no} from the odd number of sources at a large distance in a direction θ is then

Linear Arrays with Nonuniform Amplitude Distributions.

$$E_{n_o} = 2A_0 + 2A_1 \cos\psi + 2A_2 \cos 2\psi + \cdots + 2A_k \cos\left(\frac{n_o - 1}{2}\psi\right) \tag{4}$$

Now for this case let

$$2k + 1 = n_o$$

where $k = 0, 1, 2, 3, \ldots$. Then (4) becomes

$$\boxed{E_{n_o} = 2\sum_{k=0}^{k=N} A_k \cos\left(2k\frac{\psi}{2}\right) \qquad \textit{\textbf{Odd number Fourier series}}} \tag{5}$$

$$N = (n_o - 1)/2$$

The Dolph-Tchebyscheff Optimum Distribution

- consider only the case of the broadside type of array, i.e., where δ = 0. Thus,
- $\psi = d_r \sin\vartheta$ (7)

Now by de Moivre's theorem,

$$e^{jm\psi/2} = \cos m\frac{\psi}{2} + j\sin m\frac{\psi}{2} = \left(\cos\frac{\psi}{2} + j\sin\frac{\psi}{2}\right)^m \tag{8}$$

On taking real parts of (8) we have

$$\cos m\frac{\psi}{2} = \mathrm{Re}\left(\cos\frac{\psi}{2} + j\sin\frac{\psi}{2}\right)^m \tag{9}$$

Expanding (9) as a binomial series gives

$$\begin{aligned}\cos m\frac{\psi}{2} &= \cos^m\frac{\psi}{2} - \frac{m(m-1)}{2!}\cos^{m-2}\frac{\psi}{2}\sin^2\frac{\psi}{2} \\ &\quad + \frac{m(m-1)(m-2)(m-3)}{4!}\cos^{m-4}\frac{\psi}{2}\sin^4\frac{\psi}{2} - \cdots\end{aligned} \tag{10}$$

The Dolph-Tchebyscheff Optimum Distribution

P

$$
\left.\begin{array}{ll}
m = 0 & \cos m\dfrac{\psi}{2} = 1 \\
m = 1 & \cos m\dfrac{\psi}{2} = \cos\dfrac{\psi}{2} \\
m = 2 & \cos m\dfrac{\psi}{2} = 2\cos^2\dfrac{\psi}{2} - 1 \\
m = 3 & \cos m\dfrac{\psi}{2} = 4\cos^3\dfrac{\psi}{2} - 3\cos\dfrac{\psi}{2} \\
m = 4 & \cos m\dfrac{\psi}{2} = 8\cos^4\dfrac{\psi}{2} - 8\cos^2\dfrac{\psi}{2} + 1 \\
\text{etc.} &
\end{array}\right\} \qquad (11)
$$

The Dolph-Tchebyscheff Optimum Distribution

Now let

$$x = \cos\frac{\psi}{2} \tag{12}$$

whereupon the equations of (11) become

$$\left.\begin{aligned} &\cos m\frac{\psi}{2} = 1 && \text{when } m = 0 \\ &\cos m\frac{\psi}{2} = x && \text{when } m = 1 \\ &\cos m\frac{\psi}{2} = 2x^2 - 1 && \text{when } m = 2 \\ &\text{etc.} \end{aligned}\right\} \tag{13}$$

The polynomials of (13) are called Tchebyscheff polynomials, which may be designated in general by

$$T_m(x) = \cos m\frac{\psi}{2} \tag{14}$$

The Dolph-Tchebyscheff Optimum Distribution

For particular values of m, the first eight Tchebyscheff polynomials are

$$\left.\begin{aligned}
T_0(x) &= 1 \\
T_1(x) &= x \\
T_2(x) &= 2x^2 - 1 \\
T_3(x) &= 4x^3 - 3x \\
T_4(x) &= 8x^4 - 8x^2 + 1 \\
T_5(x) &= 16x^5 - 20x^3 + 5x \\
T_6(x) &= 32x^6 - 48x^4 + 18x^2 - 1 \\
T_7(x) &= 64x^7 - 112x^5 + 56x^3 - 7x
\end{aligned}\right\} \tag{15}$$

We note in (15) that the degree of the polynomial is the same as the value of m.

The roots of the polynomials occur when $\cos m(\psi/2) = 0$ or when

$$m\frac{\psi}{2} = (2k-1)\frac{\pi}{2} \tag{16}$$

where $k = 1, 2, 3, \ldots$

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The roots of x, designated x', are thus

$$x' = \cos\left[(2k-1)\frac{\pi}{2m}\right] \tag{17}$$

Binomial Amplitude Distribution Arrays

Binomial Amplitude Coefficients are defined by

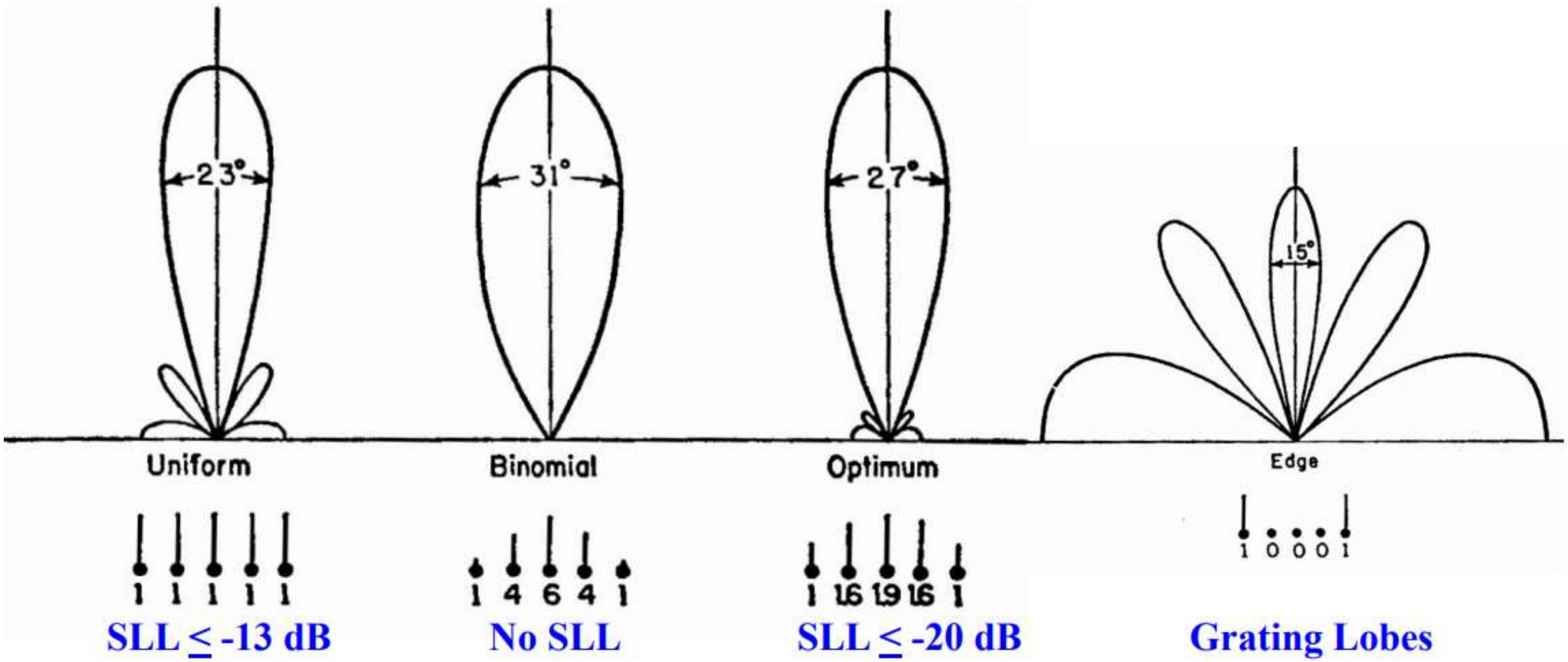
$$(1+x)^{m-1} = 1 + \frac{(m-1)x}{1!} + \frac{(m-1)(m-2)x^2}{2!} + \dots$$

$m = 1$	1
$m = 2$	1 1
$m = 3$	1 2 1
$m = 4$	1 3 3 1
$m = 5$	1 4 6 4 1
$m = 6$	1 5 10 10 5 1

No side lobe level but broad beamwidth

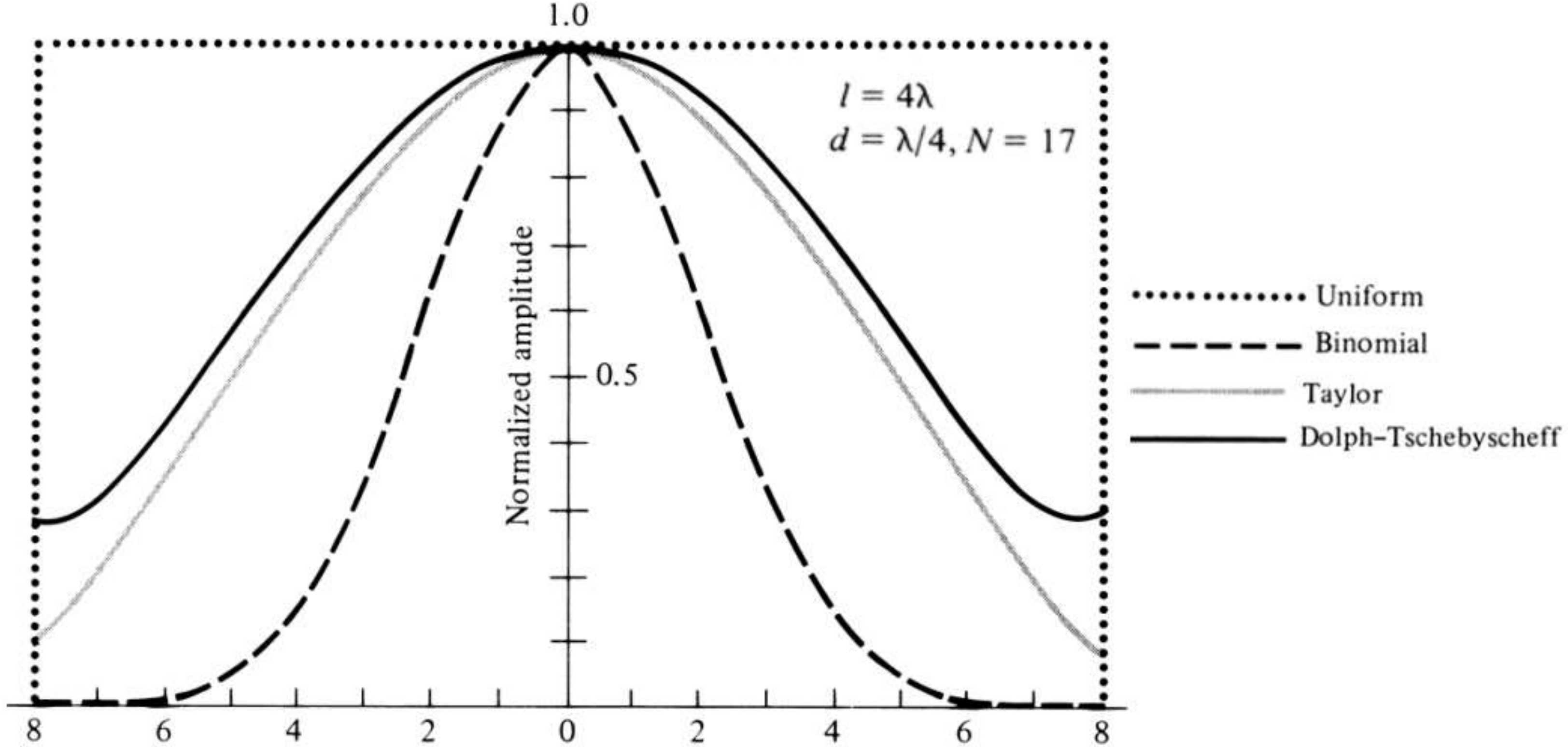
→ Gain decreases (practically not used)

Radiation Pattern of Broadside Arrays with Non-Uniform Amplitude (5 elements with spacing = λ/2, Total Length = 2 λ)



All 5 sources are in same phase but relative amplitudes are different

Non-Uniform Amplitude Distribution



YAGI-UDA ARRAY ANTENNA

- Yagi - Uda arrays
- BSA with non-uniform amplitude distribution - General considerations and Binomial arrays

- **The Yagi-Uda Array Story**

- Shintaro Uda, an assistant professor at Tohoku University, had not turned 30 when he conducted experiments on the use of parasitic reflector and director elements in 1926, published a series of 11 articles (from March 1926 to July 1929) in the Journal of the Institute of Electrical Engineers of Japan titled “On the Wireless Beam of Short Electric Waves.” (Uda-1).

- He measured patterns and gains with a single parasitic reflector, a single parasitic director and with a reflector and as many as 30 directors.

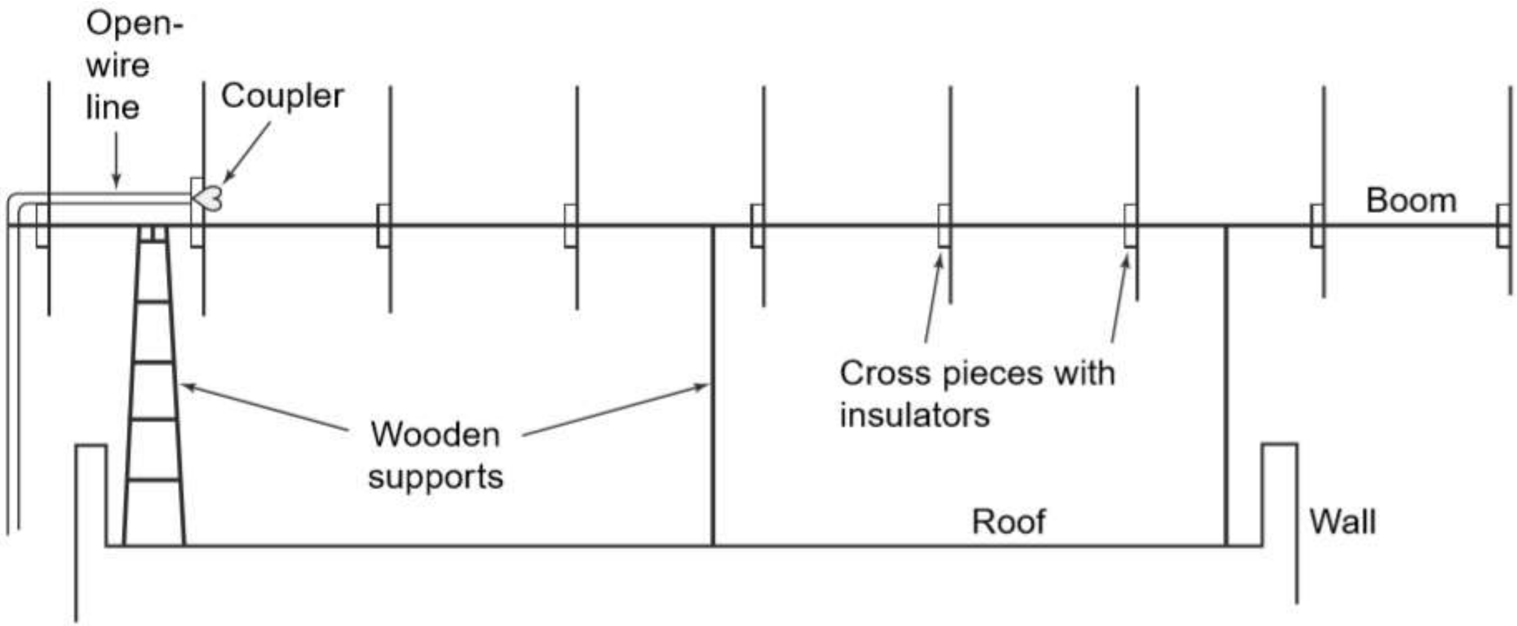


Figure Shintaro Uda's experimental antenna with 1 reflector and 7 directors on the roof of his laboratory at Tohoku University for vertically polarized transmission tests during 1927 and 1928 over land and sea paths up to 135 km using a wavelength $\lambda = 4.4$ m. The horizontal wooden boom supporting the array elements is 15 m long.

Contd.,

- One of his many experimental arrays is shown in Fig. Above, he found the **highest gain with the reflector about λ/2 in length and spaced about λ/4 from the driven element, while the best director lengths were about 10 percent less than λ/2 with optimum spacings about λ/3**.

- The patterns were measured in the **near field**, these lengths and spacings agree remarkably well with optimum values determined since then by further experimental and computer techniques.

- After **George H. Brown** demonstrated the advantages of close spacing, the reflector-to-driven-element spacings were reduced.

Contd.,

- **Hidetsugu Yagi, professor of electrical engineering at Tohoku University and 10 years Uda's senior**, presented a paper with Uda at the Imperial Academy on the "Projector of the Sharpest Beam of Electric Waves" in 1926, and in the same year they both presented a paper before the Third Pan-Pacific Congress in Tokyo titled "On the Feasibility of Power Transmission by Electric Waves."
- The narrow beams of short waves produced by the guiding action of the multi director periodic structure, which they called a "**wave canal**," had encouraged them to suggest using it for **short-wave power transmission**, an idea now being considered for **beaming solar power to the earth from a space station or from earth to a satellite**.

Contd.,

- It is reported that Professor Yagi had received a **substantial grant** from Sendai businessman Saito Zenuemon which supported the antenna research done by **Uda with Yagi's collaboration**.
- In 1928 Yagi toured the United States presenting talks before Institute of Radio Engineers sections in New York, Washington and Hartford, and in the same yearYagi published his now famous article on "Beam Transmission of Ultra Short Waves" in the Proceedings of the IRE (Yagi-1).
- Although Yagi noted that Uda had already published 9 papers on the antenna and acknowledged that Uda's ingenuity was mainly responsible for its successful development, the antenna soon came to be called "a Yagi."

Contd.,

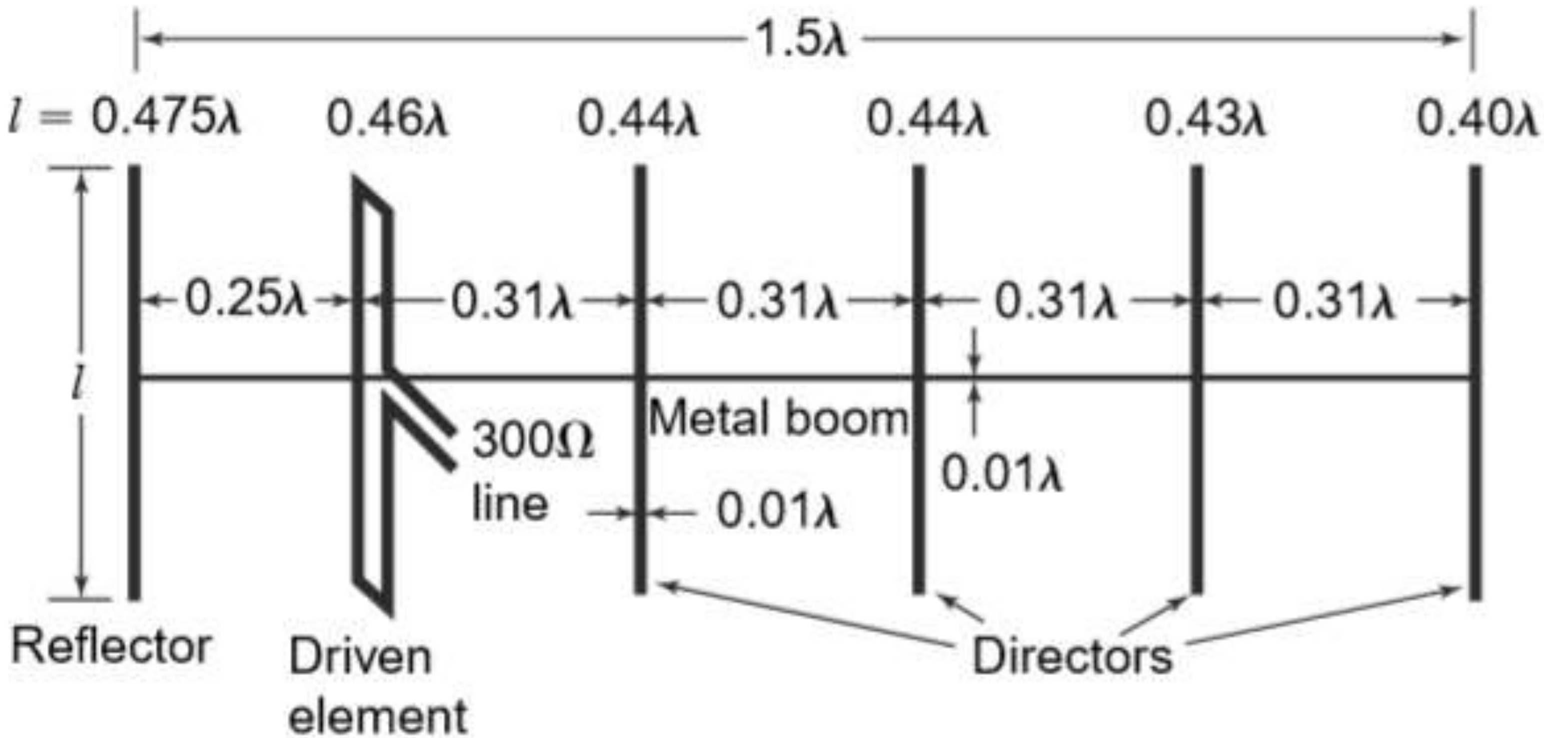


Figure Modern-version 6-element Yagi-Uda antenna with dimensions. It has a maximum directivity of about 12 dBi at the center of a bandwidth of 10 percent at half-power.

Contd.,

- In deference to Uda's contributions, we refer to the array as a Yagi-Uda antenna.

- Uda has summarized his researches on the antenna in two data-packed books (Uda-2, 3).

- A typical modern-version 6-element Yagi-Uda antenna is shown in Fig. It consists of a driven element (folded $\lambda/2$dipole) fed by a 300Ω 2-wire transmission line (twin line), a reflector and 4 directors.

- The antenna provides a gain of about 10 dBi (maximum) with a bandwidth at half-power of 10 percent.

Contd.,

- By adjusting lengths and spacings appropriately (tweeking), the dimensions can be optimized, producing an increase in gain of another decibel (Chen-1, 2; Viezbicke-1). However, the dimensions are critical.

- The inherently narrow bandwidth of the Yagi-Uda antenna can be broadened to 1.5 to 1 by lengthening the reflector to improve operation at low frequencies and shortening the directors to improve high-frequency operation sacrificing gain of as much as 5 dB.

(a) HPBW = 44° in plane of elements, HPBW = 64° in plane perpendicular to elements (from pattern).
(b) AR = infinite (pure linear polarization).
(c) Gain = 9.4 dBi by pattern integration.
(d) Pattern is shown in Fig.

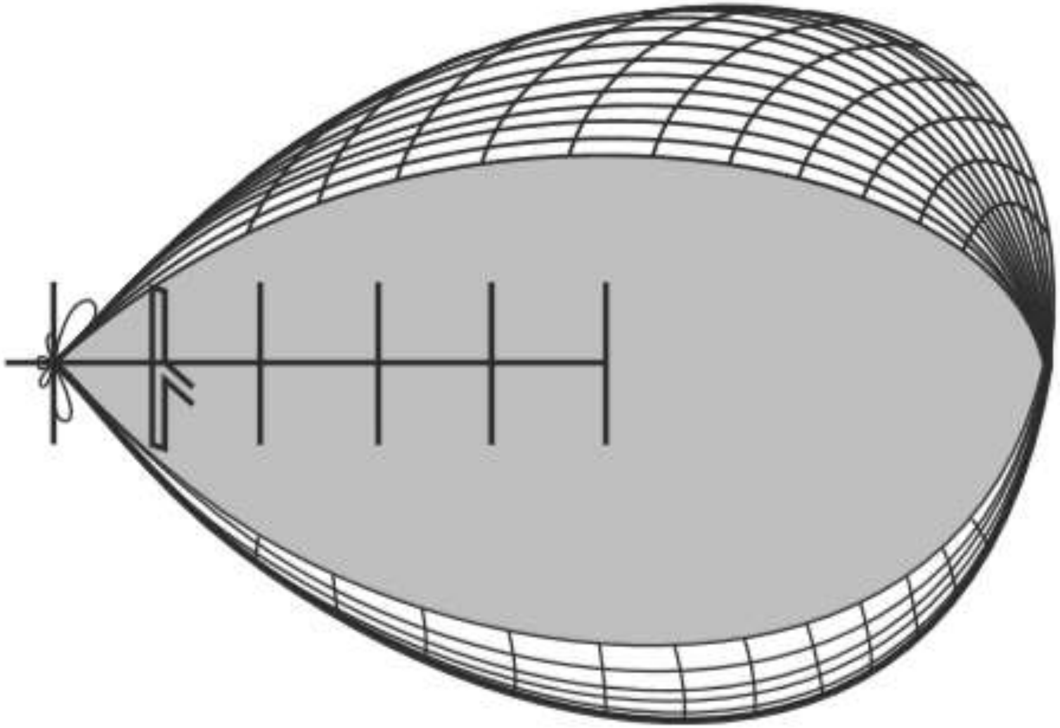


Figure Power pattern of the Yagi-Uda array of Fig. 8–31. The narrower pattern is in the plane of the elements.

Contd.,

- In the 1926,Dr.Shintaro Uda and Dr.Hidetsugu Yagi of the Tohoku imperial university invented a directional antenna system consisting of an array of coupled parallel diploes that is one of the most brilliant antenna designs, simple to construct and can achieve gain>10dB.

- It operates in HF to UHF Bands (about 3MHz to 3GHz and covers 40 to 60Km.

- It is a directional antenna system consisting of an array of coupled parallel dipoles commonly found on roof tops of houses as a terrestrial TV antenna.

Contd.,

- Yagi-uda antenna is an electromagnetic device that collects radio waves, once the antenna tuned to a particular frequency will resonate to a radio signal of the same frequency.

- It has three elements
  - Driven element
  - Reflector
  - Directors

Contd.,

- **DRIVEN ELEMENT:**

- It is the active element made of metal and has a feed point where the feed line is attached from the transmitter to the Yagi to perform the transfer of power or supply the charge from the transmitter to the antenna.

- It will be "resonant" when its electrical length is 1/2 of the wavelength of the frequency applied to its feed point.

- The feed point is on the center of the driven element.

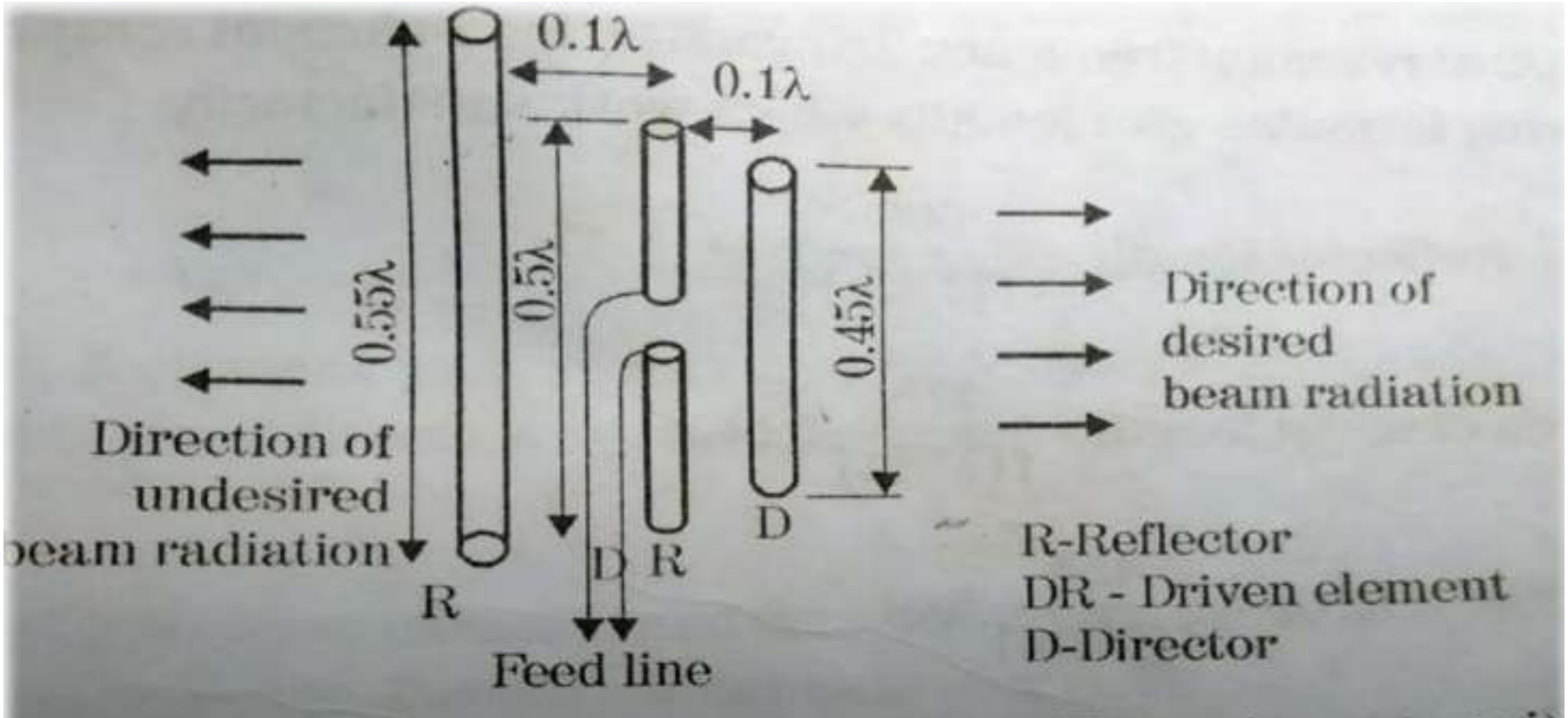
- **DIRECTOR:**

- The director is the passive element and shortest of the parasitic elements, this end is aimed at the receiving station.
- It is resonant slightly higher in frequency than the driven element, and its length will be about 5% shorter, progressively than the driven element.
- The directors lengths can vary, depending upon the director spacing, the number of directors used in the antenna, the desired pattern, pattern bandwidth and element diameter.
- The amount of gain is directly proportional to the length of the antenna array and not by the number of directors used.

- **REFLECTOR:**

- The reflector is the passive element that is placed at the rear of the driven element (The dipole) which directs the radiated field to add up in the desired direction i.e., along the direction of directors.
- It's resonant frequency is lower, and its length is approximately 5% longer than the driven element.
- It's length will vary depending on the spacing and the element diameter.
- The spacing of the reflector will be between 0.1λ and 0.25λ.
- It's spacing will depend upon the gain, bandwidth, F/B ratio, and side lobe pattern requirements of the final antenna design.

Contd.,



(a) Yagi-uda antenna (Three element yagi)

Contd.,

- Reflector here derives it's main Power from a driver , it reduces the signal strength in it's own direction and thus reflects the radiation towards the driver and directors.
- The driven element is where the signal is intercepted by the receiving equipment and has the cable attached that takes the received signal to the receiver
- The radiator and driver can be placed more closer to increase the radiation length towards the directors.

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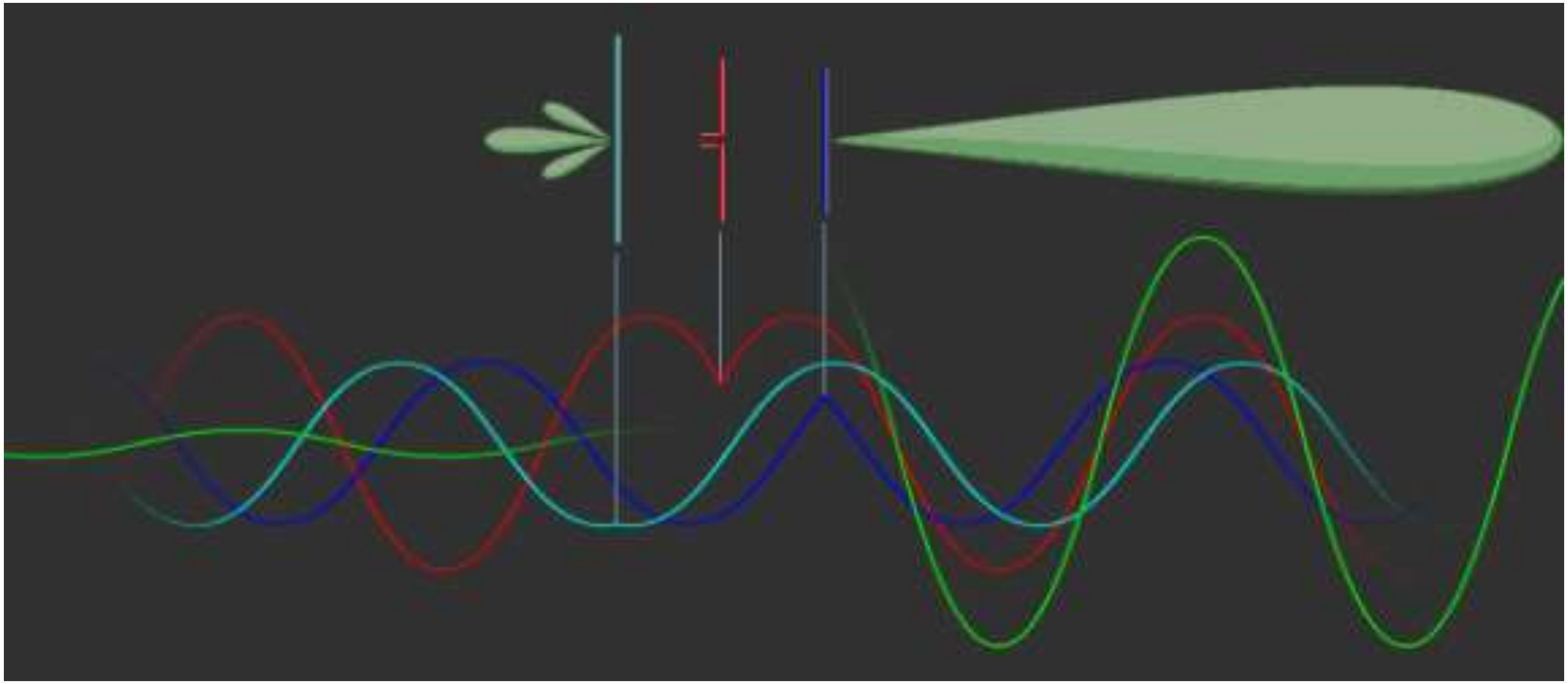


Contd.,

- Yagi Uda antenna is an array of a driven element and one or more parasitic elements. They are not connected directly to Tx line but Electrically coupled
- Parasitic elements are arranged collinearly and parallel to each other elements. Current flow in the driven element induces voltage in parasitic elements.
- The phase and current flows depends up on the spacing and reactance of the elements. The REFLECTOR length is 5% more than the Driven element ($\geq \lambda/2$) Leads to Inductive and the DIRECTOR length is 5% less than the Driven element ($\leq \lambda/2$) Leads to Capacitive
- Spacing are in order of 0.1λ to 0.15λ.

Contd.,

- **Design** For Three Element Yagi-Uda Antenna

  Reflector Length = 500/f(Mhz) Feet

  Driven element Length=475/f(Mhz) Feet

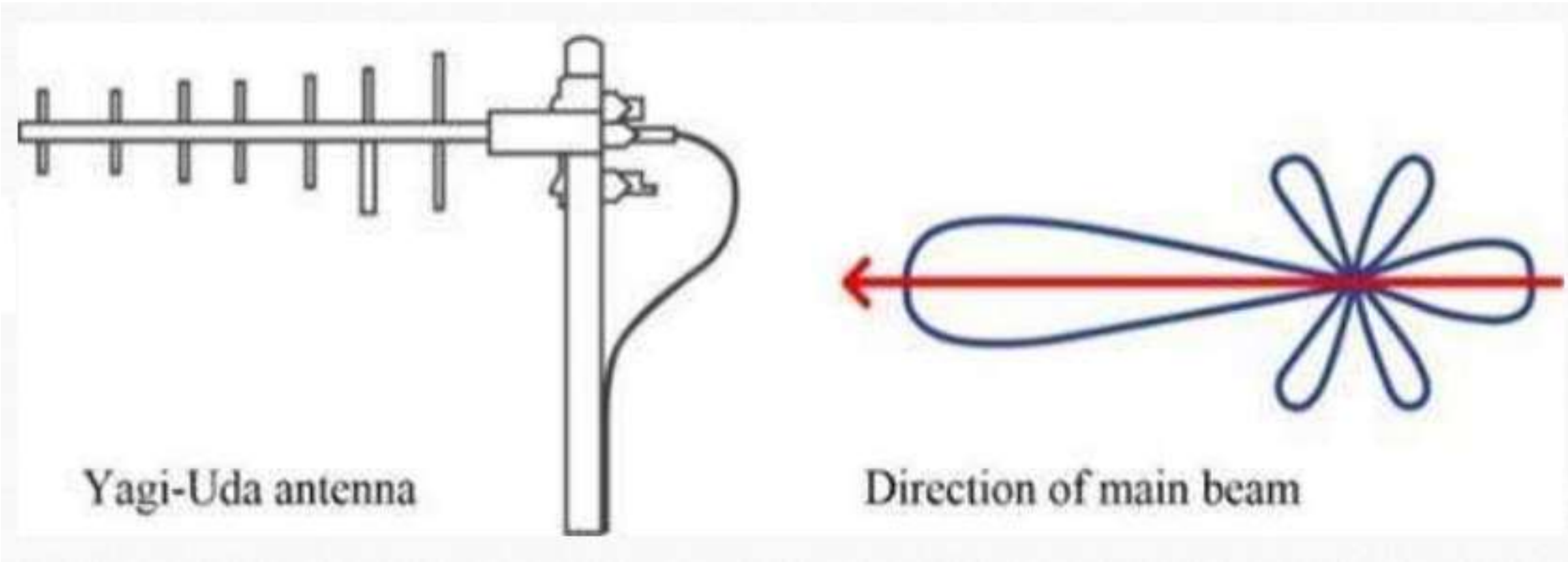
  Director Length =455/f(Mhz) Feet

  - If the number of directors are increased 5% decrease in length from previous director has to be ensured

  Spacing are in order of 0.1λ to 0.15λ.

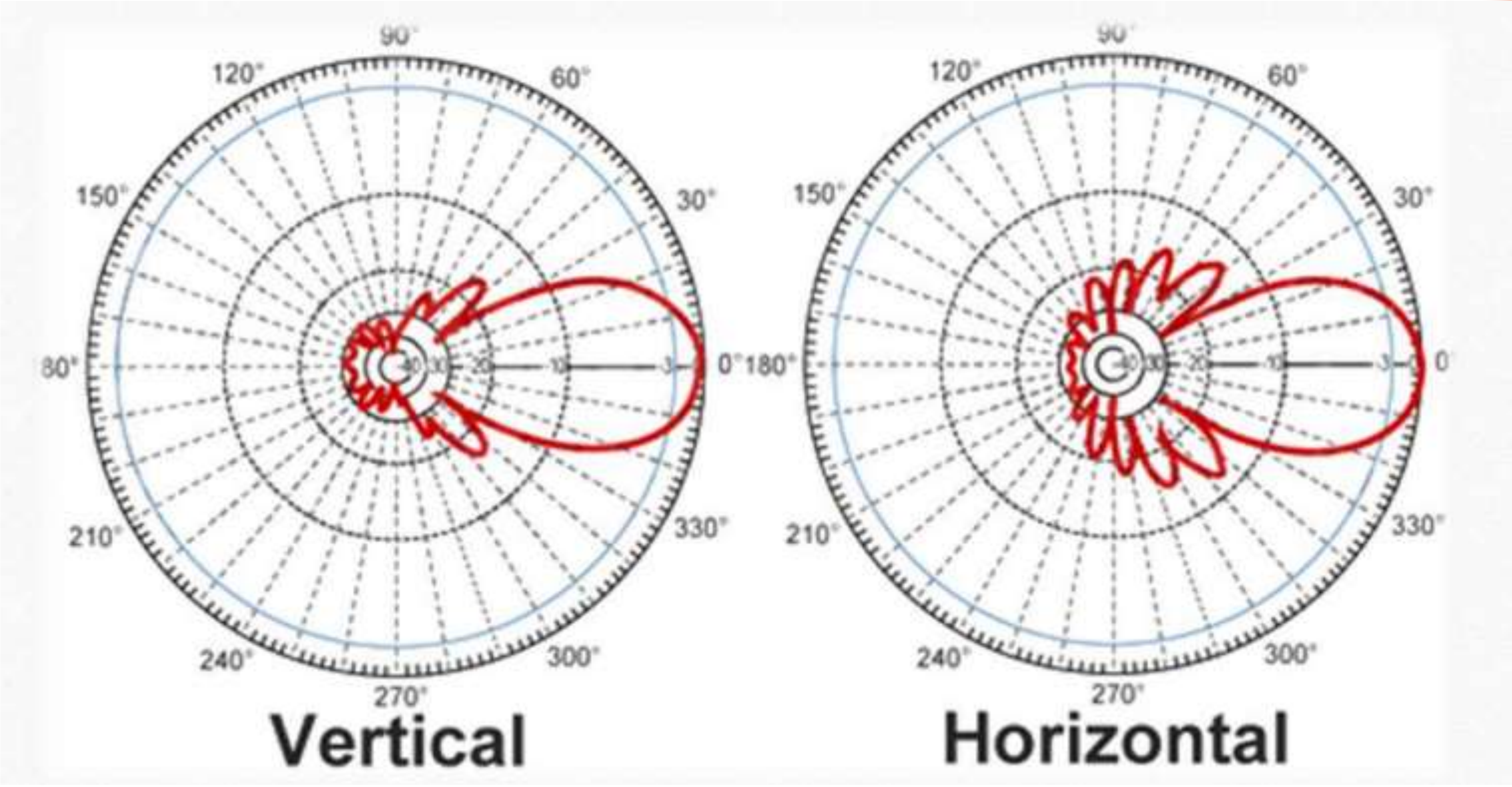
- The antenna can be optimised to either reduce number of spurious side lobes caused by radiation in the direction of the reflector or to produce the maximum level of forward gain.

- **Radiation Pattern**



Contd.,

- **Radiation Pattern**



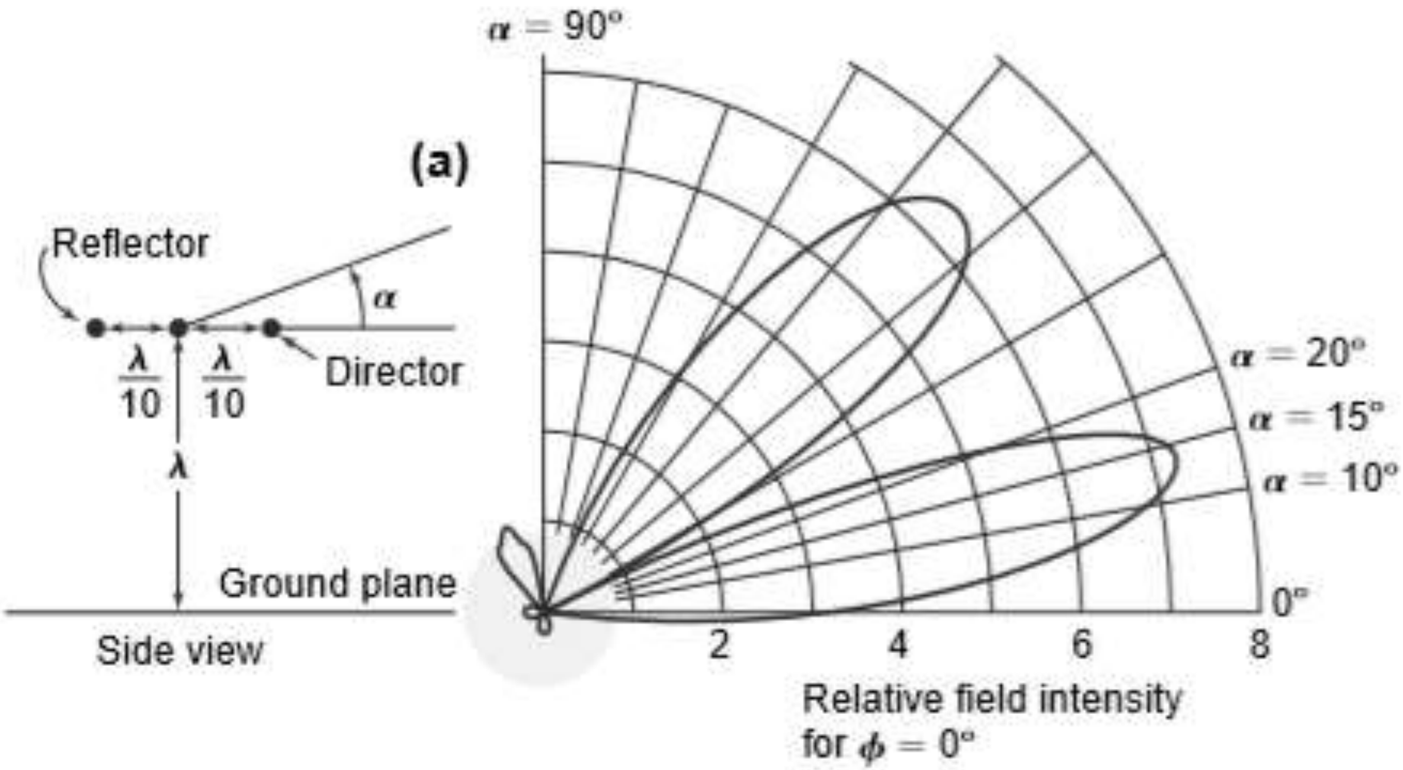
Yagi Uda Arrays

- Experimentally measured field patterns of a horizontal three-element array situated 1λ above a square horizontal ground plane about 13 λ on a side.

- The gain at $\alpha = 15^{\circ}$ for this array at a height of 1 λ is about 5 dB with respect to a single λ/2 dipole antenna at the same height.

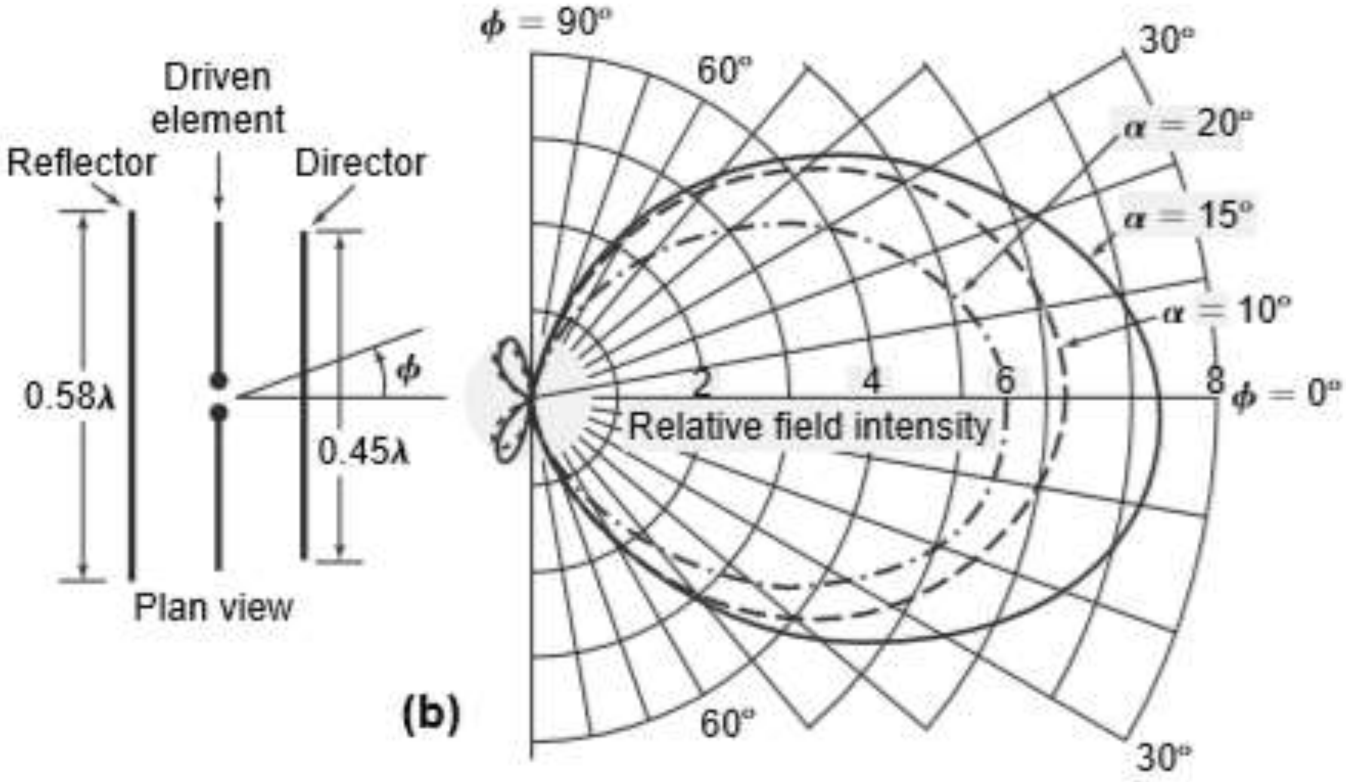
- A parasitic array of this type with closely spaced elements has a small driving-point radiation resistance and a relatively narrow bandwidth.

Contd.,



- The vertical plane pattern is shown in Fig. a shows that the finite size of the ground plane there is radiation at negative elevation angles. This phenomenon is characteristic of antennas with finite ground planes, the radiation at negative angles being largely the result of currents on the edges of the ground plane or beneath it.

Contd.,



- The azimuthal patterns at elevation angles α = 10,15 and 20◦ are shown in Fig.b.

Contd.,

- **Advantages**
  - Unidirectional beam of moderate Directivity i.e., directional antenna
  - Gain of order of 8db or front to back ratio of about 20 db i.e., moderate gain of 7dB
  - Beam antenna—3 element configuration
  - Super directivity or gain antenna due to its high gain and beam width per unit area of array
  - Further elements increase ---directivity increases
  - Construction is simple
  - Less cost and simple to feed
  - Can be used at HF
  - Adjustable Front to Back Ratio

Contd.,

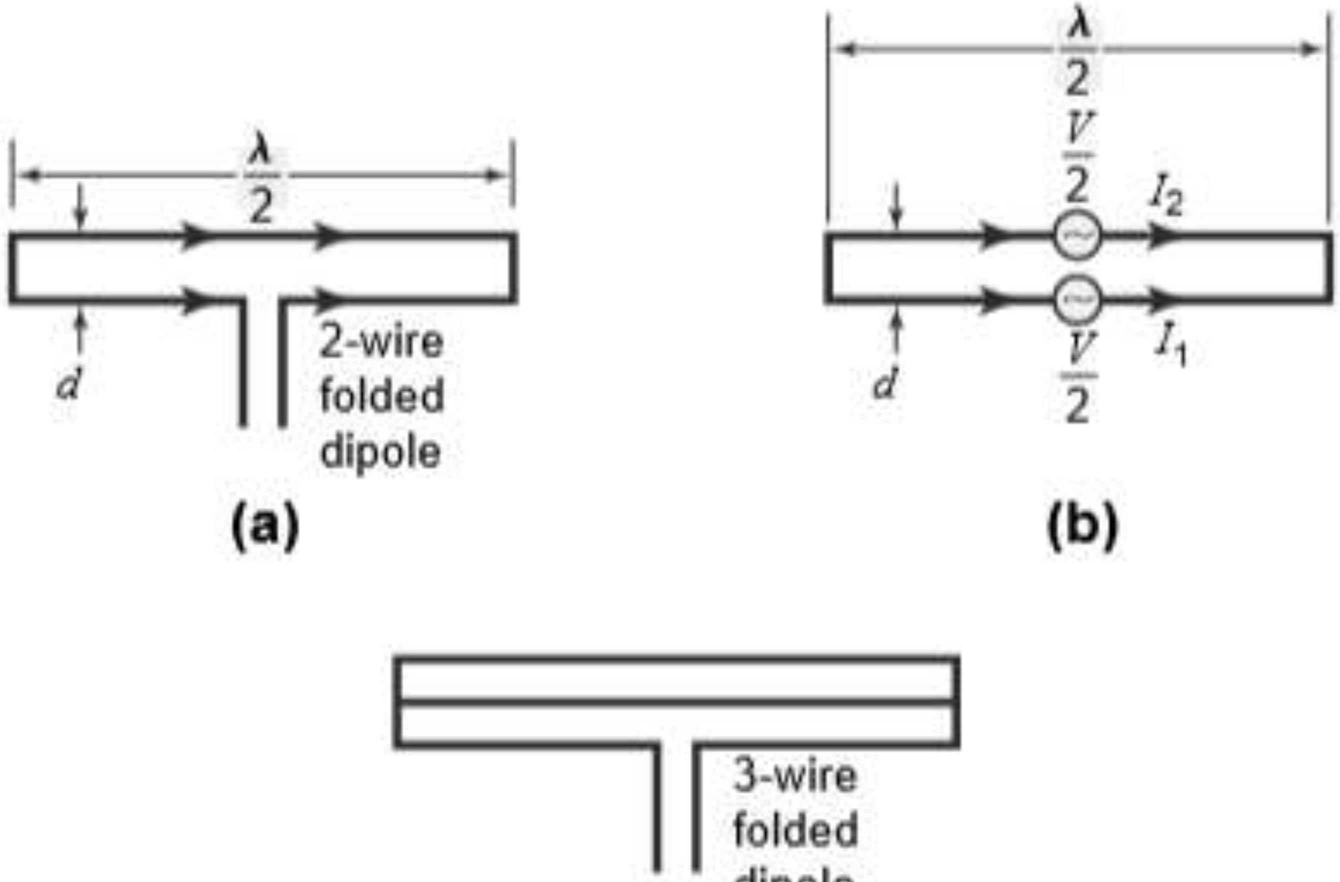
- **Disadvantages**
  - Gain is not much more
  - Need more driven elements for longer distances
  - Bandwidth is Limited
- **Remedy**
  - Broadband Planar Quasi Yagi antenna for higher Band Width
- **Applications**
  - Yagi-uda antenna is a unidirectional antenna used for television receiver's as they provide better tuning due to large bandwidth and decent gain.

- **Folded Dipole** and Its Characteristics
- Array with parasitic element

Folded dipoles & their characteristics

- The ultra close-spaced type of array used to match 2-wire transmission line of 300Ω to 600Ω impedance is called a folded dipole.
- It is a 2-wire folded λ/2 dipole, consists 2 closely spaced λ/2 elements connected together at the outer ends. The currents in the elements are substantially equal and in phase.
- Assuming that both conductors of the dipole have the same diameter, the approximate value of the terminal impedance may be deduced very simply as
- Let the emf V applied to the antenna terminals be divided between the 2 dipoles as in Fig. b. Then

Contd.,



$\frac{\lambda}{2}$
d
2-wire folded dipole
$\frac{V}{2}$
I_2
I_1
3-wire folded dipole

(a) (b) (c)

$$\frac{V}{2} = I_1 Z_{11} + I_2 Z_{12}$$

Contd.,

where

$$I_1 = \text{current at terminals of dipole 1}$$
$$I_2 = \text{current at terminals of dipole 2}$$

$$Z_{11} = \text{self-impedance of dipole 1}$$
$$Z_{12} = \text{mutual impedance of dipoles 1 and 2}$$

Since $I_1 = I_2$

$$V = 2I_1(Z_{11} + Z_{12})$$

- Since the dipoles are closely placed, d is of the order of $\lambda/100$, $Z_{12} \simeq Z_{11}$. then the terminal impedance of the folded dipole is

$$Z = \frac{V}{I_1} \simeq 4Z_{11}$$

- If Z_{11} is considered to be dipole impedance as 73Ω, then the impedance of 2-wire Folded dipole is $Z_{FD} = 4 \times 73\Omega = 292\Omega$.

- The above design can be generalized for N-Wired Folded Dipole as

$$Z_{FD} = N^2 \times Z_D = N^2 \times Z_{IN}$$

Contd.,

For equal dimensions of the dipoles,

$$I_1 = I_2 = I_3 = I$$

So

$$\frac{V}{3} = I\,(3Z_{11})$$

$$\frac{V}{I} = 3 \times 3Z_{11} = 9z_{11}$$

$$= 9 \times 73 = 657\,\Omega$$

- For the 3-wire Folded Dipole,

  $Z_{FD} = 3^2 \times 73\Omega = 657\Omega$

Contd.,

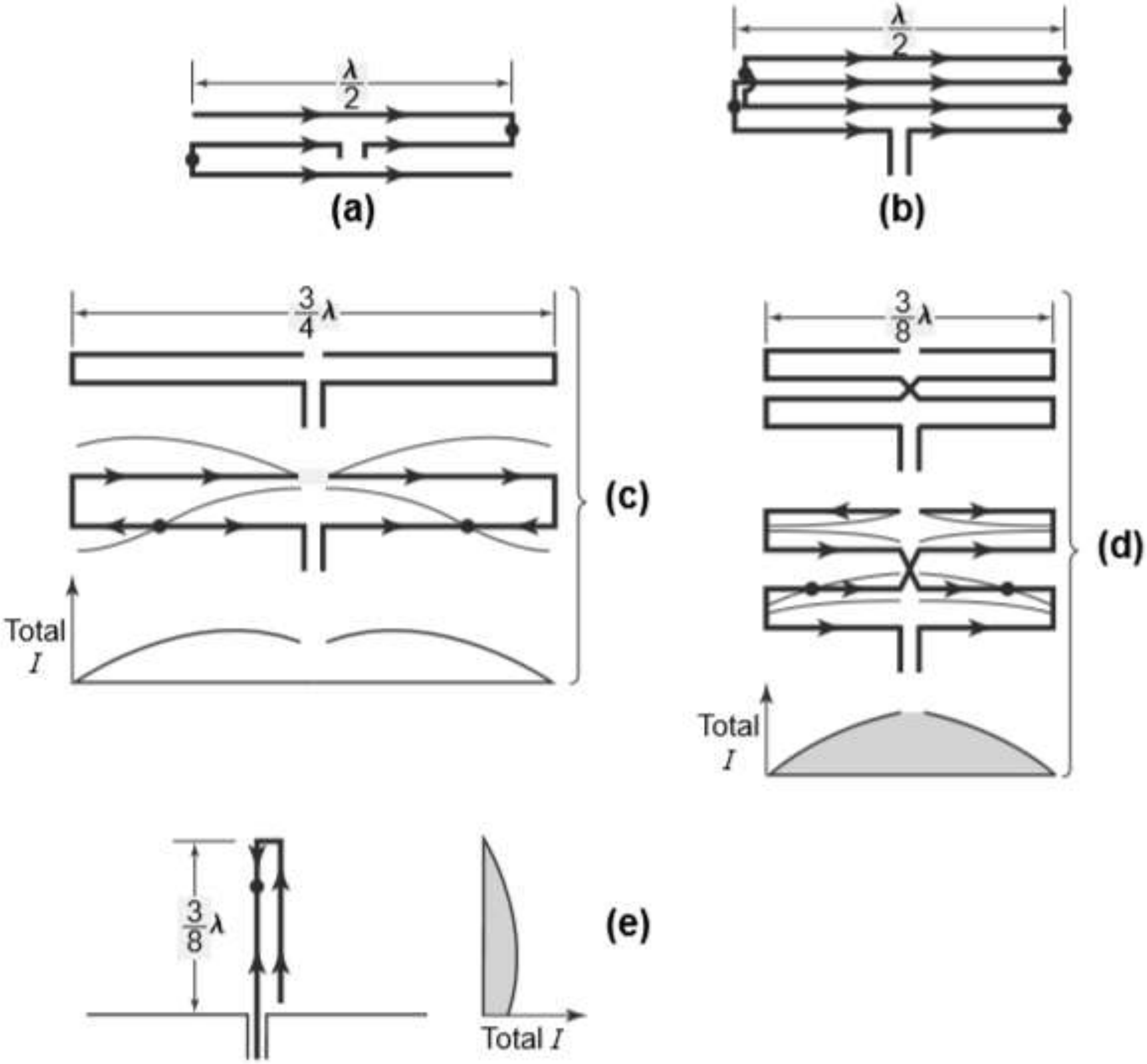


Fig. (a) Three-wire folded λ/2 dipole, (b) 4-wire folded λ/2 dipole,

(c) 2-wire 3λ/4 antenna(Z=450Ω),

(d) 4-wire 3λ/8 antenna (Z=225Ω) and

(e) 2-wire 3λ/8 stub antenna. (Z=225Ω)

Arrows indicate instantaneous current directions and dots indicate current minimum points.

Contd.,

The impedance of the dipole depends on

1. spacing between dipoles and
2. radius of the dipoles.

Case 1 If the radii of the dipoles are r_1 and r_2, then

$$Z_i = 73\left(1+\frac{r_2}{r_1}\right)^2 \Omega$$

Case 2 If the radii of the dipoles are r_1 and r_2 and d is the spacing between the elements, then

$$Z_i = 73\left[1+\frac{\log\frac{d}{r_1}}{\log\frac{d}{r_2}}\right]^2$$

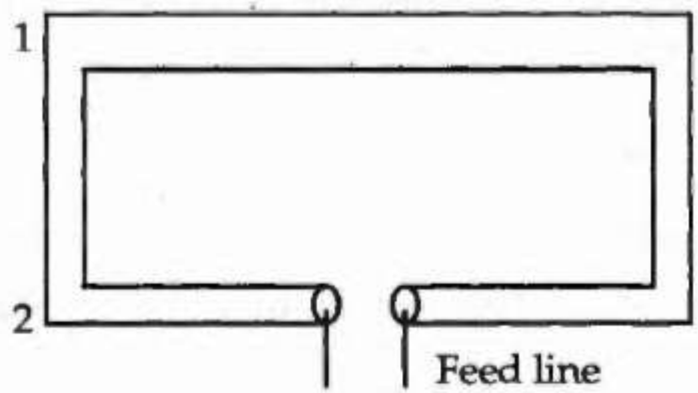
$$R_r = Z_i = 73 \times b$$

Contd.,

where b = impedance transformation ratio which is given by

$$b = \left[1 + \frac{\log \frac{d}{r_1}}{\log \frac{d}{r_2}} \right]^2$$

The Dimension of one element can be changed to obtain a desired resistance as



If the Diameter of arm2 is larger than arm1 then Z decreases, else increases.

Types of Folded dipoles

1. Unequal conductor folded dipoles
2. Multi-conductor folded dipoles

Fig. Folded Dipole

Design of Folded Dipole Antenna

- The length of Folded Dipole is

$$L_a = \frac{145}{f\,(MHz)}$$

- The length of coil to match 75Ω cable is

$$L_b = 0.8 \times L_a$$

- The length of coil to match 50Ω Cable is

$$L_b = 0.66 \times L_a$$

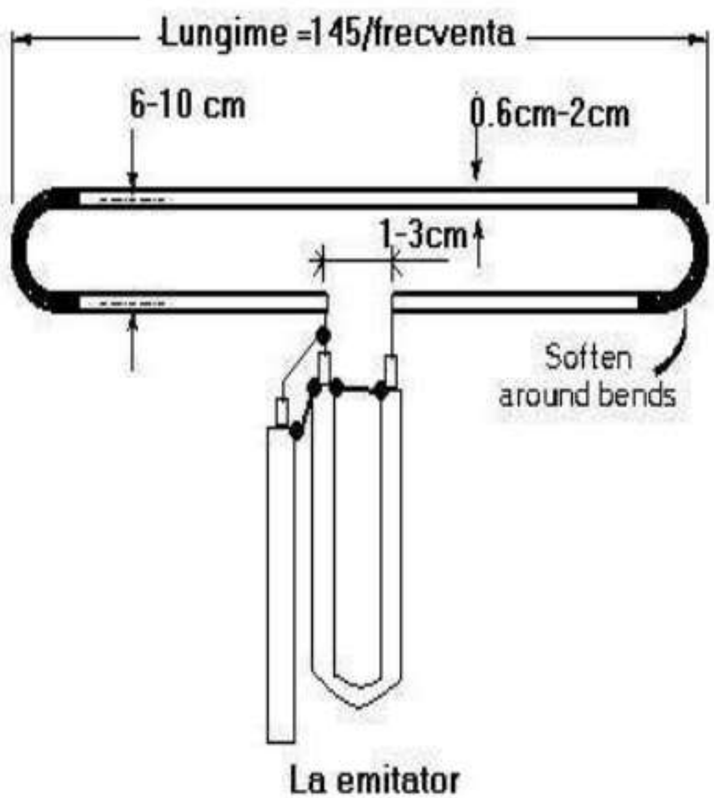
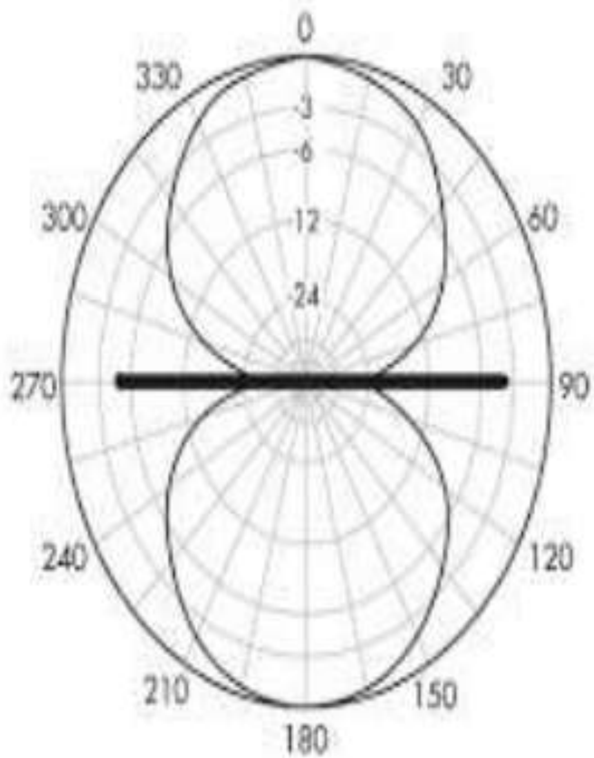


Fig. Folded Dipole Design

Properties of Folded Dipole

- Ease of construction
- Structural Rigidity
- Wider Bandwidth than Half Wave Dipole
- Impedance matched to 2-wire transmission line of 300 to 600 Ω.
- Radiation Pattern Same as Dipole – Omnidirectional.

Radiation Pattern



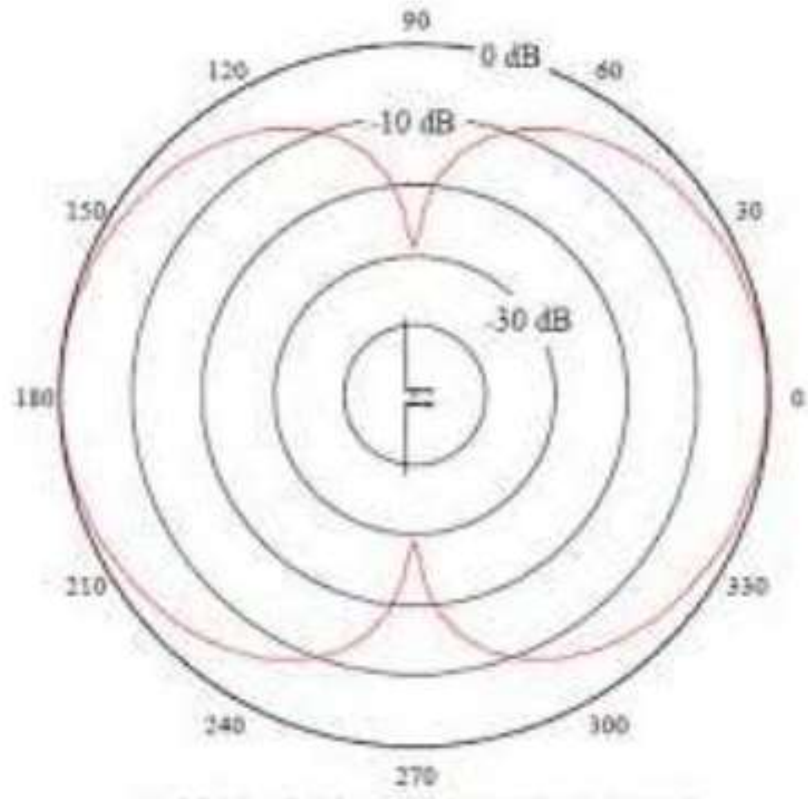


Fig. Radiation Pattern

Contd.,

- **Advantages**
  - Same radiation characteristics as that of Dipole Antenna
  - High input Impedance (300 ohms) i.e., better impedance matching characteristics
  - Higher bandwidth
  - Ease and Low Constructional Cost
  - Easy to Fabricate
  - Acts as built-in reactance compensation network
  - Less reflection for Odd Harmonics (or Low SWR) for Fundamental Frequency.

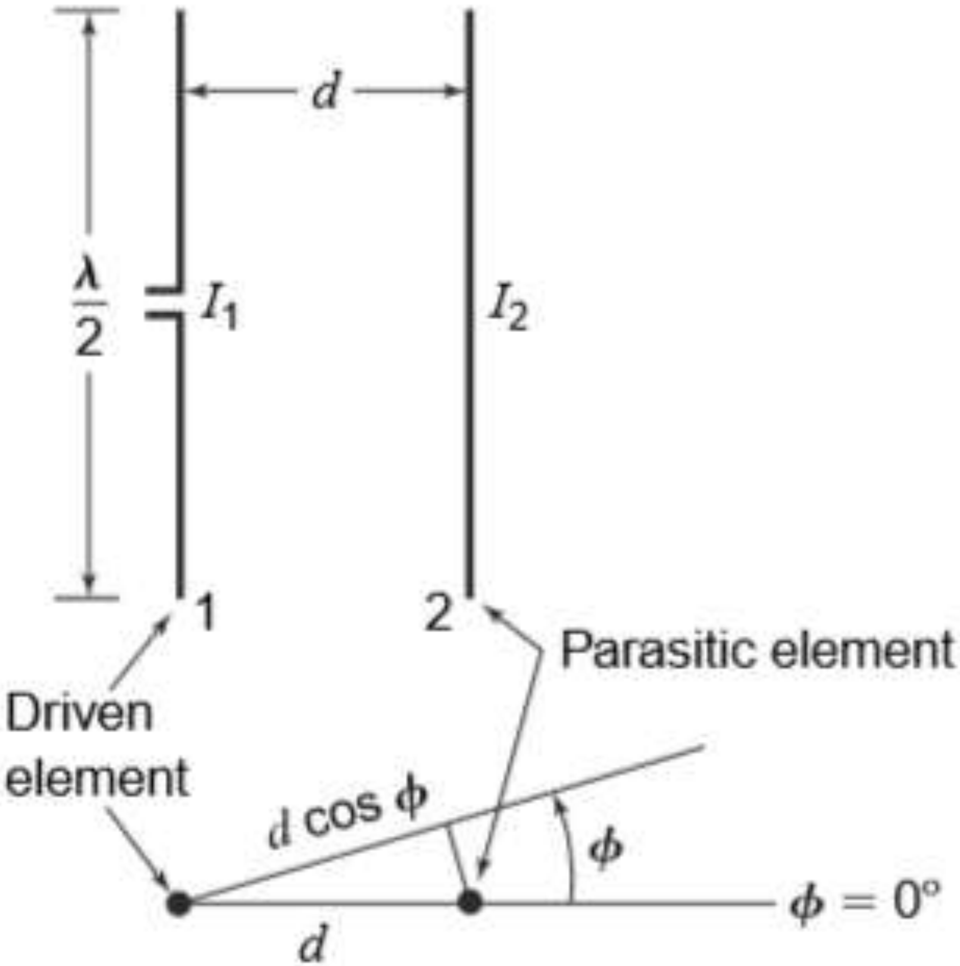
- **Disadvantages**
  - The currents on each wire will begin to cancel each other out on even multiples of the *cut* frequency, so a 40 meter folded dipole should not be used on 14 MHz.

  - On other bands even though the signal may cancel broad side to the antenna, We'll find that there is actually gain! This occurs about 45 degrees off broad side to the antenna. And this might make for interesting contacts.

Contd.,

- **Applications**
  - Active element for Yagi-Uda Antenna for TV antennas
  - Impedance Matching Device
  - Extension of Transmission Line
  - Used for receiving antenna in domestic Television in Yagi-Uda Antenna
  - VHF FM Broadcast Antennas

Arrays with parasitic elements



- Array with one driven dipole element and one parasitic element
- Antennas can also be constructed with "parasitic elements" in which currents are induced by the fields from a driven element.
- Such elements do not have transmission line connection.
- Suppose that both elements are vertical so that the azimuth angle ϕ.

Contd.,

- The circuit relations for the elements are

$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$

$$0 = I_2 Z_{22} + I_1 Z_{12}$$

$$I_2 = -I_1 \frac{Z_{12}}{Z_{22}} = -I_1 \frac{|Z_{12}| \underline{/\tau_m}}{|Z_{22}| \underline{/\tau_2}} = -I_1 \left| \frac{Z_{12}}{Z_{22}} \right| \underline{/\tau_m - \tau_2}$$

$$I_2 = I_1 \left| \frac{Z_{12}}{Z_{22}} \right| \underline{/\xi}$$

where $\xi = \pi + \tau_m - \tau_2$, in which

$$\tau_m = \arctan \frac{X_{12}}{R_{12}}$$

$$\tau_2 = \arctan \frac{X_{22}}{R_{22}}$$

where

$$R_{12} + jX_{12} = Z_{12} = \text{mutual impedance of elements 1 and 2, } \Omega$$
$$R_{22} + jX_{22} = Z_{22} = \text{self-impedance of the parasitic element, } \Omega$$

The electric field intensity at a large distance from the array as a function of ϕ is

$$E(\phi) = k\left(I_1 + I_2 \angle d_r \cos\phi\right)$$

where $d_r = \beta d = \dfrac{2\pi}{\lambda} d$

$$E(\phi) = kI_1\left(1 + \left|\frac{Z_{12}}{Z_{22}}\right| \angle \xi + d_r \cos\phi\right)$$

the driving-point impedance Z_1 of the driven element, we get

$$Z_1 = Z_{11} - \frac{Z_{12}^2}{Z_{22}} = Z_{11} - \frac{|Z_{12}^2| \angle 2\tau_m}{|Z_{22}| \angle \tau_2}$$

The real part of Z_1 is

$$R_1 = R_{11} - \left|\frac{Z_{12}^2}{Z_{22}}\right| \cos(2\tau_m - \tau_2)$$

Adding a term for the effective loss resistance, if any is present, we have

$$R_1 = R_{11} + R_{1L} - \left|\frac{Z_{12}^2}{Z_{22}}\right| \cos(2\tau_m - \tau_2)$$

For a power input P to the driven element,

$$I_1 = \sqrt{\frac{P}{R_1}} = \sqrt{\frac{P}{R_{11} + R_{1L} - |Z_{12}^2/Z_{22}| \cos(2\tau_m - \tau_2)}}$$

Contd.,

- The electric field intensity at a large distance from the array as a function of ϕ is

$$E(\phi) = k\sqrt{\frac{P}{R_{11} + R_{1L} - |Z_{12}^2/Z_{22}|\cos(2\tau_m - \tau_2)}}\left(1 + \left|\frac{Z_{12}}{Z_{22}}\right| \angle \xi + d_r \cos\phi\right)$$

- For a power input P to a single vertical $\lambda/2$ element the electric field intensity at the same distance is

$$E_{\mathrm{HW}}(\phi) = kI_0 = k\sqrt{\frac{P}{R_{00} + R_{0L}}} \qquad (\mathrm{V\,m^{-1}})$$

where

R_{00} = self-resistance of single $\lambda/2$ element, Ω
R_{0L} = loss resistance of single $\lambda/2$ element, Ω

Contd.,

- The gain in field intensity (as a function of ϕ) of the array with respect to a single λ/2 antenna with the same power input is given by (assume $R_{00} = R_{11}$ and let $R_{0L} = R_{1L}$)

$$G_f(\phi)\left[\frac{A}{\mathrm{HW}}\right] = \sqrt{\frac{R_{11}+R_{1L}}{R_{11}+R_{1L}-|Z_{12}^2/Z_{22}|\cos(2\tau_m-\tau_2)}}\left(1+\left|\frac{Z_{12}}{Z_{22}}\right|\angle \xi + d_r\cos\phi\right)$$

- If Z22 is made very large by detuning the parasitic element (i.e., by making X_{22} large), the above equation reduces to unity, i.e., the field of the array becomes the same as the single λ/2 dipole comparison antenna.

- Brown analyzed arrays with a single parasitic element for various values of parasitic element reactance (X_{22}) and was the first to point out that spacings of less than λ/4 were desirable.

Contd.,

- The magnitude of the current in the parasitic element and its phase relation to the current in the driven element depends on its tuning.

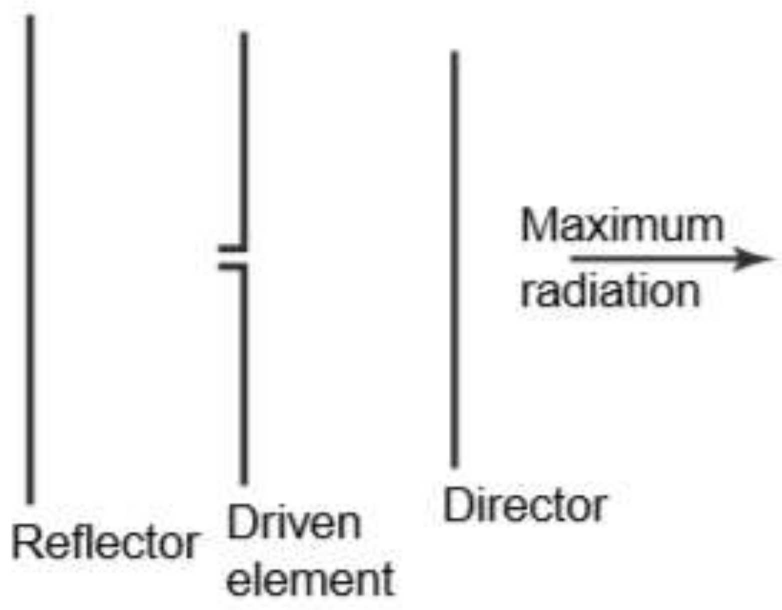
- The parasitic element may have a fixed length of $\lambda/2$, the tuning being accomplished by inserting a lumped reactance in series with the antenna at its center point.

- Alternatively, the parasitic element may be continuous and the tuning accomplished by adjusting the length. This method is often simpler in practice but is more difficult for analysis.

- **When the $\lambda/2$ parasitic element is inductive (longer than its resonant length) it acts as a reflector.**

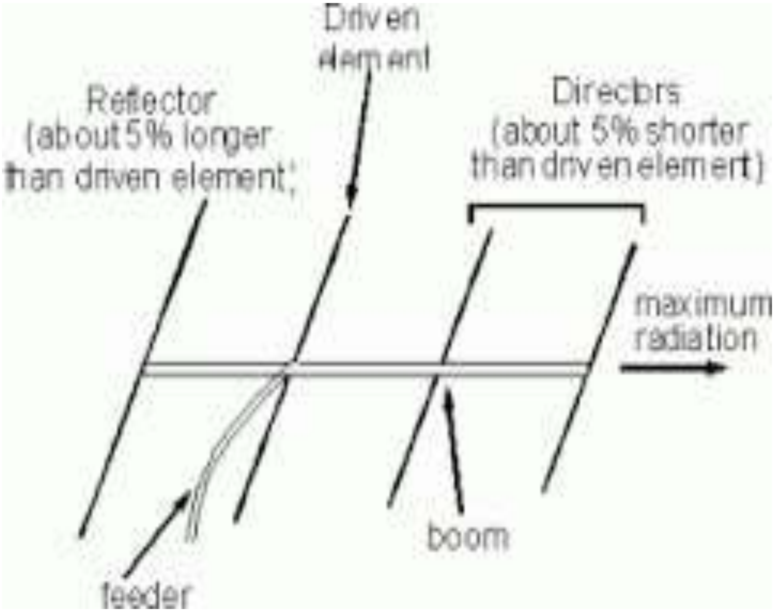
- **When it is capacitive (shorter than its resonant length) it acts as a director.**

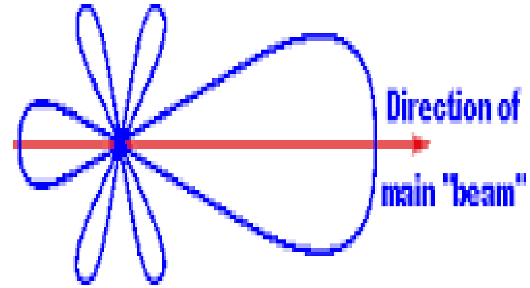
- Arrays may be constructed with both a reflector and a director called as **Yagi Uda Array Antenna**.



Yagi-Uda Array

- A Yagi-Uda array is an example of a parasitic array. Any element inan array which is not connected to the source (in the case of a transmitting antenna) or thereceiver (in the case of a receiving antenna) is defined as a parasitic element. A parasitic array isany array which employs parasitic elements. The general form of the N-element Yagi-Uda array isshown below.
- Driven element - usually a resonant dipole or folded dipole.), folded dipoles are employed as driven elements to increase the array input impedance
- Reflector - slightly longer than the driven element so that it isinductive (its current lags that of the driven element).Approximately 5 to 10 % longer than the driven element.
- Director - slightly shorter than the driven element so that it iscapacitive (its current leads that of the driven element).Approximately 10 to 20 % shorter than the driven element), not necessarily uniform





- **Advantages**
- 1. Lightweight, Low cost 2.Simple construction
- 3.Unidirectional beam (front-to-back ratio) 4.Increased directivity over other simple wire antennas
- 5.Practical for use at HF (3-30 MHz), VHF (30-300 MHz), andUHF (300 MHz - 3 GHz) Reflector spacing 0.1 to 0.258

Folded dipoles & their characteristics

- The ultra close-spaced type of array used to match 2-wire transmission line of 300Ω to 600Ω impedance is called a folded dipole.

- It is a 2-wire folded $\lambda/2$ dipole, consists 2 closely spaced $\lambda/2$ elements connected together at the outer ends. The currents in the elements are substantially equal and in phase.

- Assuming that both conductors of the dipole have the same diameter, the approximate value of the terminal impedance may be deduced very simply as

- Let the emf V applied to the antenna terminals be divided between the 2 dipoles as in Fig. b. Then

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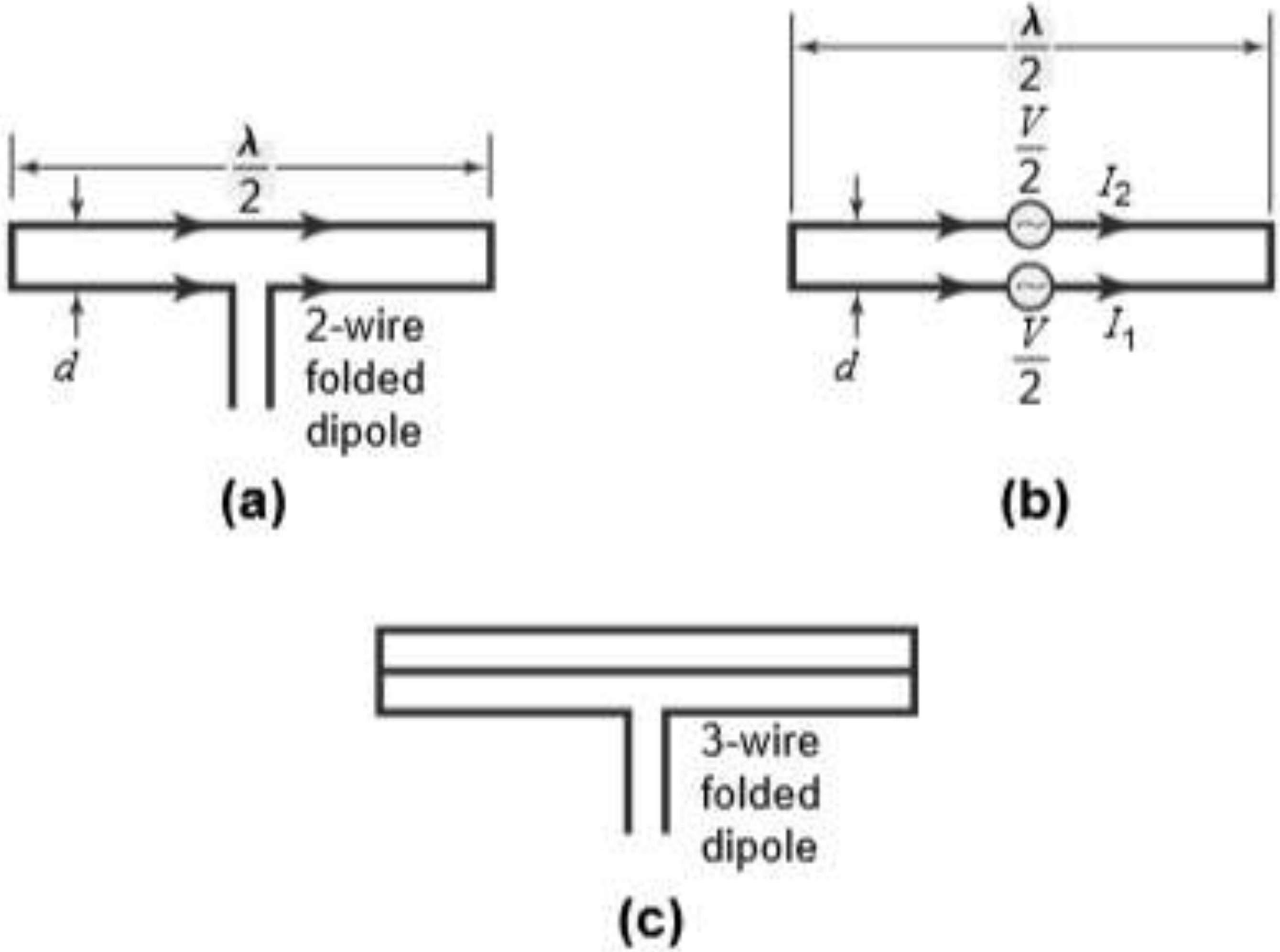


λ
2
d
2-wire
folded
dipole
(a)
λ
2
V
2
I2
I1
V
2
d
(b)
3-wire
folded
dipole
(c)

$$\frac{V}{2} = I_1 Z_{11} + I_2 Z_{12}$$

Contd.,

where

$$I_1 = \text{current at terminals of dipole 1}$$
$$I_2 = \text{current at terminals of dipole 2}$$

$$Z_{11} = \text{self-impedance of dipole 1}$$
$$Z_{12} = \text{mutual impedance of dipoles 1 and 2}$$

Since $I_1 = I_2$

$$V = 2I_1(Z_{11} + Z_{12})$$

- Since the dipoles are closely placed, d is of the order of $\lambda/100$, $Z_{12} \simeq Z_{11}$. then the terminal impedance of the folded dipole is

$$Z = \frac{V}{I_1} \simeq 4Z_{11}$$

- If Z_{11} is considered to be dipole impedance as 73Ω, then the impedance of 2-wire Folded dipole is $Z_{FD} = 4 \times 73\Omega = 292\Omega$.

- The above design can be generalized for N-Wired Folded Dipole as

$$Z_{FD} = N^2 \times Z_D = N^2 \times Z_{IN}$$

Contd.,

For equal dimensions of the dipoles,

$$I_1 = I_2 = I_3 = I$$

So $$\frac{V}{3} = I\,(3Z_{11})$$

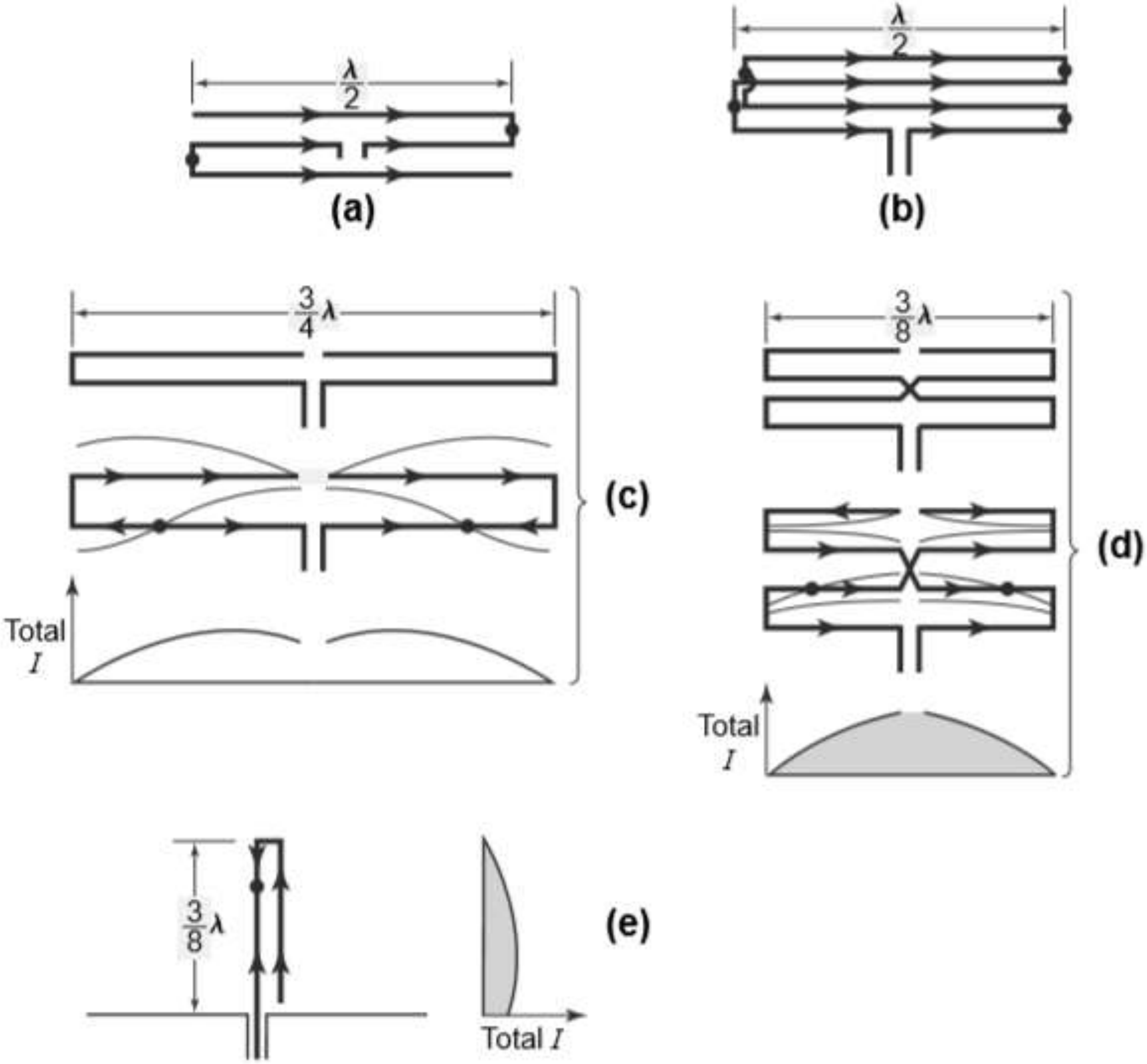
$$\frac{V}{I} = 3 \times 3Z_{11} = 9z_{11}$$

$$= 9 \times 73 = 657\,\Omega$$

- For the 3-wire Folded Dipole,

  $Z_{FD} = 3^2 \text{ x } 73\Omega = 657\Omega$

Contd.,



- Fig. (a) Three-wire folded λ/2 dipole, (b) 4-wire folded λ/2 dipole,

  (c) 2-wire 3λ/4 antenna(Z=450Ω),

  (d) 4-wire 3λ/8 antenna (Z=225Ω) and

  (e) 2-wire 3λ/8 stub antenna. (Z=225Ω)

- Arrows indicate instantaneous current directions and dots indicate current minimum points.

Contd.,

The impedance of the dipole depends on

1. spacing between dipoles and
2. radius of the dipoles.

Case 1 If the radii of the dipoles are r_1 and r_2, then

$$Z_i = 73\left(1+\frac{r_2}{r_1}\right)^2 \Omega$$

Case 2 If the radii of the dipoles are r_1 and r_2 and d is the spacing between the elements, then

$$Z_i = 73\left[1+\frac{\log\frac{d}{r_1}}{\log\frac{d}{r_2}}\right]^2$$

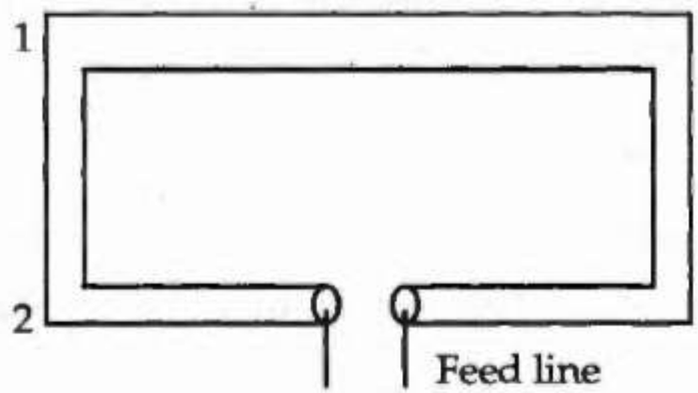
$$R_r = Z_i = 73 \times b$$

where b = impedance transformation ratio which is given by

Contd.,

$$b = \left[1 + \frac{\log \frac{d}{r_1}}{\log \frac{d}{r_2}} \right]^2$$

The Dimension of one element can be changed to obtain a desired resistance as



If the Diameter of arm2 is larger than arm1 then Z decreases, else increases.

Types of Folded dipoles

1. Unequal conductor folded dipoles
2. Multi-conductor folded dipoles

Fig. Folded Dipole

Design of Folded Dipole Antenna

- The length of Folded Dipole is

$$L_a = \frac{145}{f\,(MHz)}$$

- The length of coil to match 75Ω cable is

$$L_b = 0.8 \times L_a$$

- The length of coil to match 50Ω Cable is

$$L_b = 0.66 \times L_a$$

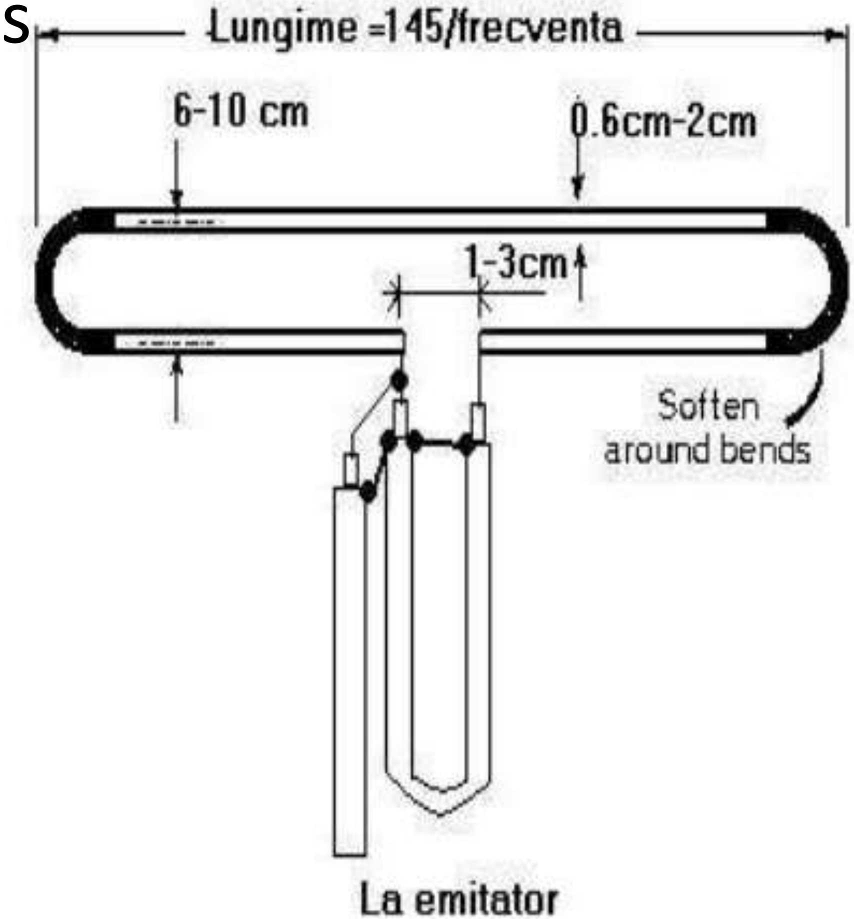
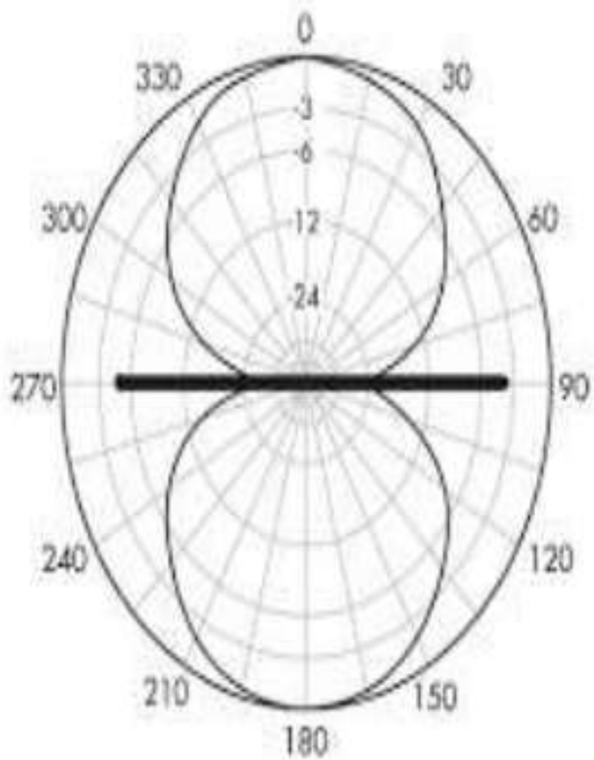


Fig. Folded Dipole Design

Properties of Folded Dipole

- Ease of construction
- Structural Rigidity
- Wider Bandwidth than Half Wave Dipole
- Impedance matched to 2-wire transmission line of 300 to 600 Ω.
- Radiation Pattern Same as Dipole – Omnidirectional.

Radiation Pattern



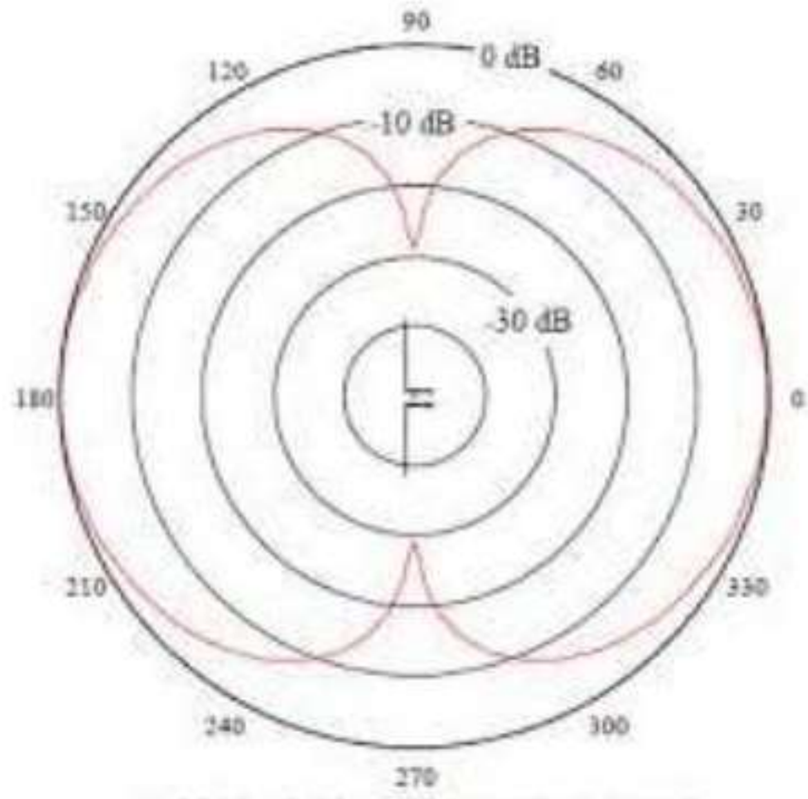


Fig. Radiation Pattern

Contd.,

- **Advantages**
  - Same radiation characteristics as that of Dipole Antenna
  - High input Impedance (300 ohms) i.e., better impedance matching characteristics
  - Higher bandwidth
  - Ease and Low Constructional Cost
  - Easy to Fabricate
  - Acts as built-in reactance compensation network
  - Less reflection for Odd Harmonics (or Low SWR) for Fundamental Frequency.

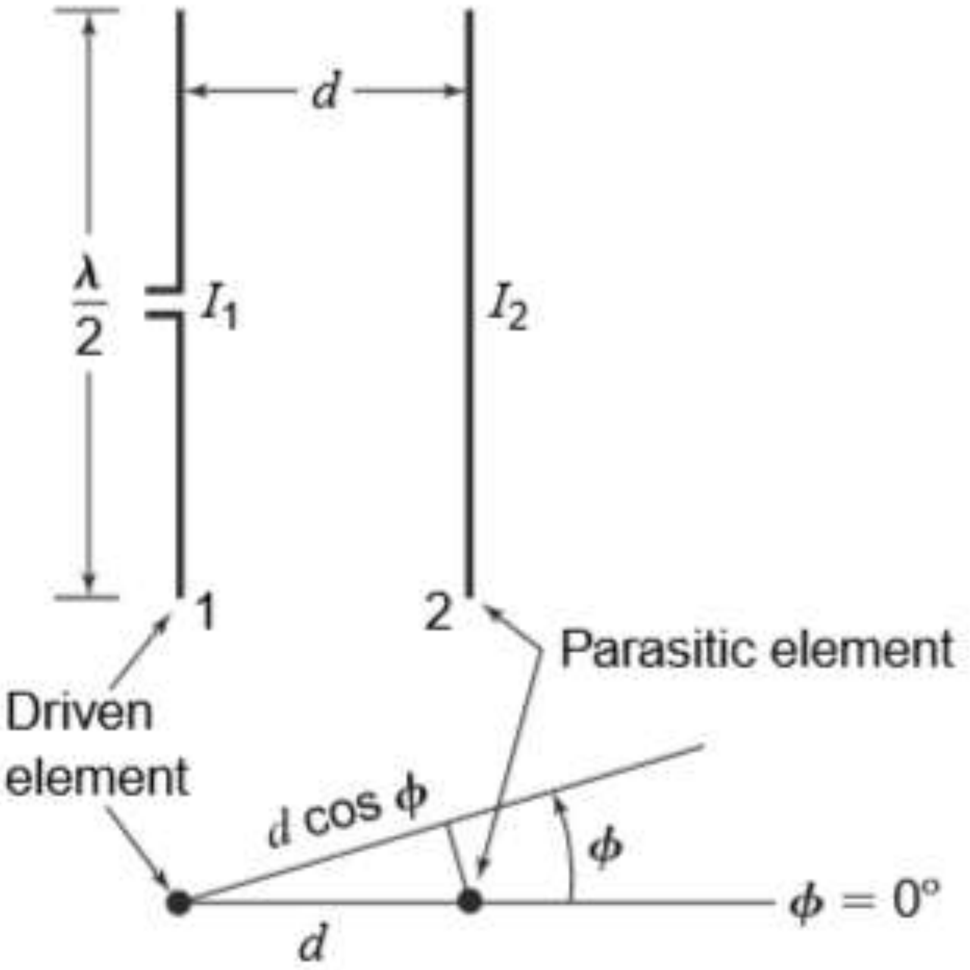
- **Disadvantages**
  - The currents on each wire will begin to cancel each other out on even multiples of the *cut* frequency, so a 40 meter folded dipole should not be used on 14 MHz.

  - On other bands even though the signal may cancel broad side to the antenna, We'll find that there is actually gain! This occurs about 45 degrees off broad side to the antenna. And this might make for interesting contacts.

Contd.,

- **Applications**
  - Active element for Yagi-Uda Antenna for TV antennas
  - Impedance Matching Device
  - Extension of Transmission Line
  - Used for receiving antenna in domestic Television in Yagi-Uda Antenna
  - VHF FM Broadcast Antennas

Arrays with parasitic elements



- Array with one driven dipole element and one parasitic element

- Antennas can also be constructed with “parasitic elements” in which currents are induced by the fields from a driven element.

- Such elements do not have transmission line connection.

- Suppose that both elements are vertical so that the azimuth angle ϕ.

- The circuit relations for the elements are

$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$
$$0 = I_2 Z_{22} + I_1 Z_{12}$$
$$I_2 = -I_1 \frac{Z_{12}}{Z_{22}} = -I_1 \frac{|Z_{12}| \angle \tau_m}{|Z_{22}| \angle \tau_2} = -I_1 \left| \frac{Z_{12}}{Z_{22}} \right| \angle \tau_m - \tau_2$$

$$I_2 = I_1 \left| \frac{Z_{12}}{Z_{22}} \right| \angle \xi$$

where $\xi = \pi + \tau_m - \tau_2$, in which

$$\tau_m = \arctan \frac{X_{12}}{R_{12}}$$
$$\tau_2 = \arctan \frac{X_{22}}{R_{22}}$$

where

$$R_{12} + jX_{12} = Z_{12} = \text{mutual impedance of elements 1 and 2, } \Omega$$
$$R_{22} + jX_{22} = Z_{22} = \text{self-impedance of the parasitic element, } \Omega$$

The electric field intensity at a large distance from the array as a function of ϕ is

$$E(\phi) = k\left(I_1 + I_2 \angle d_r \cos\phi\right)$$

where $d_r = \beta d = \frac{2\pi}{\lambda} d$

$$E(\phi) = kI_1\left(1 + \left|\frac{Z_{12}}{Z_{22}}\right| \angle \xi + d_r \cos\phi\right)$$

the driving-point impedance Z_1 of the driven element, we get

$$Z_1 = Z_{11} - \frac{Z_{12}^2}{Z_{22}} = Z_{11} - \frac{|Z_{12}^2| \angle 2\tau_m}{|Z_{22}| \angle \tau_2}$$

The real part of Z_1 is

$$R_1 = R_{11} - \left| \frac{Z_{12}^2}{Z_{22}} \right| \cos(2\tau_m - \tau_2)$$

Adding a term for the effective loss resistance, if any is present, we have

$$R_1 = R_{11} + R_{1L} - \left| \frac{Z_{12}^2}{Z_{22}} \right| \cos(2\tau_m - \tau_2)$$

For a power input P to the driven element,

$$I_1 = \sqrt{\frac{P}{R_1}} = \sqrt{\frac{P}{R_{11} + R_{1L} - |Z_{12}^2/Z_{22}| \cos(2\tau_m - \tau_2)}}$$

Contd.,

- The electric field intensity at a large distance from the array as a function of ϕ is

$$E(\phi) = k\sqrt{\frac{P}{R_{11} + R_{1L} - |Z_{12}^2/Z_{22}|\cos(2\tau_m - \tau_2)}}\left(1 + \left|\frac{Z_{12}}{Z_{22}}\right| \underline{/\xi + d_r \cos\phi}\right)$$

- For a power input P to a single vertical $\lambda/2$ element the electric field intensity at the same distance is

$$E_{\mathrm{HW}}(\phi) = kI_0 = k\sqrt{\frac{P}{R_{00} + R_{0L}}} \qquad (\mathrm{V\ m^{-1}})$$

where

R_{00} = self-resistance of single $\lambda/2$ element, Ω
R_{0L} = loss resistance of single $\lambda/2$ element, Ω

Contd.,

- The gain in field intensity (as a function of ϕ) of the array with respect to a single λ/2 antenna with the same power input is given by (assume $R_{00} = R_{11}$ and let $R_{0L} = R_{1L}$)

$$G_f(\phi)\left[\frac{A}{HW}\right] = \sqrt{\frac{R_{11}+R_{1L}}{R_{11}+R_{1L}-|Z_{12}^2/Z_{22}|\cos(2\tau_m-\tau_2)}}\left(1+\left|\frac{Z_{12}}{Z_{22}}\right|\angle \xi + d_r\cos\phi\right)$$

- If Z22 is made very large by detuning the parasitic element (i.e., by making X_{22} large), the above equation reduces to unity, i.e., the field of the array becomes the same as the single λ/2 dipole comparison antenna.

- Brown analyzed arrays with a single parasitic element for various values of parasitic element reactance (X_{22}) and was the first to point out that spacings of less than λ/4 were desirable.

Contd.,

- The magnitude of the current in the parasitic element and its phase relation to the current in the driven element depends on its tuning.

- The parasitic element may have a fixed length of $\lambda/2$, the tuning being accomplished by inserting a lumped reactance in series with the antenna at its center point.

- Alternatively, the parasitic element may be continuous and the tuning accomplished by adjusting the length. This method is often simpler in practice but is more difficult for analysis.

- **When the $\lambda/2$ parasitic element is inductive (longer than its resonant length) it acts as a reflector.**

- **When it is capacitive (shorter than its resonant length) it acts as a director.**