# MODULE- IV
# FIR FILTER DESIGN

Filters designed by considering only finite number of samples of impulse response are called FIR filters. FIR digital filter design does not involves the design of analog filter, which means here directly we can design required digital filter.

Ex: For the following system having difference equation

y (n) = 2 x (n) + 3 x(n – 1) + 4 x(n – 2)

Determine impulse response

To obtain impulse response of the system, replace x(n) by δ(n) and y(n) by h(n)

⇒ h (n) = 2 δ(n) + 3 δ(n – 1) + 4 δ(n – 2)

⇒ h (0) = 2 δ(0) + 3 δ(– 1) + 4 δ(– 2) = 2

⇒ h (1) = 2 δ(1) + 3 δ(1) + 4 δ(– 1) = 3

⇒ h (2) = 2 δ(2) + 3 δ(1) + 4 δ(0) = 4

⇒ h (n) = {2, 3, 4}

This particular system produces only three samples (Finite), therefore given system is known as FIR system or FIR filter. Absolute sum of impulse response is "2+3+4 = 9", i.e. impulse response is absolutely summable, and therefore blindly we can say that all FIR filters are stable.

## SYMMETRIC AND ANTI-SYMMETRIC FIR FILTERS:

An FIR filter of length $M$ with input $x(n)$ and output $y(n)$ is described by the difference equation

$$y(n) = b_0 x(n) + b_1 x(n-1) + \cdots + b_{M-1} x(n - M + 1)$$

$$= \sum_{k=0}^{M-1} b_k x(n-k)$$

A filter characterized by its transfer function

$$H(z) = \sum_{k=0}^{M-1} h(k) z^{-k}$$

$$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + \cdots + h(M-2)z^{-(M-2)} + h(M-1)z^{-(M-1)}$$

$$= z^{-(M-1)/2} \left\{ h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{(M-3)/2} h(n) \left[ z^{(M-1-2k)/2} \pm z^{-(M-1-2k)/2} \right] \right\}, \quad M \text{ odd}$$

$$= z^{-(M-1)/2} \sum_{n=0}^{(M/2)-1} h(n)[z^{(M-1-2k)/2} \pm z^{-(M-1-2k)/2}], \qquad M \text{ even}$$

It is straightforward to show that the frequency response of an FIR filter antisymmetric unit sample response can be expressed as

$$H(\omega) = H_r(\omega)e^{j[-\omega(M-1)/2 + \pi/2]}$$

Where

$$H_r(\omega) = 2 \sum_{n=0}^{(M-3)/2} h(n) \sin \omega \left( \frac{M-1}{2} - n \right), \qquad M \text{ odd}$$

$$H_r(\omega) = 2 \sum_{0}^{(M/2)-1} h(n) \sin \omega \left( \frac{M-1}{2} - n \right), \qquad M \text{ even}$$

If the unit sample response is anti-symmetric
$$h\ (M-1)/2=0$$

## DESIGN OF LINEAR PHASE FIR DIGITAL FILTERS USING WINDOWING TECHNIQUES:

**Design Steps:**
Following steps were required to design FIR Digital LPF / HPF / BPF / BSF.

Step 1 :
Choose the desired ideal filter frequency response $H_d\ (j\omega) = |H_d\ (j\omega)| \angle H_d(j\omega)$ .
For an ideal distortion less filter magnitude is constant and phase is linear over required band of frequencies $\Rightarrow |H_d(j\omega)| = 1$, and $\angle H_d(j\omega) = -\omega\alpha$
$\Rightarrow H_d(j\omega) = e^{-j\omega\alpha}$, Over required band of frequencies
$= 0$, for an un wanted band of frequencies.
Where, $\omega$ is frequency and $\alpha$ is constant
Step 2 :
Determine the desired filter impulse response $h_d(n)$ using IDTFT

$$h_d(n) = \frac{1}{\omega_s} \int_{-\omega_s/2}^{\omega_s/2} H_d(j\omega)\, e^{j\omega nT}\, d\omega$$

Where,
$\omega_s = 2\pi fs$: Sampling frequency in rad/sec.
$T = 1/fs$: Sampling period in sec.
For normal practice $\omega_s = 2\pi$ and $T = 1$ sec, then

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(j\omega)\, e^{j\omega n}\, d\omega$$

Step 3 :
In general $h_d\ (n)$ contains infinite number of samples, to design FIR filter consider only finite number of samples. To get finite number of samples of impulse response h (n), multiply $h_d\ (n)$ by a finite duration sequence called window w (n).
H (n) = $h_d$ (n) w (n)

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(j\omega) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha} e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega(n-\alpha)} \, d\omega$$

$$= \frac{1}{2\pi} \frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)} \Bigg|_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi} \frac{e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}}{j(n-\alpha)}$$

$$= \frac{1}{2\pi} \frac{2j \sin[\omega_c(n-\alpha)]}{j(n-\alpha)}$$

$$= \begin{cases} \dfrac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}, & \text{if } n \neq \alpha \\[2mm] \dfrac{\omega_c}{\pi}, & \text{if } n = \alpha \end{cases}$$

In general $h_d(n)$ contains infinite number of samples

## HIGH PASS FILTER:

High pass filter allows only high frequencies over the range $-\pi \leq \omega \leq -\omega_c$ and $\omega_c \leq \omega \leq \pi$ and attenuate all other low frequencies. Magnitude frequency response of an ideal and practical high pass filter as shown below
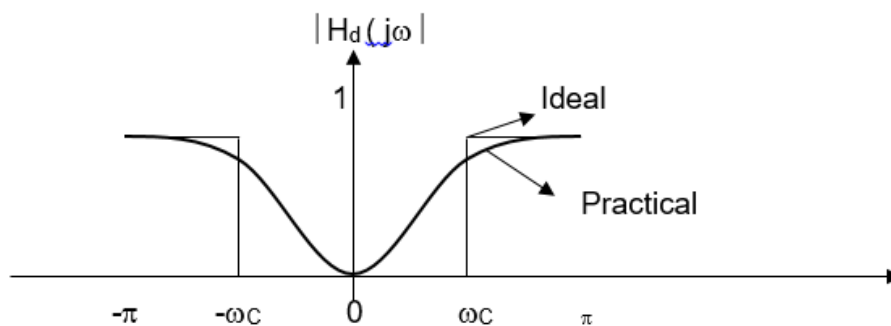


$$H_d(j\omega) = e^{-j\omega\alpha}; \qquad -\pi \leq \omega \leq -\omega_c \ \& \ \omega_c \leq \omega \leq \pi.$$
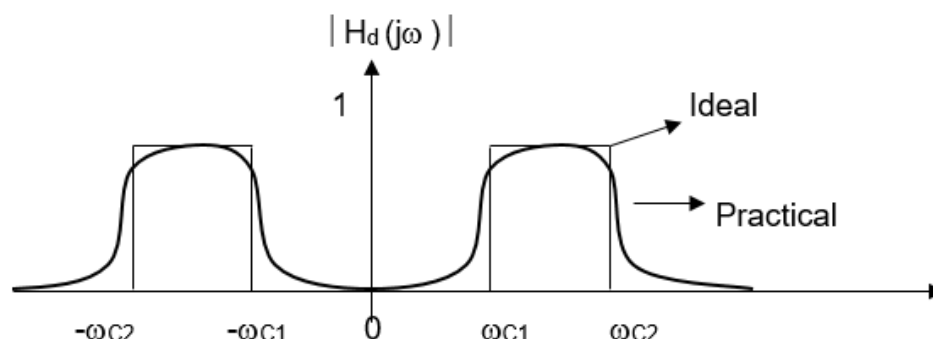$$= 0; \qquad\qquad -\omega_c \leq \omega \leq \omega_c$$

Impulse response of a desired filter can be defined as

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(j\omega) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \left( \int_{-\pi}^{-\omega_c} e^{-j\omega\alpha} e^{j\omega n} \, d\omega + \int_{\omega_c}^{\pi} e^{-j\omega\alpha} e^{j\omega n} \, d\omega \right)$$

$$= \frac{1}{2\pi} \left( \int_{-\pi}^{-\omega_c} e^{j\omega(n-\alpha)} \, d\omega + \int_{\omega_c}^{\pi} e^{j\omega(n-\alpha)} \, d\omega \right)$$

$$= \frac{1}{2\pi} \left( \frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)} \Big|_{-\pi}^{-\omega_c} + \frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)} \Big|_{\omega_c}^{\pi} \right)$$

$$= \frac{1}{2\pi} \left( \frac{e^{-j\omega_c(n-\alpha)} - e^{-j\pi(n-\alpha)}}{j(n-\alpha)} + \frac{e^{j\pi(n-\alpha)} - e^{j\omega_c(n-\alpha)}}{j(n-\alpha)} \right)$$

$$= \frac{1}{2\pi} \left( \frac{e^{j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)} - \left( e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)} \right)}{j(n-\alpha)} \right)$$

$$= \frac{1}{2\pi} \left( \frac{2j\sin[\pi(n-\alpha)] - 2j\sin[\omega_c(n-\alpha)]}{j(n-\alpha)} \right)$$

$$= \left( \frac{\sin[\pi(n-\alpha)] - \sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)} \right)$$

$$= \begin{cases} \dfrac{\sin[\pi(n-\alpha)] - \sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}, & \text{if } n \neq \alpha \\[2ex] 1 - \dfrac{\omega_c}{\pi}, & \text{if } n = \alpha \end{cases}$$

In general $h_d(n)$ contains infinite number of samples

## BAND PASS FILTER:

Band p[ass filter allows only a certain band of frequencies over the range $-\omega_{C2} \leq \omega \leq -\omega_{C1}$ and $\omega_{C1} \leq \omega \leq \omega_{C2}$ and attenuate all other un wanted band of frequencies. Magnitude frequency response of an ideal and practical band pass filter as shown below.

$$H_d(j\omega) = e^{-j\omega\alpha}; \quad -\omega_{C2} \leq \omega \leq -\omega_{C1} \ \& \ \omega_{C1} \leq \omega \leq \omega_{C2}$$
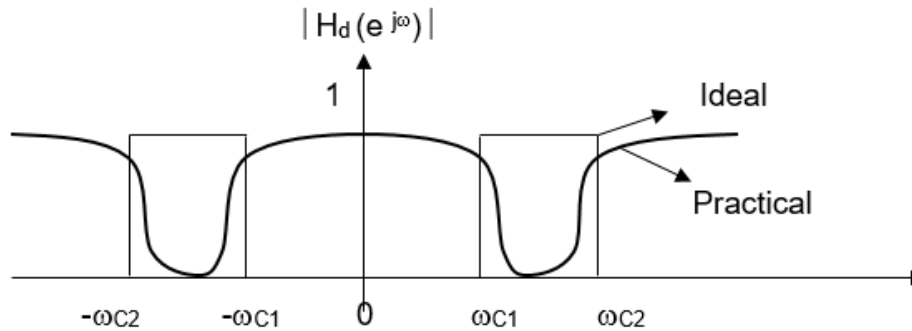$$= 0; \quad -\omega_{C1} \leq \omega \leq \omega_{C1}$$

Impulse response of a desired filter can be defined as

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(j\omega) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \left( \int_{-\omega_{c2}}^{-\omega_{c1}} e^{-j\omega\alpha} e^{j\omega n} \, d\omega + \int_{\omega_{c1}}^{\omega_{c2}} e^{-j\omega\alpha} e^{j\omega n} \, d\omega \right)$$

$$= \frac{1}{2\pi} \left( \int_{-\omega_{c2}}^{-\omega_{c1}} e^{j\omega(n-\alpha)} \, d\omega + \int_{\omega_{c1}}^{\omega_{c2}} e^{j\omega(n-\alpha)} \, d\omega \right)$$

$$= \frac{1}{2\pi} \left( \frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)} \Big|_{-\omega_{c2}}^{-\omega_{c1}} + \frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)} \Big|_{\omega_{c1}}^{\omega_{c2}} \right)$$

$$= \frac{1}{2\pi} \left( \frac{e^{-j\omega_{c1}(n-\alpha)} - e^{-j\omega_{c2}(n-\alpha)}}{j(n-\alpha)} + \frac{e^{j\omega_{c2}(n-\alpha)} - e^{j\omega_{c1}(n-\alpha)}}{j(n-\alpha)} \right)$$

$$= \frac{1}{2\pi} \left( \frac{e^{j\omega_{c2}(n-\alpha)} - e^{-j\omega_{c2}(n-\alpha)} - \left( e^{j\omega_{c1}(n-\alpha)} - e^{-j\omega_{c1}(n-\alpha)} \right)}{j(n-\alpha)} \right)$$

$$= \frac{1}{2\pi} \left( \frac{2j\sin[\omega_{c_2}(n-\alpha)] - 2j\sin\omega_{c_1}(n-\alpha)]}{j(n-\alpha)} \right)$$

$$= \left( \frac{\sin[\omega_{c_2}(n-\alpha)] - \sin[\omega_{c_1}(n-\alpha)]}{\pi(n-\alpha)} \right)$$

$$= \begin{cases} \dfrac{\sin[\omega_{c_2}(n-\alpha)] - \sin[\omega_{c_1}(n-\alpha)]]}{\pi(n-\alpha)}, & \text{if } n \neq \alpha \\[2mm] \dfrac{\omega_{c_2} - \omega_{c_1}}{\pi}, & \text{if } n = \alpha \end{cases}$$

In general $h_d(n)$ contains infinite number of samples

## BAND STOP FILTER:

Band stop filter allows entire band of frequencies over the range $-\pi \leq \omega \leq -\omega_{C2}$ & $-\omega_{C1} \leq \omega \leq \omega_{C1}$ & $\omega_{C2} \leq \omega \leq \pi$ and attenuate only a certain un wanted band of frequencies. Magnitude frequency response of an ideal and practical band stop filter as shown below.

$$H_d(e^{j\omega}) = e^{-j\omega\alpha}; \qquad -\pi \le \omega \le -\omega_{C2} \ \& \ -\omega_{C1} \le \omega \le \omega_{C1} \& \ \omega_{C2} \le \omega \le \pi.$$
$$= 0; \qquad -\omega_{C2} \le \omega \le -\omega_{C1} \ \& \ \omega_{C1} \le \omega \le \omega_{C2}$$

Impulse response of a desired filter can be defined as

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(j\omega) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \left( \int_{-\pi}^{-\omega_{c2}} e^{-j\omega\omega} e^{j\omega n} \, d\omega + \int_{-\omega_{c1}}^{\omega_{c1}} e^{-j\omega\omega} e^{j\omega n} \, d\omega + \int_{\omega_{c2}}^{\pi} e^{-j\omega\omega} e^{j\omega n} \, d\omega \right)$$

$$= \frac{1}{2\pi} \left( \int_{-\pi}^{-\omega_{c2}} e^{j\omega(n-\alpha)} \, d\omega + \int_{-\omega_{c1}}^{\omega_{c1}} e^{j\omega(n-\alpha)} \, d\omega + \int_{\omega_{c2}}^{\pi} e^{j\omega(n-\alpha)} \, d\omega \right)$$

$$= \frac{1}{2\pi} \left( \left. \frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)} \right|_{-\pi}^{-\omega_{c2}} + \left. \frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)} \right|_{-\omega_{c1}}^{\omega_{c1}} + \left. \frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)} \right|_{\omega_{c2}}^{\pi} \right)$$

$$= \frac{1}{2\pi} \left( \frac{e^{-j\omega_{c2}(n-\alpha)} - e^{-j\pi(n-\alpha)}}{j(n-\alpha)} + \frac{e^{j\omega_{c1}(n-\alpha)} - e^{-j\omega_{c1}(n-\alpha)}}{j(n-\alpha)} + \frac{e^{j\pi(n-\alpha)} - e^{j\omega_{c2}(n-\alpha)}}{j(n-\alpha)} \right)$$

$$= \frac{1}{2\pi} \left( \frac{e^{j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)} + e^{j\omega_{c1}(n-\alpha)} - e^{-j\omega_{c1}(n-\alpha)} - \left( e^{j\omega_{c2}(n-\alpha)} - e^{-j\omega_{c2}(n-\alpha)} \right)}{j(n-\alpha)} \right)$$

$$= \frac{1}{2\pi} \left( \frac{2j \sin[\pi(n-\alpha)] + 2j \sin[\omega_{c1}(n-\alpha)] - 2j \sin[\omega_{c2}(n-\alpha)]}{j(n-\alpha)} \right)$$

$$= \left( \frac{\sin[\pi(n-\alpha)] + \sin[\omega_{c1}(n-\alpha)] - \sin[\omega_{c2}(n-\alpha)]}{\pi(n-\alpha)} \right)$$

$$= \begin{cases} \dfrac{\sin[\pi(n-\alpha)] + \sin[\omega_{c1}(n-\alpha)] - \sin[\omega_{c2}(n-\alpha)]}{\pi(n-\alpha)}, & \text{if } n \ne \alpha \\[4mm] 1 - \dfrac{\omega_{c2} - \omega_{c1}}{\pi}, & \text{if } n = \alpha \end{cases}$$
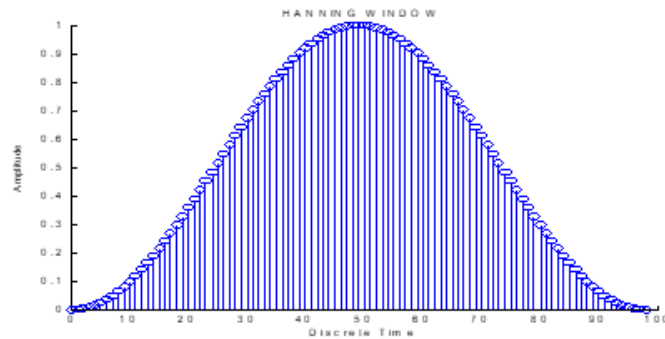
In general $h_d(n)$ contains infinite number of samples

**TYPES OF WINDOWS:**
Window technique is used to obtain finite number of samples of impulse response h (n) from infinite number of samples of desired filter impulse response $h_d$ (n). Various types of windows used in FIR filter design as given below.
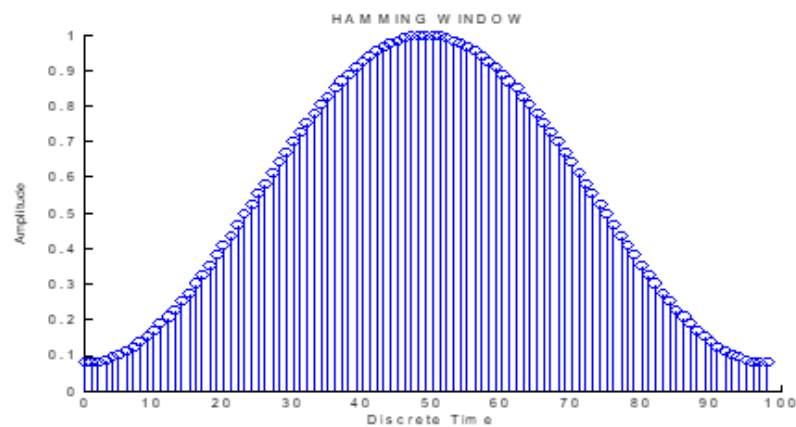
## HANNING WINDOW:

$$w\,(n) = 0.5 - 0.5\,\text{Cos}\left(\frac{2n\pi}{N-1}\right);\ 0 \le n \le N - 1.$$
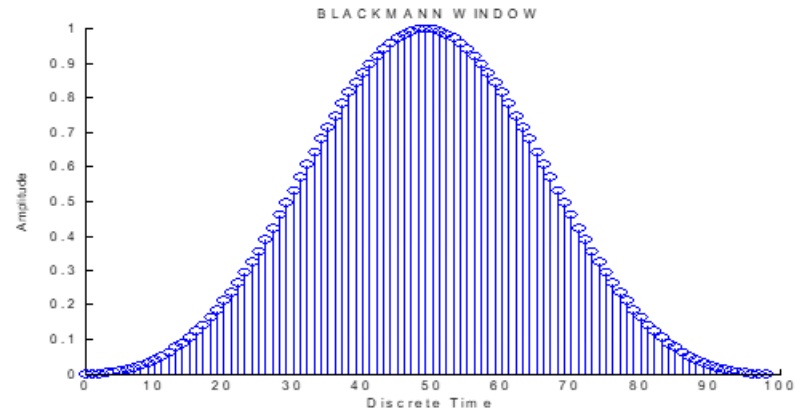


## HAMMING WINDOW:

$$w(n) = 0.54 - 0.46\,\text{Cos}\left(\frac{2n\pi}{N-1}\right);\ 0 \le n \le N - 1.$$
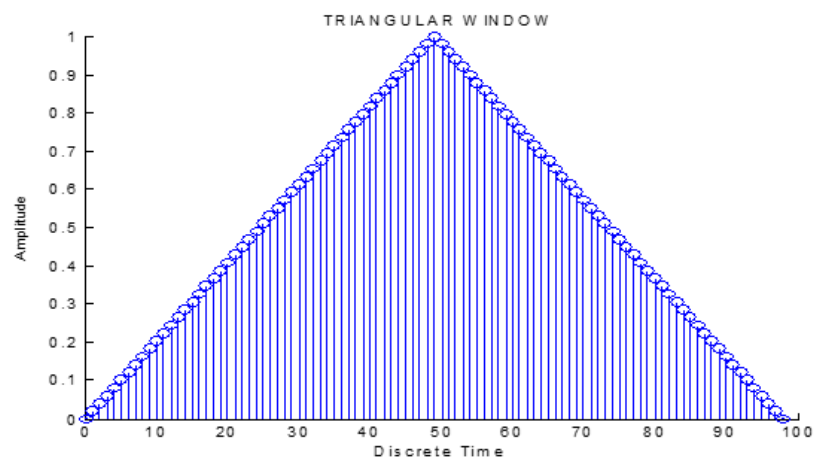
## BLACKMAN WINDOW:

$$w(n) = 0.42 - 0.5 \cos\left(\frac{2n\pi}{N-1}\right) + 0.08 \cos\left(\frac{4n\pi}{N-1}\right); \ 0 \leq n \leq N-1.$$



## TRIANGULAR (BARTLET) WINDOW :

$$w(n) = 1 - \frac{2\left|n - \frac{N-1}{2}\right|}{N-1}; \ 0 \leq n \leq N-1.$$

# LINEAR PHASE CHARACTERISTICS OF FIR FILTERS:

For a distortion less filter having frequency response $H_d(j\omega)$

$$\Rightarrow |H_d(j\omega)| = \text{Constant (k) and}$$

$$\Rightarrow \angle H_d(j\omega) = \text{Linear } (-\omega\alpha)$$

We know that

$$H_d(j\omega j = |H_d(j\omega j| \angle H_d(j\omega j$$

$$\sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n} = k\angle -\omega\alpha$$

$h_d(n) = h(n)$, over the range 0 to N – 1.

$$\Rightarrow \sum_{n=0}^{N-1} h(n) e^{-j\omega n} = k \, e^{-j\omega\alpha}$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n) [\cos(\omega n) - j\sin(\omega n)] = k [\cos(\omega\alpha) - j\sin(\omega\alpha)]$$

$$\Rightarrow \sum_{n=0}^{N-1} [h(n)\cos(\omega n) - j h(n)\sin(\omega n)] = k\cos(\omega\alpha) - j k\sin(\omega\alpha)$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\cos(\omega n) - j\sum_{n=0}^{N-1} h(n)\sin(\omega n) = k\cos(\omega\alpha) - j k\sin(\omega\alpha)$$

Compare both real and imaginary parts both side

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\cos(\omega n) = k\cos(\omega\alpha) \text{---------(1)}$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\sin(\omega n) = k\sin(\omega\alpha) \text{------------(2)}$$

Apply (2) / (1)

$$\Rightarrow \frac{\displaystyle\sum_{n=0}^{N-1} h(n)\sin(\omega n)}{\displaystyle\sum_{n=0}^{N-1} h(n)\cos(\omega n)} = \frac{k\sin(\omega\alpha)}{k\cos(\omega\alpha)}$$

$$\Rightarrow \cos(\omega\alpha)\sum_{n=0}^{N-1} h(n)\sin(\omega n) = \sin(\omega\alpha)\sum_{n=0}^{N-1} h(n)\cos(\omega n)$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\sin(\omega n)\cos(\omega\alpha) = \sum_{n=0}^{N-1} h(n)\cos(\omega n)\sin(\omega\alpha)$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)[\sin(\omega n)\cos(\omega\alpha) - \cos(\omega n)\sin(\omega\alpha)] = 0$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\sin[\omega(n-\alpha)] = 0$$

$$\Rightarrow h(0)\sin[\omega(0-\alpha)] + h(1)\sin[\omega(1-\alpha)] + \dots\dots$$

$$\dots\dots\dots\dots\dots\dots\dots + h(N-1)\sin[\omega(N-1-\alpha)] + h(N-1)\sin[\omega(N-1-\alpha)] = 0$$

Above sum is zero only when

$h(0) = h(N-1)$ and $\sin[\omega(0-\alpha)] = -\sin[\omega(N-1-\alpha)]$ ,

$h(1) = h(N-2)$ and $\sin[\omega(1-\alpha)] = -\sin[\omega(N-2-\alpha)]$, so on.

$\Rightarrow \underline{h}(n) = h(N-1)$ and $\omega(0-\alpha) = -\omega(N-1-\alpha) \Rightarrow -\alpha = -(N-1-\alpha)$

$\Rightarrow h(n) = h(N-1)$ and $2\alpha = (N-1)$

$\Rightarrow h(n) = h(N-1)$ is the condition for linear phase FIR filters and constant $\alpha = (N-1)/2$

## Design of linear phase FIR filter using Frequency sampling technique

In the frequency-sampling method for FIR filter design, we specify the desired frequency response $H_d(\omega)$ at a set of equally spaced frequencies, namely

$$\omega_k = \frac{2\pi}{M}(k+\alpha), \qquad k = 0, 1, \dots, \frac{M-1}{2}, \qquad M \text{ odd}$$

$$k = 0, 1, \dots, \frac{M}{2} - 1, \qquad M \text{ even}$$

$$\alpha = 0 \quad \text{or} \quad \tfrac{1}{2}$$

$$H(\omega) = \sum_{n=0}^{M-1} h(n)e^{-j\omega n}$$

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$$H(k+\alpha) \equiv H\left(\frac{2\pi}{M}(k+\alpha)\right)$$

$$H(k+\alpha) \equiv \sum_{n=0}^{M-1} h(n)e^{-j2\pi(k+\alpha)n/M}, \qquad k = 0, 1, \ldots, M-1$$

**Example:**

Determine the coefficients of a linear-phase FIR filter of length $M = 15$ which has a symmetric unit sample response and a frequency response that satisfies the conditions

$$H_r\left(\frac{2\pi k}{15}\right) = \begin{cases} 1, & k = 0, 1, 2, 3 \\ 0.4, & k = 4 \\ 0, & k = 5, 6, 7 \end{cases}$$

Solution:

Since h(n) is symmetric and the frequencies are selected corresponding to the case α=0

$$G(k) = (-1)^k H_r\left(\frac{2\pi k}{15}\right), \qquad k = 0, 1, \ldots, 7$$

The result of this computation is

$$h(0) = h(14) = -0.014112893$$

$$h(1) = h(13) = -0.001945309$$

$$h(2) = h(12) = 0.04000004$$
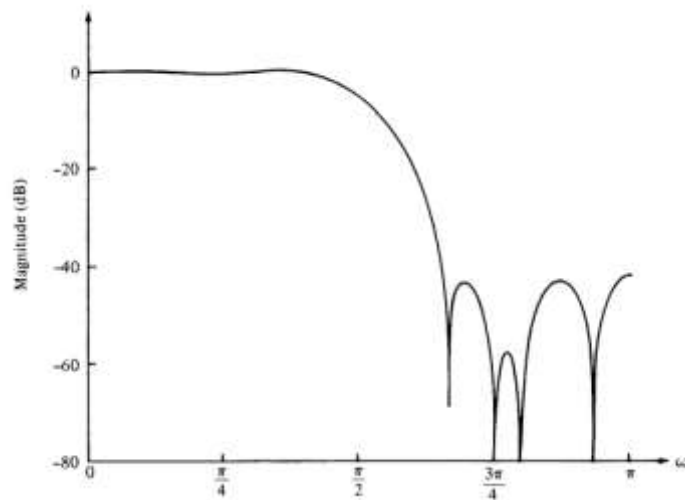
$$h(3) = h(11) = 0.01223454$$

$$h(4) = h(10) = -0.09138802$$

$$h(5) = h(9) = -0.01808986$$

$$h(6) = h(8) = 0.3133176$$

$$h(7) = 0.52$$

The frequency response characteristics of the filter is shown below.

## Comparison of IIR and FIR filters:

| Basis for Comparison | FIR Filter | IIR Filter |
|---|---|---|
| Stands for | Finite Impluse Response | Infinite Impulse Response |
| Nature | Non-recursive | Recursive |
| Computational Efficiency | Less | Comparatively more |
| Usage | Difficult | Quite easy |
| Feedback | Absent | Present |
| Stability | More | Less |
| Requirement to generate current output | Present and past samples of input. | Present and past samples of input along with past output. |
| Delay offered | High | Comparatively lower |
| Transfer function | Only zeros are present. | Both poles and zeros are present. |
| Memory requirement | More | Less |
| Sensitivity | Less | Comparatively more |
| Resolution offered at low frequencies | Less | More |
| Controllability | Easy | Quite Difficult |
| **Basis for Comparison** | **FIR Filter** | **IIR Filter** |
| Stands for | Finite Impluse Response | Infinite Impulse Response |
| Nature | Non-recursive | Recursive |