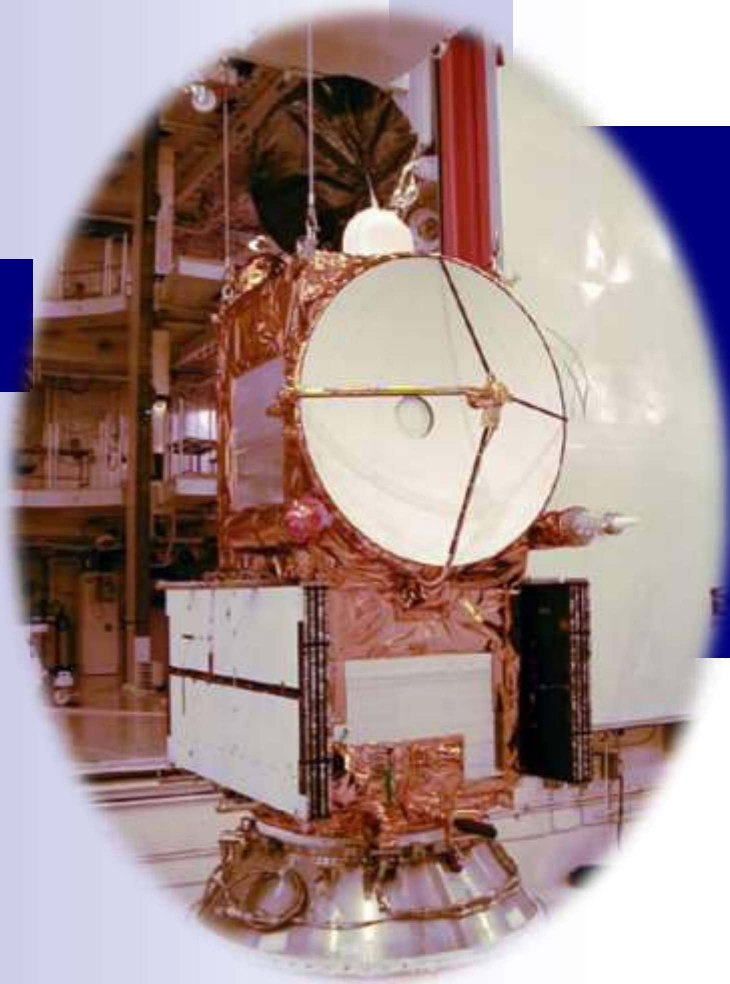


22EC102014

ANTENNAS AND PROPAGATION

MODULE 1: ANTENNA BASICS AND THIN LINEAR WIRE ANTENNAS

Introduction, Radiation mechanism, Antenna parameters patterns, Beam Area, Radiation Intensity, Beam Efficiency, Directivity-Gain-Resolution, Antenna Apertures, Effective height; Antenna Field Zones, Friis transmission equation, Retarded potentials, Radiation from small electric dipole, Quarter wave monopole and half wave dipole Current distributions, Field components, Radiated power, Radiation resistance, Beam width, Directivity, Effective area and Effective height.

INTRODUCTION

- An antenna is defined by Webster's Dictionary as " a usually metallic device (as a rod or wire) for radiating or receiving radio waves."
- The *IEEE Standard Definitions of Terms for Antennas* (IEEE Std 145-1983) defines the antenna or aerial as "a means for the radiating or receiving radio waves."
- The antenna is the **transitional structure between free-space and a guiding device**.

INTRODUCTION

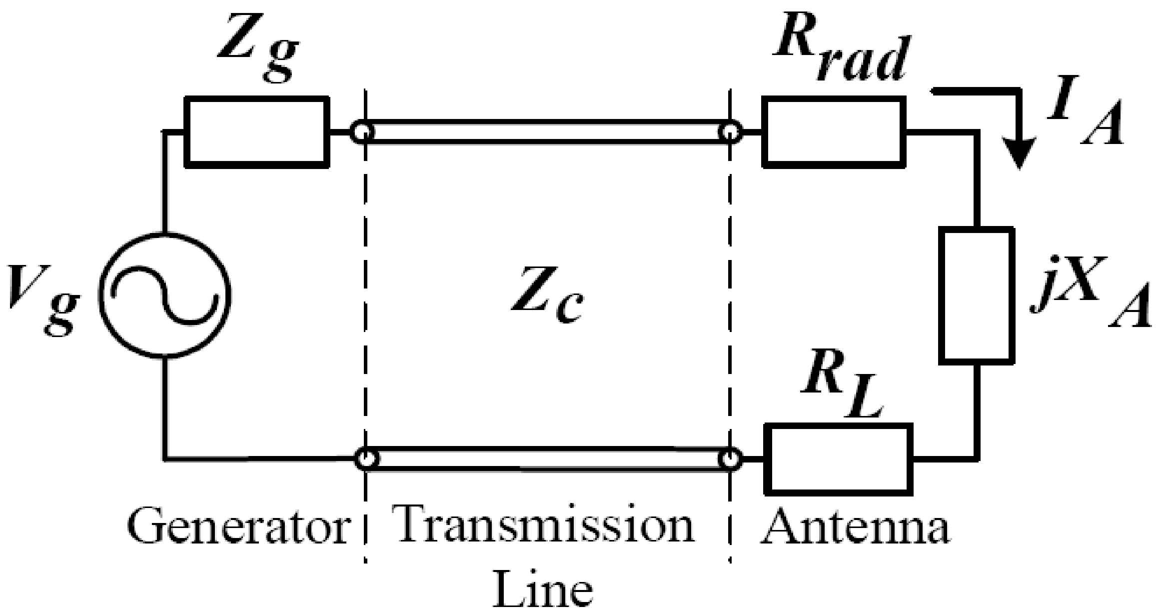
- *The antenna (aerial, EM radiator) is a device, which radiates or receives electromagnetic waves.*

- The antenna is the transition between a guiding device (transmission line, waveguide) and free space (or another usually unbounded medium).

- Its main purpose is ***to convert the energy of a guided wave into the energy of a free space wave (or vice versa) as efficiently as possible, while in the same time the radiated power has a certain desired pattern of distribution in space***.

INTRODUCTION

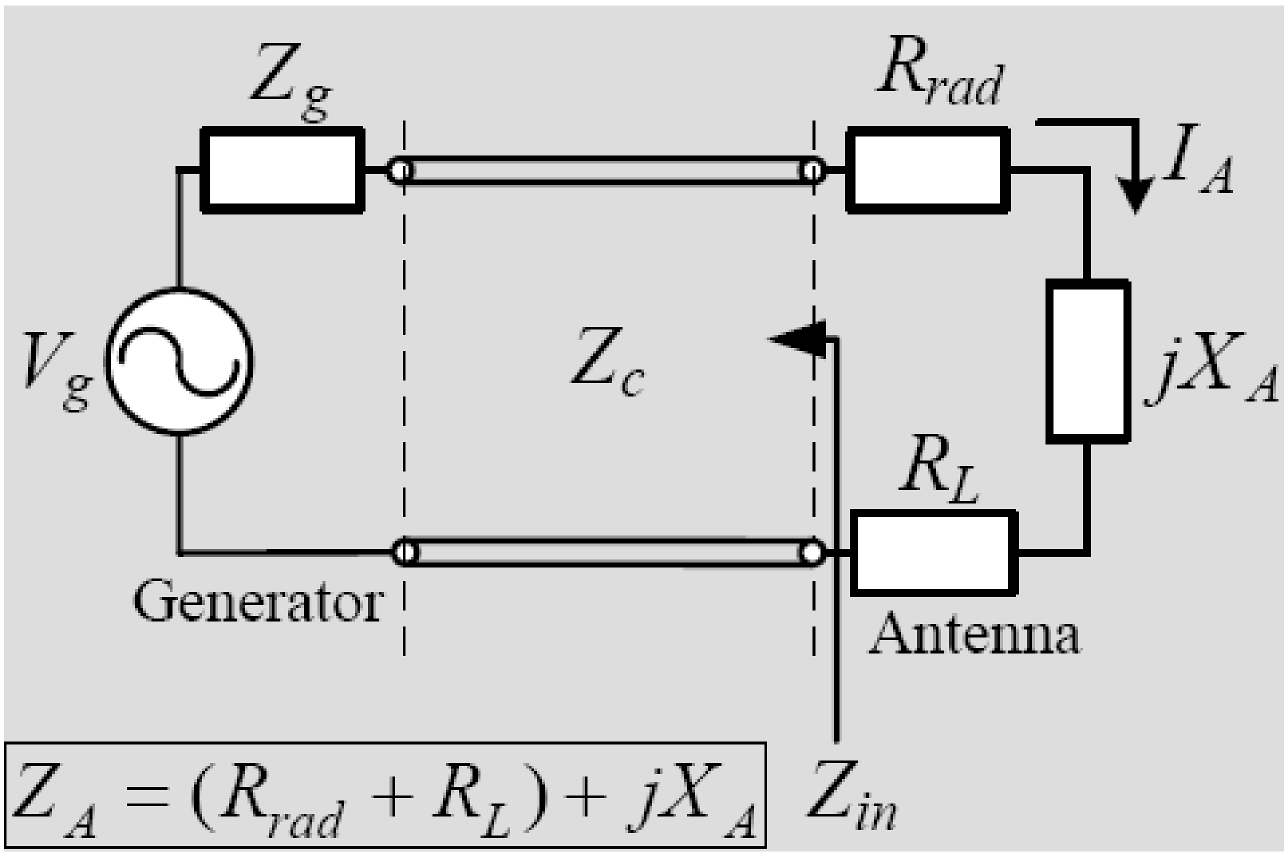
- transmission-line Thevenin equivalent circuit of a radiating (transmitting) system



V_g - voltage-source generator (transmitter);
Z_g - impedance of the generator (transmitter);
R_{rad} - radiation resistance (related to the radiated power as $P_{rad} = I^2_A \cdot R_{rad}$)
R_L - loss resistance (related to conduction and dielectric losses);
jX_A - antenna reactance.
Antenna impedance: $Z_A=(R_{rad}+R_L)+jX_A$

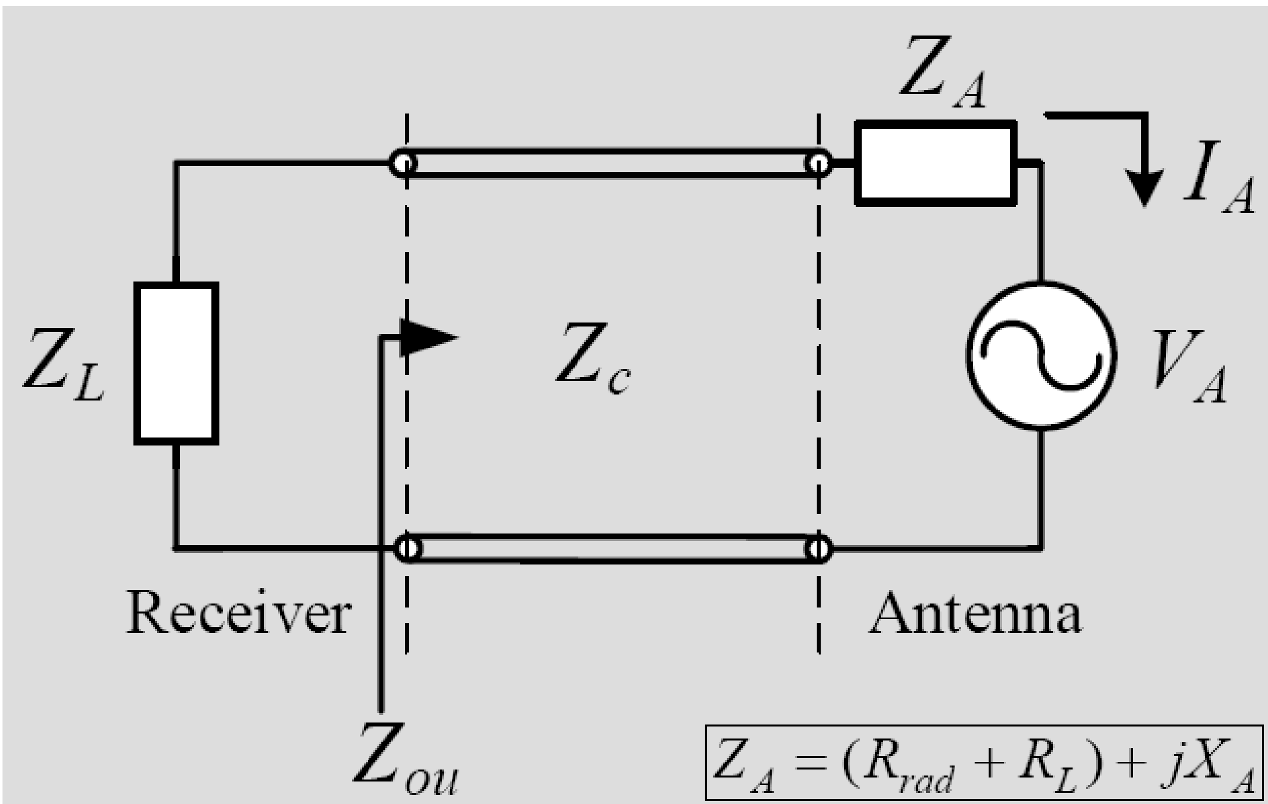
INTRODUCTION

- transmission-line Thevenin equivalent of transmitting antenna

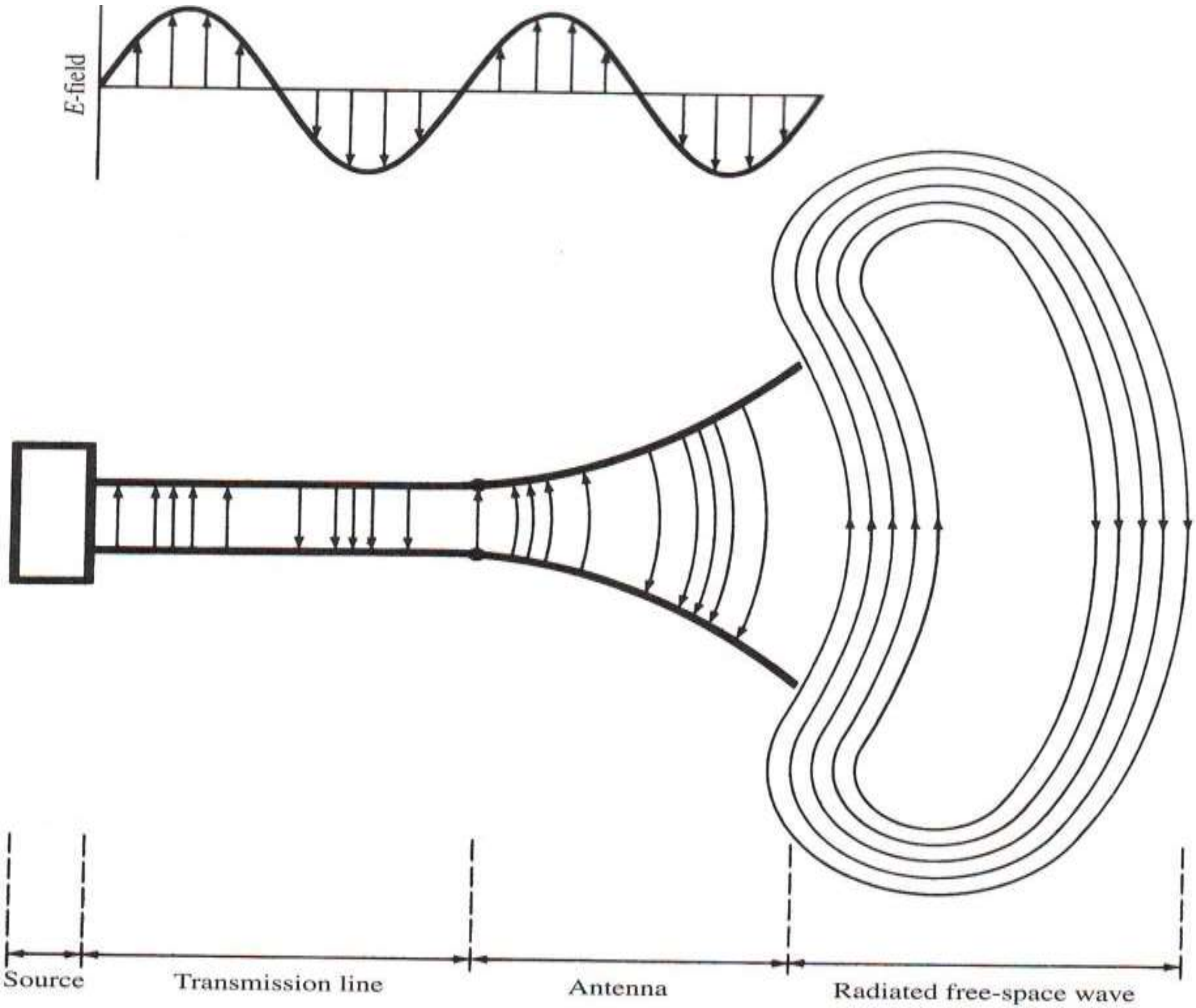


INTRODUCTION

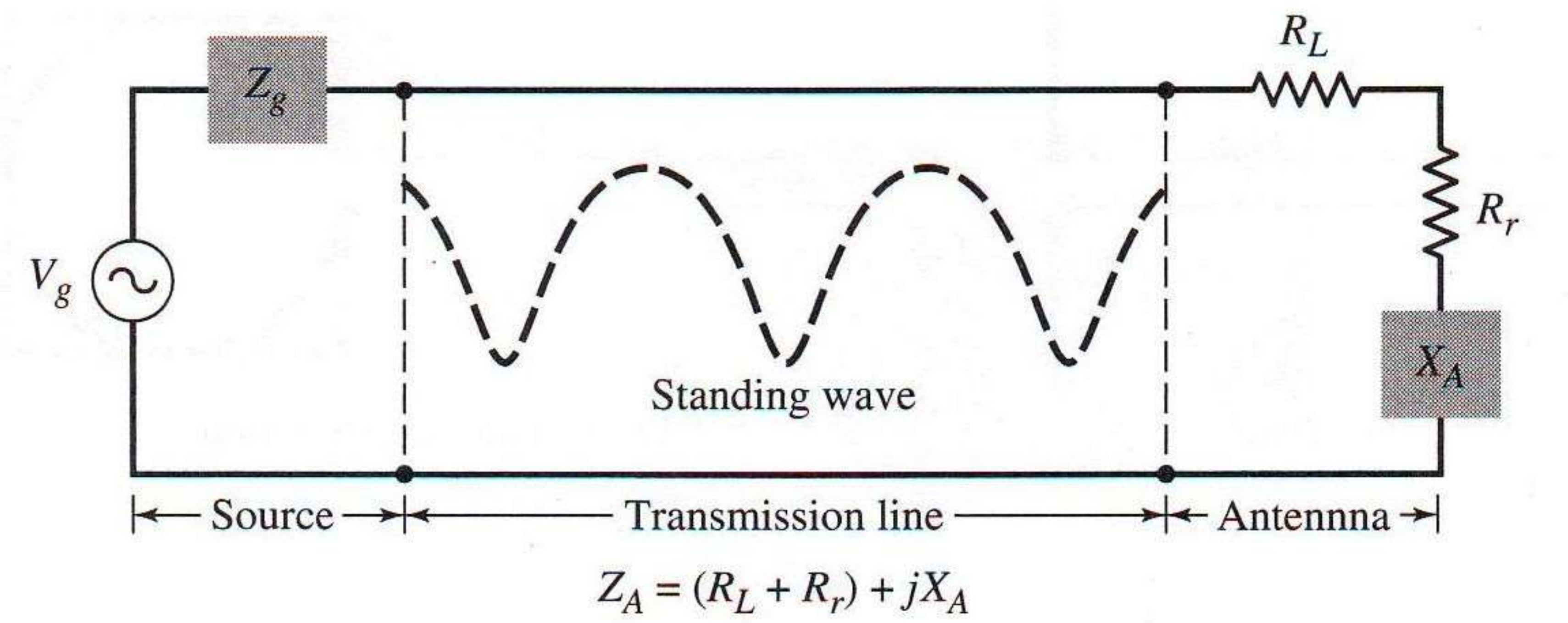
- transmission-line Thevenin equivalent of receiving antenna



INTRODUCTION



INTRODUCTION



BRIEF HISTORICAL NOTES

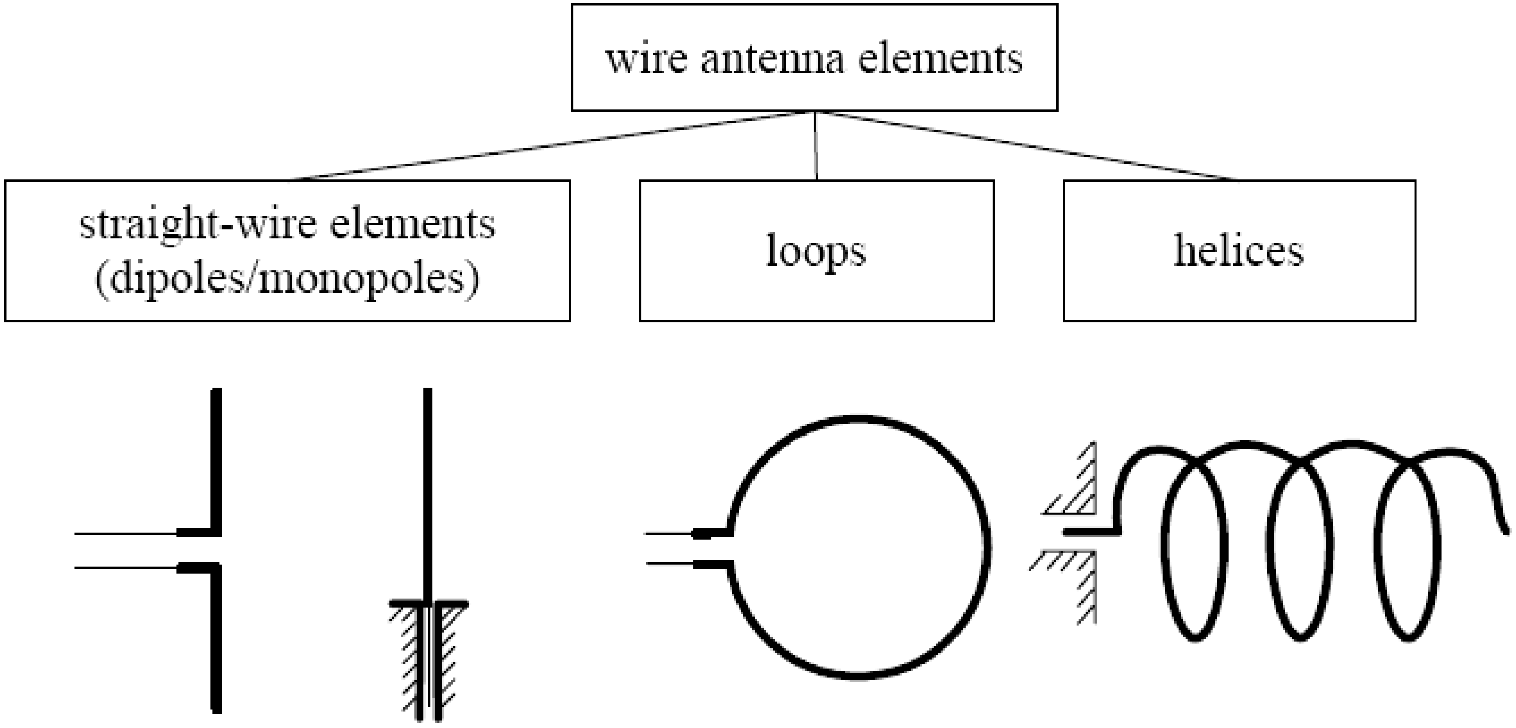
- **James C. Maxwell** develops mathematical model of electromagnetism, "*A Treatise on Electricity and Magnetism*", 1873.
- **Heinrich R. Hertz** demonstrates in 1886 the first wireless EM wave system.
- **Alexander Popov** sends a message from a Russian Navy ship 30 miles out in sea, all the way to his lab in St. Petersburg, 1875.
- **Guglielmo Marconi** performs the first transatlantic transmission from Poldhu in Cornwall, England, to Newfoundland, Canada.

BRIEF HISTORICAL NOTES

- The beginning of 20th century (until WW2): boom in wire antenna technology and in wireless communications (up to UHF, over thousands of kilometers), DeForest triode
- WW2 marks a new era in wireless communications and antenna technology: new microwave generators (magnetron and klystron). New antenna types: waveguide apertures, horns, reflectors, etc.

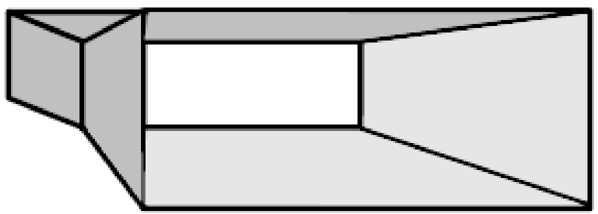
REVIEW OF ANTENNA TYPES

- Single elements
  - Wire antenna

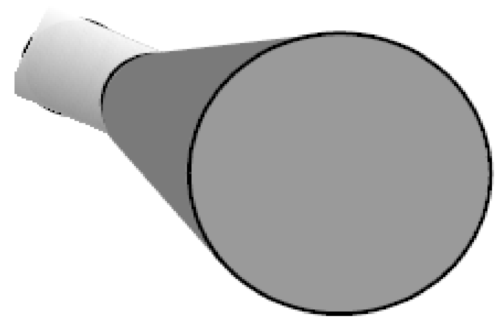


REVIEW OF ANTENNA TYPES

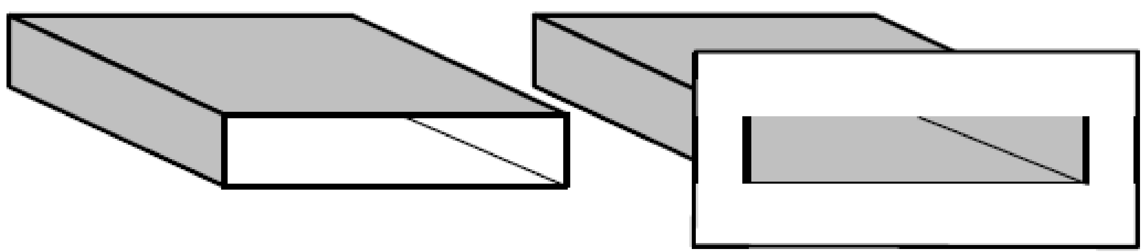
- Single elements
  - aperture antennas – mostly microwave (1 to 20 GHz), high power



rectangular horn



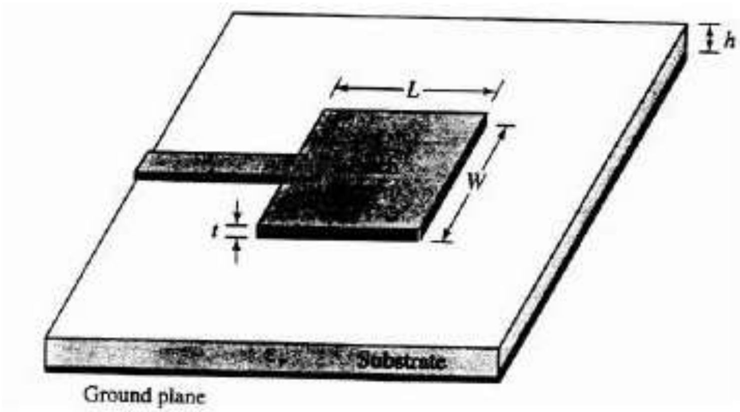
conical horn



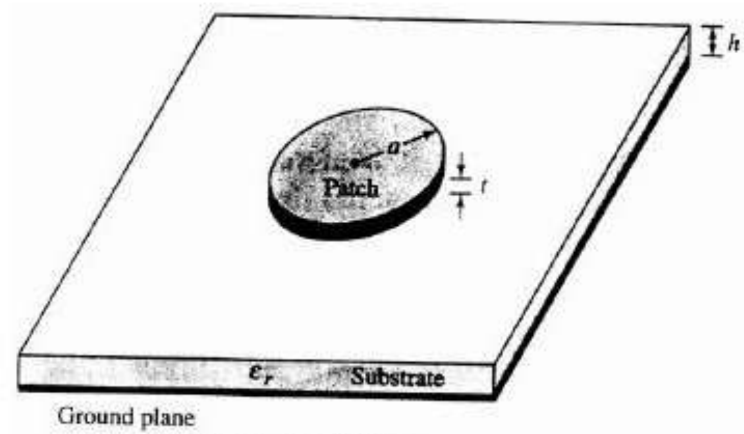
open-end waveguides

REVIEW OF ANTENNA TYPES

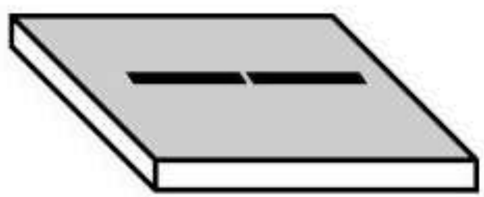
- Single elements
  - Single patches



rectangular patch



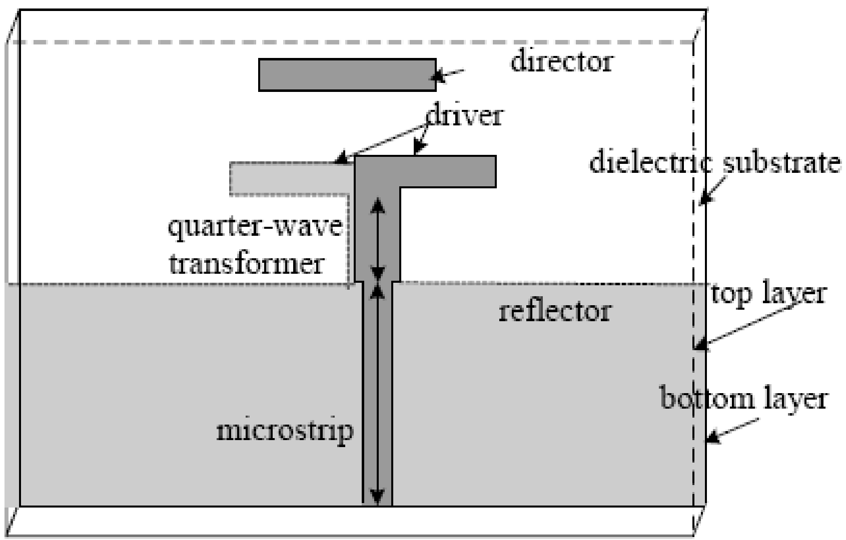
circular patch

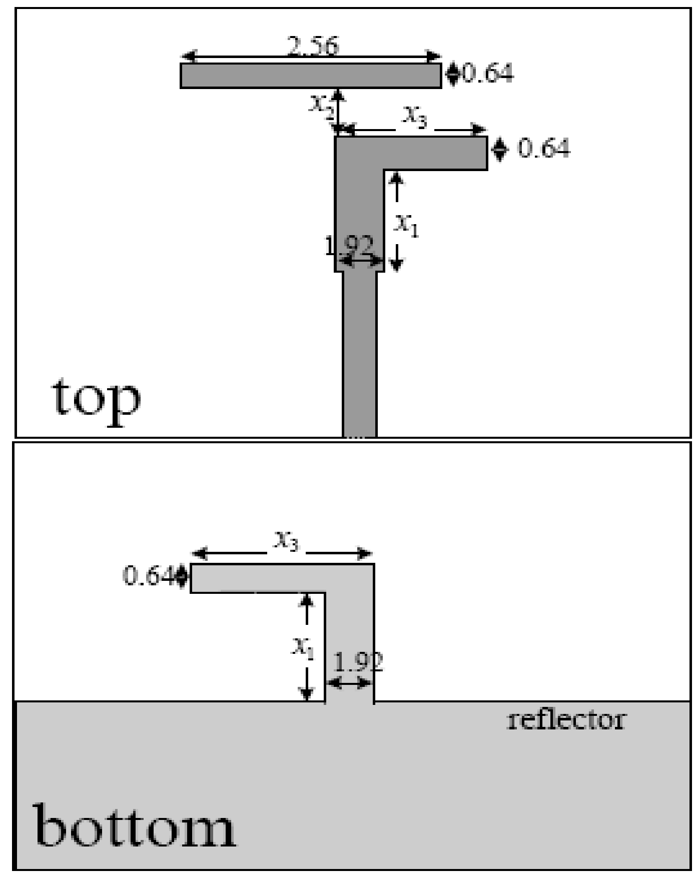


printed dipole

REVIEW OF ANTENNA TYPES

- Single elements
  - Printed patches

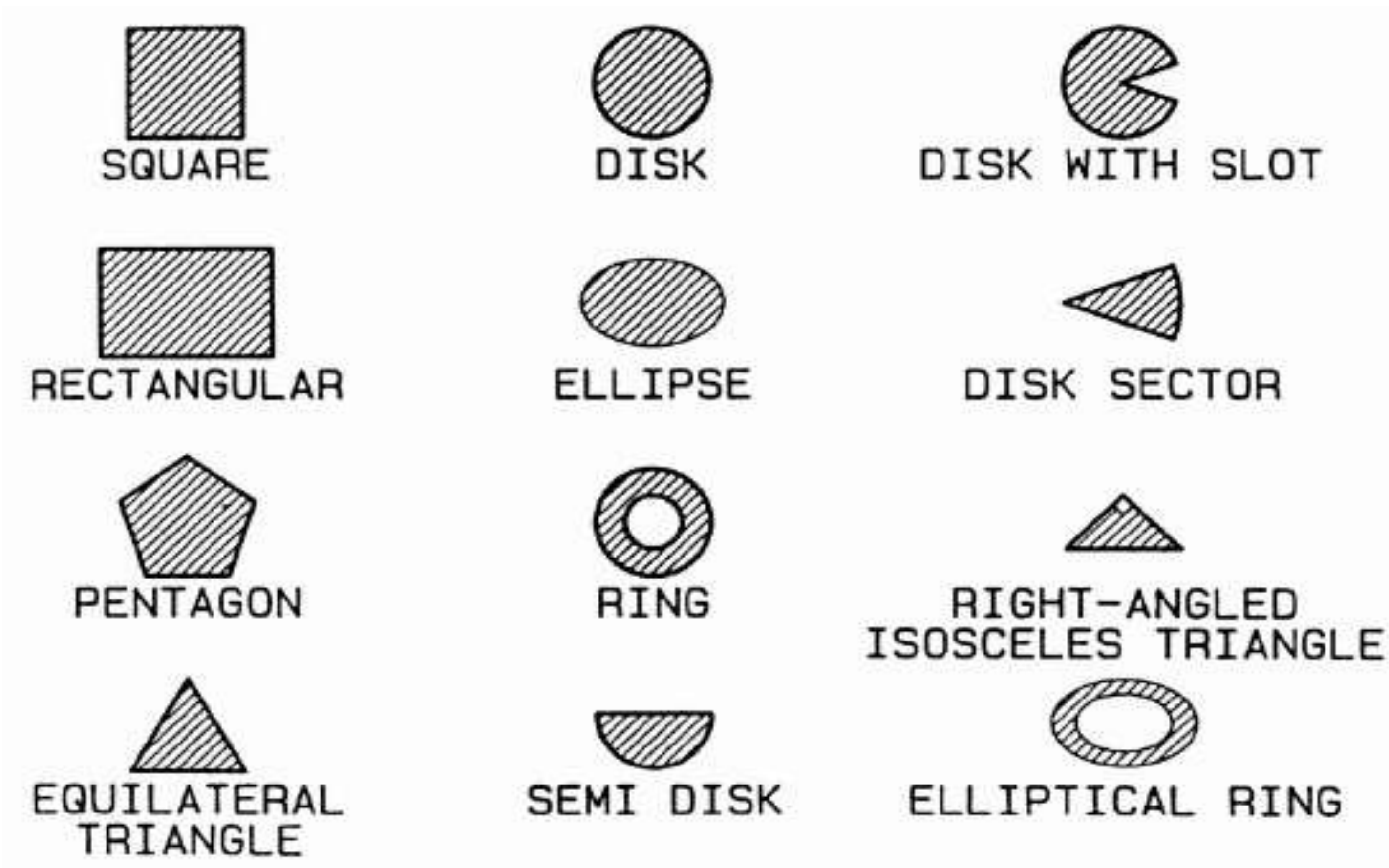




double-layer printed Yagi antenna with electromagnetically coupled feed

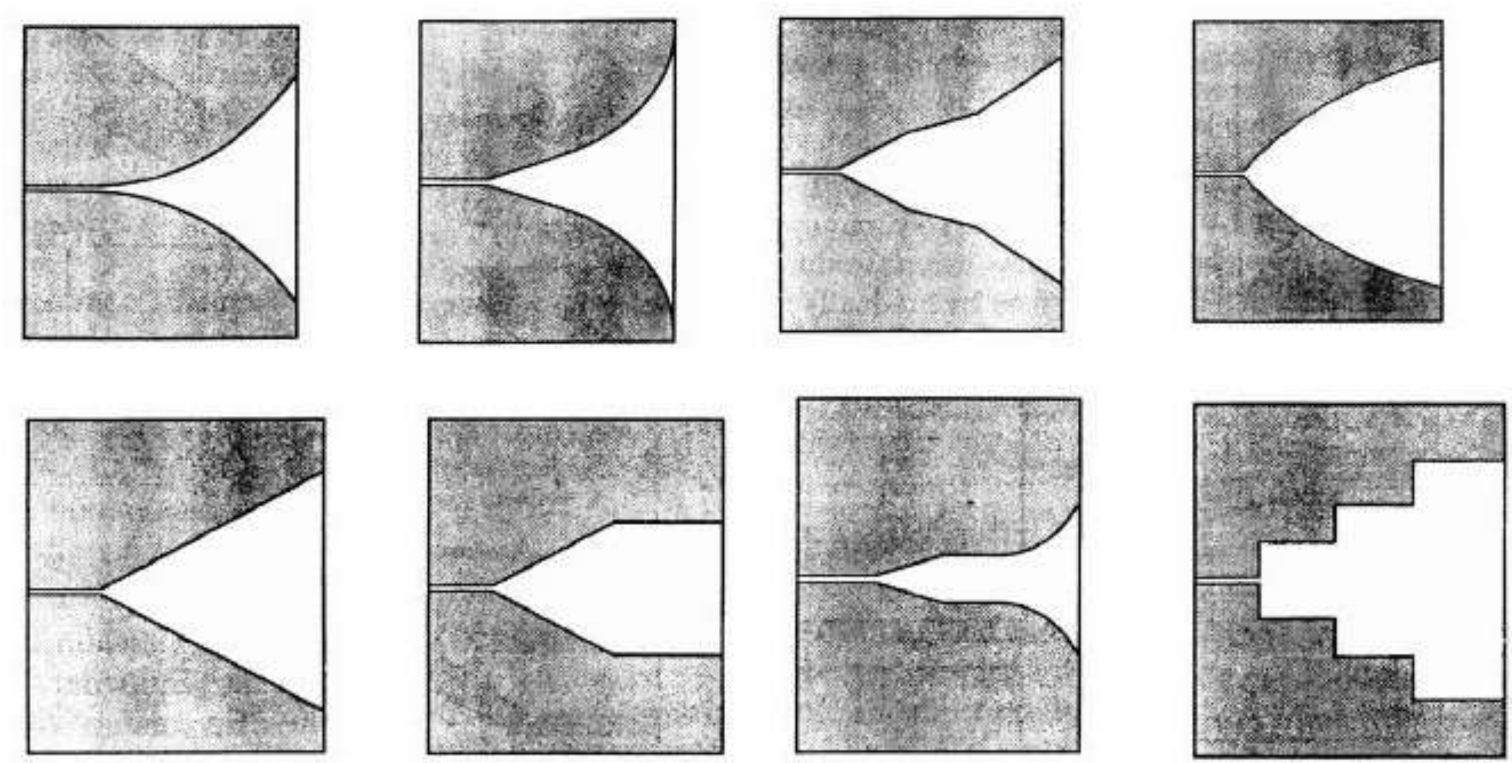
REVIEW OF ANTENNA TYPES

- Single elements
  - Printed patch shapes



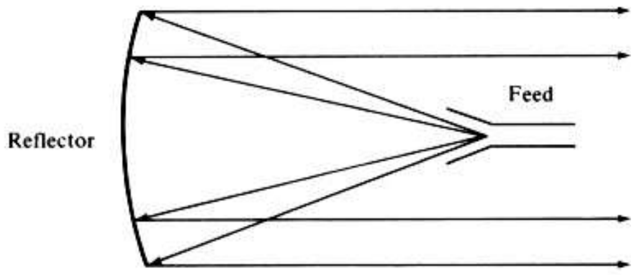
REVIEW OF ANTENNA TYPES

- Single elements
  - printed slot antennas – typical range above 10 GHz

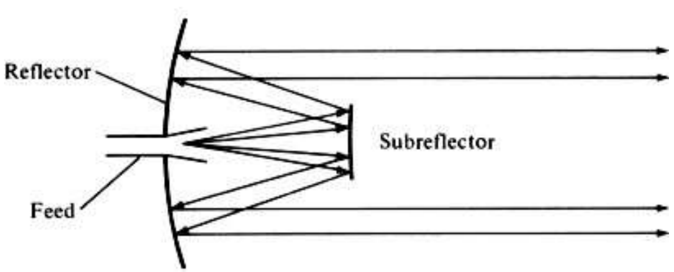


REVIEW OF ANTENNA TYPES

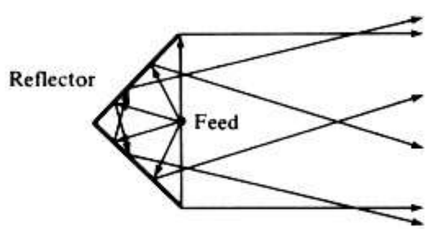
- Single elements
  - reflector antennas – high directivity, electrically large



(a) Parabolic reflector with front feed



(b) Parabolic reflector with Cassegrain feed



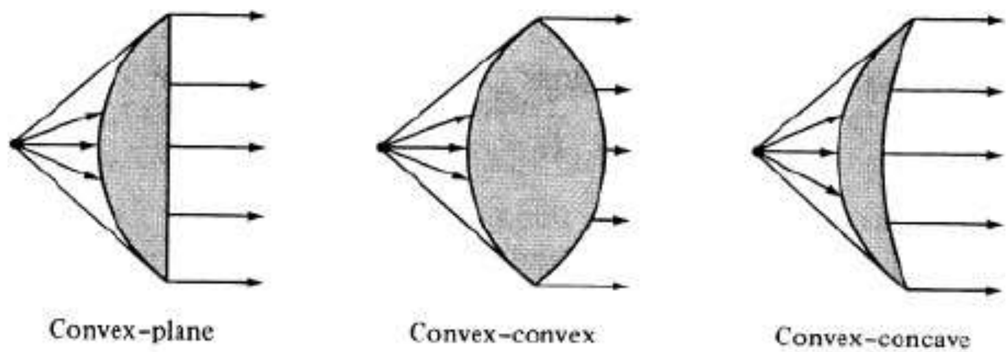
(c) Corner reflector

REVIEW OF ANTENNA TYPES

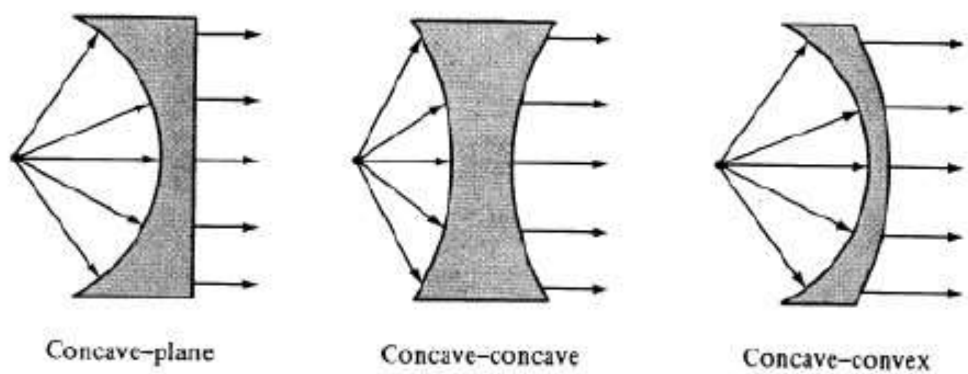
- Single elements
  - reflector antennas –   radio-astronomy

REVIEW OF ANTENNA TYPES

- Single elements
  - lens antennas – infrared and optical



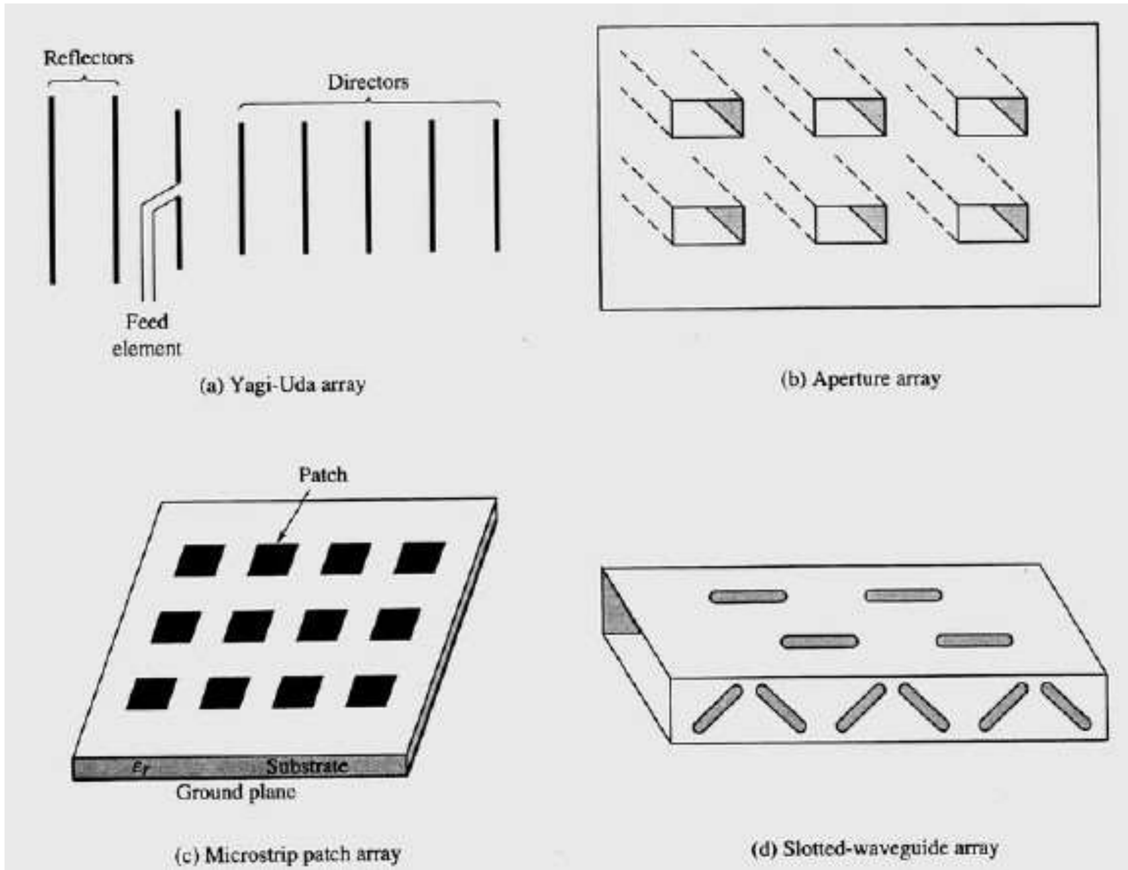
(a) Lens antennas with index of refraction $n > 1$



(b) Lens antennas with index of refraction $n < 1$

REVIEW OF ANTENNA TYPES

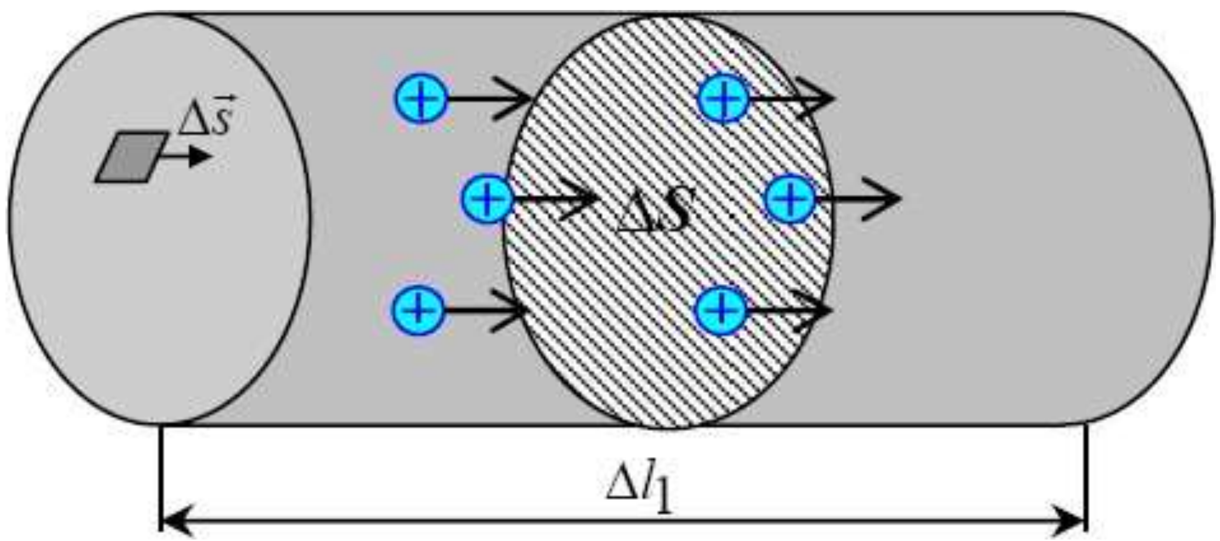
- array



- electronic scanning (phased arrays)
- beam shaping (nulls, maxima, beamwidth)

RADIATION MECHANISM

- Radiation is produced by accelerated or decelerated charge (time-varying current element)
- ***Definition***: A current element ($I\Delta l$), $A \times m$, is a filament of length Δl and current I.



RADIATION MECHANISM

- Assume the existence of a piece of a very thin wire where electric currents can be excited.
- The current $\boldsymbol{i}$ flowing through the wire cross-section ΔS is defined as the amount of charge passing through ΔS in 1 second:

$$i = \rho \cdot \Delta S \cdot \Delta l_1 = \rho \cdot \Delta S \cdot v \quad \mathrm{A} \qquad [1.1]$$

$\boldsymbol{\rho}$ (C/m^3) is the electric charge volume density,
$\boldsymbol{v}$ (m/s) is the velocity of the charges normal to the cross-section,
$\boldsymbol{\Delta l_1}$ (m/s) is the distance traveled by a charge in 1 second.

RADIATION MECHANISM

Equation [1.1] can be also written as:

$$J = \rho \cdot v_z \quad \text{A/m}^2 \qquad [1.2]$$

Where J is the electric current density.

The product $\rho_1 = \rho . \Delta S$ is the charge per unit length (charge line density) along the wire.

$$i = v \cdot \rho_l \quad \text{A} \qquad [1.3]$$

So that

$$\frac{di}{dt} = \rho_l \frac{dv}{dt} = \rho_l \cdot a \quad \text{A/s} \qquad [1.4]$$

RADIATION MECHANISM

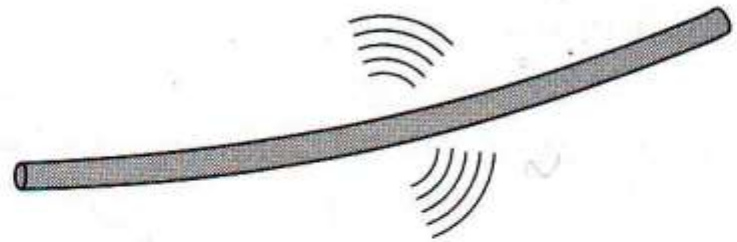
- where a (m/s^2) is the acceleration of the charge.
- The time-derivative of a current source would then be proportional to the amount of charge q enclosed in the volume of the current element and to its acceleration:

$$\Delta l \frac{di}{dt} = \Delta l \cdot \rho_l \cdot a = q \cdot a \qquad \text{A x m/s} \qquad [1.5]$$

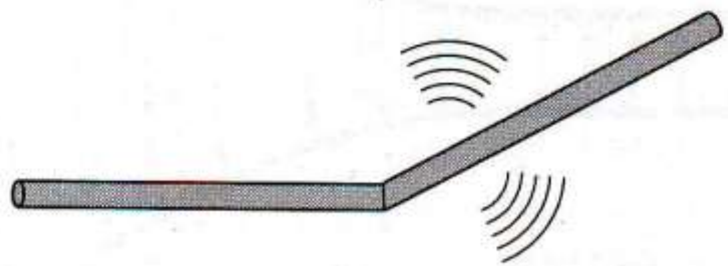
RADIATION MECHANISM

- To accelerate/decelerate charges, one needs sources of electromotive force and/or discontinuities of the medium in which the charges move.
- Such discontinuities can be bends or open ends of wires, change in the electrical properties of the region, etc.
- In summary:
  - If charge is not moving, current is zero → no radiation.
  - If charge is moving with a uniform velocity → no radiation.
  - If charge is accelerated due to electromotive force or due to discontinuities, such as termination, bend, curvature → radiation occurs.

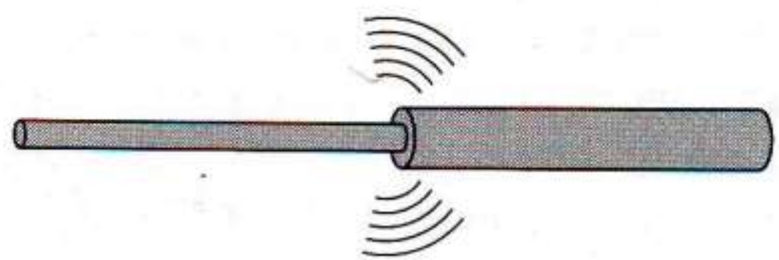
RADIATION MECHANISM



(a) Curved

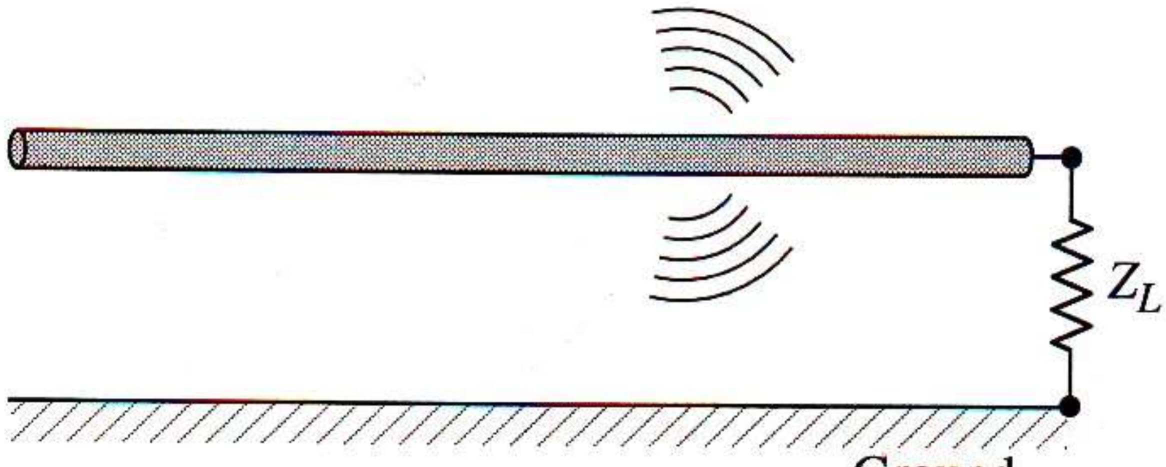


(b) Bent

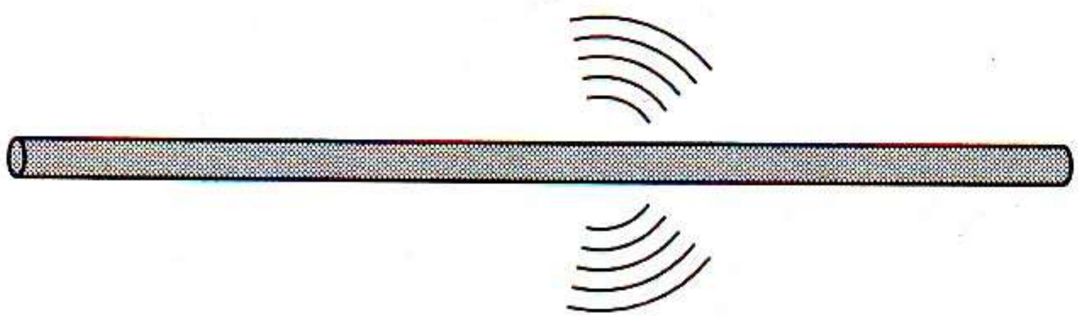


(c) Discontinuous

RADIATION MECHANISM



(d) Terminated



(e) Truncated

FUTURES CHALLENGES

- Integration problem
  - Monolithic MIC technology
  - Meta-materials, etc.
- Innovative antenna designs
  - Smart antenna
  - Reconfigurable antenna, etc.
- New applications
  - WLAN, UWB, GPS, etc.
- Complicated design
  - Higher frequencies
  - Low profile
  - Higher gain, etc.

BASIC ANTENNA PARAMETERS

INTRODUCTION

- To describe the performance of an antenna, definitions of various parameters are necessary.
- Some of the parameters are interrelated and not all of them need be specified for complete description of the antenna performance.
- IEEE Standard Definitions of Terms for Antennas (IEEE Std 145 – 1983)

RADIATION PATTERN

- An antenna radiation pattern or antenna pattern is defined as "a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates.
- In most cases, the radiation pattern is determined in the far-field region and is represented as a function of the directional coordinates.
- Radiation properties include power flux density, radiation intensity, field strength, directivity phase or polarization."

RADIATION PATTERN

- Representation of the radiation properties of the antenna as a function of angular position
- ***Power pattern***: the trace of the angular variation of the received/radiated power at a constant radius from the antenna
- ***Amplitude field pattern***: the trace of the spatial variation of the magnitude of electric (magnetic) field at a constant radius from the antenna.
- Often the *field* and *power* pattern are normalized with respect to their maximum value, yielding *normalized field* and *power patterns*.
- The power pattern is usually plotted on a logarithmic scale or more commonly in decibels (*dB*)

RADIATION PATTERN

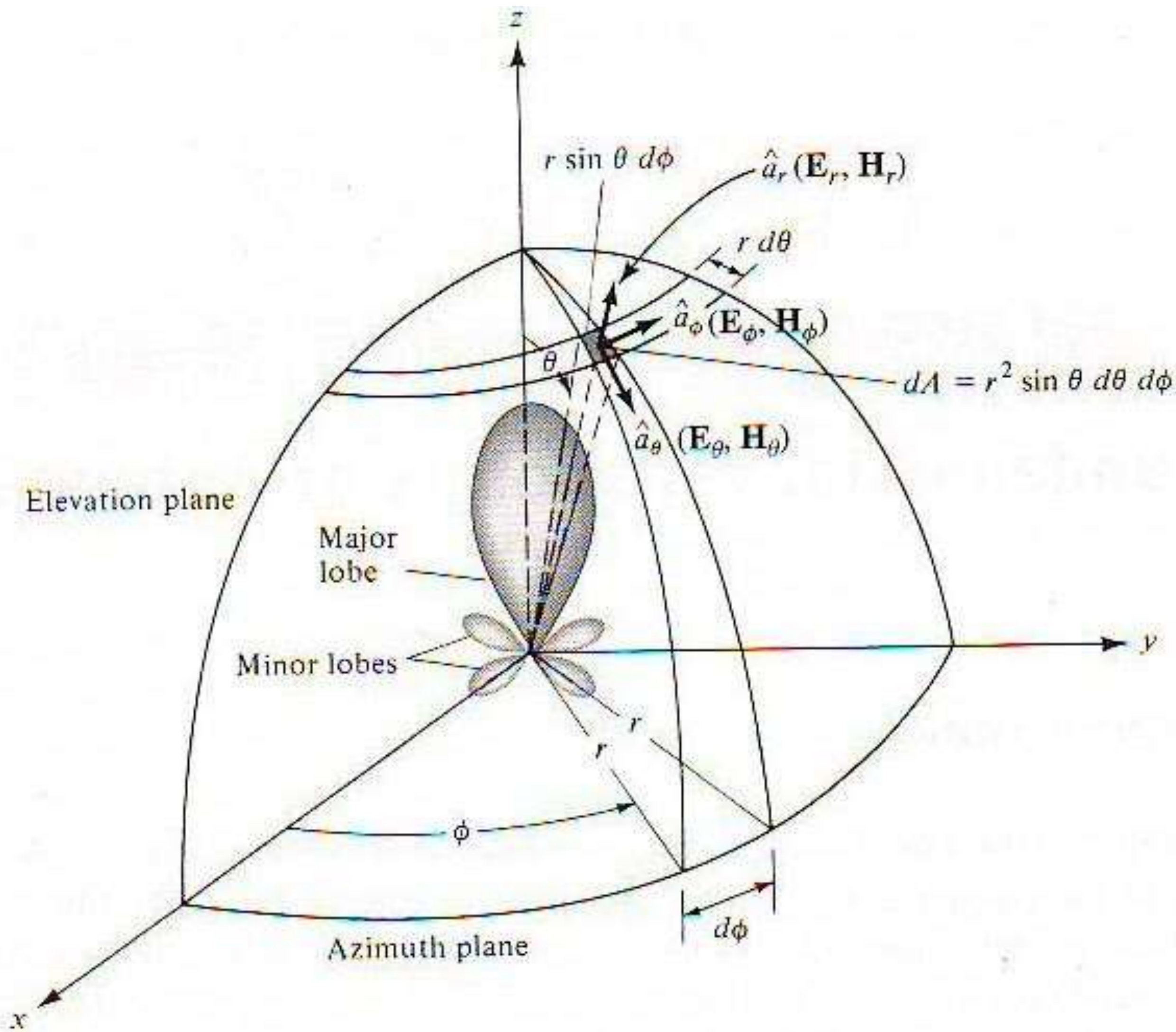
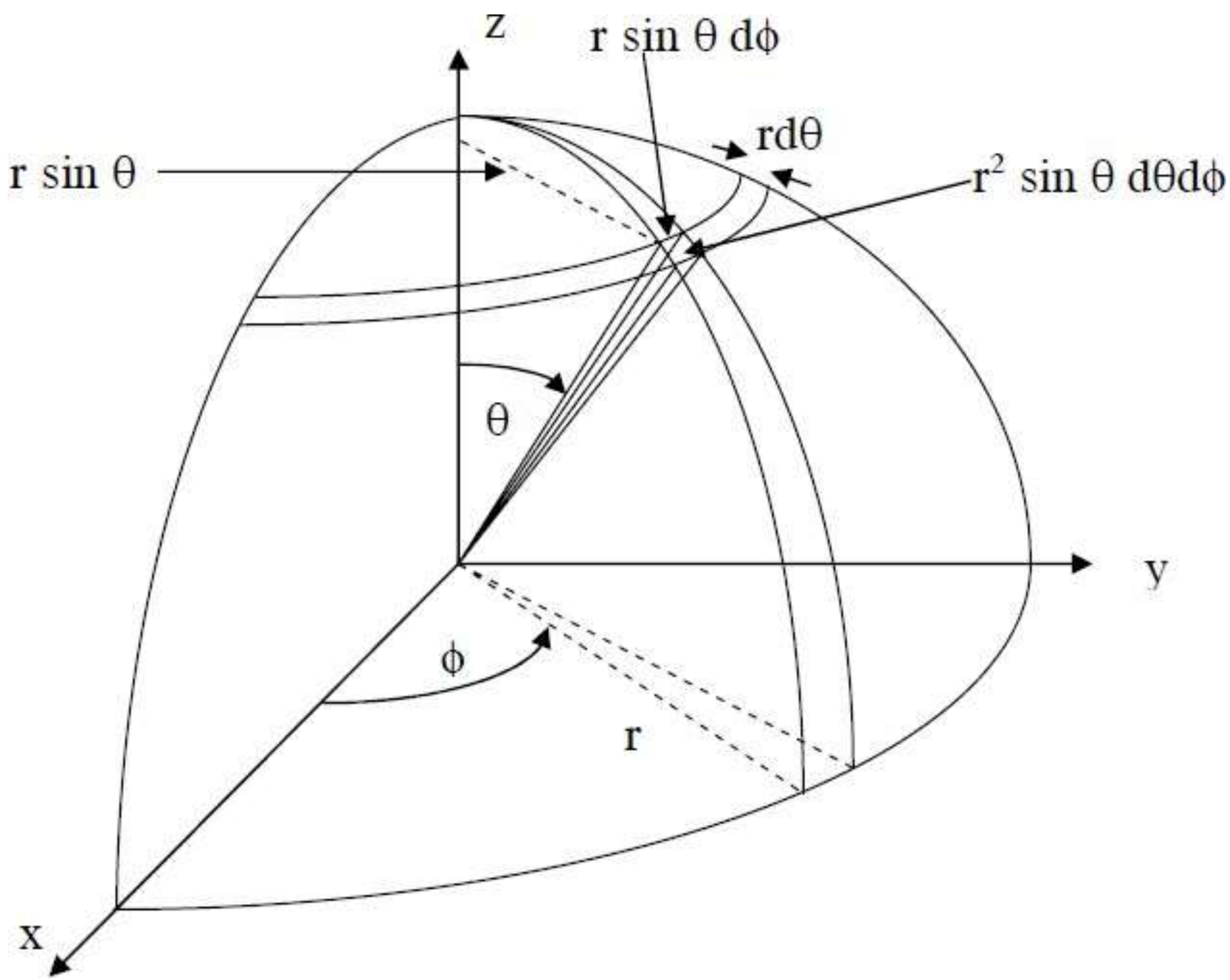


Figure 1.1: coordinate system for antenna analysis

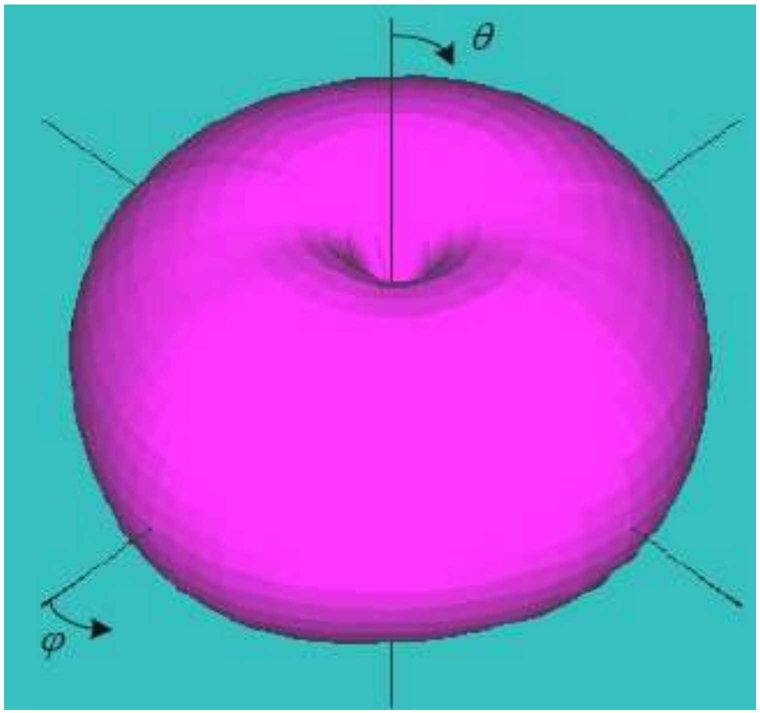
RADIATION PATTERN



RADIATION PATTERN

- For an antenna, the
  - *Field* pattern (*in linear scale*) typically represents a plot of the magnitude of the electric or magnetic field as a function of the angular space.
  - *Power* pattern (*in linear scale*) typically represents a plot of the square of the magnitude of the electric or magnetic field as a function of the angular space.
  - *Power* pattern (*in dB*) represents the magnitude of the electric or magnetic field, in decibels, as a function of the angular space.
- **Note**: *the power pattern and the amplitude field pattern are the same when computed and plotted in dB.*

RADIATION PATTERN



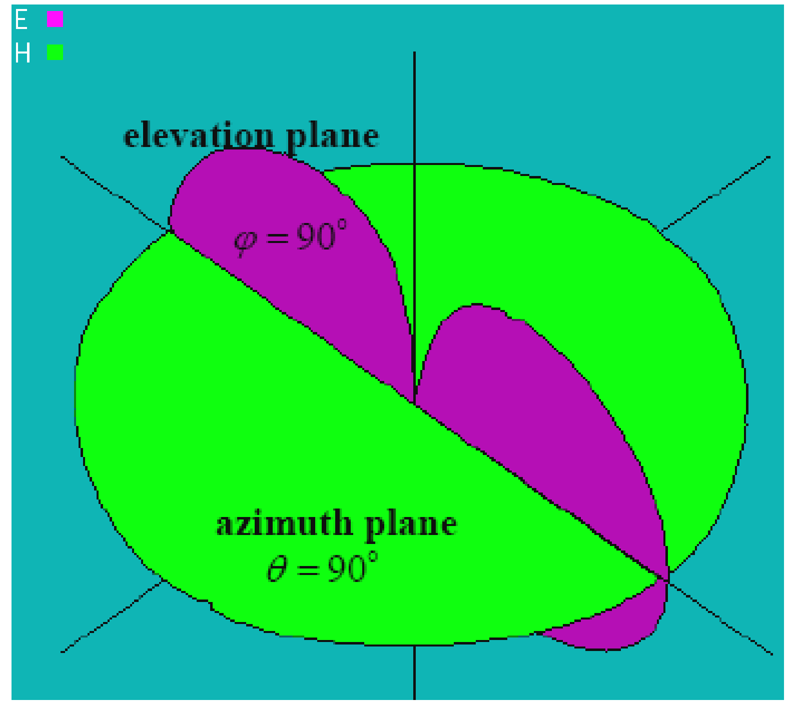


Figure 1.2: 3-D and 2-D patterns

RADIATION PATTERN

Plotting the pattern:

The trace of the pattern is obtained by setting the length of the radius-vector

$$\left|\vec{r}(\theta,\varphi)\right| \qquad [1.6]$$

Corresponding to the (θ,φ) [1.7]

Point of the radiation pattern proportional to the strength of the field

$$\left|\vec{E}(\theta,\varphi)\right| \qquad [1.8]$$

(in the case of an amplitude field pattern) or proportional to the power density

$$\left|\vec{E}(\theta,\varphi)\right|^2 \qquad [1.9]$$

(in the case of a power pattern).

RADIATION PATTERN

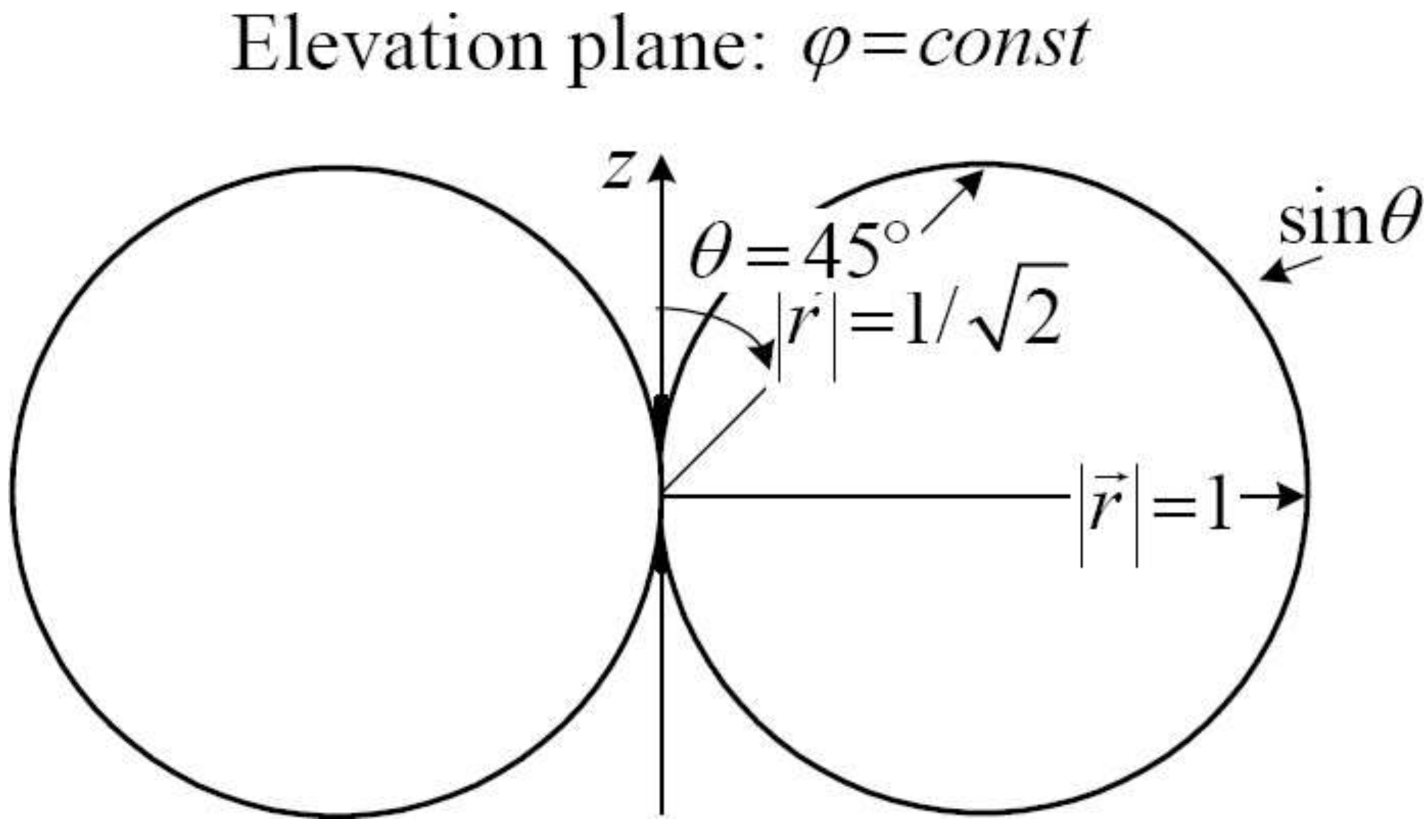


Figure 1.3: plotting the pattern

RADIATION PATTERN

- In Fig. 1.4, the plus (+) and the minus (-) signs in the lobes *indicate the relative polarization of the amplitude between the various lobes, which changes (alternates) as the nulls are crossed*.
- To find the points where the pattern achieves its half-power (-3 dB points), relative to the maximum value of the pattern:
  - Field pattern at 0.707 value of its maximum.
  - Power pattern (in linear scale) at its 0.5 value of its maximum.
  - Power pattern (in dB) at -3 dB value of its maximum.
- The angular separation between the two half-power points is referred to as *HPBW*.

RADIATION PATTERN

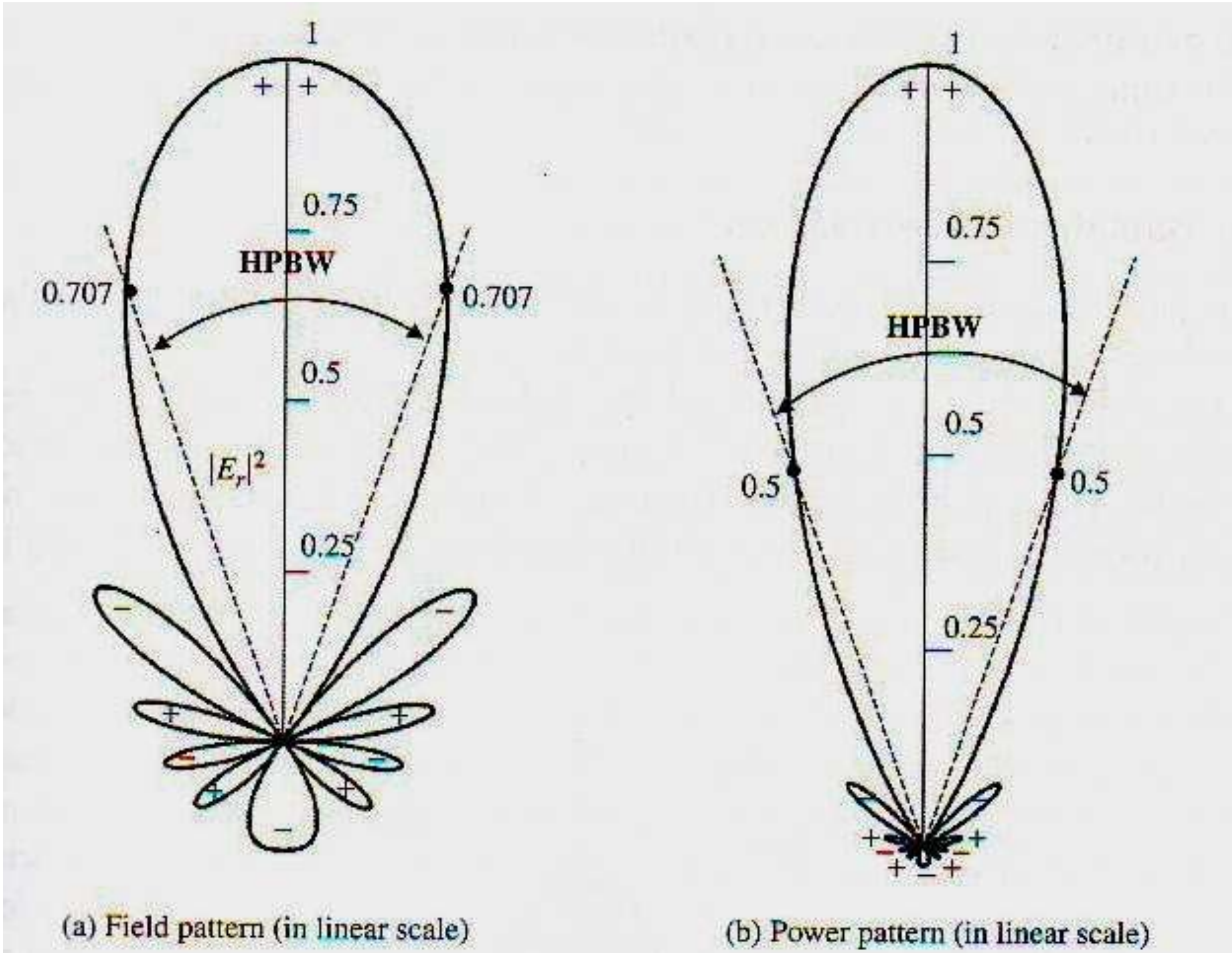


Figure 1.4: two-dimensional normalized field pattern of a 10-element linear array with a spacing of d = 0.25λ

RADIATION PATTERN

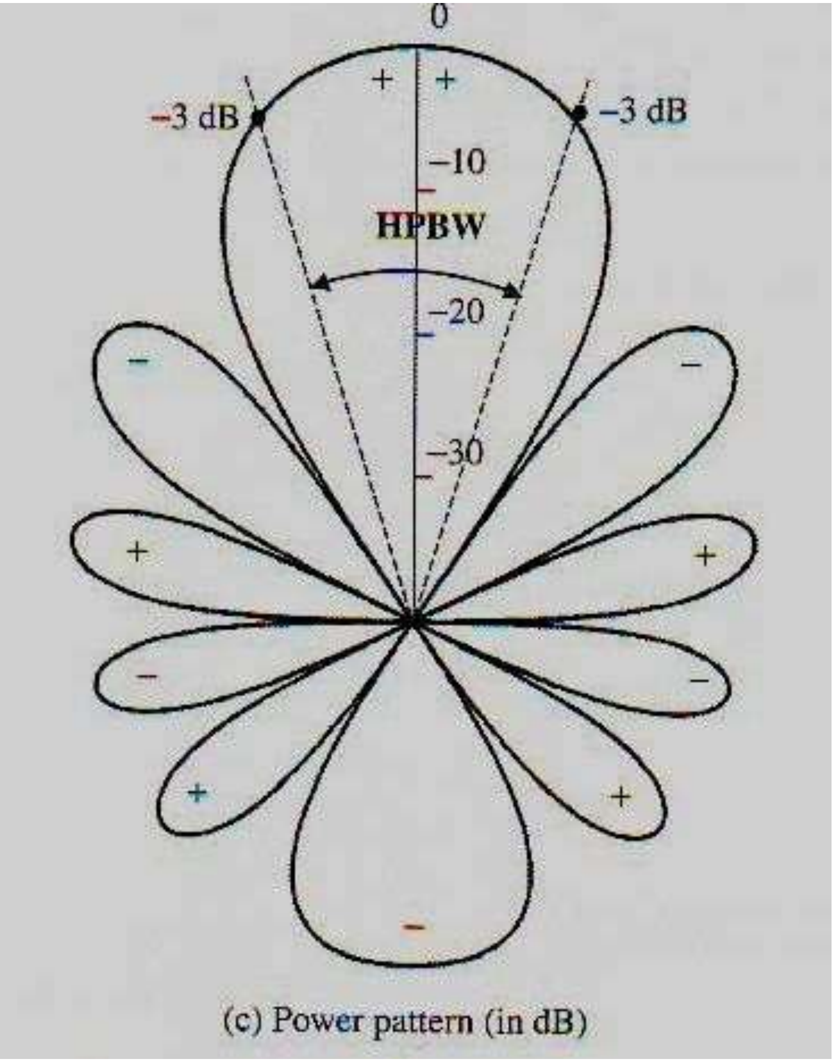


Figure 1.4: two-dimensional normalized field pattern of a 10-element linear array with a spacing of $d = 0.25\lambda$

RADIATION PATTERN

- All three patterns yield the same angular separation between the two half-power points, which referred as *HPBW* (as illustrate in Figure 1.4)

- Practical: the three dimensional pattern is measured and recorded in a series of two-dimensional-patterns.

RADIATION PATTERN LOBES

- Various parts of radiation pattern are referred to as *lobes*.
- Various lobes can be sub-classified in *major* or *main*, *minor*, *side* and *back* lobes. (see Fig. 1.5)
- Radiation lobes is a "portion of the radiation pattern bounded by regions of relatively weak radiation intensity."

RADIATION PATTERN LOBES

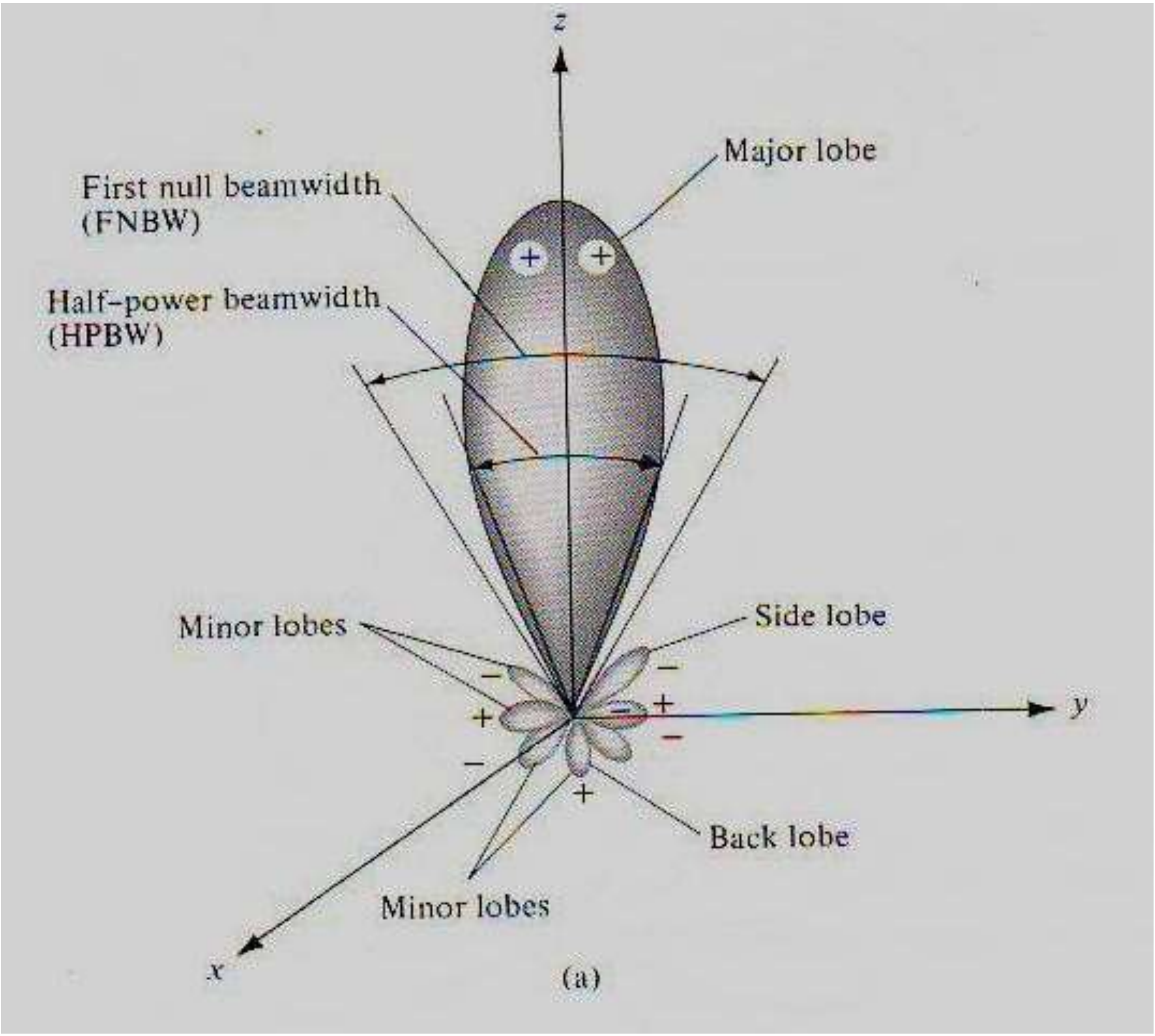


Figure 1.5: Radiation lobes and beamwidths of an antenna pattern

RADIATION PATTERN LOBES

- A *major lobe* (also called *main beam*) is defined as "the radiation lobe containing the direction of maximum radiation".
- A *minor lobe* is any lobe except major lobe.
- A *side lobe* is "a radiation lobe in any direction other than intended lobe". Usually a side lobe is adjacent to the main lobe and occupies the hemisphere in the direction of the main beam.
- A *back lobe* is "a radiation lobe whose axis makes an angle of approximately 180º with respect to the beam of the antenna".
- Minor lobes usually represent radiation in undesired directions, and they should be minimized.
- Side lobes are normally the largest of minor lobes.

RADIATION PATTERN LOBES

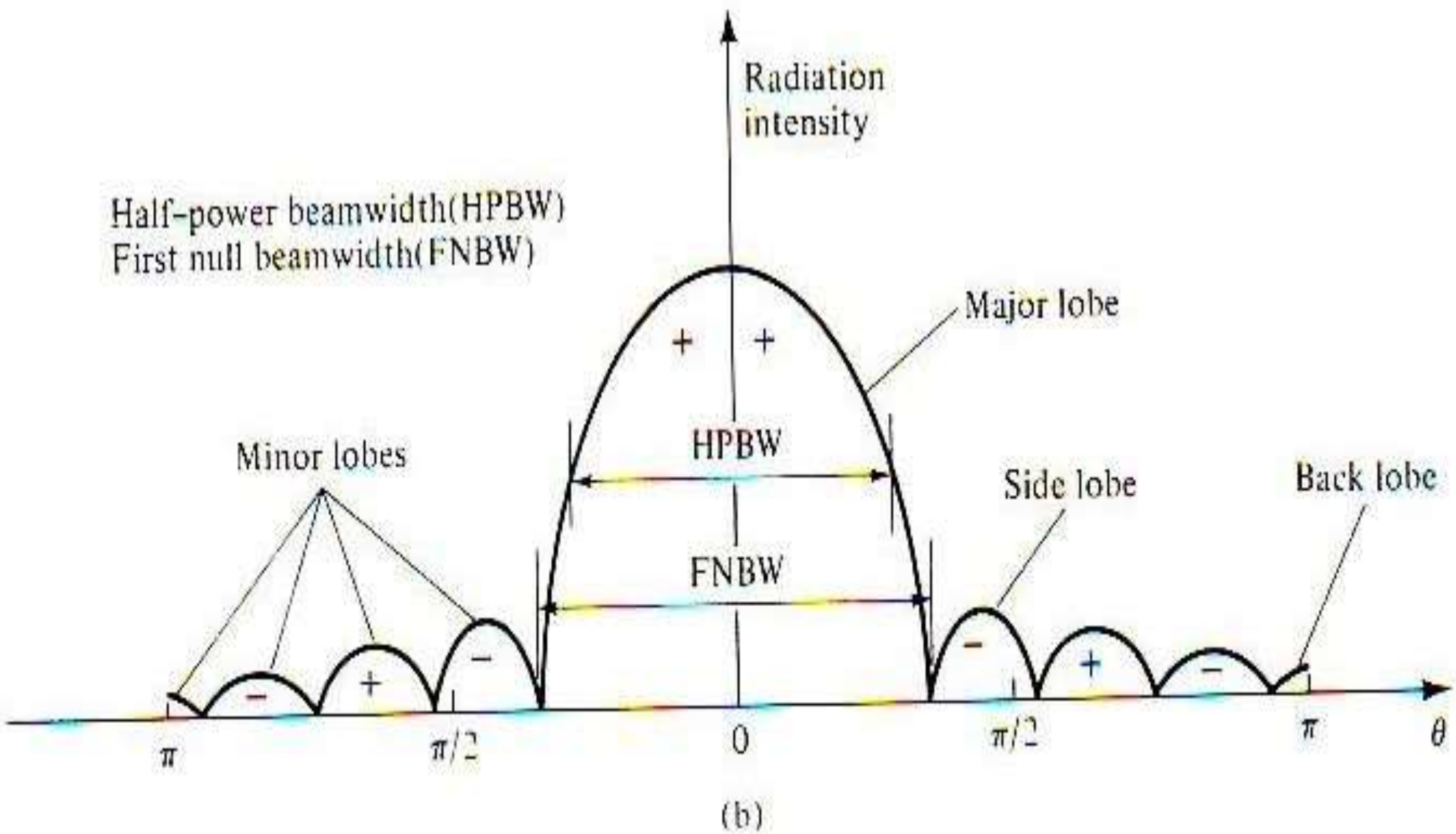


Figure 1.6: Linear plot of power pattern and its associated lobes and beamwidths.

RADIATION PATTERN LOBES

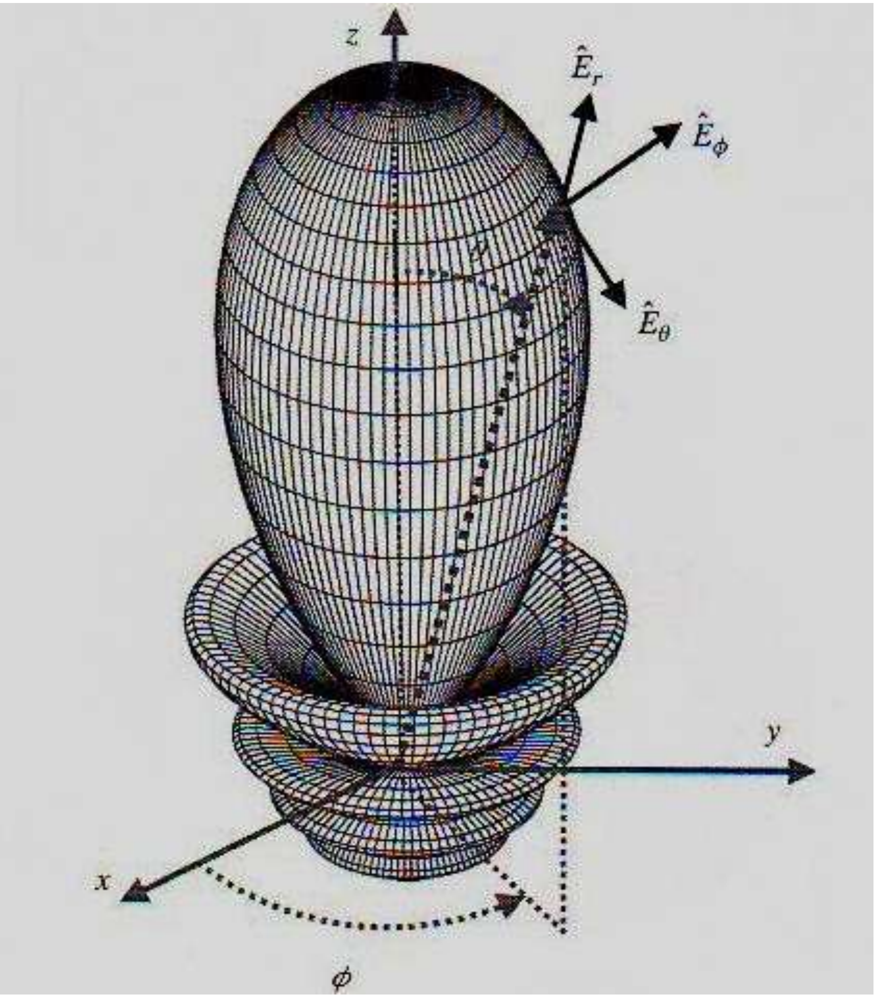


Figure 1.7: Normalized three-dimensional amplitude field pattern (in linear scale) of a 10-element linear array antenna with a uniform spacing of 0.25λ and progressive phase shift (β = -0.6π) between the elements.

ISOTROPIC, DIRECTIONAL & OMNIDIRECTIONAL PATTERN

- ***Isotropic pattern*** is the pattern of an antenna having *equal radiation in all directions*. This is an ideal (not physically achievable) concept. However, it is used to define other antenna parameters. It is represented simply by a sphere whose center coincides with the location of the isotropic radiator.
- ***Directional antenna*** is an antenna, which radiates (receives) much *more efficiently in some directions than in others*. Usually, this term is applied to antennas whose directivity is much higher than that of a half wavelength dipole.
- ***Omnidirectional antenna*** is an antenna, *which has a non-directional pattern in a given plane*, and *a directional pattern in any orthogonal plane* (e.g. single-wire antennas).

ISOTROPIC, DIRECTIONAL & OMNIDIRECTIONAL PATTERN

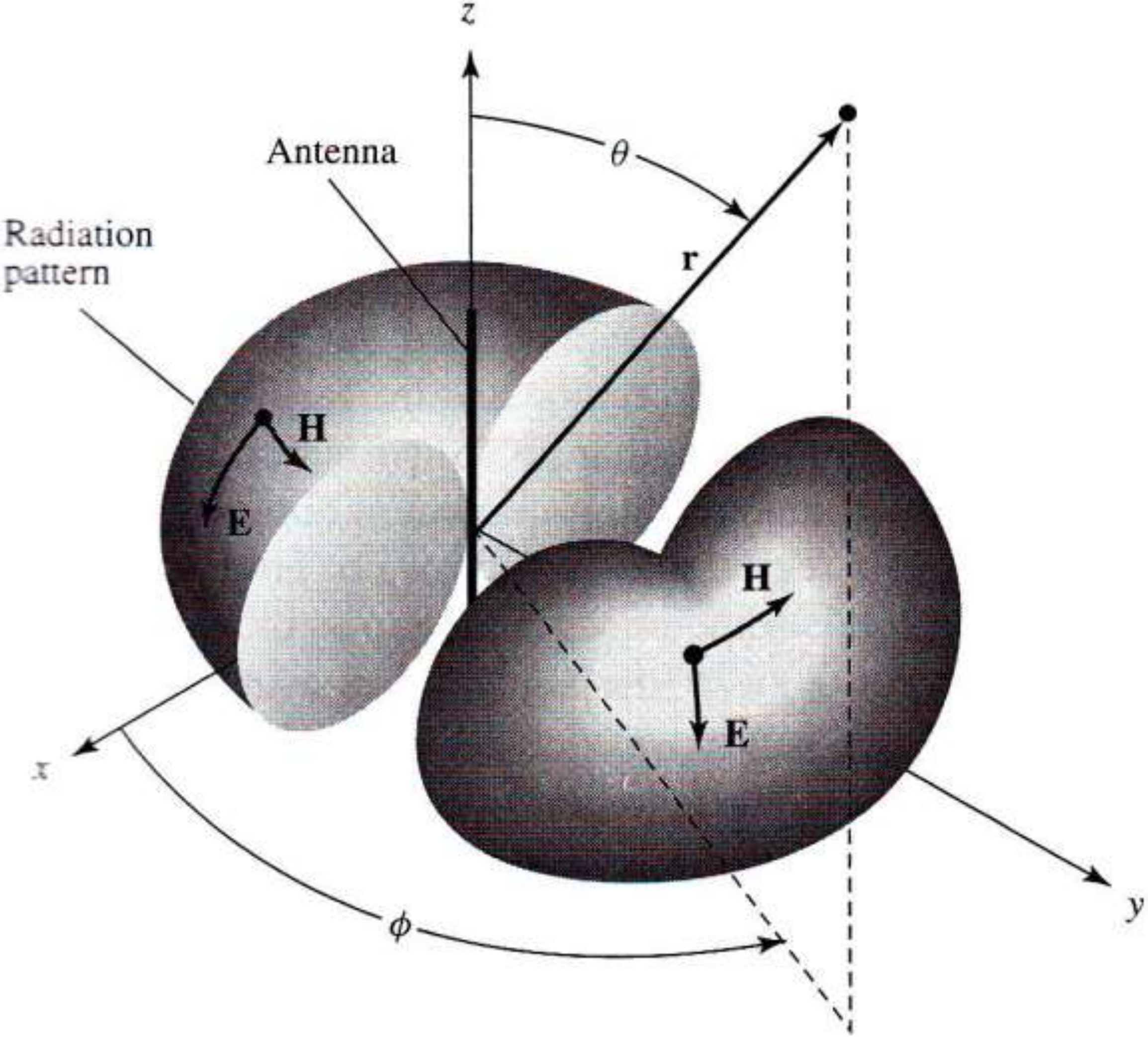


Figure 1.8: Omnidirectional Antenna Pattern.

ISOTROPIC, DIRECTIONAL & OMNIDIRECTIONAL PATTERN

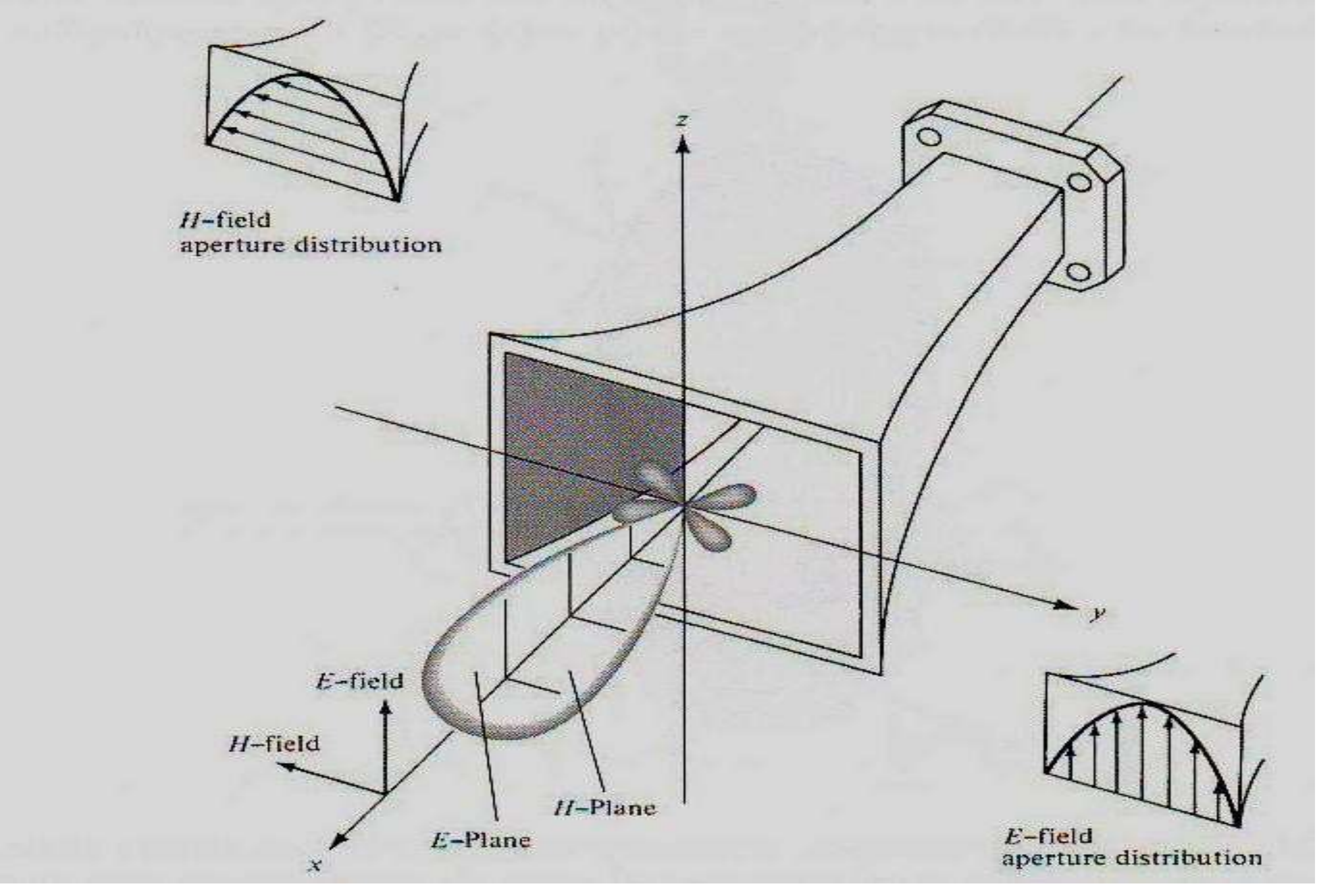


Figure 1.9: Principle E- and H-plane patterns for a pyramidal horn antenna.

PRINCIPAL PATTERNS

- ***Principal patterns*** are the 2-D patterns of linearly polarized antennas, measured in the ***E-plane*** (a plane parallel to the $|\bar{E}|$ vector and containing the direction of maximum radiation)
- the ***H-plane*** (a plane parallel to the $\vec{H}$ vector, orthogonal to the *E*-plane, and containing the direction of maximum radiation).

PATTERN BEAMWIDTH

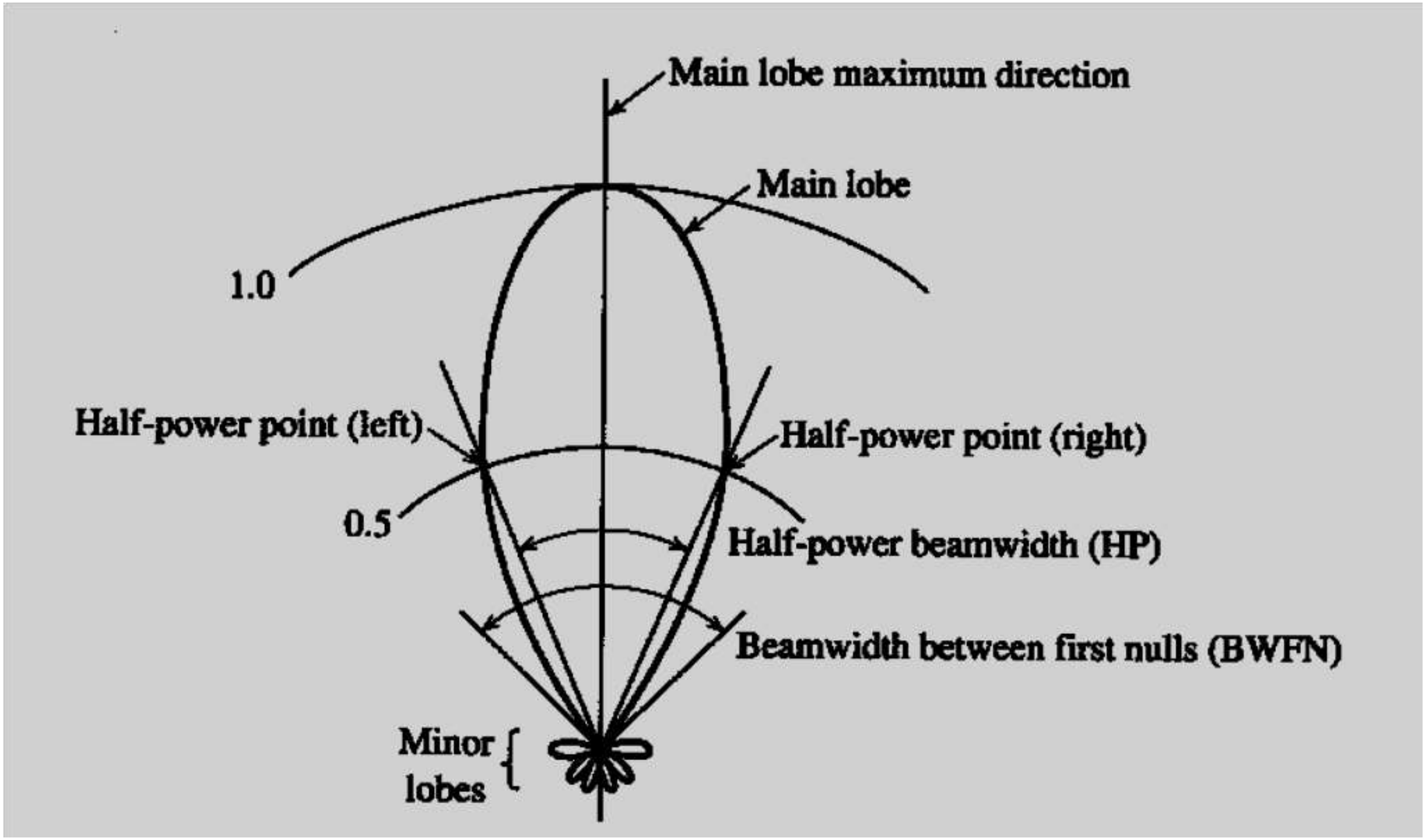
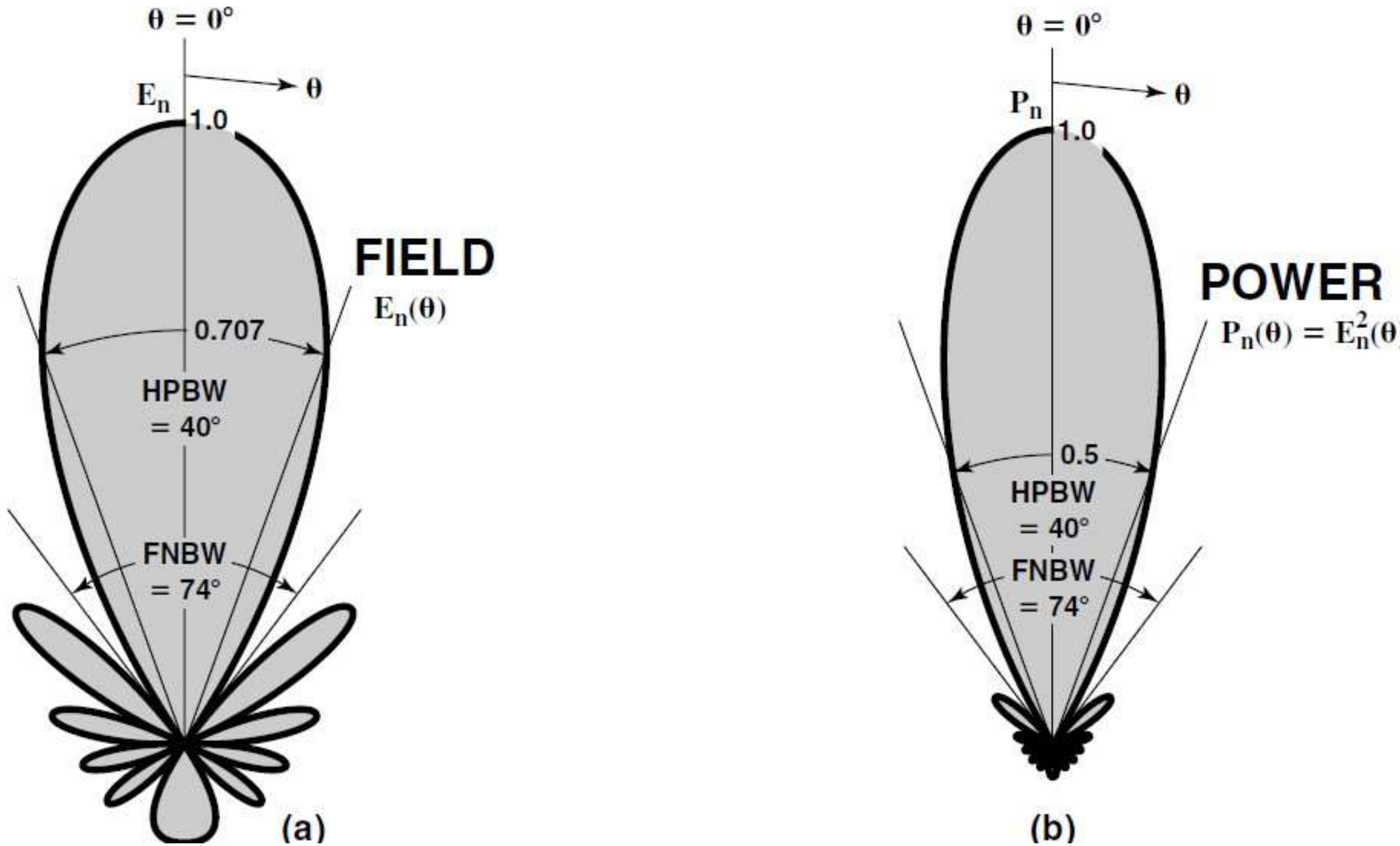
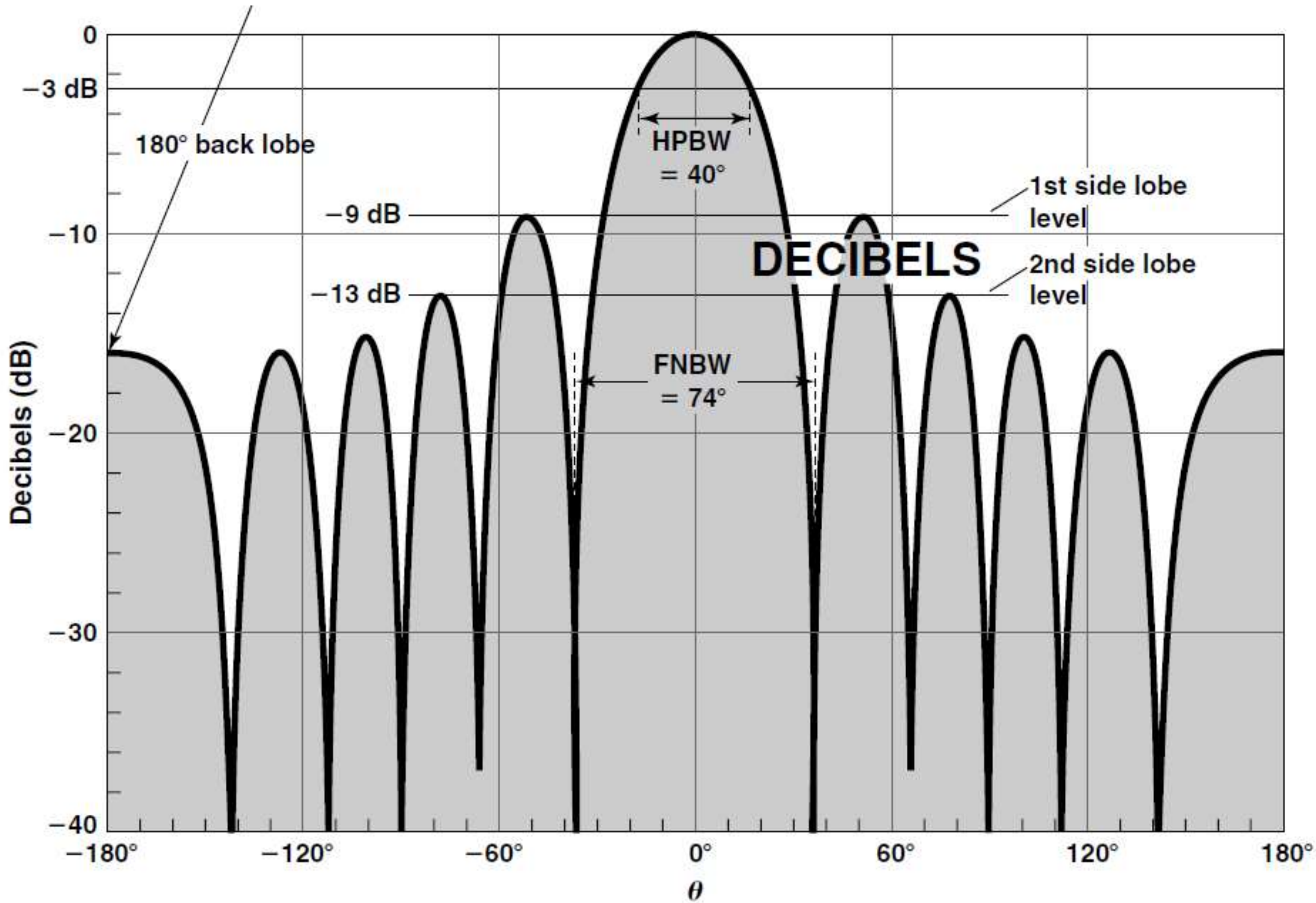


Figure 1.10: Pattern beamwidth.

PATTERN BEAMWIDTH

- ***Half-power beamwidth*** (HPBW) is the angle between two vectors, originating at the pattern's origin and passing through these points of the major lobe where the radiation intensity is half its maximum.
- ***First-null beamwidth*** (FNBW) is the angle between two vectors, originating at the pattern's origin and tangent to the main beam at its base.
- Often, it is true that FNBW≈2·HPBW


θ = 0°
θ
E_n
1.0
FIELD
$E_n(\theta)$
0.707
HPBW
= 40°
FNBW
= 74°
(a)
θ = 0°
θ
P_n
1.0
POWER
$P_n(\theta) = E_n^2(\theta)$
0.5
HPBW
= 40°
FNBW
= 74°
(b)



0
−3 dB
180° back lobe
HPBW
= 40°
1st side lobe
level
−9 dB
−10
DECIBELS
2nd side lobe
level
−13 dB
FNBW
= 74°
Decibels (dB)
−20
−30
−40
−180°
−120°
−60°
0°
60°
120°
180°
θ

Example

- The normalized radiation intensity of an antenna is represented by

$$U(\theta) = \cos^2(\theta)\cos^2(3\theta), \quad (0 \le \theta \le 90^\circ, \quad 0^\circ \le \phi \le 360^\circ)$$

Find

a. half-power beamwidth HPBW (*in radians and degrees)*

b. first-null beamwidth FNBW (*in radians and degrees)*

- ***Solution***

Since the U(θ) represents the power pattern, to find the half-power beamwidth you set the function equal to half of its maximum, or

$$U(\theta)|_{\theta=\theta_h} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_h} = 0.5 \Rightarrow \cos\theta_h \cos 3\theta_h = 0.707$$

$$\theta_h = \cos^{-1}\left(\frac{0.707}{\cos 3\theta_h}\right)$$

Since this is an equation with transcendental functions, it can be solved iteratively . After a few iterations, it is found that

So $\theta h = 14.32°$

Since the function $U(\theta)$ *is symmetrical about the maximum at* $\theta = 0$, *then* the HPBW is HPBW = $2\theta h = 28.65°$.

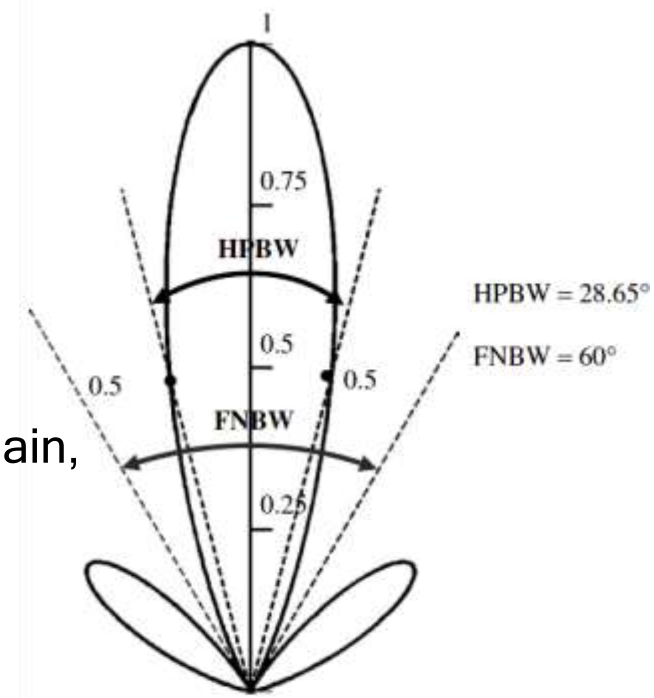
b) To find the first-null beamwidth (FNBW), you set the $U(\theta)$ *equal to zero, or*

$$U(\theta)|_{\theta=\theta_n} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_n} = 0$$

This leads to two solutions for θ_n.

$$\cos\theta_n = 0 \Rightarrow \theta_n = \cos^{-1}(0) = \frac{\pi}{2}\ radians = 90^\circ$$

$$\cos 3\theta_n = 0 \Rightarrow \theta_n = \frac{1}{3}\cos^{-1}(0) = \frac{\pi}{6}\ radians = 30^\circ$$



The one with the smallest value leads to the FNBW. Again, because of the symmetry of the pattern, the FNBW is

$$\text{FNBW} = 2\theta_n = \frac{\pi}{3}\ radians = 60^\circ$$

H.W

1-Find the half-power beamwidth (HPBW) and first-null beamwidth (FNBW), in radians and degrees, for the following normalized radiation intensities:

$$\left.\begin{array}{ll} \text{(a) } U(\theta) = \cos\theta & \text{(b) } U(\theta) = \cos^2\theta \\ \text{(c) } U(\theta) = \cos(2\theta) & \text{(d) } U(\theta) = \cos^2(2\theta) \\ \text{(e) } U(\theta) = \cos(3\theta) & \text{(f) } U(\theta) = \cos^2(3\theta) \end{array}\right\} (0 \le \theta \le 90^\circ, 0 \le \phi \le 360^\circ)$$

2-Find the half-power beamwidth (HPBW) and first-null beamwidth (FNBW), in radians and degrees, for the following normalized radiation intensities:

$$\left.\begin{array}{l} \text{(a) } U(\theta) = \cos\theta\cos(2\theta) \\ \text{(b) } U(\theta) = \cos^2\theta\cos^2(2\theta) \\ \text{(c) } U(\theta) = \cos(\theta)\cos(3\theta) \end{array}\right\} (0 \le \theta \le 90^\circ, 0 \le \phi \le 360^\circ)$$

Field Regions

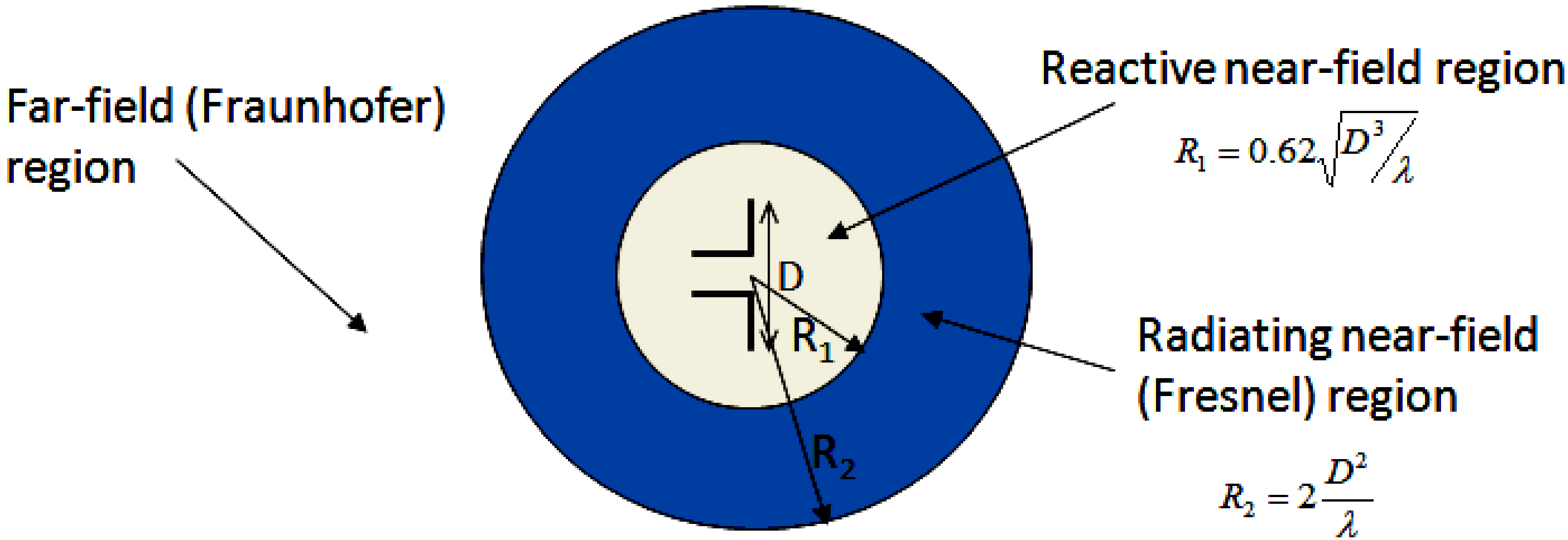


Figure 1.11: Field regions of an antenna.

FIELD REGIONS

- Reactive near-field region is defined as "*that portion of the near-field region immediately surrounding the antenna wherein the reactive field predominates*".

$$R < 0.62\sqrt{D^3/\lambda} \qquad [1.10]$$

λ is the wavelength

D is the largest dimension of the antenna

- For a very short dipole, or equivalent radiator the outer boundary is commonly taken to exist at a distance $\lambda/2\pi$ from the antenna surface.

FIELD REGIONS

- Radiating near-field region (Fresnel) is defined as "*that region of the field of an antenna between the reactive near-field region and the far-field region wherein radiation fields predominate and wherein the angular field distribution is dependent upon the distance from the antenna*".

$$0.62\sqrt{D^3/\lambda} \leq R < 2D^2/\lambda \qquad [1.11]$$

λ is the wavelength

D is the largest dimension of the antenna

- If the antenna has a dimension that is not large compared to wavelength, this region may not exist.

FIELD REGIONS

- Far-field region (Fraunhofer) is defined as "*that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna*".

$$R > \frac{2D^2}{\lambda} \qquad [1.12]$$

λ is the wavelength

D is the largest dimension of the antenna

- The far-field patterns of certain antennas, such as multibeam reflector antennas are sensitive to variations in phase over their apertures.

FIELD REGIONS

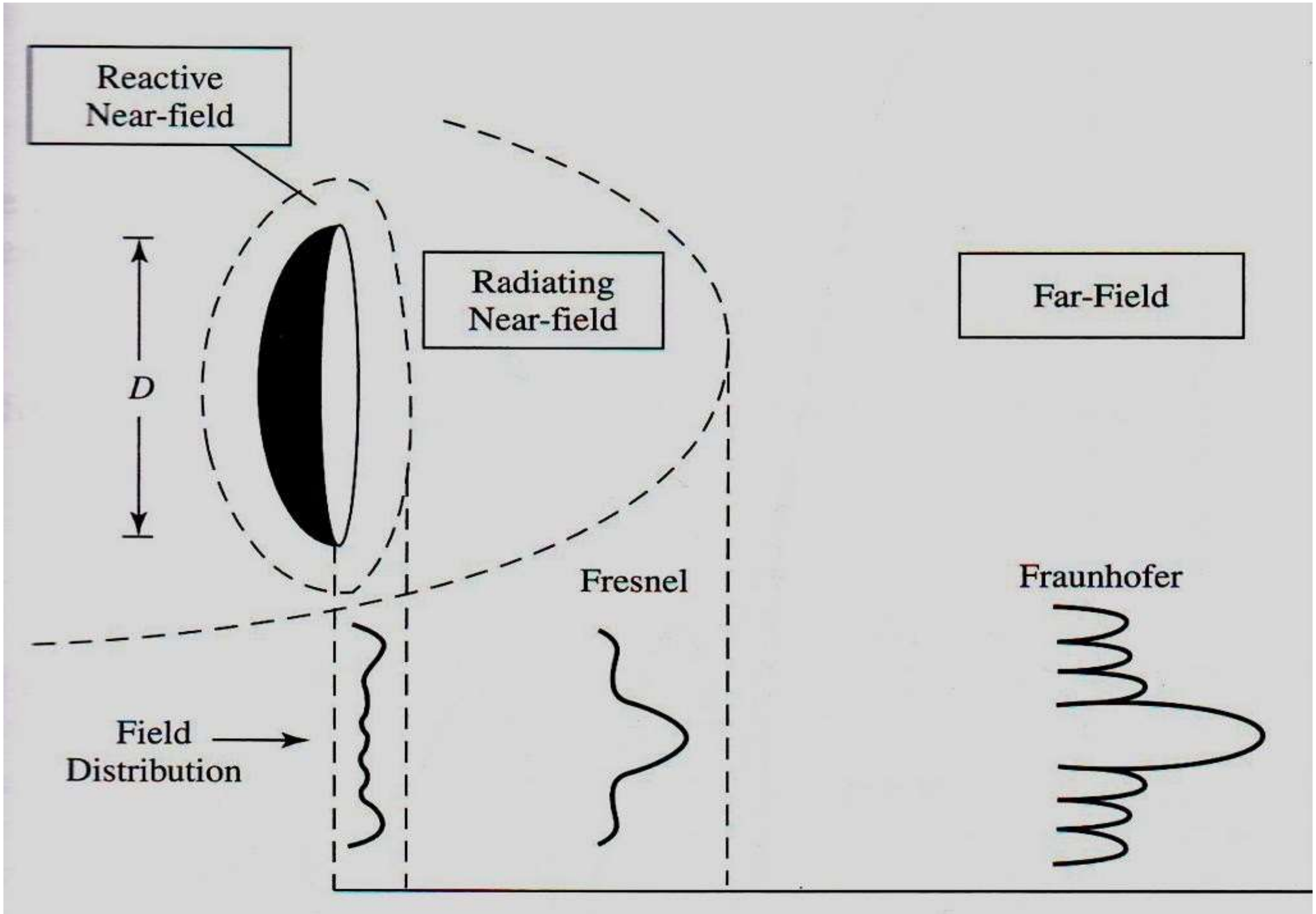


Figure 1.12: Typical changes of antenna amplitude pattern shape from reactive near field toward the far field.

RADIAN & STERADIAN

The measure of a plane angle is a *radian*.

One radian is defined as the plane angle with its vertex at the center of a circle of radius *r* that is subtended by an arc whose length is *r*.

$$C = 2\pi r \qquad [1.13]$$

There are 2π rad (2π*r*/*r*) in a full circle.

RADIAN & STERADIAN

The measure of a solid angle is a *steradian*.

One steradian is defined as the solid angle with its vertex at the center of a circle of a sphere radius *r* that is subtended by a spherical surface area equal to that of square which each side of a length *r*.

$$A = 4\pi r^2 \qquad [1.14]$$

There are 4π sr ($4\pi r^2/r^2$) in a closed sphere.

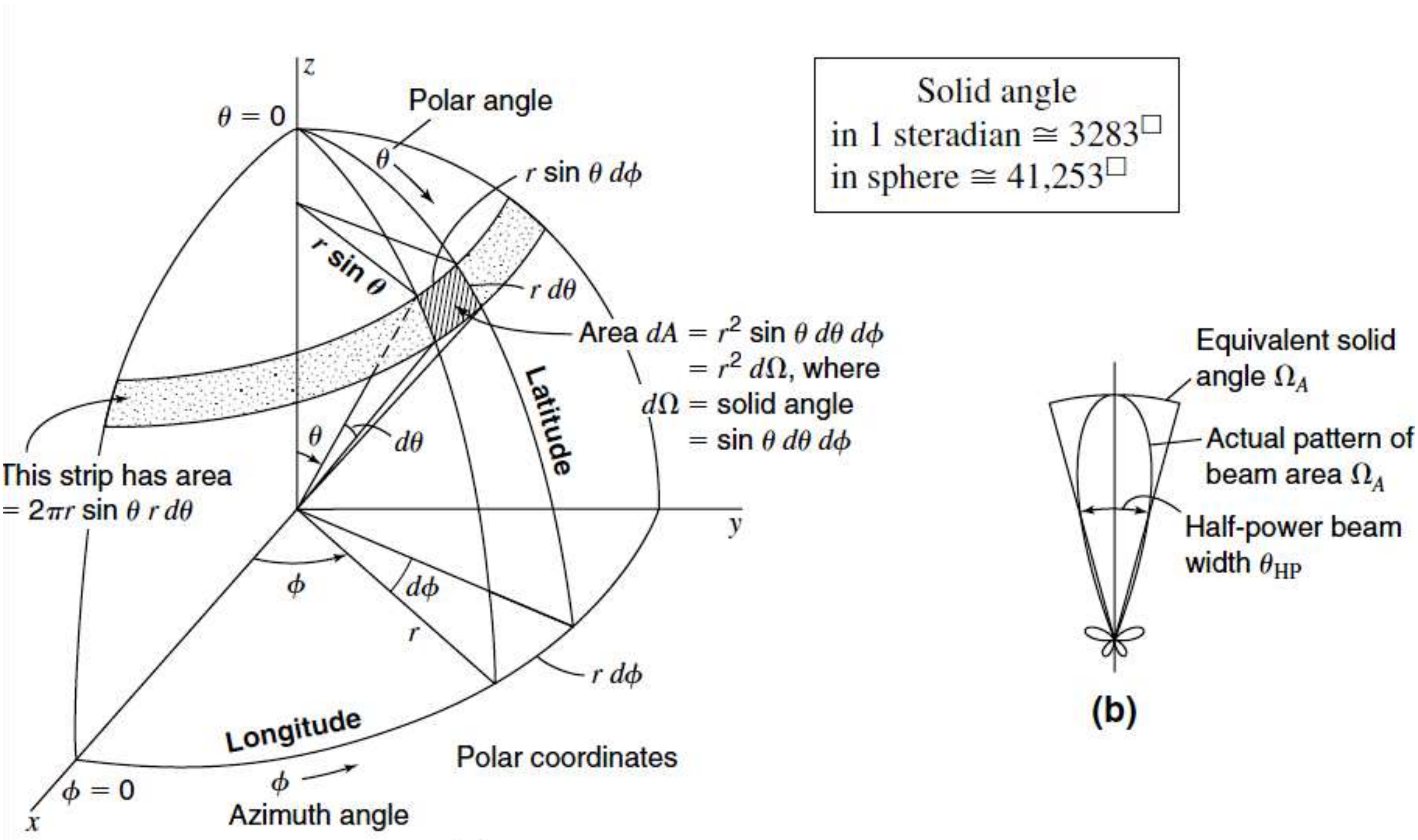
RADIAN & STERADIAN

The infinitesimal area *dA* on the surface of a sphere of radius r, is shown in Fig. 1.22 is given by:

$$dA = r^2 \sin\theta d\theta d\phi \qquad (m^2) \qquad [1.15]$$

Therefore, the element of solid angle *dΩ* of a sphere can be written as:

$$d\Omega = \frac{dA}{r^2} = \sin\theta d\theta d\phi \qquad (sr) \qquad [1.16]$$



z
θ = 0
Polar angle
θ
r sin θ dφ
r sin θ
r dθ
Area dA = r² sin θ dθ dφ
= r² dΩ, where
dΩ = solid angle
= sin θ dθ dφ
Latitude
θ
dθ
This strip has area
= 2πr sin θ r dθ
y
φ
dφ
r
r dφ
Longitude
Polar coordinates
φ = 0
φ
Azimuth angle
x
Solid angle
in 1 steradian ≅ 3283□
in sphere ≅ 41,253□
Equivalent solid
angle Ω_A
Actual pattern of
beam area Ω_A
Half-power beam
width θ_HP
(b)

$$\text{Area of sphere} = 2\pi r^2 \int_0^{\pi} \sin\theta \, d\theta = 2\pi r^2[-\cos\theta]_0^{\pi} = 4\pi r^2$$

where 4π = solid angle subtended by a sphere, sr

Thus,

$$1 \text{ steradian} = 1 \text{ sr} = (\text{solid angle of sphere})/(4\pi)$$

$$= 1 \text{ rad}^2 = \left(\frac{180}{\pi}\right)^2 (\text{deg}^2) = 3282.8064 \text{ square degrees}$$

Therefore,

$$4\pi \text{ steradians} = 3282.8064 \times 4\pi = 41{,}252.96 \cong 41{,}253 \text{ square degrees} = 41{,}253^{\square}$$

= solid angle in a sphere

RADIAN & STERADIAN

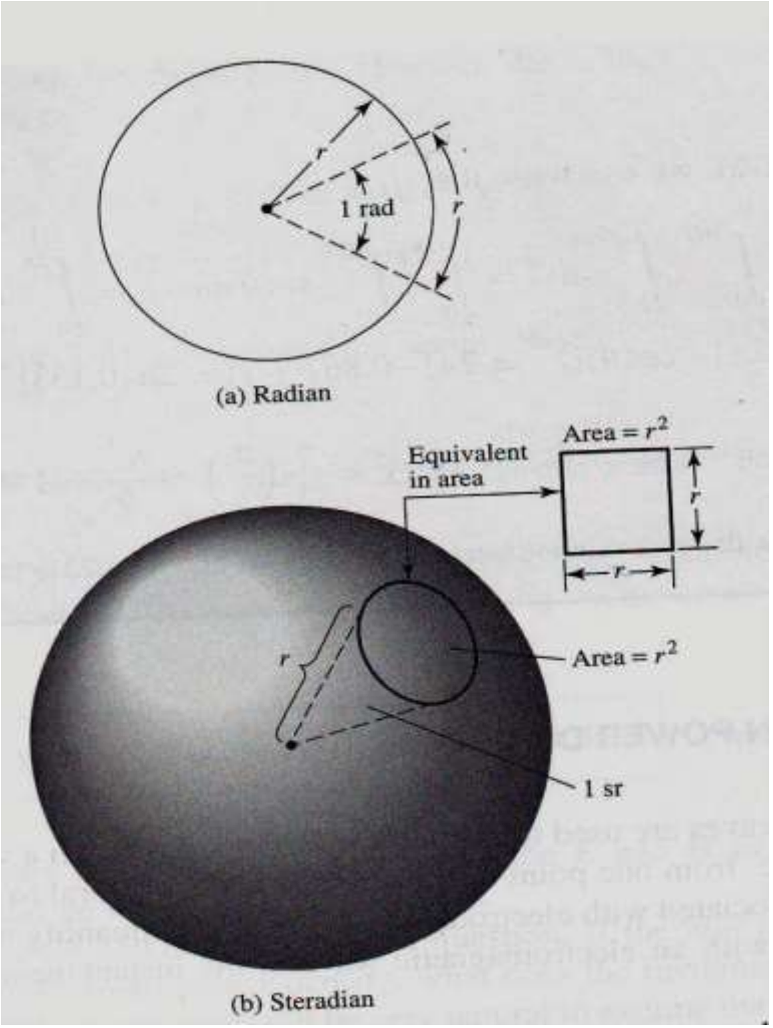


Figure 1.14: Geometrical arrangements for defining a radian and a steradian

Beam Area

The beam area or *beam solid angle or A of an antenna is given by the integral* of the normalized power pattern P(over a sphere (4*π sr*)

$$\Omega_A = \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} P_n(\theta, \phi) \sin\theta \, d\theta \, d\phi$$

where $d\Omega = \sin\theta \, d\theta \, d\varphi$, *sr*

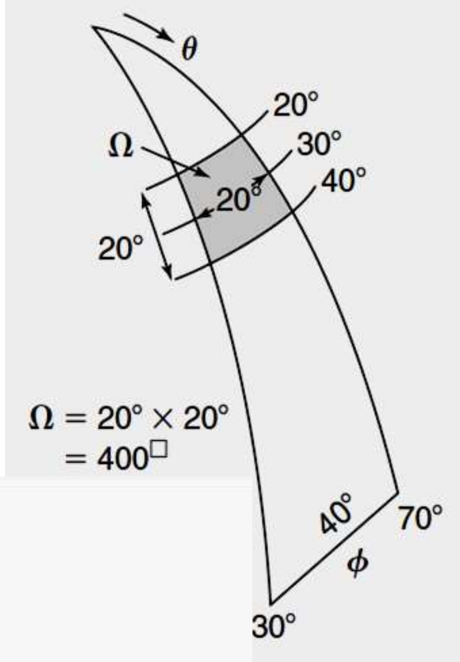
The *beam area of an antenna can often be described approximately in terms of the* angles subtended by the ***half-power points*** *of the main lobe in the two principal planes.*

$$\boxed{\textbf{\textit{Beam area}} \cong \Omega_A \cong \theta_{HP}\phi_{HP} \qquad (\text{sr})}$$

where *θHP and φHP are the half-power beamwidths (HPBW) in the two principal planes,* minor lobes being neglected.

Example(BOOK)

Find the number of square degrees in the solid angle *on a spherical surface that is* between *θ = 20◦ and θ = 40◦ (or 70◦ and 50◦ north latitude) and between φ = 30◦ and φ = 70◦ (30◦ and 70◦ east longitude).*



$$\Omega = \int_{30^\circ}^{70^\circ} d\phi \int_{20^\circ}^{40^\circ} \sin\theta \, d\theta = \frac{40}{360} 2\pi [-\cos\theta]_{20}^{40}$$

$$= 0.222\pi \times 0.173 = 0.121 \text{ steradians} \qquad \text{(sr)}$$

$$= 0.121 \times 3283 = 397 \text{ square degrees} = 397^{\square} \qquad \textit{Ans.}$$

The solid angle Ω shown in the sketch may be *approximated* as the product of two angles $\Delta\theta = 20^\circ$ and $\Delta\phi = 40^\circ \sin 30^\circ = 40^\circ \times 0.5 = 20^\circ$ where 30° is the median θ value of latitude. Thus, $\Omega = \Delta\theta\, \Delta\phi = 20^\circ \times 20^\circ = 400^{\square}$, which is within 3/4% of the answer given above.

RADIATION POWER DENSITY

Electromagnetic waves are used to transport information through a wireless medium or a guiding structure, from one point to other. It is then natural to assume that power and energy are associated with electromagnetic fields. *The quantity used to describe the power associated with an electromagnetic wave is the instantaneous Poynting vector* defined as:

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} \qquad [1.17]$$

$\mathbf{P}$ = instantaneous Poynting vector (W/m^2)
$\mathbf{E}$ = instantaneous electric-field intensity (V/m)
$\mathbf{H}$ = instantaneous magnetic-field intensity (A/m)

RADIATION POWER DENSITY

The total power crossing a closed surface can be obtained by integrating the normal component of Poynting vector over the entire surface. In equation form:

$$\mathbf{P} = \iint_S \mathbf{W}.ds = \iint_S \mathbf{W}.\mathbf{\tilde{n}}\, da \qquad W \qquad [1.18]$$

P = instantaneous total power (W)

ñ = unit vector normal to the surface

da = infinitesimal area of the closed surface (m^2)

RADIATION POWER DENSITY

For applications of time varying fields, it is often more desirable to find the average power density which is obtained by integrating the instantaneous Poynting vector over one period. For time-harmonic variations of the form **$e^{j\omega t}$**. So, the complex fields **E** and **H** which are related to their instantaneous counterparts ***E*** and ***H***.

$$\mathbf{E}(x, y, z; t) = \mathrm{Re}\left[\mathbf{E}(x, y, z)e^{j\omega t}\right] \qquad [1.19]$$

$$\mathbf{H}(x, y, z; t) = \mathrm{Re}\left[\mathbf{H}(x, y, z)e^{j\omega t}\right] \qquad [1.20]$$

RADIATION POWER DENSITY

By using the equation [1.19] and [1.20] and the identity (as shown in page 39), The time average Poynting vector (average power density) can be written as:

$$p_{av}(x,y,z) = [\mathbf{p}(x,y,z;t)]_{av} = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^{*}\right] \quad (\mathrm{W/m^2}) \qquad [1.21]$$

The ½ factor appears in equation [1.21] because **E** and **H** fields represent peak values and it should omitted for RMS values.

RADIATION POWER DENSITY

The imaginary part of equation [1.21] represent the reactive (stored) power density associated with the electromagnetic fields. So, The power density associated with the electromagnetic fields of an antenna in its far-field region is predominately real and will be referred to as ***radiation density***. The average power radiated by an antenna:

$$P_{rad} = P_{av} = \oiint_S W_{rad} \cdot ds = \oiint_S W_{av} \cdot \hat{n} da$$

[1.22]

$$= \frac{1}{2} \oiint_S \mathrm{Re}\left(\mathbf{E} \times \mathbf{H}^*\right) \cdot ds$$

RADIATION POWER DENSITY

An isotropic radiator is an ideal source that radiates equally in all directions. Isotropic radiator have the symmetric radiation, which there is no function of the spherical coordinate angles *θ* and *Φ*. Thus the total power radiated is given by:

$$P_{rad} = \oiint_S W_0 \cdot ds = \int_0^{2\pi}\int_0^{\pi} \left[\hat{a}_r W_0(r)\right] \cdot \left[\hat{a}_r r^2 \sin\theta \; d\theta d\phi\right] \qquad [1.23]$$

$$= 4\pi r^2 W_0$$

The power density:

$$W_0 = \hat{a}_r W_0 = \hat{a}_r \left(\frac{P_{rad}}{4\pi r^2}\right) \quad (\text{W/m}^2) \qquad [1.24]$$

RADIATION INTENSITY

Radiation intensity in a given direction is defined as “the power radiated from an antenna per unit solid angle.” The radiation intensity is a far-field parameter.

$$U = r^2 P_{rad} \qquad [1.25]$$

U = radiation intensity (W/unit solid angle)

The total power obtained by the radiation intensity:

$$P_{rad} = \oint\!\!\oint_{\Omega} U \, d\Omega = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi \qquad [1.26]$$

DIRECTIVITY

The directivity of an antenna is defined as "*the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. The average radiation intensity is equal to the total power radiated by the antenna divided by 4π. If the direction is not specified, the direction of maximum radiation intensity is implied*."

The directivity of a non-isotropic source is equal to the ratio of its radiation intensity in a given direction over that of an isotropic source.

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} \qquad [1.27]$$

DIRECTIVITY

If the direction is not specified, it implies the direction of maximum radiation intensity (maximum directivity) expressed as:

$$D_{\max} = D_0 = \frac{U|_{\max}}{U_0} = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{rad}} \qquad [1.28]$$

D = directivity (dimensionless)
D_0 = maximum directivity (dimensionless)
U = radiation intensity (W / unit solid angle)
U_{max} = maximum radiation intensity (W / unit solid angle)
U_0 = radiation intensity of isotropic source (W / unit solid angle)
P_{rad} = total radiated power (W)

Approximate Directivity

$$\boxed{D = \frac{40{,}000}{\theta^{\circ}_{\mathrm{HP}}\phi^{\circ}_{\mathrm{HP}}} \qquad \textit{Approximate directivity}}$$

$\theta^{\circ}_{\mathrm{HP}}$ = half-power beamwidth in one principal plane

$\phi^{\circ}_{\mathrm{HP}}$ = half-power beamwidth in other principal plane

ANTENNA EFFICIENCY

- The total efficiency of the antenna e_t is used to estimate the *total loss of energy at the input terminals* of the antenna and within the antenna structure. It includes all mismatch losses and the dielectric/conduction losses (described by the ***radiation efficiency*** e as defined by the IEEE Standards)

- The overall efficiency:

$$et = e_r e_{cd} e_d \qquad [1.29]$$

e_t = total efficiency (dimensionless)

e_r = reflection = $e_r = 1 - |\Gamma|^2$ (mismatch) efficiency (dimensionless)

e_{cd} $e_{cd} = \dfrac{R_{rad}}{R_{cd} + R_{rad}}$ = conduction efficiency (dimensionless)

$$R_{cd} = \frac{l}{2\pi b}\sqrt{\frac{\omega\mu_o}{2\sigma}}$$

e_d = dielectric efficiency (dimensionless)

Γ = voltage reflection coefficient at the input terminals of the antenna.

ANTENNA EFFICIENCY

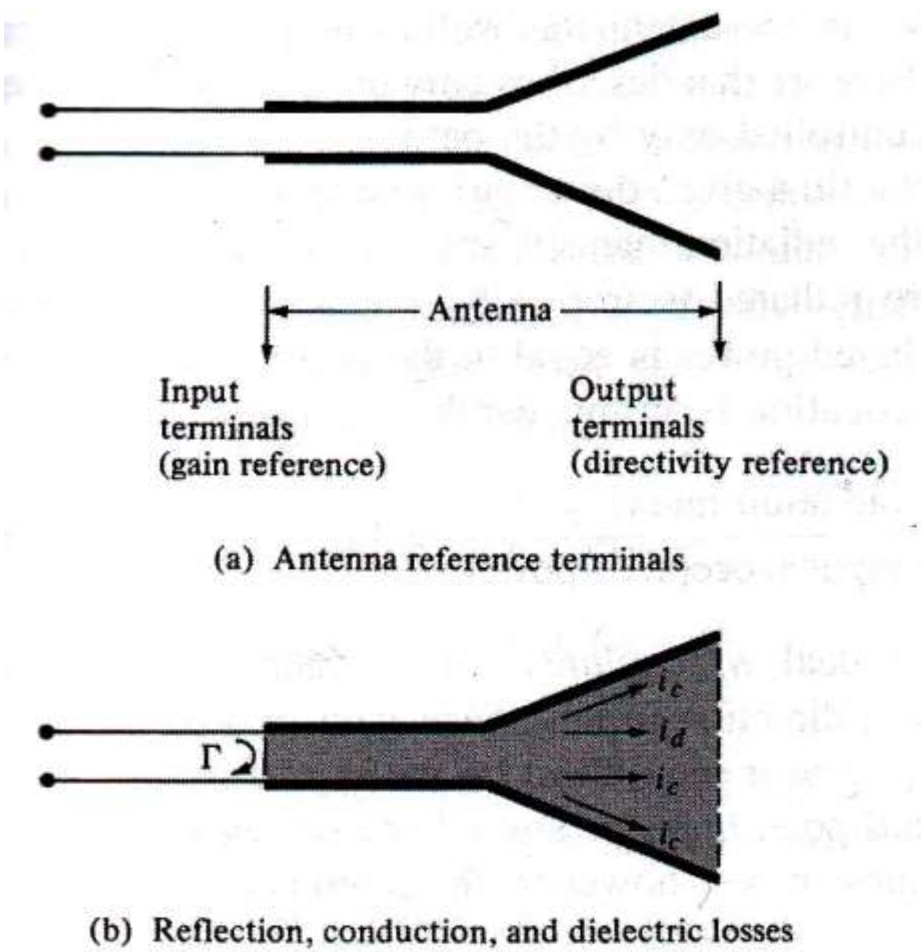


Figure 1.16: Reference terminals and losses of an antenna

ANTENNA GAIN

The gain G of an antenna is the ratio of the radiation intensity *U* in a given direction and the radiation intensity that would be obtained, if the power fed to the antenna were radiated isotropically.

$$G(\theta,\phi) = 4\pi \frac{U(\theta,\phi)}{P_{in}} \qquad [1.30]$$

in most cases, we deal with *relative gain* which defined as "the ratio of the power gain in a given direction to the power gain of a reference antenna in its referenced direction". The power input must be same for both antennas. The reference antenna usually a dipole, horn, or any other antenna whose gain can be calculated or it is known.

ANTENNA GAIN

- In most cases, the reference antenna is a *lossless isotropic source*.
- *When the direction is not stated, the power gain is usually taken in the direction of maximum radiation*.

A half-wavelength dipole antenna, with an input impedance of 73Ω is to be connected to a generator and transmission line with an output impedance of 50Ω. Assume the antenna is made of copper wire 2.0 mm in diameter and the operating frequency is 10.0 GHz. Assume the radiation pattern of the antenna is

$$U(\theta,\phi) \approx B_o \sin^3(\theta)$$

Find the overall gain of this antenna

SOLUTION

First determine the directivity of the antenna

$$D(\theta,\varphi) = 4\pi \frac{U(\theta,\varphi)}{P_{rad(tot)}}$$

$$D(\theta,\varphi) = 4\pi \frac{B_o \sin^3(\theta)}{B_0\left(\frac{3\pi^2}{4}\right)} = \frac{16}{3\pi}\sin^3(\theta)$$

$$D_0 = D_{\max} = \frac{16}{3\pi} = 1.697$$

Next step is to determine the efficiencies

$$e_t = e_r e_{cd}$$

$$e_r = (1 - |\Gamma|^2) = (1 - \left|\frac{73-50}{73+50}\right|^2) = 0.965$$

$$e_{cd} = \frac{R_{rad}}{R_{cd} + R_{rad}}$$

$$R_{cd} = \frac{l}{2\pi b}\sqrt{\frac{\omega\mu_o}{2\sigma}} = \frac{0.015}{2\pi(0.001)}\sqrt{\frac{2\pi 10\cdot 10^9 \cdot 4\pi 10^{-7}}{2\cdot 5.7\cdot 10^7}} = 0.0628\Omega$$

$$e_{cd} = \frac{73}{73+0.0628} = 0.9991$$

$$e_t = e_r e_{cd} = 0.965\cdot 0.9991 = 0.964$$

Next step is to determine the gain

$$G(\theta,\varphi) = e_r e_{cd} D(\theta,\varphi)$$

$$G(\theta,\varphi) = 0.964\frac{16}{3\pi}\sin^3(\theta)$$

$$G_0 = G_{max} = 0.964\frac{16}{3\pi} = 1.636$$

$$G_0(dB) = 10\log_{10}(1.636) = 2.14\,dB$$

BEAM EFFICIENCY

- The ***beam efficiency*** is *the ratio of the power radiated in a cone of angle $2\theta_1$ and the total radiated power*. The angle $2\theta_1$ can be generally any angle, but usually this is the first-null beam width.

$$BE = \frac{\int_0^{2\pi}\int_0^{\Theta_1} U(\theta,\varphi)\sin\theta \, d\theta \, d\varphi}{\int_0^{2\pi}\int_0^{\pi} U(\theta,\varphi)\sin\theta \, d\theta \, d\varphi} \qquad [1.31]$$

FREQUENCY BANWIDTH

- This is the *range of frequencies, within which the antenna characteristics* (input impedance, pattern) conform to certain specifications.
- Antenna characteristics, which should conform to certain requirements, might be: input impedance, radiation pattern, beamwidth, polarization, side-lobe level, gain, beam direction and width, radiation efficiency. Usually, separate bandwidths are introduced: impedance bandwidth, pattern bandwidth, etc.

FREQUENCY BANWIDTH

- The FBW of broadband antennas is expressed as the ratio of the upper to the lower frequencies, where the antenna performance is acceptable:

$$FBW = \frac{f_{\max}}{f_{\min}} \qquad [1.32]$$

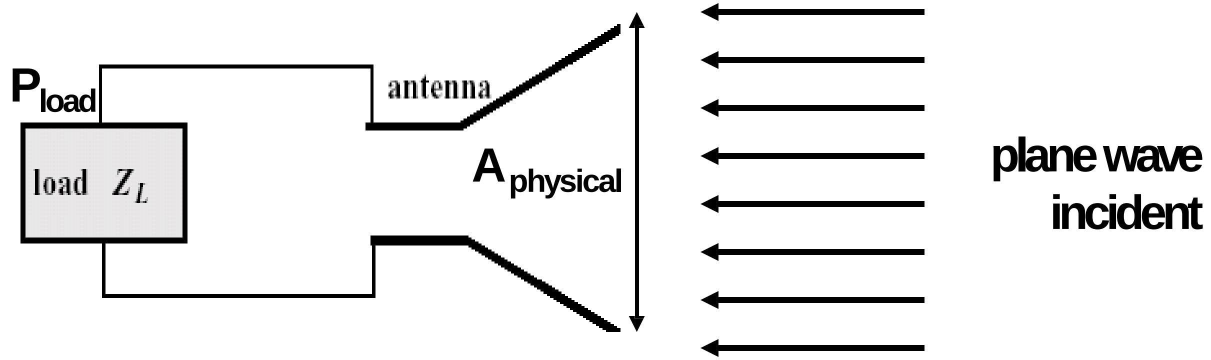
For narrowband antennas, the FBW is expressed as a percentage of the frequency difference over the center frequency:

$$FBW = \frac{f_{\max} - f_{\min}}{f_0} \times 100\% \qquad [1.33]$$

$$f_0 = \left(f_{\max} + f_{\min}\right)/2 \qquad f_0 = \sqrt{f_{\max}\, f_{\min}}$$

Effective Aperture

The ***effective antenna aperture*** is the ratio of the available power at the terminals of the antenna to the power flux density of a plane wave incident upon the antenna, which is polarization matched to the antenna. If there is no specific direction chosen, the direction of maximum radiation intensity is implied.



Antenna Effective Area A_e

- Measure of the effective absorption area presented by an antenna to an incident plane wave.
- Depends on the antenna gain and wavelength

$$A_e = \frac{\lambda^2}{4\pi} G(\theta, \varphi) \ [\mathrm{m}^2]$$

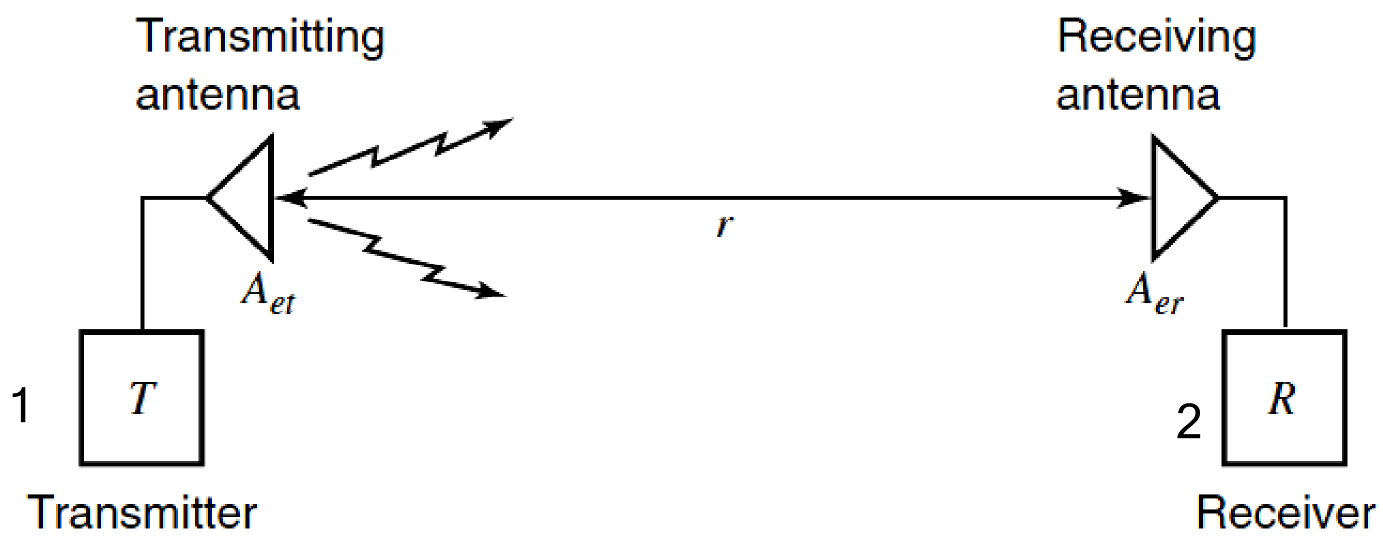
Aperture efficiency: $\varepsilon_{ap} = A_e / A_p$

A_p: physical area of antenna's aperture, square meters

THE RADIO COMMUNICATION LINK

The power received over a radio communication link " as showing in the figure below" by assuming lossless, matched antennas, "PLF=1" let the transmitter feed a power *Pt to* a transmitting antenna of effective aperture A_{et}.

At a distance *r a receiving antenna of* effective aperture A_{er} *Intercepts some of the power radiated by the transmitting antenna* and delivers it to the receiver *R.*



Assuming the transmitting antenna is isotropic, the power per unit area available at the receiving antenna is

$$S_r = \frac{P_t}{4\pi r^2} \qquad (\mathrm{W})/_{m2}$$

If the antenna has gain G_t *yield*

$$S_r = \frac{P_t G_t}{4\pi r^2} \qquad (\mathrm{W})/_{m2}$$

So power collected by the lossless, matched receiving antenna of effective aperture A_{er} *is*

$$P_r = S_r A_{er} = \frac{P_t G_t A_{er}}{4\pi r^2} \qquad (\mathrm{W})$$

The gain of the transmitting antenna can be expressed as

$$G_t = \frac{4\pi A_{et}}{\lambda^2}$$

Substituting this G_t in P_r yields the *Friis transmission formula*

$$\frac{P_r}{P_t} = \frac{A_{er} A_{et}}{r^2 \lambda^2} \qquad \text{(dimensionless)} \qquad \textbf{\textit{Friis transmission formula}}$$

دجوي الو ةقباقتملا نيئاوهلا ةلاح يف ةلداعملا هذه مدختست فب مدع او
فارحنا

Or

$$\boxed{\frac{P_2}{P_1} = \left(\frac{\lambda}{4\pi r}\right)^2 G_1(\theta_1, \phi_1) G_2(\theta_2, \phi_2)}$$

ل θ, φ فارحنا دجوي اذا هدختست (ةيوازب)
ةلادك يئاوهلا وا ماطسملا لسرملا

Where where *r, θ1, φ1 are the local spherical coordinates for antenna 1*
And *θ2, φ2 are the local spherical coordinates for antenna 2*

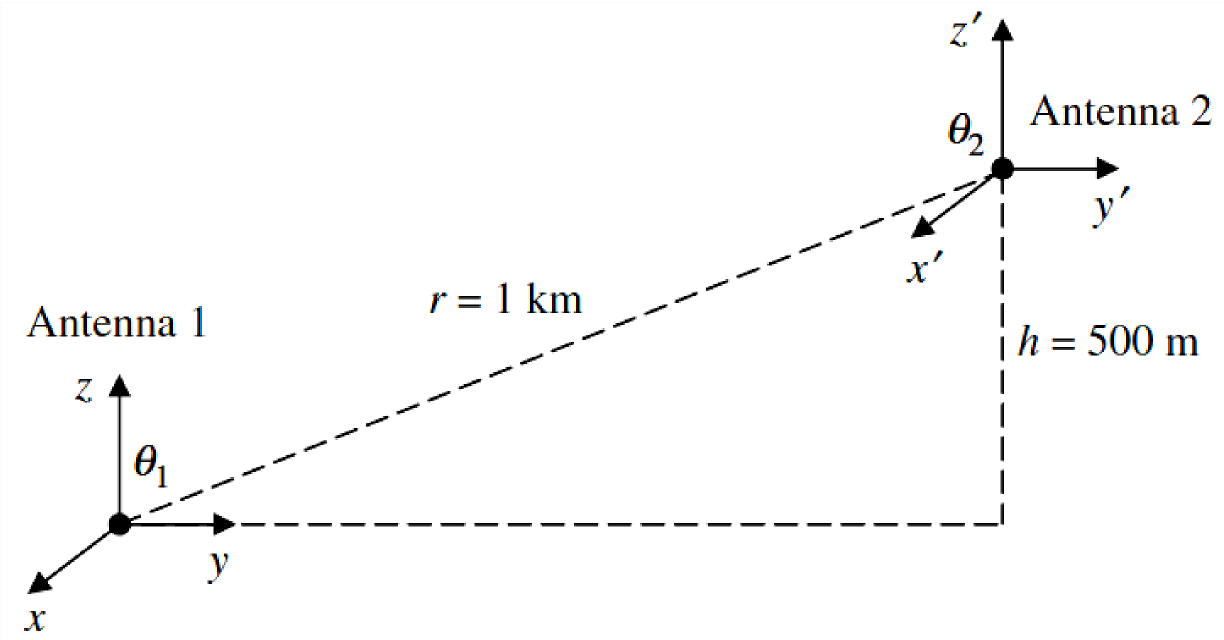
where

P_r = received power, W
P_t = transmitted power, W
A_{et} = effective aperture of transmitting antenna, m^2
A_{er} = effective aperture of receiving antenna, m^2
r = distance between antennas, m
λ = wavelength, m

بلكلا لاثم عجار 2-11.1

Example

Calculate the receive power *P2 at antenna 2 if the transmi* power *P_1 = 1 kW. The transmitter gain is* $G_1(\theta_1, \phi_1) = \sin^2(\theta_1)$ *and the* receiver gain is $G_2(\theta_2, \phi_2) = \sin^2(\theta_2)$ *The operating frequency is 2 GHz. Use* the Fig. below to solve the problem. The *y-z and y-z planes are coplanar*



Solution

The geometry indicates $\theta_1 = 60^\circ$ and $\theta_2 = 120^\circ$.

$$P_2 = P_1 \left(\frac{\lambda}{4\pi r}\right)^2 G_1(\theta_1, \phi_1) G_2(\theta_2, \phi_2)$$

$$= P_1 \left(\frac{\lambda}{4\pi r}\right)^2 \sin^2(\theta_1) \sin^2(\theta_2)$$

$$= 10^3 \left(\frac{15 \text{ cm}}{4\pi 10^3}\right)^2 \sin^2(60) \sin^2(120)$$

$$= 80.1 \text{ nW}$$

ملاحظة في حالة ال gain معطى بالرقم dB مثلا (6 dB) يجب تحويله الى الديسمل حتى تستطيع استخدامه في معادلة Friis

2-11-1. Received power and the Friis formula.

What is the maximum power received at a distance of 0.5 km over a free-space 1 GHz circuit consisting of a transmitting antenna with a 25 dB gain and a receiving antenna with a 20 dB gain? The gain is with respect to a lossless isotropic source. The transmitting antenna input is 150 W.

***Solution*:**

$$\lambda = c/f = 3\times 10^8/10^9 = 0.3\text{ m}, \qquad A_{et} = \frac{D_t\lambda^2}{4\pi}, \qquad A_{er} = \frac{D_r\lambda^2}{4\pi}$$

$$P_r = P_t\frac{A_{et}A_{er}}{r^2\lambda^2} = P_t\frac{D_t\lambda^2 D_r\lambda^2}{(4\pi)^2 r^2\lambda^2} = 150\,\frac{316\times 0.3^2\times 100}{(4\pi)^2\,500^2} = 0.0108\text{ W} = 10.8\text{ mW}$$

H.W

Q1

Two lossless, polarization-matched antennas are aligned for maximum radiation between them, and are separated by a distance of 50λ. The antennas are matched to their transmission lines and have directivities of 20 dB. Assuming that the power at the input terminals of the transmitting antenna is 10 W, find the power at the terminals of the receiving antenna.

Q2

Repeat Problem 1 for two antennas with 30 dB directivities and separated by 100λ. The power at the input terminals is 20 W.

Q3

Transmitting and receiving antennas operating at 1 GHz with gains of 20 and 15 dB, respectively, are separated by a distance of 1 km. Find the power delivered to the load when the input power is 150 W. Assume the PLF = 1.

Hertzian Dipole

Is an infinitesimal current element (*I dl*). *Although such a* current element does not exist in real life

طول الهوائي

$$\ell \ll \lambda$$

الطول الموجي
للتردد الذي
يعمل به الهوائي

max current

$$I(z) = I = I_o e^{j\omega t}$$

$$A_z = \frac{\mu}{4\pi r} \int_v \frac{J(t)}{} \, dv$$

$J(t)$: Is the current density at time (t) in the volume dv

where

So

$$A_z = \int_{l} \frac{\mu I_o e^{j\omega t} \, dl}{4\pi r} \Rightarrow \int_{-\frac{L}{2}}^{\frac{L}{2}}$$

التكامل على طول الهوائي

$\because A_\theta = -A_z \sin\theta$, $A_r = A_z \cos\theta$

$A_\phi = 0$

اويرمز لها (dl)

$$\therefore A_\theta = \frac{-\mu I_o (L)}{4\pi r} e^{j\omega t} * \sin\theta$$

$$A_r = \frac{\mu I_o L}{4\pi r} e^{j\omega t} * \cos\theta$$

$$A_z = \int_{v} \frac{\mu}{4\pi r} J(t - \frac{r}{c}) \, dV$$

where $A = \int \frac{\mu I [t - r/c] \, dL}{4\pi r}$

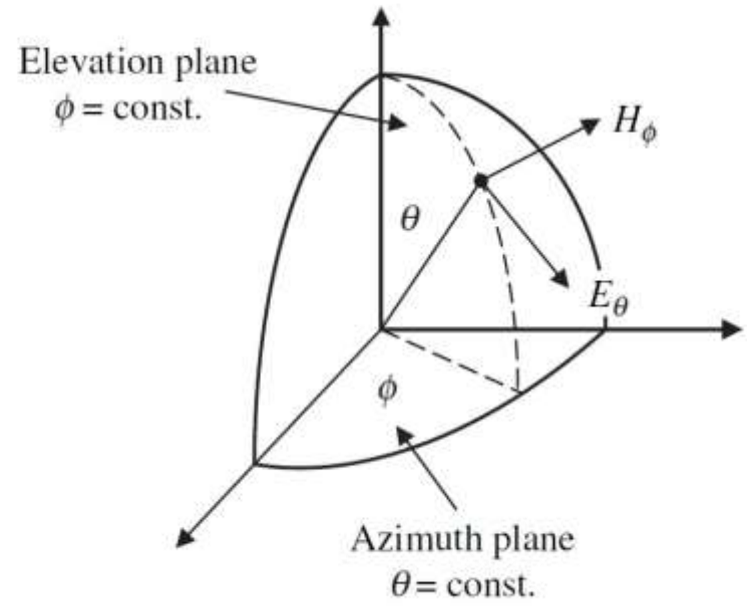
retarded time

is retarded vector magnetic potential

$$\therefore A_\theta = \frac{-\mu I_o L}{4\pi r} e^{j\omega (t - \frac{r}{c})} \cdot \cos\theta$$

And

$$A_r = \frac{\mu I_o L}{4\pi r} e^{j\omega(t - \frac{r}{c})} \cdot \cos\theta$$



$\because$ $B = \nabla \times A$, where $B = \mu H \Rightarrow H = \frac{1}{\mu} B$

$\therefore$ $H = \frac{1}{\mu} \nabla \times A$

$$H = \frac{1}{\mu} \begin{vmatrix} \frac{a_r}{r^2 \sin\theta} & \frac{a_\theta}{r \sin\theta} & \frac{a_\phi}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin\theta A_\phi \end{vmatrix}$$

sph. Coor.

Since $H_r = H_\theta = 0$

$$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right)\hat{\boldsymbol{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol{\phi}}$$

H_ϕ

$H_r = H_\theta = 0$

$$H\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(\frac{rA_\theta}{\mu}\right) - \frac{\partial}{\partial\theta}\left(\frac{A_r}{\mu}\right)\right]$$

$$= \frac{1}{r}\left[\frac{1}{4\pi}\left(-jL \sin\theta\, I_o\, e^{j\omega(t-\frac{r}{c})} * \left(\frac{-w}{c}\right)\right) + \frac{1}{4\pi r}\left(I_o L e^{j\omega(t-\frac{r}{c})} * \sin\theta\right)\right]$$

$\therefore$

$$H\phi = \frac{I_o L \sin\theta}{4\pi}\left[\frac{j\omega\, e^{j(t-\frac{r}{c})}}{cr} + \frac{e^{j\omega(t-\frac{r}{c})}}{r^2}\right]$$

The first term is the radiation field and the second term is the induction field. So the radiation field only $\frac{1}{r}$ term

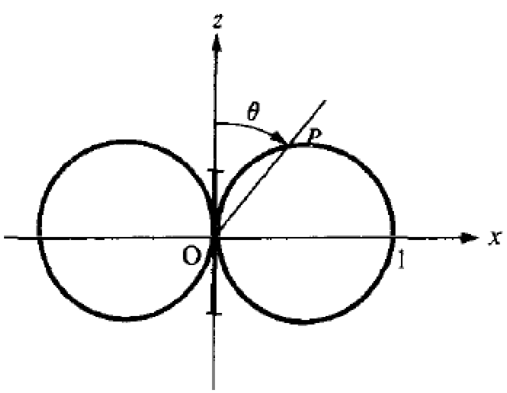
$$\boxed{H\phi = \frac{j I_o * L \sin\theta\, \beta}{2\lambda r}\, e^{j\omega(t-\frac{r}{c})} \quad \cdots\cdots\cdots (1)}$$

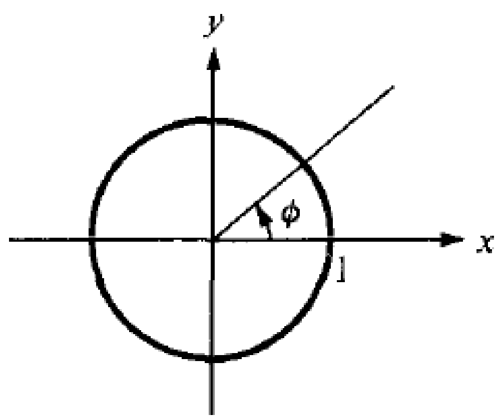
Since $E_\theta = \eta * H_\phi = 120\pi H_\phi$

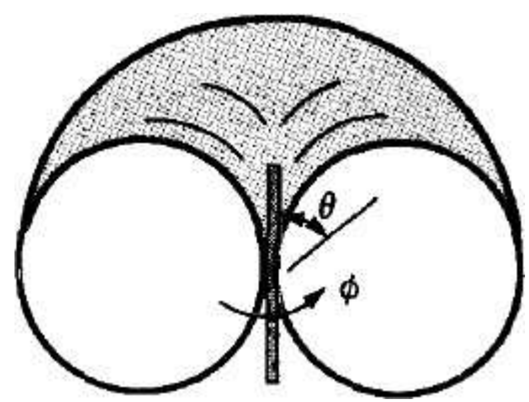
$$\therefore \quad E_\theta = \frac{j60\pi I_o L \beta \sin\theta}{\lambda r} e^{j\omega(t - \frac{r}{c})}$$

$$\therefore \boxed{|E_\theta| = \frac{\beta 60\pi I_o L \sin\theta}{\lambda r}} \quad \text{- - - -} \quad V/m \quad \text{- - -} \quad (2)$$

Where $E = E_m \sin\theta \Rightarrow \underbrace{E_m}_{max} = \frac{60\pi I_o L}{\lambda r}$







Power Radiated and radiation resistance of Hertzian Dipole

The time-average power density is obtained as

$$\mathcal{P}_{ave} = \frac{1}{2}\,\mathrm{Re}\,(\mathbf{E}_s \times \mathbf{H}_s^*) = \frac{1}{2}\,\mathrm{Re}\,(E_{\theta s} H_{\phi s}^* \,\mathbf{a}_r)$$
$$= \frac{1}{2}\,\eta |H_{\phi s}|^2 \,\mathbf{a}_r$$

Substituting $H\phi$ yields the time-average radiated power as

$$P_{rad} = \int \mathcal{P}_{ave} \cdot d\mathbf{S}$$
$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{I_o^2 \eta \beta^2 \,dl^2}{32\pi^2 r^2} \sin^2\theta \; r^2 \sin\theta \, d\theta \, d\phi$$
$$= \frac{I_o^2 \eta \beta^2 \,dl^2}{32\pi^2}\, 2\pi \int_0^{\pi} \sin^3\theta \, d\theta$$

$$\boxed{\begin{aligned}\int_0^{\pi} \sin^3\theta \, d\theta &= \int_0^{\pi} (1 - \cos^2\theta)\, d(-\cos\theta) \\ &= \left.\frac{\cos^3\theta}{3} - \cos\theta \right|_0^{\pi} = \frac{4}{3}\end{aligned}}$$

Where $\beta^2 = 4\pi^2/\lambda^2$ So the power radiated is become

$$P_{rad} = \frac{I_o^2 \pi\eta}{3}\left[\frac{dl}{\lambda}\right]^2$$

If free space is the medium of propagation $\eta = 120\pi$ $\Rightarrow$ $P_{rad} = 40\pi^2\left[\frac{dl}{\lambda}\right]^2 I_o^2$

This power is equivalent to the power dissipated in a fictitious resistance Rrad by Current

وهمية

$I = I_o \cos wt$ *that is* $P_{rad} = I_{rms}^2 R_{rad}$

$$\boxed{P_{rad} = \frac{1}{2} I_o^2 R_{rad}} \Leftrightarrow \boxed{P_{rad} = 40\pi^2\left[\frac{dl}{\lambda}\right]^2 I_o^2}$$

where I rms is the root-mean-square value of I .

$$R_{rad} = \frac{2P_{rad}}{I_o^2}$$

So

$$R_{\text{rad}} = 80\pi^2 \left[\frac{dl}{\lambda}\right]^2$$

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Hertzain Dipole ةمواقم لاسرلاا ل

The resistance ***Rrad*** is a characteristic property of the Hertzian dipole antenna and is called its **radiation resistance**

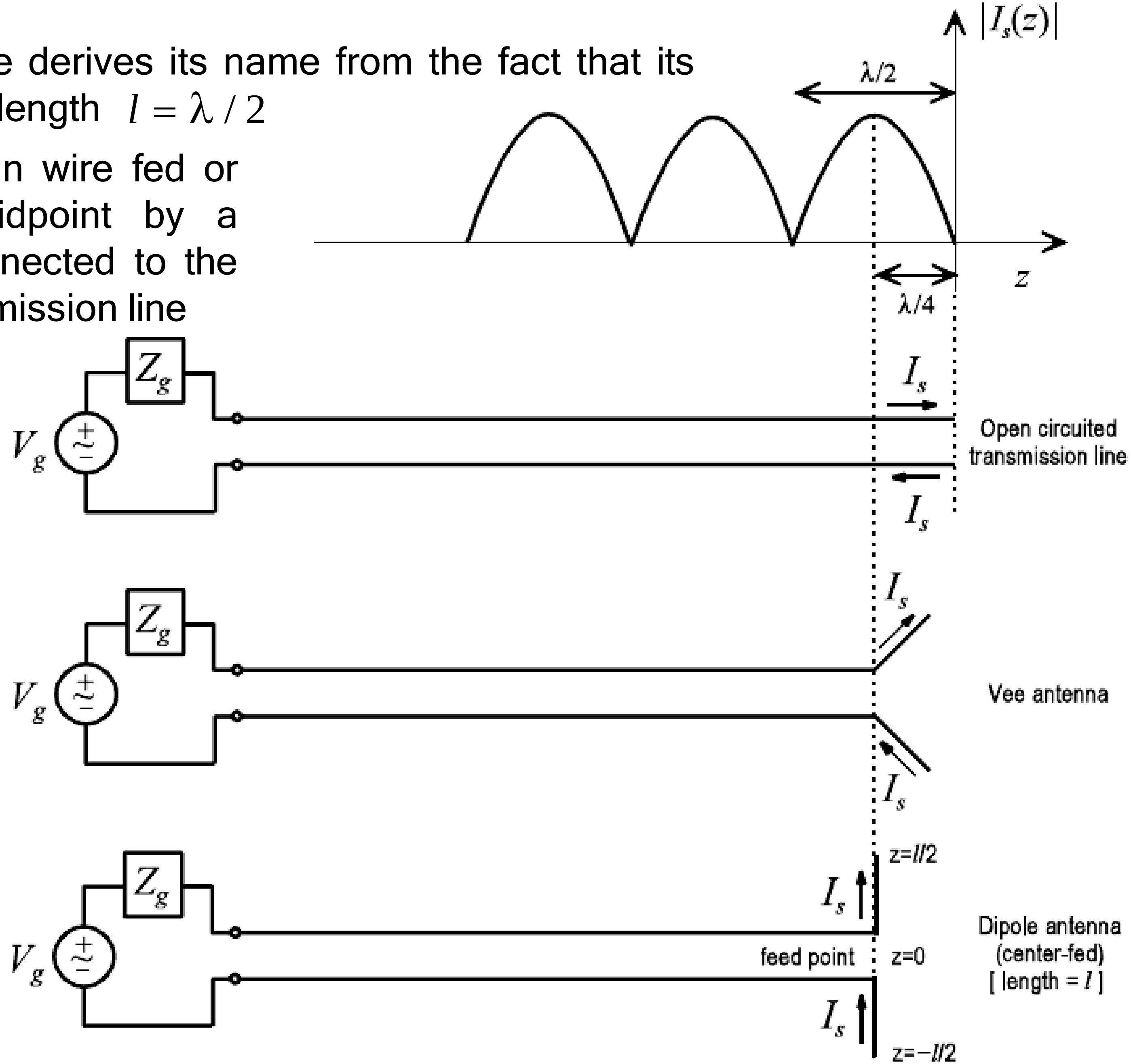
Note that the Hertzian dipole is assumed to be infinitesimally small $dl \leq \lambda/10$

لا طرش **Hertzian dipole** ىلا دحاو لقا هلوطنوكي نا
يجوملا لوطلا نم ةرشع

Half wave Dipole

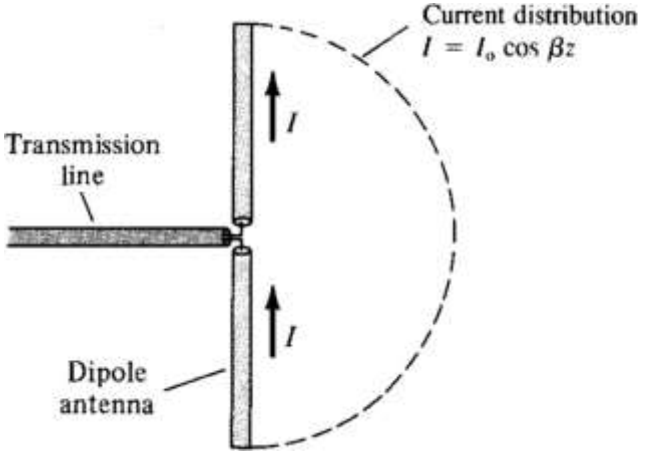
The half-wave dipole derives its name from the fact that its length is half a wavelength $l = \lambda / 2$

-It consists of a thin wire fed or excited at the midpoint by a voltage source connected to the antenna via a transmission line



The field due to the dipole can be easily obtained if we consider it as consisting of a chain of Hertzian dipoles

The magnetic vector potential at ***P*** due to a differential length *dl(= dz)* of the dipole carrying a phasor current $I_s = I_o \cos \beta z$



$$dA_{zs} = \frac{\mu I_o \cos \beta z \, dz}{4\pi r'} e^{-j\beta r'}$$

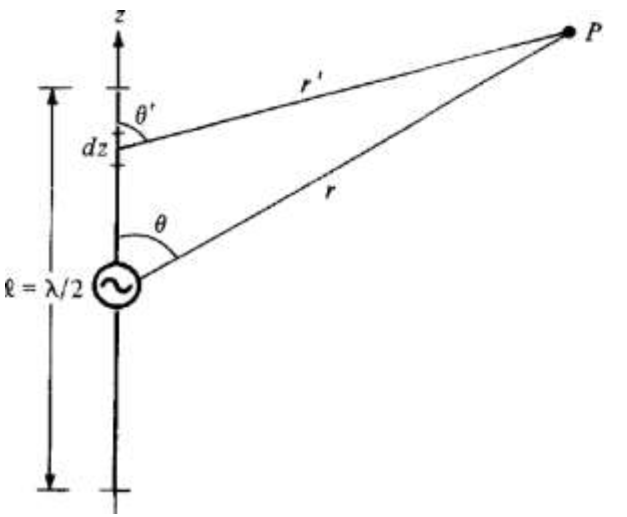
if

$$r >> l$$

$$r - r' = z \cos \varphi$$

$$or$$

$$r' = r - z \cos \varphi$$



So, we may suppose $r' \approx r$ in the denominator where the magnitude of the distance is needed.

For the phase term in the numerator the difference between *B*r and *B*r' is significant, so we replace r' by (r - z cosθ) not r.

So we maintain the cosine term in the exponent while neglecting it in the denominator

$$A_{zs} = \frac{\mu I_o}{4\pi r} \int_{-\lambda/4}^{\lambda/4} e^{-j\beta(r - z\cos\theta)} \cos \beta z \, dz$$

$$= \frac{\mu I_o}{4\pi r} e^{-j\beta r} \int_{-\lambda/4}^{\lambda/4} e^{j\beta z \cos\theta} \cos \beta z \, dz$$

By this propriety

$$\int e^{az} \cos bz \, dz = \frac{e^{az}(a \cos bz + b \sin bz)}{a^2 + b^2}$$

$$A_{zs} = \frac{\mu I_o e^{-j\beta r} e^{j\beta z \cos\theta}}{4\pi r} \frac{(j\beta \cos\theta \cos \beta z + \beta \sin \beta z)}{-\beta^2 \cos^2\theta + \beta^2} \Bigg|_{-\lambda/4}^{\lambda/4}$$

Since $\beta = 2\pi/\lambda$ or $\beta\, \lambda/4 = \pi/2$ and $-\cos^2\theta + 1 = \sin^2\theta$,

$$A_{zs} = \frac{\mu I_o e^{-j\beta r}}{4\pi r\beta^2 \sin^2\theta}[e^{j(\pi/2)\cos\theta}(0+\beta) - e^{-j(\pi/2)\cos\theta}(0-\beta)]$$

Using the identity $e^{jx} + e^{-jx} = 2\cos x$, we obtain

$$A_{zs} = \frac{\mu I_o e^{-j\beta r}\cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r\beta \sin^2\theta}$$

Since

$$\mathbf{B}_s = \mu\mathbf{H}_s = \nabla \times \mathbf{A}_s \text{ and } \nabla \times \mathbf{H}_s = j\omega\varepsilon\mathbf{E}_s$$

$$\boxed{H_{\phi s} = \frac{jI_o e^{-j\beta r}\cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r\sin\theta}, \qquad E_{\theta s} = \eta H_{\phi s}}$$

Notice that the radiation term of $H_{\phi s}$ and $E_{\theta s}$ are in time phase and orthogonal

Power Calculation

$$\mathcal{P}_{\text{ave}} = \frac{1}{2}\eta\,|H_{\phi s}|^2\,\mathbf{a}_r$$

$$= \frac{\eta I_o^2 \cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{8\pi^2 r^2 \sin^2\theta}\,\mathbf{a}_r$$

The time-average radiated power can be determined as

$$P_{\text{rad}} = \int \mathcal{P}_{\text{ave}} \cdot d\mathbf{S}$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{\eta I_o^2 \cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{8\pi^2 r^2 \sin^2\theta}\, r^2 \sin\theta\, d\theta\, d\phi$$

$$= \frac{\eta I_o^2}{8\pi^2}\, 2\pi \int_0^{\pi} \frac{\cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}\, d\theta$$

For free space $\eta = 120\pi$

$$= 30\, I_o^2 \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\, d\theta$$

Since $$\int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\, d\theta = \int_{\pi/2}^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\, d\theta$$

So

$$P_{\text{rad}} = 60 I_o^2 \int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right) d\theta}{\sin\theta}$$

$x2$

Changing variables, $u = \cos\theta$, and using partial fraction reduces

$$P_{\text{rad}} = 60 I_o^2 \int_0^1 \frac{\cos^2 \frac{1}{2}\pi u}{1 - u^2}\, du$$

$$\frac{1}{1-u^2} = \frac{1}{2}\left(\frac{1}{1+u} + \frac{1}{1-u}\right)$$

$$= 30 I_o^2 \left[\int_0^1 \frac{\cos^2 \frac{1}{2}\pi u}{1 + u}\, du + \int_0^1 \frac{\cos^2 \frac{1}{2}\pi u}{1 - u}\, du \right]$$

Replacing $1 + u$ with v in the first integrand and $1 - u$ with v in the second results

$$P_{\text{rad}} = 30I_o^2 \left[\int_0^1 \frac{\sin^2 \frac{1}{2}\pi v}{v}\, dv + \int_1^2 \frac{\sin^2 \frac{1}{2}\pi v}{v}\, dv \right]$$

$$\cos(90-\theta) = \sin\theta$$

$$\cos(90+\theta) = -\sin\theta$$

$$= 30I_o^2 \int_0^2 \frac{\sin^2 \frac{1}{2}\pi v}{v}\, dv$$

Changing variables, $w = \pi v$, yields

$$P_{\text{rad}} = 30I_o^2 \int_0^{2\pi} \frac{\sin^2 \frac{1}{2} w}{w}\, dw$$

$$\sin^2\theta = \frac{1}{2}\left(1-\cos(2\theta)\right)$$

$$= 15I_o^2 \int_0^{2\pi} \frac{(1 - \cos w)}{w}\, dw$$

$$= 15I_o^2 \int_0^{2\pi} \left[\frac{w}{2!} - \frac{w^3}{4!} + \frac{w^5}{6!} - \frac{w^7}{8!} + \cdots \right] dw$$

$$= 15I_o^2 \int_0^{2\pi} \left[\frac{w}{2!} - \frac{w^3}{4!} + \frac{w^5}{6!} - \frac{w^7}{8!} + \cdots \right] dw$$

$$\text{since } \cos w = 1 - \frac{w^2}{2!} + \frac{w^4}{4!} - \frac{w^6}{6!} + \frac{w^8}{8!} - \cdot$$

$$P_{\text{rad}} = 15I_o^2 \left[\frac{(2\pi)^2}{2(2!)} - \frac{(2\pi)^4}{4(4!)} + \frac{(2\pi)^6}{6(6!)} - \frac{(2\pi)^8}{8(8!)} + \cdots \right]$$

$$\simeq 36.56\, I_o^2$$

$$\boxed{R_{\text{rad}} = \frac{2P_{\text{rad}}}{I_o^2} = 73\ \Omega}$$

Note the significant increase in the radiation resistance of the half-wave dipole over that of the Hertzian dipole

The total input impedance Zin of the antenna is the impedance seen at the terminals of the antenna and is given by

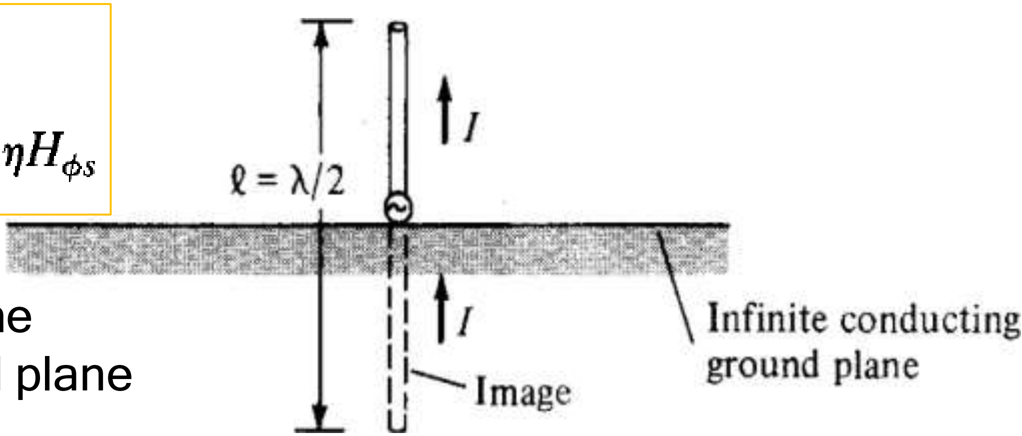
$$Z_{in} = R_{in} + jX_{in} \Longrightarrow Z_{in} = 73 + j42.5\ \Omega \text{ for a dipole length } \ell = \lambda/2$$

QUARTER-WAVE MONOPOLE ANTENNA $l = \lambda / 4$

The quarter-wave monopole antenna consists of one-half of a half-wave dipole antenna located on a conducting ground plane as in Figure .The monopole antenna is perpendicular to the plane, which is usually assumed to be infinite and perfectly conducting. It is fed by a coaxial cable connected to its base

Where

$$H_{\phi s} = \frac{jI_o e^{-j\beta r} \cos\left(\frac{\pi}{2} \cos\theta\right)}{2\pi r \sin\theta}, \qquad E_{\theta s} = \eta H_{\phi s}$$



The integration of monopole only over the hemispherical surface above the ground plane
$0 < \theta < \pi/2$

Hence, the monopole radiates only half as much power as the dipole with the same current, so

$$P_{\mathrm{rad}} \simeq 18.28\, I_{\mathrm{o}}^{2}$$

$$R_{\mathrm{rad}} = \frac{2P_{\mathrm{rad}}}{I_{\mathrm{o}}^{2}}$$

$$\boxed{R_{\mathrm{rad}} = 36.5\ \Omega}$$

Where the total input impedance for a λ/4 monopole

$$Z_{\mathrm{in}} = 36.5 + j21.25\ \Omega$$