MODULE 2: FLUID KINEMATICS AND DYNAMICS

INTRODUCTION

Fluid kinematics refers to the features of a fluid in motion. It only deals with the motion of fluid particles without taking into account the forces causing the motion. Considerations of velocity, acceleration, flow rate, nature of flow and flow visualization are taken up under fluid kinematics.

A fluid motion can be analyzed by one of the two alternative approaches, called **Lagrangian** and **Eulerian**.

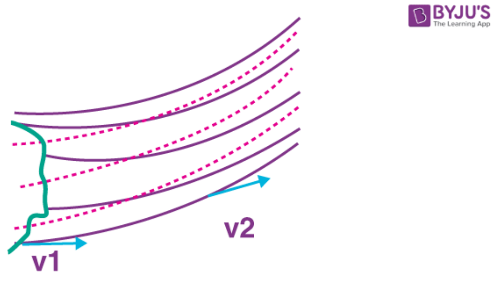
In Lagrangian approach, a particle or a fluid element is identified and followed during the course of its motion with time.

Difficulty in tracing a fluid particle (s) makes it nearly impossible to apply the Lagrangian approach. The alternative approach, called Eulerian approach consists of observing the fluid by setting up fixed stations (sections) in the flow field.

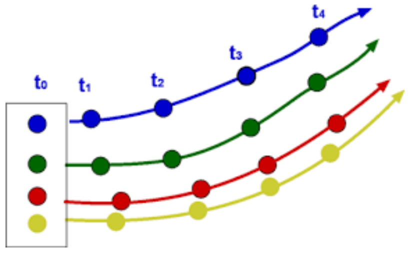
Types of lines:

There are three types of lines such as stream line, path line and streak line.

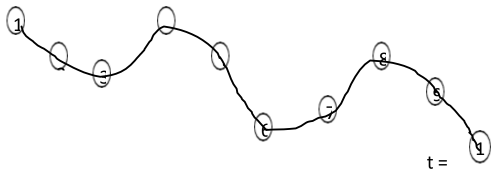
Streamlines are defined as the path taken by particles of fluid under steady flow conditions. If we represent the flow lines as curves, then the tangent at any point on the curve gives the direction of the fluid velocity at that point.



Pathlines are the trajectories that individual fluid particles follow. These can be thought of as "recording" the path of a fluid element in the flow over a certain period. The direction the path takes will be determined by the streamlines of the fluid at each moment in time.



Streak line is that imaginary line that connects all the fluid particles that has gone through a point/section over a period of time in a fluid



motion.

Stream tube:

It is an imaginary tube formed by stream line on its surface such that the flow only enters the tube   from one side and leaves it on the   other side only.   No flow takes place across the stream tube. This concept will help in the analysis of fluid motion.

Classification of flows

Steady and unsteady flows:

A flow is said to be steady if the properties (P) of the fluid and flow do not changewith time (t) at any section or point in a fluid flow is known as steady flow

A flow is said to be unsteady if the properties (P) of the fluid and flow change with time (t) at any section or point in a fluid flow is known as unsteady flow.

Uniform and non-uniform flows:

A flow is said to be uniform if the properties (P) of the fluid and flow do not change (with direction) over a length of flow considered along the flow at any instant.

A flow is said to be non-uniform if the properties (P) of the fluid and flow change (with direction) over a length of flow considered along the flow at any instant.

One, two and three dimensional flows:

Flow is said to be direction and will one-dimensional if the properties vary only along one axis / be constant with respect to other two directions of a three-

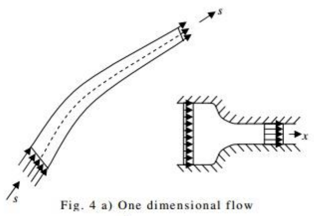


Fig. 4 a) One dimensional flow

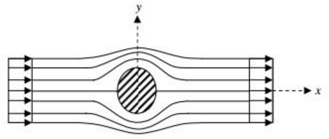


Fig. 4 b) Two dimensional flow

Flow is said to be ***two-dimensional*** if the properties vary only along two axes / directions and will be constant with respect to other direction of a three-dimensional axis system.

Flow is said to be ***three-dimensional*** if the properties vary along all the axes / directions of a three-dimensional axis system.

4. *Laminar and Turbulent flows:*

When the flow occurs like sheets or laminates and the fluid elements flowing in a layer does not mix with other layers, then the flow is said to be laminar. The Reynolds number (R_e) for the flow will be less than 2000.

$$R_e = \frac{\rho v D}{\mu}$$

When the flow velocity increases, the sheet like flow gets mixed up and the fluid elements mix with other layers there by causing turbulence. There will be eddy currents generated and flow reversal takes place. This flow is said to be Turbulent.

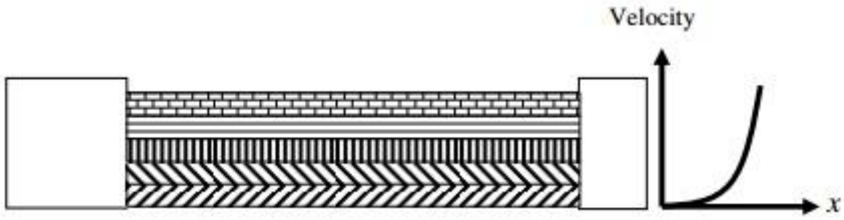


Fig. 5 Laminar flow

The Reynolds number for the flow will be greater than 4000.

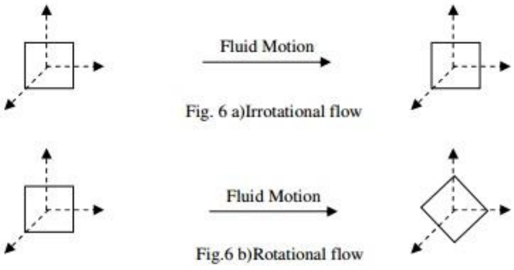
For flows with Reynolds number between 2000 to 4000 is said to be transition flow.

5. *Compressible and Incompressible flows:*

Flow is said to be ***Incompressible*** if the fluid density does not change (constant) along the flow direction and is ***Compressible*** if the fluid density varies along the flow direction

6. ***Rotational and Irrotational flows:***

Flow is said to be ***Rotational*** if the fluid elements does not rotate about their own axis as they move along the flow and is ***Rotational*** if the fluid elements rotate along their axis as they move along the flow direction.



7. Critical, Sub-critical and Super-critical flows:

Froude's Number

It is the ratio of the inertia forces to gravity forces and mathematically

$$F_e = \frac{V}{\sqrt{gd}},$$

If the Froude's number is ONE, the flow is ***critical***, Less than ONE, ***Sub-critical*** and Greater than ONE, ***Super-critical.***

Rate of flow or Discharge (Q):

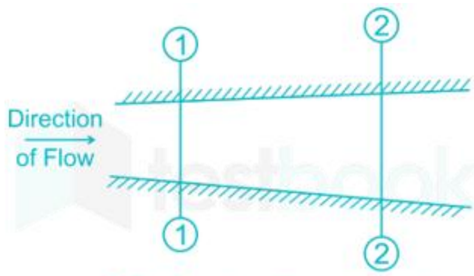
Rate of flow or discharge is said to be the quantity of fluid flowing per second across a section of a flow. Rate of flow can be expressed as mass rate of flow or volume rate of flow. Accordingly

Mass rate of flow = Mass of fluid flowing across a section / time Rate of flow = Volume of fluid flowing across a section / time

Equation of Continuity

The equation proves the law of conservation of mass in fluid dynamics. Also, if the fluid is incompressible, the density will remain constant for steady flow.

$$\rho A v = \text{constant}$$



Fluid flowing through a pipe

Velocity potential function and stream function are two scalar functions that help study whether the given fluid flow is rotational or irrotational. Both the functions provide a specific Laplace equation. The fluid flow can be rotational or irrotational flow based on whether it satisfies the Laplace equation or not.

Velocity potential function is a scalar function, whose negative derivative, with respect to any direction, gives the velocity component in that direction.

$$u = -\frac{\partial \varphi}{\partial x};\ v = -\frac{\partial \varphi}{\partial y};\ w = -\frac{\partial \varphi}{\partial z}; \quad \text{Eq.2}$$

Here, **u**, **v**, and **w** are the velocity components of the fluid flow along x, y, and z directions.

Stream function is a scalar function of space and time whose derivative with respect to any direction would give the velocity component at right angles to that direction. It is represented by ‘psi”, where

$$\Psi = f(x, y) \quad \text{Eq.8}$$

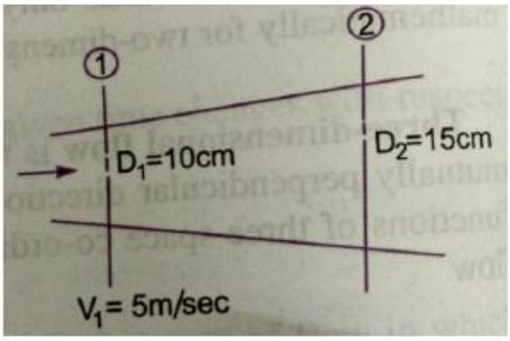
$$\frac{\delta \psi}{\delta x} = v;\ \frac{\delta \psi}{\delta y} = -u; \quad \text{Eq.9}$$

Flownet and its uses

A grid obtained by drawing a series of streamlines ψ and equipotential lines Φ is known as a flow net. Flow net provides a simple graphical technique for studying two-dimensional irrotational flows, when the mathematical calculation is difficult and cumbersome.

Flownet is used in analysis of seepage flow below the structure, to determine the uplift pressure and exit gradient.

1. The diameter of a pipe at sections 1 and 2 are 10 cm and 15cms respectively. Find the discharge through pipe, if the velocity of water flowing through the pipe at section 1 is 5m/sec. determine the velocity at section 2.



Given:

At section 1,

D_1 = 10cms = 0.1m

$$A_1 = \frac{\pi}{4}(D_1{}^2) = \frac{\pi}{4}(0.1)^2 = 0.007254\ m^2$$

V_1 = 5m/sec

At section 2, D_2 = 15cms =0.15m

$$A_2 = \frac{\pi}{4}(0.15)^2 = 0.01767\ m^2$$

Discharge through pipe $Q = A_1 \times V_1$

$= 0.007854 \times 5 = 0.03927$ m^3/sec

We have $A_1 V_1 = A_2 V_2$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.007854}{0.01767} \times 5.0 = 2.22 m/s$$

2. Water is flowing through a pipe of 5cm dia. Under a pressure of 29.43N/cm2 and with mean velocity of 2 m/sec. find the total head or total energy per unit weight of water at a cross-section, which is 5m above datum line.

Given: dia. Of pipe = 5cm = 0.05m

Pressure P = 29.43N/cm^2 = 29.43 x 10^{4}N/m^2
Velocity V = 2 m/sec
Datum head Z = 5m

Total head = Pressure head + Kinetic head + Datum head

$$\text{Pressure head} = \frac{p}{\rho g} = \frac{(29.43 \times 10^4)}{(1000 \times 9.81)} = 30m$$

$$\text{Kinetic head} = \frac{V^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204m$$

Datum head = Z = 5m

$$\frac{p}{\rho g} + \frac{V^2}{2g} + Z = 30 + 0.204 + 5 = 35.204m$$

Total head = 35.204m

FLUID DYNAMICS

The branch of science which deals with study of fluids in motion with considering forces is known as fluid dynamics. The various forces are considered for analysis of fluids in motion is gravity force, pressure force, viscous force, turbulent force and compressible force.

Euler's Equation of motion

Euler's equation of motion is founded on the fundamental premise of Newton's second law of motion. As a result, we can put the following equation here: Net force in the direction of S over the fluid element = mass of the fluid element x acceleration in the direction of S.

Bernoulli's Equation of motion

Assumptions made in the derivation of Bernoulli's equation:

(i). The fluid is ideal, i.e., Viscosity is zero. (ii). The flow is steady (iii). Te flow is incompressible. (iv). The flow is irrotational.

Bernoulli's theorem states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are:

Pressure Energy = $p / \rho g$

Kinetic energy = $v2 / 2g$

Datum Energy = z

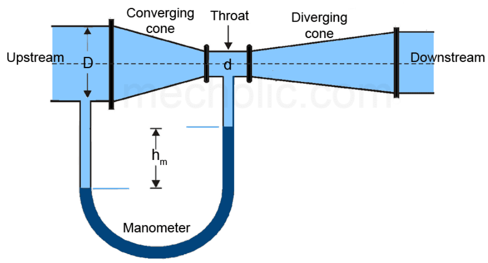
The mathematically, Bernoulli's theorem is written as

$$(p/w) + (v2 / 2g) + z = \text{Constant.}$$

Practical Applications of Bernoulli's theorem

- Venturimeter
- Orificemeter
- Pitot tube

A venturimeter is a measuring or also considered as a meter device that is usually used to measure the flow of a fluid in the pipe. A Venturi meter may also be used to increase the velocity of any type fluid in a pipe at any particular point. It basically works on the principle of Bernoulli's Theorem.



A Venturimeter consists of the following

- Converging cone or Diameter (the area is decreasing).
- Throat Diameter (the area is constant).
- Diverging cone (the area is increasing).

Converging Cone

When water flows through this cone the area is decreasing, therefore, the speed of flowing water increases and pressure decreases.

Throat Diameter

When water flows through this cone the area remains constant therefore the speed of flowing water and pressure remain constant.

Diverging Cone

When water flows through this cone the area is increasing, therefore, the speed of flowing water decreases and pressure decreases.

Types of Venturimeter

There are three different **types of venturimeter** which are as follows

- Horizontal Venturimeter
- Vertical Venturimeter
- Inclined Venturimeter

The formulae to calculate discharge through venturimeter is given below.

$$A_1 v_1 = A_2 v_2 \Rightarrow v_1 = \frac{A_2 v_2}{A_1}$$

Hence
$$h = \frac{v_2^{\,2}}{2g}\left[\frac{A_1^2 - A_2^2}{A_1^2}\right]$$

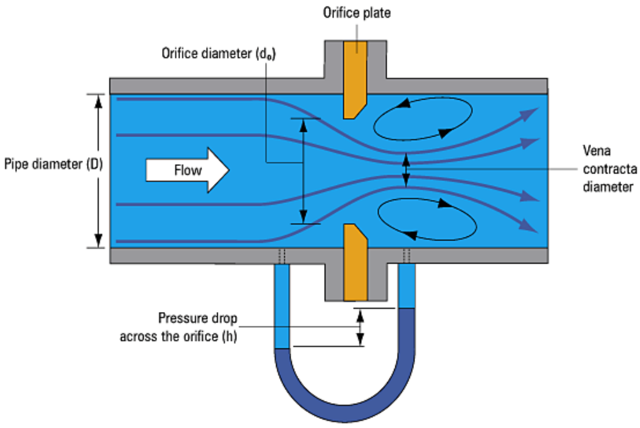
$$\Rightarrow v_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Discharge
$$Q = A_1 v_1 = A_2 h$$

$$\Rightarrow Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Orificemeter

An orificemeter is a piece of equipment used to measure the flow rate of a gas or a fluid. It mainly consists of an orifice plate, an orifice plate housing, and a meter tube.



Orificemeter consists of following four Parts:

1. Inlet Suction
2. Orifice Plate
3. Flow Conditioner and
4. Outlet section

The name inlet section means the fluid will enter into the orifice meter through the inlet section.

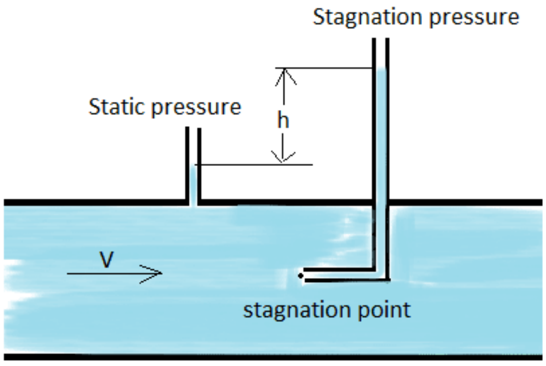
The orifice plate is situated between the inlet and outlet and the plate is used to generate pressure drop that will enable the flow rate. The orifice plate construction: It is thin size having one hole from that the water will pass.

The flow conditioner is used to increase the linear flow in the inlet section of the meter tube. The flow conditioner is installed nearly the inlet section of the meter tube

The outlet section, the pressure of the fluid is being discharged and determined.

Pitot tube

The Pitot static tube is mainly used for making temporary measurements of flow, although it is also used in some instances for permanent flow monitoring. It measures the local velocity of flow at a particular point within a pipe rather than the average flow velocity as measured by other types of flowmeter.



Pitot tube is frequently used in industrial settings to measure the velocity of liquids and gases. It is also used to measure the speed of an aircraft and a boat. Pitot tube is used to determine the local velocity at a specific point in the flow stream. It helps in calculating the pressure energy of a stopped fluid.

Pitot Tube operates on a fundamental principle of fluid mechanics, i.e. when a fluid stops flowing, all of its kinetic energy is converted into pressure energy.

Vortex flows

There are two types of vortices: free vortices and forced vortices. A free vortex is formed, for example, when water flows out of a vessel through a central hole in the base. No external force is required to rotate the fluid, and the degree of rotation is dependent upon the initial disturbance.

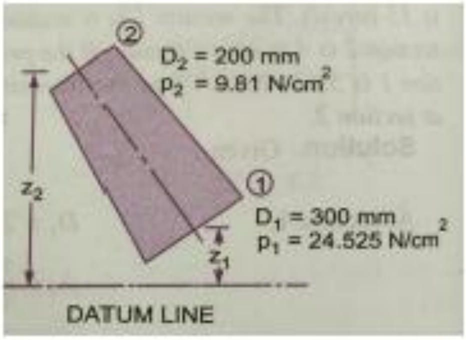
Momentum equation and its applications

The momentum equation is used to determine the resultant force exerted on the boundaries of a flow passage by a stream of flowing fluid as the flow changes its direction or the magnitude of velocity or both.

Following are the practical application of momentum equation:

(1) Flow though bend pipes
(2) Jet propulsion and propellers
(3) Fluid flow though stationary and moving plates or vanes.
(4) Non-uniform flow through sadden enlarged pipes.
(5) Hydraulic jump in open channels

Q1.The flow of water through a pipe, with diameters of 300mm at the bottom end and 200mm at the upper end, exhibits a pressure intensity of 24.525N/cm² at the bottom and 9.81N/cm² at the upper end. To ascertain the variance in datum head, given a flow rate of 40 liters per second, requires applying fluid dynamics principles such as Bernoulli's equation and the continuity equation.



Given:

section 1 D_1= 300mm =0.3m

$$A_1 = \left(\frac{\pi}{4}\right) \times (0.3)^2 A_1 = 0.07065 \text{ m}^2$$

P_1= 24.525N/cm^2 = 24.525 × 10^{4}N/m^2

Section 2 D_2 = 200mm 0.2m

$$A_2 = \left(\frac{\pi}{4}\right)) \times (0.2)^2 = 0.0314 \text{ m}^2$$

P_2 = 9.81N/cm^2 = 9.81 × 10^4 N/m^2

Rate of flow Q = 40 lit/Sec =40/1000 = 0.04 m^3/sec

$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.04}{0.07065} = 0.566 \text{m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.04}{00.0314} = 1.274 \text{m/sec}$$

Applying Bernoulli's equation at sections 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^{\,2}}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^{\,2}}{2g} + Z_2$$

$$\frac{(24.525 \times 10^4)}{(1000 \times 9.81)} + \frac{(0.566)^2}{(2 \times 9.81)} + Z_1 = \frac{(9.81 \times 10^4)}{(1000 \times 9.81)} + \frac{(1.274)^2}{(2 \times 9.81)} + Z_2$$

$$25 + 0.32 + Z_1 = 10 + 1.623 + Z_2$$

$$Z_2 - Z_1 = 25.32 - 11.623 = 13.697 \text{ or say } 13.70\text{m}$$

The difference in datum head = $Z_2 - Z_1$ = 13.70m

Tutorial Questions

1. A pipeline (Pipe 1) with a diameter of 450 mm bifurcates into two pipes (Pipe 2 and Pipe 3) with diameters of 300 mm and 200 mm, respectively. If the average velocity in the 450 mm diameter pipe is 3 m/s, determine (i) the discharge through the 450 mm diameter pipe, and (ii) the velocity in the 200 mm diameter pipe, given the velocity in the 300 mm diameter pipe is 2.5 m/s.

2. A pipeline named ABC spans 200 m and ascends on a slope of 1 in 40. The section AB, with a length of 100 m and a diameter of 100 mm, enlarges suddenly at B to a diameter of 200 mm, remaining consistent for the remaining length BC (100 m). With a flow rate of 0.03 m^3/s pumped in at the lower end A and discharged at the upper end C into a closed tank, and a supply end pressure of 200 kN/m^2, determine the pressure at point C.

3. In a pipe line, a 45° reducing bend is incorporated, with diameters at the inlet and outlet being 40 cm and 20 cm, respectively. Calculate the force exerted by water on the bend if the pressure intensity at the inlet is 21.58 N/cm^2. The water flow rate is 500 liters/s.

4. Explain the distinctions among streamline, pathline, streakline, and stream tube.