MODULE-V

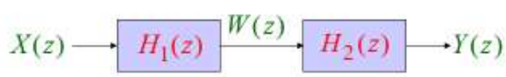
REALIZATION OF DISCRETE-TIME SYSTEMS

STRUCTURAL REALIZATION OF IIR SYSTEMS

The causal IIR digital filters we are concerned with in this course are characterized by a real rational transfer function of or, equivalently by a constant coefficient difference equation
From the difference equation representation, it can be seen that the realization of the causal IIR digital filters requires some form of feedback

DIRECT FORM STRUCTURE:

$$H(z) = \frac{P(z)}{D(z)} = \frac{p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3}}{1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}}$$

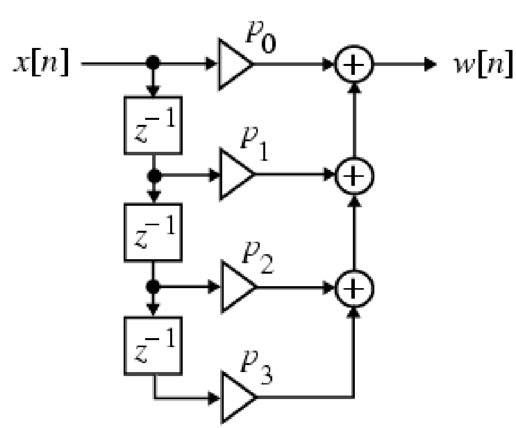


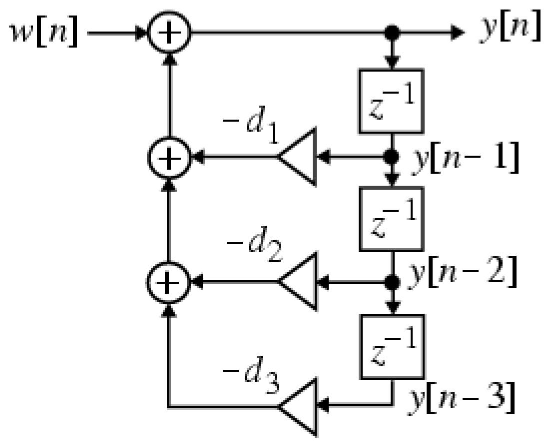
where

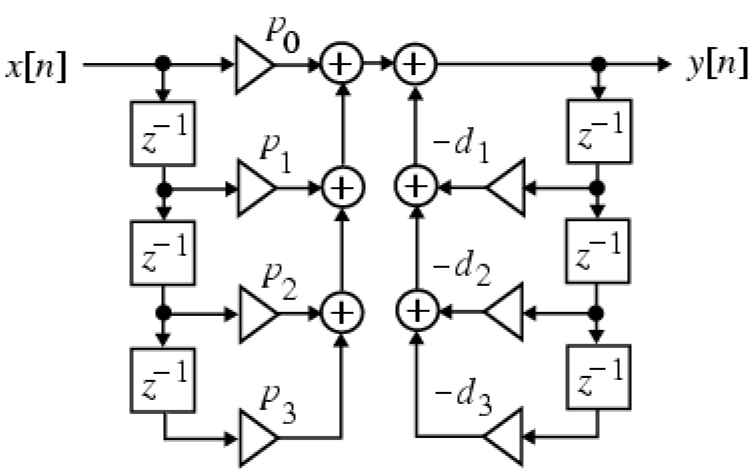
$$H_1(z) = \frac{W(z)}{X(z)} = P(z) = p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3}$$

$$H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{D(z)} = \frac{1}{1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}}$$

$$w[n] = p_0 x[n] + p_1 x[n-1] + p_2 x[n-2] + p_3 x[n-3]$$





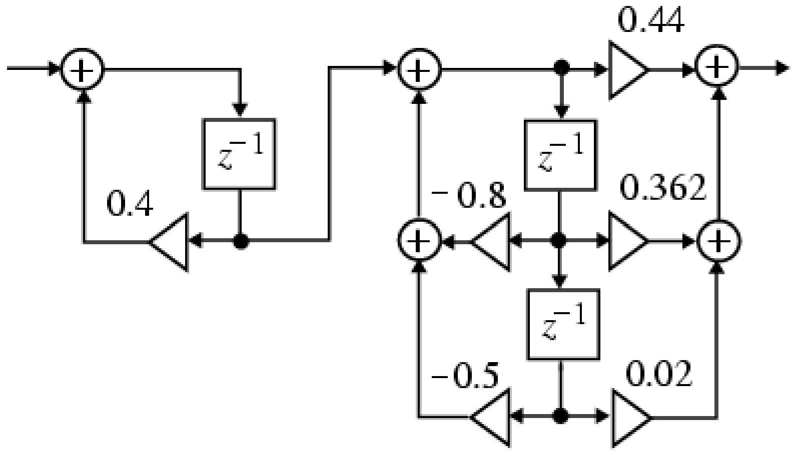


CASCADE FORM STRUCTURE:

$$H(z) = \frac{P(z)}{D(z)} = \frac{P_1(z)P_2(z)P_2(z)}{D_1(z)D_2(z)D_3(z)}$$

$$H(z) = \frac{0.44z^{-1} + 0.362z^{-2} + 0.02z^{-3}}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$

$$=\left(\frac{0.44+0.362z^{-1}+0.02z^{-2}}{1+0.8z^{-1}+0.5z^{-2}}\right)\left(\frac{z^{-1}}{1-0.4z^{-1}}\right)$$

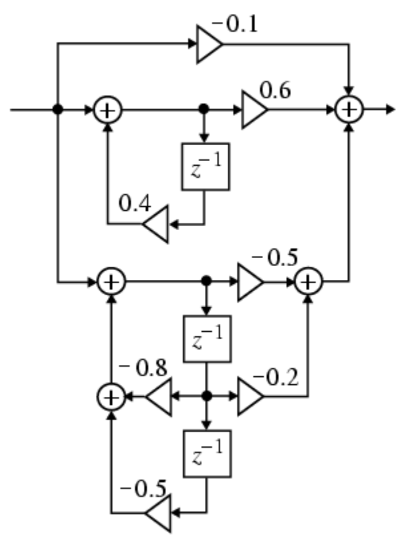


PARALLEL FORM STRUCTURE:

Realize the following system using parallel form

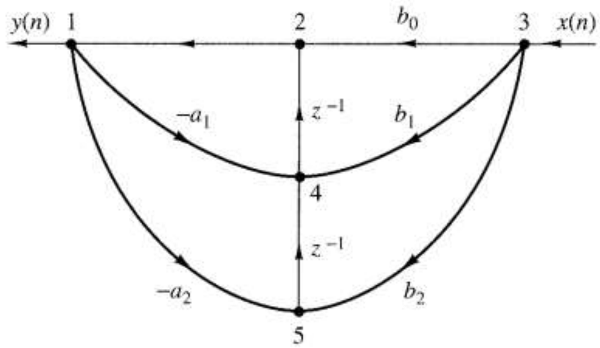
$$H(z)=\frac{0.44z^{-1}+0.362z^{-2}+0.02z^{-3}}{1+0.4z^{-1}+0.18z^{-2}-0.2z^{-3}}$$

$$H(z)=-0.1+\frac{0.6}{1-0.4z^{-1}}+\frac{-0.5-0.2z^{-1}}{1+0.8z^{-1}+0.5z^{-2}}$$

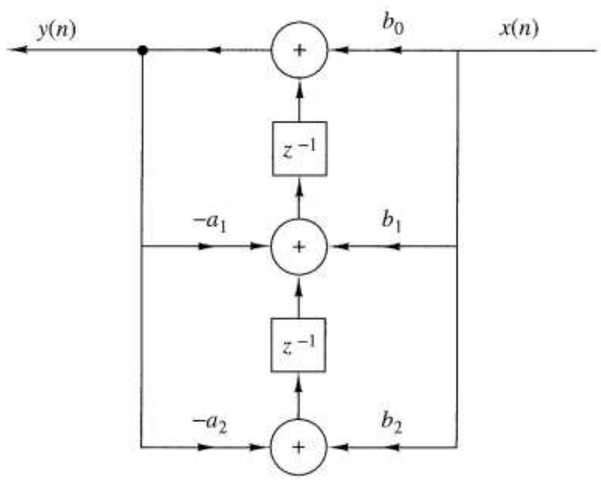


TRANSPOSED FORM:

Apply transposition theorem on direct form –II structure. First reverse all signal flow directions.second change nodes into adders and nodes into adders, finally change input and output.



(a)



(b)

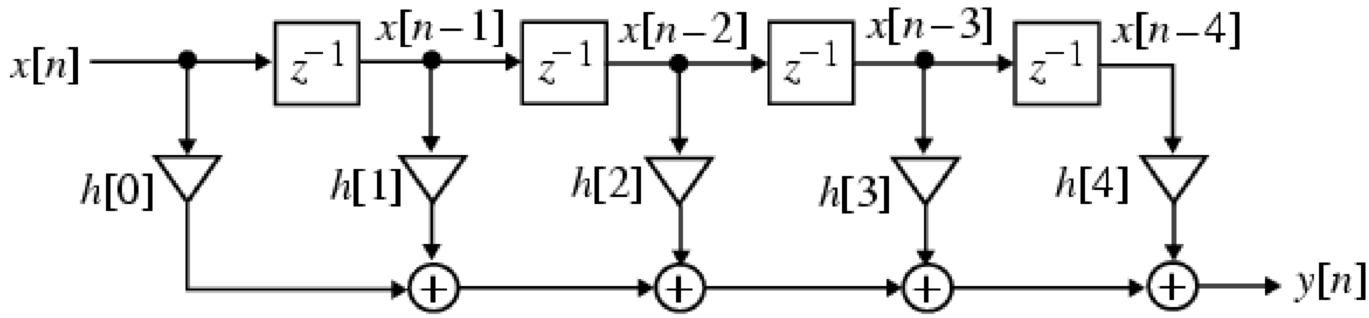
STRUCTURAL REALIZATION OF FIR SYSTEMS:

DIRECT FORM REALIZATION:

An analysis of this structure yields

Y(n)=h(0)+h(1)+h(2)x(n-2)+ h(3)x(n-3)+ h(4)x(n-4)

Which is precisely of the form of the convolution sum description.

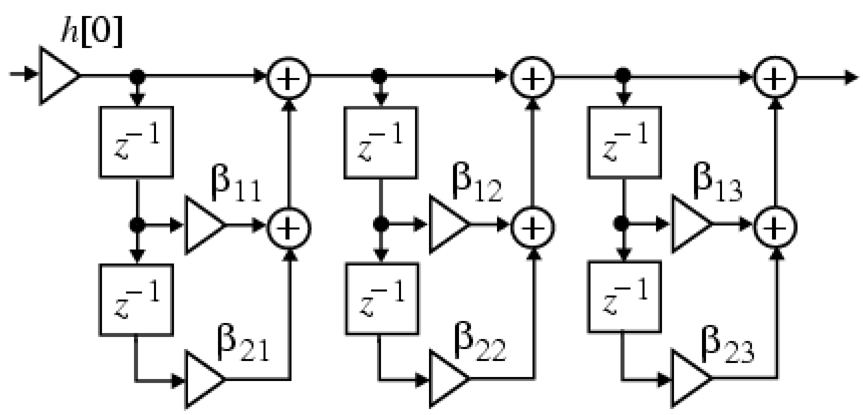


CASCADE-FORM STRUCTURE:

A higher-order FIR transfer function can also be realized as a cascade of second-order FIR sections and possibly a first-order section.

$$H(z) = h[0]\prod_{k=1}^{K}(1+\beta_{1k}z^{-1}+\beta_{2k}z^{-2})$$

A cascade realization for N = 6 is shown below



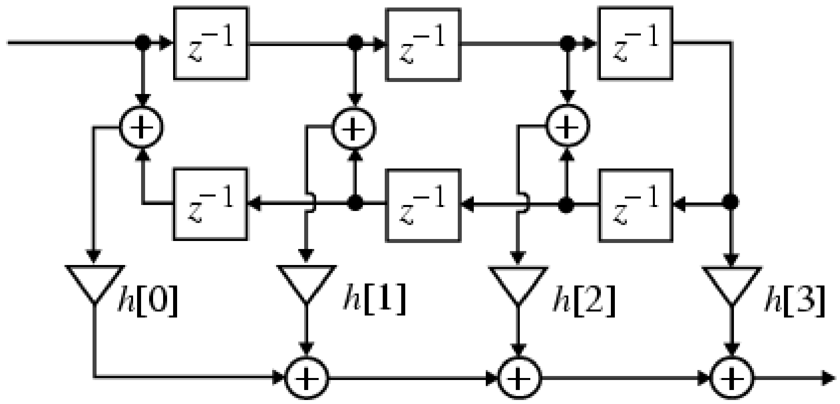
LINEAR PHASE STRUCTURE:

The symmetry (or anti symmetry) property of a linear-phase FIR filter can be exploited to reduce the number of multipliers into almost half of that in the direct form implementation.

Consider a length-7 Type 1 FIR transfer function with a symmetric impulse response:

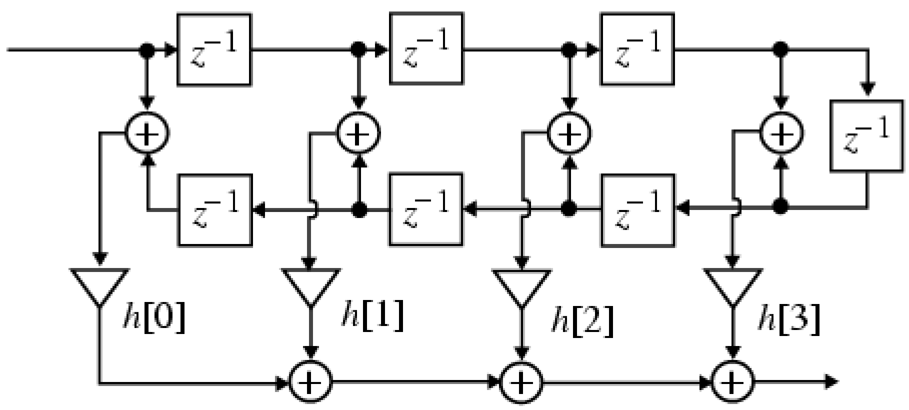
$$H(Z)=h(0)+h(1)z^{-1}+h(2)z^{-2}+h(3)z^{-3}+h(2)z^{-4}+h(1)z^{-5}+h(0)z^{-6}$$

$$H(Z)=h(0)(1+ z^{-6})+ h(1)(z^{-1}+ z^{-5}) + h(2)(z^{-2}+ z^{-4}) + h(3) z^{-3}$$

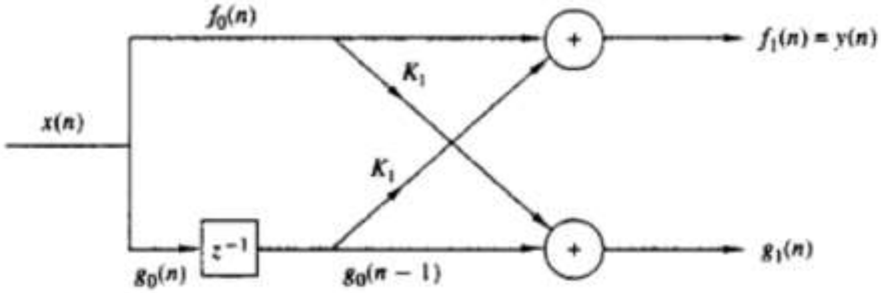


For example, a length-8 Type 2 FIR transfer function can be expressed as

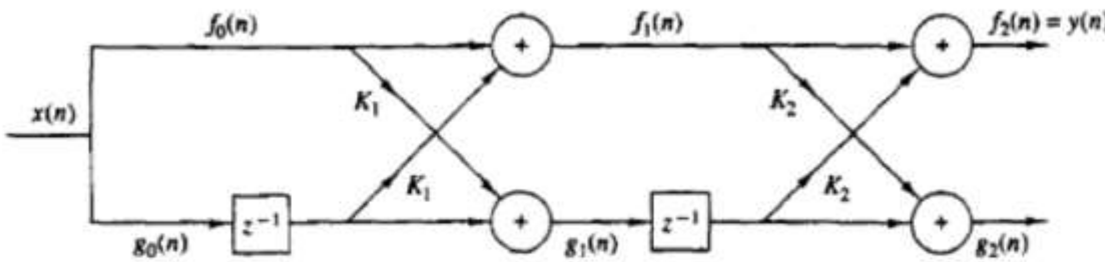
$$H(Z)=h(0)(1+ z^{-7})+ h(1)(z^{-1}+ z^{-6}) + h(2)(z^{-2}+ z^{-5}) + h(3) (z^{-3}+ z^{-4})$$



LATTICE STRUCTURE



$$f_0(n) = g_0(n) = x(n)$$
$$f_1(n) = f_0(n) + K_1 g_0(n-1) = x(n) + K_1 x(n-1)$$
$$g_1(n) = K_1 f_0(n) + g_0(n-1) = K_1 x(n) + x(n-1)$$



$$f_2(n) = x(n) + K_1 x(n-1) + K_2[K_1 x(n-1) + x(n-2)]$$
$$= x(n) + K_1(1+K_2)x(n-1) + K_2 x(n-2)$$

The general form of lattice structure for m stage is given by'

$$f_0(n) = g_0(n) = x(n)$$
$$f_m(n) = f_{m-1}(n) + K_m g_{m-1}(n-1) \qquad m = 1, 2, \ldots, M-1$$
$$g_m(n) = K_m f_{m-1}(n) + g_{m-1}(n-1) \qquad m = 1, 2, \ldots, M-1$$