Module-I:

EQUILIBRIUM OF SYSTEM OF COPLANAR CONCURRENT FORCES

Mechanics

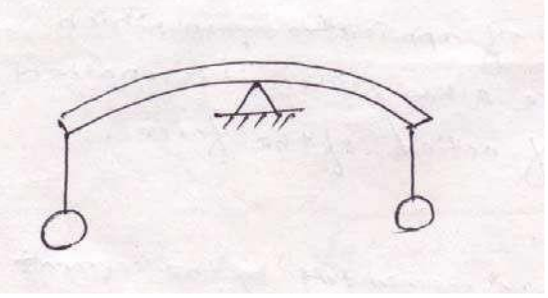
It is defined as that branch of science, which describes and predicts the conditions of rest or motion of bodies under the action of forces. Engineering mechanics applies the principle of mechanics to design, taking into account the effects of forces.

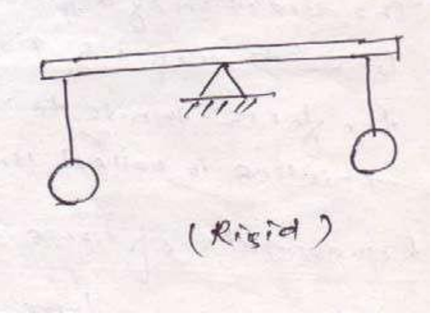
Statics

Statics deal with the condition of equilibrium of bodies acted upon by forces.

Rigid body

A rigid body is defined as a definite quantity of matter, the parts of which are fixed in position relative to each other. Physical bodies are never absolutely but deform slightly under the action of loads. If the deformation is negligible as compared to its size, the body is termed as rigid.



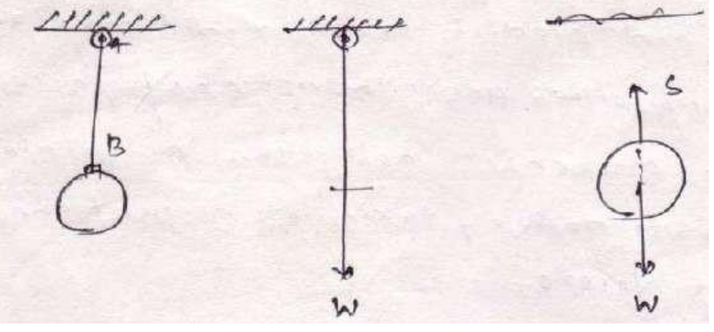


Force

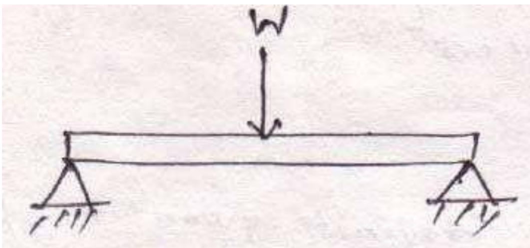
Force may be defined as any action that tends to change the state of rest or motion of a body to which it is applied.

The three quantities required to completely define force are called its specification or characteristics. So the characteristics of a force are:

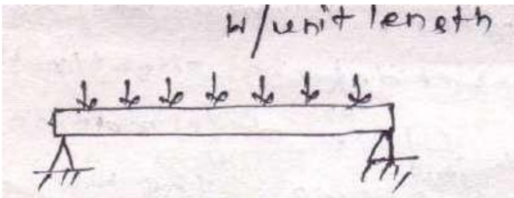
1. Magnitude
2. Point ofapplication
3. Direction ofapplication



Concentrated force/point load



Distributed force

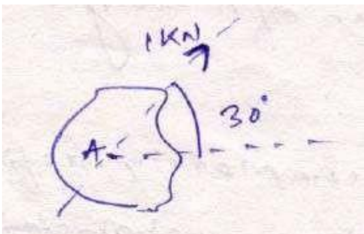


Line of action of force

The direction of a force is the direction, along a straight line through its point of application in which the force tends to move a body when it is applied. This line is called line of action of force.

Representation of force

Graphically a force may be represented by the segment of a straight line.

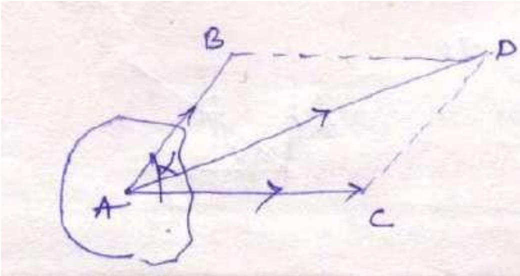


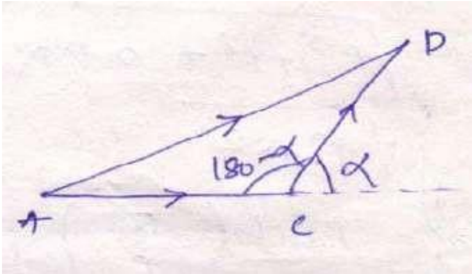
Composition of two forces

The reduction of a given system of forces to the simplest system that will be its equivalent is called the problem of composition of forces.

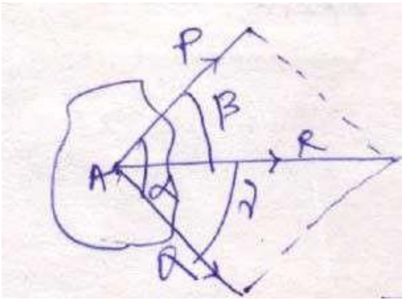
Parallelogram law

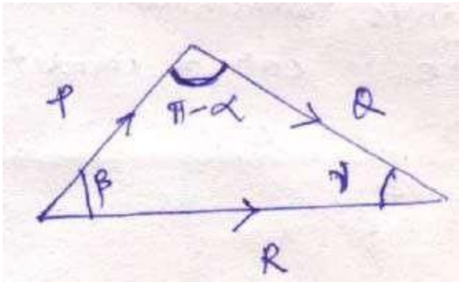
If two forces represented by vectors AB and AC acting under an angle α are applied to a body at point A. Their action is equivalent to the action of one force, represented by vector AD, obtained as the diagonal of the parallelogram constructed on the vectors AB and AC directed as shown in thefigure.





Force AD is called the resultant of AB and AC and the forces are called its components.





$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos\alpha\right)}$$

Now applying triangle law

$$\frac{P}{Sin\gamma} = \frac{Q}{Sin\beta} = \frac{R}{Sin(\pi - \alpha)}$$

Special cases

Case-I: If α = 0˚

$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos0\right)} = \sqrt{(P+Q)^2} = (P+Q)$$

P Q R

R = P+Q

Case- II: If α = 180˚

$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos180\right)} = \sqrt{(P^2 + Q^2 - 2PQ)} = \sqrt{(P-Q)^2} = (P-Q)$$

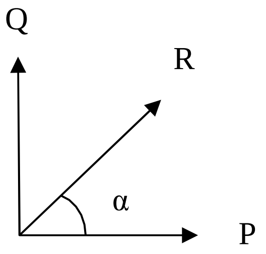
Q

P

Case-III: If $\alpha = 90^\circ$

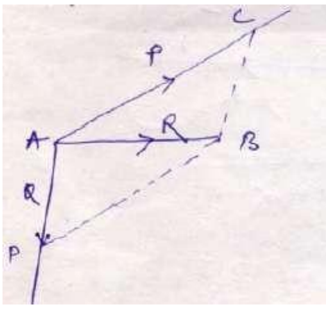
$$R = \sqrt{\left(P^2+Q^2+2PQ\times Cos90\right)} = \sqrt{P^2+Q^2}$$

$\alpha = \tan^{-1}(Q/P)$



Resolution of a force

The replacement of a single force by a several components which will be equivalent in action to the given force is called resolution of a force.



Action and reaction

Often bodies in equilibrium are constrained to investigate the conditions.

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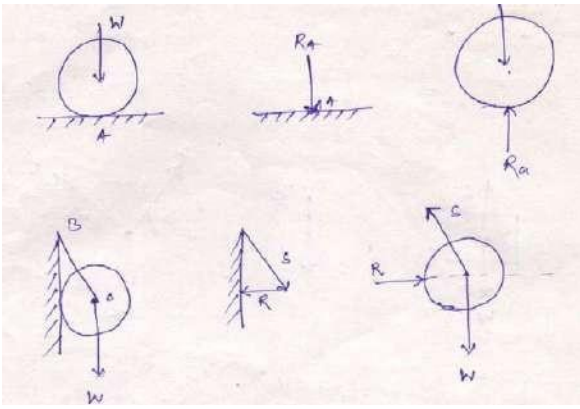
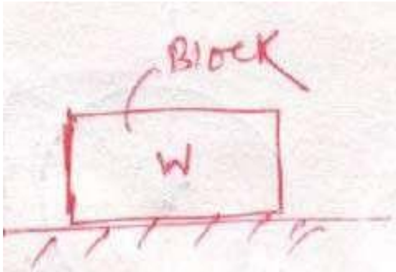
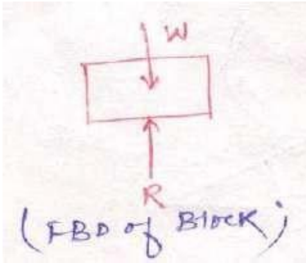


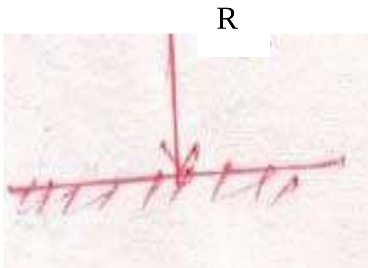
Free body diagram

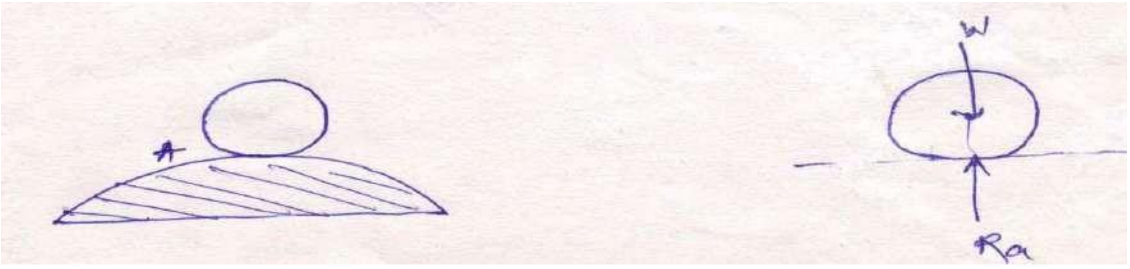
Free body diagram is necessary to investigate the condition of equilibrium of a body or system. While drawing the free body diagram all the supports of the body are removed and replaced with the reaction forces acting on it.

1. Draw the free body diagrams of the followingfigures.

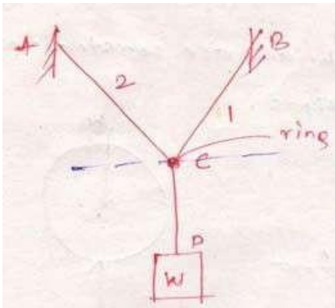


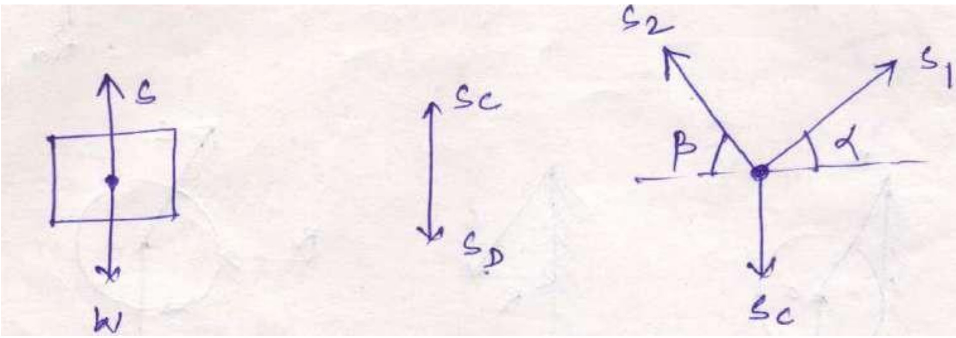




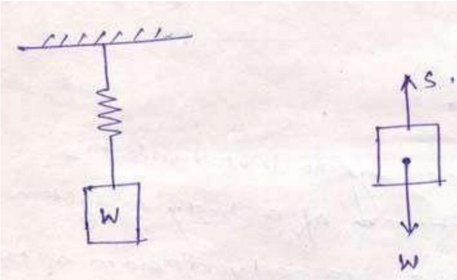


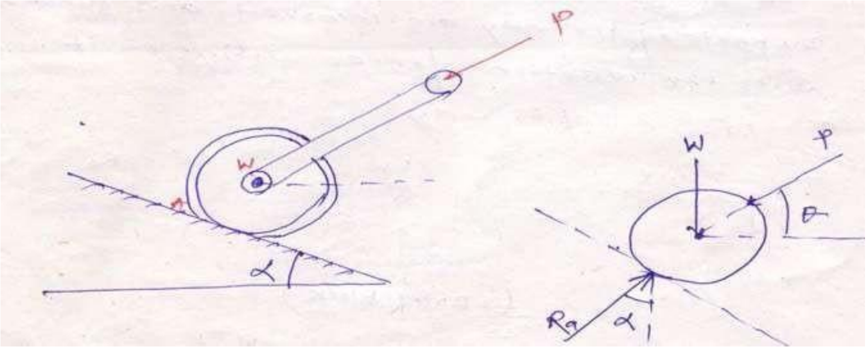
2. Draw the free body diagram of the body, the string CD and thering.





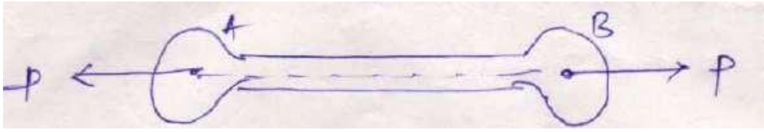
3. Draw the free body diagram of the following figures.



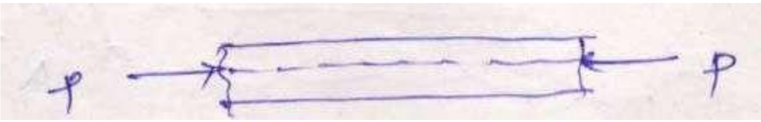


Equilibrium of colinear forces:

Equilibrium law: Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.



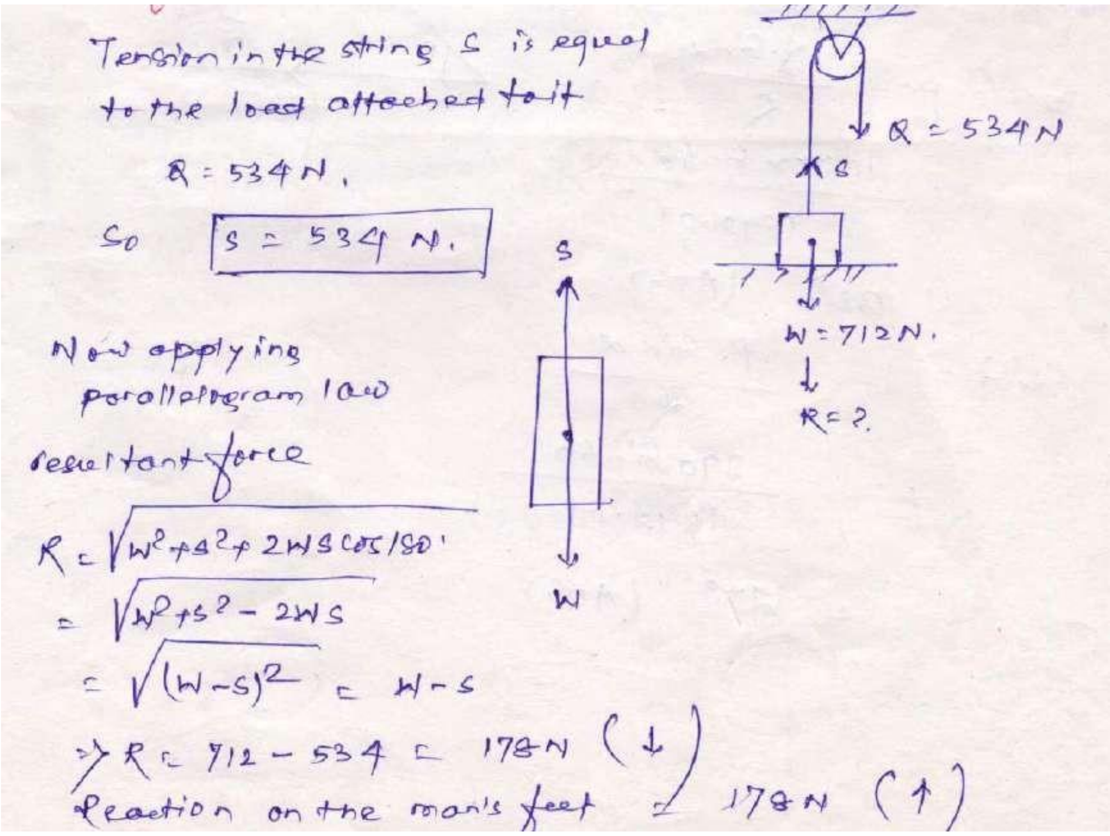
(tension)



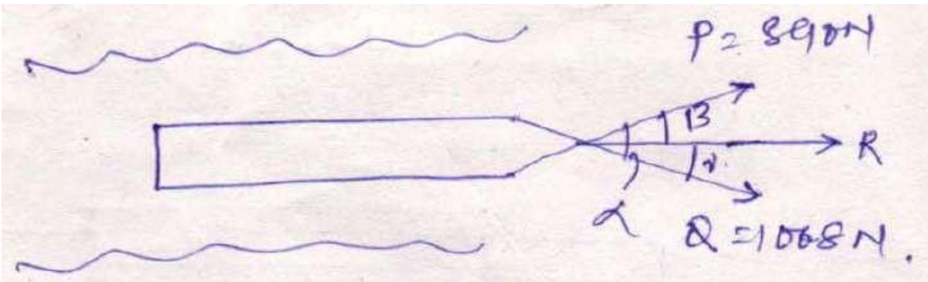
(compression)

Superposition and transmissibility

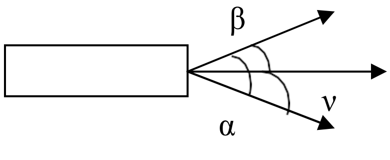
Problem 1: A man of weight W = 712 N holds one end of a rope that passes over a pulley vertically above his head and to the other end of which is attached a weight Q = 534 N. Find the force with which the man's feet press against the floor.



Problem 2: A boat is moved uniformly along a canal by two horses pulling with forces P = 890 N and Q = 1068 N acting under an angle $\alpha = 60^\circ$. Determine the magnitude of the resultant pull on the boat and the angles β and ν.



P = 890 N, $\alpha = 60^\circ$
Q = 1068 N

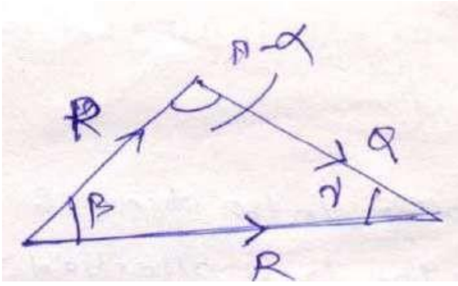


$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$= \sqrt{890^2 + 1068^2 + 2 \times 890 \times 1068 \times 0.5}$$

$$= 1698.01 N$$

$$\frac{Q}{\sin\beta} = \frac{P}{\sin v} = \frac{R}{\sin(\pi - \alpha)}$$



$$\sin\beta = \frac{Q\sin\alpha}{R}$$

$$= \frac{1068 \times \sin 60^\circ}{1698.01}$$

$$= 33^\circ$$

$$\sin v = \frac{P\sin\alpha}{R}$$

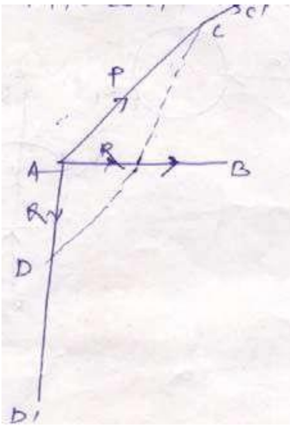
$$= \frac{890 \times \sin 60^\circ}{1698.01}$$

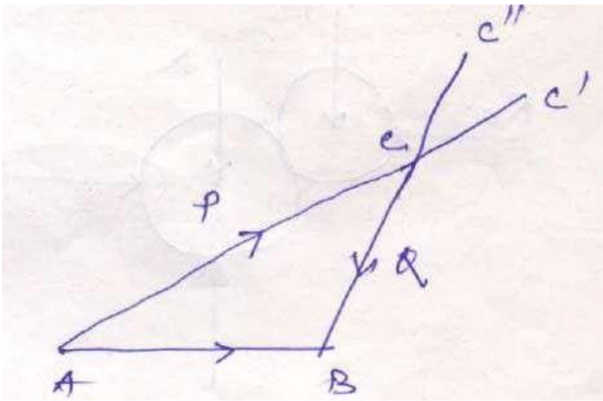
$$= 27^\circ$$

Resolution of a force

Replacement of a single force by several components which will be equivalent in action to the given force is called the problem of resolution of aforce.

By using parallelogram law, a single force R can be resolved into two components P and Q intersecting at a point on its line of action.





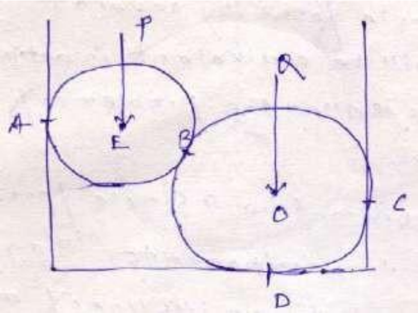
Equilibrium of collinear forces:

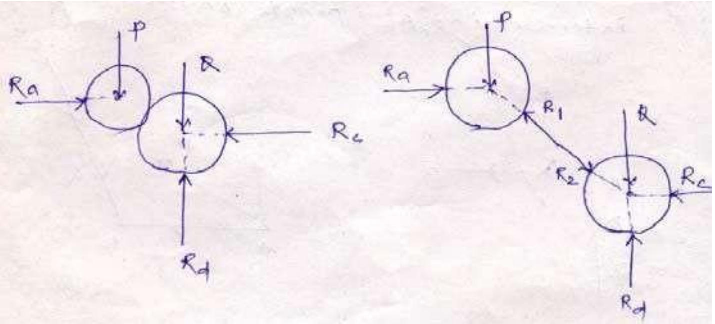
Equilibrium law: Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.

Law of superposition

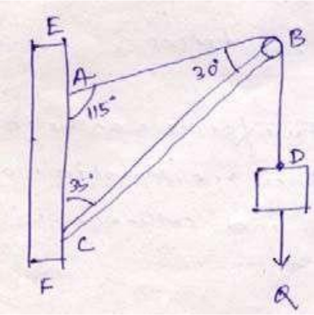
The action of a given system of forces on a rigid body will no way be changed if we add to or subtract from them another system of forces in equllibrium.

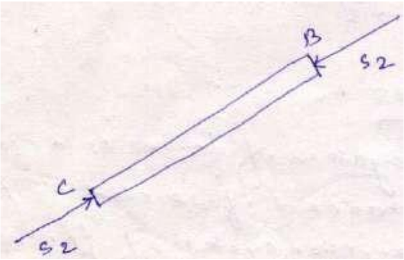
Problem 3: Two spheres of weight P and Q rest inside a hollow cylinder which is resting on a horizontal force. Draw the free body diagram of both the spheres, together and separately.

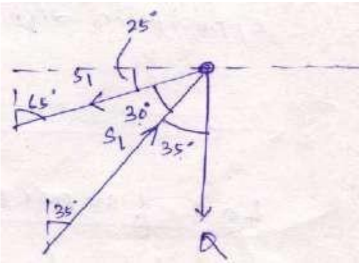




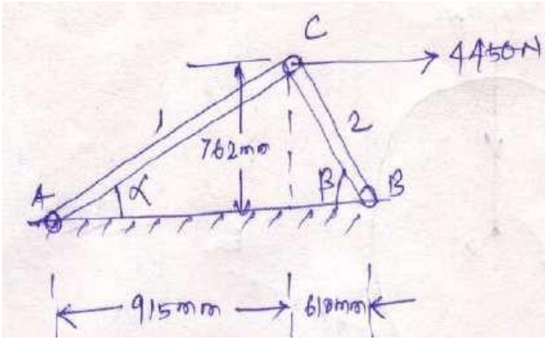
Problem 4: Draw the free body diagram of the figure shown below.

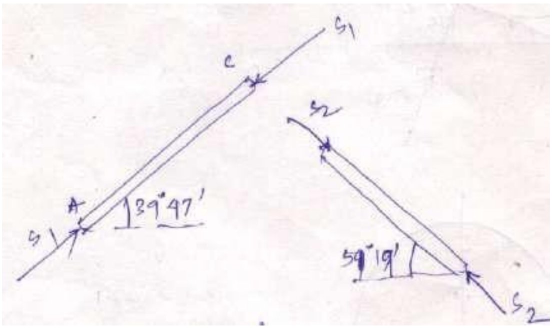




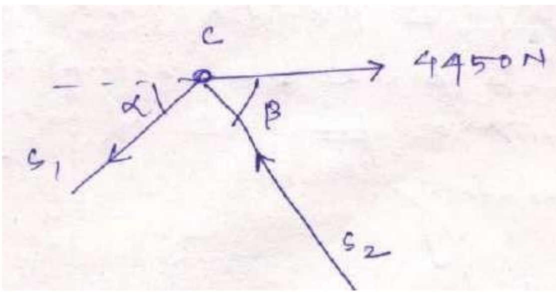


Problem 5: Determine the angles α and β shown in the figure.

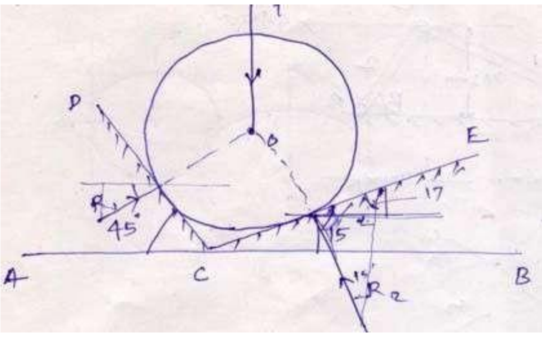


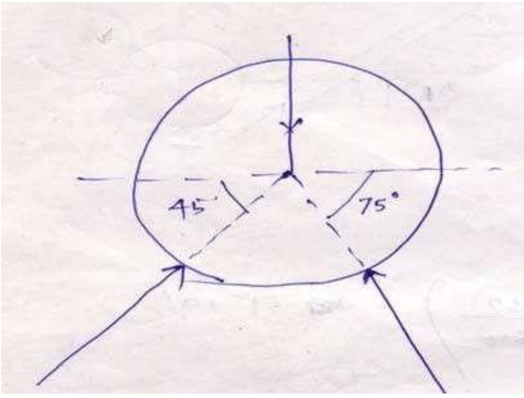


$$\alpha = \tan^{-1}\left(\frac{762}{915}\right)$$
$$= 39^{\circ}47'$$
$$\beta = \tan^{-1}\left(\frac{762}{610}\right)$$
$$= 51^{\circ}19'$$

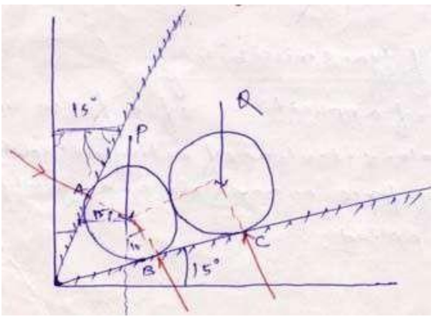


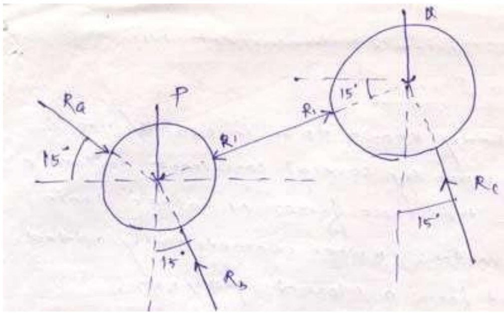
Problem 6: Find the reactions R_1 and R_2.



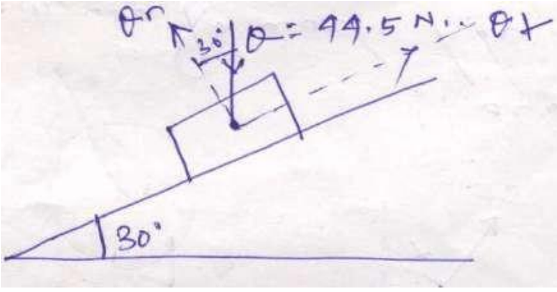


Problem 7: Two rollers of weight P and Q are supported by an inclined plane and vertical walls as shown in the figure. Draw the free body diagram of both the rollers separately.

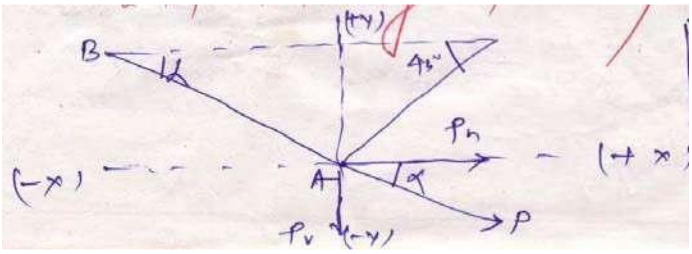




Problem 8: Find θ_n and θ_t in the following figure.



Problem 9: For the particular position shown in the figure, the connecting rod BA of an engine exert a force of P = 2225 N on the crank pin at A. Resolve this force into two rectangularcomponents$P_h$and$P_v$horizontallyandverticallyrespectivelyatA.

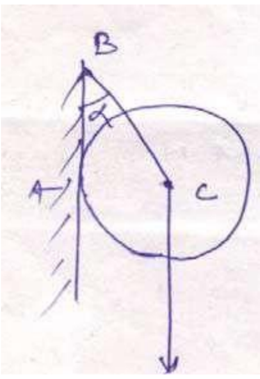


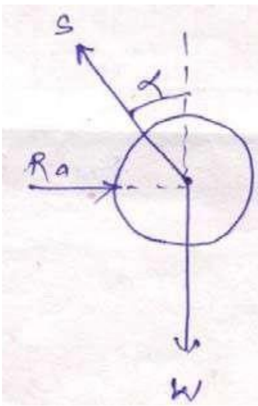
P_h = 2081.4 N
P_v = 786.5 N

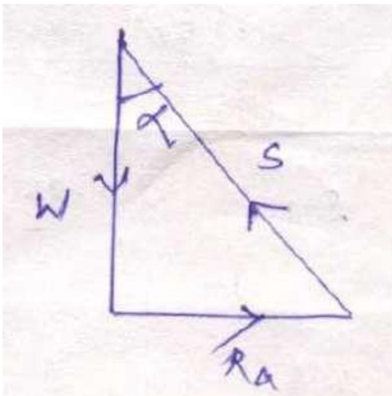
Equilibrium of concurrent forces in a plane

- If a body known to be in equilibrium is acted upon by several concurrent, coplanar forces, then these forces or rather their free vectors, when geometrically added must form a closedpolygon.

- This system represents the condition of equilibrium for any system of concurrent forces in aplane.





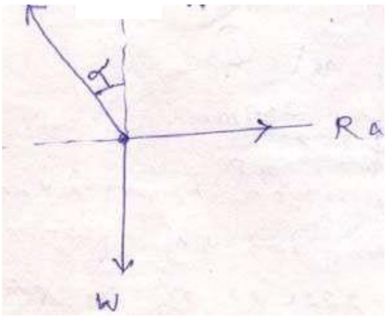


W

$R_a = w\tan\alpha$

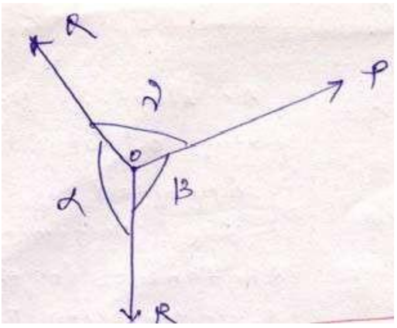
$S = w\sec\alpha$

S

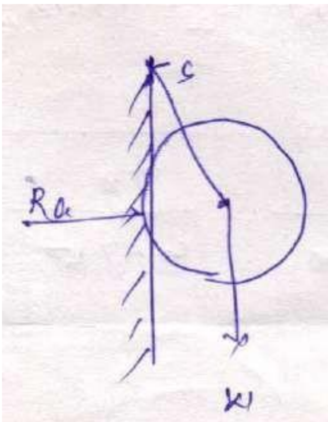


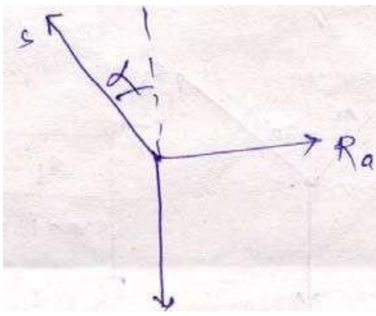
Lami’s theorem

If three concurrent forces are acting on a body kept in an equllibrium, then each force is proportional to the sine of angle between the other two forces and the constant of proportionality issame.



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\upsilon}$$

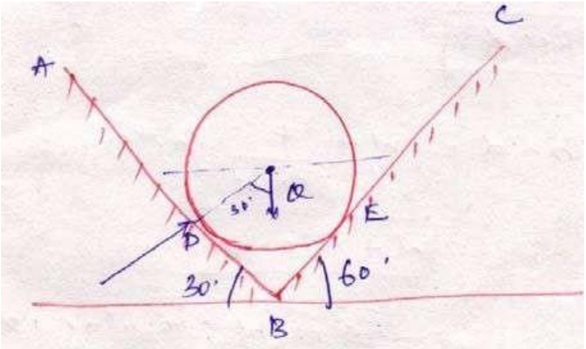


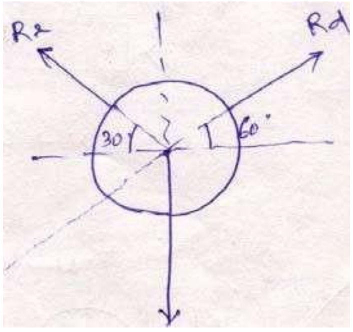


W

$$\frac{S}{\sin 90} = \frac{R_a}{\sin(180-\alpha)} = \frac{W}{\sin(90+\alpha)}$$

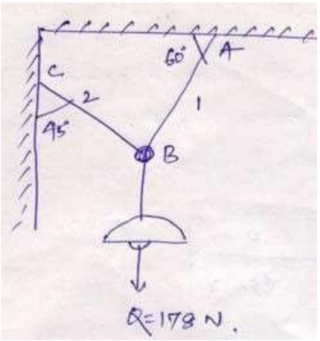
Problem: A ball of weight Q = 53.4N rest in a right angled trough as shown in figure. Determine the forces exerted on the sides of the trough at D and E if all the surfaces are perfectlysmooth.

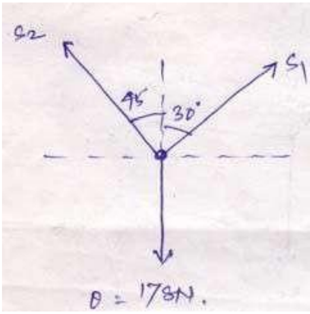




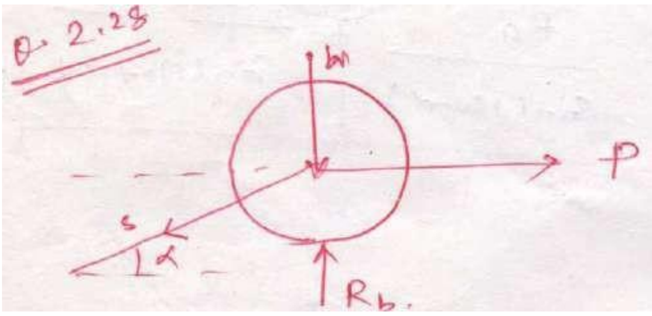
W

Problem: An electric light fixture of weight Q = 178 N is supported as shown in figure. Determine the tensile forces S_1 and S_2 in the wires BA and BC, if their angles of inclination aregiven.





$$\frac{S_1}{\sin 135} = \frac{S_2}{\sin 150} = \frac{178}{\sin 75}$$

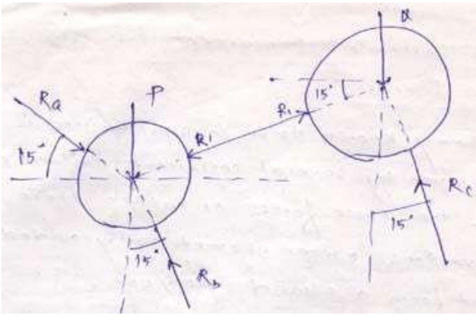


$S_1 \cos\alpha = P$

$\boxed{S = P\sec\alpha}$

$$R_b = W + S \sin \alpha$$
$$= W + \frac{P}{\cos\alpha} \times \sin \alpha$$
$$= W + P \tan \alpha$$

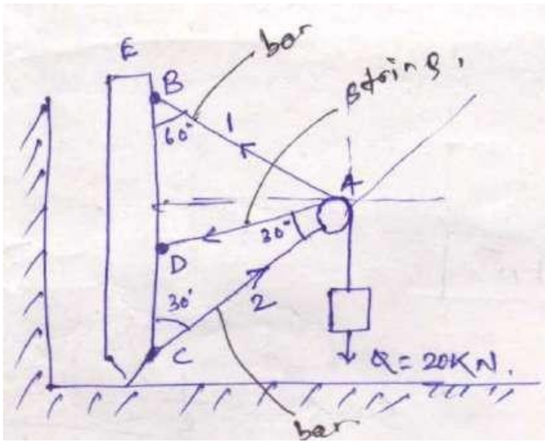
Problem: A right circular roller of weight W rests on a smooth horizontal plane and is held in position by an inclined bar AC. Find the tensions in the bar AC and vertical reaction R_b if there is also a horizontal force P is active.

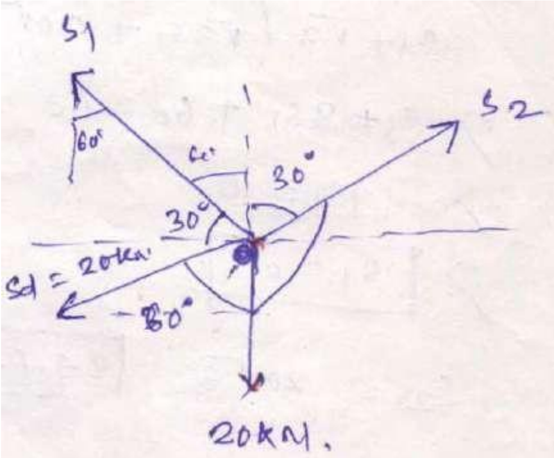


Theory of transmissiibility of a force:

The point of application of a force may be transmitted along its line of action without changing the effect of force on any rigid body to which it may be applied.

Problem:





$$\sum X = 0$$

$$S_1 \cos 30 + 20 \sin 60 = S_2 \sin 30$$

$$\frac{\sqrt{3}}{2} S_1 + 20 \frac{\sqrt{3}}{2} = \frac{S_2}{2}$$

$$\frac{S_2}{2} = \frac{\sqrt{3}}{2} S_1 + 10\sqrt{3}$$

$$S_2 = \sqrt{3} S_1 + 20\sqrt{3} \qquad (1)$$

$$\sum Y = 0$$

$$S_1 \sin 30 + S_2 \cos 30 = S_d \cos 60 + 20$$

$$\frac{S_1}{2} + S_2 \frac{\sqrt{3}}{2} = \frac{20}{2} + 20$$

$$\frac{S_1}{2} + \frac{\sqrt{3}}{2} S_2 = 30$$

$$S_1 + \sqrt{3} S_2 = 60 \qquad (2)$$

Substituting the value of S_2 in Eq.2, we get

$$S_1 + \sqrt{3}\left(\sqrt{3} S_1 + 20\sqrt{3}\right) = 60$$

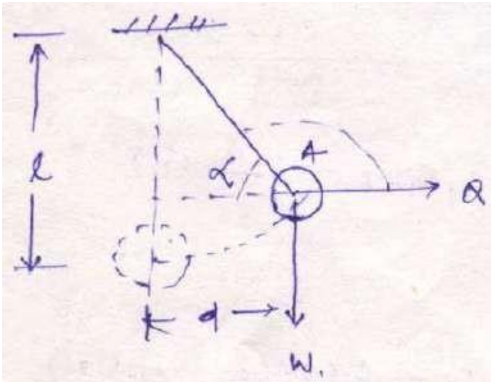
$$S_1 + 3S_1 + 60 = 60$$

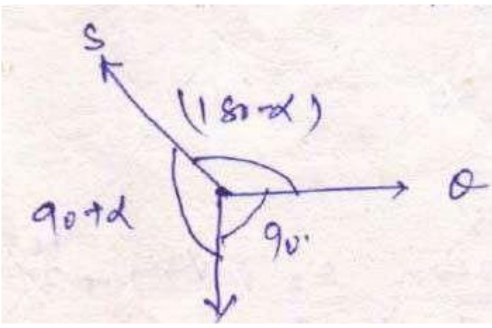
$$4S_1 = 0$$

$$S_1 = 0KN$$

$$S_2 = 20\sqrt{3} = 34.64KN$$

Problem: A ball of weight W is suspended from a string of length l and is pulled by a horizontal force Q. The weight is displaced by a distance d from the vertical position as shown in Figure. Determine the angle α, forces Q and tension in the string S in the displacedposition.





W

$$\cos\alpha = \frac{d}{l}$$

$$\alpha = \cos^{-1}\left(\frac{d}{l}\right)$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\Rightarrow \sin\alpha = \sqrt{(1-\cos^2\alpha)}$$

$$= \sqrt{1-\frac{d^2}{l^2}}$$

$$= \frac{1}{l}\sqrt{l^2-d^2}$$

Applying Lami's theorem,

$$\frac{S}{\sin 90} = \frac{Q}{\sin(90+\alpha)} = \frac{W}{\sin(180-\alpha)}$$

$$\frac{Q}{\sin(90+\alpha)}=\frac{W}{\sin(180-\alpha)}$$

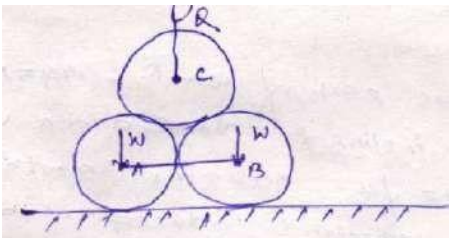
$$\Rightarrow Q=\frac{W\cos\alpha}{\sin\alpha}=\frac{W\left(\frac{d}{l}\right)}{\frac{1}{l}\sqrt{l^2-d^2}}$$

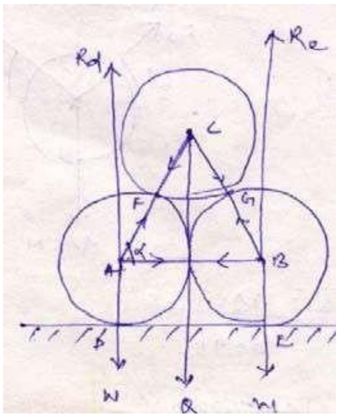
$$\Rightarrow Q=\frac{Wd}{\sqrt{l^2-d^2}}$$

$$S=\frac{W}{\sin\alpha}=\frac{W}{\frac{1}{l}\sqrt{l^2-d^2}}$$

$$=\frac{Wl}{\sqrt{l^2-d^2}}$$

Problem: Two smooth circular cylinders each of weight W = 445 N and radius r = 152 mm are connected at their centres by a string AB of length l = 406 mm and rest upon a horizontal plane, supporting above them a third cylinder of weight Q = 890 N and radius r = 152 mm. Find the forces in the string and the pressures produced on the floor at the point of contact.





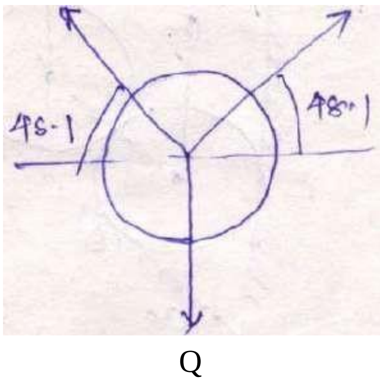
$$\cos\alpha=\frac{203}{304}$$

$$\Rightarrow \alpha=48.1^\circ$$

$$\frac{R_g}{\sin 138.1}=\frac{R_e}{\sin 138.1}=\frac{Q}{83.8}$$

$$\Rightarrow R_g=R_e=597.86N$$

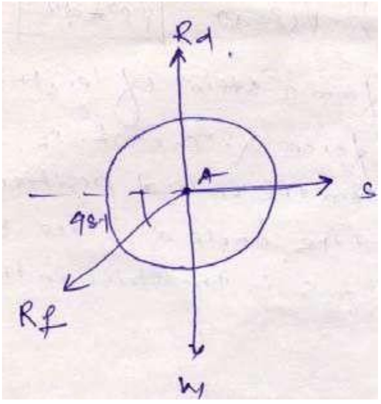
R_g R_f



Q

Resolving horizontally

$$\sum X = 0$$
$$S = R_f \cos 48.1$$
$$= 597.86 \cos 48.1$$
$$= 399.27N$$

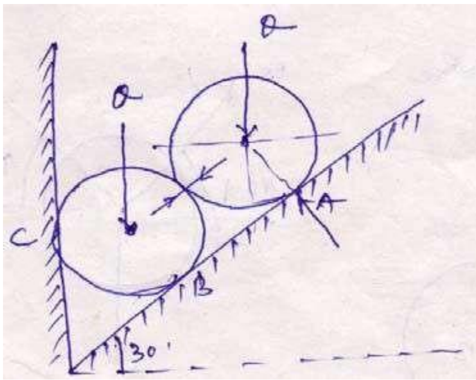


Resolving vertically

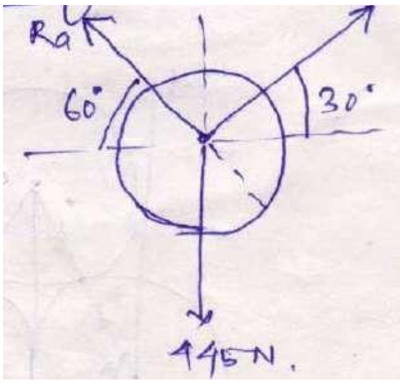
$$\sum Y = 0$$
$$R_d = W + R_f \sin 48.1$$
$$= 445 + 597.86 \sin 48.1$$
$$= 890N$$

$$R_e = 890N$$
$$S = 399.27N$$

Problem: Two identical rollers each of weight Q = 445 N are supported by an inclined plane and a vertical wall as shown in the figure. Assuming smooth surfaces, find the reactions induced at the points of support A, B and C.



S



$$\frac{R_a}{\sin 120} = \frac{S}{\sin 150} = \frac{445}{\sin 90}$$

$$\Rightarrow R_a = 385.38N$$
$$\Rightarrow S = 222.5N$$

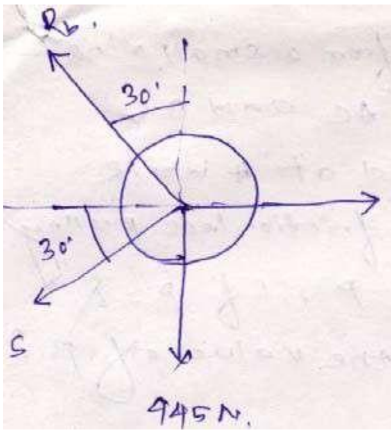
Resolving vertically

$\sum Y = 0$

$R_b \cos 60 = 445 + S \sin 30$

$\Rightarrow R_b \frac{\sqrt{3}}{2} = 445 + \frac{222.5}{2}$

$\Rightarrow R_b = 642.302N$



R_c

Resolving horizontally

$\sum X = 0$

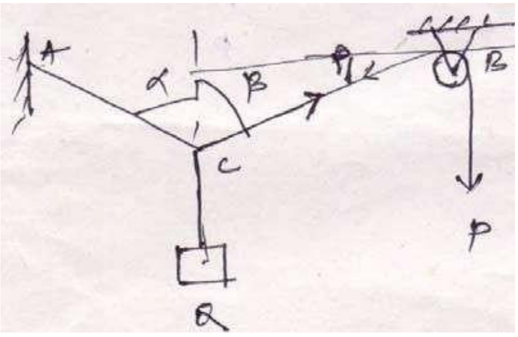
$R_c = R_b \sin 30 + S \cos 30$

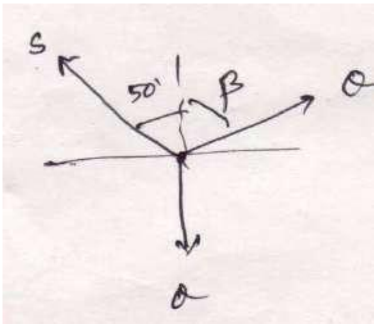
$\Rightarrow 642.302 \sin 30 + 222.5 \cos 30$

$\Rightarrow R_c = 513.84N$

Problem:

A weight Q is suspended from a small ring C supported by two cords AC and BC. The cord AC is fastened at A while cord BC passes over a frictionless pulley at B and carries a weight P. If P = Q and α = 50˚, find the value of β.





Resolving horizontally

$\sum X = 0$

$S \sin 50 = Q \sin \beta$ (1)

Resolving vertically

$\sum Y = 0$

$S \cos 50 + Q \sin \beta = Q$

$\Rightarrow S \cos 50 = Q(1 - \cos \beta)$

Putting the value of S from Eq. 1, weget

$S\cos 50 + Q\sin\beta = Q$

$\Rightarrow S\cos 50 = Q(1-\cos\beta)$

$\Rightarrow Q\dfrac{\sin\beta}{\sin 50}\cos 50 = Q(1-\cos\beta)$

$\Rightarrow \cot 50 = \dfrac{1-\cos\beta}{\sin\beta}$

$\Rightarrow 0.839\sin\beta = 1-\cos\beta$

Squaring both sides,

$0.703\sin^2\beta = 1+\cos^2\beta - 2\cos\beta$

$0.703(1-\cos^2\beta) = 1+\cos^2\beta - 2\cos\beta$

$0.703 - 0.703\cos^2\beta = 1+\cos^2\beta - 2\cos\beta$

$\Rightarrow 1.703\cos^2\beta - 2\cos\beta + 0.297 = 0$

$\Rightarrow \cos^2\beta - 1.174\cos\beta + 0.297 = 0$

$\Rightarrow \beta = 63.13^\circ$