

Power System Analysis

UNIT – II

Load Flow Studies or Power Flow Studies: -

Power flow analysis or load flow analysis is the determination of active power, reactive power, voltage and current at various points in PS operating under normal steady state or static condition. The mathematical formation of PFS results in a set of algebraic non-linear equations lot of calculation work is involved in solution of these equations and calculations and very tedious and time consuming.

Earlier LF studies are made by AC N/W analyzer now-a-days we are using digital computer because of greater flexibility. Economical and quicker operation.

Necessity of Load Flow studies: -

The following are the information is necessary for the following functions.

1. for planning, control and continuous evaluation of current performance in the existing N/W.
2. For analyze the effectiveness of alternative plans of system expansion to meet increased load demand or in designing a new system.
3. To know the effect of interconnected load, new generating stations and transformer Lines before they are installed.
4. For determining the most favorable location and suitable size for power capacitors both for improvement of PF and improving the bus voltages.
5. For determining the optimal way of generating capacity.
6. Power flow studies are required to determine the optimal allocation of loads among various stations in the load.
7. The effect of disturbances which may result in system voltages can be minimized by proper free fault load flow analysis
8. Power flow studies are required at various stages of transient or dynamic stability analysis.

Data Required for Load Flow Studies: -

A bus is a node at which one or more lines, one or more loads and generations are connected. The four quantities at the bus are real power, reactive power, magnitude of bus voltage and phase angle of bus voltage out of 4 quantities, 2 are specified, the other 2 quantities are determined through load

flow studies. If all the 4 quantities at every bus in the system are known the real and reactive power flows in the transformer line can be determined.

Depending upon the quantities have been specified, the buses are classified into,

S.NO	Type of Bus	Quantities specified	Quantities to be calculated
1.	PQ Bus	P_P, Q_P	$ V_P , \delta_P$
2.	PV Bus	$P_P, V_P $	Q_P, δ_P
3.	Slack Bus	$ V_P , \delta_P$	P_P, Q_P

Formation of Bus Admittance Matrix: -

Consider a 3-Bus system shown in fig. Let S_{GP} denotes the 3 ϕ complex generated power flowing into p-bus and S_{LP} be the 3 ϕ complex power flowing out of p-bus.

$$\left. \begin{array}{l} S_{GP} = P_{GP} + jQ_{GP} \\ S_{LP} = P_{LP} + jQ_{LP} \end{array} \right\} P, Q \rightarrow \text{active and reactive powers}$$

At each bus generated and load power can be combined so that 3 ϕ complex power flowing into p-bus can be return as

$$\begin{aligned} S_P &= S_{GP} - S_{LP} \\ &= P_{GP} + jQ_{GP} - P_{LP} - jQ_{LP} \\ &= (P_{GP} - P_{LP}) + j(Q_{GP} - Q_{LP}) \end{aligned}$$

Fig. a

Fig. B

In fig (b) S_1, S_2, S_3 are the 3 ϕ complex powers flowing into the buses.

I_1, I_2, I_3 : currents flowing into buses each transformer line is represented by a π -ckt.

At node 1: -

$$\begin{aligned} I_1 &= I_{10} + I_{12} + I_{13} \\ &= y_{10} V_1 + (V_1 - V_2) y_{12} + (V_1 - V_3) y_{13} \\ &= (y_{10} + y_{12} + y_{13}) V_1 + y_{12} \cdot V_2 - Y_{13} \cdot V_3 \\ I_1 &= y_{11} V_1 + y_{12} \cdot V_2 + y_{13} V_3 \quad \rightarrow \quad (1) \end{aligned}$$

Where $y_{11} = y_{10} + y_{12} + y_{13}$

$$Y_{12} = - y_{12}$$

$$Y_{13} = - y_{13}$$

At node 2: -

$$\begin{aligned} I_2 &= I_{20} + I_{21} + I_{23} \\ &= V_{20} \cdot V_2 + (V_2 - V_1) y_{21} + (V_2 - V_3) y_{23} \\ &= - y_{21} \cdot V_1 + (y_{20} + y_{21} + y_{23}) V_2 - y_{23} \cdot V_3 \\ I_2 &= y_{21} \cdot V_1 + y_{22} \cdot V_2 + y_{23} \cdot V_3 \quad \rightarrow \quad (2) \end{aligned}$$

Where $y_{21} = -y_{12}$

$$Y_{22} = y_{20} + Y_{20} + Y_{23}$$

$$Y_{23} = -Y_{32}$$

At node 3: -

$$I_3 = I_{30} + I_{31} + I_{32}$$

$$I_3 = Y_{31} V_1 + Y_{32} V_2 + Y_{33} V_3 \rightarrow (3)$$

Where $Y_{31} = -Y_{13}$

$$Y_{32} = -Y_{23}$$

$$Y_{33} = Y_{31} + Y_{32} + Y_{30}$$

Expressing equations (1), (2), (3) in matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$I_{\text{Bus}} = [Y_{\text{Bus}}] V_{\text{Bus}}$$

$$I_P = \sum_{q=1}^n Y_{PQ} \cdot V_q \text{ Where } P=1,2,\dots,n$$

Y_{PP} : - self admittance (or) driving PF admittance at P-node & equals to the sum of the admittances connected to Pth Bus.

Y_{pq} : - mutual admittance or transfer admittance between 'P & Q' nodes and equals to the negative of the sum of all the admittances connected between 'P & q' buses.

Formation of Static Load Flow Equations: -

The current entering into pth bus of n-bus system is given by

$$I_P = Y_{P1} \cdot V_1 + Y_{P2} \cdot V_2 + \dots + Y_{Pn} \cdot V_n$$

$$= \sum_{q=1}^n Y_{Pq} \cdot V_q$$

$$\text{Let } V_q = |V_q| \angle \delta_q \text{ \& } V_p = |V_p| \angle \delta_p$$

$$Y_{pq} = |Y_{pq}| \angle \delta_{pq}$$

$$I_P = \sum_{q=1}^n |Y_{Pq}| |V_q| \angle \delta_{pq} + \delta_q$$

The complex power injected into the pth bus is

$$S_P = P_P + j Q_P$$

$$= V_P \cdot I_P^*$$

$$S_P = |V_P| \sum_{q=1}^n |Y_{Pq}| |V_q| \angle \delta_P - \delta_q - \delta_{pq}$$

$$*P_P = |V_P| \sum_{q=1}^n |Y_{Pq}| |V_q| \cos (\delta_P - \delta_q - \delta_{pq}) \quad \rightarrow \quad (1)$$

$$Q_P = |V_P| \sum_{q=1}^n |Y_{Pq}| |V_q| \sin (\delta_P - \delta_q - \delta_{pq}) \quad \rightarrow \quad (2)$$

Where $q = 1, 2, 3 \dots n$.

The equations (1) & (2) are called as static load flow equations. These 2 equations represents $2n$ power flow equations at each bus we have 4 variables resulting in $4n$ variables. In order to find out the solution it is necessary to specify 2 variables at each bus. Therefore no. of unknown variables reduces to $2n$. the solution of these $2n$ variables is calculated by numerical methods equations (1) & (2) are non linear equations. These equations can be solved by iterative techniques which employ successive approximation & finally converge to a solution.

Methods of Load Flow Solution: -

Commonly used load flow methods are

1. Gauss – seidel method
2. Newton Raphson method

The desirable features of an ideal load flow methods are as follows: -

1. High speed (i.e) fast in convergence
2. Minimal storage
3. Simplicity & easy of programming
4. High Reliable in nature

The most commonly used methods of load flow study are mentioned above

Gauss – Seidel Method: -

(a) Without PV Bus: -

To start the GS method assume that out of n buses, one bus is slack bus and the remaining $(n-1)$ buses are PQ buses. Initially we assume the magnitude

and phase angles at these n-1 buses & ϕ update these voltages at every step as iteration. The total current entering into Pth bus of an 'n' bus is given by

$$I_P = Y_{P1}V_1 + Y_{P2}V_2 + \dots + Y_{Pn}V_n$$

$$= \sum_{q=1}^n Y_{Pq} \cdot V_q \quad \rightarrow \quad (1)$$

The complex power injected into Pth bus is

$$S_P = P_P + j Q_P = V_P \cdot I_P^*$$

$$S_P^* = P_P - j Q_P = V_P^* \cdot I_P$$

$$I_P = \frac{P_P - j Q_P}{V_P^*} \quad \rightarrow \quad (2)$$

From equations (1),

$$I_P = Y_{PP} \cdot V_P + \sum_{q \neq 1}^n Y_{Pq} \cdot V_q$$

$$\frac{P_P - j Q_P}{V_P^*} = Y_{PP} \cdot V_P + \sum_{q \neq 1}^n Y_{Pq} \cdot V_q$$

$$Y_{PP} \cdot V_P = \frac{P_P - j Q_P}{V_P^*} - \sum_{q \neq 1}^n Y_{Pq} \cdot V_q$$

$$V_P = \frac{1}{Y_{PP}} \left[\frac{P_P - j Q_P}{V_P^*} - \sum_{q \neq 1}^n Y_{Pq} \cdot V_q \right] \quad \rightarrow \quad (3)$$

Where P = 2,3,----- n

Since bus (1) is a slack bus the equation (3) represents a set of (n-1) equations for P=2,3,-----n. which are solved simultaneously for V_2, V_3, \dots, V_n .

In order to save computer time it is appropriate to right equation (3) in the following form.

$$\text{Let } A_P = \frac{P_P - j Q_P}{Y_{PP}}$$

$$\& B_{Pq} = \frac{Y_{Pq}}{Y_{PP}} \quad P = 2,3, \dots, n$$

$$q = 1,2, \dots, n$$

The values of A_p & B_{pq} are computed once in the beginning and they are used in every iteration.

In general for Pth bus the voltage at the (k+1)th iteration can be written

$$\text{as } VP^{K+1} = \frac{A_P}{(V_P^K)^*} - \sum_{\substack{q=1 \\ q \neq P}}^n B_{pq} \cdot V_q \quad \rightarrow \quad (4)$$

$$P = 2, 3, \dots, n$$

In order to improve the rate of convergence at every step of iteration we use the most updated values of the bus voltages to compute the new values of the bus voltages (i.e) newly calculated voltages, V_P^{K+1} immediately replaces V_P^K and is used in the subsequent equations. Therefore the above equation (4) can be written as

$$V_P^{K+1} = \frac{1}{Y_{PP}} \left[\frac{Pp - JQp}{(V_P^K)^*} - \sum_{q=1}^{P-1} Y_{pq} \cdot V_q^{K+1} - \sum_{q=P+1}^n Y_{pq} \cdot V_Q^K \right] \quad \rightarrow \quad (5)$$

Now equation (4) can be written as

$$V_P^{K+1} = \frac{A_p}{(V_P^K)^*} - \sum_{q=1}^{P-1} B_{pq} \cdot V_q^{K+1} - \sum_{q=P+1}^n B_{pq} \cdot V_Q^K \quad \rightarrow \quad (6)$$

$$P = 2, 3, \dots, n$$

$$Q = 1, 2, \dots, n$$

The iterative process is continued till changes in the bus voltages in the successive iterations becomes less than prespecified tolerance level for all buses.

$$|\Delta V_P^{K+1}| = V_P^{K+1} - V_P^K < \sum \quad \rightarrow \quad (7)$$

$$\sum = 0.01 \text{ to } 0.001$$

Calculation of Slack Bus Power: -

When all the bus voltages have been determined S_P^* is obtained by.

$$S_P^* = V_P^* \cdot I_P = P_P - JQ_P$$

$$= V_P^* \sum_{Q=1}^N Y_{PQ} \cdot V_Q$$

Conjugate of S_P^* gives S_P (i.e) power at slack bus.

Calculation of Line Flows & Losses: -

This is the last step of load flow analysis where in the power flows on the various lines of N/W are calculated. Consider the line connected between 'P & q' the line is represented by finite as shown in fig. The current I_{pq} & complex power S_{pq} can be found as

$$I_{pq} = I_{pq1} + I_{pq0}$$

$$= (V_p - V_q) Y_{pq1} + V_p \cdot Y_{pq0}$$

$$S_{pq} = P_{pq} + j Q_{pq} = V_p \cdot I_{pq}^*$$

$$S_{pq} = V_p \{ (V_p^* - V_q^*) Y_{pq}^* + V_p^* \cdot Y_{pq0}^* \}$$

Similarly the power fed from q bus into the line is given by

$$S_{qp} = V_q (V_q^* - V_p^*) Y_{qp}^* + V_q V_q^* Y_{qp0}^*$$

Thus power flow over all the lines can be calculated. The power loss in the line connecting 'p & q' buses is given by $S_{pq} + S_{qp}$. & the total transformer. Loss is the sum of all losses over all the lines.

(b) With PV buses: -

Some of the buses in a P.S are voltage controlled busses where P & V are specified. Q & δ are unknown.

Let the buses be numbered as

$$P = 1 \rightarrow \text{Slack bus}$$

$$P=2,3,\dots,m \rightarrow \text{P-V buses}$$

$$P = m+1, \dots, n \rightarrow \text{P-Q buses}$$

Here the voltage magnitude is to be maintained at specified value (i.e) $|V_p|$ spec the values of Q & δ are to be updated in every iteration.

$$\text{Let } I_p = Y_{p1} V_1 + Y_{p2} V_2 + \dots + Y_{pn} V_n$$

$$= \sum_{q=1}^n Y_{pq} V_q ; P=1, 2, \dots, n$$

Complex power $S_p^* = P_p - j Q_p = V_p^* \cdot I_p$

$$I_p = \frac{P_p - j Q_p}{V_p^*}$$

$$\frac{P_p - j Q_p}{V_p^*} = \sum_{q=1}^n Y_{pq} V_q$$

$$P_p - j Q_p = V_p^* \sum_{q=1}^n Y_{pq} V_q$$

The reactive power at any bus (pth bus) is given by

$$Q_p = -\text{Im} [V_p^* \sum_{q=1}^n Y_{pq} V_q] \quad \rightarrow \quad (1)$$

Where Im \rightarrow imaginary part

In general

$$Q_p^{K+1} = -\text{Im} \left\{ V_p^* \sum_{q=1}^n Y_{pq} V_q^{K+1} + (V_p^K)^* \sum_{q=1}^n Y_{pq} V_q^K \right\} \quad \rightarrow \quad (2)$$

The revised value of ΔP is obtained by δp^{K+1} angle of V_p^{K+1} .

$$\delta p^{K+1} \text{ angle of } \left[\frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_p^K)^*} - \sum_{q=1}^{P-1} Y_{pq} \cdot V_q^{K+1} - \sum_{q=P+1}^n Y_{pq} \cdot V_q^K \right] \right]$$

\rightarrow At PV bus there is a limits for Q (i.e) Q_{\max} & Q_{\min} . After calculation of Q from equation (2) we have to check for limits. $Q_{\min} \leq Q_p^{K+1} \leq Q_{\max}$. If Q_p^{K+1} is with in the limits then by using Q_p^{K+1} & $|V_p|$ spec calculate V_p^{K+1} & δp^{K+1} .

Now put $V_p^{K+1} = |V_p|_{\text{spec}}$ & retain the phase angle δp^{K+1} & move to the next bus. If Q_p^{K+1} is greater than Q_{\max} then put $Q_p^{K+1} = Q_{\max}$ and the at the Pth bus as a PQ bus & continue the calculations similar to PQ bus.

If $Q_p^{K+1} < Q_{\min}$ then put $Q_p^{K+1} = Q_{\min}$ & this bus as PQ bus & continue the calculations similar to that of PQ bus.

\rightarrow In order to improve the rate of convergence in gauss seidel method we employ "acceleration factor α " the acceleration factor to be employed are system dependent the normal value of acceleration factor will be balance 1.4 to 1.6 the voltage corrections during consecutive iterations will be modified as.

$$|V_{pacc}^{K+1}| = V_p^K + \alpha (V_p^{K+1} - V_p^K)$$

α = acceleration factor

wrong selection of α may result in slower convergence & some times may result in divergence in solution.

(1). The following is the system data for a load flow solution the line admittance is

Bus code	Admittance
1-2	2-j8
1-3	1-j4
2-3	0.66 – j 2.664
2-4	1- j4
3-4	2 – j8

The active & reactive powers are

Bus	code	P	Q	V	Remarks
1	-	-	1.06	$\angle 0$	Slack bus
2	0.5	0.2	-	-	PQ
3	0.4	0.3	-	-	PQ
4	0.3	0.1	-	-	PQ

Determine the voltages at the end of first iteration using gauss siedel method.

Take AF $\alpha = 1.6$

Solv: -

(1) Assume 1+j0 as voltage for buses 2,3,4

$$(2) \quad Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

$$Y_{11} = Y_{12} + Y_{13} = 3 - j12$$

$$Y_{12} = -2 + j8; Y_{13} = -1 + j4; Y_{14} = 0$$

$$Y_{21} = -2 + j8; Y_{22} = Y_{23} + Y_{24} = 3.666 - j14.664 + Y_{12}$$

$$Y_{33} = Y_{13} + Y_{23} + Y_{34} = 3.66 - j14.664j$$

$$Y_{44} = Y_{24} + Y_{34} = 3 - j12$$

$$Y_{31} = -1 + j4; Y_{32} = -0.66 + j2.664$$

$$Y_{Bus} = \begin{bmatrix} 3 - j12 & -2 + j8 & -1 + j4 & 0 \\ -2 + j8 & 3.66 - j14.664 & -0.66 + j2.66 & -1 + j4 \\ -1 + j4 & -0.66 + j2.66 & 3.66 - j14.66 & -2 + j8 \\ 0 & -1 + j4 & -2 + j8 & 3 - j12 \end{bmatrix}$$

(3) K=0, P=2, q=1,2----

$$V_P^{K+1} = \frac{1}{Y_{PP}} \left[\frac{P_P - JQ_P}{(V_P^K)^*} - \sum_{q=1}^{P-1} Y_{Pq} V_{Pq}^{K+1} - \sum_{q=P+1}^n Y_{Pq} \cdot V_q^K \right]$$

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - JQ_2}{(V_2^K)^*} - \sum_{q=1} Y_{21} V_2^1 - Y_{23} \cdot V_3^0 - Y_{24} \cdot V_4^0 \right]$$

$$V_3^1 = \frac{1}{Y_{33}} \left[\frac{P_3 - JQ_3}{(V_3^0)^*} - Y_{31} V_3^1 - Y_{32} \cdot V_2^0 - Y_{34} \cdot V_4^0 \right]$$

$$V_4^1 = \frac{1}{Y_{44}} \left[\frac{P_4 - JQ_4}{(V_4^0)^*} - Y_{41} V_3^1 - Y_{42} \cdot V_3^1 - Y_{43} \cdot V_3^1 \right]$$

$$\setminus * V_2^1 = \frac{1}{3.66 - j14.664} \left[\frac{0.5 - j0.2}{1} - (-2 + j8) \right] * /$$

Note: - For load buses P & Q values are -Ve & for generator buses P & Q are +Ve.

$$V_2^1 = \frac{1}{3.66 - j14.664} \left[\frac{-0.5 - j0.2}{1} - (-2 + j8)(1.06 \angle 0^\circ) + (0.66 - j2.66)X1 + (1 - j4)X1 \right]$$

$$= 0.016 + 0.064j [-0.5 + j0.2 + 2.12 - 8.48j + 0.66 - j2.66 + 1 - j4]$$

$$= 0.016 + 0.064j [3.28 - 14.94j]$$

$$= 1.00864 - 0.02912j$$

$$= 1.009 \angle -1.653$$

$$V_2^1 \neq V_{P(acc)}^{K+1} = V_P^K + \alpha [V_P^{K+1} - V_P^K]$$

$$V_2^1(acc) = V_2^0 + \alpha [V_2^1 - V_2^0]$$

$$= 1 + 1.6 [1.00864 - 0.02912j - 1]$$

$$= 1 + 0.13824 - 0.0465j$$

$$= 1.138 - j0.0465$$

$$V_3^1 = \frac{1}{3.66 - j14.664} \left[\frac{-0.4 - j0.3}{1} - (-1 - j4)(1.06j) + (0.66 - j2.66)X(1.018 - j0.0465) + (2 - j8)X1 \right]$$

$$= (0.016 + 0.0642j) [-0.4 + j0.3 + 1.06 - 4.24j + 0.54819 - 2.73857j + 2 - j8]$$

$$= (0.016 + 0.0642j) (3.20819 - 14.6785j)$$

$$= 0.9936 - 0.0288j$$

$$V_4^1 = \frac{1}{3-j12} \left[\frac{-0.3-j0.1}{1} - (1-j4)(1.0189-j0.0465) + (2-j8)X(1.01728-j0.448) \right]$$

$$V_3^1 (acc) = V_3^0 + \alpha [V_3^1 - V_3^0]$$

$$= 1+1.6 [0.9936 - 0.028j - 1]$$

$$= 1+ 0.01728 - 0.0448 j$$

$$= 1.01728 - 0.0448 j \quad 0.9894 - j \quad 0.4679$$

$$V_4^1 = (0.0196+0.07843j) [-0.3+j0.1+0.8329-4.1221j-8.851j-1.7644]$$

$$= (0.0196+j0.07843) (-1.2315 - 12.8731j)$$

$$= 0.9854 - 0.0688 j$$

$$V_4^1 (acc) = V_4^0 + \alpha [V_4^1 - V_4^0]$$

$$= 1+1.6[0.9854 - 0.0688j -1]$$

$$= 0.97664 - 0.10349 j$$

$$K=1; P=2; q=1,2,3,4$$

$$V_2^2 = \frac{1}{y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^1)^*} \right]$$

(2) In the above problem bus 2 taken as generator bus with $V_2 = 1.04$. the reactive power constrain is $0.1 \leq Q_2 \leq 1.0$ determine the voltages with a flat voltage profile of assuming the acceleration factor $\alpha=1$.

Bus 2: -

$$Q_P = -\text{Im} \left[(V_P^K)^* \sum_{q=1}^{p-1} Y_{Pq} \cdot V_q^{K+1} - (V_P^K)^* \sum_{q=p}^n y_{Pq} \cdot V_q^K \right]$$

$$P=2. K=0, q=1,2,3,4$$

$$Q_2 = -\text{Im} \left[(V_2^0)^* \sum_q Y_{21} \cdot V_1^1 + (V_2^0)^* y_{22} \cdot V_2^0 + (V_2^0)^* y_{23} \cdot V_3^0 + (V_2^0)^* y_{24} \cdot V_4^0 \right]$$

$$= -\text{Im} [(V_2^0)^* [Y_{21} \cdot V_1^1 + y_{22} \cdot V_2^0 + y_{23} \cdot V_3^0 + y_{24} \cdot V_4^0]]$$

$$= -\text{Im} [1.04(-2+j8) \times 1.06 + (3.66-j14.664) \times 1.04 - (0.66-j2.66) \times 1 - (1-j4) \times 1]$$

$$= -\text{Im} [1.04 (-5.9264 - 23.73056j + 0.66 - j2.66 + 1-j4)]$$

$$= -\text{Im} (1.04 (-4.2604 + 17.06656j))$$

$$= 0.1108$$

Q2 lies within the limits. Therefore V_2 will be taken as $V_2(\text{spec})$ & phase angle as in this iteration.

P_2 is a generator bus therefore P_2 & Q_2 are taken as +ve the value of P_2 is specified & Q_2 as the value calculated above.

$$V_P^{k+1} = \frac{1}{Y_{PP}} \left[\frac{P_P - jQ_P}{(V_P^K)^*} - \sum_{q=1}^{p-1} Y_{Pq} V_q^{K+1} - \sum_{q=p+1}^n Y_{pq} \cdot V_q^K \right]$$

$$V_2^{k+1} = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} \cdot V_3^0 - Y_{24} V_4^0 \right]$$

$$= \frac{1}{3.66 - j14.664} \left[\frac{0.5 - 0.1108j}{1.04} + (2 - j8) \times 1.06 + (0.66 - j2.66) \times 1 + (1 - j4) \times 1 \right]$$

$$= 0.01602 + 0.064195j [4.26076 - 15.2505j]$$

$$= 1.04726 + 0.029206j$$

$$= 1.04796 \angle 1.5969$$

$$= \delta_2^{0+1} = 1.5969$$

$$V_{2\text{acc}}^1 = 1.04 \angle 1.5969 = 1.03959 + 0.02898j$$

$$V_3^1 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31} V_1^1 - Y_{32} \cdot V_2^1 - Y_{34} V_4^0 \right]$$

$$= (0.016 + 0.0642j) \left[\frac{-0.4 + j0.3}{1} + (1-j4) \times 1.06 + (0.66-j2.66) \right]$$

$$(1.03959 + 0.02898j) + (2 - j8) \times 1].$$

$$= (0.016 + 0.0642j) (3.423216 - 14.6862j)$$

$$= 0.99773 - 0.01565j$$

$$V_4^1 = (0.0196 + 0.07843j) \left[\frac{-0.3 + j0.1}{1} + (1-j4) \times (1.03959 + 0.2898j) + \right.$$

$$(2 - j8) \times (0.99773 - j0.01565)$$

$$= (0.0196 + j0.07843) [2.72577 - 12.04252j]$$

$$= 0.99792 - 0.022251j$$

(3). In the above problem the reactive power constrain on a generator bus is $0.2 \leq Q_2 \leq 1.0$ solve the voltages at the end of first iteration $\alpha = 1$.

Solv: - $Q_2 = 0.11082$

Since Q_2 calculated corresponding to $V_2=1+j0$ is 0.11082 which is less than minimum specified.

Fix $Q_2 = Q_{2\min} = 0.2$ & the bus is treated as a load bus for this iteration. The generator bus to be treated as load bus means that the specified quantities P & Q and unknown values are V & δ . But P & Q are to be taken as +Ve as in the case of generator bus.

$$\begin{aligned}
 V_2^{K+1} &= \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_3^0)^{\phi}} - Y_{21}V_1^1 - Y_{23} \cdot V_3^0 - Y_{24} V_4^0 \right] \\
 &= \frac{1}{3.66 - j14.664} \left[\frac{0.5 + j0.2}{1.04} + (2-j8) \times 1.06 + (0.666-j2.664) \times 1 + (1-j4) \times 1 \right] \\
 &= (0.01602 + 0.6-j0.64195j) [4.266769 - 15.3363j] \\
 &= 1.052867 + 0.02822 j \\
 V_{2acc}^1 &= 1.0532 \angle 1.5353 \\
 \delta_{12}^1 &= 1.5353
 \end{aligned}$$

$$\begin{aligned}
 V_3^1 &= \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{(V_3^*)^*} - Y_{31}V_1^1 - Y_{32} \cdot V_2^1 - Y_{34} V_4^0 \right] \\
 &= (0.01602 + 0.0642195j) \left[\frac{-0.4 + j0.3}{1} + (1-j4) \times 1.06 + (0.66-j2.66) \times \right. \\
 &\quad \left. (1.052867 + 0.02822 j) + (2 - j8) \times 1 \right] \\
 &= (0.01602 + 0.064195 j) [3.43007 - 14.7262 j] \\
 &= 1.00029 - 0.01575 j \\
 V_4^1 &= \frac{1}{Y_{44}} \left[\frac{P_4 - jQ_4}{(V_4^*)^*} - Y_{42}V_2^1 - Y_{43} \cdot V_3^1 \right] \\
 &= \frac{1}{3-j12} \left[\frac{0.3 + j0.1}{1} + (1-j4) \times (1.052867 + 0.02822 j) + (2-j8) \times (1.00029 \right. \\
 &\quad \left. - 0.01572 j) \right] \\
 &= (0.0196 + 0.07843 j) [2.740567 - 12.1175] \\
 &= 1.004 - j 0.02255
 \end{aligned}$$

→ The following fig. Shows a 3 bus P.S N/W

Bus 1 → $V = 1.05 \angle 0^\circ$ p.u

Bus 2 → $V = 1$ p.u, P.G = 3 p.u

Bus 3 → $P_L = 4$ p.u, $Q_L = 2$ p.u

Carryout load flow solution by gauss siedel method at the end of first iteration.

Neglect the limits on reactive power generation.

$$Y_{\text{Bus}} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -5.833j & 2.5j & 3.33j \\ 2.5j & -5.833j & 3.33j \\ 3.33j & 3.33j & -6.667j \end{bmatrix}$$

Bus 2: -

$$Q_2 = -\text{Im} \left[V_P^* \sum_{q=1}^{p-1} Y_{Pq} V_q^{K+1} + (V_P^K)^* \sum_{q=P}^n Y_{Pq} V_q^K \right]$$

$$Q_2 = -\text{Im} \left[(V_2^*)^0 [Y_{23} V_2^0 + Y_{21} V_1^1 + Y_{22} V_2^0] \right]$$

$$= -\text{Im} [1 \times [3.33j \times 1 + 2.5j \times 1.05]]$$

$$= -5.883 - 0.122$$

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^*)^0} - Y_{21} V_2^1 - Y_{23} V_3^0 \right]$$

$$= \frac{1}{-5.833j} \left[\frac{3 - 0.122j}{1} - 2.5j \times 1.05 + 3.33j \times 1 \right]$$

$$= 0.171438j [3 - 6.077j] = 1.04183 + 0.514314j$$

$$= 1.16186 \angle 26.27^\circ.$$

$$V_2^1 = 1 \angle 26.27^\circ = 0.89672 + 0.4426j$$

$$V_3^1 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{(V_3^*)^0} - Y_{32} V_2^1 - Y_{31} V_1^1 \right]$$

$$\begin{aligned}
&= \frac{1}{-6.667j} \left[\frac{-0.4 + 2j}{1} - 3.33j \times (0.89672 + 0.4426j) - 3.33j \times 1.05 \right] \\
&= 0.14999j [-2.526142 - 4.482578j] \\
&= 0.67234 - 0.378896j = 0.7717 \angle -29.4^\circ
\end{aligned}$$

(4). Consider a 3-bus system shown in fig. The line important are given in p.u & line charging is neglected. Take bus 1 as slack bus with voltage $V = 1.05 \angle 0^\circ$. at bus (2) $P_{G2} = 25\text{MW}$, $Q_{G2} = 15\phi \text{ MVAR}$, $P_{L2} = 50 \text{ MW}$, $Q_{L2} = 25 \text{ MVAR}$ & at bus 3, $P_{L3} = 60 \text{ MW}$ & $Q_{L3} = 30 \text{ MVAR}$. Assume 100 MVA as base MVA carryout one iteration of load flow using Gauss Siedel method & $\alpha = 1.4$

Solv: -

At Bus (2)

$$P_2 = P_{G2} - P_{L2}$$

$$= 25 - 50 = -25 \text{ MW}$$

$$Q_2 = Q_{G2} - Q_{L2}$$

$$= 15 - 25 = -10 \text{ MVAR}$$

$$Y_{11} = \frac{1}{0.05 + j0.25} + \frac{1}{0.03 + j0.15}$$

$$= 2.0513 - 10.2564j$$

$$Y_{12} = -\frac{1}{0.05 + j0.25} = -0.76923 + 3.84615j$$

$$\begin{aligned}
P_2 \text{ (in p.u)} &= -\frac{25}{100} = -0.25 \text{ p.u} \\
Q_2 &= \frac{-10}{100} = -0.1 \text{ p.u}
\end{aligned}
\left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} \text{Assuming Pf} = 1 \\ \text{KVA} = \text{KW} \times \text{Cos}\phi \end{array}$$

$$P_3 = \frac{60}{100} = 0.6 \text{ p.u}; \quad Q_3 = \frac{30}{100} = 0.3 \text{ p.u}$$

$$Y_{\text{Bus}} = \begin{bmatrix} 2.0513 - 10.2564j & -0.76923 + 3.84615j & -1.282051 + 6.4102j \\ -0.76923 + 3.84615j & 1.730769 - 8.6538j & -0.96154 + 4.80769j \\ -1.282051 + 6.4102j & -0.96154 + 4.80769j & 2.2435 - 11.2179j \end{bmatrix}$$

$$V_2^1 = \frac{1}{Y_{22}} \left[-Y_{23}V_3^0 - Y_{21}V_1^1 + \frac{P_2 - jQ_2}{(V_2^*)^0} \right]$$

$$= \frac{1}{1.730769 - 8.6538j} \left[\frac{-0.25 + 0.1j}{1} - (-0.96154 + 4.80769j)(-0.76923 + 3.84615j) \times 1.05 \right]$$

$$= (0.0222 + 0.1111 j) [1.519232 - 8.7462 j]$$

$$= 1.005429 - 0.02537 j$$

$$V_{2(\text{acc})}^1 = 1 + 1.4 [1.005429 - 0.02537 j - 1 j] = 1.0076 - 0.03552 j$$

$$\begin{aligned} V_3^1 &= \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{(V_3^*)^0} - Y_{31} V_1^1 - Y_{32} V_2^1 \right] \\ &= \frac{1}{2.2435 - 11.2179 j} \left[\frac{-0.6 + 0.3 j}{1} + (1.282051 - 6.4102 j) \times 1.05 \right. \\ &\quad \left. + (0.96154 - 4.80769 j) \times (1.0076 - 0.03552 j) \right] \\ &= (0.017142 + 0.085715 j) [1.54423 - 11.303909 j] \\ &= 0.995829 - 0.061496 j \\ V_{3(\text{acc})}^1 &= 1 + 1.4 (0.995829 - 0.061496 j - 1) \\ &= 0.99416 - 0.086094 j \end{aligned}$$

(5). A 2 bus system is shown in fig. At bus 1 $V_1 = 1 \angle 0^\circ$. The complex power generation $S_{g1} = 1 + j1$; $S_{D1} = 0.5 + j1$, at bus 2 $S_{g2} = 0 + j1.0$; $S_{D2} = 0.5 + j1$ find V_2 at the end of 2nd iteration by GS method. Assuming the line connecting the buses have $Z_L = j 0.5 \text{ p.u}$ & shunt sustenance is neglected.

Ans: -

$$P_1 = P_{g1} = P_{D1}$$

$$Q_1 = Q_{g1} = Q_{D1}$$

$$S_1 = S_{g1} - S_{D1} = 1 + j - 0.5 - j = 0.5 \text{ p.u}$$

$$S_2 = S_{g2} - S_{D2} = 0 + j - 0.5 - j = -0.5 \text{ p.u}$$

$$Y_{\text{Bus}} = \begin{bmatrix} -2j & +2j \\ +2j & -2j \end{bmatrix}$$

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^*)^0} - Y_{21} V_1^1 \right] = \frac{1}{-2j} \left[\frac{-0.5}{1} - 2j \times 1 \right]$$

$$= +0.5 j (+0.5 - 2j) = 1 - 0.25 j$$

$$V_2^2 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^*)^1} - Y_{21} V_1^1 \right]$$

$$V_2^{11} = \left[\frac{+0.5}{+1 + 0.25j} - 2j \times 1 \right] \times \frac{1}{-2j}$$

$$= 0.5 j [-0.4706 - 0.11765 j - 2j]$$

$$= -1 + 0.294125 J + 0.941175 - 0.2353 J$$

(6). A 2 bus system has been shown in fig. Determine the voltage at bus 2 by as method after 2nd iteration.

$$Y_{11} = Y_{22} = 1.6 \angle -80^\circ \text{ p.u}$$

$$Y_{21} = Y_{12} = 1.9 \angle 100^\circ \text{ p.u}$$

$$V_1 = 1.1 \angle 0^\circ.$$

Ans: -

$$Y_{Bus} = \begin{bmatrix} 0.2778 - 1.5756j & -0.3299 + 1.87113j \\ -0.3299 + 1.87113j & 0.2778 - 1.5756j \end{bmatrix}$$

$$P_2 = 0.5, Q_2 = 0.3 \text{ \& } P_1 = 1.1, Q_1 = 0.2 \text{ p.u}$$

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^*)^0} - Y_{21} \cdot V_1^1 \right]$$

$$= \frac{1}{0.2778 - 1.5756j} \left[\frac{-0.5 + j0.3}{1} + (0.3299 - 1.87113j) \times 1.1 \right]$$

$$= (0.10853 + 0.6155j) [-0.13711 - 1.758243j]$$

$$= 1.06732 - 0.2752j$$

$$V_2^2 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^*)^1} - Y_{21} \cdot V_1^1 \right]$$

$$= (0.10853 + 0.6155j) \left[\frac{-0.5 + j0.3}{1.06732 + 0.2752j} + (0.3299 - 1.87113j) \times 1.1 \right]$$

$$= (0.10853 + 0.6155j) (-0.14433 - 1.90795j)$$

$$= 1.034 - 0.187645j$$

(7). For the N/W shown in fig. Obtain complex bus bar voltages at the end of first iteration using GS method. Bus 1 is a slack bus with voltage $1 \angle 0^\circ$ consider

$$P_2 + jQ_2 = -5.96 + j1.46, P_3 = 6.02 \text{ \& } |V_3| = 1.02$$

(7). The data for 2-bus system is given below S_{G1} , S_{D1} , is unknown $V_1 = 1.0 \angle 0^\circ$; $S_1 =$ to be determined. $S_{G2} = 0.25 + j Q_{G2}$; $S_{D2} = 1 + j 0.5$ p.u 2 buses are connected by a transformer line for p.u reactance of 0.5 p.u find Q_2 & angle of V_2 , neglect shunt susceptance of the tie line assume $V_2 = 1$ perform 2 iterations using GS method.

Ans: -

$$V_1 = 1.0 \angle 0^\circ$$

$$V_2 = 1 \text{ p.u}$$

$$Q_2^1 = -\text{Im} \left\{ (V_2^*)^0 [Y_{21} \cdot V_1^1 + Y_{22} \cdot V_2^0] \right\}$$

$$= \text{Im} [1 \times +2j + -2j \cdot 1]$$

$$= - \text{p.u}$$

$$P_2 = P_{G2} - P_{D2} = 0.25 - 1 = -0.75 \text{ p.u}$$

$$Q_2 = Q_{G2} - Q_{D2} = Q_{G2} - 0.5$$

$$Q_{G2} = -0.5 \text{ p.u}$$

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^*)^0} - Y_{21} \cdot V_1^1 \right]$$

$$= \frac{1}{-2j} \left[\frac{-0.75 - j0}{1} - (2j) \times 1 \right]$$

$$= +0.5j (+0.75 - 2j)$$

$$= 1 - 0.375j = 1.068 \angle -20.556^\circ$$

$$P_{\text{u}} = 0.9363 - 0.35112j$$

$$Y_{\text{Bus}} = \begin{bmatrix} -2j & +2j \\ 2j & -2j \end{bmatrix}$$

$$Q_2^2 = -\text{Im} \left\{ (V_2^*)^1 [Y_{21} \cdot V_1^2 + Y_{22} \times V_2^1] \right\}$$

$$= -\text{Im} \left\{ (1 + 0.375j) [2j \times 1 + -2j \times (1 - 0.375j)] \right\}$$

$$= -\text{Im} \left\{ (1 + 0.375j) (-0.70224 + 0.1274j) \right\}$$

$$= 0.12734 \text{ P.U}$$

$$V_2^2 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^*)^0} - Y_{21} \cdot V_1^2 \right]$$

$$= \frac{1}{-2j} \left[\frac{-0.75 - 0.28125j}{0.93 + 0.375j} + (-2j) \times 1 \right]$$

$$= 0.92 + 9 - 0.3735 \text{ J}$$

$$= 1.00025 \angle -21.95$$

$$\delta^2 = -21.95$$

$$V_2^2 = 1 \angle -21.95$$

Advantages of GS method: -

1. Time required for iteration is less.

Disadvantages: -

1. Depending upon slack bus no of iteration will increase

Advantages of GS method: -

1. Simplicity
2. Less computer memory requirement
3. Less computational time for iteration.

Disadvantages: -

1. Slow rate of convergence therefore it require more no of iterations.
2. Increase of iteration with of buses.
3. Effect of convergence due to the choice of slack bus.

Due to these disadvantages this method is used only for the systems having small no. of iterations.

4. What are PV buses how are they handle in the LFS

1. (a). Explain clearly the classification of buses in load flow problem.

- (b). Derive static load flow equations.

2. (a). Explain with a flow chart the computational procedure for load flow solution using gauss seidel method

3. How do you determine LF voltage when reactive power states is not specified.

NEWTON RAPHSON METHOD: -

NR method is quadratic in rate of convergence but GS is linear convergence method.

The load flow equations are a set of non-linear algebraic equations. Hence, numerical iterative techniques is required. For obtaining solution to them the NR method has a quadratic rate of convergence, where as Gauss, Gauss Seidel methods have a linear rate of convergence.

Advantages: -

1. More accuracy & surety of convergence.
2. Only about 3 iterations are required as compared to more than 25 or 50 in as method.
3. NO. of iterations is almost independent of system size.
4. This method is insensitive to slack bus selection, regulating transformer etc.
5. It does not require any α

Disadvantages: -

1. The solution technique is difficult.
2. Calculations in each iteration are more computational time per iteration is large.
3. This computer memory requirement is large.
4. This method can be used for large system.

→ NR method is an iterative method which approximates the an iterative method which approximates the set of non linear simultaneous equations to a set of linear simultaneous equation by using taylor series expansion & the terms are limited to first approximation.

Mathematical background about NRM Analysis: -

Let us consider a set of non-linear equations. Let the unknown variables be (x_1, x_2, \dots, x_n) and the specified quantities be (y_1, y_2, \dots, y_n) . These are related by a set of non-linear equations.

$$\left. \begin{array}{l} Y_1 = f_1(x_1, x_2, \dots, x_n) \\ Y_2 = f_2(x_1, x_2, \dots, x_n) \\ | \\ Y_n = f_n(x_1, x_2, \dots, x_n) \end{array} \right\} \rightarrow (1)$$

To solve these equations let the initial solution for the values of x s are $(x_{10}, x_{20}, \dots, x_{n0})$ the initial solution will not satisfy the set of equations exactly. Assume $\Delta x_1^0, \Delta x_2^0, \dots, \Delta x_n^0$ are the corrections required for the initial solution method to satisfy the above equations.

$$\begin{aligned}
y_1 &= f_1(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots, x_n^0 + \Delta x_n^0) \\
y_2 &= f_2(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots, x_n^0 + \Delta x_n^0) \\
&| \\
&| \\
y_n &= f_n(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots, x_n^0 + \Delta x_n^0)
\end{aligned}$$

R.H.S of these equations can be expanded using Taylor series expansion about the initial solution.

$$\begin{aligned}
y_1 &= f_1(x_1^0, x_2^0, \dots, x_n^0) + \frac{\partial f_1}{\partial x_1} \bigg|_{x^0} \Delta x_1^0 + \frac{\partial f_1}{\partial x_2} \bigg|_{x^0} \Delta x_2^0 + \dots + \frac{\partial f_1}{\partial x_n} \bigg|_{x^0} \Delta x_n^0 + \phi_0 \\
y_2 &= f_2(x_1^0, x_2^0, \dots, x_n^0) + \frac{\partial f_2}{\partial x_1} \bigg|_{x^0} \Delta x_1^0 + \frac{\partial f_2}{\partial x_2} \bigg|_{x^0} \Delta x_2^0 + \dots + \frac{\partial f_2}{\partial x_n} \bigg|_{x^0} \Delta x_n^0 + \phi_0 \\
&| \\
y_n &= f_n(x_1^0, x_2^0, \dots, x_n^0) + \frac{\partial f_n}{\partial x_1} \bigg|_{x^0} \Delta x_1^0 + \frac{\partial f_n}{\partial x_2} \bigg|_{x^0} \Delta x_2^0 + \dots + \frac{\partial f_n}{\partial x_n} \bigg|_{x^0} \Delta x_n^0 + \phi_0
\end{aligned}$$

Where ϕ_0 involves higher order derivatives and higher powers of Δx^s hence they are neglected according to NR method.

All these equations are written in matrix form then we get,

$$\begin{bmatrix} y_1 = f_1(x_1^0, x_2^0, \dots, x_n^0) \\ y_2 = f_2(x_1^0, x_2^0, \dots, x_n^0) \\ | \\ y_n = f_n(x_1^0, x_2^0, \dots, x_n^0) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ | \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ | \\ \Delta x_n^0 \end{bmatrix}$$

$$[D] = [J] [C]$$

Where D = Diff between specified quantities and calculated quantities.

J = Jaccobian Matrix

C = (Correction Matrix) Solution for matrix

$$C = [J]^{-1} [D]$$

The next better solution is obtained by

$$\begin{aligned}
x_1^0 &= x_1^0 + \Delta x_1^0 ; & x_2^0 &= x_2^0 + \Delta x_2^0 & \dots & x_n^0 &= x_n^0 + \Delta x_n^0 \\
\downarrow & & \downarrow & & & & \\
\end{aligned}$$

Initial Value

Incremental Value

This process is repeated until 2 successive values for each x_1 differ only by a specified tolerance, which is very small value.

Application of NR method to the solution of LF equations: -

For the application of NR method the load flow equations have to be formulated in a suitable form. The formation can be either in rectangular or polar form.

Formation in Rectangular Coordinates: -

Case I: -

Consider 'n' bus system there is only one generator bus which is taken as a slack bus and all other buses will be considered as PQ buses.

Let the voltages & admittances are expressed in rectangular form as,

$$V_p \angle \delta_p = e_p + j \delta_p f_p$$

$$V_q \angle \delta_q = e_q + j \delta_q f_q$$

$$Y_{pq} = G_{pq} - j B_{pq}$$

Where e_p = real part of voltage V_p .

f_p = imaginary part of voltage V_p

G_{pq} = Conductance

B_{pq} = Susceptance

Power injected at any bus 'P' = $P_p - jQ_p$

$$P = V_p^* \cdot I_p.$$

$$I_p = \sum_{q=1}^n y_{pq} \cdot V_q = \sum_{q=1}^n (G_{pq} - jB_{pq}) (e_q + jf_q)$$

$$\begin{aligned}
P_P - JQ_P &= V_P^* \left\{ \sum_{q=1}^n (G_{pq} \cdot e_q - JB_{pq}) (e_q + Jf_q) \right\} \\
&= (e_q - Jf_q) \left\{ \sum_{q=1}^n (G_{pq} \cdot e_q + B_{pq} \cdot f_q) (-JB_{pq} \cdot e_q + JG_{pq} f_q) \right\} \\
P_P &= e_P \sum_{q=1}^n (G_{pq} \cdot e_q + B_{pq} \cdot f_q) + f_P \sum_{q=1}^n (-B_{pq} \cdot e_q + G_{pq} f_q) \\
Q_P &= f_P \sum_{q=1}^n (G_{pq} \cdot e_q + B_{pq} \cdot f_q) + e_P \sum_{q=1}^n (B_{pq} \cdot e_q + G_{pq} f_q) \\
P_P &= \sum_{q=1}^n \left\{ (G_{pq} \cdot e_q + B_{pq} \cdot f_q) + f_P (G_{pq} \cdot e_q - B_{pq} \cdot e_q) \right\} \longrightarrow (1) \\
Q_P &= \sum_{q=1}^n \left\{ (G_{pq} \cdot e_q + B_{pq} \cdot f_q) - e_P (G_{pq} \cdot e_q - B_{pq} \cdot e_q) \right\} \longrightarrow (2) \\
V_P &= e_P + J f_P \Rightarrow V_P^2 = e_P^2 + f_P^2 \longrightarrow (3)
\end{aligned}$$

The equations (1), (2) & (3) are load flow equations in rectangular coordinates.

The real & imaginary powers P_P & Q_P are known. The real & imaginary components of voltages are unknown for all buses except slack bus.

Let P_2, P_3, \dots, P_n are the specified real powers Q_2, Q_3, \dots, Q_n be the specified reactive powers of (n-1) buses. The unknown variables are real part of bus voltages e_2, e_3, \dots, e_n and imaginary part of bus voltage f_2, f_3, \dots, f_n now the matrix equation for this P.S is written as,

$$D = [J] C$$

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ | \\ \Delta P_n \\ | \\ \Delta Q_2 \\ \Delta Q_3 \\ | \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_1} & \frac{\partial P_2}{\partial e_2} & \dots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_1} & \dots & \frac{\partial P_2}{\partial f_n} \\ \frac{\partial P_3}{\partial e_1} & \frac{\partial P_3}{\partial e_2} & \dots & \frac{\partial P_3}{\partial e_n} & \frac{\partial P_3}{\partial f_1} & \dots & \frac{\partial P_3}{\partial f_n} \\ | & | & & | & | & & | \\ \frac{\partial P_n}{\partial e_1} & \frac{\partial P_n}{\partial e_2} & \dots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_1} & \dots & \frac{\partial P_n}{\partial f_n} \\ | & | & & | & | & & | \\ \frac{\partial Q_2}{\partial e_1} & \frac{\partial Q_2}{\partial e_2} & \dots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_1} & \dots & \frac{\partial Q_2}{\partial f_n} \\ \frac{\partial Q_3}{\partial e_1} & \frac{\partial Q_3}{\partial e_2} & \dots & \frac{\partial Q_3}{\partial e_n} & \frac{\partial Q_3}{\partial f_1} & \dots & \frac{\partial Q_3}{\partial f_n} \\ | & | & & | & | & & | \\ \frac{\partial Q_n}{\partial e_1} & \frac{\partial Q_n}{\partial e_2} & \dots & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_1} & \dots & \frac{\partial Q_n}{\partial f_n} \end{bmatrix} \begin{bmatrix} \Delta e_1 \\ \Delta e_2 \\ | \\ \Delta e_n \\ \Delta f_1 \\ \Delta f_2 \\ | \\ \Delta f_n \end{bmatrix}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

$$\text{Where } J_1 = \frac{\partial P_p}{\partial e}; \quad J_2 = \frac{\partial P_p}{\partial f}; \quad J_3 = \frac{\partial Q_p}{\partial e}; \quad J_4 = \frac{\partial Q_p}{\partial f}$$

Case II: - Both PQ & PV buses: -

When the system has both PQ buses and PV buses. First bus is selected as a slack bus and the buses 2 to m are PQ buses and the buses M+1 to n are called as PV buses. Let p_2, p_3, \dots, p_n are the specified real powers of n-1 buses & Q_2, Q_3, \dots, Q_m are the specified reactive powers are of the load buses. Let $|V_{m+1}|, |V_{m+2}|, \dots, V_n$ are the specified voltage magnitudes of generator buses. The unknown e_2, e_3, \dots, e_n and imaginary of bus voltages f_2, f_3, \dots, f_n .

Now the matrix equation...

$D = [J]$ C can be written as,

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ | \\ | \\ \Delta P_n \\ \Delta Q_2 \\ \Delta Q_3 \\ | \\ \Delta Q_m \\ |\Delta V_{m+1}|^2 \\ | \\ |\Delta V_{Pn}|^2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \frac{\partial P_2}{\partial f_n} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \frac{\partial P_3}{\partial e_n} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} & \frac{\partial P_3}{\partial f_n} \\ | & | & | & | & | & | \\ \frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & \frac{\partial P_n}{\partial f_n} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & \frac{\partial Q_2}{\partial f_n} \\ \frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & \frac{\partial Q_3}{\partial e_n} & \frac{\partial Q_3}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} & \frac{\partial Q_3}{\partial f_n} \\ | & | & | & | & | & | \\ \frac{\partial Q_m}{\partial e_2} & \frac{\partial Q_m}{\partial e_3} & \frac{\partial Q_m}{\partial e_n} & \frac{\partial Q_m}{\partial f_2} & \frac{\partial Q_m}{\partial f_3} & \frac{\partial Q_m}{\partial f_n} \\ \frac{\partial |V_{M+1}|^2}{\partial e_2} & \frac{\partial |V_{M+1}|^2}{\partial e_3} & \frac{\partial |V_{M+1}|^2}{\partial e_n} & \frac{\partial |V_{M+1}|^2}{\partial f_2} & \frac{\partial |V_{M+1}|^2}{\partial f_3} & \frac{\partial |V_{M+1}|^2}{\partial f_n} \\ | & | & | & | & | & | \\ \frac{\partial V_n^2}{\partial e_2} & \frac{\partial V_n^2}{\partial e_3} & \frac{\partial V_n^2}{\partial e_n} & \frac{\partial V_n^2}{\partial f_2} & \frac{\partial V_n^2}{\partial f_3} & \frac{\partial V_n^2}{\partial f_n} \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ | \\ | \\ \Delta e_n \\ \Delta f_2 \\ \Delta f_3 \\ | \\ \Delta f_n \end{bmatrix}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta V_P^2 \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \\ J_5 & J_6 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix}$$

$$\text{Where } J_1 = \frac{\partial P}{\partial e}; \quad J_2 = \frac{\partial P}{\partial f}; \quad J_3 = \frac{\partial Q}{\partial e}; \quad J_4 = \frac{\partial Q}{\partial f}; \quad J_5 = \frac{\partial |V_P|^2}{\partial e}; \quad J_6 = \frac{\partial |V_P|^2}{\partial f}$$

Derivation of Jaccobians: -

$$J_1 = \frac{\partial P}{\partial e}$$

$$P_P = \sum_{q=1}^n \left\{ e_p (G_{pq} \cdot e_q + B_{pq} \cdot f_q) + f_p (G_{pq} \cdot e_q - B_{pq} \cdot e_q) \right\}$$

$$Q_P = \sum_{q=1}^n \left\{ f_p (G_{pq} \cdot e_q + B_{pq} \cdot f_q) - e_p (G_{pq} \cdot e_q - B_{pq} \cdot e_q) \right\}$$

$$|V_P|^2 = e_P^2 + f_P^2$$

$$J_1 = \frac{\partial P}{\partial e} :-$$

$$\frac{\partial P_P}{\partial e_q} \text{ (half diagonal elements)}$$

$$\sum_{q=1}^n (G_{pq} \cdot e_q - B_{pq} \cdot f_q) q \neq P$$

$$\frac{\partial P_P}{\partial e_P} (\text{Diagonal terms})$$

$$\text{Put } q = p$$

$$P_P = \sum_{q=1}^n \left\{ e_P (G_{pq} \cdot e_q + B_{pq} \cdot f_q) + f_P (G_{pq} \cdot f_P - B_{pq} \cdot e_q) \right\}$$

$$\frac{\partial P_P}{\partial e_P} = 2e_P \cdot G_{PP} + B_{PP} \cdot f_P - B_{PP} \cdot f_P = 2e_P \cdot G_{PP} + \sum_{\substack{q=1 \\ q=1}}^n (G_{Pq} \cdot e_q + B_{pq} \cdot f_q)$$

$$\frac{\partial P_P}{\partial e_P} = 2e_P \cdot G_{PP} + \sum_{\substack{q=1 \\ q=1}}^n (G_{Pq} \cdot e_q + B_{pq} \cdot f_q)$$

$$J_2 = \frac{\partial P}{\partial t} : -$$

$$\text{Half diagonalelements} = \frac{\partial P_P}{\partial f_P}$$

$$= \sum_{q=1}^n \left\{ G_{pq} \cdot f_q + B_{pq} \cdot e_q \right\} B_{pq} \cdot e_P + 2G_{PP} \cdot f_P - B_{PP} \cdot e_P$$

$$= 2G_{PP} \cdot f_P + \sum_{q=1}^n \left\{ G_{pq} \cdot f_q + B_{pq} \cdot e_q \right\}$$

$$J_3 = \frac{\partial Q}{\partial e} : -$$

$$\text{Half diagonalelements} = \frac{\partial Q_P}{\partial e_q}$$

$$= \sum_{\substack{q=1 \\ q=1}}^n (G_{pq} \cdot f_q + B_{pq} \cdot e_q)$$

$$\text{Diagonal elements} = \frac{\partial Q_P}{\partial e_P}$$

$$(Q_P)_{q=P} = \sum_{q=1}^n \left\{ f_P (G_{pq} \cdot e_q + B_{pq} \cdot f_q) - e_P (G_{PP} \cdot F_P - B_{PP} \cdot e_P) \right\}$$

$$\frac{\partial Q_P}{\partial e_P} = 2e_P \cdot G_{PP} + 2B_{PP} \cdot e_P - f_P B_{PP} + \sum_{q=1}^n (G_{pq} \cdot f_q - B_{pq} \cdot e_q)$$

$$= 2e_P \cdot G_{PP} + \sum_{q=1}^n (G_{pq} \cdot f_q - B_{pq} \cdot e_q)$$

$$J_4 = \frac{\partial Q}{\partial f} : -$$

$$\frac{\partial Q_P}{\partial f_P} = 2 B_{PP} \cdot f_P + \sum_{q=1}^n (G_{Pq} \cdot e_q + B_{Pq} \cdot f_q)$$

$$\frac{\partial Q_P}{\partial f_q} = \sum_{\substack{q=1 \\ q \neq P}}^n (B_{Pq} \cdot f_P - e_P \cdot G_{Pq})$$

$$J_5 = \frac{\partial |V_P|^2}{\partial e} : -$$

$$\frac{\partial |V_P|^2}{\partial e} = 2 e_P$$

$$\frac{\partial |V_P|^2}{\partial e_q} = 0$$

$$J_6 = \frac{\partial |V_P|^2}{\partial f} : -$$

$$\frac{\partial |V_P|^2}{\partial f_P} = 2 f_P; \frac{\partial |V_P|^2}{\partial f_Q} = 0$$

Let P_{Spe} , Q_{spe} & $|V_{\text{spec}}|^2$ be the specified quantities at bus P and P_P^0 , Q_P^0 and $|V_P^0|^2$ are the calculated values.

$$\Delta P_P = P_{\text{spec}} - P_P^0$$

$$\Delta Q_P = Q_{\text{spec}} - Q_P^0$$

$$|\Delta V_P|^2 = |V_{\text{Spec}}|^2 - |V_P^0|^2.$$

After calculated the Jaccobian element and L.H.S column (difference matrix) the

voltage vector $\begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix}$ can be calculated by using.

$$\begin{bmatrix} \Delta e^0 \\ \Delta f^0 \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P_P^0 \\ \Delta Q_P^0 \\ |\Delta V_P^0|^2 \end{bmatrix}$$

$$e_P^1 = e_P^0 + \Delta e_P^0; f_P^1 = f_P^0 + \Delta f_P^0$$

These Values will be used in the next iteration. In general new better estimates for the bus voltages is $e_P^{K+1} = e_P^K + \Delta e_P^K$ & $f_P^{K+1} = f_P^K + \Delta f_P^K$. The process is

repeated till the magnitude of the largest element in the residual column vector is less than the specified value.

(1) The load flow data for a sample system are given below. The voltage magnitude at bus 2 is maintained at 1.04 p.u. The maximum and minimum reactive power limits of the generator bus are 0.35 p.u & 0 p.u. Determine the set of LF equations at the end of first iteration using NR method in rectangular coordination.

Bus Data

<u>Line Data</u>	<u>Impedance</u>	<u>Bus code</u>	<u>Generation Load</u>				<u>Assumed voltage</u>
Bus Code			MN	MVAR	MW	MVAR	
			P _G	Q _G	P ₂	Q ₂	
1 - 2	0.08 + J 0.24	1	0	0	0	0	1.06+J0.0
1 - 3	0.02 + J 0.06	2	0.2	0	0	0	1.04
2 - 3	0.06 + J 0.18	3	0	0	0.6	0.25	1

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$Y_{11} = Y_{12} + y_{13} = 6.25 - 18.75 J$$

$$Y_{12} = - Y_{12} = - 1.25 + 3.75 J$$

$$Y_{13} = - 5 + 15 J$$

$$Y_{Bus} = \begin{bmatrix} 6.25 - 18.75J & -1.25 + 3.75J & -5 + 15J \\ (G_{11} - JB_{11}) & (G_{12} - JB_{12}) & (G_{13} - JB_{13}) \\ -1.25 + J3.751 & 2.917 - J8.75 & -1.667 + J5 \\ (G_{21} - JB_{21}) & (G_{22} - JB_{22}) & (G_{23} - JB_{23}) \\ -5 + 15J & -1.667 + J5 & 6.667 - 20J \\ (G_{31} - JB_{31}) & (G_{32} - JB_{32}) & (G_{33} - JB_{33}) \end{bmatrix}$$

Bus 2: -

$$P_P = \sum_{q=1}^n \left\{ (G_{pq} \cdot e_q + B_{pq} \cdot f_q) + f_P (G_{pq} \cdot f_q - B_{pq} \cdot e_q) \right\}$$

$$Q_P = \sum_{q=1}^n \left\{ f_P (G_{pq} \cdot e_q + B_{pq} \cdot f_q) - e_P (G_{pq} \cdot f_q - B_{pq} \cdot e_q) \right\}$$

$$\begin{aligned}
P_2 &= \sum_{q=1}^n \left\{ e_P (G_{pq} \cdot e_q) \right\} \\
&= e_2 (G_{21} \cdot e_1 + G_{22} \cdot e_2 + G_{23} \cdot e_3) \\
&= 1.04(-1.25 \times 1.06 + 2.917 \times 1.04 - 1.667 \times 1) \\
&= 0.0433
\end{aligned}$$

$$\begin{aligned}
Q_P &= \sum_{q=1}^n \left\{ e_P B_{pq} \cdot e_q \right\} \\
Q_2 &= e_2 (B_{21} \cdot e_1 + B_{22} \cdot e_2 + B_{23} \cdot e_3) \\
&= 1.04(-3.751 \times 1.06 + 8.75 \times 1.04 - 5 \times 1) \\
&= 0.12889
\end{aligned}$$

Bus 3:-

$$\begin{aligned}
P_3 &= e_3 (G_{31} \cdot e_1 + G_{32} \cdot e_2 + G_{33} \cdot e_3) \\
&= 1(-5 \times 1.06 - 1.667 \times 1.04 + 6.667 \times 1) \\
&= -0.36668
\end{aligned}$$

$$\begin{aligned}
Q_3 &= e_3 (B_{31} \cdot e_1 + B_{32} \cdot e_2 + B_{33} \cdot e_3) \\
&= 1(-15 \times 1.06 - 5 \times 1.04 + 20 \times 1) \\
&= -1.1
\end{aligned}$$

$$\Delta P_2 = P_{2\text{spec}} - P_{2\text{cal}} = 0.2 - 0.0433 = 0.1567$$

$$\Delta P_3 = 0.6 + 0.36668 = -0.2332$$

$$\Delta Q_2 = 0 - 0.13 = -0.13; \Delta Q_3 = -0.25 - 0 = -0.25$$

Procedure: - (Algorithm for NR method in RC): -

1. Assume a suitable solution for all buses except the slack bus. Let $V_p = 1+j0$ for $P=1,2,\dots,n$ except $P \neq S$.
2. Set convergence criterion = ϵ if the largest absolute of residues is $<\epsilon$. the process is terminated otherwise the process is repeated
3. Set iteration count $K=0$
4. Set bus count $P=0$
5. Check if P is a slack bus if yes go to step (10).
6. Calculate the real & reactive powers P_{pq} Q_p .
7. Evaluate $\Delta P^k = P_{\text{spec}}^k - P_{\text{cal}}^k$
8. Check if the bus is a generator bus if yes, compare the Q_p^k with the limits. If it exceeds the limit fix the reactive power generation to the

corresponding limit & treat the bus as a load bus for that iterations go to next step. If the limit is not violated evaluate the voltage residue.

$$\Delta V_P^2 = \Delta V_{PSpec}^2 - \Delta V_{pcas}^2 \text{ \& go to step (10)}$$

9. Evaluate $\Delta Q_P^K = Q_{PSpec}^K - Q_{Pcal}^K$.

10. Advance the bus count by 1 step $P = P+1$ & check if all the buses have been accounted if not go to step. 5

11. Determine the largest absolute value of the residue.

12. If the largest absolute value of residual $< \epsilon$ go to step 17.

13. Evaluate the elements of jacobian matrix.

14. Calculate the voltage increments Δe_P^K & Δf_P^K . Calculate the new bus voltages.

$$e_P^{K+1} = e_P^K + \Delta e_P^K \text{ \& } f_P^{K+1} = f_P^K + \Delta f_P^K$$

15. Evaluate for all the voltages.

16. Advance iteration count $K=K+1$ & go to step 4.

17. Evaluate bus & line powers & print the results.

→ The load flow data for sample P.S are given below the voltage at bus 2 is to be maintained at 1.04. The maximum and minimum Q at bus 2 are 0.35 p.u & 0.0 p.u. Determine the set of LF equations at the end of 1st iteration by using NR method in rec coordinates are given below:

