UNIT 3

Game Theory and Inventory Models

UNIT III
Game Theory

Introduction: The definition given by William G. Nelson runs as follows: "Game theory, more properly the theory of games of strategy, is a mathematical method of analyzing a conflict.
The alternative is not between this decision or that decision, but between this strategy or that strategy to be used against the conflicting interest".

In the perception of Robert Mockler, "Game theory is a mathematical technique helpful in making decisions in situations of conflicts, where the success of one part depends at the expense of others, and where the individual decision maker is not in complete control of the factors influencing the outcome".

According to von Neumann and Morgenstern, "The 'Game' is simply the totality of the rules which describe it. Every particular instance at which the game is played – in a particular way – from beginning to end is a 'play'. The game consists of a sequence of moves, and the play of a sequence of choices".

According to Edwin Mansfield, "A game is a competitive situation where two or more persons pursue their own interests and no person can dictate the outcome. Each player, an entity with the same interests, make his own decisions. A player can be an individual or a group".

Assumptions for a Competitive Game:

Game theory helps in finding out the best course of action for a firm in view of the anticipated counter moves from the competing organizations. A competitive situation is a competitive game if the following properties hold,
1. The number of competitors is finite, say N.
2. A finite set of possible courses of action is available to each of the N competitors.
3. A play of the game results when each competitor selects a course of action from the set of courses available to him. In game theory we make an important assumption that all the players select their courses of action simultaneously. As a result, no competitor will be in a position to know the choices of his competitors.
4. The outcome of a play consists of the particular courses of action chosen by the individual players. Each outcome leads to a set of payments, one to each player, which may be either positive, or negative, or zero.

Managerial Applications of the Games Theory:

The techniques of game theory can be effectively applied to various managerial problems as detailed below:
1) Analysis of the market strategies of a business organization in the long run.
2) Evaluation of the responses of the consumers to a new product.
3) Resolving the conflict between two groups in a business organization.
4) Decision making on the techniques to increase market share.
5) Material procurement process.
6) Decision making for transportation problem.
7) Evaluation of the distribution system.

Concepts in the Theory:

Players: The competitors or decision makers in a game are called the players of the game.

Strategies: The alternative courses of action available to a player are referred to as his strategies.

Pay off: The outcome of playing a game is called the payoff to the concerned player.

Optimal Strategy: A strategy by which a player can achieve the best pay off is called the optimal strategy for him.

Zero-sum game: A game in which the total payoffs to all the players at the end of the game is zero is referred to as a zero-sum game.

Non-zero-sum game: Games with "less than complete conflict of interest" are called non-zero sum games. The problems faced by a large number of business organizations come under this category. In such games, the gain of one player in terms of his success need not be completely at the expense of the other player.

Payoff matrix: The tabular display of the payoffs to players under various alternatives is called the payoff matrix of the game.

Pure strategy: If the game is such that each player can identify one and only one strategy as the optimal strategy in each play of the game, then that strategy is referred to as the best strategy for that Player and the game is referred to as a game of pure strategy or a pure game

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Mixed strategy: If there is no one specific strategy as the 'best strategy' for any player in a game, then the game is referred to as a game of mixed strategy or a mixed game. In such a game, each player has to choose different alternative courses of action from time to time.

N-person game: A game in which N-players take part is called an N-person game.

Maxi min-Mini max Principle: The maximum of the minimum gains is called the maxi min value of the game and the corresponding strategy is called the maxi min strategy. Similarly, the minimum of the maximum losses is called the mini max value of the game and the corresponding strategy is called the mini max strategy. If both the values are equal, then that would guarantee the best of the worst results.

Negotiable or cooperative game: If the game is such that the players are taken to cooperate on any or every action which may increase the payoff of either player, then we call it a negotiable or cooperative game.

Non-negotiable or non-cooperative game: If the players are not permitted for coalition, then werefer to the game as a non-negotiable or non-cooperative game.

Saddle point: A saddle point of a game is that place in the payoff matrix where the maximum of the row minima is equal to the minimum of the column maxima. The payoff at the saddle point is called **the value of the game** and the corresponding strategies are called the **pure strategies.**

Dominance: One of the strategies of either player may be inferior to at least one of the remaining ones. The superior strategies are said to dominate the inferior ones.

Types of Games:

There are several classifications of a game. The classification may be based on various factors such as the number of participants, the gain or loss to each participant, the number of Strategies available to each participant, etc. Some of the important types of games are enumerated below.

Two person games and n-person games: In two person games, there are exactly two players and each competitor will have a finite number of strategies. If the number of players in a game exceeds two, then we refer to the game as n-person game.

Zero sum game and non-zero sum game: If the sum of the payments to all the players in a game is zero for every possible outcome of the game, then we refer to the game as a zero sum game. If the sum of the payoffs from any play of the game is either positive or negative but not zero, then the game is called a non-zero sum game

Games of perfect information and games of imperfect information: A game of perfect information is the one in which each player can find out the strategy that would be followed by his opponent. On the other hand, a game of imperfect information is the one in which no player can know in advance what strategy would be adopted by the competitor and a player has to proceed in his game with his guess works only.

Games with finite number of moves / players and games with unlimited number of moves: A game with a finite number of moves is the one in which the number of moves for each player is limited before the start of the play. On the other hand, if the game can be continued over an extended period of time and the number of moves for any player has no restriction, then we call it a game with unlimited number of moves.

Constant-sum games: If the sum of the game is not zero but the sum of the payoffs to both players in each case is constant, then we call it a constant sum game. It is possible to reduce such a game to a zero sum game.

2x2 two person game and 2xn and mx2 games: When the number of players in a game is two and each player has exactly two strategies, the game is referred to as 2x2 two-person game. A game in which the first player has precisely two strategies and the second player has three or more strategies is called an 2xn game. A game in which the first player has three or more strategies and the second player has exactly two strategies is called an mx2 game.

3x3 and large games: When the number of players in a game is two and each player has exactly three strategies, we call it a 3x3 two person game. Two-person zero sum games are said to be larger if each of the two players has 3 or more choices. The examination of 3x3 and larger games is involving difficulties. For such games, the technique of linear programming can be used as a method of solution to identify the optimum strategies for the two players.

Non-constant games: Consider a game with two players. If the sum of the payoffs to the two players is not constraint in all the plays of the game, then we call it a non-constant game. Such games are divided into negotiable or cooperative games and non-negotiable or non-cooperative games.

Two-person zero sum games: A game with only two players, say player A and player B, is called a two-person zero sum game if the gain of the player A is equal to the loss of the player B, so that the total sum is zero.

Payoff matrix: When players select their particular strategies, the payoffs (gains or losses) can be represented in the form of a payoff matrix. Since, the game is zero sum, the gain of one playeris equal to the loss of other and vice-versa. Suppose A has m strategies and B has n strategies.

Consider the following payoff matrix. Player A wishes to gain as large a payoff **aij** as possible while player B will do his best to reach as small a value ij a as possible where the gain to player B and loss to player A be (-**aij**).

The amount of payoff, i.e., V at an equilibrium point is known as the **value of the game**. The optimal strategies can be identified by the players in the long run.

Fair game: The game is said to be fair if the value of the game V = 0.

Player B's strategies

$$\text{Player A's strategies}\quad \begin{array}{c} \\ A_1 \\ A_1 \\ \vdots \\ Am \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & \cdots & B_n \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{array}$$

Assumptions for two-person zero sum game:

For building any model, certain reasonable assumptions are quite necessary. Some assumptions for building a model of two-person zero sum game are listed below.

- Each player has available to him a finite number of possible courses of action. Sometimes the set of courses of action may be the same for each player. Or, certain courses of action may be available to both players while each player may have certain specific courses of action which are not available to the other player.
- Player A attempts to maximize gains to himself. Player B tries to minimize losses to himself.
- The decisions of both players are made individually prior to the play with no communication between them.
- The decisions are made and announced simultaneously so that neither player has an advantage resulting from direct knowledge of the other player's decision.
- Both players know the possible payoffs of themselves and their opponents.

Mini max andMaxi min Principles

The selection of an optimal strategy by each player without the knowledge of the competitor's strategy is the basic problem of playing games. The objective of game theory is to know how these players must select their respective strategies, so that they may optimize their payoffs. Such a criterion of decision making is referred to as mini max-maxi min principle. This principle in games of pure strategies leads to the best possible selection of a strategy for both players.

For example, if player A chooses his i^{th} strategy, then he gains at least the payoff min **aij**, which is minimum of the i^{th} row elements in the payoff matrix. Since his objective is to maximize his payoff, he can choose strategy i so as to make his payoff as large as possible. i.e., a payoff which is not less than

Similarly, player B can choose j^{th} column elements so as to make his loss not greater than

$$\min_{1\le j\le n} \max_{1\le i\le m} a_{ij} .$$

If the maxi min value for a player is equal to the mini max value for another player, i.e.

$$\max_{1\le i\le m} \min_{1\le j\le n} a_{ij} = V = \min_{1\le j\le n} \max_{1\le i\le m} a_{ij}$$

then the game is said to have a saddle point (equilibrium point) and the corresponding strategies are called optimal strategies. If there are two or more saddle points, they must be equal.

Problem:

Solve the game with the following pay-off matrix.

Player B

Strategies

$$\text{Player A Strategies} \quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \left[\begin{array}{ccccc} -2 & 5 & -3 & 6 & 7 \\ 4 & 6 & 8 & -1 & 6 \\ 8 & 2 & 3 & 5 & 4 \\ 15 & 14 & 18 & 12 & 20 \end{array}\right] \end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	-3
2	-1
3	2
4	12

Maximum of {-3, -1, 2, 12} = 12

Next consider the maximum of each column.

Column	Maximum Value
1	15
2	14
3	18
4	12
5	20

Minimum of {15, 14, 18, 12, 20}= 12

We see that the maximum of row minima = the minimum of the column maxima. So the game has a saddle point. The common value is 12. Therefore the value V of the game = 12.

Interpretation: In the long run, the following best strategies will be identified by the two players

The best strategy for player A is strategy 4.

The best strategy for player B is strategy IV.

The game is favorable to player A.

Problem 2: Solve the game with the following pay-off matrix

$$\text{Player X Strategies} \quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \text{Strategies} \\ \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \left|\begin{array}{ccccc} 9 & 12 & 7 & 14 & 26 \\ 25 & 35 & 20 & 28 & 30 \\ 7 & 6 & -8 & 3 & 2 \\ 8 & 11 & 13 & -2 & 1 \end{array}\right| \end{array}$$

Solution: First consider the minimum of each row.

Row	Minimum Value
1	7
2	20
3	-8
4	-2

Maximum of {7, 20, –8, -2} = 20

Next consider the maximum of each column.

Column	Maximum Value
1	25
2	35
3	20
4	28
5	30

Minimum of {25, 35, 20, 28, 30}= 20

It is observed that the maximum of row minima and the minimum of the column maxima are equal. Hence the given the game has a saddle point. The common value is 20.This indicates that the value V of the game is 20.

Interpretation: The best strategy for player X is strategy 2.
The best strategy for player Y is strategy III.
The game is favorable to player A.

Problem :

Solve the following game:

Player B

Strategies

$$\text{Player A Strategies} \quad \begin{array}{c|cccc|} & I & II & III & IV \\ \hline 1 & 1 & -6 & 8 & 4 \\ 2 & 3 & -7 & 2 & -8 \\ 3 & 5 & -5 & -1 & 0 \\ 4 & 3 & -4 & 5 & 7 \\ \hline \end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	-6
2	-8
3	-5
4	-4

Maximum of {-6, -8, -5, -4} = -4

Next consider the maximum of each column

Column	Maximum Value
1	5
2	-4
3	8
4	7

Minimum of {5, -4, 8, 7}= - 4

Since the max {row minima} = min {column maxima}, the game under consideration has a saddle point. The common value is –4. Hence the value of the game is –4.

Interpretation.

The best strategy for player A is strategy 4.

The best strategy for player B is strategy II. Since the value of the game is negative, it is concluded that the game is favorable to player B.

Games with no Saddle point:

2 x 2 zero-sum game When each one of the first player A and the second player B has exactly two strategies, we have a 2 x 2 game.

Motivating point First let us consider an illustrative example**.**

Problem :

Examine whether the following 2 x 2 game has a saddle point

$$\begin{array}{cc} & \textbf{Player B} \\ \textbf{Player A} & \begin{bmatrix} 3 & 5 \\ 4 & 2 \end{bmatrix} \end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	3
2	2

Maximum of {3, 2} = 3

Next consider the maximum of each column.

Column	Maximum Value
1	4
2	5

Minimum of {4, 5}= 4

We see that max {row minima} and min {column maxima} are not equal. Hence the game has no saddle point

Method of solution of a 2x2 zero-sum game without saddle point**:** Suppose that a 2x2 game has no saddle point. Suppose the game has the following pay-off matrix.

Player B

Strategy

$$\textbf{Player A Strategy} \quad \begin{array}{|cc|} \hline a & b \\ c & d \\ \hline \end{array}$$

Since this game has no saddle point, the following condition shall hold:

$$Max\{Min\{a,b\}, Min\{c,d\}\} \neq Min\{Max\{a,c\}, Max\{b,d\}\}$$

In this case, the game is called a mixed game. No strategy of Player A can be called the best strategy for him. Therefore, A has to use both of his strategies. Similarly, no strategy of Player B can be called the best strategy for him and he has to use both of his strategies.

Let *p* be the probability that Player A will use his first strategy. Then the probability that Player A will use his second strategy is 1-p.If Player B follows his first strategy. Expected value of the pay-off to Player A.

Expected value of the pay-off to Player A

$$= \left\{ \begin{array}{c} \text{Expected value of the pay-off to Player A} \\ \text{arising from his first strategy} \end{array} \right\} + \left\{ \begin{array}{c} \text{Expected value of the pay-off to Player A} \\ \text{arising from his second strategy} \end{array} \right\}$$

$$= ap + c(1-p) \longrightarrow (1)$$

In the above equation, note that the expected value is got as the product of the corresponding values of the pay-off and the probability.

If Player B follows his second strategy

$$\left. \begin{array}{c} \text{Expected value of the} \\ \text{pay-off to Player A} \end{array} \right\} = bp + d(1-p) \qquad (2)$$

If the expected values in equations (1) and (2) are different, Player B will prefer the minimum of the two expected values that he has to give to player A. Thus, B will have a pure strategy.

This contradicts our assumption that the game is a mixed one. Therefore, the expected values of the pay-offs to Player A in equations (1) and (2) should be equal. Thus, the condition

$$\begin{aligned}
ap + c(1-p) &= bp + d(1-p) \\
ap - bp &= (1-p)[d-c] \\
p(a-b) &= (d-c) - p(d-c) \\
p(a-b) + p(d-c) &= d-c \\
p(a-b+d-c) &= d-c \\
p &= \frac{d-c}{(a+d)-(b+c)} \\
1-p &= \frac{a+d-b-c-d+c}{(a+d)-(b+c)} \\
&= \frac{a-b}{(a+d)-(b+c)}
\end{aligned}$$

$$\left\{\begin{matrix}\text{The number of times A} \\ \text{will use first strategy}\end{matrix}\right\} : \left\{\begin{matrix}\text{The number of times A} \\ \text{will use second strategy}\end{matrix}\right\} = \frac{d-c}{(a+d)-(b+c)} : \frac{a-b}{(a+d)-(b+c)}$$

The expected pay-off to Player A

$$\begin{aligned}
&= ap + c(1-p) \\
&= c + p(a-c) \\
&= c + \frac{(d-c)(a-c)}{(a+d)-(b+c)} \\
&= \frac{c\{(a+d)-(b+c)\} + (d-c)(a-c)}{(a+d)-(b+c)} \\
&= \frac{ac + cd - bc - c^2 + ad - cd - ac + c^2)}{(a+d)-(b+c)} \\
&= \frac{ad - bc}{(a+d)-(b+c)}
\end{aligned}$$

Therefore, the value V of the game is

$$\frac{ad - bc}{(a+d)-(b+c)}$$

To find the number of times that B will use his first strategy and second strategy:
Let the probability that B will use his first strategy be *r*. Then the probability that B will use his second strategy is *1-r*.

When A use his first strategy

The expected value of loss to Player B with his first strategy = *ar*
The expected value of loss to Player B with his second strategy = *b(1-r)*
Therefore, the expected value of loss to B = ar + b(1-r) $\longrightarrow$ (3)

When A use his second strategy
The expected value of loss to Player B with his first strategy = cr
The expected value of loss to Player B with his second strategy = d(1-r)

Therefore, the expected value of loss to B = cr + d(1-r) $\longrightarrow$ (4)

If the two expected values are different then it results in a pure game, which is a contradiction. Therefore, the expected values of loss to Player B in equations (3) and (4) should be equal. Hence, the condition

$$ar + b(1-r) = cr + d(1-r)$$

$$ar + b - br = cr + d - dr$$

$$ar - br - cr + dr = d - b$$

$$r(a - b - c + d) = d - b$$

$$r = \frac{d-b}{a-b-c+d}$$

$$= \frac{d-b}{(a+d)-(b+c)}$$

Problem:

Solve the following game

$$\begin{array}{cc} & Y \\ X & \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix} \end{array}$$

Solution:

First consider the row minima

Row	Minimum Value
1	2
2	1

Maximum of {2, 1} = 2

Next consider the maximum of each column

Column	Maximum Value
1	4
2	5

We see that Max {row minima} == min {column maxima}

So the game has no saddle point. Therefore it is a mixed game.

We have a = 2, b = 5, c = 4 and d = 1.

Let *p* be the probability that player X will use his first strategy. We have

$$p = \frac{d-c}{(a+d)-(b+c)}$$
$$= \frac{1-4}{(2+1)-(5+4)}$$
$$= \frac{-3}{3-9}$$
$$= \frac{-3}{-6}$$
$$= \frac{1}{2}$$

The probability that player X will use his second strategy is $1\text{-}p = 1\text{-}\frac{1}{2} = \frac{1}{2}$.

Value of the game V = $\frac{ad-bc}{(a+d)-(b+c)} = \frac{2-20}{3-9} = \frac{-18}{-6} = 3$.

Let r be the probability that Player Y will use his first strategy. Then the probability that Y will use his second strategy is $(1\text{-}r)$. We have

$$r = \frac{d-b}{(a+d)-(b+c)}$$
$$= \frac{1-5}{(2+1)-(5+4)}$$
$$= \frac{-4}{3-9}$$
$$= \frac{-4}{-6}$$
$$= \frac{2}{3}$$
$$1-r = 1-\frac{2}{3} = \frac{1}{3}$$

Interpretation.

$$\text{p} : (1\text{-p}) = \frac{1}{2} : \frac{1}{2}$$

Therefore, out of 2 trials, player X will use his first strategy once and his second strategy once.

$$\text{r} : (1\text{-r}) = \frac{2}{3} : \frac{1}{3}$$

Therefore, out of 3 trials, player Y will use his first strategy twice and his second strategy once.

The Principle of Dominance:

In the previous lesson, we have discussed the method of solution of a game without a saddle point. While solving a game without a saddle point, one comes across the phenomenon of the dominance of a row over another row or a column over another column in the pay-off matrix of the game. Such a situation is discussed in the sequel.In a given pay-off matrix A, we say that the ith row dominates the k^{th} row if

$$a_{ij} \geq a_{kj} \quad \text{for all } j = 1,2,\ldots,n$$

and

$$a_{ij} > a_{kj} \quad \text{for at least one } j.$$

In this case, the player B will loose more by choosing the strategy for the q^{th} column than by choosing the strategy for the p^{th} column. So he will never use the strategy corresponding to the q^{th} column. When dominance of a row (or a column) in the pay-off matrix occurs, we can delete a row (or a column) from that matrix and arrive at a reduced matrix. This principle of dominance can be used in the determination of the solution for a given game.

Let us consider an illustrative example involving the phenomenon of dominance in a game.

Problem :

Solve the game with the following pay-off matrix:

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{cccc} I & II & III & IV \end{array} \\ \text{Player A} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 4 & 2 & 3 & 6 \\ 3 & 4 & 7 & 5 \\ 6 & 3 & 5 & 4 \end{bmatrix} \end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	2
2	3
3	3

Maximum of {2, 3, 3} = 3

Next consider the maximum of each column.

Column	Maximum Value
1	6
2	4
3	7
4	6

Minimum of {6, 4, 7, 6}= 4

The following condition holds:
Max {row minima} $\neq$ min {column maxima}
Therefore we see that there is no saddle point for the game under consideration.
Compare columns II and III.

Column II	Column III
2	3
4	7
3	5

We see that each element in column III is greater than the corresponding element in column II. The choice is for player B. Since column II dominates column III, player B will discard his strategy 3.Now we have the reduced game

$$\begin{array}{cc} & \begin{array}{ccc} I & II & IV \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 4 & 2 & 6 \\ 3 & 4 & 5 \\ 6 & 3 & 4 \end{bmatrix} \end{array}$$

For this matrix again, there is no saddle point. Column II dominates column IV. The choice is for player B. So player B will give up his strategy 4
The game reduces to the following:

$$\begin{array}{cc} & \begin{array}{cc} I & II \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 4 & 2 \\ 3 & 4 \\ 6 & 3 \end{bmatrix} \end{array}$$

This matrix has no saddle point.
The third row dominates the first row. The choice is for player A. He will give up his strategy 1 and retain strategy 3. The game reduces to the following

$$\begin{bmatrix} 3 & 4 \\ 6 & 3 \end{bmatrix}$$

Again, there is no saddle point. We have a 2x2 matrix. Take this matrix as $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then we have a = 3, b = 4, c = 6 and d = 3. Use the formulae for p, 1-p, r, 1-r and V.

$$\begin{aligned} p &= \frac{d-c}{(a+d)-(b+c)} \\ &= \frac{3-6}{(3+3)-(6+4)} \\ &= \frac{-3}{6-10} \\ &= \frac{-3}{-4} \\ &= \frac{3}{4} \end{aligned}$$

$$1-p = 1-\frac{3}{4} = \frac{1}{4}$$

$$\begin{aligned} r &= \frac{d-b}{(a+d)-(b+c)} \\ &= \frac{3-4}{(3+3)-(6+4)} \\ &= \frac{-1}{6-10} \\ &= \frac{-1}{-4} \\ &= \frac{1}{4} \end{aligned}$$

$$1-r = 1-\frac{1}{4} = \frac{3}{4}$$

The value of the game

$$\begin{aligned} V &= \frac{ad-bc}{(a+d)-(b+c)} \\ &= \frac{3x3-4x6}{-4} \\ &= \frac{-15}{-4} \\ &= \frac{15}{4} \end{aligned}$$

Thus, $X = \left(\frac{3}{4}, \frac{1}{4}, 0, 0\right)$ and $Y = \left(\frac{1}{4}, \frac{3}{4}, 0, 0\right)$ are the optimal strategies.

Method of convex linear combination :

A strategy, say s, can also be dominated if it is inferior to a convex linear combination of several other pure strategies. In this case if the domination is strict, then the strategy s can be deleted. If strategy s dominates the convex linear combination of some other pure strategies, then one of the pure strategies involved in the combination may be deleted. The domination will be decided as per the above rules. Let us consider an example to illustrate this case.

Problem:

Solve the game with the following pay-off matrix for firm A:

$$\begin{array}{cc} & \text{Firm B} \\ & \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \text{Firm A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} & \begin{bmatrix} 4 & 8 & -2 & 5 & 6 \\ 4 & 0 & 6 & 8 & 5 \\ -2 & -6 & -4 & 4 & 2 \\ 4 & -3 & 5 & 6 & 3 \\ 4 & -1 & 5 & 7 & 3 \end{bmatrix} \end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	-2
2	0
3	-6
4	-3
5	-1

Maximum of {-2, 0, -6, -3, -1} = 0

Next consider the maximum of each column.

Column	Maximum Value
1	4
2	8
3	6
4	8
5	6

Minimum of { 4, 8, 6, 8, 6}= 4

Hence,

Maximum of {row minima}$\neq$ minimum of {column maxima}.

So we see that there is no saddle point. Compare the second row with the fifth row. Each element in the second row exceeds the corresponding element in the fifth row. Therefore, A_2 dominates A_5 . The choice is for firm A. It will retain strategyA_2 and give up strategy A_5 .Therefore the game reduces to the following.

$$\begin{array}{c c} & \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} 4 & 8 & -2 & 5 & 6 \\ 4 & 0 & 6 & 8 & 5 \\ -2 & -6 & -4 & 4 & 2 \\ 4 & -3 & 5 & 6 & 3 \end{bmatrix} \end{array}$$

Compare the second and fourth rows. We see that A_2 dominates A_4 . So, firm A will retain the strategy A_2 and give up the strategy A_4 . Thus the game reduces to the following:

$$\begin{array}{c c} & \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 4 & 8 & -2 & 5 & 6 \\ 4 & 0 & 6 & 8 & 5 \\ -2 & -6 & -4 & 4 & 2 \end{bmatrix} \end{array}$$

Compare the first and fifth columns. It is observed that B1 dominates B5. The choice is for firm B. It will retain the strategy B_1 and give up the strategy B_5 . Thus the game reduces to the Following

$$\begin{array}{c c} & \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 4 & 8 & -2 & 5 \\ 4 & 0 & 6 & 8 \\ -2 & -6 & -4 & 4 \end{bmatrix} \end{array}$$

Compare the first and fourth columns. We notice that B1 dominates B4. So firm B will discard the strategy B_4 and retain the strategy B_1 . Thus the game reduces to the following

$$\begin{array}{c c} & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 4 & 8 & -2 \\ 4 & 0 & 6 \\ -2 & -6 & -4 \end{bmatrix} \end{array}$$

For this reduced game, we check that there is no saddle point. Now none of the pure strategies of firms A and B is inferior to any of their other strategies. But, we observe that convex linear combination of the strategies B_2 and B_3dominates B_1 , i.e. the averages of payoffs due to strategies B_2 and B_3,

$$\left\{\frac{8-2}{2}, \frac{0+6}{2}, \frac{-6-4}{2}\right\} = \{3,3,-5\}$$

dominate B_1 . Thus B_1 may be omitted from consideration. So we have the reduced matrix

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cc} B_2 & B_3 \end{array} \\ \begin{bmatrix} 8 & -2 \\ 0 & 6 \\ -6 & -4 \end{bmatrix} \end{array}$$

Here, the average of the pay-offs due to strategies A_1 and A_2 of firm A, i.e.

$\left\{\frac{8+0}{2}, \frac{-2+6}{2}\right\} = \{4, 2\}$ dominates the pay-off due to A_3. So we get a new reduced 2x2 pay-off matrix

Firm B's strategy

$$\text{Firm A's strategy} \qquad \begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} B_2 & B_3 \end{array} \\ \begin{bmatrix} 8 & -2 \\ 0 & 6 \end{bmatrix} \end{array}$$

We have a = 8, b = -2, c = 0 and d = 6.

$$\begin{aligned} p &= \frac{d-c}{(a+d)-(b+c)} \\ &= \frac{6-0}{(6+8)-(-2+0)} \\ &= \frac{6}{16} \\ &= \frac{3}{8} \end{aligned}$$

$$1-p = 1-\frac{3}{8} = \frac{5}{8}$$

$$\begin{aligned} r &= \frac{d-b}{(a+d)-(b+c)} \\ &= \frac{6-(-2)}{16} \\ &= \frac{8}{16} \\ &= \frac{1}{2} \end{aligned}$$

$$1-r = 1-\frac{1}{2} = \frac{1}{2}$$

. Value of the game

$$V = \frac{ad - bc}{(a+d)-(b+c)}$$
$$= \frac{6x8 - 0x(-2)}{16}$$
$$= \frac{48}{16} = 3$$

So the optimal strategies are

$$A = \left\{\frac{3}{8}, \frac{5}{8}, 0, 0, 0\right\} \text{ and } B = \left\{0, \frac{1}{2}, \frac{1}{2}, 0, 0\right\}.$$

The value of the game = 3. Thus the game is favourable to firm A.

Problem:
For the game with the following pay-off matrix, determine the saddle point

Player B

$$\text{Player A} \quad \begin{array}{c c} & \begin{array}{cccc} I & II & III & IV \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 2 & -1 & 0 & -3 \\ 1 & 0 & 3 & 2 \\ -3 & -2 & -1 & 4 \end{bmatrix} \end{array}$$

Solution:

$$\begin{array}{c c} & \textit{Column II} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{array}{|c|} -1 \\ 0 \\ -2 \end{array} \end{array} \qquad \begin{array}{c} \textit{Column III} \\ \begin{array}{|c|} 0 \\ 3 \\ -1 \end{array} \end{array} \qquad \begin{array}{r} 0 > -1 \\ 3 > \ \ 0 \\ -1 > -2 \end{array}$$

The choice is with the player B. He has to choose between strategies II and III. He will lose more in strategy III than in strategy II, irrespective of what strategy is followed by A. So he will drop strategy III and retain strategy II. Now the given game reduces to the following game.

$$\begin{array}{c c} & \begin{array}{ccc} I & II & IV \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 2 & -1 & -3 \\ 1 & 0 & 2 \\ -3 & -2 & 4 \end{bmatrix} \end{array}$$

Consider the rows and columns of this matrix

Row minimum:

I Row	:	-3	
II Row	:	0	Maximum of $\{-3, 0, -3\} = 0$
III Row	:	-3	

Column maximum:

I Column	:	2	
II Column	:	0	Minimum of $\{2, 0, 4\} = 0$
III Column	:	4	

Interpretation: No player gains and no player loses. i.e., The game is not favourable to any player. i.e. It is a fair game.

Problem:

Solve the game

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 4 & 8 & 6 \\ 6 & 2 & 10 \\ 4 & 5 & 7 \end{bmatrix} \end{array}$$

Solution:

First consider the minimum of each row

Row	Minimum
1	4
2	2
3	4

Maximum of $\{4, 2, 4\} = 4$

Next, consider the maximum of each column.

Column	Maximum
1	6
2	8
3	10

Minimum of $\{6, 8, 10\} = 6$

Since Maximum of { Row Minima} and Minimum of { Column Maxima } are different, it follows that the given game has no saddle point.

Denote the strategies of player A by A_1, A_2 , A_3 . Denote the strategies of player B by B_1 ,B_2 ,B_3 .

Compare the first and third columns of the given matrix.

$$\begin{array}{cc} B_1 & B_3 \\ \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix} & \begin{bmatrix} 6 \\ 10 \\ 7 \end{bmatrix} \end{array}$$

The pay-offs in B_3 are greater than or equal to the corresponding pay-offs in B_1 . The player B has to make a choice between his strategies 1 and 3. He will lose more if he follows strategy 3 rather than strategy 1. Therefore he will give up strategy 3 and retain strategy 1. Consequently, the given game is transformed into the following game:

$$\begin{array}{cc} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 4 & 8 \\ 6 & 2 \\ 4 & 5 \end{bmatrix} \end{array}$$

Compare the first and third rows of the above matrix.

$$\begin{array}{cc} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_3 \end{array} & \begin{bmatrix} 4 & 8 \\ 4 & 5 \end{bmatrix} \end{array}$$

The pay-offs in A_1 are greater than or equal to the corresponding pay-offs in A_2 . The player A has to make a choice between his strategies 1 and 3. He will gain more if he follows strategy 1 rather than strategy 3. Therefore he will retain strategy 1 and give up strategy 3.Now the given game is transformed into the following game.

$$\begin{array}{cc} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 4 & 8 \\ 6 & 2 \end{bmatrix} \end{array}$$

It is a 2x2 game. Consider the row minima

Row	Minimum
1	4
2	2

Maximum of {4, 2} = 4

Next, consider the maximum of each column

Column	Maximum
1	6
2	8

Minimum of {6, 8} = 6

Maximum {row minima} and Minimum {column maxima } are not equal.Therefore, the reduced game has no saddle point. So, it is a mixed game

Take $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 6 & 2 \end{bmatrix}$. We have a = 4, b = 8, c = 6 and d = 2.

The probability that player A will use his first strategy is p. This is calculated as

$$p = \frac{d-c}{(a+d)-(b+c)}$$

$$= \frac{2-6}{(4+2)-(8+6)}$$

$$= \frac{-4}{6-14}$$

$$= \frac{-4}{-8} = \frac{1}{2}$$

The probability that player B will use his first strategy is r. This is calculated as

$$r = \frac{d-b}{(a+d)-(b+c)}$$

$$= \frac{2-8}{-8}$$

$$= \frac{-6}{-8}$$

$$= \frac{3}{4}$$

Value of the game is V. This is calculated as

$$V = \frac{ad-bc}{(a+d)-(b+c)}$$

$$= \frac{4x2-8x6}{-8}$$

$$= \frac{8-48}{-8}$$

$$= \frac{-40}{-8} = 5$$

Interpretation

Out of 3 trials, player A will use strategy 1 once and strategy 2 once. Out of 4 trials, player B will use strategy 1 thrice and strategy 2 once. The game is favorable to player A.

Problem:

Solve the game with the following pay-off matrix.**(Dividing a game into sub-games)**

$$\text{Player A} \quad \begin{array}{c} \\ I \\ II \\ III \end{array} \begin{array}{c} \text{Player B} \\ \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -4 & 6 & 3 \\ -3 & 3 & 4 \\ 2 & -3 & 4 \end{bmatrix} \end{array}$$

Solution:

First, consider the row minima.

Row	Minimum
1	-4
2	-3
3	-3

Maximum of {-4, -3, -3} = -3

Next, consider the column maxima.

Column	Maximum
1	2
2	6
3	4

Minimum of {2, 6, 4} = 2

We see that Maximum of { row minima} $\neq$ Minimum of { column maxima}.

So the game has no saddle point. Hence it is a mixed game. Compare the first and third Columns.

$$\begin{array}{ccc} I\ Column & III\ Column & \\ \begin{bmatrix} -4 \\ -3 \\ 2 \end{bmatrix} & \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix} & \begin{array}{r} -4 \le 3 \\ -3 \le 4 \\ 2 \le 4 \end{array} \end{array}$$

We assert that Player B will retain the first strategy and give up the third strategy. We get the following reduced matrix

$$\begin{bmatrix} -4 & 6 \\ -3 & 3 \\ 2 & -3 \end{bmatrix}$$

We check that it is a game with no saddle point.

Sub games : Let us consider the 2x2 sub games. They are:

$$\begin{bmatrix} -4 & 6 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} -4 & 6 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 3 \\ 2 & -3 \end{bmatrix}$$

First, take the sub game

$$\begin{bmatrix} -4 & 6 \\ -3 & 3 \end{bmatrix}$$

Compare the first and second columns. We see that $-4 \le 6$ and $-3 \le 3$. Therefore, the game reduces to $\begin{bmatrix} -4 \\ -3 \end{bmatrix}$. Since $-4 < -3$, it further reduces to -3.

Next, consider the sub game

$$\begin{bmatrix} -4 & 6 \\ 2 & -3 \end{bmatrix}$$

We see that it is a game with no saddle point. Take a = -4, b = 6, c = 2, d = -3. Then the value of the game is

$$V = \frac{ad - bc}{(a+d)-(b+c)}$$
$$= \frac{(-4)(-3)-(6)(2)}{(-4+3)-(6+2)}$$
$$= 0$$

Next, take the sub game $\begin{bmatrix} -3 & 3 \\ 2 & -3 \end{bmatrix}$. In this case we have a = -3, b = 3, c = 2 and d = -3. The value of the game is obtained as

$$V = \frac{ad - bc}{(a+d)-(b+c)}$$
$$= \frac{(-3)(-3)-(3)(2)}{(-3-3)-(3+2)}$$
$$= \frac{9-6}{-6-5} = -\frac{3}{11}$$

Let us tabulate the results as follows:

Sub game	Value
$\begin{bmatrix} -4 & 6 \\ -3 & 3 \end{bmatrix}$	-3
$\begin{bmatrix} -4 & 6 \\ 2 & -3 \end{bmatrix}$	0
$\begin{bmatrix} -3 & 3 \\ 2 & -3 \end{bmatrix}$	$-\frac{3}{11}$

The value of 0 will be preferred by the player A. For this value, the first and third strategies of A correspond while the first and second strategies of the player B correspond to the value 0 of the game. So it is a fair game.

<u>Graphical solution of a 2x2 game with no saddle point:</u>

Problem:

Consider the game with the following pay-off matrix.

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix} \end{array}$$

Solution: First consider the row minima.

Row	Minimum
1	2
2	1

Maximum of {2, 1} = 2.

Next, consider the column maxima.

Column	Maximum
1	4
2	5

Minimum of {4, 5} = 4.

We see that Maximum { row minima} |== Minimum { column maxima }
So, the game has no saddle point. It is a mixed game.
Equations involving probability and expected value:
Let p be the probability that player A will use his first strategy.
Then the probability that A will use his second strategy is **1--p**.
Let E be the expected value of pay-off to player A.
When B uses his first strategy
The expected value of pay-off to player A is given by

$$\begin{aligned} E &= 2p + 4(1-p) \\ &= 2p + 4 - 4p \\ &= 4 - 2p \qquad \longrightarrow \qquad (1) \end{aligned}$$

When B uses his second strategy
The expected value of pay-off to player A is given by

$$\begin{aligned} E &= 5p + 1(1-p) \\ &= 5p + 1 - p \\ &= 4p + 1 \qquad \longrightarrow \qquad (2) \end{aligned}$$

Consider equations (1) and (2). For plotting the two equations on a graph sheet, get some points on them as follows: E = -2p+4 E = 4p+1

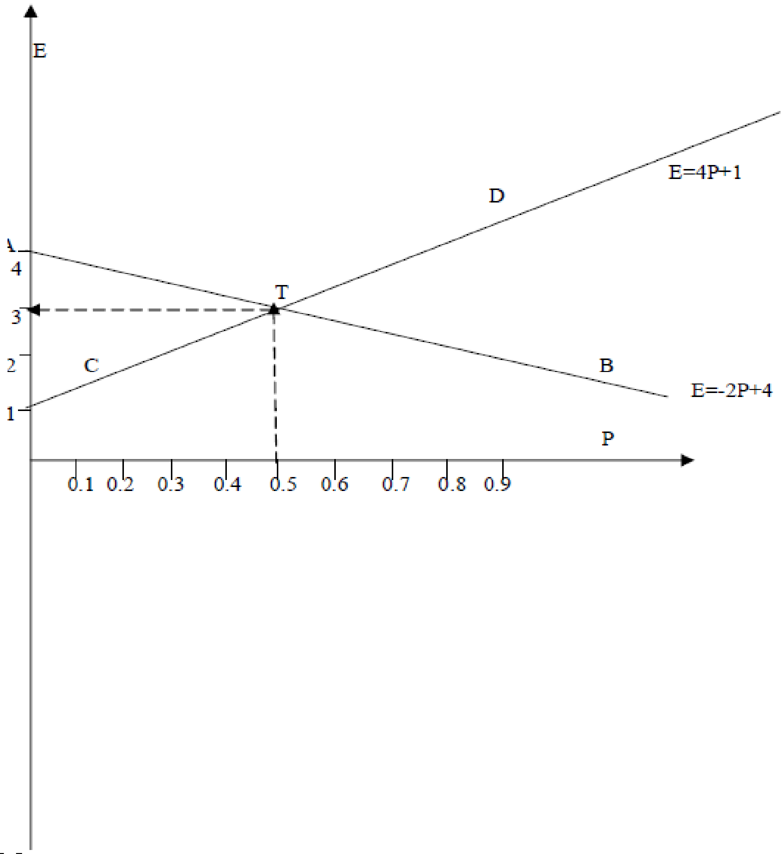
p	0	1	0.5
E	4	2	3

p	0	1	0.5
E	1	5	3

Graphical solution:

Procedure: Take probability and expected value along two rectangular axes in a graph sheet. Draw two straight lines given by the two equations (1) and (2). Determine the point of intersection of the two straight lines in the graph. This will give the common solution of the two equations (1) and (2). Thus we would obtain the value of the game.

Represent the two equations by the two straight lines AB and CD on the graph sheet. Take the point of intersection of AB and CD as T. For this point, we have p = 0.5 and E = 3. Therefore, the value V of the game is 3.



Problem:

Solve the following game by graphical method.

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} -18 & 2 \\ 6 & -4 \end{bmatrix} \end{array}$$

Solution:

First consider the row minima.

Row	Minimum
1	- 18
2	- 4

Maximum of {-18, - 4} = - 4.

Next, consider the column maxima.

Column	Maximum
1	6
2	2

Minimum of {6, 2} = 2.

We see that Maximum {row minima}== Minimum { column maxima }So, the game has no saddle point. It is a mixed game. Let p be the probability that player A will use his first strategy.Then the probability that A will use his second strategy is 1- p.

When B uses his first strategy. The expected value of pay-off to player A is given by

$$E = -18p + 6(1-p)$$
$$= -18p + 6 - 6p$$
$$= -24p + 6$$

E = -24 p + 6

p	0	1	0.5
E	6	-18	-6

$$E = 2p - 4(1-p)$$
$$= 2p - 4 + 4p$$
$$= 6p - 4$$

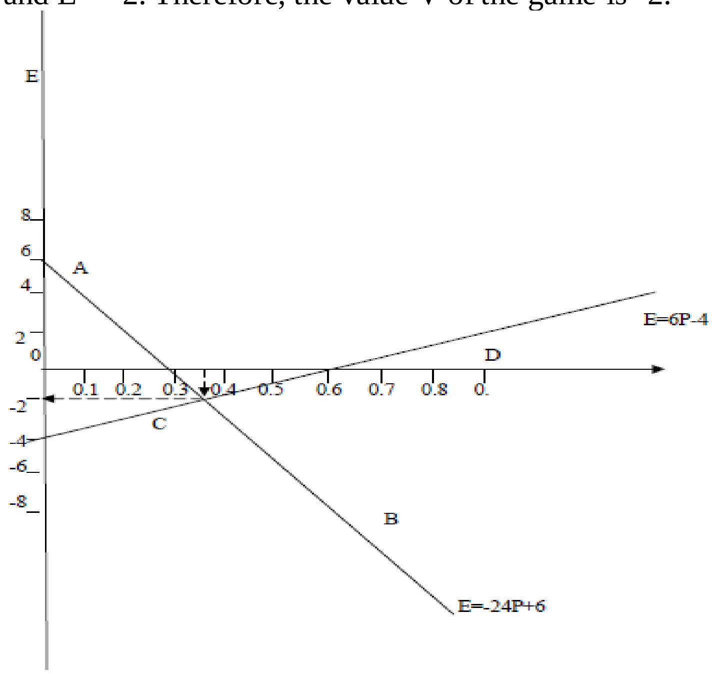
E = 6p-4

p	0	1	0.5
E	-4	2	-1

Graphical solution:

Take probability and expected value along two rectangular axes in a graph sheet. Draw two straight lines given by the two equations (1) and (2). Determine the point of intersection of thetwo straight lines in the graph. This will provide the common solution of the two equations (1)and (2). Thus we would get the value of the game.

Represent the two equations by the two straight lines AB and CD on the graph sheet. Take thepoint of intersection of AB and CD as T. For this point, we have p = 1/ 3 and E = -2. Therefore, the value V of the game is -2.



Inventory

Introduction :

Simply inventory is a stock of physical assets. The physical assets have some economic value, which can be either in the form of material, men or money. Inventory is also called as an idle resource as long as it is not utilized. Inventory may be regarded as those goods which are procured, stored and used for day to day functioning of the organization.

Inventory can be in the form of physical resource such as raw materials, semi-finished goods used in the process of production, finished goods which are ready for delivery to the consumers, human resources, or financial resources such as working capital etc.

Thus, inventory control is the technique of maintaining stock items at desired levels. In other words, inventory control is the means by which material of the correct quality and quantity is made available as and when it is needed with due regard to economy in the holding cost, ordering costs, setup costs, production costs, purchase costs and working capital.

Objectives of Inventory : Inventory has the following main objectives:

- To supply the raw material, sub-assemblies, semi-finished goods, finished goods, etc. to its users as per their requirements at right time and at right price.
- To maintain the minimum level of waste, surplus, inactive, scrap and obsolete items.
- To minimize the inventory costs such as holding cost, replacement cost, breakdown cost and shortage cost.
- To maximize the efficiency in production and distribution.
- To maintain the overall inventory investment at the lowest level
- To treat inventory as investment which is risky? For some items, investment may lead to higher profits and for others less profit.

Inventory Conversion Diagram: The socks at input are called raw materials whereas thestocks at the output are called products. The stocks at the conversion process may be called finished or semi-finished goods or sometimes may be raw material depending on the requirement of the product at conversion process, where the input and output are based on the market situations of uncertainty, it becomes physically impossible and economically impractical for each stock item to arrive exactly where it is required and when it is required.

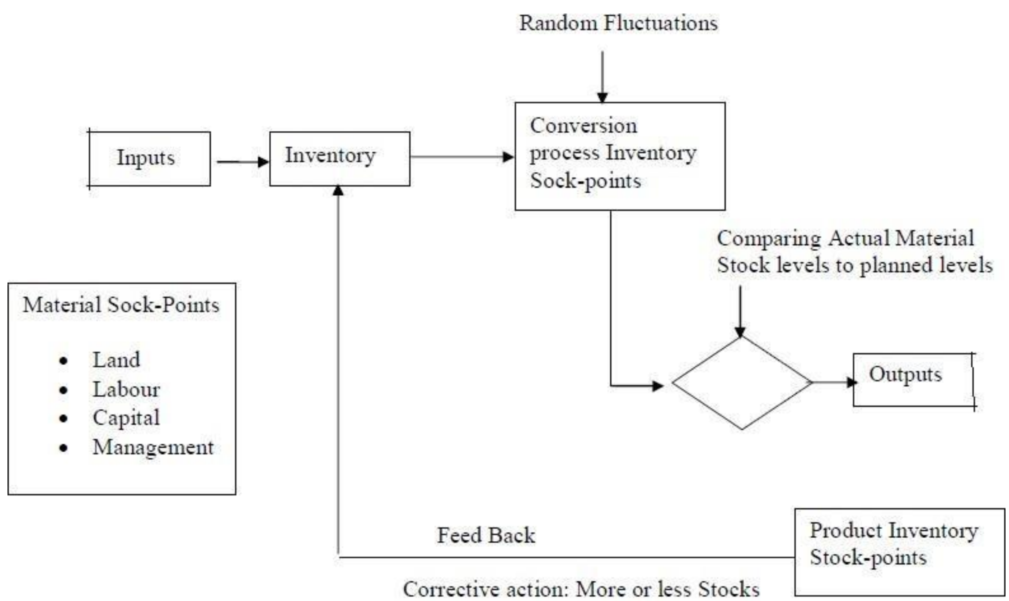


Fig: Materials Conversion Process

Role of Inventory:

Inventories play an essential and pervasive role in any organization because they make itpossible:

- To meet unexpected demand
- To achieve return on investment
- To order largest quantities of goods, components or materials from the suppliers at Advantageous prices
- To provide reasonable customer service through supplying most of the requirements from Stock without delay
- To avoid economically impractical and physically impossible delivering/getting right Amount of stock at right time of required
- To maintain more work force levels
- To facilitate economic production runs
- To advantage of shipping economies
- To smooth seasonal or critical demand
- To facilitate the intermittent production of several products on the same facility
- To make effective utilization of space and capital
- To meet variations in customer demand
- To take the advantage of price discount
- To hedge against price increases
- To discount quantity

Basic Functions of Inventory :

The important basic function of inventory is

- Increase the profitability- through manufacturing and marketing support. But zero inventory manufacturing- distribution system is not practically possible, so it is important to remember that each rupee invested in inventory should achieve a specific goal.

The other inventory basic functions are

- Geographical Specialization
- Decoupling
- Balancing supply and demand and
- Safety stock

Factors affecting inventory

1.) **Inventory or stock cost:-**
There are several:

i) Purchase/Production cost– cost of purchasing a unit of item

ii) Ordering/Acquisition/Set-up cost – costs related to acquisition of purchased items i.e. those of getting an item to a firm's store e.g. transport, loading and off-loading, inspection.

iii) Inventory carrying/ holding costs – costs associated with holding a given level of inventory e.g. warehousing, spoilage, security, pilferage, administrative, insurance, depreciation.

iv) Stock-out cost/ shortage costs – incurred due to a delay in meeting demand or inability to meet demand at all because of shortage of stock loss of future sales, cost associated with future replenishment.

2. **Order cycle** – the time period between placements of 2 successive orders.

3. **Lead time** – time between placing an order and actual replenishment of item. Also referred to as procurement time.

4. **Time horizon** – this is the period over which the inventory level will be controlled.

5. **Maximum stock** – the level beyond which stocks should not be allowed to rise.

6. **Minimum stock level/buffer stock/safety stock** – level below which stock should not be allowed to fall. It is the additional stock needed to allow for delay in delivery or for any higher than expected demand that may arise due to lead time.

7. **Reorder level** – point at which purchased order must be sent to supplier for the supply of more stock. The level of stock at which further replenishment order should be placed.

8. **Reorder quantity** – the quantity of the replacement order.

ROP (Reorder Point) = Daily Demand X Lead Time

$$ROP = D/T \times T_L$$

Note that Demand is on daily basis

9. **Average stock level**

$$\text{Average stock level} = \frac{\text{Minimum stock level} + \text{Maximum stock level}}{2}$$

10. **Physical stock** – no. of items physically in stock at any given time.

11. **Stock replenishment** – rate at which items are added to the inventory.

12. **Free stock** – the physical stock plus the outstanding replenishment orders minus the unfulfilled requirements.

13. **Economic order quantity (EOQ)** – the quantity at which the cost of having stocks isminimum.

14. **Economic batch quantity (EBQ)** – quantity of stock within the enterprise. Company orders from within its own warehouses unlike in EOQ where it is ordered from elsewhere.

15. Demand:-

- Customer's demand, size of demand, rate of demand and pattern of demand isimportant
- Size of demand = no. of items demanded per period
- Can be deterministic (Static or dynamic) or probabilistic (governed by discrete or continuous probability distribution)
- The rate of demand can be variable or constant
- Pattern reflects items drawn from inventory -instantaneous (at beginning or end) or gradually at uniform rate

There are four major elements of inventory costs that should be taken for analysis, such as

(1) Item cost, Rs. C/item.
(2) Ordering cost, Rs. Co/order.
(3) Holding cost Rs. Ch/item/unit time.
(4) Shortage cost Rs. Cs/item/Unit time.

(1) Item Cost (C)

This is the cost of the item whether it is manufactured or purchased. If it is
manufactured, it includes such items as direct material and labor, indirect materials and labor and overhead expenses. When the item is purchased, the item cost is the purchase price of 1 unit. Let it be denoted by Rs. C per item.

(2) Purchasing or Setup or Acquisition or Ordering Cost (Co)

Administrative and clerical costs are involved in processing a purchase order,
expediting, follow up etc., It includes transportation costs also. When a unit is manufactured, the unit set up cost includes the cost of labor and materials used in the set up and set up testing and training costs. This is denoted by Rs. Co per set up or per order

Inventory holding cost (Ch): If the item is held in stock, the cost involved is the item carrying or holding cost. Some of the costs included in the unit holding cost are
(1) Taxes on inventories
(2) Insurance costs for inflammable and explosive items,
(3) Obsolescence,
(4) Deterioration of quality, theft, spillage and damage to times,
(5) Cost of maintaining inventory records.
This cost is denoted by Rs. Ch/item/unit time. The unit of time may be days, months, weeksor years.

Shortage Cost (Cs): The shortage cost is due to the delay in satisfying demand (due to wrong planning); but the demand is eventually satisfied after a period of time. Shortage cost is not considered as the opportunity cost or cost of lost sales. The unit shortage cost includes such items as,
(1) Overtime requirements due to shortage,
(2) Clerical and administrative expenses.
(3) Cost of expediting.
(4) Loss of goodwill of customers due to delay.
(5) Special handling or packaging costs.
(6) Lost production time.
This cost is denoted by Rs. Cs per item per unit time of shortage.

The basic deterministic inventory models

1. EOQ Model with Uniform Demand
2. EOQ Model with Different rates of Demands in different cycles
3. EOQ Model with Shortages (backorders) allowed
4. EOQ Model with Uniform Replenishment

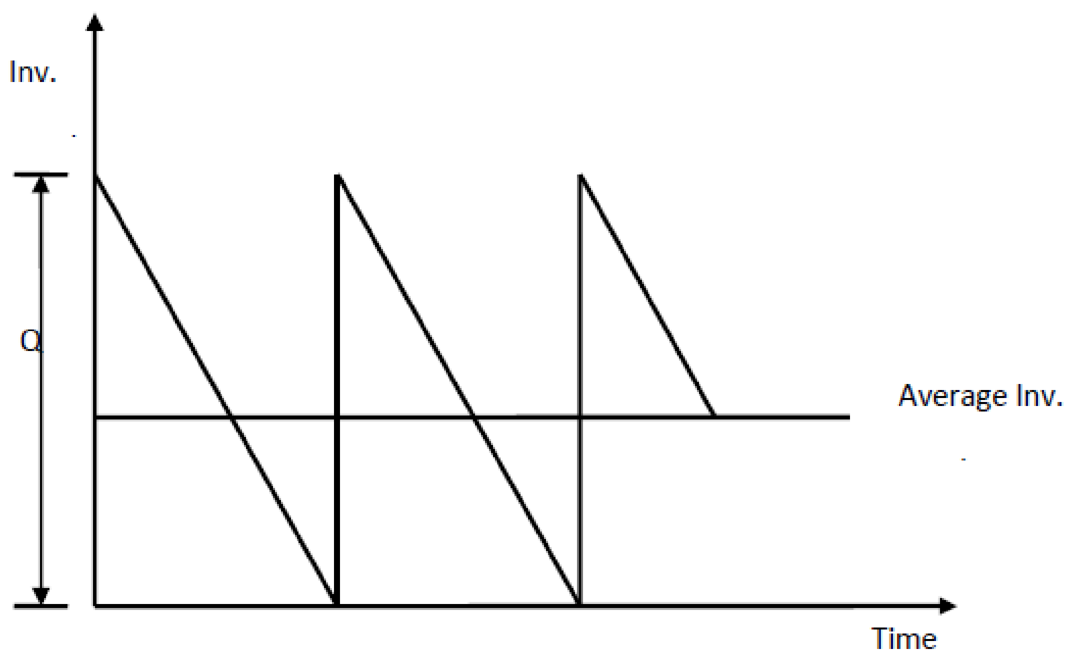
Notations used:-
Q = number of units per order
Q* = economic order quantity or optimal no. of units per order to minimize total cost D = annual demand requirement (units per year)
C = cost of 1 unit of item
C0 = ordering (preparation or set-up) cost of each order
Ch = Cc = holding or carrying cost per unit per period of time T = length of time between two successive orders
N = no. of orders or manufacturing runs per yearTC = Total Inventory cost
The optimal order quantity (EOQ) is at a point where the ordering cost = holding cost

Model 1- EOQ Model with Uniform Demand

Policy: Whenever the inventory level is 0, order Q items
Objective: Choose a Q that will minimize total Inventory Cost
The behavior of inventory at hand with respect to time is illustrated below:
1. No stock-out is allowed.
2. Quantity discounts are not allowed – purchase price is constant.
3. Lead time is known and fixed.
This is the ordering quantity which minimizes the balance of cost between inventory holding cost and ordering costs
.

It is based on the following assumptions:

1. A known constant stock holding cost.
2. A known constant ordering cost.
3. The rate of demand is known (is deterministic).
4. A known constant price per unit.
5. Inventory replenishment is done instantaneously
6. No stock-out is allowed.
7. Quantity discounts are not allowed – purchase price is constant.
8. Lead time is known and fixed.



1. **Annual ordering cost**

$$Annual\ ordering\ cost = (no.\ of\ orders\ placed\ per\ year) \times (ordering\ cost\ per\ order)$$

$$= \left(\frac{Annual\ Demand}{no.\ of\ units\ in\ each\ order} \right) \times (order\ cost\ per\ order)$$

$$= \frac{D}{Q} \times C_o \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

2. **Annual holing (or carrying) cost**

$$Annual\ ordering\ cost = (Average\ inventory\ level) \times (carrying\ cost\ per\ order)$$

$$= \frac{Q}{2} \times C_h \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

3. **Equating (1) and (2) above**

Since the minimum TC occurs at the point where the ordering cost and the inventory carrying costs are equal, we equate the 2 equations above.

$$\frac{D}{Q} \times C_o = \frac{Q}{2} \times C_h$$

Solve for Q

$$2DC_o = Q^2 C_h$$

$$Q^2 = \frac{2DC_o}{C_h}$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

$$EOQ = \sqrt{\frac{2DC_o}{C_h}}$$

1. Inventory holding or carrying costs are often expressed as annual percentage(s) of the unit costor price. C_o or C_h as % of unit cost or price

I = annual inventory carrying charge (cost) as 1% of priceC_h = **IC** where C is the unit price of inventory item

$$EOQ = Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

2. Total cost is sum of annual C_h and annual ordering cost.

$$TC = \frac{D}{Q}.C_o + \frac{Q}{2}.C_h$$

Put value of Q* in TC,

$$TVC = \sqrt{2DC_oC_h}$$

Problem:

A supplier is required to deliver 20000 tons of raw materials in one year to a large manufacturing organization. The supplier maintains his go-down to store the material received from various resources. He finds that cost of inventory holding is 30 paisa per ton per month. His cost for ordering the material is Rs. 400. One of the conditions of the supplier contract from the manufacturing organization is that the contract will be terminated in the event of supply not being maintained as a schedule. Determine (1) in what lot size is the supplier should produce the material for minimum total associated cost of inventory? (2) At what time interval should he procure the material? It may be assume that replacement of inventory is instantaneous

Solution:

Given Date

D = 20000 tons

T = 12 months

C_h = Rs. 0.30 per tons per months

C_o = Rs. 400

(1) Economic order quantity

$$EOQ = \sqrt{\frac{2DC_o}{C_h}}$$

$$= \sqrt{\frac{2(12000)(400)}{0.30 \times 12}}$$

$$Q^* = 2108\ tons$$

(2) Tine interval

$$t_{co} = \frac{Q^*}{D}$$

$$= \frac{2108}{20000}$$

$$= 1.26 \text{ month}$$

Problem :

In the above example, if there is (i) 10 per cent increase in holding cost or (ii) 10 percent increase in ordering cost, in each case determine the optimal lot size and corresponding minimum total expected cost of inventory. Comment the result.

Ans:-

(i) $C_h' = 1.1\ C_h$

$$EOQ = \sqrt{\frac{2DC_o}{C_h'}}$$

$$= \sqrt{\frac{2(12000)(400)}{1.1 \times 0.30 \times 12}}$$

$$Q^* = 2010\ tons$$

$$TAC = \sqrt{2C_h' C_o D}$$

$$= \sqrt{2 \times 1.1 \times 0.30 \times 400 \times 12000}$$

$$= 7960\ \text{Rs.}$$

(ii) $C_o' = 1.1\ C_o$

$$EOQ = \sqrt{\frac{2DC_o'}{C_h}}$$

$$= \sqrt{\frac{2(12000)(400)(1.1)}{0.30 \times 12}}$$

$$Q^* = 2211\ tons$$

$$TAC = \sqrt{2C_h C_o' D}$$

$$= \sqrt{2 \times 0.30 \times 1.1 \times 400 \times 12000}$$

$$= 7960\ \text{Rs.}$$

Problem :

A certain item costs Rs. 250 per ton. The monthly requirement is 5 tons and each time the stock is replenished, there is an order cost of Rs. 120. The cost of carrying inventory has been estimated at 10% of the value of the stock per year. What is the optimal order quantity? If lead time is 3 months, determine the re order point. At what intervals the order should be placed?

Solution: -

Given Data

C = Rs. 250 per ton C_o = Rs. 120

$C_h = 250 \times 0.1$ = Rs. 25 per ton per year D = 5×12 = 60 tons

T_L = 3 months

$$EOQ = \sqrt{\frac{2DC_o}{C_h}}$$

$$= \sqrt{\frac{2(60)(120)}{25}}$$

$$Q^* = 24\ tons$$

$$t_{co} = \frac{Q^*}{D}$$

$$= \frac{24}{60}$$

$$= 0.4 \text{ year or } 4.8 \text{ months}$$

$$Q_R = \frac{D}{T} \times T_L$$

$$= \frac{60}{12} \times 3$$

$$= 15 \text{ tons}$$

Problem :

A manufacturer has to supply his customers with 1200 units of his product per annum. Theinventory carrying cost amounts to ₹ 1.2 per unit. The set-up cost per run is ₹ 160. Find:

i) EOQ

ii) Minimum average yearly cost

iii) Optimum no of orders per year

iv) The optimum time between orders (optimum period of supply per optimum order)

Solution:

i) Economic order quantity

$$EOQ = \sqrt{\frac{2DC_o}{C_h}}$$

$$= \sqrt{\frac{2(1200)(160)}{1.2}}$$

$$= 565.69 \text{ or } 566\ units$$

ii) Minimum average yearly cost

$$TC = \frac{D}{Q}.C_o + \frac{Q}{2}.C_h$$

$$TC(Q^*) = \frac{DC_o}{Q^*} + \frac{Q^*C_h}{2}$$

$$= \frac{1200(160)}{566} + \frac{566(1.2)}{2}$$

$$= 339.22 + 339.6$$

$$= \text{Rs } 678.82 \text{ or Rs } 679$$

iii) Optimum no. of orders per year (N^*)

$$N^* = \frac{Demand}{EOQ}$$

$$= \frac{1200}{566}$$

$$= 2.1 \; orders \Rightarrow 3 \; orders$$

iv) Optimum time between orders

$$T^* = \frac{no.\ of\ working\ days\ in\ a\ year}{N^*}$$

$$= \frac{365}{3}$$

$$= 122$$

Problem:

The annual demand per item is 6400 units. The unit cost is ₹ 12 and the inventory carrying charges 25% per annum. If the cost of procurement is ₹ 300 determine:

i) EOQ
ii) No. of orders per year
iii) Time between 2 consecutive orders
iv) Optimum cost

Ans:-

i) EOQ

$$EOQ = \sqrt{\frac{2DC_o}{C_h}}$$

$$= \sqrt{\frac{2(6400)(300)}{(0.25)(12)}}$$

$$= 1131 \; units$$

ii) N^*

$$N^* = \frac{Demand}{EOQ}$$

$$= \frac{6400}{1131}$$

$$= 5.65 \; orders \Rightarrow 6 \; orders$$

iii) Time between 2 consecutive orders

$$T^* = \frac{no.\ of\ working\ days\ in\ a\ year}{N^*}$$

$$= \frac{365}{5.65}$$

$$= 64.60$$

(OR)

$$T^* = \frac{EOQ}{Demand} \times 12\ months$$

$$= \frac{1131}{6400} \times 12\ months$$

$$= 2\ months\ 4\ days$$

Model 2- EOQ Model with Different Rates of Demand

Assumptions of this model are same as those of model 1 except

Demand rate is different in different cycles. The total demand D is specified as demand during time horizon T

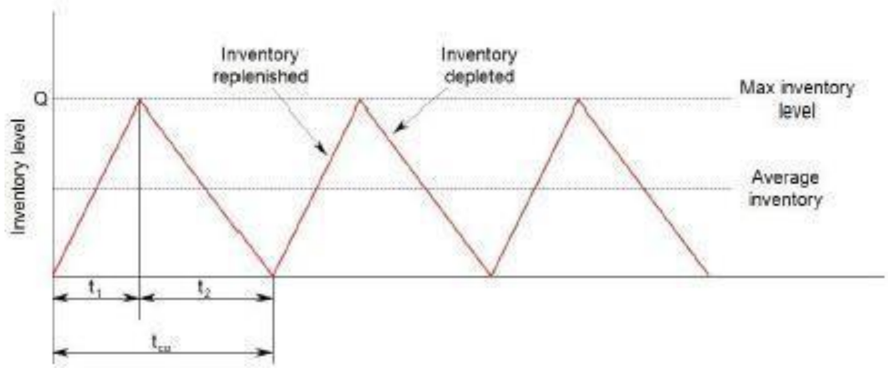
$$Holding\ Cost = \frac{Q}{2} \times \left(\frac{d-r}{d}\right)$$

$$Order\ /\ set\ \text{up cost} = \frac{D \times C_o}{Q}$$

$$EOQ = Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{d}{d-r}}$$

$$TVC = \sqrt{2rC_oC_h\left(\frac{d-r}{d}\right)}$$

$$t_{co} = \frac{Q^*}{r}$$



Problem:

A manufacturing company needs 4000 units of material every month. The delivery system from the supplier is so scheduled that once delivery commences the materials is received at the rate of 6000 units per month. The cost of processing purchase order is Rs. 600 and the inventory carrying cost is 30 paisa per unit per month. Determine the optimal lot size and interval at which the order is to be placed. What is maximum inventory during a cycle?

Solution:-

Given Data

Co = Rs. 600

Ch = Rs. 0.30 per unit per monthd = 6000 units per months

r = 4000 units per months

Optimal lot size

$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{d}{d-r}}$$

$$= \sqrt{\frac{2 \times 4000 \times 600}{0.30}} \times \sqrt{\frac{6000}{6000-4000}}$$

$$= 6928 \text{ units}$$

Interval time

$$t_{co} = \frac{Q^*}{r}$$

$$= \frac{6928}{4000}$$

$$= 1.732 \text{ months}$$

Maximum inventory Q_{max}

$$Q_{max} = Q^* - rt_1$$

Where, $t_1 = \frac{Q^*}{d}$

$$= \frac{6928}{6000}$$

$$= 1.154 \text{ months}$$

$$Q_{max} = 6928 - 4000 \times 1.154$$

$$= 2309.33 \text{ units}$$

Problem:

The demand for a certain item is 150 units per week. No shortages are to be permitted. Holding cost is 5 paisa per unit per week. Demand can be met either by manufacturing or purchasing.

With each source the data are as follows

	Manufacture	Purchase
Item cost Rs./ Unit	10.50	12
Set up/ Ordering cost Rs. / Order or set up	90	20
Replenishment rate units / week	260	Infinite
Lead time in weeks	4	10

Determine (a) the minimum cost procurement source and its economic advantage over its alternative resource, (b) E.O.Q. or E.B.Q. as per the source selected, (c) the minimum procurement level (Re-order point)

Solution:-

For manufacture

Given DataCo = Rs. 90

Ch = Rs. 0.05 per unit per monthd = 260 units per weeks

r = 150 units per weeksT_L = 4 week

C = Rs. 10.50 per unit

$$TVC = \sqrt{2rC_oC_h(\frac{d-r}{d})} + C \times r$$

$$= \sqrt{2 \times 150 \times 90 \times 0.05 \times (\frac{260-150}{260})} + 10.50 \times 150$$

$$= 1598.9 \text{ Rs. per week}$$

For purchase Given Data

Co = Rs. 20

Ch = Rs. 0.05 per unit per monthr = 150 units per weeks

TL = 4 week

C = Rs. 12 per unit

$$TVC = \sqrt{2rC_oC_h} + C \times r$$

$$= \sqrt{2 \times 150 \times 20 \times 0.05} + 12 \times 150$$

$$= 1817.32 \text{ Rs. per week}$$

(a) Minimum TC is 1598.9 per week for manufacture

(b) E.B.Q

$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{d}{d-r}}$$

$$= \sqrt{\frac{2\times 90\times 150}{0.05}} \times \sqrt{\frac{260}{260-150}}$$

$$= 1129.76 \text{ units}$$

(c) Re-order point

$$Q_R = r \times T_L$$

$$= 150 \times 4$$

$$= 600 \text{ units}$$

Model 3- EOQ Model with Shortages (backorders) allowed

Assumptions of this model are same as those of model 1 except Shortages is allowed.

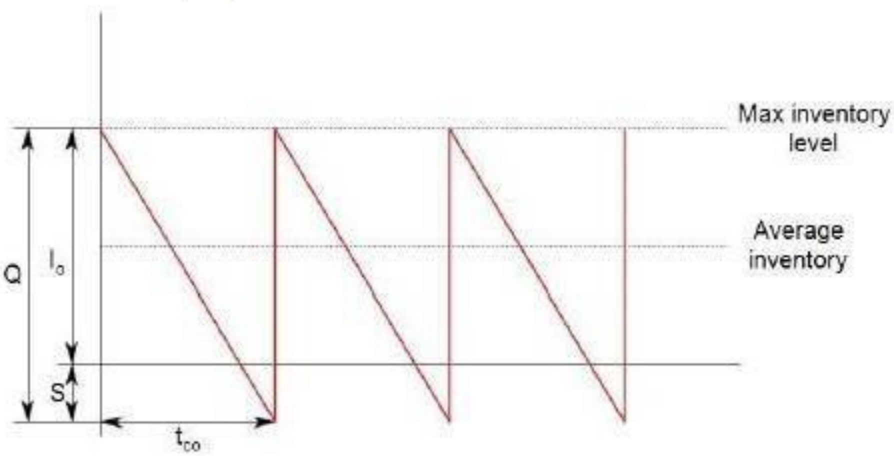
$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{C_h + C_s}{C_s}}$$

$$TVC = \sqrt{2rC_oC_h(\frac{C_s}{C_h + C_s})}$$

$$t_{co} = \frac{Q^*}{r}$$

Initial stock I_o

$$I_o = Q^* \times (\frac{C_s}{C_h + C_s})$$



Problem:

A tractor manufacturing company has entered in to a contract with M/s Auto Diesel for delivering 30 engines per day. M/s Auto Diesel has committed that for every day's delay in delivery; there will be penalty of delayed supply at the rate of Rs. 100 per engine per day. M/s Auto Diesel has the inventory holding cost of Rs. 600 per engine per month. Assume replenishment of engines as instantaneous and ordering cost as Rs. 15000. What should be initial inventory level and what should be ordering quantity for minimum associated cost of inventory?At what interval procurement should be made?

Solution:-

Given Data

Co = Rs. 15000

Ch = Rs. 600 per engine per month

Cs = 100*30 =3000 per engine per monthr = 30*30 = 900 engine per month

$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{C_h + C_s}{C_s}}$$

$$= \sqrt{\frac{2 \times 900 \times 15000}{600}} \times \sqrt{\frac{600 + 3000}{3000}}$$

$$= 233 \text{ engines}$$

$$I_o = Q^* \times (\frac{C_s}{C_h + C_s})$$

$$= 233 \times (\frac{3000}{600 + 3000})$$

$$= 195 \text{ engines}$$

$$t_{co} = \frac{Q^*}{r}$$

$$= \frac{233}{30}$$

$$= 7.76 \text{ days}$$

Problem:

In above example, find out optimum order quantity if shortage is not permitted. Compare this with the value of obtained in above example and comment on the result.

Ans:-

$$Q^* = \sqrt{\frac{2rC_o}{C_h}}$$

$$= \sqrt{\frac{2 \times 900 \times 15000}{600}}$$

$$= 212 \text{ engines}$$

$\Rightarrow$ If shortage is not permitted, EOQ is reducing 233 engines per order to 212 engines per order.

Model 4- EOQ Model with Uniform Replenishment

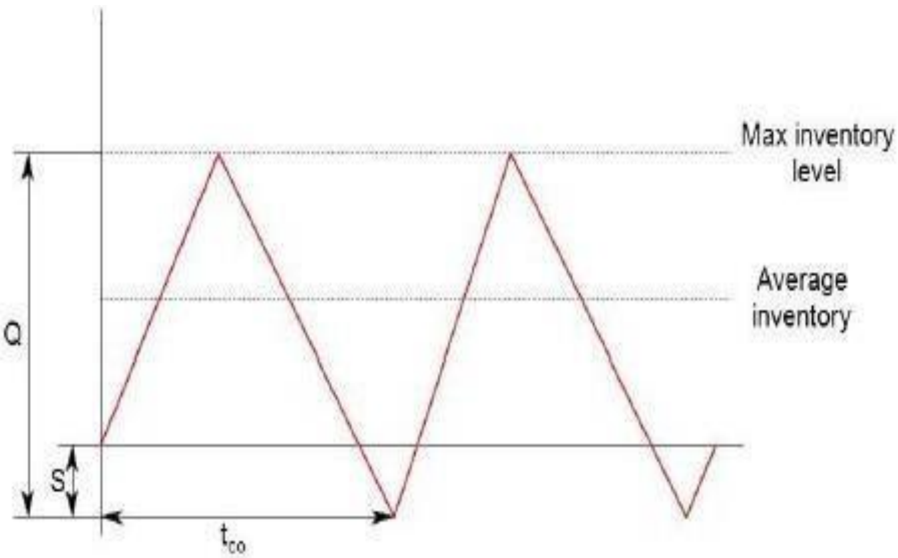
Assumptions of this model are same as those of model 1 except Demand is variable and Shortages is allowed.

$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{d}{d-r}} \times \sqrt{\frac{C_h + C_s}{C_s}}$$

$$TVC = \sqrt{2rC_oC_h\left(\frac{d-r}{d}\right)\left(\frac{C_s}{C_h+C_s}\right)}$$

$$t_{co} = \frac{Q^*}{r}$$

Initial stock I_o $$I_o = Q^* \times \left(\frac{C_s}{C_h+C_s}\right) \times \left(\frac{d-r}{d}\right)$$



Problem :

The demand for an item in a company is 18000 units per year and the company can produce the item at a rate of 3000 units per month. The set up cost is Rs. 500 per set up and the annual inventory holding cost is estimated at 20 percent of the investment in average inventory. The cost of one unit short is Rs. 20 per year. Determine, (i) Optimal production batch quantity, (ii) Optimum cycle time and production time, (iii) Maximum inventory level in the cycle, (iv) Maximum shortage permitted and (v) Total associated cost per year. The cost of the items is Rs. 20 per unit.

Solution:-

Given Data Co = Rs. 500

Ch = 0.20*20 = Rs. 4 per unit per year Cs = Rs. 20 per unit per year r = 18000 unit per year d = 3000*12 = 36000 unit per year

$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{d}{d-r}} \times \sqrt{\frac{C_h + C_s}{C_s}}$$
$$= \sqrt{\frac{2 \times 18000 \times 500}{4}} \times \sqrt{\frac{36000}{36000-18000}} \times \sqrt{\frac{4+20}{20}}$$
$$= 3286 \text{ units}$$

$$t_{co} = \frac{Q^*}{r}$$
$$= \frac{3286}{18000}$$
$$= 0.18255 \text{ year}$$
$$= 2.19 \text{ months}$$

$$t_{po} = \frac{Q^*}{d}$$
$$= \frac{3286}{36000}$$
$$= 0.091277 \text{ year}$$
$$= 1.09 \text{ months}$$

$$I_o = Q^* \times (\frac{C_s}{C_h + C_s}) \times (\frac{d-r}{d})$$
$$= 3286 \times (\frac{20}{4+20}) \times (\frac{36000-18000}{36000})$$
$$= 1369 \text{ units}$$

$$S = Q^* \times (\frac{C_h}{C_h + C_s}) \times (\frac{d-r}{d})$$
$$= 3286 \times (\frac{4}{4+20}) \times (\frac{36000-18000}{36000})$$
$$= 273.88 \text{ units}$$

$$TC = \sqrt{2rC_oC_h(\frac{d-r}{d})(\frac{C_s}{C_h+C_s})} + C \times r$$

$$TC = \sqrt{2 \times 18000 \times 500 \times 4 \times (\frac{36000-18000}{36000})(\frac{20}{4+20})} + 20 \times 18000$$

$$= 365477.22$$

Model 5- EOQ Model with Quantity Discounts :

Quantity discounts occur in numerous situations where suppliers provide an incentive for large order quantities by offering a lower purchase cost when items are ordered in larger lots or quantities. In this section we show how the EOQ model can be used when quantity discounts areavailable.

EOQ without discounts

$$Q^* = \sqrt{\frac{2rC_o}{C_h}}$$

- EOQ with discounts

$$Q_o^* = \frac{(C_d \times r + Q^* \times i \times C)}{i(C - C_d)}$$

- Max Net Saving with discounts

$$X_{max} = \frac{(C_d \times r + Q^* \times i \times C)^2}{2 \times i \times r(C - C_d)} - C_o$$

Problem:

A wholesale dealer in bearings purchases 30000 bearings annually at intervals and order size suitable to him. The price is Rs. 150 per bearing. The manufacturing company offers the dealer a discount of Rs. 7 per bearing for the order size larger than earlier. The reorder cost is Rs. 40 and the inventory carrying cost amounts to 20 percent of the investment in purchase price. Decide the optimum order size for special discount offer purchase and the maximum benefit he can derive from this order.

Solution:-

Given Data Co = Rs. 40 Ch = 0.20*150 = Rs. 30 per unit per year r = 30000 unit per year C = Rs 150 per unit Cd = Rs 7 per unit i = 0.20

$$Q^* = \sqrt{\frac{2rC_o}{C_h}}$$

$$= \sqrt{\frac{2 \times 30000 \times 40}{30}}$$

$$= 283 \text{ units}$$

$$Q_o^* = \frac{(C_d \times r + Q^* \times i \times C)}{i(C - C_d)}$$

$$= \frac{(7 \times 30000 + 283 \times 0.2 \times 150)}{0.2(150 - 7)}$$

= 7640 Units

$$X_{max} = \frac{(C_d \times r + Q^* \times i \times C)^2}{2 \times i \times r(C - C_d)} - C_o$$

$$= \frac{(7 \times 30000 + 283 \times 0.2 \times 150)^2}{2 \times 0.2 \times 30000(150 - 7)} - 40$$

= 27775

Model 6- Probabilistic Inventory Models

The inventory models that we have discussed thus far have been based on the assumption that the demand rate is constant and deterministic throughout the year. We developed minimum-cost order quantity and reorder-point policies based on this assumption. In situations where the demand rate is not deterministic, models have been developed that treat demand as probability distribution. In this section we consider a single-period inventory model with probability demand. The single-period inventory model refers to inventory situations in which one order is placed for the product; at the end of the period, the product has either sold out, or there is a surplus of unsold items that will be sold for a salvage value. The single-period inventory model is applicable in situations involving seasonal or perishable items that cannot be carried in inventory and sold in future periods. Seasonal clothing (such as bathing suits and winter coats) is typically handled in a single-period manner. In these situations, a buyer places one preseason order for each item and then experiences a stock out or hold a clearance sale on the surplus stock at the end of the season. No items are carried in inventory and sold the following year.

Newspapers are another example of a product that is ordered one time and is either sold or not sold during the single period. While newspapers are ordered daily, they cannot be carried in inventory and sold in later periods. Thus, newspaper orders may be treated as a sequence of single-period models; that is, each day or period is separate, and a single-period inventory decision must be made each period (day). Since we order only once for the period, the only inventory decision we must make is how much of the product to order at the start of the period. Because newspaper sales are an excellent example of a single-period situation, the single-periodinventory problem is sometimes referred to as the newsboy problem.

Optimum stock level

$$P_r = \frac{C_s}{C_s + C_h}$$

Problem:

A large industrial campus has decided to have its diesel generator system for street lighting, security illumination and round the clock process systems. The generator needs a tailor made for each other control unit which cost Rs. 18000 per number when ordered with the total equipment of diesel generator. A decision needs to be taken whether additional numbers of this unit should be ordered along with equipment, and if so, how many units should be ordered? These control units, though tropicalized and considered quite reliable, are known to have failed from time to time and history of failures of similar equipment give the following probability of failure. It is found that if the control unit fails, the entire generator system comes to a grinding halt. When control unit fails and a spare unit is not available it is estimated that the cost rush order procurement, including the associated cost of the downtime is Rs. 50000 per unit. Considering that any investment in inventory is the cost of inventory, decide how many spare units should be ordered along with the original order. Determine total associated cost for each no. of spare unit

No. of units having failed and hence No. of spare Required	0	1	2	3	4	5	6
Probability	0.6	0.2	0.1	0.05	0.03	0.02	0

Ans:-

⇒ Let us consider the elementary approach as the population of demand varies only from 0 to 5.

⇒ I = 0,

$$TAC = C_s \times 1 \times P_1 + C_s \times 2 \times P_2 + C_s \times 3 \times P_3 + C_s \times 4 \times P_4 + C_s \times 5 \times P_5$$

$$= 50000(0.2 + 2 \times 0.1 + 3 \times 0.05 + 4 \times 0.03 + 5 \times 0.02)$$

$$= 38500$$

$\Rightarrow$ I = 1,

$$TAC = C_h \times 1 \times P_0 + C_s \times 0 \times P_1 + C_s \times 1 \times P_2 + C_s \times 2 \times P_3 + C_s \times 3 \times P_4 + C_s \times 4 \times P_5$$

$$= 18000 \times 0.6 + 50000 \times 0.1 + 50000 \times 2 \times 0.05 + 50000 \times 3 \times 0.03 + 50000 \times 4 \times 0.02$$

$$= 29300$$

$\Rightarrow$ I = 2,

$$TAC = C_h \times 2 \times P_0 + C_h \times 1 \times P_1 + C_s \times 0 \times P_2 + C_s \times 1 \times P_3 + C_s \times 2 \times P_4 + C_s \times 3 \times P_5$$

$$= 18000 \times 2 \times 0.6 + 18000 \times 0.2 + 50000 \times 1 \times 0.05 + 50000 \times 2 \times 0.03 + 50000 \times 3 \times 0.02$$

$$= 33700$$

$\Rightarrow$ I = 3,

$$TAC = C_h \times 3 \times P_0 + C_h \times 2 \times P_1 + C_h \times 1 \times P_2 + C_s \times 0 \times P_3 + C_s \times 1 \times P_4 + C_s \times 2 \times P_5$$

$$= 18000 \times 3 \times 0.6 + 18000 \times 2 \times 0.2 + 18000 \times 1 \times 0.05 + 50000 \times 1 \times 0.03 + 50000 \times 2 \times 0.02$$

$$= 44900$$

$\Rightarrow$ I = 4,

$$TAC = C_h \times 4 \times P_0 + C_h \times 3 \times P_1 + C_h \times 2 \times P_2 + C_h \times 1 \times P_3 + C_s \times 0 \times P_4 + C_s \times 1 \times P_5$$

$$= 18000 \times 4 \times 0.6 + 18000 \times 3 \times 0.2 + 18000 \times 2 \times 0.05 + 18000 \times 1 \times 0.03 + 50000 \times 1 \times 0.02$$

$$= 59500$$

$\Rightarrow$ I = 5,

$$TAC = C_h \times 5 \times P_0 + C_h \times 4 \times P_1 + C_h \times 3 \times P_2 + C_h \times 2 \times P_3 + C_h \times 1 \times P_4 + C_s \times 0 \times P_5$$

$$= 18000 \times 5 \times 0.6 + 18000 \times 4 \times 0.2 + 18000 \times 3 \times 0.05 + 18000 \times 2 \times 0.03 + 18000 \times 1 \times 0.02$$

$$= 76140$$

Cumulative Probability Table

No. of units	$\sum_0^s P_r$
0	0.6
1	0.8
2	0.9
3	0.95
4	0.98
5	1.00
6	1.00

$$P_r = \frac{C_s}{C_s + C_h}$$

$$= \frac{50000}{50000 + 18000}$$

$$= 0.7352$$

As I = 1, total associated cost is minimum, 1 spare units should be ordered along with theoriginal order. Pr is between 0.6 and 0.8 ; $0.6 \le 0.7352 \le 0.8$

Optimum stock level is 1.

Problem: In the above problem as regular purchase price of control unit is almost one third of the estimated rush order associated cost of one unit. The management decides to buy two spare units with the first order. Having decided that, the management would like to know for what range of actual values of shortage cost, the decision is justified

Ans:-

$\Rightarrow$ For I = 2 to be optimum

$$P_{r1} < \frac{C_s}{C_s + C_h} < P_{r2}$$

$$\Rightarrow P_{r1} < \frac{C_s}{C_s + C_h}$$

$$0.8 < \frac{C_s}{C_s + 6000}$$

$$24000 < C_s$$

$$\Rightarrow \frac{C_s}{C_s + C_h} < P_{r2}$$

$$\frac{C_s}{C_s + 6000} < 0.9$$

$$C_s < 54000$$

$\Rightarrow$ Value of C_s is between 24000 and 54000

Problem -

Probabilistic demand of sweets in a large chain of sweet marts is rectangular between 1000 kg and 1400 kg. Profit per kg of fresh sweet sold is Rs. 14.70. If sweet is not sold fresh, next day it can be sold at a loss of Rs. 2.30 per kg. Determine the optimum stock to have fresh sweet on hand every day.

Ans:- Given Data Co = Rs. 14.70 per kg Ch = Rs 2.30 per kg Range = 1400-1000 = 400

$$f(r) = \frac{1}{Range} = \frac{1}{400}$$

$$\int_{1000}^{I_0} f(r)\,dr = \frac{C_s}{C_s + C_h}$$

$$\int_{1000}^{I_0} \frac{1}{400}\,dr = \frac{14.70}{14.70 + 2.30}$$

$$\frac{1}{400}(I_o - 1000) = \frac{14.70}{17}$$

$$I_o = 1.346 \text{ kg}$$

Problem :

A newspaper boy buys daily papers from vendor and gets commission of 4 paisa for each paper sold. As he is always demanding large number in a lot, he has agreed to pay 3 paisa per each copy returned unsold. He has the past experience of the demand (its probability) as under.

23 (0.01), 24 (0.03), 25 (0.06), 26(0.10), 27(0.20), 28(0.25), 29(0.15), 30(0.10), 31(0.05), 32(0.05)

How many papers should he lift from vendor for minimum associated cost?

Solution:-

Given Data

Co = 4 paisa per paperCh = 3 paisa per paper

Cumulative Probability Table

No. of units	$\sum_{0}^{s} P_r$
23	0.01
24	0.04
25	0.10
26	0.20
27	0.40
28	0.65
29	0.80
30	0.90
31	0.95
32	1.00

$$P_r = \frac{C_s}{C_s + C_h}$$
$$= \frac{4}{4+3}$$
$$= 0.571$$

$\Rightarrow$ P_r is between 0.6 and 0.8

$0.4 < 0.571 < 0.65$

$\Rightarrow$ Optimum stock level is 28 newspapers.

ABCAnalysys:

ABC analysis is an inventory categorization method which consists in dividing items into three categories (A, B, C):

- A being the most valuable items,
- B bieng Inter class item
- C being the least valuable ones.

This method aims to draw managers' attention on the critical few (A-items) not on the trivial many (C-items)

The ABC approach states that a company should rate items from A to C, basing its ratings on the following rules:

A-items are goods which annual consumption value is the highest; the top 70-80% of the annual consumption value of the company typically accounts for only 10-20% of total inventory items.
B-items are the interclass items, with a medium consumption value; those 15-25% of annual consumption value typically accounts for 30% of total inventory items.
C-items are, on the contrary, items with the lowest consumption value; t he lower 5% of the annual consumption value typically accounts for 50% of total inventory items .

	Percentage of items	Percentage value of annual usage	
Class A items	About 20%	About 80%	Close day to day control
Class B items	About 30%	About 15%	Regular review
Class C items	About 50%	About 5%	Infrequent review