Module 3: Bevel, Worm, and Cross Helical Gears

1. Straight Bevel Gears

Introduction & Applications

- Bevel gears are used when power needs to be transmitted at an angle, typically 90°.
- Common applications:
  - Automotive differentials
  - Power tools
  - Industrial machinery
  - Aerospace and railway systems

Tooth Terminology

- **Pitch Cone:** The imaginary cone forming the pitch surface of the bevel gear.
- **Base Cone:** The cone formed at the base of the gear teeth.
- **Face Width:** The length of the teeth along the face of the cone.
- **Addendum & Dedendum:** The radial distance from the pitch surface to the tip/root of the tooth.
- **Pressure Angle:** The angle between the direction of the force exerted by the tooth and the tangent to the pitch cone.

Tooth Forces and Stresses

- Bevel gears experience **three types of forces**:
  - **Tangential Force (F_t):** Contributes to power transmission.
  - **Radial Force (F_r):** Acts perpendicular to the gear shaft.
  - **Axial (Thrust) Force (F_a):** Acts along the gear shaft.
- **Stress Analysis:**
  - Bending stresses (Lewis Equation for strength calculation)
  - Contact stresses (Hertzian stress calculation for durability)

Equivalent Number of Teeth

- Used to transform bevel gear calculations into equivalent spur gear calculations for ease of design.
- **Formula:** Ze=Zcos⁡3αZ_e = \frac{Z}{\cos^3 \alpha}Ze=cos3αZ where **Z** is the actual number of teeth and **α** is the pitch cone angle.

Estimating Dimensions of a Pair of Straight Bevel Gears

- **Module (m):** Determined based on torque transmission needs.
- **Pitch Angle (δ):** Calculated using the formula tan⁡δ=Z2Z1\tan \delta = \frac{Z_2}{Z_1}tanδ=Z1Z2 where $\mathbf{Z_1}$ and $\mathbf{Z_2}$ are the number of teeth on the pinion and gear.
- **Cone Distance (R):** R=mZ2sin⁡δR = \frac{m Z}{2 \sin \delta}R=2sinδmZ
- **Face Width (b):** Typically 1/3 to 1/4 of the cone distance.

2. Worm Gears

Introduction & Working Principle

- Worm gears consist of a **worm (screw-like shaft)** and a **worm wheel**.
- Used for **high-speed reduction** (ratios up to 100:1).
- Applications:
    - Lifting mechanisms (e.g., cranes)
    - Conveyor systems
    - Automotive steering systems

Merits & Demerits

✔ Advantages:

- High speed reduction in a single stage
- Self-locking capability (prevents back-driving)
- Smooth and quiet operation

✖ Disadvantages:

- Low efficiency (due to sliding friction)
- High heat generation requiring lubrication
- Requires high-quality materials

Thermal Capacity & Material Selection

- **Thermal capacity:** Heat generated during operation must be dissipated.
- **Materials:**
    - **Worm:** Hardened steel or case-hardened alloy steel
    - **Worm Wheel:** Bronze (reduces friction)

Forces and Stresses in Worm Gears

- **Forces Acting on Worm Gear:**
    - Axial force

    - Radial force
    - Tangential force (for torque transmission)
- **Stress Analysis:**
    - Bending stresses on worm wheel teeth
    - Shear stresses on the worm
    - Surface fatigue (due to sliding contact)

Efficiency of Worm Gears

- Efficiency depends on the **lead angle (λ)** and the **coefficient of friction (μ)**.
- Formula for efficiency: η=tan⁡λtan⁡(λ+ϕ)\eta = \frac{\tan \lambda}{\tan (\lambda + \phi)}η=tan(λ+ϕ)tanλ where **φ** is the friction angle.
- **Typical efficiency range:** 40-85% (higher for larger lead angles).

Estimating the Size of the Worm Gear Pair

- **Lead angle calculation:** λ=tan⁡−1(Lπdw)\lambda = \tan^{-1} \left(\frac{L}{\pi d_w} \right)λ=tan−1(πdwL) where **L** is the lead of the worm, and **d_w** is the worm diameter.
- **Worm wheel dimensions:** dw=mZwd_w = m Z_wdw=mZw where **Z_w** is the number of teeth on the worm wheel.
- **Center distance (C):** C=dw+dg2C = \frac{d_w + d_g}{2}C=2dw+dg where **d_g** is the worm gear diameter.

3. Cross Helical Gears

Introduction & Applications

- Used in cases where **shafts intersect at angles other than 90°**.
- Applications:
    - Textile machinery
    - Robotics
    - Light-duty power transmission

Terminology & Design Parameters

- **Helix Angle (β):** The angle between the gear tooth and the axis of rotation.
- **Normal Module (m_n):** Module in the normal plane, used to define gear size.
- **Transverse Module (m_t):** mt=mncos⁡βm_t = \frac{m_n}{\cos \beta}mt=cosβmn
- **Lead Angle (λ):** Determines the amount of axial movement per revolution.

Forces in Cross Helical Gears

- **Tangential Force (F_t):** Contributes to power transmission.

- **Axial Force (F_a):** Induces thrust load.
- **Radial Force (F_r):** Acts perpendicular to the shaft.

Estimating the Size of Cross Helical Gear Pairs

- **Pitch Circle Diameter (d):** d=Zmtcos⁡βd = \frac{Z m_t}{\cos \beta}d=cosβZmt
- **Center Distance (C):** C=d1+d22C = \frac{d_1 + d_2}{2}C=2d1+d2 where **d_1** and **d_2** are the pitch diameters of the pinion and gear.