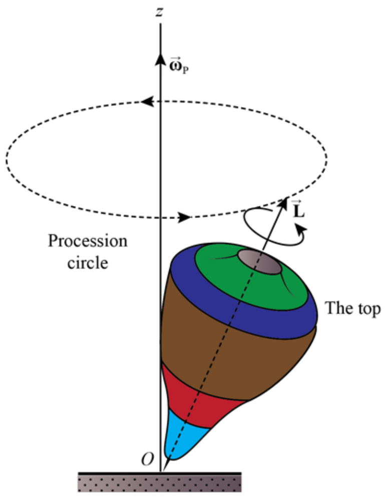
UNIT-II

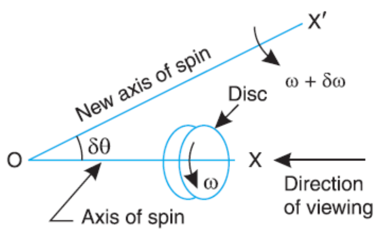
GYROSCOPE & TURNING MOMENT DIAGRAMS

Gyroscope:

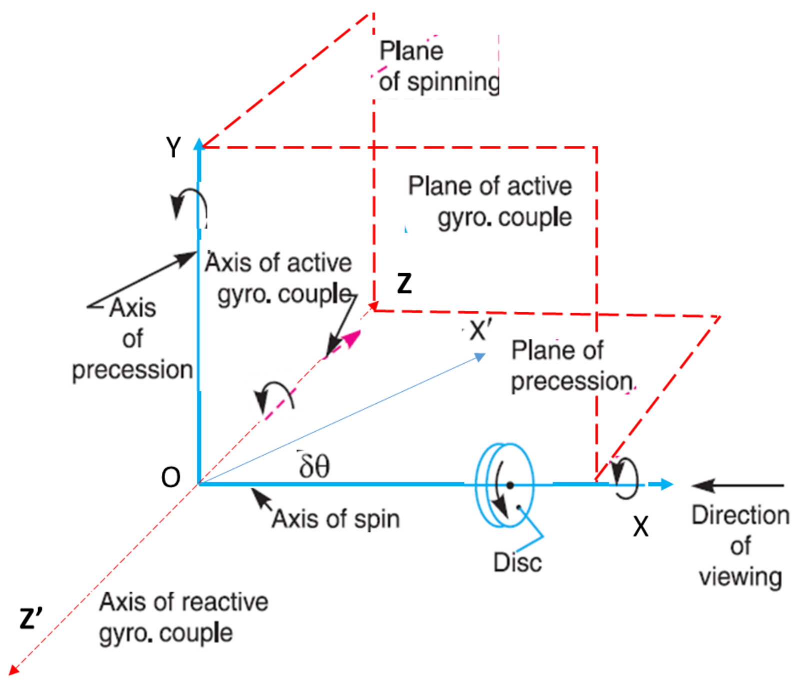
The axis of precession is perpendicular to the plane in which the axis of spin is going to rotate. If the angular velocity of the disc remains constant at all positions of the axis of spin, then $d\theta/dt$ is zero; and thus α_c is zero. If the angular velocity of the disc changes the direction, but remains constant in magnitude, then angular acceleration of the disc is given by,

The angular acceleration $\alpha_c = \omega.{}^{d\theta}/_{dt} = \omega.\omega_p$ is known as *gyroscopic acceleration*





Gyroscopic couple



Since the plane in which the disc is rotating is parallel to the plane *YOZ,* therefore it is called ***plane of spinning***

The horizontal plane *XOZ* is called ***plane of precession.***

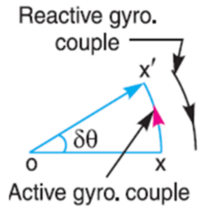
OY is the ***axis of precession.***

I = Mass moment of inertia of the disc about OX, and $\therefore$ Angular momentum of the disc $= I.\omega$

ω = Angular velocity of the disc.

$\therefore$ Change in angular momentum $= \vec{ox'} - \vec{ox} = \vec{xx'} = \vec{ox}.\delta\theta$ ...(in the direction of $\vec{xx'}$)

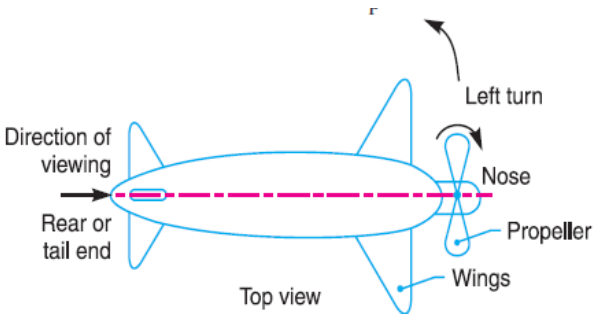
$= I.\ \omega.\delta\theta$ and rate of change of angular momentum $= I.\omega \times \dfrac{\delta\theta}{dt}$

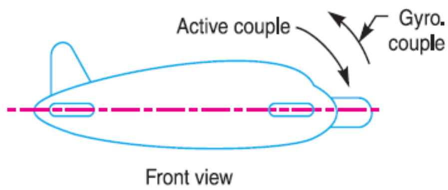


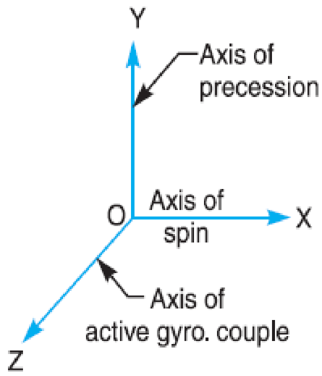
- Since the rate of change of angular momentum will result by the application of a couple to the disc, therefore the couple applied to the disc causing precession

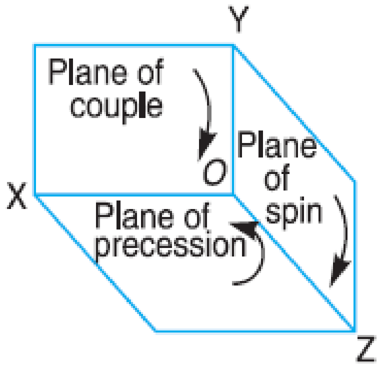
$$C = \underset{\delta t \to 0}{\text{Lt}}\ I.\omega \times \frac{\delta\theta}{\delta t} = I.\omega \times \frac{d\theta}{dt} = I.\omega.\omega_P \qquad ...\left(\because \frac{d\theta}{dt} = \omega_P\right)$$

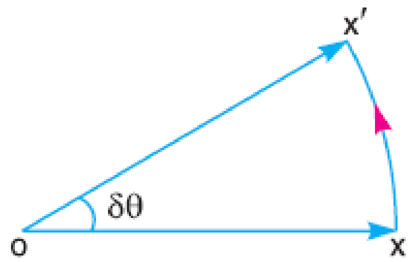
Gyroscopic Couple- In case of Aeroplan



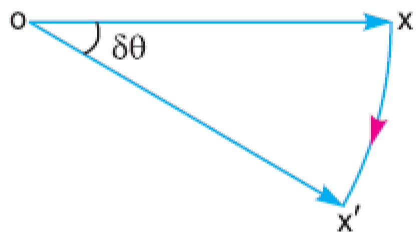








(*a*) Aeroplane taking left turn.



(*b*) Aeroplane taking right turn.

from the rear or tail end propeller rotates in the clockwise direction and the aeroplane takes a turn to the left Turn	Raise the nose and dip the tail of the aeroplane
Right turn	Dip the nose and raise the tail
engine or propeller rotates in anticlockwise direction Left turn	Dip the nose and raise the tail of the aeroplane
Right turn	Raise the nose and dip the tail of the aeroplane
propeller rotates in clockwise direction Left turn	Raise the tail and dip the nose of the aeroplane.
Right turn	Raise the nose and dip the tail of the aeroplane

A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it process about the vertical axis?

given $d = 300 \text{ mm or r} = 150\text{mm} = 0.15\text{mm} \;\; \text{m} = 5\text{kg} \;\; \text{l} = 600\text{mm} \; = 0.6\text{m}$

$\text{N} = 300 \text{ r.p.m or } \omega = 2\pi \times \frac{360}{60} = 31.42\frac{\text{rad}}{\text{s}} \text{ I} = \text{m}.\frac{\text{r}^2}{2} = 5.\frac{0.15^2}{2}$

$$= 0.056\text{kg} - \text{m}^2$$

$$C = m.g.l = 5 \times 981 \times 0.6 = 29.43N - m$$

$$\omega_\rho = speeed\ of\ precession$$

$$29.43 = I.\omega.\omega_\rho = \; 0.056 \times 31.42 \times \omega_\rho \; = 1.76\omega_\rho$$

$$\omega_\rho = \frac{29.43}{1.76} = 16.7\text{rad/s}$$

A uniform disc of 150 mm diameter has a mass of 5 kg. It is mounted centrally in bearings which maintain its axle in a horizontal plane. The disc spins about it axle with a constant speed of 1000 r.p.m. while the axle precesses uniformly about the vertical at 60 r.p.m. The directions of rotation are as shown in Fig. If the distance between the bearings is 100 mm, find the resultant reaction at each bearing due to the mass and gyroscopic effects.

Give,

$d = 150\text{mm or } r = 75\text{mm} = 0.75\text{mm, M} = 5\text{kg, N} = 1000\text{rpm or, } \omega = 2\pi\frac{1000}{60}$

$$x = 104.7\frac{\text{rad}}{\text{sN}_\text{P}} = 60\frac{\text{rad}}{\text{s}}\text{or}$$

$$\omega_\rho = 2\pi\frac{60}{60} = \frac{6.284\text{rad}}{\text{s}}$$

$\text{x} = 100\text{mm} = 0.1\text{m I} = \text{m}\frac{\text{r}^2}{2} = 5\,\frac{0.075^2}{2} = 0.014\text{kgm}^2$

$$C = I \times \omega \times \omega_{\rho} = 0.014 \times 104.7 \times 6.284 = 9.2\text{N} - \text{m}$$

$$F = \frac{C}{X} = \frac{9.2}{0.1} = 92\text{N}$$

$$R_A = R_B$$

$$\frac{5}{2} = 2.5\text{Kg} = 2.5 \times 9.81 = 24.5\text{N}$$

$$R_{A1}\ and\ R_{B1}$$

$$R_{A1} = F + R_A = 92 + 24.5 = 116.5\text{N(upwards)}$$

$$R_{B1=}F - R_B = 92 - 24.5$$

$$= 67.5\text{N(downwards)}$$

An plane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it

Given

$R = 50\text{m}$, v= $\frac{2000\text{km}}{\text{hr}} = \frac{55.6\text{m}}{\text{sm}} = 4000\text{kg}$

$k = 0.3\text{mN} = 2400\text{rpm or } \omega = 2\pi \times \frac{2400}{60} = 251 rad/s$

$I = \text{m.k}^2 = 400 \times 0.3^2 = 36\text{kg} - \text{m}^2$

$$\omega_{\rho} = \frac{V}{R} = \frac{55.6}{50} = 1.11\text{rad/s}$$

$$C = I \times \omega \times \omega_{\rho} = 36 \times 251.4 \times 1.11 = 100.46\text{N} - \text{m}$$

Slide 12

$$\theta = \phi \sin(\omega_1.t)$$

$$\omega_1 = angular\ velocity\ of\ S.H.M$$

$$= \frac{2\pi}{time\ period\ of\ S.H.Min\ sec}$$

$$= \frac{\frac{2\pi}{t_p} rad}{s}$$

$$\cos\ (\omega_1.t) = 1$$

$$\omega_{\rho} = \frac{d\theta}{dt} = \frac{d}{dt}\big(\phi \sin(\omega_1.t)\big) \quad = \phi\omega_1 \cos(\omega_1.t)$$

$$\omega_{pmax} = \phi.\omega_1 = \phi \times \frac{2\pi}{t_p}$$

$$C_{max} = I.\omega.\omega_{pmax}$$

$$\alpha = \frac{d^2\theta}{dt^2} = -\phi(\omega_1)^2$$

$$\sin(\omega_1.t)\omega_1.t = 1$$

$$\alpha_{max} = (\omega_1)^2$$

The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steer to the left in a curve of 75 m radius

Given

$m = 8t = 8000kg$ $k = 0.6m$ $N = 1800rpm$ or $\omega = 2\pi \times \frac{1800}{60} = \frac{188.5rad}{s}$

$v = \frac{100km}{h} = \frac{27.8m}{s}$ $r = 75m$

$$I = m.k^2 = 8000(0.6)^2 = 2880kg - m^2$$

$$\omega_\rho = \frac{v}{r} = \frac{27.8}{75} = 0.37rad/s$$

$$C = I \times \omega \times \omega_\rho = 2880 \times 188.5 \times 1.37 = 200.866n - m$$

***The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship: 1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h. 2. when the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees*.**

Given

$m = 3500kg, k = 0.45m, N = 3000rpm$ or $\omega = 2\pi \times \frac{3000}{60} = 314.2rad/s$

Given

$v = \frac{km}{h}$=10m/s $r = 100m$

$$I = m.k^2 = 3500(0.45)^2 = 708.75kg - m^2$$

$$\omega_\rho = \frac{v}{r} = \frac{10}{100} = 0.1rad/s$$

$C = I \times \omega \times \omega_\rho = 708.75 \times 314.2 \times 0.1$

$$= 22270n - m$$

Given

$t_p = 40s\ 2\phi = 12,\ \phi = \frac{12}{2} = 6 = 6 \times \frac{\pi}{180} = 0.105rad$

$$\omega_1 = \frac{2\pi}{t_p} = \frac{2\pi}{40} = 0.157rad$$

$$\omega_\rho = \phi.\omega_1 = 0.15 \times 0.157 = 0.0165\ rad/s$$

$$C = I \times \omega \times \omega_\rho = 708.75 \times 314.2 \times 0.0165 = 3675N - m$$

A ship propelled by a turbine rotor which has a mass of 5 tonnes and a speed of 2100 r.p.m. The rotor has a radius of gyration of 0.5 m and rotates in a clockwise direction when viewed from the stern. Find the gyroscopic effects in the following conditions: 1. The ship sails at a speed of 30 km/h and steers to the left in a curve having 60 m radius. 2. The ship pitches 6 degree above and 6 degree below the horizontal position. The bow is descending with its maximum velocity. The motion due to pitching is simple harmonic and the periodic time is 20 seconds. 3. The ship rolls and at a certain instant it has an angular velocity of 0.03 rad/s clockwise when viewed from stern. Determine also the maximum angular acceleration during pitching. Explain how the direction of motion due to gyroscopic effect is determined in each case.

Given

$$m = 5t = 5000kg$$

$$N = 2100rpm \text{ or } \omega = 2\pi \times \frac{2100}{60} = 220rad/s$$

K=0.5m

Given

$$v = 30\frac{km}{h}{=}8.33m/s,\ r = 60m,\ \omega_\rho = \frac{v}{r} = \frac{8.33}{60} = 0.14rad/s$$

$$I = m.k^2 = 5000(0.5)^2 = 1250kg - m^2$$

$$C = I \times \omega \times \omega_\rho = 1250 \times 220 \times 0.14 = 38500n - m$$

Given

$t_p = 20s,\ \phi = 6 = 6 \times \frac{\pi}{180} = 0.105rad$

$$\omega_1 = \frac{2\pi}{t_p} = \frac{2\pi}{20} = 0.3142rad$$

$$\omega_{\rho max} = \phi.\omega_1 = 0.105 \times 0.3142 = 0.033\ rad/s$$

$$C_{max} = I \times \omega \times \omega_{\rho max} = 1250 \times 220 \times 0.033 = 9075N - m$$

$\omega_\rho = \frac{0.03rad}{s},\ C = I \times \omega \times \omega_\rho,\ 1250 \times 220 \times 0.03 = 8250N - M$

$\alpha_{max} = \phi.\omega^2 = 0105(0.3142)^2 = 0.01rad/r^2$

The turbine rotor of a ship has a mass of 2000 kg and rotates at a speed of 3000 r.p.m. clockwise when looking from a stern. The radius of gyration of the rotor is 0.5 m. Determine the gyroscopic couple and its effects upon the ship when the ship is steering to the right in a curve of 100 m radius at a speed of 16.1 knots (1 knot = 1855 m/hr). Calculate also the torque and its effects when the ship is pitching in simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 50 seconds and the total angular displacement between the two extreme positions of pitching is 12°. Find the maximum acceleration during pitching motion.

Given

$m = 2000\text{kg}, \text{k} = 0.5\text{m}, \text{N} = 3000\text{rpm or } \omega = 2\pi \times \frac{3000}{60} = 314.2 rad/s$

$\text{v} = 16.1\text{knots} = 16.1 \times \frac{1855}{3600} = 8.3\text{m/s}, \text{r} = 100\text{m}$

$$\text{I} = \text{m.k}^2 = 2000(0.5)^2 = 500\text{kg} - \text{m}^2$$

$$\omega_\rho = \frac{\text{v}}{\text{r}} = \frac{8.3}{10} = 0.083\text{rad/s}$$

$$\text{C} = \text{I} \times \omega \times \omega_\rho = 500 \times 314.2 \times 0.083 = 13.040\text{n} - \text{m}$$

Given

$\text{t}_\text{p} = 50\text{s}, 2\phi = 12, \phi = \frac{12}{2} = 6 = 6 \times \frac{\pi}{180} = 0.105\text{rad}$

$$\omega_1 = \frac{2\pi}{\text{t}_\text{p}} = \frac{2\pi}{50} = 0.1257\text{rad}$$

$\omega_{\rho\text{max}} = \phi.\omega_1 = 0.15 \times 0.1257 = 0.0132 \text{ rad/s}$

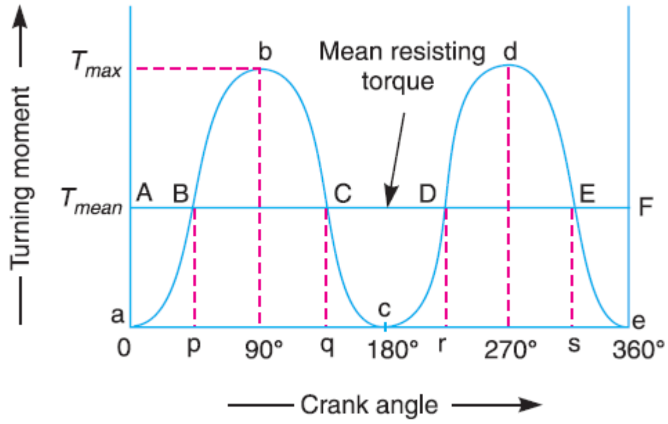
$$\text{C}_{\text{max}} = \text{I} \times \omega \times \omega_{\rho\text{max}} = 500 \times 314.2 \times 0.0132 = 2074\text{N} - \text{m}$$

$$\alpha_{\text{max}} = \phi.\omega^2 = 0.105(0.1257)^2 = 0.00166\text{rad/r}^2$$

Turning Moment Diagrams-TMD- *crank effort diagram*

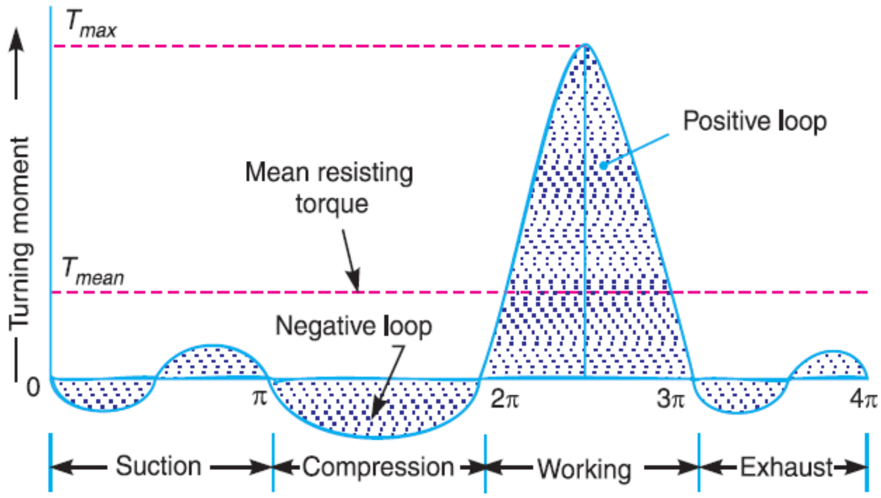
Graphical representation of the turning moment or crank-effort for various positions of the crank. It is plotted on Cartesian co-ordinates, in which the turning moment is taken as the ordinate and crank angle as abscissa.

Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine

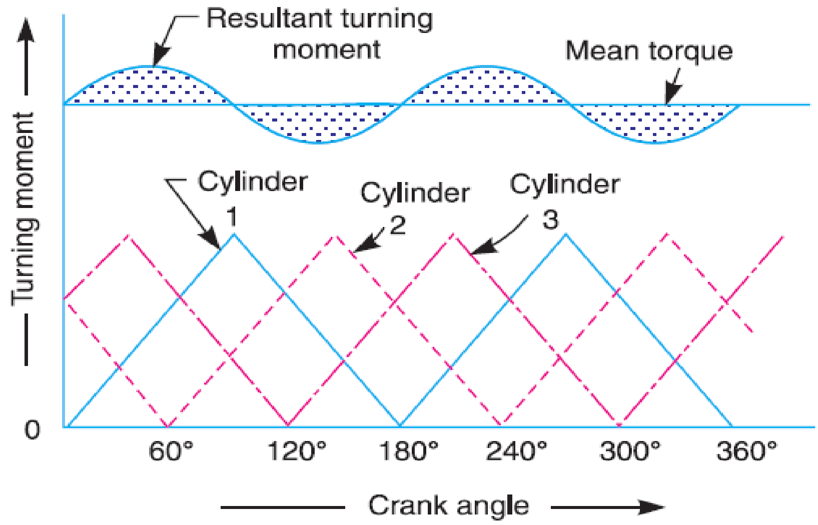


$$T = F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

IC Engine and multi cylinder engine

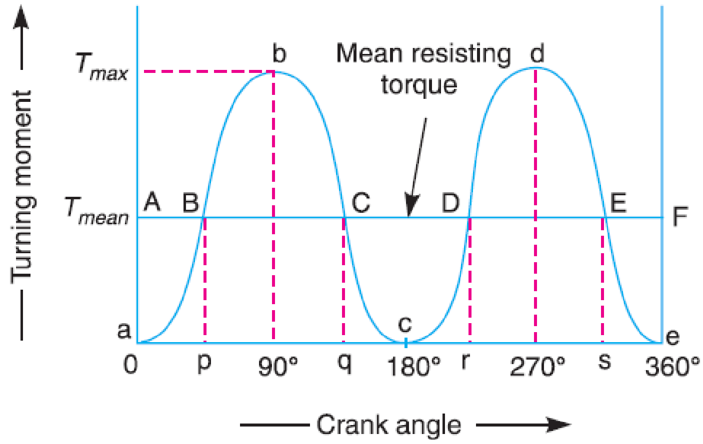


Turning Moment Diagram for a Multi-cylinder Engine

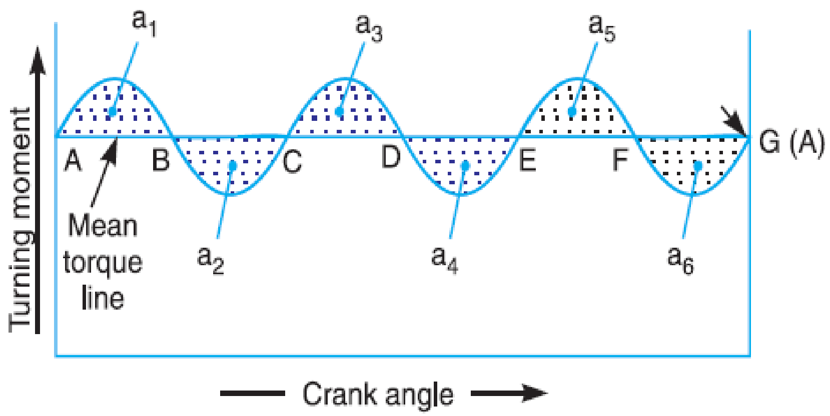


Fluctuation of Energy

*The variations of energy above and below the mean resisting torque line are called **fluctuations of energy**. The areas BbC, CcD, DdE, etc. represent fluctuations of energy.*



Evaluation of Max Fluctuation of Energy



A TMD for a multi-cylinder engine is shown by a wavy curve in Fig. The horizontal line AG represents the mean torque line. Let a1, a3, a5 be the areas above the mean torque line and a2, a4 and a6 be the areas below the mean torque line.

$$T = F_P \times r(\sin(\theta) + \frac{\sin(2\theta)}{\sqrt[2]{n^2 - \sin(\theta)^2}})$$

Energy At different stages

$$A = E$$

$$B = E + a_1$$

$$C = E + a_1 - a_2$$

$$D = E + a_1 - a_2 + a_3$$

$$E = E + a_1 - a_2 + a_3 - a_4$$

$$F = E + a_1 - a_2 + a_3 - a_4 + a_5$$

$$G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$$

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

Coefficient of Fluctuation of Energy

Ratio of the maximum fluctuation of energy to the work done per cycle

S.No.	*Type of engine*	*Coefficient of fluctuation of energy (C_E)*
1.	Single cylinder, double acting steam engine	0.21
2.	Cross-compound steam engine	0.096
3.	Single cylinder, single acting, four stroke gas engine	1.93
4.	Four cylinders, single acting, four stroke gas engine	0.066
5.	Six cylinders, single acting, four stroke gas engine	0.031

ean Torque

$$T_{mean} = \frac{p \times 60}{2\pi N} = \frac{p}{\omega}$$

$$\omega = \frac{2\pi N}{60}$$

$$work\ done\ per\ cycle = \frac{p \times 60}{n}$$

Fluctuation of speeds $C_s = \frac{N_1 - N_2}{N}$

$$= \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega}$$

$$= \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} = \frac{V_1 - V_2}{V} = \frac{2(V_1 - V_2)}{V_1 + V_2}$$

Steadiness constant $m = \frac{1}{C_S} = \frac{N}{N_1 - N_2}$

$$N = \frac{N_1 + N_2}{2}$$

$$\omega = \frac{\omega_1 - \omega_2}{\omega}$$

$C_s = \frac{N_1 - N_2}{N}$ OR$= \frac{\omega_1 - \omega_2}{\omega}$

$$E = \frac{1}{2} \times I\omega^2 = \frac{1}{2} \times m.k^2.\omega^2$$

$$= \frac{1}{2} \times I(\omega_1)^2 - \frac{1}{2} \times I(\omega_2)^2$$

$$= \frac{1}{2} \times I[(\omega_1)^2 - (\omega_2)^2] \qquad (\omega = \frac{\omega_1 - \omega_2}{\omega})$$

$$= \frac{1}{2} \times I(\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= \mathrm{I}.\omega(\omega_1 - \omega_2) \ldots\ldots.(1)$$

$$= \mathrm{I}.\omega^2 \left(\frac{\omega_1 - \omega_2}{\omega}\right) = \mathrm{I}.\omega^2.C_s = m.k^2.\omega^2.C_s \ldots..(I = m.k^2) \ldots\ldots(2)$$

$$= 2.\mathrm{E}.C_s(in\ N - m\ or\ joules) \ldots..(E = \frac{1}{2} \times I.\omega) \ldots..(3)$$

$$\Delta \mathrm{E} = \mathrm{m.R^2}.\omega^2.C_s = m.v^2.C_s$$

$$\mathrm{v} = \omega.\mathrm{R}$$

$$\Delta \mathrm{E} = \mathrm{I}.\frac{2\pi \mathrm{N}}{60}\left(\frac{2\pi \mathrm{N_1}}{60} - \frac{2\pi \mathrm{N_2}}{60}\right)$$

$$= \frac{4\pi^2}{3600}.\mathrm{I.N(N_1 - N_2)}$$

$$= \frac{\pi^2}{900}.m.k^2.N(\mathrm{N_1 - N_2})$$

$$= \frac{\pi^2}{900}.m.k^2.N^2.C_s \ldots\ldots(C_s = \frac{\mathrm{N_1 - N_2}}{N})$$

The mass of flywheel of an engine is 6.5 tonnes and the radius of gyration is 1.8 metres. It is found from the turning moment diagram that the fluctuation of energy is 56 kN-m. If the mean speed of the engine is 120 r.p.m., find the maximum and minimum speeds. M=6.5

T=6500kg

K=1.8m

$$\Delta \mathrm{E} = 56\mathrm{kn - m}$$

$$= 56 \times 10^3 n - m$$

N=120RPM

$$56 \times 10^3 = \frac{\pi^2}{900}.m.k^2.N(\mathrm{N_1 - N_2})$$

$$= \frac{\pi^2}{900} \times 6500(1.8)^2 \times 120(\mathrm{N_1 - N_2})$$

$$= 27715(\mathrm{N_1 - N_2})$$

$$\mathrm{N_1 - N_2} = \frac{56 \times 10^3}{27715} = 2\ rpm\ \ldots..(1)$$

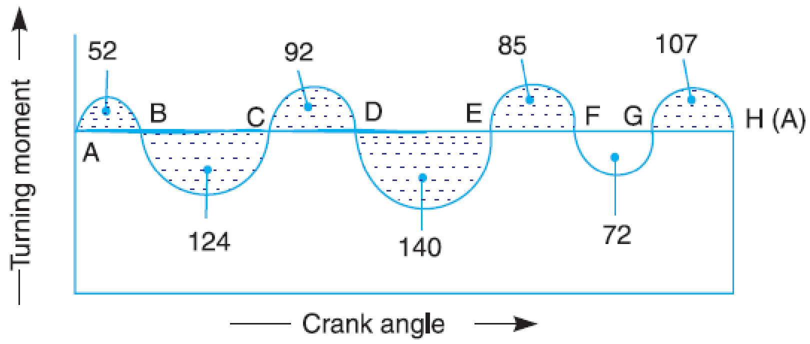
$$120 = \frac{\mathrm{N_1 + N_2}}{2}\ or\ \mathrm{N_1 + N_2}$$

$$= 120 \times 2 = 240 rpm$$

$$from\ equation\ (1) and (2)$$

$$N_1 = 121 rpm \quad N_2 = 119 rpm$$

The turning moment diagram for a multicylinder engine has been drawn to a scale 1 mm = 600 N-m vertically and 1 mm = 3° horizontally. The intercepted areas between the output torque curve and the mean resistance line, taken in order from one end, are as follows : + 52, – 124, + 92, – 140, + 85, – 72 and + 107 mm2, when the engine is running at a speed of 600 r.p.m. If the total fluctuation of speed is not to exceed ± 1.5% of the mean, find the necessary mass of the flywheel of radius 0.5 m.



$N = 600 \text{rpm or } \omega = 2\pi \times \frac{600}{60} = 62.84 \text{ rad/s } r = 0.5\text{m}$

$$C_s = \frac{\omega_1 - \omega_2}{\omega} = 0.03$$

$$\omega_1 - \omega_2 = 3\%\omega = 0.03\omega$$

$$1\text{mm} = 3 = 3 \times \frac{\pi}{180} = \frac{\pi}{60} rad$$

$$1\text{mm}^2 = 600 \times \frac{\pi}{60} = 31.42 N - m$$

$$B = E + 52$$

$$C = E + 52 - 124 = E - 72$$

$$D = E + 72 + 92 = E + 20$$

$$E = E + 20 - 140 = E - 120$$

$$F = E - 120 + 85 = E - 35$$

$$G = E - 35 - 72 = E - 107$$

$$H = E - 107 + 107 = E = energy\ at\ A$$

$$= (E + 52) - (E - 120) = 172 = 172 \times 31.42 = 5404 \text{N} - \text{m}$$

$$\Delta E = m. R^2. \omega^2. C_s$$

$$5404 = m. R^2. \omega^2. C_s$$

$$= m. (0.5)^2. (62.84)^2. (0.03) = 29.6m$$

$$\mathrm{m} = \frac{5404}{29.6} = 183\mathrm{kg}$$

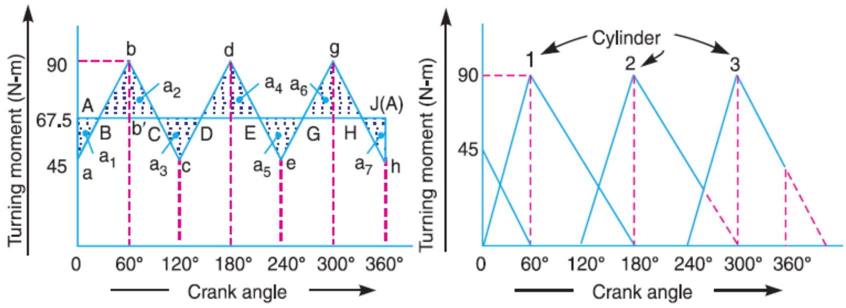
A three cylinder single acting engine has its cranks set equally at 120° and it runs at 600 r.p.m. The torque-crank angle diagram for each cycle is a triangle for the power stroke with a maximum torque of 90 N-m at 60° from dead centre of corresponding crank. The torque on the return stroke is sensibly zero. Determine : 1. power developed. 2. coefficient of fluctuation of speed, if the mass of the flywheel is 12 kg and has a radius of gyration of 80 mm, 3. coefficient of fluctuation of energy, and 4. maximum angular acceleration of the flywheel.

Answers: $N = 600\mathrm{rpm}$ or $\omega = 2\pi \times \frac{600}{60}$ =62.84 rad/s

$\mathrm{T_{max}} = 90\mathrm{N-m}, \mathrm{m} = 12\mathrm{kg}, \mathrm{k} = 80\mathrm{mm} = 0.08\mathrm{m}$

$3 \times \frac{1}{2} \times \pi \times 90 = 424\mathrm{N-m}, \mathrm{T_{mean}} = \frac{\mathrm{workdone/cycle}}{\mathrm{crank}\frac{angle}{\mathrm{cycle}}} = \frac{424}{2\pi} = 67.5\mathrm{N-m}$

power developed $= \mathrm{T_{mean}} \times \omega = 67.5 \times 62.84 = 4240W = 4.24kw$



$$a_1 = AREAof\ triangleAaB = \frac{1}{2} \times AB \times Aa$$

$$= \frac{1}{2} \times \frac{\pi}{6} \times (67.5 - 45) = 5.89N - m = a_7$$

$$a_2 = AREAof\ triangleAaB = \frac{1}{2} \times BC \times bb'$$

$$= \frac{1}{2} \times \frac{\pi}{3} \times (90 - 67.5) = 11.78N - m = a_3 = a_4 = a_5 = a_6$$

A=E

B=E+5.89

C= E-5.89+11.78= E+5.89

D= E+5.89-11.78= E-5.89

E= E-5.89+11.78= E+5.89

G= E+5.89-11.78= E-5.89

H= E-5.89+11.78= E+5.89

J= E+5.89-11.78= E=ENERGY at A

=E+5.89= E-5.89

$\triangle E$ = E+5.89= E-5.89=11.78N-m

11.78= $\mathrm{m.R^2.\omega^2}.C_s = 12 \times (0.08)^2 \times (62.84)^2 \times C_s = 303.3C_s$

$$C_s = \frac{11.78}{303.3} = 0.04$$

$$C_E = \frac{max.fluctuation\ of\ energy}{work\ done\ /cycle} = \frac{11.78}{424} = 0.0278$$

$$T_{max} - T_{mean} = I.\alpha = m.k^2.\alpha$$

$$90 - 67.5 = 12 \times (0.08)^2 \times \alpha = 0.077\alpha$$

$$\alpha = \frac{90 - 67.5}{0.077} = 292rad/s$$

Problem on TMD

Given

$a_1 = 0.45 \times 10^{-3} m^2$

$a_2 = 1.7 \times 10^{-3} m^2$

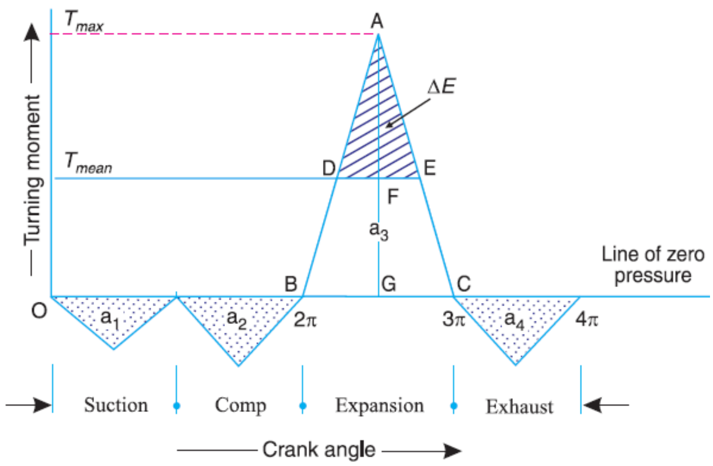
$a_3 = 6.8 \times 10^{-3} m^2$

$a_4 = 0.65 \times 10^{-3} m^2$

$N_1 = 202rpm$

$N_2 = 198rpm$

$R = 1.2m$



$$netarea = a_3 - (a_1 + a_2 + a_4) = 6.8 \times 10^{-3} - (0.45 \times 10^{-3} + 1.7 \times 10^{-3} + 0.65 \times 10^{-3}) = 4 \times 10^{-3} m^2$$

$$1m^2 = 3N - m = 3 \times 10^6 N - m$$

$$net\ work\ done\ per\ cycle = 4 \times 10^{-3} \times 3 \times 10^6 N - m \ldots .(1)$$

$$= T_{mean} \times 4\pi N - m$$

$$T_{mean} = FG = \frac{12 \times 10^6}{4\pi} = 955N - m \ldots\ldots(2)$$

$$= a_3 \times energy\ scale = 6.8 \times 10^{-3} \times 3 \times 10^6 = 20.4 \times 10^3 N - m \ldots\ldots(3)$$

$$= \frac{1}{2} \times BC \times AG = \frac{1}{2} \times \pi \times AG = 1.57 \times AG \ldots\ldots(4)$$

$$from\ equ(3) and (4) AG = \frac{20.4 \times 10^3}{1.57} = 12985N - m$$

$$T_{excess} = AF = AG - FG = 12985 - 955 = 12030N - m$$

$\frac{\mathrm{DE}}{\mathrm{BC}} = \frac{\mathrm{AF}}{\mathrm{AG}}$ OR

$$\mathrm{DE} = \frac{\mathrm{AF}}{\mathrm{AG}} \times \mathrm{BC} = \frac{12030}{12985} \times \pi = 2.9\mathrm{rad}$$

$$\Delta \mathrm{E} = \text{area of } \Delta \mathrm{ADE}$$

$$= \frac{1}{2} \times \mathrm{DE} \times \mathrm{AF} = \frac{1}{2} \times 2.9 \times 12030 = 17444N - m$$

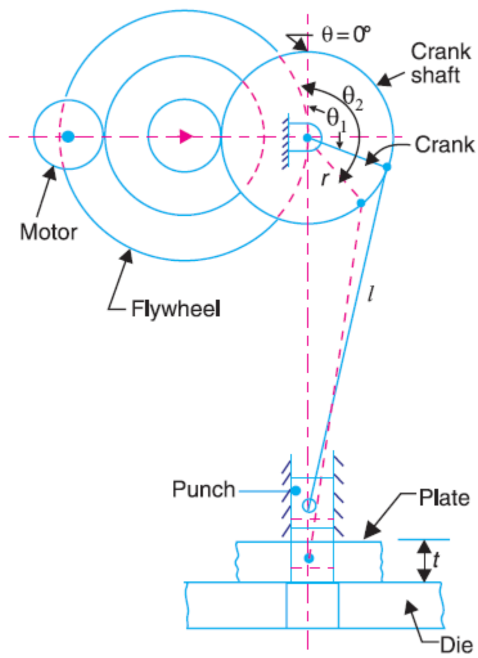
$$= \frac{N_1 + N_2}{2} = \frac{202 + 198}{2} = 200RPM$$

$$17444 = \frac{\pi^2}{900} \times m.R^2.N(N_1 - N_2) = \frac{\pi^2}{900} \times 1.2^2 \times 200(202 - 198) = 12.63m$$

$$m = \frac{17444}{12.36} = 1381kg$$

Flywheel in Punching Press

The flywheel can also be used to perform the same function when the torque is constant and the load varies during the cycle.



Let E1 be the energy required for punching a hole. This energy is determined by the size of the hole punched, the thickness of the material and the physical properties of the material.

Fs = Area sheared × Ultimate shear stress

It is assumed that as the hole is punched, the shear force decreases uniformly from maximum value to zero.

Thus, maximum fluctuation of energy, $\Delta E = E_1 - E_2$

The values of θ_1 and θ_2 may be determined only if the crank radius *(r)*, length of connecting rod *(l)* and the relative position of the job with respect to the crankshaft axis are known.

Problem on Flywheel

Given

$$T_1 = (5000 + 500sin(\theta))N - m$$

$$T_2 = \left(5000 + 600sin(2\theta)\right)N$$

$$-m00\mathrm{kg} = 0.4\mathrm{mN} = 150\mathrm{rpm}$$

$$\omega = \frac{2\pi 150}{60} = \frac{15.71\mathrm{rad}}{\mathrm{s}}$$

$$= T_2 - T_1 = \left(500 + 600\sin(2\theta)\right) - \left(5000 + 500\sin(\theta)\right)$$

$$= 600\sin(2\theta) - 500\sin(\theta)$$

$$600\sin(2\theta) = 500\sin(\theta)\ or$$

$$1.2\sin(2\theta) = \sin(\theta)$$

$$1.2 \times \sin(\theta)\cos(\theta) = \sin(\theta)\ or 2.4\sin(\theta)\cos(\theta) = \sin(\theta)$$

$$\sin(\theta) = 0\ or \cos(\theta) = \frac{1}{2}4 = 0.4167$$

$$\sin(\theta) = 0, \theta = 180, and 360$$

$$\theta_\mathrm{A} = 0, \theta_\mathrm{C} = 180, \theta_\mathrm{E} = 360$$

$$\cos(\theta) = 0.4167, \theta = 65.4\ and\ 294.6$$

$$\theta_\mathrm{B} = 65.4\mathrm{and}\theta_\mathrm{D} = 2946$$

$$\Delta\mathrm{E} = \int_{180}^{294.6} (T_2 - T_1)\mathrm{d}\theta = \int_{180}^{294.6} \left(\left(500 + 600\sin(2\theta)\right) - \left(5000 + 500\sin(\theta)\right)\right)\mathrm{d}\theta$$

$$= \left[-\frac{600\cos(2\theta)}{2} + 500\cos(\theta)\right] = 1204\mathrm{N} - \mathrm{m}$$

$$1204 = \mathrm{m}.\mathrm{k}^2.\omega^2.\mathrm{C_s} = 500 \times 0.4^2 \times 15.71^2 \times \mathrm{C_s} = 19744\mathrm{C_s}$$

$$\mathrm{C_s} = \frac{1204}{19744} = 0.061$$

$$T = (T_2 - T_1) = \left(\left(500 + 600\sin(2\theta)\right) - \left(5000 + 500\sin(\theta)\right)\right) = 600\sin(2\theta) - 500\sin(\theta) \ldots (1)$$

$$\frac{d}{d\theta}\left(600\sin(2\theta) - 500\sin(\theta)\right) = 0$$

$$1200\cos(2\theta) - 500\cos(\theta) = 0$$

$$12\cos(2\theta) - 5\cos(\theta) = 0$$

$$12(2\cos^2\theta - 1) - 5\cos(\theta) = 0$$

$$24\cos^2\theta - 5\cos(\theta) - 12 = 0$$

$$\cos(\theta) = \frac{5 + \sqrt{25 + 4 \times 12 \times 24}}{2 \times 24}$$

$$\theta = 35 \text{ or } 127.6$$

$$T_{max} = 600\sin(70) - 500\sin(35) = 277N - m$$

$$T_{min} = 600\sin(127.6) - 500\sin(127.6) = -976N - m$$

$$\alpha_{max} = \frac{T_{max}}{I} = \frac{277}{500 \times 0.4^2} = \frac{3.46rad}{s}$$

$$\alpha_{min} = \frac{T_{max}}{I} = \frac{976}{500 \times 0.4^2} = 12.2rad/s$$

A punching press is driven by a constant torque electric motor. The press is provided with a flywheel that rotates at maximum speed of 225 r.p.m. The radius of gyration of the flywheel is 0.5 m. The press punches 720 holes per hour; each punching operation takes 2 second and requires 15 kN-m of energy. Find the power of the motor and the minimum mass of the flywheel if speed of the same is not to fall below 200 r. p. m

Given

$$N_1 = 225rpm \;\; k = 0.5m \;\; , holepunched = 720perhr, E_1 = 15k - n = 15 \times 10^3 n - m$$

$$N_2 = 200rpm = 15 \times 10^3 \times \frac{720}{3600} = 3000n - \frac{m}{s} = 3000W = 3kw$$

$$E_2 = 3000 \times 2 = 6000n - m$$

$$\triangle E = E_1 - E_2 = 15 \times 10^3 - 6000 = 9000n - m$$

$$N = \frac{N_1 + N_2}{2} = \frac{225 + 200}{2} = 212.5rpm$$

$$9000 = \frac{\pi^2}{900} \times m \times (0.5)^2 \times 212.5 \times (225 - 200) = 14.565mm$$

$$= \frac{9000}{14.565} = 618kg$$

Amachinepunching 38mm holes in 32mm thick plate requires 7N of energy persq. Mmof sheared area, and unches onehole in every10seconds.

Calculatethepowerofthemotorrequired.Themeanspeedoftheflywheelis25metrespersecond.Thepunchhasastrokeof100mm.Findthemassoftheflywheelrequired,ifthetotalfluctuationofspeedisnottoexceed3%ofthemeanspeed.Assumethatthemotorsuppliesenergytothemachineatuniformrate

Given

D=38m, t=32mm, $E_1 = 7n - m/mm^2$, sheared area v=25m/s, s=100mm, $\mathrm{v}_1 - \mathrm{v}_2 = 0.03v$

$$A = \pi d.t = \pi \times 38 \times 32 = 3820mm^2$$

$$E_1 = 7 \times 3820 = 26740n - m$$

$$= \frac{26740}{10} = 2674n - \frac{m}{s}$$

$$= 2674W = 2.674kw$$

$$\frac{10}{2 \times 100} \times 32 = 1.6s$$

$$E_2 = 2674 \times 1.6 = 4278n - m$$

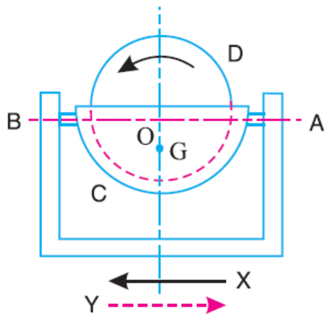
$$\triangle \mathrm{E} = E_1 - E_2 = 2674 - 4278 = 22462n - m$$

$$C_s = \frac{\mathrm{v}_1 - \mathrm{v}_2}{v} = 0.03$$

$$22462 = m.v^2.C_s = m.(25)^2.(0.03) = 18.75m$$

$$m = \frac{22462}{18.75} = 1198kg$$

Problem on Gyrowheel



Given

$$m_1 = 0.5kg\,, k = 20mm = 0.02m\,, OG = h = 10mm = 0.01m$$

$$m_2 = 0.3kg\ n = 3000rpm\ or\ \omega = \frac{2\pi.3000}{60} = \frac{314.2rad}{s}, v = \frac{15m}{s}, R = 50m$$

$$I = m_1.k^2 = 0.5(0.02)^2 = 0.0002kg - m^2$$

$$\omega_\rho = \frac{v}{r} = \frac{15}{50} = 0.3rad/s$$

$$C_1 = I\omega\omega_\rho \cos(\theta) = 0.0002 \times 314.2 \times 0.3\cos(\theta)n - m = 0.019\cos(\theta)n - m$$

$$C_2 = \frac{m_2 . v^2}{R} \times h\cos(\theta) = \frac{(0.3).(15)^2}{50} \times 0.01\cos(\theta)n - m = 0.0135\cos(\theta)n - m$$

$$= C_1 - C_2 = 0.019\cos(\theta) - 0.0135\cos(\theta) = 0.0055\cos(\theta)n - m$$

$$\mathrm{w}_2 = \mathrm{m}_2 . \mathrm{g.h.}\sin(\theta)$$

$$= 0.3 \times 9.81 \times 0.01\sin(\theta)n - m$$

$$= 0.029\sin(\theta)n - m$$

$$0.0055\cos(\theta)n - m = 0.029\sin(\theta)n - m$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{0.0055}{0.029} = 0.1896$$

$$\theta = 10.74$$

$$= C_1 + C_2 = 0.019\cos(\theta) + 0.0135\cos(\theta) = 0.0325\cos(\theta)n - m$$

$$0.0325\cos(\theta) = 0.029\sin(\theta)$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{0.0325}{0.029} = 1.1207\theta = 48.26$$

$$\theta = 48.26$$