MODULE 1

Linear Programming Problem

MODULE 1
LINER PROGRAMMING PROBLEM

Operation Research is a relatively new discipline. The contents and the boundaries of the OR are not yet fixed. Therefore, to give a formal definition of the term Operations Research is a difficult task. The OR starts when mathematical and quantitative techniques are used to substantiate the decision being taken. The main activity of a manager is the decision making. In our daily life we make the decisions even without noticing them. The decisions are taken simply by common sense, judgment and expertise without using any mathematical or any other model in simple situations. Operations Research tools are not from any one discipline. takes tools from different discipline such as Mathematics, Statistics, Economics, Psychology, Engineering etc, and Combines these tools to make a new set of knowledge for Decision Making.

DEFINITION of OR :

According to the Operational Research Society of Great Britain "Operational Research is the attack of modern science on complex problems arising in the direction and management of large systems of Men, Machines, Materials and Money in Industry, Business, Government and Defense. Its distinctive approach is to develop a Scientific model of the system, Incorporating measurements of factors such as Change and Risk, with which to predict and compare the outcomes of alternative Decisions, Strategies or Controls. The purpose is to help management determine its policy and actions scientifically".

Stages of Development of Operations Research: Main six stages of OR are given below:

Stage	Process activity(s)	Process Output
I	Site visits, Conferences,Observations, Research	Sufficient information and support to proceed
II	Define: Use, Objectives, Limitations	Clear grasp of need for and natureof solution requested
III	Define inter relationships, formulateequations, Use known O.R. Model, Search alternate Model	Models that work under stated environmental constraints
IV	Analyze: internal-external data, Facts Collectoptions Use computer data banks	Sufficient inputs to operate andtest model
V	Test the model, find limitations, Update themodel	Solution(s) that support currentorganizational goals
VI	Resolve behavioral issues, Sell the idea, Give explanations, Management involvement	improved working and Management support for longer run operation of model

Tools and Techniques:

The common frequently used tools/techniques are mathematical procedures, cost analysis, electronic computation. However, operations researchers given special importance to the development and the use of techniques like

- Linear Programming
- Game Theory
- Decision Theory
- Queuing Theory
- Inventory Models and Simulation

In addition to the above techniques, some other common tools are

- Non-Linear Programming
- Integer Programming
- Dynamic Programming
- Sequencing Theory
- Markov Process,
- Network Scheduling (PERT/CPM),
- Symbolic Model,
- Information theory, and
- Value theory.

Linear Programming: This is constrained optimization technique, which optimize some criterion within some constraints. In Linear programming the objective function (profit, loss or return on investment) and constraints are linear. There are different methods available to solve linear programming

Game Theory : This is used for making decisions under conflicting situations where there are one or more players/opponents. In this the motive of the players are dichotomized. The success of one player tends to be at the cost of other players and hence they are in conflict.

Decision Theory : It is concerned with making decisions under conditions of complete certainty about the future outcomes and under conditions such that we can make some probability about what will happen in future.

Queuing Theory : This is used in situations where the queue is formed (for example customers waiting for service, aircrafts waiting for landing, jobs waiting for processing in the computer system, etc). The objective here is minimizing the cost of waiting without increasing the cost of servicing.

Inventory Model: It makes a decision that minimize total inventory cost. This model successfully reduces the total cost of purchasing, carrying, and out of stock inventory.

Simulation : It is a procedure that studies a problem by creating a model of the process involved in the problem and then through a series of organized trials and error solutions attempt to determine the best solution. Sometimes this is a difficult/time consuming procedure. Simulation is used when actual experimentation is not feasible or solution of model is not possible.

Non-linear Programming : This is used when the objective function and the constraints are not linear in nature. Linear relationships may be applied to approximate non-linear constraints but limited to some range, because approximation becomes poorer as the range is extended. Thus, the non-linear programming is used to determine the approximation in which a solution lies and then the solution is obtained using linear methods

Dynamic Programming : It is a method of analyzing multistage decision processes. In this each elementary decision depends on those preceding decisions and as well as external factors.

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Applications of Operations Research

Accounting:

- Assigning audit teams effectively
- Credit policy analysis
- Cash flow planning
- Developing standard costs
- Establishing costs for byproducts
- Planning of delinquent account strategy

Construction:

- Project scheduling, monitoring and control
- Determination of proper work force
- Deployment of work force
- Allocation of resources to projects

Facilities Planning:

- Factory location and size decision
- Estimation of number of facilities required
- Hospital planning
- International logistic system design
- Transportation loading and unloading
- Warehouse location decision

Finance:

- Building cash management models
- Allocating capital among various alternatives
- Building financial planning models
- Investment analysis
- Portfolio analysis
- Dividend policy making

Manufacturing:

- Inventory control
- Marketing balance projection
- Production scheduling
- Production smoothing

Marketing:

- Advertising budget allocation
- Product introduction timing
- Selection of Product mix
- Deciding most effective packaging alternative

Organizational Behavior / Human Resources:

- Personnel planning
- Recruitment of employees
- Skill balancing
- Training program scheduling
- Designing organizational structure more effectively

Limitations of Operations Research

Distance between O.R. specialist and Manager: Operations Researchers job needs a mathematician or statistician, who might not be aware of the business problems. Similarly, a manager is unable to understand the complex nature of OR. Thus there is a big gap between the two personnel

Magnitude of Calculations : The aim of the O.R. is to find out optimal solution taking into consideration all the factors. In this modern world these factors are enormous and expressing them in quantitative model and establishing relationships among these require voluminous calculations, which can be handled only by machines

Money and Time Costs : The basic data are subjected to frequent changes, incorporating these changes into the operations research models is very expensive. However, a fairly good solution at present may be more desirable than a perfect operations research solution available in future or after some time

Non-quantifiable Factors : When all the factors related to a problem can be quantifiable only then OR provides solution otherwise not. The non-quantifiable factors are not incorporated in OR models. Importantly O.R. models do not take into account emotional factors or qualitative factors.

Implementation: The implementation of decisions is a delicate task. This task must take into account the complexities of human relations and behavior and in some times only the psychological factors.

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Linear Programming

- It is a special and versatile technique which can be applied to Advertising, Distribution, Refinery Operations, Investment, Transportation analysis and Production.
- It is useful not only in industry and business but also in non-profit sectors such as Education, Government, Hospital, and Libraries
- The linear programming method is applicable in problems characterized by the presence of decision variables.
- The objective function and the constraints can be expressed as linear functions of the decision variables.
- The decision variables are in some sense, controllable inputs to the system being modeled.
- An objective function represents some principal objective criterion or goal that measures the effectiveness of the system such as maximizing profits or productivity, or minimizing cost or consumption
- There is always some practical limitation on the availability of resources like Man, Material, Machine, or Time for the system.
- These constraints are expressed as linear equations involving the decision variables.

L PP Formulation

- The L PP formulation is illustrated through a product mix problem. The product mix problem occurs in an industry where it is possible to manufacture a variety of products.
- A product has a certain margin of profit per unit, and uses a common pool of limited resources. In this case the linear programming technique identifies the products combination which will maximize the profit subject to the availability of limited resource constraints

Problem 1 : An Industry is manufacturing two types of products P_1 and P_2.The profits per Kg of the two products are Rs.30 and Rs.40 respectively. These two products require processing in three types of machines. The following table shows the available machine hours per day and the time required on each machine to produce one Kg of P_1 and P_2. Formulate the problem in the form of linear programming model

Profit/Kg	**P_1 (Rs.30)**	**P_2 (Rs.40)**	**Total available Machine (hours/day)**
Machine 1	**3**	**2**	**600**
Machine 2	**3**	**5**	**800**
Machine 3	**5**	**6**	**1100**

Solution:

Constraints in this Problem are of "less than or equal to" type

Introduce the decision variable as follows

Let x_1 = amount of P_1

x_2 = amount of P_2

In order to maximize profits, we establish the objective function as

$$30X_1 + 40X_2$$

Since one Kg of P_1 requires 3 hours of processing time in machine 1 while the corresponding requirement of P_2 is 2 hours. So, the first constraint can be expressed as

$$3X_1 + 2X_2 \leq 600$$

Similarly, corresponding to machine 2 and 3 the constraints are

$$3X_1 + 5X_2 \leq 800$$

$$5X_1 + 6X_2 \leq 1100$$

In addition to the above there is no negative production, which may be represented algebraically as $X_1 \geq 0$; $X_2 \geq 0$

Thus, the product mix problem in the linear programming model is as follows:

$$\textbf{Maximize : } 30X_1 + 40X_2$$
$$\textbf{Subject to : } 3X_1 + 2X_2 \leq 600$$
$$3X_1 + 5X_2 \leq 800$$
$$5X_1 + 6X_2 \leq 1100$$
$$X_1 \geq 0, X_2 \geq 0$$

Graphical Analysis of Linear Programming:

In this we use two models

1. Maximization Problems
2. Minimization problems

Maximization Problem :-

$$\text{Maximize } Z = 30X_1 + 40X_2$$
$$\text{Subject to } 3X_1 + 2X_2 \le 600$$
$$3X_1 + 5X_2 \le 800$$
$$5X_1 + 6X_2 \le 1100 \text{ and}$$
$$X_1 \ge 0,\ X_2 \ge 0$$

Solution:-

$$Z = 30X_1 + 40X_2$$

Let us consider Equation (1)

$$3X_1 + 2X_2 \le 600$$

i.e.,Put $X_1 = 0$, $X_2 = 0$, Equation

(1)

We can get point = (0, 0)

Put $X_1 = 0$, in Equation (1) $X_2 = 300$

and $X_2 = 0$, in Equation (1) $X_1 = 200$

We can get points = (0, 300) and (200,0)

Let us consider Equation (2) i.e.,

$$3X_1 + 5X_2 \le 800$$

Put $X_1 = 0$, in Equation (2) $X_2 = 160$ and $X_2 = 0$, in Equation (2) $X_1 = 266.66$

We can get points = (0, 160) and (266.66, 0)

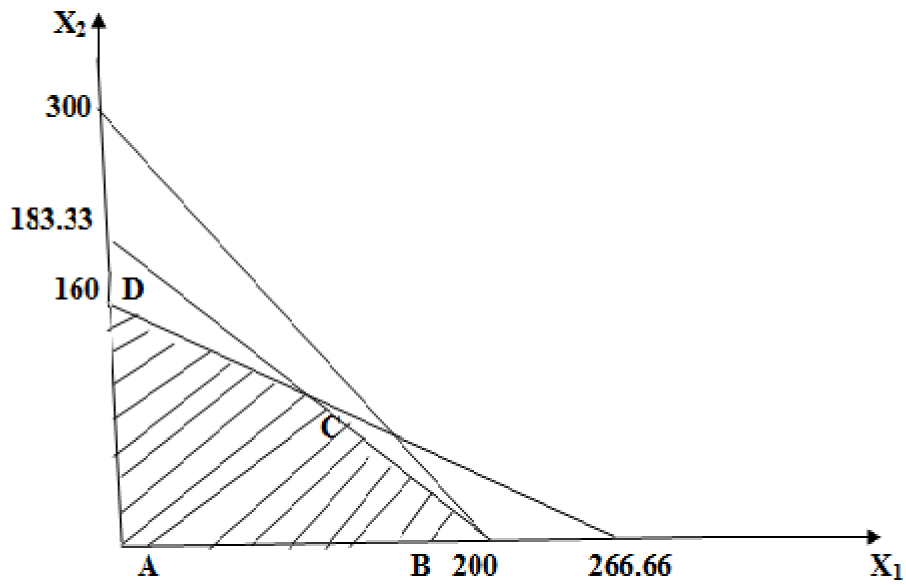
Let us consider Equation (3) i.e.,

$$5X_1 + 6X_2 \le 1100$$

Put $X_1 = 0$, in Equation (3) $X_2 = 183.33$ and $X_2 = 0$, in Equation (3) $X_1 = 220$

We can get points = (0, 183.33) and (200, 0)

A, B, C, and D is feasible Region



A, B, C, and D is feasible Region

With help of graph from the Feasible Region we calculate Z_{MAX} Value

Key Terms:

Objective Function: Is a linear function of the decision variables representing the objective of the manager/decision maker.

Constraints: Are the linear equations or inequalities arising out of practical limitations.

Decision Variables: Are some physical quantities whose values indicate the solution.

Feasible Solution: Is a solution which satisfies all the constraints (including the non-negative) presents in the problem.

Feasible Region: Is the collection of feasible solutions.

Multiple Solutions: Are solutions each of which maximize or minimize the objective function.

Unbounded Solution: Is a solution whose objective function is infinite.

Infeasible Solution: Means no feasible solution.

SIMPLEX METHOD

The Linear Programming with two variables can be solved graphically. The graphical method of solving. Linear programming problem is of limited application in the business problems as the number of variables is substantially large.

If the linear programming problem has larger number of variables, the suitable method for solving is Simplex Method.

The simplex method is an iterative process, through which it reaches ultimately to the minimum or maximum value of the objective function.

The simplex method also helps the decision maker/manager to identify the following:

- Redundant Constraints
- Multiple Solutions
- Unbounded Solution
- Infeasible Problem

Basics of Simplex Method

The basic of simplex method is explained with the following linear programming problem.

Maximize: $60X_1 + 70X_2$

Subject to: $2X_1 + X_2 \leq 300;$
$3X_1 + 4X_2 \leq 509;$
$4X_1 + 7X_2 \leq 812;$
$X_1, X_2 \geq 0$

Solution:

First, introduce the variables $S_3, S_4, S_5 \geq 0$

So that the constraints become equations,thus,

$$2X_1 + X_2 + 1S_3 + 0S_4 + 0S_5 = 300$$
$$3X_1 + 4X_2 + 0S_3 + 1S_4 + 0S_5 = 509$$
$$4X_1 + 7X_2 + 0S_3 + 0S_4 + 1S_5 = 812$$

Corresponding to the three constraints, the variables S_3, S_4, S_5 are called as slack variables.
Now, the system of equation has three equations and five variables.
There are two types of Solutions they are

- Basic Solutions
- Basic Feasible Solutions

Basic Solution

Equate any two variables to zero in the above system of equations, and will have three variables. Such system of three equations with three variables is solvable such a solution is called as basic solution.
For example suppose we take $X_1 = 0$ and $X_2 = 0$, the solution of the system with remaining three variables is $S_3 = 300$, $S_4 = 509$ and $S_5 = 812$, this is a basic solution
The variables S_3, S_4, and S_5 are known as basic variables where as the variables x1, x2 are known as non-basic variables.
The number of basic solution of a linear programming problem is depends on the presence of the number of constraints and variables
For example if the number of constraints is "m" and the number of variables including the slack variables "n" there are at most basic solution.

Basic Feasible Solution

A basic solution of a linear programming problem is called as basic feasible solutions if it is feasible it means all the variables are non-negative
The solution $S_3 = 300$, $S_4 = 509$ and $S_5 = 812$ is a basic feasible solution.
The number of basic feasible solution of a linear programming problem is depends on the presence of the number of constraints and variables.

For example if the number of constraints is m and the number of variables including the slack variables is n then there are at most basic feasible solutions

Every basic feasible solution is an extreme point of the convex set of feasible solutions and every extreme point is a basic feasible solution of the set of given constraints.
It is impossible to identify the extreme points geometrically if the problem has several variables but the extreme points can be identified using basic feasible solutions. Since one the basic feasible solution will maximize or minimize the objective function, the searching of extreme points can be carry out starting from one basic feasible solution to another.
The Simplex Method provides a systematic search so that the objective function increases in the cases of maximization progressively until the basic feasible solution has been identified where the objective function is maximized.

Problem:
Consider the following linear programming problem

Maximize :$60X_1 + 70X_2$

Subject to: $2X_1 + X_2 + 1S_3 + 0S_4 + 0S_5 = 300;$

$3X_1 + 4X_2 + 0S_3 + 1S_4 + 0S_5 = 509;$

$4X_1 + 7X_2 + 0S_3 + 0S_4 + 1S_5 = 812$

and $X_1, X_2, S_3, S_4, S_5 \geq 0$

Solution:
In this problem the slack variables S_3, S_4, and S_5 provide a basic feasible solution from which the simplex computation starts. i.e., $S_3==300$, $S_4=509$ and $S_5=812$. This result follows because of the special structure of the columns associated with the slacks.
If z represents profit, then $z = 0$ corresponding to this basic feasible solution.
We represent by C_B the coefficient of the basic variables in the objective function and by X_B the numerical values of the basic variable.
So that the numerical values of the basic variables are: $X_{B1}=300$, $X_{B2}=509$, $X_{B3}=812$.
The profit $Z = 60X_1+70X_2$

$\Rightarrow Z - 60X_1 - 70X_2 = 0.$

Table 1: The simplex computation starts with the first compact standard simplex tableas given below:

C_B	Basic Variables	C_j XB	60 x_1	70 x_2	0 s_3	0 s_4	0 s_5
0	s_3	300	2	1	1	0	0
0	s_4	509	3	4	0	1	0
0	s_5	812	4	7	0	0	1
	z		-60	-70	0	0	0

“In the objective function the coefficients of the variables are $C_{B1} = C_{B2} = C_{B3} = 0$. The topmost row of the Table 1 denotes the coefficient of the variables X_1, X_2, S_3, S_4, S_5 of the objective function respectively. The column under x_1 indicates the coefficient of x_1 in the three equations respectively. Similarly, the remaining column also formed. As $z = 60X_1+70X_2$, It is observed that if either X_1 or X_2, which is currently non-basic is included as a basic variable so that the profit will increase. Since the coefficient of X_2 is higher we choose X_2 to be included as a basic variable in the next iteration. An equivalent criterion of choosing a new basic variable can be obtained the last row of Table 1 i.e. corresponding to “Z” Since the entry corresponding to X_2 is smaller between the two negative values, X_2 will be included as a basic variable in the next iteration”. However, with three constraints there can be only three basic variables. Thus, by bringing X_2 a basic variable one of the existing basic variables becomes non- basic. For identifying it,

Consider the first equation i.e. $2X_1 + X_2 + 1S_3 + 0S_4 + 0S_5 = 300$

From this equation $2X_1+S_3 = 300 - X_2$

But $X_1 = 0$. Hence, in order that $S_3 \geq 0$, $300 - X_2 \geq 0$, i.e. $X_2 \leq 300$

Similarly consider the second equation i.e.

$$3X_1 + 4X_2 + S_4 = 509$$

From this equation $3X_1+S_4 = 509 - 4X_2$

But, $X_1=0$. Hence, in order that $S_4 \geq 0$ $509 - 4x2 \geq 0$ i.e. $X_2 \leq 509/9$

Similarly consider the third equation i.e.

$$4X_1 + 7X_2 + S_5 = 812$$

From this equation $4X_1+S_5 = 812-7X_2$

But $X_1 = 0$. Hence, in order that $S_5 \geq 0$

$$812 - 7X_2 \geq 0 \quad \text{i.e. } X_2 \leq 812 / 7$$

Therefore,

$$X_2 \leq 300, \quad X_2 \leq 509/9, \quad X_2 \leq 812/7$$

Thus $X_2 = \text{Min} (X_2 \leq 300, X_2 \leq 509/9, X_2 \leq 812/7)$ it means

$X_2 = \text{Min} (X_2 \leq 300/1, X_2 \leq 509/9, X_2 \leq 812/7) = 116$

Therefore $X_2 = 116$ If $X_2 = 116$, you may be note from the third equation

$$7X_2+S_5 = 812 \qquad \text{i.e. } S_5 = 0$$

Thus, the variable S_5 becomes non-basic in the next iteration.

So that the revised values of the other two basic variables are

$$S_3=300-X_2 = 184 \qquad S_4 = 509 - 4 \times 116 = 45$$

Refer to Table 1, we obtain the elements of the next Table i.e. Table 2 using the following rules:

1. Allocate the quantities which are negative in the Z- row. Suppose if all the quantities the inclusion of any non-basic variable will not increase the value of the objective

function. Hence the present solution maximizes the objective function. If there are more than one negative values, we choose the variable as a basic variable corresponding to which the Z value is least as this is likely to increase the more profit.

2. Let X_j be the incoming basic variable and the corresponding elements of the j^{th} row column be denoted by Y_{1j}, Y_{2j} and Y_{3j} respectively. If the present values of the basic variables are X_{B1}, X_{B2} and X_{B3} respectively, then we can compute.
   Min [X_{B1} / Y_{1j}, X_{B2} / Y_{2j}, X_{B3} / Y_{3j}] for Y_{1j}, Y_{2j}, $Y_{3j} > 0$.

   Note that if any $Y_{ij} \leq 0$, this need not be included in the comparison. If the minimum occurs Corresponding to X_{Br} / Y_{rj} then the r^{th} basic variable will become non-basic in the next iteration.

3. Using the following rules the Table 2 is computed from the Table 1.
   i.) The revised basic variables are S_3, S_4 and X_2. Accordingly, we make $C_{B1}= 0$, $C_{B2} = 0$ and $C_{B3} = 70$.
   ii.) As X_2 is the incoming basic variable we make the coefficient of x2 one by dividing Each element of row -3 by 7. Thus, the numerical value of the element corresponding to X_1 is 4/7, corresponding to S_5 is 1/ 7 in Table 2.
   iii.) The incoming basic variable should appear only in the third row. So, multiply the third-row of Table 2 by 1 and subtract it from the first-row of Table 1 element by element. Thus, the element corresponding to X2 in the first-row of Table 2 is 0. Therefore, the element corresponding to X_1 is (2 - 1 x 4) / 7= 10 / 7 and the element corresponding to S_5 is (0 – 1 x 1 / 7) = -1/7 In this way we obtain the elements of the first and the second row in Table 2.

In Table 2 the numerical values can also be calculated in a similar way

C_B	Basic Variables	C_j / X_B	60 x_1	70 x_2	0 s_3	0 s_4	0 s_5
0	s_3	184	10/7	0	1	0	-1/7
0	s_4	45	5/7	0	0	1	-4/7
70	x_2	116	4/7	1	0	0	1/7
	zj-cj		-140/7	0	0	0	70/7

Let C_{B1}, C_{B2}, C_{B3} be the coefficients of the basic variables in the objective function.
For example, in Table 2 $C_{B1}=0$, $C_{B2}=0$ and $C_{B3}=70$. Suppose corresponding to a variable J, the quantity Z_j is defined as $Z_j= C_{B1}, Y_1+ C_{B2}, Y_{2j} + C_{B3} Y_{3j}$.
Then the Z-row can also be represented as $Z_j - C_j$.

For example: $Z_1 - C_1 = (10/7 \times 0) + (5/7 \times 0) + (70 \times 4/7) - 60 = -140/7$

$Z_5 - C5 = (-1/7 \times 0) - (4/7 \times 0) + (1/7 \times 70) - 0 = 70/7$

1. Now we apply rule (1) to Table 2. Here the only negative Zj - Cj is Z_1-C_1 = -140/7
Hence X_1 should become a basic variable at the next iteration.
2. Compute the minimum of the ratio

$$\text{Min} \left(\frac{184}{10/7}, \frac{45}{5/7}, \frac{116}{4/7} \right) = \text{Min} \left(\frac{644}{5}, 63, 203 \right) = 63$$

This minimum occurs corresponding to S_4, it becomes a non basic variable in next iteration
3. Like Table 2, the Table 3 is computed using the rules (i), (ii), (iii) as described above.

C_B	Basic Variables	C_j / X_B	60 x_1	70 x_2	0 s_3	0 s_4	0 s_5
0	s3	94	0	0	1	-2	1
60	x1	63	1	0	0	7/5	-4/5
70	x2	80	0	1	0	-4/5	3/5
	z_j-c_j		0	0	0	28	-6

1. $Z_5 - C_5 < 0$ should be made a basic variable in the next iteration.
2. Now compute the minimum ratios

$$\text{Min} \left(\frac{94}{1}, \frac{80}{3/5} \right) = 94$$

Note: Since Y_5 = -4/5 < 0, the corresponding ratio is not taken for comparison.
The variable s3 becomes non basic in the next iteration
3. From the Table 3, Table 4 is calculated following the usual steps.

C_B	Basic Variables	C_j / X_B	60 x_1	70 x_2	0 s_3	0 s_4	0 s_5
0	s_5	94	0	0	1	-2	1
60	x_1	691/5	1	0	4/5	-1/5	0
70	x_2	118/5	0	1	-3/5	2/5	0
	z_j-c_j		0	0	6	16	0

Note that $z_j - c_j \geq 0$ for all j, so that the objective function can't be improved any further.
Thus, the objective function is maximized for $x1 = 691/5$ and $x2=118/5$ and
The maximum value of the objective function is 9944

Two Phase and Big M-Method

The simplex method was applied to linear programming problems with less than or equal to (≤) type constraints. Thus, there we could introduce slack variables which provide an initial basic feasible solution of the problem.

Generally, the linear programming problem can also be characterized by the presence of both less than or equal to "(≤)' type or 'greater than or equal to "(≥)' type constraints.

In such case it is not always possible to obtain an initial basic feasible solution using slack variables.

The greater than or equal to type of linear programming problem can be solved by using the following methods:

1. Two Phase Method
2. Big M- Method

Problem:

Solve the following problem with Two phase method Minimize $12.5X_1 + 14.5X_2$

Subject to:

$X_1 + X_2 \geq 2000$ $0.4X_1 + 0.75X_2 \geq 1000$ $0.075X_1 + 0.1X_2 \leq 200$ $X_1, X_2 \geq 0$

Solution:

Here the objective function is to be minimized; the values of X_1 and X_2 which minimized this objective function are also the values which maximize the revised objective function i.e. **Maximize: 12.5 $X_1 - 14.5X_2$**

We can multiply the second and the third constraints by 100 and 1000 respectively for the Convenience of calculation.

Thus, the revised linear programming problem is:

Maximize $-12.5X_1 - 14.5X_2$

Subject to:

$$X_1 + X_2 \geq 2000$$
$$40X_1 + 75X_2 \geq 100000$$
$$75X_1 + 100X_2 \leq 200000 \text{ and } X_1, X_2 \geq 0$$

Now we convert the two inequalities by introducing surplus variables S_3 and S_4 respectively. The third constraint is changed into an equation by introducing a slack variable S_5.

Thus, the linear programming problem becomes as

Maximize: $12.5X_1 - 14.5X_2$

Subject to: $X_1 + X_2 - S_3 = 2000$

$$40X_1 + 75X_2 - S_4 = 100000$$
$$75X_1 + 100X_2 + S_5 = 200000 \quad X_1, X_2, S_3, S_4, S_5 \geq 0$$

Even though the surplus variables can convert greater than or equal to type constraints into equations they are unable to provide initial basic variables to start the Simplex method calculation.

Introducing two more additional variables A_6 and A_7 called as artificial variableto facilitate the calculation of an initial basic feasible solution.

In this method the calculation is carried out in Two phases hence two-phase method

Phase I - In this phase we will consider the following linear programming problem

Maximize: $-A_6 - A_7$

Subject to: $X_1 + X_2 - S_3 + A_6 = 2000$

$40X_1 + 75X_2 - S_4 + A_7 = 100000$

$75X_1 + 100X_2 + S_5 = 200000$

and $X_1, X_2, S_3, S_4, S_5, A_6, A_7 \geq 0$

The initial basic feasible solution of the problem is $A_6 = 2000$, $A_7 = 100000$ and $S_5 = 200000$.

As the minimum value of the objective function of the Phase –I, is zero at the end of the Phase –I, calculation both A6 and A7 become zero.

C_B	Basic variables	C_j X_B	0 x_1	0 x_2	0 s_3	0 s_4	0 s_5	-1 A_6	-1 A_7
-1	A_6	2000	1	1	-1	0	0	1	0
-1	A_7	100000	40	75	0	-1	0	0	1
0	S_5	200000	75	100	0	0	1	0	0
		z_j-c_j	-41	-76	1	1	0	0	0

Table 1

Here X_2 becomes a basic variable and A_7 becomes non basic variable in the next iteration. It is no longer considered for re-entry in the table.

C_B	Basic variables	C_j X_B	0 x_1	0 x_2	0 s_3	0 s_4	0 s_5	-1 A_6
-1	A_6	2000/3	7/15	0	-1	1/75	0	1
0	X_2	4000/3	8/15	1	0	-1/75	0	0
0	S_5	200000/3	65/3	0	0	4/3	1	0
		z_j-c_j	-1/15	0	1	-1/75	0	0

TABLE 2

Then X_1 becomes a basic variable and A_6 becomes a non basic variable in the next iteration.

C_B	Basic variables	C_j X_B	0 x_1	0 x_2	0 s_3	0 s_4	0 s_5
0	x_1	10000/7	1	0	-15/7	1/35	0
0	x_2	4000/7	0	1	8/7	-1/35	0
0	s_5	250000/7	0	0	325/7	16/21	1
		z_j-c_j	0	0	0	0	0

Table 3

The calculation of Phase - I end at this stage. Note that, both the artificial variable have been removed and also found a basic feasible solution of the problem.
The basic feasible solution is:

$$X_1 = 10000/7$$
$$X_2 = 4000/2$$
$$S_5 = 250000/7$$

Phase II

The initial basic feasible solution obtained at the end of the Phase I calculation is used as the initial basic feasible solution of the problem, the original objective function is introduced and the usual simplexprocedure is applied to solve the linear programming problem.

C_B	Basic variables	C_j X_B	-25/2 x_1	-29/2 x_2	0 s_3	0 s_4	0 s_5
-25/2	x_1	10000/7	1	0	-15/7	1/35	0
-29/2	x_2	4000/7	0	1	8/7	-1/35	0
0	s_5	250000/7	0	0	325/7	5/7	1
		z_j-c_j	0	0	143/14	2/35	0

Table 1

In this Table 1 all $Z_j - C_j \geq 0$ the current solution maximizes the revised objective function.
Thus, the solution of the problem is:

$$X_1 = 10000/7 = 1428$$
$$X_2 = 4000/7 = 571.4$$

and The Minimum Value of the objective function is: 26135.3

Big –M Method

In this method also we need artificial variables for determining theinitial basic feasible solution

$$\text{Maximize: } -12.5X_1 - 14.5X_2$$

Subject to:
$$X_1 + X_2 - S_3 = 2000$$
$$40X_1 + 75X_2 - S_4 = 100000$$
$$75X_1 + 100X_2 + S_5 = 200000$$
$$\text{and } X_1, X_2, S_3, S_4, S_5 \geq 0.$$

Introduce the artificial variables A6 and A7 in order to provide basic feasible solution in the second and third constraints.

The objective function is revised using a large positive number say M,
Thus, instead of the original problem, consider the following problem i.e.,

$$\text{Maximize: } -12.5\,X_1 - 14.5X_2 - M\,(A_6 + A_7)$$
$$\text{Subject to: } X_1 + X_2 - S_3 + A_6 = 2000$$
$$40X_1 + 75X_2 - S_4 + A_7 = 100000$$
$$75X_1 + 100X_2 + S_5 = 200000$$

and $X_1, X_2, S_3, S_4, S_5, A_6, A_7 \geq 0$

The coefficient of A_6 and A_7 are large negative number in the objective function.
Since the objective function is to be maximized in the optimum solution, the artificial variables will be zero.

Therefore, the basic variable of the optimum solution variable other than theartificial variables and hence is a basic feasible solution of the original problem.The successive calculation of simplex tables is as follows

C_B	Basic variables	C_j X_B	-12.5 x_1	-14.5 x_2	0 s_3	0 s_4	0 s_5	-M A_6	-M A_7
-M	A_6	2000	1	1	-1	0	0	1	0
-M	A_7	100000	40	75	0	-1	0	0	1
0	S_5	200000	75	100	0	0	1	0	0
		z_j-c_j	-41M +12.5	-76M +14.5	M	M	0	0	0

Table 1

Since M is a large positive number, the coefficient of M in the Zj – Cj row would decide the entering basic variable. As -76M < - 41M, X_2 becomes a basic variable in the next iteration replacing A_7. The artificial variable A_7 can't be re-entering as basic variable.

C_B	Basic variables	C_j X_B	-12.5 x_1	-14.5 x_2	0 s_3	0 s_4	0 s_5	-M A_6
-M	A_6	2000/3	7/15	0	-1	1/75	0	1
-14.5	X_2	4000/3	8/15	1	0	-1/75	0	0
0	S_5	200000/3	65/3	0	0	4/3	1	0
		z_j-c_j	-7/15M +143/30	0	M	-M/75 +29/150	0	0

Table 2

Now X_1becomes a basic variable replacing A_6, Like A_7 the variable A_6 also artificial variable so it can't be re-entering in the table.

C_B	Basic variables	C_j X_B	-12.5 x_1	-14.5 x_2	0 s_3	0 s_4	0 s_5
-12.5	x_1	10000/7	1	0	-15/7	1/35	0
-14.5	x_2	4000/7	0	1	8/7	-1/35	0
0	s_5	250000/7	0	0	325/7	16/21	1
		z_j-c_j	0	0	143/14	2/35	0

Table 3

Hence The optimum solution of the problem is

$X_1 = 10000/7$

$X_2 = 4000/7$ and

The Minimum Value of the Objective Function is: 26135.3