The Strategic Role of the Weight of Evidence in Abduction

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1 Peirce on Balance and Weight

While the idea of evidential weight is closely associated with Keynes' discussion in *A Treatise*, Peirce was the first to bring our attention to its importance.[1] Like Keynes, Peirce was keenly aware of the distinction between the balance and the weight of the evidence. While Peirce develops the ideas on behalf of the Bayesian of his time in order to demonstrate its shortcoming, Peirce brings about formal develop of Bayesian ideas that come to be influential.[2] His development of the notion of the balance of the evidence is especially important. We already discussed Keynes' view on the notion, which is identified as the magnitude of a probability. Peirce's idea is a bit more involved—this is partly due to his concern with the viability of a based epistemology based on partial belief, while Keynes simply assumes a Russellian epistemology that serves as the philosophical springboard for his interpretation of probability.

[1] Charles S. Peirce, "The Probability of Induction," in *Writings of Charles S. Peirce: A Chronological Edition, Volume 3: 1872–1878* (Indiana University Press, 1986).

[2] Technically they were followers of Laplace, who developed many Bayesian ideas.

Peirce takes that the balance of the evidence as an *epistemological* thesis about the justification of belief that has its root in Humean epistemology.[3] In *An Enquiry Concerning Human Understanding*, Hume makes his famous argument against the existence of miracle by appealing to the concept of probability. Hume argues that due to the fact that the only evidence we have regarding miracle is witness testimony, whether or not miracle exists can only be determined probabilistically by determining the balance of the evidence:

> All probability, then, supposes an opposition of experiments and observations, where the one side is found to overbalance the other, and to produce a degree of evidence, proportioned to the superiority[4]

The idea is that probability should allow us to combine evidence for both sides of the argument, and determine which is favored by all the evidence we have. In addition, Hume thought that the balance of the evidence is not just a comparative measure, but also a quantitative one:

> In all cases, we must balance the opposite experiments, where they are opposite, and deduct the smaller number from the greater, in order to know the exact force of the superior evidence.[5]

Peirce thought that the best case for Bayesianism is to combine Hume's epistemology with the probability theory developed by Laplace. Peirce proposes a measurement of the belief H on evidence E, we should take the odds of the probability of H

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[3] Charles S. Peirce, *The Essential Peirce, Volume 2: Selected Philosophical Writings (1893-1913)*, ed. Peirce Edition Project (Indiana University Press, 1998), 76.

[4] David Hume, *An Enquiry Concerning Human Understanding and Other Writings* (Cambridge University Press, 2007), 97.

[5] Hume, 98.

conditional on E versus its probability conditional on $\neg E$. In other words,

$$\frac{P(H|E)}{P(\neg H|E)}$$

Which can be algebraically decomposed into

$$\frac{P(E|H)P(H)}{P(E|\neg H)P(\neg H)}$$

Using Principle of Indifference, which will be discussed below, the above becomes $\frac{H}{\neg H} = 1$, the balance of evidence is simply $\frac{P(E|H)}{P(E|\neg H)}$, which is an expression of what we now call the Bayes Factor. Peirce then proposes that Hume's idea can be captured by taking the log of the ratio. Since

$$\frac{P(E|H)P(H)}{P(E|\neg H)P(\neg H)} = log(P(E|H)) - log(P(E|\neg H))$$

. A positive value means the evidence is in favor of H and negative against it. It also captures Hume's idea that the strengths of two independent pieces of evidence should be combinable. Suppose E_1 and E_2 are independent. So, $P(E_1 \wedge E_2|H) = P(E_1|H)P(E_2|H)$. If we take the logarithm of the product, the product becomes $log(P(E_1|H)) + log(P(E_2|H))$. This captures the idea that adding two pieces of independent evidence together should increase the intensity of our belief. Some modern Bayesians were so impressed with Peirce's account that they incorporated Peirce's ideas into their own frameworks. I. J. Good refers to Peirce as a precursor of his account.[6] Branden Fitelson cites Peirce as the inspiration of his Bayesian account

[6] I. J. Good, "An Error by Peirce Concerning Weight of Evidence," *Journal of Statistical Computation and Simulation* 13, no. 2 (1981): 155–57.

of independent evidence.[7]

However, Peirce argues that the balance of the evidence alone cannot be satisfactory capture our state of belief. Peirce illustrates the idea roughly as follows: imagine two urns A and B with unknown proportions of black and white balls. Suppose you sample (with replacement) 100 balls from the urn A and find 50 black balls and 50 white balls. Justifiably, you infer that the proportion of black balls in A - call it θ_A is about 0.5. You then decide to sample from B, but this time you only manage to draw 4 samples, 2 of which are black balls. Your best estimate for θ_B is 0.5. At this point, I offer you another chance to draw from one of the urns, and if you manage to draw a black ball from that urn, you get \$100. Which urn would you pick?

Clearly, $\theta_B = \theta_A$, but it is unclear if we should regard them with indifference, for a simple reason: the evidence for $\theta_A = 0.5$ is more substantial than the evidence for $\theta_B = 0.5$.

What Peirce has in mind, it should be clear, is precisely what Keynes later calls the weight of evidence. However, while Keynes was hesitant in affirming the necessity of evidential weight, Peirce thinks its role is indispensable:

> to express the proper state of our belief, not one number but two are requisite, the first depending on the inferred probability, the second on the amount of knowledge on which that probability is based.[8]

For Peirce„ the weight of evidence is tightly connected to the notion of the *precision* of a probability, as he suggests it should be based on what he calls probable

[7] Branden Fitelson, "A Bayesian Account of Independent Evidence with Applications," *Proceedings of the Philosophy of Science Association* 2001, no. 3 (2001): S123.

[8] Peirce, "The Probability of Induction," 296.

error, which is a precursor to the modern Frequentist notion of confidence interval.[9] This is evidenced by Peirce's example above—it anticipated what is now known as the *Ellsburg Paradox*, an empirical phenomenon in which human subjects are shown to be sensitive to difference between precise(a point value) and imprecise(an interval) probabilities.[10] Peirce was impressed by Boole's way of deriving indefinite probabilities by using arbitrary constants.[11] Boole's view is seen as the precursor of modern theory of imprecise probability.[12] It is then no surprise that Peirce is cognizant of the nuance issues surrounding the precision of probability.

Peirce, furthermore, has a dynamic view on the weight of evidence. Unlike Keynes, Peirce does not seem to think the weight of evidence *always* increase when new relevant evidence is introduced; because, he thinks that the weight of evidence is tied to the probability's "liability to be changed by further experience."[13] So—to use the terminology from chapter 1—when our belief is resilient, the weight of the evidence will not change even if new evidence is introduced.

2 The Problem of Total Evidence

Peirce's insight is closely to a puzzle between evidential weight and expected utility noted by Keynes—on one hand, expected utility seems to be entirely independent of the weight of evidence, since utility is only discounted by the magnitude of its

[9] Ian Hacking, *The Taming of Chance* (Cambridge University Press, 1990).

[10] Daniel Ellsberg, "Risk, Ambiguity, and the Savage Axioms," *The Quarterly Journal of Economics* 75, no. 4 (1961): 643–69.

[11] Charles S. Peirce, "Harvard Lecture Vi, 1865," in *Writings of Charles S. Peirce: A Chronological Edition, Volume 1: 1857–1866* (Indiana University Press, 1982), 239.

[12] Peter Walley, *Statistical Reasoning with Imprecise Probabilities*, 1991, 43.

[13] Peirce, "The Probability of Induction," 295.

probability, without any regard to is weight. On the other hand, however, proponents of expected utility theory often implicitly assumes the importance of the weight of evidence. Bernoulli, for instance, suggests that rationality demands the utilization of all evidence available to us. Keynes reasons that this implies that it's always rational to get more evidence, but then it raises another critical question about whether or not one could ever be rational in refusing new evidence.[14] If the answer for the former question is positive, and the latter question negative, then we have to conclude that rationality dictates us that we should never stop looking for more evidence.

Keynes does not make the jump from "using all the evidence" to "get all the evidence" clear. Nevertheless, this problem is revisited many years later in an exchange between Ayer and Carnap. In his *Logical Foundation of Probability*, Carnap restates Bernoulli's maxim as "the requirement of total evidence".

> *Requirement of total evidence*: in the application of inductive logic to a given knowledge situation, the total evidence available must be taken as basis for determining the degree of confirmation.[15]

Aver, in response to Carnap, raises the Keynesian question: should "total evidence" include relevant evidence that I do not yet have in possession?[16] The answer must be "yes", Ayer argues. If finding the truth value of some proposition P could potentially sway the balance of my evidence, then I should definitely acquire it. Thus the principle of total evidence seems to suggest that I am also rationally compelled to consider some evidence I do not yet have.

[14]John Maynard Keynes, *A Treatise on Probability* (Macmillan; Co., Limited., 1921), 84–85.

[15]Rudolf Carnap, *Logical Foundations of Probability* (Chicago]University of Chicago Press, 1950), 211.

[16]A. J. Ayer, *Probability and Evidence* (Macmillan, 1972), 56.

But Ayer points out that this cannot be the whole picture: taken as a rule of rationality, this means we should never stop acquiring unless we are certain that we have acquired all available evidence. This, however, assumes that we know what evidence is available, but it could often be unrealistic to expect to know how much evidence we *do not* currently have.

Ayer argues that this is only a symptom of a deeper problem about logical nature of probability and its bearing on the rationality of our evidential practice. The logical interpretation of probability, held by both Carnap and Keynes, takes probability as a logical relation between propositions, so within this picture, inductive rationality is a matter of having the right degrees of belief between premises and conclusion—this is analogous to the idea deductive rationality is perceive correctly whether the conclusion follows necessarily from the premises.

Logic, however, does not care about how much evidence we have; it only cares about the relation between our propositions. This problem affects the subjective interpretation of probability as well. As Leonard Savage points out, probability could reveal to us the incoherence within the web of our belief, but it cannot tell us how to resolve it.

> According to the personalistic view, the role of the mathematical theory of probability is to enable the person using it to detect inconsistencies in his own real or envisaged behavior. It is also understood that, having detected an inconsistency, he will remove it. An inconsistency is typically removable in many different ways, among which the theory gives no guidance for

choosing.[17]

We encountered a version of this problem in chapter 1, in which we considered Keynes' definition of the weight of evidence in terms of the conditional relevance. Once again, the weight of evidence seems directly relevant to inductive reasoning, yet it cannot be easily situated in the probabilistic framework. The notion of resiliency, which we discussed in detail in chapter 1, does not seem to do any better. While it captures the expression of the weight of evidence, it does very little in illuminating on how it dedicates the rationality of our decisions. Both Joyce and Skyrms are silent on this.

The *utility* of evidence, Keynes suggests, is the key of the solution: he suggests that often getting evidence for a belief low in evidential weight will "probably produce the greatest amount of good", but the situation is opposite when the evidence for the belief is weighty—"there clearly comes a point when it is no longer worth while to spend trouble"[18] Thus, for a hypothesis of interest H, the same evidence E generates different amount of utility relative to the amount of information we already have for H. If an agent has almost no information about H, gathering more information would generate the most utility, but for the same evidence, the demand might to low, because the agent might already have enough information about H, so getting more evidence would yield very little to no utility.

If this is right, the importance of the weight of evidence lies not purely in the *amount* of evidence, but how much we have relative to how much we *need*. In the *Treatise*, Keynes does switch implicitly these two way of thinking of evidential weight—

[17] Leonard J. Savage, *The Foundations of Statistics* (Dover, 1954), 57.
[18] Keynes, *A Treatise on Probability*, 84–85.

sometimes refers the weight of evidence a balance "between the absolute amounts of relevant knowledge and of relevant ignorance respectively"[19] In a later chapter of the *Treatise*, he also calls weight "the degree of completeness of the information"[20] These remarks suggest that weight is about how much evidence we *need* as much as how much we *do* know.

3 The Value of Evidence in Light of the Weight of Evidence

The puzzle about the utility of evidence, and its bearing on the rationality of the gathering of evidence, has been addressed Ramsey on an unpublished note. Interestingly, however, he has proven essentially the opposite conclusion reached by Ayer and Keynes: Ramsey shows that we should always look more more evidence, because we can never be worse off from doing it. How can this be?

Ramsey's argument is roughly that, *if* we assume an agent to be a perfect Bayesian and that new information does not cost anything, then she will never be no worse off getting new evidence.[21] In fact, she is guaranteed to be *better* off as long as the new evidence will tell her something new. A perfect Bayesian agent is someone who studiously updates her opinions based on Bayes' rule and then act by choosing the action that maximize her expected utility. Note that this assumes two things:

[19] Keynes, 78.
[20] Keynes, 357.
[21] F. P. Ramsey, "Weight or the Value of Knowledge," *British Journal for the Philosophy of Science* 41, no. 1 (1990): 1–4, also see I. J. Good, "On the Principle of Total Evidence," *British Journal for the Philosophy of Science* 17, no. 4 (1966): 319–21 and Savage, *The Foundations of Statistics*, sec 6.2.

first, for any decision problem she faces, there is always going at least one course of action that maximizes her expected utility, and second, as Skyrms points out, this also implies that the agent knows that she will always *stays* being perfectly Bayesian in the future.

I will make use of an intuitive example rather than reproducing the proof here.[22] Suppose we have three hypotheses about the content of an urn in front of us:

1. H_b: 90 black balls and 10 white balls
2. H_w: 10 white balls and 90 black balls
3. H_n: 50 white balls and 50 black balls.

We then start by assuming $P(H_b) = P(H_w) = P(H_n) = 1/3$. Suppose we win \$1 by picking the correct hypothesis. Our expected payoff for choosing each hypothesis would be the same at 1/3. Nevertheless, we are allow to sample with replacement as many times as we wish. Should we get more evidence? Yes, according to Ramsey, we should.

To begin, at this point, the probability of getting a black ball is the same as getting a white ball. Let E_b be "a black ball is drawn" and E_w for white balls. So:

$$P(E_b) = P(H_b)P(E_b|H_b) + P(H_w)P(E_b|H_w) + P(H_n)P(E_b|H_n)$$

$$= 1/3(0.9) + 1/3(0.1) + 1/3(0.5) = 0.5$$

And $P(E_w) = 1 - P(E_b) = 0.5$. So, in the event of drawing a black ball from the urn,

[22] This example is adapted from Isaac Levi, "The Weight of Argument," in *Fundamental Uncertainty: Rationality and Plausible Reasoning*, ed. Silva Marzetti Dall'Aste Brandolini and Roberto Sczzieri (Palgrave MacMilan, 2011), 39–58

we would update our belief like so:

$$P(H_b|E_b) = \frac{P(H_b)P(E_b|H_b)}{P(E_b)} = \frac{1/3(0.9)}{0.5} = 0.6$$

Similarly, applying the calculation on the other hypotheses, we get:

$$P(H_w|E_b) = 0.067$$

$$P(H_n|E_b) = 0.333$$

Similar argument can be made assuming E_w, that is, a white ball is chosen. In that case $P(H_w|E_w) = 0.6$. If we were an ideal Bayesian agent, we should pick H_b if E_b, and pick H_w if E_w. Since an ideal Bayesian would choose the option that maximizes our expected utility, in either case the expected value after drawing from the urn once is 0.6, which is an improvement, since before drawing our expected utility is 1/3 for all options. The net gain in expected utility would be $0.6 - 0.33 = 0.27$, is referred to as *the value of information* in the decision theory literature.[23]

It turns out that we would be even better off if we were to draw from the urn again. Suppose the first draw yields a black ball. So now we have one piece of evidence in hand. Let us refer to our state of belief after the first draw as $H'_b, E'_b, ..$ and so on. For instance, $P(H'_b) = P(H_b|E_b)$ and $P(E'_b) = P(E'_b|E_b)$. One notable change is that $P(E'_b) = 0.7132$ and $P(E'_w) = 0.2868$. If we draw again and get a black ball, this means:

[23]Howard Raiffa and Robert Schlaifer, *Applied Statistical Decision Theory* (Harvard, 1964), 89–90. For a more digestible presentation see Robert Winkler, *An Introduction to Bayesian Inference and Decision* (Probabilistic Publishing, 2010) sec.6.3.

$$P(H_b'|E_b') = 0.757$$

$$P(H_w'|E_b') = 0.009$$

$$P(H_n'|E_b') = 0.233$$

If a white ball were to be drawn:

$$P(H_b'|E_w') = 0.21$$

$$P(H_w'|E_w') = 0.21$$

$$P(H_n'|E_w') = 0.58$$

Thus, if for the second sample we get a black ball, we would choose b since it has the maximum expected utility at 0.757, and if we get a white ball, we choose n with the expected value at 0.58. So, the expected utility, if we were to draw from the urn again, is: $0.7132(0.757) + 0.2867(0.58) = 0.706$, which is an improvement over just drawing once. The net gain is $0.706 - 0.6 = 0.106$. Ramsey's proof shows that we can keep on getting more evidence and we will never be worse off. In fact, we will be better off as long as there is evidence out there we do not yet have.

What should we make of Ramsey's proof? There are two issues involved here. The first is Keynes' observation that evidence can have a diminishing return, so relevant evidence does not increase weight, and the other is how the weight of evidence bears on the rationality of our action, especially when it comes to the gathering evidence.

Ramsey's note provides a good answer for the former, but not the latter.

To begin, Ramsey's contribution here is a way for us to think about the relationship between the *weight* of the evidence in possession and the *value* of the potential new evidence.[24] Ramsey clearly thought is the value of evidence E for hypothesis H as something along the line of the difference between the prior expected utility $EU(H)$ and the posterior $EU(H|E)$.

For instance, we saw that in the example above, the posterior expected utility of the first draw was 0.27 higher than our prior expected utility, and we saw a net gain of 0.106 in expected utility if we were to draw again after drawing a black ball, so we see that the first piece of evidence has a higher value than the second one. What Ramsey's proof demonstrates is that new evidence has a diminishing return—I get a "bigger bang for the buck" for my evidence gathering endeavor when I have less evidence. In light of this, Keynes' example of the balance of the evidence unchanged by the introduction of new evidence is then somewhat incomplete. This explains one of Keynes' puzzle about worthiness of our endeavor to get more evidence in light of the evidence we have in possession.

However, the broader normative question is still unanswered: how should the weight of evidence guide the rationality of our action? To be sure, I do not question that given some assumptions, Ramsey's result will necessarily follow: the same result is proven by both Good and Leonard Savage, so there is no doubt that the result will holds if the assumptions are granted, but that's a big *if*—we have to question if how

[24]As noted, this is essentially the idea of the value of information in decision theory, but, as a historical note, Ramsey, inspired by Keynes' puzzle about the weight of evidence, has anticipated this development by many years.

often these assumptions actually hold.

Ramsey probably understood that information was rarely free. However, Ramsey might have interpreted Keynes' puzzle not as a *decision problem about evidence* but a question regarding its intrinsic value. We saw that Ayer essentially posed the same question to Carnap. History essentially repeated itself when I.J. Good puts forth essentially the answer to Ayer. Interestingly, Good interprets Ayer's as questioning "why... we should bother to make new observations."[25] So, Good seem to think what is needed is a justification for getting new evidence *in general*. Ramsey might have interpret Keynes in the same way. With respect to this version of the problem, the proof makes perfect sense, since it demonstrates that all things being equal we usually end up with better expected utility by considering more evidence. But Keynes' question was about reconciling the general duty to get more evidence and the intuition that evidence gathering for a belief of interest is not always a worthy endeavor.

The scenario we imagined quickly breaks down once we starts to introduce some sort of cost. It was assumed in the example that it costs us neither money nor time to draw from the urn, but suppose it costs us 25 cents for each sample. This means that we would be gaining only $0.27 - 0.25 = 0.02$ in expected payoff for the first draw, and the second draw would definitely not be worth the additional 25 cents. Or suppose that one dollar is not worthy any endeavor that lasts longer than 15 seconds, and it takes 30 seconds to draw from the urn. As soon as minimally realistic assumptions are introduced, Ramsey's result no longer holds.

Cost might also enter into consideration in different forms, e.g., computational

[25]Irving J. Good, *Good Thinking: The Foundations of Probability and Its Applications* (Univ Minnesota Pr, 1983), 178.

cost or memory. In the same context, Savage ponders over an interesting case that introduces yet another dimension of the problem: consider a very ill person, who is given the option to find out with no cost if the disease she has is mortal. Savage points out that an argument can be made that in this case refusing information could be rational. The thought is that the patient may decide that, based on an assessment of her own personality, she would live the rest of her remaining life in agony if she were to find out that her disease is very serious, whereas she could live relatively happily without knowing. Savage's response is that in this case the information is not really free; it has a *psychological* cost.[26]

Savage's response was intended to paint a counterexample as an explainable exception to the idea that it's always rational to get free evidence, but this sort of implicit cost is the norm, not an exception: at the minimal, any endeavor to seek more evidence will cost us at least time, and the loss of time is the loss of opportunity. Cost might also enter into consideration in different forms, e.g., computational cost or memory. In light of this, Ramsey's demonstration—the value of evidence depends on how much evidence we have and how much evidence there is to get—does not ease Keynes' concern at all, but it makes the question of evidence gathering more pressing: I might not be getting the most out of my evidential endeavor if it turns out that I could be much better off by examining a different hypothesis, for the very set up for Ramsey's proof requires us to have already chosen *one* hypothesis of interest. But in reality, we often have competing interests. The question is not whether I will get some utility out of gathering evidence for hypotheses in which I have interest, but how I should go about doing it, given the limitation of resource.

[26]Savage, *The Foundations of Statistics*, 107.

4 Abduction

I believe that Keynes' puzzle about the weight of evidence and its relation to utility is best analyzed in terms of C. S. Peirce's distinction between abductive, deductive, and inductive reasoning. In particular, whether or not we ought to increase the weight of evidence for a particular belief is an *abductive* question, not an inductive one.

Peirce has a tripartite classification of scientific reasoning: abduction is "the first starting of a hypothesis and the entertaining of it."[27] Once the hypothesis is chosen, deductive reasoning is used to tease out the implications of the main hypothesis, a process that often breaks it down into smaller testable ones. Induction, exemplified by statistical and probabilistic reasoning, is then carried out to verify the main or an implied hypothesis.

A quick sketch of this picture reveals that abduction, at least for Peirce, is *not* inference to the best explanation, even though the two are often confused.[28] Roughly speaking, inference to the best explanation is the thesis that a hypothesis can be justified by how well it *explains* the evidence we have. Abduction, on the other hand, is not even always about the explanatory power of the hypothesis, even though it can often be. While Peirce's view on abduction evolves throughout his life, he is consistent in holding that abduction is the preparatory step the inquirer takes before the experiment or observation. For instance, in an early paper which still refer to abduction as "hypothesis," Peirce says that the abduced hypothesis 'should be

[27]Charles S. Peirce, *Collected Papers of Charles Sanders Peirce* (Cambridge: Harvard University Press, 1931), 6.525.

[28]Mcauliffe, "How Did Abduction Get Confused with Inference to the Best Explanation?" *Transactions of the Charles S. Peirce Society* 51, no. 3 (2015): 300–319.

distinctly put as a question, before making the observations which are to test its truth.'[29]

Peirce characterizes abduction as the *creation*, or *selection* of the hypothesis, that accounts for the observation of some surprising facts. This must be understood in the context of the richness of Peirce's epistemology: inquiry—the practice of knowledge—is an interplay between doubt and belief. Doubt and belief are distinguished by their psychological and practical differences: psychologically, doubt is an uneasy and unstable state, while belief is characterized by imperturbability—a sense of stability of the mind.[30] Practically, a belief expresses itself as as a stable habit—an established second nature, while the instability of doubt prompts us to seek satisfaction through reaching a state of belief. The degree of a belief, in turn, expresses itself in terms of the decisiveness of action.[31]

Aductive reasoning plays an important in this picture of our epistemic life, because it is a direct reaction to the irradiation of doubt. Peirce points out that naturally this is where explanatory virtues tend to matter a lot, since the first rational response is to formulate a hypothesis that explains the occurrence of this surprising fact.

[29]Charles S. Peirce, "Deduction, Induction, and Hypothesis," in *Writings of Charles S. Peirce: A Chronological Edition, Volume 3: 1872–1878* (Indiana University Press, 1986), 331.

[30]Charles S. Peirce, "The Fixation of Belief," in *Writings of Charles S. Peirce: A Chronological Edition, Volume 3: 1872–1878* (Indiana University Press, 1966), 247.

[31]This is not to say Peirce believes in subjective probability. While Peirce has indicated that beliefs can be measured quantitatively, as far as the interpretation of probability goes, he is an objective propensity theorist.

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