

An important key to a Peircean interpretation of the weight of evidence is Peirce's tripartite classification of reasoning. As is well known, Peirce is responsible for coining the phrase "abduction", which is distinguished from deduction and induction. Peirce's understanding deduction and induction is very close to the modern understanding of the distinction. Deduction considers the necessary consequences between propositions, so a derivation in, say, predicate logic and theorem proving would be considered as deduction by Peirce. However, it is important to note that the necessity is a property of the *relationship* and not of the conclusion. In other words, a deductive argument can sometimes have a conclusion that is only *probably* true. An example would be deriving a probability based on a binomial distribution.(???) We know that, from probability theory, for a random variable with binary outcomes—success or failure, for instance—the probability of getting  $k$  success out of  $n$  trials, given the probability of a single success, is

$$\binom{n}{k} p^k (1-p)^{n-k}$$

Thus, suppose we have  $n = 5$  fair coins, so  $p = 0.5$ . The probability of exactly  $x = 3$  of the coin turning up heads is  $\binom{5}{3} 0.5^3 (1-0.5)^{2} = 0.3125$ . This, again, is a necessary consequence, so, even though the conclusion itself is probable, the *inference* is necessary, and is, therefore, deductive.

Induction, on the other hand, involves inference that is neither necessary nor justified by the relation between propositions.<sup>1</sup> Peirce tends to emphasize induction's role as

However, unlike how it is represented in most modern logic textbooks, Peirce does not conflate the deductive/inductive distinction with the certain/probable belief distinction. In other words, Peirce does not think that deduction can only concern inferences between propositions that are fully believed or assumed—some deductive conclusions are only probably true, even though they follow as a necessary consequence. Peirce calls these *probable deductions*.(???) They are Peirce's probabilistic extension of Aristotle's categorical syllogisms. For instance, a simple probable deduction is as follows:

1. The proportion  $\rho$  of the  $M$ 's are  $P$ 's.
2.  $S$  is an  $M$ .
3. It follows, with probability  $\rho$ , that  $S$  is a  $P$ .

It is clearly analogous to a categorical syllogism in which one derive a particular affirmative about a property of  $S$  by virtue of  $S$  being a member of  $M$ , which has the property  $P$ . The only difference here is that the proportion  $\rho$  is introduced. Peirce's idea is that even though in a probable deduction, even though  $S$  is not necessarily  $P$ , one is entitled to infer that  $S$  is  $P$  with the probability of  $\rho$ . Since

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<sup>1</sup>This is the familiar conclusion of Hume's problem of induction; however, Peirce in general expresses very little interest in the skeptical problem of induction—his perspective is one of a practicing scientist, who concerns himself with the reliability of the inductive inference involved in scientific inquiry. As far as Peirce is concerned, yhat induction has been indispensable and reliable already justifies its role in inquiry. (EP2 97)

this conclusion is a necessary consequence, it must be deductively derived. A more complex example of probable deduction mentioned by Peirce explicitly is the calculation of a binomial probability based on the probability mass function. We know that, from basic probability, for a random variable with only two outcomes, success or failure, the probability of getting  $k$  success out of  $n$  trials, given the probability of a single success, is

$$\binom{n}{k} p^k (1-p)^{n-k}$$

Thus, suppose we have  $n = 5$  fair coins, so  $p = 0.5$ . The probability of exactly  $x = 3$  of the coin turning up heads is  $\binom{5}{3} 0.5^3 (1-0.5)^{2} = 0.3125$ . This, again, is a necessary consequence, so, even though the conclusion itself is probable, the *inference* is necessary, so it is deductive. There is also *statistical* variant called *statistical deduction*. It is slightly more involved, as it takes the Weak Law of Large Number as a premise, for an argument. Roughly speaking, it is the inference that, if we draw a n random samples,  $S_{1:n}$ , from  $M$ , then it necessarily follows that the proportion of  $S$ 's that are  $P$ 's is also approximately  $\rho$ .

Peirce's notion of probable deduction is relevant to our earlier discussion regarding Peirce's criticism of conceptualism. Part of the difficulty in interpreting Peirce's positive ideas about degrees of belief is that in that context it is not clear whether Peirce believes in them, or he was simply setting up a *Reductio Ad Absurdum* argument. However, it is clear that Peirce thinks that the conclusion of a probable deduction is essentially a partial belief justified deductively—Peirce even explicitly suggests that a deductively derived probable deduction should have a degree of confidence corresponding to its probability, based on the odd-ratio definition of degrees of belief he proposed in his reconstruction of conceptualism. As Issac Levi points out, Peirce's degree of confidence "has all the earmarks of what many contemporaries would call a subjective probability including the disposition to take risks."(???)

This is also relevant to our discussion of Keynes in the last chapter, for Peirce's notion of probable deduction has a strong affinity to Keynes' idea of probability as logical relations. Recall that for Keynes, there is a distinction between subjective degrees of belief and *rational* degrees of belief, the latter of which is supposed to correspond to the objective degree of partial entailment between the premises and conclusion.

In any case, we can see that deduction, as Peirce sees it, is characterized by its relational and necessary nature. In contrast, Probable induction is helpful in bringing out how Peirce distinguish induction from deduction. Earlier, we saw that in probable deductions, the proportion of the population or the probability of success is always given in the premises. In contrast, *induction* occurs when this particular premise is instead the conclusion to be confirmed.

1.  $X_1, \dots, X_n$  are sampled randomly from the population  $M$ .
2. The proportion  $\rho$  of  $X_1, \dots, X_n$  is  $P$ .

3. Hence,

That is, we draw samples from the population with the unknown parameter, and from the data we infer the

His view on the matter is expansive, and it evolved throughout his life. Abduction involves