Them's the proofs

Before making any attempt at the test, you should do at least half of the problems here. You should be comfortable with proving any of these in order to pass to test.

Deduction Task Credit You can receive one task credit by doing 3 problems and show to class. Rules:

1. Only 1 problem per class.
2. Complete accuracy is not needed for credit - just make a honest effort. But I reserve the right to refuse credit if it's clear that you are basically winging it.
3. Must stay for the whole class period to get credit.
4. Must email me the day before class meeting and I will assign you a problem.
5. Space is limited. Priority will be given to students who need to practice and credit.

1 Basic Proofs

1. $\{A \wedge B, B \rightarrow C\} \vdash A \wedge (B \wedge C)$
2. $\{(P \vee R) \wedge (S \vee R), \neg R \wedge Q\} \vdash P \wedge (Q \vee R)$
3. $\{(X \wedge Y) \rightarrow Z, X \wedge W, W \rightarrow Y\} \vdash Z$
4. $\{A \vee (B \vee G), A \vee (B \vee H), \neg A \wedge \neg B\} \vdash G \wedge H$
5. $\{P \wedge (Q \wedge \neg R), R \vee T\} \vdash T \vee S$
6. $\{((A \rightarrow D) \vee B) \vee C, \neg C, \neg B, A\} \vdash D$
7. $\{A \vee \neg\neg B, \neg B \vee \neg C, C \vee A, \neg A\} \vdash D$
8. $\{P \leftrightarrow (Q \leftrightarrow R), P, P \rightarrow R\} \vdash Q$
9. $\{A \rightarrow (B \rightarrow C), A, B\} \vdash C$
10. $\{(X \vee A) \rightarrow \neg Y, Y \vee (Z \wedge Q), X\} \vdash Z$
11. $\{A \wedge (B \wedge C), A \wedge D, B \wedge E\} \vdash D \wedge (E \wedge C)$
12. $\{A \rightarrow B, B \rightarrow C, C \rightarrow A, B, \neg A\} \vdash D$
13. $\{\neg A \wedge B, A \vee P, A \vee Q, B \rightarrow R\} \vdash P \wedge (Q \wedge R)$

2 Conditional Proofs

1. $\{X \rightarrow (A \wedge B), B \rightarrow Y, B \rightarrow A\} \vdash X \rightarrow Y$
2. $\{\neg W \wedge \neg E, Q \rightarrow D\} \vdash (W \vee Q) \rightarrow (E \vee D)$
3. $\{(A \wedge B) \rightarrow D, D \rightarrow (X \wedge Y), C \rightarrow Z\} \vdash A \wedge (B \wedge C) \rightarrow X \wedge (Y \wedge Z)$

3 Reductio Ad Absurdum

1. $A \to (B \vee (C \vee D)) \vdash \neg[A \wedge (\neg B \wedge (\neg C \wedge \neg D))]$
2. $P \to Q \vdash \neg Q \to \neg P$
3. $P \wedge Q \vdash \neg(P \to \neg Q)$
4. $(P \wedge Q) \to (R \vee S), \neg(R \vee S) \vdash \neg(P \wedge Q)$
5. $\neg(P \to Q) \vdash P \wedge \neg Q$

4 Theorems

Modus Tollens $\{A \to B, \neg B\} \vdash \neg A\}$

Hypothetical Syllogism $\{A \to B, B \to C\} \vdash A \to C$

Idempotence of disjunction $\{A \vee A\} \vdash A$

Idempotence of conjunction $\{A\} \vdash A \wedge A$

Weakening $\{A\} \vdash B \to A$

Argument by Cases 1 $\{X \vee Y, X \to Z, Y \to Z\} \vdash Z$

Argument by Cases 2 $\{X \vee Y, X \vdash Z, Y \vdash Z\} \vdash Z$

Biconditional Introduction $\{X \vdash Y, Y \vdash X\} \vdash X \leftrightarrow Y$

Peirce's Law $((A \to B) \to A) \vdash A$

5 Logical Laws

1. DeMorgan's Law
2. Contraposition
3. Material Conditional
4. Association
5. Distribution
6. Exportation: $P \to (Q \to R) \equiv (P \wedge Q) \to R$

6 Argument by Cases

1. $A \vee B \vdash B \vee A$
2. $A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$
3. $(A \vee B) \wedge (B \to C) \vdash A \vee C$
4. $(\neg H \vee M), \neg M \to \neg C \vdash (H \vee C) \to M$
5. $(A \wedge B) \vee (A \wedge C) \vdash A \wedge (B \vee C)$
6. $(S \wedge J) \vee (\neg S \wedge \neg J) \vdash (S \leftrightarrow J)$
7. $K \to (F \vee C), J \to (C \vee D), \neg C \vdash \neg(F \vee D) \to \neg(K \vee J)$

7 Tautologies

1. $A \to (A \vee B)$
2. $A \to (B \to A)$
3. $A \to (B \ \to (A \wedge B))$
4. $(A \wedge B) \to ((A \vee C) \wedge (B \vee C))$
5. $(A \leftrightarrow B) \to (A \to B)$
6. $(B \to \neg B) \leftrightarrow \neg B$
7. $A \vee \neg A$
8. $\neg\neg\neg\neg\neg(A \wedge \neg A)$
9. $A \vee ((\neg A \vee B) \wedge (\neg A \vee C))$
10. $((A \wedge B) \to (B \wedge A)) \wedge (\neg(A \wedge B) \to \neg(B \wedge A))$
11. $(A \to (B \wedge C)) \leftrightarrow ((\neg B \vee \neg C) \to \neg A)$
12. $(P \to [P \to Q]) \to (P \to Q)$
13. $(P \vee \neg P) \wedge (Q \leftrightarrow Q)$
14. $(P \wedge \neg P) \vee (Q \leftrightarrow Q)$
15. $(A \vee B) \to (\neg B \to A)$
16. $M \vee \neg(M \wedge N)$
17. $[H \to (O \to N)] \to [(H \wedge O) \to N]$
18. $(D \to B) \to \{(D \to T) \to [D \to (B \wedge T)]\}$
19. $(K \to F) \to [\neg F \to \neg(K \wedge P)]$
20. $[(F \vee G) \to (P \wedge Q)] \to (\neg Q \to \neg F)$
21. $[L \to (M \to N)] \to [(L \to M) \to (L \to N]$
22. $[(S \vee T) \to F] \to \{[(F \vee G) \to H] \to (S \to H)\}$
23. $(I \wedge \neg J) \vee [(J \wedge K) \vee \neg(K \wedge I)]$
24. $[[C \wedge (A \vee D)] \vee \neg(C \wedge F)) \vee \neg(A \wedge \neg G)$
25. $((H \wedge F) \to C) \wedge \neg(H \to (F \to C))$
26. $(\neg(G \vee Q) \wedge (K \to G)) \wedge \neg(P \vee \neg K)$
27. $(A \leftrightarrow B) \leftrightarrow (\neg A \leftrightarrow B)$

8 Logical Equivalence

1. $(P \to R) \land (Q \to R) \equiv_{\vdash} (P \lor Q) \to R$
2. $(P \to (Q \lor R)) \equiv_{\vdash} (P \to Q) \lor (P \to R)$
3. $(P \leftrightarrow Q) \equiv_{\vdash} \neg P \leftrightarrow \neg Q$
4. $\neg(P \leftrightarrow Q) \equiv_{\vdash} (P \leftrightarrow \neg Q)$
5. $P \equiv_{\vdash} (P \lor (Q \land P))$

9 Contradiction

1. $B, \neg(A \to B)$
2. $A \leftrightarrow \neg B, B \leftrightarrow C, A \leftrightarrow C$
3. $\neg(A \to B), \neg(B \to C)$
4. $\neg(A \to A)$
5. $(A \leftrightarrow \neg(A \leftrightarrow A)), A$

10 Derivation w/ Quantifiers

1. $\{\forall x(Mx \leftrightarrow Nx), Ma \land \exists x Rxa\} \vdash \exists x Nx$
2. $\{\forall x(\neg Mx \lor Ljx), \forall x(Bx \to Ljx), \forall x(Mx \lor Bx)\} \vdash \forall x Ljx$
3. $\forall x(Cx \land Dt) \vdash \forall x Cx \land Dt$
4. $\exists x(Cx \lor Dt) \vdash \exists x Cx \lor Dt$
5. $\forall x Fx \vdash \forall y Fy$
6. $Fb, Gb \vdash \exists x(Fx \land Gx)$
7. $\forall x \forall y Hxy \vdash \exists x \exists y Hxy$
8. $\forall x(Fx \leftrightarrow Gx), \forall y(Gy \leftrightarrow Hy) \vdash \forall x(Fx \leftrightarrow Hx)$
9. $\forall y Hyy, \exists z Bz \vdash \exists(Bx \land Hxx)$
10. $\forall x Fx \vdash \exists y(\neg Fy \to Gy)$
11. $\exists x Fxa, \exists y Fby, \forall x \forall y(Fxy \to Fyx) \vdash \exists x Fax \land \exists y Fyb$
12. $\forall x \forall y(Fxy \to (Gx \to \neg Gx)) \vdash \forall x \forall y Fxy \to \forall z \neg Gz$
13. $\exists y Gya, \forall z(Gza \to Gaz) \vdash \exists x Gax$

11 Tautologies w/ Quantifiers

1. $\forall x Fx \vee \neg \forall x Fx$
2. $\forall z(Fz \rightarrow (Fz \vee Gz))$
3. $\exists x Fx \rightarrow \exists x(Fx \vee Gx)$
4. $\exists x \forall y Fxy \rightarrow \exists x \exists y Fxy$
5. $\forall x(Ax \rightarrow Bx) \rightarrow \forall x(Bx \vee \neg Ax)$
6. $\forall x(Ax \rightarrow Bx) \vee \exists x Ax$
7. $\exists x(Fx \rightarrow \forall y Fy)$
8. $\forall x \forall y \forall z Gxyz \rightarrow \forall x \forall y \forall z(Gxyz \rightarrow Gzyx)$

12 ABANDON HOPE, ALL YE WHO ENTER HERE

1. $\vdash \neg \exists x \neg Ax \leftrightarrow \forall x Ax$
2. $\vdash \neg \forall x \neg Ax \leftrightarrow \exists x Ax$
3. $\vdash \forall x \exists y(Ax \vee By) \leftrightarrow \exists y \forall x(Ax \vee By)$
4. $\forall x(Ax \wedge \exists y \neg Bxy) \equiv_{\vdash} \neg \exists x(\neg Ax \vee \forall y(Bxy \wedge Bxy))$