

## Them's the proofs

Before making any attempt at the test, you should do at least half of the problems here. You should be comfortable with proving any of these in order to pass to test.

**Deduction Task Credit** You can receive one task credit by doing 3 problems and show to class. Rules:

1. Only 1 problem per class.
2. Complete accuracy is not needed for credit. But I reserve the right to refuse credit if it's clear that you are basically winging it.
3. Must stay for the whole class period to get credit.
4. Must email me the day before class meeting and I will assign you a problem.
5. Space is limited. Priority will be given to students who need to practice and credit.

**Basic Proofs** Turn in problem 12 for grade. The rest are for practice.

1.  $\{A \wedge B, B \rightarrow C\} \vdash A \wedge (B \wedge C)$
2.  $\{(P \vee R) \wedge (S \vee R), \neg R \wedge Q\} \vdash P \wedge (Q \vee R)$
3.  $\{(X \wedge Y) \rightarrow Z, X \wedge W, W \rightarrow Y\} \vdash Z$
4.  $\{A \vee (B \vee G), A \vee (B \vee H), \neg A \wedge \neg B\} \vdash G \wedge H$
5.  $\{P \wedge (Q \wedge \neg R), R \vee T\} \vdash T \vee S$
6.  $\{((A \rightarrow D) \vee B) \vee C, \neg C, \neg B, A\} \vdash D$
7.  $\{A \vee \neg \neg B, \neg B \vee \neg C, C \vee A, \neg A\} \vdash D$
8.  $\{P \leftrightarrow (Q \leftrightarrow R), P, P \rightarrow R\} \vdash Q$
9.  $\{A \rightarrow (B \rightarrow C), A, B\} \vdash C$
10.  $\{(X \vee A) \rightarrow \neg Y, Y \vee (Z \wedge Q), X\} \vdash Z$
11.  $\{A \wedge (B \wedge C), A \wedge D, B \wedge E\} \vdash D \wedge (E \wedge C)$
12.  $\{A \rightarrow B, B \rightarrow C, C \rightarrow A, B, \neg A\} \vdash D$
13.  $\{\neg A \wedge B, A \vee P, A \vee Q, B \rightarrow R\} \vdash P \wedge (Q \wedge R)$

**Conditional Proofs** Turn in problem 3 and 4 for grade

1.  $\{X \rightarrow (A \wedge B), B \rightarrow Y, B \rightarrow A\} \vdash X \rightarrow Y$
2.  $\{\neg W \wedge \neg E, Q \rightarrow D\} \vdash (W \vee Q) \rightarrow (E \vee D)$
3.  $\{(A \wedge B) \rightarrow D, D \rightarrow (X \wedge Y), C \rightarrow Z\} \vdash A \wedge (B \wedge C) \rightarrow X \wedge (Y \wedge Z)$

**Reductio Ad Absurdum** Turn in problem 6 and 7 for grade

1.  $A \rightarrow (B \vee (C \vee D)) \vdash \neg[A \wedge (\neg B \wedge (\neg C \wedge \neg D))]$
2.  $P \rightarrow Q \vdash \neg Q \rightarrow \neg P$
3.  $P \wedge Q \vdash \neg(P \rightarrow \neg Q)$
4.  $(P \wedge Q) \rightarrow (R \vee S), \neg(R \vee S) \vdash \neg(P \wedge Q)$
5.  $\neg(P \rightarrow Q) \vdash P \wedge \neg Q$

### 1 Theorems

**Modus Tollens**  $\{A \rightarrow B, \neg B\} \vdash \neg A$

**Hypothetical Syllogism**  $\{A \rightarrow B, B \rightarrow C\} \vdash A \rightarrow C$

**Idempotence of disjunction**  $\{A \vee A\} \vdash A$

**Idempotence of conjunction**  $\{A\} \vdash A \wedge A$

**Weakening**  $\{A\} \vdash B \rightarrow A$

**Argument by Cases 1**  $\{X \vee Y, X \rightarrow Z, Y \rightarrow Z\} \vdash Z$

**Argument by Cases 2**  $\{X \vee Y, X \vdash Z, Y \vdash Z\} \vdash Z$

**Biconditional Introduction**  $\{X \vdash Y, Y \vdash X\} \vdash X \leftrightarrow Y$

**Peirce's Law**  $((A \rightarrow B) \rightarrow A) \vdash A$

### 2 Logical Laws

1. DeMorgan's Law
2. Contraposition
3. Material Conditional
4. Association
5. Distribution
6. Exportation:  $P \rightarrow (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$

### 3 Argument by Cases

1.  $A \vee B \vdash B \vee A$
2.  $A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$
3.  $(A \vee B) \wedge (B \rightarrow C) \vdash A \vee C$
4.  $(\neg H \vee M), \neg M \rightarrow \neg C \vdash (H \vee C) \rightarrow M$
5.  $(A \wedge B) \vee (A \wedge C) \vdash A \wedge (B \vee C)$
6.  $(S \wedge J) \vee (\neg S \wedge \neg J) \vdash (S \leftrightarrow J)$
7.  $K \rightarrow (F \vee C), J \rightarrow (C \vee D), \neg C \vdash \neg(F \vee D) \rightarrow \neg(K \vee J)$

**4 Tautologies**

1.  $A \rightarrow (A \vee B)$
2.  $A \rightarrow (B \rightarrow A)$
3.  $A \rightarrow (B \rightarrow (A \wedge B))$
4.  $(A \wedge B) \rightarrow ((A \vee C) \wedge (B \vee C))$
5.  $(A \leftrightarrow B) \rightarrow (A \rightarrow B)$
6.  $(B \rightarrow \neg B) \leftrightarrow \neg B$
7.  $A \vee \neg A$
8.  $\neg\neg\neg\neg(A \wedge \neg A)$
9.  $A \vee ((\neg A \vee B) \wedge (\neg A \vee C))$
10.  $((A \wedge B) \rightarrow (B \wedge A)) \wedge (\neg(A \wedge B) \rightarrow \neg(B \wedge A))$
11.  $(A \rightarrow (B \wedge C)) \leftrightarrow ((\neg B \vee \neg C) \rightarrow \neg A)$
12.  $(P \rightarrow [P \rightarrow Q]) \rightarrow (P \rightarrow Q)$
13.  $(P \vee \neg P) \wedge (Q \leftrightarrow Q)$
14.  $(P \wedge \neg P) \vee (Q \leftrightarrow Q)$
15.  $(A \vee B) \rightarrow (\neg B \rightarrow A)$
16.  $M \vee \neg(M \wedge N)$
17.  $[H \rightarrow (O \rightarrow N)] \rightarrow [(H \wedge O) \rightarrow N]$
18.  $(D \rightarrow B) \rightarrow \{(D \rightarrow T) \rightarrow [D \rightarrow (B \wedge T)]\}$
19.  $(K \rightarrow F) \rightarrow [\neg F \rightarrow \neg(K \wedge P)]$
20.  $[(F \vee G) \rightarrow (P \wedge Q)] \rightarrow (\neg Q \rightarrow \neg F)$
21.  $[L \rightarrow (M \rightarrow N)] \rightarrow [(L \rightarrow M) \rightarrow (L \rightarrow N)]$
22.  $[(S \vee T) \rightarrow F] \rightarrow \{[(F \vee G) \rightarrow H] \rightarrow (S \rightarrow H)\}$
23.  $(I \wedge \neg J) \vee [(J \wedge K) \vee \neg(K \wedge I)]$
24.  $[[C \wedge (A \vee D)] \vee \neg(C \wedge F)] \vee \neg(A \wedge \neg G)$
25.  $((H \wedge F) \rightarrow C) \wedge \neg(H \rightarrow (F \rightarrow C))$
26.  $(\neg(G \vee Q) \wedge (K \rightarrow G)) \wedge \neg(P \vee \neg K)$
27.  $(A \leftrightarrow B) \leftrightarrow (\neg A \leftrightarrow B)$

### 5 Logical Equivalence

1.  $(P \rightarrow R) \wedge (Q \rightarrow R) \equiv_{\vdash} (P \vee Q) \rightarrow R$
2.  $(P \rightarrow (Q \vee R)) \equiv_{\vdash} (P \rightarrow Q) \vee (P \rightarrow R)$
3.  $(P \leftrightarrow Q) \equiv_{\vdash} \neg P \leftrightarrow \neg Q$
4.  $\neg(P \leftrightarrow Q) \equiv_{\vdash} (P \leftrightarrow \neg Q)$
5.  $P \equiv_{\vdash} (P \vee (Q \wedge P))$

### 6 Contradiction

1.  $B, \neg(A \rightarrow B)$
2.  $A \leftrightarrow \neg B, B \leftrightarrow C, A \leftrightarrow C$
3.  $\neg(A \rightarrow B), \neg(B \rightarrow C)$
4.  $\neg(A \rightarrow A)$
5.  $(A \leftrightarrow \neg(A \leftrightarrow A)), A$

### 7 Derivation w/ Quantifiers

1.  $\{\forall x(Mx \leftrightarrow Nx), Ma \wedge \exists xRxa\} \vdash \exists xNx$
2.  $\{\forall x(\neg Mx \vee Ljx), \forall x(Bx \rightarrow Ljx), \forall x(Mx \vee Bx)\} \vdash \forall xLjx$
3.  $\forall x(Cx \wedge Dt) \vdash \forall xCx \wedge Dt$
4.  $\exists x(Cx \vee Dt) \vdash \exists xCx \vee Dt$
5.  $\forall xFx \vdash \forall yFy$
6.  $Fb, Gb \vdash \exists x(Fx \wedge Gx)$
7.  $\forall x\forall yHxy \vdash \exists x\exists yHxy$
8.  $\forall x(Fx \leftrightarrow Gx), \forall y(Gy \leftrightarrow Hy) \vdash \forall x(Fx \leftrightarrow Hx)$
9.  $\forall yHy, \exists zBz \vdash \exists(Bx \wedge Hxx)$
10.  $\forall xFx \vdash \exists y(\neg Fy \rightarrow Gy)$
11.  $\exists xFxa, \exists yFby, \forall x\forall y(Fxy \rightarrow Fyx) \vdash \exists xFax \wedge \exists yFyb$
12.  $\forall x\forall y(Fxy \rightarrow (Gx \rightarrow \neg Gx)) \vdash \forall x\forall yFxy \rightarrow \forall z\neg Gz$
13.  $\exists yGya, \forall z(Gza \rightarrow Gaz) \vdash \exists xGax$

## 8 Tautologies w/ Quantifiers

1.  $\forall x Fx \vee \neg \forall x Fx$
2.  $\forall z (Fz \rightarrow (Fz \vee Gz))$
3.  $\exists x Fx \rightarrow \exists x (Fx \vee Gx)$
4.  $\exists x \forall y Fxy \rightarrow \exists x \exists y Fxy$
5.  $\forall x (Ax \rightarrow Bx) \rightarrow \forall x (Bx \vee \neg Ax)$
6.  $\forall x (Ax \rightarrow Bx) \vee \exists x Ax$
7.  $\exists x (Fx \rightarrow \forall y Fy)$
8.  $\forall x \forall y \forall z Gxyz \rightarrow \forall x \forall y \forall z (Gxyz \rightarrow Gzyx)$

## 9 ABANDON HOPE, ALL YE WHO ENTER HERE

1.  $\vdash \neg \exists x \neg Ax \leftrightarrow \forall x Ax$
2.  $\vdash \neg \forall x \neg Ax \leftrightarrow \exists x Ax$
3.  $\vdash \forall x \exists y (Ax \vee By) \leftrightarrow \exists y \forall x (Ax \vee By)$
4.  $\forall x (Ax \wedge \exists y \neg Bxy) \equiv \neg \exists x (\neg Ax \vee \forall y (Bxy \wedge Bxy))$