Practice Problem for Module 3

September 25, 2016

Before making any attempt at the test, you should do at least half of the problems here. You should be comfortable with proving any of these in order to pass to test. No solutions are given for those - these are meant for you to work on your own and then with me if you run into trouble. Problems with solutions will be given for the module quiz.

Basic Proofs

Note: The following are to be proven without using rules of replacements

Part 1 Derive the following.

(1) $\{A \wedge B, B \to C\} \vdash A \wedge (B \wedge C)$

(2) $\{(P \vee R) \wedge (S \vee R), \neg R \wedge Q\} \vdash P \wedge (Q \vee R)$

(3) $\{(X \wedge Y) \to Z, X \wedge W, W \to Y\} \vdash Z$

(4) $\{A \vee (B \vee G), A \vee (B \vee H), \neg A \wedge \neg B\} \vdash G \wedge H$

(5) $\{P \wedge (Q \wedge \neg R), R \vee T\} \vdash T \vee S$

(6) $\{((A \to D) \vee B) \vee C, \neg C, \neg B, A\} \vdash D$

(7) $\{A \vee \neg\neg B, \neg B \vee \neg C, C \vee A, \neg A\} \vdash D$

(8) $\{P \leftrightarrow (Q \leftrightarrow R), P, P \to R\} \vdash Q$

(9) $\{A \to (B \to C), A, B\} \vdash C$

(10) $\{(X \vee A) \to \neg Y, Y \vee (Z \wedge Q), X\} \vdash Z$

(11) $\{A \wedge (B \wedge C), A \wedge D, B \wedge E\} \vdash D \wedge (E \wedge C)$

(12) $\{A \wedge (B \vee \neg C), \neg B \wedge (C \vee E), E \to D\} \vdash D$

(13) $\{A \to B, B \to C, C \to A, B, \neg A\} \vdash D$

(14) $\{\neg A \wedge B, A \vee P, A \vee Q, B \to R\} \vdash P \wedge (Q \wedge R)$

Conditional Proofs

Part 2 Derive the following

(1) $\{X \leftrightarrow (A \wedge B), B \leftrightarrow Y, B \to A\} \vdash X \leftrightarrow Y$

(2) $\{\neg W \wedge \neg E, Q \leftrightarrow D\} \vdash (W \vee Q) \leftrightarrow (E \vee D)$

(3) $\{B \to \neg E, A \to \neg D, D \vee (E \vee R), (R \wedge A) \to C\} \vdash A \to (B \to C)$

(4) $\{(K \to K) \to R, (R \vee M) \to N \vdash N$

(5) $\{(A \wedge B) \leftrightarrow D, D \leftrightarrow (X \wedge Y), C \leftrightarrow Z\} \vdash A \wedge (B \wedge C) \leftrightarrow X \wedge (Y \wedge Z)$

Indirect Proofs

Part 3 Derive the following using indirect derivation. You may also have to use conditional derivation. *But you must not use derived rules or rules of replacement*

(1) $A \to (B \vee (C \vee D)) \vdash \neg[A \wedge (\neg B \wedge (\neg C \wedge \neg D))]$

(2) $P \to Q \vdash \neg Q \to \neg P$

(3) $P \wedge Q \vdash \neg(P \to \neg Q)$

(4) $(P \wedge Q) \to (R \vee S), \neg(R \vee S) \vdash \neg(P \wedge Q)$

(5) $\neg(P \to Q) \vdash P \wedge \neg Q$

(6) $S \leftrightarrow T, T \vee S \vdash \neg(T \to \neg S)$

(7) $Q \to R \vdash \neg Q \vee R$

Logical Equivalence

Part 4 Prove each of the following equivalences

(1) $(P \to R) \wedge (Q \to R) \dashv\vdash (P \vee Q) \to R$

(2) $(P \to (Q \vee R)) \dashv\vdash (P \to Q) \vee (P \to R)$

(3) $(P \leftrightarrow Q) \dashv\vdash \neg P \leftrightarrow \neg Q$

(4) $\neg(P \leftrightarrow Q) \dashv\vdash (P \leftrightarrow \neg Q)$

(5) $\vdash P \leftrightarrow (P \vee (Q \wedge P))$

Part 5 Prove each of the following tautologies

(1) $\vdash (B \to \neg B) \leftrightarrow \neg B$

(2) $\vdash (P \to [P \to Q]) \to (P \to Q)$

(3) $\vdash (P \vee \neg P) \wedge (Q \leftrightarrow Q)$

(4) $\vdash (P \wedge \neg P) \vee (Q \leftrightarrow Q)$

Derived Rules

Part 6 Provide proofs using both axioms and derived rules to show each of the following.

(1) $\{C \to (E \wedge G), \neg C \to G\} \vdash G$

(2) $\{(W \vee X) \vee (Y \vee Z),\ X \to Y,\ \neg Z\} \vdash W \vee Y$

Part 7 Prove the following theorems

(1) (Idempotence of $\vee$, Idem$\vee$): $\mathcal{A} \vee \mathcal{A} \vdash \mathcal{A}$

(2) (Weakening, WK): $\mathcal{A} \vdash \mathcal{B} \to \mathcal{A}$