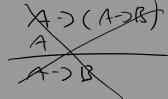
# Module 3: Derived Rules and Theorems Phil 150: Intro to Formal Logic

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## Rules of Replacement (Logical

- DeMorgan's (DeM)
- Contrapositive (Contra)
- · Commutation (Com): An B, Auß = BnA, B.A
- · Biconditional Exportation (B) AGフB = (ムラら)へ(ロラム)
- Idempotent (I)  $A \equiv A \wedge A$ ,  $A \wedge A$



#### Derived Rules of Inferences

- Argument by Cases
- ・Hypothetical Syllogism イラβ, βラ(トター)(いり)
- · Modus Tollen → B A → B A
- To sure to take a look at what exactly is meant by a proof of a derived rule.
- Also, remember which rules are derived, and which are primitive/basic.

#### Argument by Cases

- Also known as Constructive Dilemma.
- Intuitively: damned if you do; damned if you don't.
- 1 If you do it, you are damned.
  - 2 If you don't it you are damned.
  - 3 So...

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  - 3 So...
  - 4 You are damned.

### More on Argument by Cases



- Two forms one requires subproofs; one doesn't.
- · Example: A ∨ (B ∨ C) ⊢ (A ∨ B) ∨ C ← Association

## Theorems: Derivations without Premises

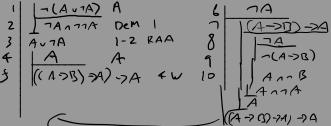
• A derivation with no premises shows all its conclusions to be logical truths, which are also called theorems.

 $\, \cdot \,$  More technically: A sentence P is a theorem if and only if

 $\{\} \vdash P.$   $\cdot \vdash ((A \to B) \to A) \to A \quad 2.$  3 4 5 6

#### Things to keep in mind

- · A theorem almost always starts with a subderivation.
- two main options:  $(RAA), (\rightarrow I)$
- (AC): advanced technique; requires the "excluded middle".
- $((A \rightarrow B) \rightarrow A) \rightarrow A$  using (RAA) and (AC).



(7(6,0), (1(->6)), 7 7(P,15)
Problems

Good for Argument by cases.

• 1 
$$(A \lor B) \land (B \to C) \vdash A \lor C$$

2 
$$(\neg H \lor M), \neg M \to \neg C \vdash (H \lor C) \to M$$

$$(A \land B) \lor (A \land C) \vdash A \land (B \lor C)$$

$$4 (S \wedge J) \vee (\neg S \wedge \neg J) \vdash (S \leftrightarrow J)$$

$$(5) K \to (F \lor C), J \to (C \lor D), \neg C \vdash \neg (F \lor D) \to \neg (K \lor J)$$

Theorems: Easier

2 
$$M \vee \neg (M \wedge N)$$

$$(3)$$
  $[H \rightarrow (O \rightarrow N)] \rightarrow [(H \land O) \rightarrow N]$ 

Theorems: Harder

$$(D \to B) \to ((D \to T) \to (D \to (B \land T)))$$

$$(K \to F) \to (\neg F \to \neg (K \land P))$$

$$(L \to (M \to N)) \to ((L \to M) \to (L \to N))$$

$$\bullet \quad \mathbf{1}\mathcal{A}((H \wedge F) \to C) \wedge \neg (H \to (F \to C))$$

$$2 \not (\neg (G \lor Q) \land (K \to G)) \land \neg (P \lor \neg K)$$

$$(3 + (A \leftrightarrow B) \leftrightarrow (\neg A \leftrightarrow B))$$