Teller 1-5: Natural Deduction

Phil 150: Intro to Formal Logic

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What is Natural Deduction?

- Things we want do with logic: validity, contradiction, logical truths
- Truth table can be too clumsy and tedious.
- Rules of Inferences
- These rules allow us to *deduce* or *derive* statements that necessarily follow.

Basic Structure of a Derivation

$X, Y \models Z$ iff $X, Y \vdash Z$ semantic

$X, Y \models Z$

vs

- The meaning of $X, Y \vdash Z$ syntactical
- Basic layout of a derivation
- Premises, justification, lines.

Conditional Elimination ($\rightarrow$ E)

Scope lines

$A, A \rightarrow B \vdash B$

1	$A \rightarrow B$	given	
2	A		
3	B	$1,2 \rightarrow E$	

Justification

- Also known as Modus Ponens (MP)
- $X \rightarrow Y$
  X
  Y

  Assumptions / Premises
- $X \rightarrow Y, X \vdash Y$
- Example: Someone being in Durham implies that this person being in North Carolina. You are in Durham. *Ergo*....
- Truth table should corroborate.

$(((A \wedge B) \vee C) \rightarrow B) \rightarrow F \quad \vdash F$

$((A \wedge B) \vee C) \rightarrow B)$

Disjunction Elimination ($\vee$ E)

$((A \wedge B) \to C)$

Z

- $$\begin{array}{|l} X \vee Y \\ \neg X \\ \hline Y \end{array}$$
- Intuitively, why is this right?

Disjunction Introduction ($\vee$ I)

$(\cup \mathrm{I})$

- $$\begin{array}{|l} X \\ \hline (X \vee Y) \end{array} \rightarrow \begin{array}{|l} X \\ \hline Y \vee X \end{array}$$
- This looks too good to be true. Intuitively, why is this right?
- X or Y can be a compound sentence.

$X = A \quad , \quad Y = (A \wedge C \wedge B \wedge E)$

$X = A \vee \overline{(A \wedge \neg A)} \leftarrow Y$

Examples

Want: C

1	$A \to B$	Given
2	$B \to C$	Given
3	A	
4	B	1, 3 $\to$E
5	C	2, 4 $\to$E

- $A \to B, B \to C, A \vdash C$
- $A \to \neg B, B \vee C, A \vdash C \vee D$
- $(M \vee \neg T) \to (A \vee J), \neg A, B \vee M, \neg A \to \neg B \vdash J \vee D$

Given want $C \vee D$

1	$A \to \neg B$	
2	$B \vee C$	
3	A	
4	$\neg B$	1, 3 $\to$E
5	C	2, 4 $\vee$E
6.	$C \vee D$	5 $\vee$E

CW.2

1. $A \vdash (A \vee B) \vee C$
2. $((P \vee Q) \to R), P \vdash R$
3. $\neg\neg A, A \to \neg C \vdash \neg C$
4. $(A \wedge B) \to C, \neg(\neg A \vee \neg C) \vdash C$
5. $B \to M, \neg M \vdash \neg B$ RAA
6. $C, C \to (A \wedge B) \vdash A \wedge (B \vee C)$
7. $\neg A \vee B, A \vdash B$

Subderivation

1 | $A \wedge B$

2 | | C

3 | | A 1 $\wedge$E

| $(A \wedge B) \wedge C$ X

← accessible

- Idea: hypothetical thinking and assuming something for the sake of argument.
- accessibility, scope lines, discharged assumptions

Intuitive Example

If WWIII occurs, we are all screwed. Here's why: surely you agree that if WWIII were to occur, nuclear weapons would be used. Also, you believe that if nuclear weapons are used, then we are all royally screwed. Now, imagine this: WWIII actually happens. What follows? Nuclear weapons are used, which leads to the scenario of us being screwed. In conclusion, if WWIII occurs, we are all screwed.

Logical Structure of the Argument

Step 1

Step one: Non-hypothetical assumptions:
(1) If WWIII occurs, nuclear weapons will be used.
(2) If nuclear weapons are used, we are all royally screwed.

Logical Structure of the Argument

Step 2

Step one: Non-hypothetical assumptions:
(1) If WWIII occurs, nuclear weapons will be used.
(2) If nuclear weapons are used, we are all royally screwed.

Step two: Hypothetical assumption introduced for the sake of argument:
(3) WWIII occurs.
(4) Nuclear weapons will be used. (MP), 1, 3
(5) We are all royally screwed. (MP), 2, 4

Logical Structure of the Argument

Step 3

Step one: Non-hypothetical assumptions:
(1) If WWIII occurs, nuclear weapons will be used.
(2) If nuclear weapons are used, we are all royally screwed.

Step two: Hypothetical assumption introduced for the sake of argument:
(3) WWIII occurs.
(4) Nuclear weapons will be used. (MP), 1, 3
(5) We are all royally screwed. (MP), 2, 4

Step three: Hypothetical scenario is discharged. But we learn that:
(6) If WWIII occurs, we are all screwed.

Subderivation example: Conditional Introduction ($\rightarrow I$)

- | n. P
  | ⋮
  | m. Q
  P $\rightarrow$ Q

- example: $P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$

1.	$P \rightarrow Q$	Given
2.	$Q \rightarrow R$	Given
3.	P	A
4.	Q	1,3 $\rightarrow E$
5.	R	2, 4 $\rightarrow E$
6.	R	2, 4 $\rightarrow E$ X
6.	$P \rightarrow R$	3-5 $\rightarrow E$

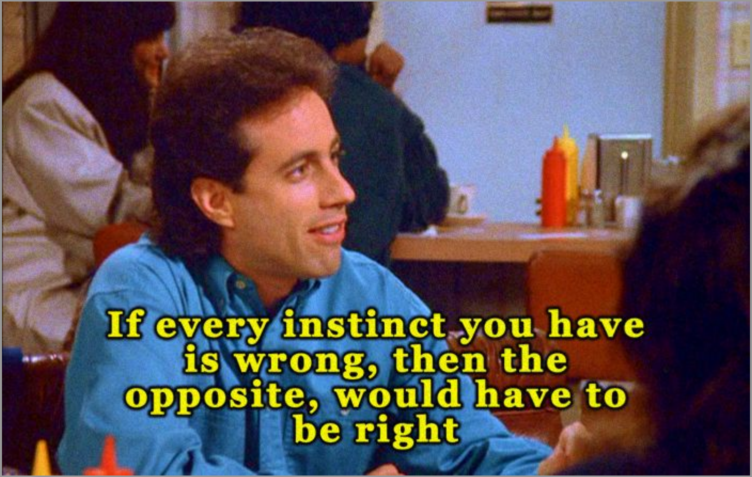
Reiteration (R)

- Simply: you are allowed reiterate anything that's accessible to you.
- Example: $A, B \vdash A \rightarrow B$

Negation Introduction ($\neg$ I)

- Also known as Reductio Ad Absurdum (RAA), that is, reduced to absurdity.
- If you make an assumption that leads to a contradiction, that must mean its logical opposite must be true.

The Costanza Principle



Structure of (RAA)

- Start a derivation with the logical opposite of what you want to prove (usually a negation), and try to find a contradiction ($X \wedge \neg X$)
- Example: $^{1)}A \to B, ^{2)}\neg B \vdash \neg A$

3.	A	Assumption
4.	B	1, 3 $\to$E
5.	$\neg B$	2 R
6.	$\neg A$	3 – 5 RAA ($\neg$I)

More Examples

Hypothetical syllogism

- $A \to B, B \to C, C \to D \vdash A \to D$

material implication

- $N \vee P \vdash \neg N \to P$

weakening

- $B \vdash A \to B$

Want: $A \to D$

1	$A \to B$	Given
2	$B \to C$	
3.	$C \to D$	
4	A	Assumption
5.	B	1, 4 $\to$E
6.	C	2, 5 $\to$E
7	D	3, 6 $\to$E
8	$A \to D$	4-7 $\to$I

CW3

1. $C \vdash A \to (B \to C)$
2. $\neg N \to S, S \to C \to N \vdash N$
3. $P \vdash ((P \to Q) \to Q)$