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Part I Basic Concepts

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Chapter 1

What Is Logic?

1.1 Introduction

Logic is a part of the study of human reason, the ability we have to think abstractly, solve problems, explain the things that we know, and infer new knowledge on the basis of evidence. Traditionally, logic has focused on the last of these items, the ability to make inferences on the basis of evidence. This is an activity you engage in every day. Consider, for instance, the game of Clue. (For those of you who have never played, Clue is a murder mystery game where players have to decide who committed the murder, what weapon they used, and where they were.) A player in the game might decide that the murder weapon was the candlestick by ruling out the other weapons in the game: the knife, the revolver, the rope, the lead pipe, and the wrench. This evidence lets the player know something they did not know previously, namely, the identity of the murderer.

In logic, we use the word "argument" to refer to the attempt to show that certain evidence supports a conclusion. This is very different from the sort of argument you might have with your family, which could involve screaming and throwing things. We are going to use the word "argument" a lot in this book, so you need to get used to thinking of it as a name for a rational process, and not a word that describes what happens when people disagree.

A logical argument is structured to give someone a reason to believe some conclusion. Here is the argument from the first paragraph written out in a way that shows its structure.

P_1: In a game of Clue, the possible murder weapons are the knife, the candlestick, the revolver, the rope, the lead pipe, and the wrench.
P_2: The murder weapon was not the knife.
P_3: The murder weapon was also not the revolver, the rope, the lead pipe, or the wrench.

C: Therefore, the murder weapon was the candlestick.

In the argument above, statements P_1–P_3 are the evidence. We call these the *premises*. The word "therefore" indicates that the final statement, marked with a C, is the *conclusion* of the argument. If you believe the premises, then the argument provides you with a reason to believe the conclusion. You might use reasoning like this purely in your own head, without talking with anyone else. You might wonder what the murder weapon is, and then mentally rule out each item, leaving only the candlestick. On the other hand, you might use reasoning like this while talking to someone else, to convince them that the murder weapon is the candlestick. (Perhaps you are playing as a team.) Either way the structure of the reasoning is the same.

We can define LOGIC then more precisely as the part of the study of reasoning that focuses on argument. In more casual situations, we will follow ordinary practice and use the word "logic" to either refer to the business of studying human reason or the thing being studied, that is, human reasoning itself. While logic focuses on argument, other disciplines, like decision theory and cognitive science, deal with other aspects of human reasoning, like abstract thinking and problem solving more generally. Logic, as the study of argument, has been pursued for thousands of years by people from civilizations all over the globe. The initial motivation for studying logic is generally practical. Given that we use arguments and make inferences all the time, it only makes sense that we would want to learn to do these things better. Once people begin to study logic, however, they quickly realize that it is a fascinating topic in its own right. Thus the study of logic quickly moves from being a practical business to a theoretical endeavor people pursue for its own sake.

In order to study reasoning, we have to apply our ability to reason to our reason itself. This reasoning about reasoning is called METAREASONING. It is part of a more general set of processes called METACOGNITION , which is just any kind of thinking about thinking. When we are pursing logic as a practical discipline, one important part of metacognition will be awareness of your own thinking, especially its weakness and biases, as it is occurring. More theoretical metacognition will be about attempting to understand the structure of thought itself.

Whether we are pursuing logical for practical or theoretical reasons, our focus is on argument. The key to studying argument is to set aside the subject being argued about and to focus on the *way* it is argued *for*. The section opened with an example that was about a game of Clue. However, the kind of reasoning used in that example was just the process of elimination. Process of elimination can be applied to any subject. Suppose a group of friends is deciding which restaurant to eat at, and there are six restaurants in town. If you could rule out five of the possibilities, you would use an argument just like the one above to decide where to eat. Because logic sets aside what an argument is about, and just looks at how it works rationally, logic is said to have CONTENT NEUTRALITY. If we say an argument is good, then the same kind of argument applied to a different topic will also be good. If we say an argument is good for solving murders, we will also say that the same kind of argument is good for deciding where to eat, what kind of disease is destroying your crops, or who to vote for.

When logic is studied for theoretical reasons, it typically is pursued as FORMAL LOGIC. In formal logic we get content neutrality by replacing parts of the argument we are studying with abstract symbols. For instance, we could turn the argument above into a formal argument like this:

P_1: There are six possibilities: A, B, C, D, E, and F.
P_2: A is false.
P_3: B, D, E, and F are also false.

C: $\therefore$ The correct answer is C.

Here we have replaced the concrete possibilities in the first argument with abstract letters that could stand for anything. We have also replaced the English word "therefore" with the symbol "$\therefore$," which means therefore. This lets us see the formal structure of the argument, which is why it works in any domain you can think of. In fact, we can think of formal logic as the method for studying argument that uses abstract notation to identify the formal structure of argument. Formal logic is closely allied with mathematics, and studying formal logic often has the sort of puzzle-solving character one associates with mathematics.

When logic is studied for practical reasons, it is typically called critical thinking. It is the study of reasoning with the goal of improving our reasoning in the real world. Sometimes people use the term "critical thinking" to simply refer to any time one is reasoning well. However, we will be using the term CRITICAL THINKING more narrowly to refer to the use of metareasoning to improve our reasoning in practical situations. Critical thinking is generally pursued as INFORMAL LOGIC, rather than formal logic. This means that we will keep arguments in ordinary language and draw extensively on your knowledge of the world to evaluate them. In contrast to the clarity and rigor of formal logic, informal logic is suffused with ambiguity and vagueness. There are problems with multiple correct answers, or where reasonable people can disagree with what the correct answer is. This is because you will be dealing with reasoning in the real world, which is messy.

You can think of the difference between formal logic and informal logic as the difference between a laboratory science and a field science. If you are studying, say, mice, you could discover things about them by running experiments in a lab, or you can go out into the field where mice live and observe them in their natural habitat. Informal logic is the field science for arguments: you go out and study arguments in their natural habitats, like newspapers, courtrooms, and scientific journal articles. Like studying mice scurrying around a meadow, the process takes patience, and often doesn't yield clear answers but it lets you see how things work in the real world. Formal logic takes arguments out of their natural habitat and performs experiments on them to see what they are capable of. The arguments here are like lab mice. They are pumped full of chemicals and asked to perform strange tasks, as it were. They live lives very different than their wild cousins. Some of the arguments will wind up looking like the "ob/ob mouse", a genetically engineered obese mouse scientists use to study type II diabetes (See Figure 1.1). These arguments will be huge, awkward, and completely unable to survive in the wild. But they will tell us a lot about the limits of logic as a process.

Our main goal in studying arguments is to separate the good ones from the bad ones. The argument about Clue we saw earlier is a good one, based on the process of elimination. It is good because it leads to truth. If I've got all the premises right, the conclusion will also be right. The textbook *Logic: Techniques of Formal Reasoning* (Kalish et al., 1980) had a nice way of capturing the meaning of logic: "logic is the study of virtue in argument." This textbook will accept this definition, with the caveat that an argument is virtuous if it helps us get to the truth.

Figure 1.1: The ob/ob mouse (left), a laboratory mouse which has been genetically engineered to be obese, and an ordinary mouse (right). Formal logic, which takes arguments out of their natural environment, often winds up studying arguments that look like the ob/ob mouse. They are huge, awkward, and unable to survive in the wild, but they tell us a lot about the limits of logic as a process. Photo from Wikimedia Commons (2006).

Logic is different from RHETORIC, which is the study of effective persuasion. Rhetoric does not look at virtue in argument. It only looks at the power of arguments, regardless of whether they lead to truth. An advertisement might convince you to buy a new truck by having a gravelly voiced announcer tell you it is "ram tough" and showing you a picture of the truck on top of a mountain, where it no doubt actually had to be airlifted. This sort of persuasion is often more effective at getting people to believe things than logical argument, but it has nothing to do with whether the truck is really the right thing to buy. In this textbook we will only be interested in rhetoric to the extent that we need to learn to defend ourselves against the misleading rhetoric of others. This will not, however, be anything close to a full treatment of the study of rhetoric.

1.2 Statement, Argument, Premise, Conclusion

So far we have defined logic as the study of argument and outlined its relationship to related fields. To go any further, we are going to need a more precise definition of what exactly an argument is. We have said that an argument is not simply two people disagreeing; it is an attempt to prove something using evidence. More specifically, an argument is composed of statements. In logic, we define a STATEMENT as a unit of language that can be true or false. To put it another way, it is some combination of words or symbols that have been put together in a way that lets someone agree or disagree with it. All of the items below are statements.

(a) *Tyrannosaurus rex* went extinct 65 million years ago.

(b) *Tyrannosaurus rex* went extinct last week.

(c) On this exact spot, 100 million years ago, a *T. rex* laid a clutch of eggs.

(d) George W. Bush is the king of Jupiter.

(e) Murder is wrong.

(f) Abortion is murder.

(g) Abortion is a woman's right.

(h) Lady Gaga is pretty.

(i) Murder is the unjustified killing of a person.

(j) The slithy toves did gyre and gimble in the wabe.

(k) The murder of logician Richard Montague was never solved.

Because a statement is something that can be true *or* false, statements include truths like (a) and falsehoods like (b). A statement can also be something that that must either be true or false, but we don't know which, like (c). A statement can be something that is completely silly, like (d). Statements in logic include statements about morality, like (e), and things that in other contexts

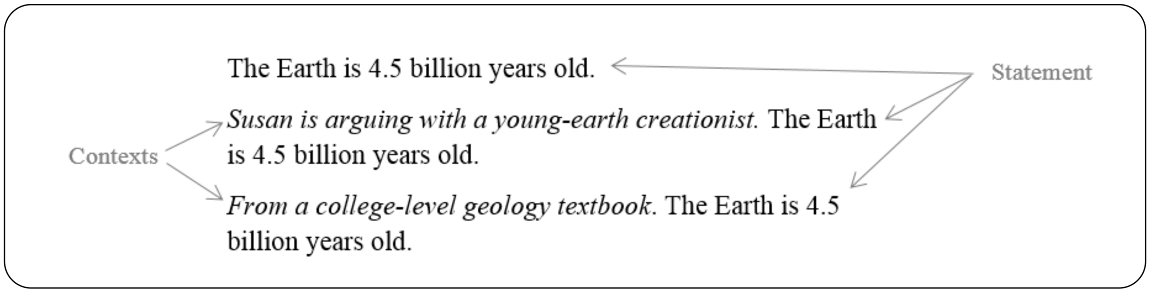


Table 1.1: A statement in different contexts, or no context.

might be called "opinions," like (f) and (g). People disagree strongly about whether (f) or (g)are true, but it is definitely possible for one of them to be true. The same is true about (h), although it is a less important issue than (f) and (g). A statement in logic can also simply give a definition, like (i). This sort of statement announces that we plan to use words a certain way, which is different from statements that describe the world, like (a), or statements about morality, like (f). Statements can include nonsense words like (j), because we don't really need to know what the statement is about to see that it is the sort of thing that can be true or false. All of this relates back to the content neutrality of logic. The statements we study can be about dinosaurs, abortion, Lady Gaga, and even the history of logic itself, as in statement (k), which is true.

We are treating statements primarily as units of language or strings of symbols, and most of the time the statements you will be working with will just be words printed on a page. However, it is important to remember that statements are also what philosophers call "speech acts." They are actions people take when they speak (or write). If someone makes a statement they are typically telling other people that they believe the statement to be true, and will back it up with evidence if asked to. When people make statements, they always do it in a context—they make statements at a place and a time with an audience. Often the context statements are made in will be important for us, so when we give examples, statements, or arguments we will sometimes include a description of the context. When we do that, we will give the context in *italics*. See Table 1.1 for examples. For the most part, the context for a statement or argument will be important in the chapters on critical thinking, when we are pursing the study of logic for practical reasons. In the chapters on formal logic, context is less important, and we will be more likely to skip it.

"Statements' in this text does *not* include questions, commands, exclamations, or sentence fragments. Someone who asks a *question* like "Does the grass need to be mowed?" is typically not claiming that anything is true or false. (Sometimes people make statements and disguise them as questions, for instance if they were trying to hint that the lawn needs to be mowed. These are generally called rhetorical questions, and we will leave the study of them to the rhetoricians.) Generally, *questions* will not count as statements, but *answers* will. "What is this course about?" is not a statement. "No one knows what this course is about," is a statement.

For the same reason *commands* do not count as statements for us. If someone bellows "Mow the grass, now!" they are not saying whether the grass has been mowed or not. You might infer that they believe the lawn has not been mowed, but then again maybe they think the lawn is fine and just want to see you exercise. Note, however, that commands are not always phrased as imperatives. "You will respect my authority" *is* either true or false—either you will or you will not—and so it counts as a statement in the logical sense.

An exclamation like "Ouch!" is also neither true nor false. On its own, it is not a statement. We will treat "Ouch, I hurt my toe!" as meaning the same thing as "I hurt my toe." The "ouch" does not add anything that could be true or false.

Finally, a lot of possible strings of words will fail to qualify as statements simply because they don't form a complete sentence. In your composition classes, these were probably referred to as sentence fragments. This includes strings of words that are parts of sentences, such as noun phrases like "The tall man with the hat" and verb phrases, like "ran down the hall." Phrases like these are missing something they need to make a claim about the world. The class of sentence fragments also includes completely random combinations of words, like "The up if blender route," which don't even have the form of a statement about the world.

Other logic textbooks describe the components of argument as "propositions," or "assertions," and we will use these terms sometimes as well. There is actually a great deal of disagreement about what the differences between all of these things are and which term is best used to describe parts of arguments. However, none of that makes a difference for this textbook. We could have used any of the other terms in this text, and it wouldn't change anything. Some textbooks will also use the term "sentence" here. We will not use the word "sentence" to mean the same thing as "statement." Instead, we will use "sentence" the way it is used in ordinary grammar, to refer generally to statements, questions, and commands.

We use statements to build arguments. An ARGUMENT is a connected series of statements designed to convince an audience of another statement. Here an audience might be a literal audience sitting in front of you at some public speaking engagement. Or it might be the readers of a book or article. The audience might even be yourself as you reason your way through a problem. Let's start with an example of an argument given to an external audience. This passage is from an essay by Peter Singer called "Famine, Affluence, and Morality" in which he tries to convince people in rich nations that they need to do more to help people in poor nations who are experiencing famine.

> *A contemporary philosopher writing in an academic journal* If it is in our power to prevent something bad from happening, without thereby sacrificing anything of comparable moral importance, we ought, morally, to do so. Famine is something bad, and it can be prevented without sacrificing anything of comparable moral importance. So, we ought to prevent famine. (Singer, 1972)

Singer wants his readers to work to prevent famine. This is represented by the last statement of the passage, "we ought to prevent famine," which is called the conclusion of the passage. The CONCLUSION of an argument is the statement that the argument is trying to convince the audience of. The statements that do the convincing are called the PREMISES. In this case, the argument has three premises: (1) "If it is in our power to prevent something bad from happening, without thereby sacrificing anything of comparable moral importance, we ought, morally, to do

Premise Indicators:	because, as, for, since, given that, for the reason that
Conclusion Indicators:	therefore, thus, hence, so, consequently, it follows that, in conclusion, as a result, then, must, accordingly, this implies that, this entails that, we may infer that

Table 1.3: Premise and Conclusion Indicators.

so"; (2) "Famine is something bad"; and (3) "it can be prevented without sacrificing anything of comparable moral importance."

Now let's look at an example of internal reasoning.

> *Jack arrives at the track, in bad weather.* There is no one here. I guess the race is not happening.

In the passage above, the words in *italics* explain the context for the reasoning, and the words in regular type represent what Jack is actually thinking to himself. This passage again has a premise and a conclusion. The premise is that no one is at the track, and the conclusion is that the race was canceled. The context gives another reason why Jack might believe the race has been canceled, the weather is bad. You could view this as another premise–it is very likely a reason Jack has come to believe that the race is canceled. In general, when you are looking at people's internal reasoning, it is often hard to determine what is actually working as a premise and what is just working in the background of their unconscious.

When people give arguments to each other, they typically use words like "therefore" and "because." These are meant to signal to the audience that what is coming is either a premise or a conclusion in an argument. Words and phrases like "because" signal that a premise is coming, so we call these PREMISE INDICATORS. Similarly, words and phrases like "therefore" signal a conclusion and are called CONCLUSION INDICATORS. The argument from Peter Singer (on page 8) uses the conclusion indicator word, "so." Table 1.3 is an incomplete list of indicator words and phrases in English.

The two passages we have looked at in this section so far have been simply presented as quotations. But often it is extremely useful to rewrite arguments in a way that makes their logical structure clear. One way to do this is to use something called "canonical form." An argument written in CANONICAL FORM has each premise numbered and written on a separate line. Indicator words and other unnecessary material should be removed from the premises. Although you can shorten the premises and conclusion, you need to be sure to keep them all complete sentences with the same meaning, so that they can be true or false. The argument from Peter Singer, above, looks like this in canonical form:

P_1: If we can stop something bad from happening, without sacrificing anything of comparable moral importance, we ought to do so.

P_2: Famine is something bad.
P_3: Famine can be prevented without sacrificing anything of comparable moral importance.

C: We ought to prevent famine.

Each statement has been written on its own line and given a number. The statements have been paraphrased slightly, for brevity, and the indicator word "so" has been removed. Also notice that the "it" in the third premise has been replaced by the word "famine," so that statements reads naturally on its own.

Similarly, we can rewrite the argument Jack gives at the racetrack, on page 9, like this:

P: There is no one at the race track.

C: The race is not happening.

Notice that we did not include anything from the part of the passage in italics. The italics represent the context, not the argument itself. Also, notice that the "I guess" has been removed. When we write things out in canonical form, we write the content of the statements, ignore information about the speaker's mental state, like "I believe" or "I guess."

One of the first things you have to learn to do in logic is to identify arguments and rewrite them in canonical form. This is a foundational skill for everything else we will be doing in this text, so we are going to run through a few examples now, and there will be more in the exercises. The passage below is paraphrased from the ancient Greek philosopher Aristotle.

> *An ancient philosopher, writing for his students* Again, our observations of the stars make it evident that the earth is round. For quite a small change of position to south or north causes a manifest alteration in the stars which are overhead. (Aristotle, c.350 BCE/1984c, 298a2-10)

The first thing we need to do to put this argument in canonical form is to identify the conclusion. The indicator words are the best way to do this. The phrase "make it evident that" is a conclusion indicator phrase. He is saying that everything else is *evidence* for what follows. So we know that the conclusion is that the earth is round. "For" is a premise indicator word—it is sort of a weaker version of "because." Thus the premise is that the stars in the sky change if you move north or south. In canonical form, Aristotle's argument that the earth is round looks like this.

P: There are different stars overhead in the northern and southern parts of the earth.

C: The earth is spherical in shape.

That one is fairly simple, because it just has one premise. Here's another example of an argument, this time from the book of Ecclesiastes in the Bible. The speaker in this part of the bible is generally referred to as The Preacher, or in Hebrew, Koheleth. In this verse, Koheleth uses both a premise indicator and a conclusion indicator to let you know he is giving reasons for enjoying life.

> *The words of the Preacher, son of David, King of Jerusalem* There is something else meaningless that occurs on earth: the righteous who get what the wicked deserve, and the wicked who get what the righteous deserve. . . . So I commend the enjoyment of life, because there is nothing better for a person under the sun than to eat and drink and be glad. (Ecclesiastes 8:14-15, New International Version)

Koheleth begins by pointing out that good things happen to bad people and bad things happen to good people. This is his first premise. (Most Bible teachers provide some context here by pointing that that the ways of God are mysterious and this is an important theme in Ecclesiastes.) Then Koheleth gives his conclusion, that we should enjoy life, which he marks with the word "so." Finally he gives an extra premise, marked with a "because," that there is nothing better for a person than to eat and drink and be glad. In canonical form, the argument would look like this.

P_1: Good things happen to bad people and bad things happen to good people.
P_2: There is nothing better for people than to eat, to drink and to enjoy life.

C: You should enjoy life.

Notice that in the original passages, Aristotle put the conclusion in the first sentence, while Koheleth put it in the middle of the passage, between two premises. In ordinary English, people can put the conclusion of their argument where ever they want. However, when we write the argument in canonical form, the conclusion goes last.

Unfortunately, indicator words aren't a perfect guide to when people are giving an argument. Look at this passage from a newspaper:

> *From the general news section of a national newspaper* The new budget underscores the consistent and paramount importance of tax cuts in the Bush philosophy. His first term cuts affected more money than any other initiative undertaken in his presidency, including the costs thus far of the war in Iraq. All told, including tax incentives for health care programs and the extension of other tax breaks that are likely to be taken up by Congress, the White House budget calls for nearly $300 billion in tax cuts over the next five years, and $1.5 trillion over the next 10 years. (Toner, 2006)

Although there are no indicator words, this is in fact an argument. The writer wants you to believe something about George Bush: tax cuts are his number one priority. The next two sentences in the paragraph give you reasons to believe this. You can write the argument in canonical form like this.

P_1: Bush's first term cuts affected more money than any other initiative undertaken in his presidency, including the costs thus far of the war in Iraq.
P_2: The White House budget calls for nearly $300 billion in tax cuts over the next five years, and $1.5 trillion over the next 10 years.

C: Tax cuts are of consistent and paramount importance of in the Bush philosophy.

The ultimate test of whether something is an argument is simply whether some of the sentences provide reason to believe another one of the sentences. If some sentences support others, you are looking at an argument. The speakers in these two cases use indicator phrases to let you know they are trying to give an argument.

A final bit of terminology for this section. An INFERENCE is the act of coming to believe a conclusion on the basis of some set of premises. When Jack in the example above saw that no one was at the track, and came to believe that the race was not on, he was making an inference. We also use the term inference to refer to the connection between the premises and the conclusion of an argument. If your mind moves from premises to conclusion, you make an inference, and the premises and the conclusion are said to be linked by an inference. In that way inferences are like argument glue: they hold the premises and conclusion together.

1.3 Arguments and Nonarguments

We just saw that arguments are made of statements. However, there are lots of other things you can do with statements. Part of learning what an argument is involves learning what an argument is not, so in this section and the next we are going to look at some other things you can do with statements besides make arguments.

The list below of kinds of nonarguments is not meant to be exhaustive: there are all sorts of things you can do with statements that are not discussed. Nor are the items on this list meant to be exclusive. One passage may function as both, for instance, a narrative and a statement of belief. Right now we are looking at real world reasoning, so you should expect a lot of ambiguity and imperfection. If your class is continuing on into the critical thinking portions of this textbook, you will quickly get used to this.

Simple Statements of Belief

An argument is an attempt to persuade an audience to believe something, using reasons. Often, though, when people try to persuade others to believe something, they skip the reasons, and give a SIMPLE STATEMENT OF BELIEF. This is a kind of nonargumentative passage where the speaker simply asserts what they believe without giving reasons. Sometimes simple statements of belief are prefaced with the words "I believe," and sometimes they are not. A simple statements of belief can be a profoundly inspiring way to change people's hearts and minds. Consider this passage from Dr. Martin Luther King's Nobel acceptance speech.

> I believe that even amid today's mortar bursts and whining bullets, there is still hope for a brighter tomorrow. I believe that wounded justice, lying prostrate on the blood-flowing streets of our nations, can be lifted from this dust of shame to reign supreme among the children of men. I have the audacity to believe that peoples everywhere can have three meals a day for their bodies, education and culture for their minds, and dignity, equality and freedom for their spirits. (King, 1964)

This actually is a part of a longer passage that consists almost entirely of statements that begin with some variation of "I believe."It is incredibly powerful oration, because the audience, feeling the power of King's beliefs, comes to share in those beliefs. The language King uses to describe how he believes is important, too. He says his belief in freedom and equality requires audacity, making the audience feel his courage and want to share in this courage by believing the same things.

These statements are moving, but they do not form an argument. None of these statements provide evidence for any of the other statements. In fact, they all say roughly the same thing, that good will triumph over evil. So the study of this kind of speech belongs to the discipline of rhetoric, not of logic.

Expository Passages

Perhaps the most basic use of a statement is to convey information. Often if we have a lot of information to convey, we will sometimes organize our statements around a theme or a topic. Information organized in this fashion can often appear like an argument, because all of the statements in the passage relate back to some central statement. However, unless the other statements are given as reasons to believe the central statement, the passage you are looking at is not an argument. Consider this passage:

> *From a college psychology textbook.* Eysenck advocated three major behavior techniques that have been used successfully to treat a variety of phobias. These techniques are modeling, flooding, and systematic desensitization. In **modeling** phobic people watch nonphobics cope successfully with dreaded objects or situations.In **flooding** clients are exposed to dreaded objects or situations for prolonged periods of time in order to extinguish their fear. In contrast to flooding, **systematic desensitization** involves gradual, client-controlled exposure to the anxiety eliciting object or situation. (Adapted from Ryckman 2007)

We call this kind of passage an expository passage. In an EXPOSITORY PASSAGE , statements are organized around a central theme or topic statement. The topic statement might look like a conclusion, but the other statements are not meant to be evidence for the topic statement. Instead, they elaborate on the topic statement by providing more details or giving examples. In the passage above, the topic statement is "Eysenck advocated three major behavioral techniques . . . ." The statements describing these techniques elaborate on the topic statement, but they are not evidence for it. Although the audience may not have known this fact about Eysenk before reading the passage, they will typically accept the truth of this statement instantly, based on the textbook's authority. Subsequent statements in the passage merely provide detail.

The name "expository passage" might make it sound like we are only talking about formal writing, but really people give information organized around topic sentences all the time. Consider this:

> *Your friend Bea is on the phone:* Kelly is driving me insane. First she told Michael that I was out when I was right there in my room, and then she ate the leftover food I was keeping for lunch today.

In this passage, "Kelly is driving me insane" acts as a topic sentence, and the other two statements provide details illustrating the topic sentence. This doesn't really count as an argument, though, because Bea probably doesn't need to convince you that Kelly is driving her insane. You take your friend's word for it as soon as she says it. Most human communication is actually like this. We assume people are telling the truth until we are given reason to doubt them. Societies that lack this basic level of trust can quickly break down because no one can cooperate enough to get basic tasks accomplished.

Deciding whether a passage is an argument or an expository passage is complicated by the fact that sometimes people argue by example:

> **Steve:** Kenyans are better distance runners than everyone else.
>
> **Monica:** Oh come on, that sounds like an exaggeration of a stereotype that isn't even true.
>
> **Steve:** What about Dennis Kimetto, the Kenyan who set the world record for running the marathon? And you know who the previous record holder was: Emmanuel Mutai, also Kenyan.

Here Steve has made a general statement about all Kenyans. Monica clearly doubts this claim, so Steve backs it up with some examples that seem to match his generalization. This isn't a very strong way to argue: moving from two examples to statement about all Kenyans is probably going to be a kind of bad argument known as a hasty generalization. (This mistake is covered in the complete version of this text in the chapter on induction) The point here however, is that Steve is just offering it as an argument.

The key to telling the difference between expository passages and arguments by example is whether there is a conclusion that they audience needs to be convinced of. In the passage from the psychology textbook, "Eysenck advocated three major behavioral techniques" doesn't really work as a conclusion for an argument. The audience, students in an introductory psychology course, aren't likely to challenge this assertion, the way Monica challenges Steve's overgeneralizing claim.

Context is very important here, too. The Internet is a place where people argue in the ordinary sense of exchanging angry words and insults. In that context, people are likely to actually give some arguments in the logical sense of giving reasons to believe a conclusion.

Narratives

Statements can also be organized into descriptions of events and actions, as in this snippet from book V of *Harry Potter*.

> But she [Hermione] broke off; the morning post was arriving and, as usual, the *Daily Prophet* was soaring toward her in the beak of a screech owl, which landed perilously close to the sugar bowl and held out a leg. Hermione pushed a Knut into its leather pouch, took the newspaper, and scanned the front page critically as the owl took off again. (Rowling, 2003)

We will use the term NARRATIVE loosely to refer to any passage that gives a sequence of events or actions. A narrative can be fictional or nonfictional. It can be told in regular temporal sequence or it can jump around, forcing the audience to try to reconstruct a temporal sequence. A narrative can describe a short sequence of actions, like Hermione taking a newspaper from an owl, or a grand sweep of events, like this passage about the rise and fall of an empire in the ancient near east:

> The Guti were finally expelled from Mesopotamia by the Sumerians of Erech (*c.* 2100), but it was left to the kings of Ur's famous third dynasty to re-establish the Sargonoid frontiers and write the final chapter of the Sumerian History. The dynasty lasted through the twenty first century at the close of which the armies of Ur were overthrown by the Elamites and Amorites (McEvedy and Woodcock, 1967).

This passage does not feature individual people performing specific actions, but it is still united by character and action. Instead of Hermione at breakfast, we have the Sumarians in Mesopotamia. Instead of retrieving a message from an owl, the conquer the Guti, but then are conquered by the Elamites and Amorites. The important thing is that the statements in a narrative are not related as premises and conclusion. Instead, they are all events which are united common characters acting in specific times and places.

1.4 Arguments and Explanations

Explanations are are not arguments, but they they share important characteristics with arguments, so we should devote a separate section to them. Both explanations and arguments are parts of reasoning, because both feature statements that act as reasons for other statements. The difference is that explanations are not used to convince an audience of a conclusion.

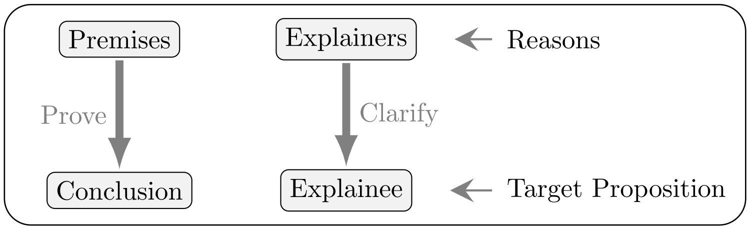


Figure 1.2: Arguments vs. Explanations.

Let's start with workplace example. Suppose you see your co-worker, Henry, removing a computer from his office. You think to yourself "Gosh, is he stealing from work?" But when you ask him about it later, Henry says, "I took the computer because I believed that it was scheduled for repair." Henry's statement looks like an argument. It has the indicator word "because" in it, which would mean that the statement "I believed it was scheduled for repairs" would be a premise. If it was, we could put the argument in canonical form, like this:

P: I believed the computer was scheduled for repair

C: I took the computer from the office.

But this would be awfully weird as an argument. If it were an argument, it would be trying to convince us of the conclusion, that Henry took the computer from the office. But you don't need to be convinced of this. You already know it—that's why you were talking to him in the first place.

Henry is giving reasons here, but they aren't reasons that try to *prove* something. They are reasons that *explain* something. When you explain something with reasons, you increase your understanding of the world by placing something you already know in a new context. You already knew that Henry took the computer, but now you know *why* Henry took the computer, and can see that his action was completely innocent (if his story checks out).

Both arguments and explanations both involve giving reasons, but the reasons function differently in each case. An EXPLANATION is defined as a kind of reasoning where reasons are used to provide a greater understanding of something that is already known.

Because both arguments and explanations are parts of reasoning, we will use parallel language to describe them. In the case of an argument, we called the reasons "premises." In the case of an explanation, we will call them EXPLAINERS. Instead of a "conclusion," we say that the explanation has an EXPLAINEE. We can use the generic term REASONS to refer to either premises or explainers and the generic term TARGET PROPOSITION to refer to either conclusions or explainees. Figure 1.2 shows this relationship.

We can put explanations in canonical form, just like arguments, but to distinguish the two, we will simply number the statements, rather than writing Ps and Cs, and we will put an E next to the line that separates explainers and exaplainee, like this:

1. Henry believed the computer was scheduled for repair
   ——— E
2. Henry took the computer from the office.

Often the same piece of reasoning can work as either an argument or an explanation, depending on the situation where it is used. Consider this short dialogue

Monica visits Steve's cubical.

Monica: All your plants are dead.

Steve: It's because I never water them.

In the passage above, Steve uses the word "because," which we've seen in the past is a premise indicator word. But if it were a premise, the conclusion would be "All Steve's plants are dead." But Steve can't possibly be trying to convince Monica that all his plants are dead. It is something that Monica herself says, and that they both can see. The "because" here indicates a reason, but here Steve is giving an explanation, not an argument. He takes something that Steve and Monica already know—that the plants are dead—and puts it in a new light by explaining how it came to be. In this case, the plants died because they didn't get water, rather than dying because they didn't get enough light or were poisoned by a malicious co-worker. The reasoning is best represented like this:

1. Steve never waters his plants.
   ——— E
2. All the plants are dead.

But the same piece of reasoning can change form an explanation into an argument simply by putting it into a new situation:

Monica and Steve are away from the office.

Monica: Did you have someone water your plants while you were away?

Steve: No.

Monica: I bet they are all dead.

Here Steve and Monica do not know that Steve's plants are dead. Monica is inferring this idea based on the premise which she learns from Steve, that his plants are not being watered. This time "Steve's plants are not being watered" is a premise and "The plants are dead" is a conclusion. We represent the argument like this:

P. Steve never waters his plants.

———

C. All the plants are dead.

In the example of Steve's plants, the same piece of reasoning can function either as an argument or an explanation, depending on the situation context where it is given. This is because the reasoning in the example of the plants is causal: the *causes* of the plants dying are given as reasons for the death, and we can appeal to causes either to explain something that we know happened or to predict something that we think might have happened.

Not all kinds of reasoning are flexible like that, however. Reasoning from authority can be used in some kinds of argument, but often makes a lousy explanation. Consider another conversation between Steve and Monica:

Monica: I saw on a documentary last night that the universe is expanding and probably will keep expanding for ever.

Steve: Really?

Monica: Yeah, Steven Hawking said so.

There aren't any indicator words here, but it looks like Monica is giving an argument. She states that the universe is expanding, and Steve gives a skeptical "really?" Monica then replies by saying that she got this information from the famous physicist Steven Hawking. It looks like Steve is supposed to believe that the universe will expand indefinitely because Hawking, an authority in the relevant field, said so. This makes for an ok argument:

P: Steven Hawking said that the universe is expanding and will continue to do so indefinitely.

C: The universe is expanding and will continue to do so indefinitely.

Arguments from authority aren't very reliable, but for very many things they are all we have to go on. We can't all be experts on everything. But now try to imagine this argument as an explanation. What would it mean to say that the expansion of the universe can be *explained* by the fact that Steven Hawking said that it should expand. It would be as if Hawking were a god, and the universe obeyed his commands! Arguments from authority are acceptable, but not ideal. Explanations from authority, on the other hand, are completely illegitimate.

In general, arguments that appeal to how the world works are more satisfying than ones which appeals to the authority or expertise of others. Compare the following pair of arguments:

(a) Jack says traffic will be bad this afternoon. So, traffic will be bad this afternoon.

(b) Oh no! Highway repairs begin downtown today. And a bridge lift is scheduled for the middle of rush hour. Traffic is going to be terrible

Even though the second passage is an argument, the reasons used to justify the conclusion could be used in an explanation. Someone who accepts this argument will also have an explanation ready to offer if someone should later ask "Traffic was terrible today! I wonder why?". This is not true of the first passage: bad traffic is not explained by saying "Jack said it would be bad." The argument that refers to the drawbridge going up is appealing to a more powerful sort of reason, one that works in both explanations and arguments. This simply makes for a more satisfying argument, one that makes for a deeper understanding of the world, than one that merely appeals to authority.

Although arguments based on explanatory premises are preferred, we must often rely on other people for our beliefs, because of constraints on our time and access to evidence. But the other people we rely on should hopefully hold the belief on the basis of an empirical understanding. And if *those* people are just relying on authority, then we should hope that at some point the chain of testimony ends with someone who is relying on something more than mere authority. In [cross ref] we'll look more closely at sources and how much you should trust them.

We just have seen that they same set of statements can be used as an argument or an explanation depending on the context. This can cause confusion between speakers as to what is going on. Consider the following case:

Bill and Henry have just finished playing basketball.

Bill: Man, I was terrible today.

Henry: I thought you played fine.

Bill: Nah. It's because I have a lot on my mind from work.

Bill and Henry disagree about what is happening—arguing or explaining. Henry doubts Bill's initial statement, which should provoke Bill to argue. But instead, he appears to plough ahead with his explanation. What Henry can do in this case, however, is take the reason that Bill offers as an explanation (that Bill is preoccupied by issues at work) and use it as a premise in an argument for the conclusion "Bill played terribly." Perhaps Henry will argue (to himself) something like this: "It's true that Bill has a lot on his mind from work. And whenever a person is preoccupied, his basketball performance is likely to be degraded. So, perhaps he did play poorly today (even though I didn't notice)."

In other situations, people can switch back and forth between arguing and explaining. Imagine that Jones says "The reservoir is at a low level because of several releases to protect the down-stream ecology." Jones might intend this as an explanation, but since Smith does not share the belief that the reservoir's water level is low, he will first have to be given reasons for believing that it is low. The conversation might go as follows:

Jones: The reservoir is at a low level because of several releases to protect the down-stream ecology.

Smith: Wait. The reservoir is low?

Jones: Yeah. I just walked by there this morning. You haven't been up there in a while?

Smith: I guess not.

Jones: Yeah, it's because they've been releasing a lot of water to protect the ecology lately.

When challenged, Smith offers evidence from his memory: he saw the reservoir that morning. Once Smith accepts that the water level is low, Jones can restate his explanation.

Some forms of explanation overlap with other kinds of nonargumentative passages. We are dealing right now with thinking in the real world, and as we mentioned on page 4 the real world is full of messiness and ambiguity. One effect of this is that all the categories we are discussing will wind up overlapping. Narratives and expository passages, for instance, can also function as explanations. Consider this passage

> *From an article on espn.go.com* Duke beat Butler 61-59 for the national championship Monday night. Gordon Hayward's half-court, 3-point heave for the win barely missed to leave tiny Butler one cruel basket short of the Hollywood ending.

On the one hand, this is clearly a narrative—retelling a sequence of events united by time, place, and character. But it also can work as an explanation about how Duke won, if the audience immediately accepts the result. 'The last shot was a miss and then Duke won' can be understood as 'the last shot was a miss and so Duke won'.

Key Terms

Argument

Canonical form

Conclusion

Conclusion indicator

Content neutrality

Critical thinking

Explainee

Explainer

Explanation

Expository passage

Formal logic

Inference

Informal logic

Logic

Metacognition

Metareasoning

Narrative

Premise

Premise indicator

Reason

Rhetoric

Simple statement of belief

Statement

Target proposition

Chapter 2

The Basics of Evaluating Argument

2.1 Two Ways an Argument Can Go Wrong

Arguments are supposed to lead us to the truth, but they don't always succeed. There are two ways they can fail in their mission. First, they can simply start out wrong, using false premises. Consider the following argument.

P_1: It is raining heavily.
P_2: If you do not take an umbrella, you will get soaked.

C: You should take an umbrella.

If premise (1) is false—if it is sunny outside—then the argument gives you no reason to carry an umbrella.The argument has failed its job. Premise (2) could also be false: Even if it is raining outside, you might not need an umbrella. You might wear a rain poncho or keep to covered walkways and still avoid getting soaked. Again, the argument fails because a premise is false.

Even if an argument has all true premises, there is still a second way it can fail. Suppose for a moment that both the premises in the argument above are true. You do not own a rain poncho. You need to go places where there are no covered walkways. Now does the argument show you that you should take an umbrella? Not necessarily. Perhaps you enjoy walking in the rain, and you would like to get soaked. In that case, even though the premises were true, the conclusion would be false. The premises, although true, do not *support* the conclusion. Back on page 12 we defined an inference, and said it was like argument glue: it holds the premises and conclusion together. When an argument goes wrong because the premises do not support the conclusion, we say there is something wrong with the inference.

Consider another example:

P_1: You are reading this book.
P_2: This is a logic book.

C: You are a logic student.

This is not a terrible argument. Most people who read this book are logic students. Yet, it is possible for someone besides a logic student to read this book. If your roommate picked up the book and thumbed through it, they would not immediately become a logic student. So the premises of this argument, even though they are true, do not guarantee the truth of the conclusion. Its inference is less than perfect.

Again, for any argument, there are two ways that it could fail. First, one or more of the premises might be false. Second, the premises might fail to support the conclusion. Even if the premises were true, the form of the argument might be weak, meaning the inference is bad.

2.2 Valid, Sound

In logic, we are mostly concerned with evaluating the quality of inferences, not the truth of the premises. The truth of various premises will be a matter of whatever specific topic we are arguing about, and, as we have said, logic is content neutral.

The strongest inference possible would be one where the premises, if true, would somehow force the conclusion to be true. This kind of inference is called valid. There are a number of different ways to make this idea of the premises forcing the truth of the conclusion more precise. Here are a few:

An argument is valid if and only if. . .

(a) it is impossible to consistently both (i) accept the premises and (ii) reject the conclusion

(b) it is impossible for the premises to be true and the conclusion false

(c) the premises, if true, would necessarily make the conclusion true.

(d) the conclusion is true in every imaginable scenario in which the premises are true

(e) it is impossible to write a consistent story (even fictional) in which the premises are true and the conclusion is false

In the glossary, we formally adopt item (b) as the definition for this textbook: an argument is VALID if and only if it is impossible for the premises to be true and the conclusion false. However, P$_1$: Lady Gaga is from Mars.

C: Lady Gaga is from the fourth planet from our sun.

Figure 2.1: A **valid** argument.

nothing will really ride on the differences between the definitions in the list above, and we can look at all of them in order to give us a sense of what logicians mean when they use the term “valid”.

The important thing to see is that all the definitions in the list above try to get at what *would* happen if the premises were true. None of them assert that the premises actually *are* true. This is why definitions (d) and (e) talk about what would happen if you somehow *pretend* the premises are true, for instance by telling a story. The argument is valid if, when you pretend the premises are true, you also have to pretend the conclusion is true. Consider the argument in Figure 2.1

The American pop star Lady Gaga is not from Mars. (She’s from New York City.) Nevertheless, if you imagine she’s from Mars, you simply have to imagine that she is from the fourth planet from our sun, because mars simply is the fourth planet form our sun. Therefore this argument is valid.

This way of understanding validity is based on what you can imagine, but not everyone is convinced that the imagination is a reliable tool in logic. That is why definitions like (c) and (b) talk about what is necessary or impossible. If the premises are true, the conclusion necessarily must be true. Alternately, it is impossible for the premises to be true and the conclusion false. The idea here is that instead of talking about the imagination, we will just talk about what can or cannot happen at the same time. The fundamental notion of validity remains the same, however: the truth of the premises would simply guarantee the truth of conclusion.

So, assessing validity means wondering about whether the conclusion would be true *if* the premises were true. This means that valid arguments can have false conclusions. This is important to keep in mind because people naturally tend to think that any argument must be good if they agree with the conclusion. And the more passionately people believe in the conclusion, the more likely we are to think that any argument for it must be brilliant. Conversely, if the conclusion is something we don’t believe in, we naturally tend to think the argument is poor. And the more we don’t like the conclusion, the less likely we are to like the argument.

But this is not the way to evaluate inferences at all. The quality of the inference is entirely independent of the truth of the conclusion. You can have great arguments for false conclusions and horrible arguments for true conclusions. Confusing the truth of the conclusion with the quality of of the inference is a mistake in logic we can call the myside fallacy. A FALLACY is any common mistake in reasoning The MYSIDE FALLACY is specifically the common mistake of evaluating an argument based merely on whether one agrees or disagrees with the conclusion. You can also think of this as the fallacy of mistaking the conclusion for the argument.

An argument is valid if it is impossible for the premises to be true and the conclusion false. This means that you can have valid arguments with false conclusions, they just have to also have false premises. Consider the example in Figure 2.2

P_1: Oranges are either fruits or musical instruments.
P_2: Oranges are not fruits.

C: Oranges are musical instruments.

Figure 2.2: A **valid** argument

The conclusion of this argument is ridiculous. Nevertheless, it follows validly from the premises. This is a valid argument. *If* both premises were true, *then* the conclusion would necessarily be true.

This shows that a valid argument does not need to have true premises or a true conclusion. Conversely, having true premises and a true conclusion is not enough to make an argument valid. Consider the example in Figure 2.3

The premises and conclusion of this argument are, as a matter of fact, all true. This is a terrible argument, however, because the premises have nothing to do with the conclusion. Imagine what would happen if Paris declared independence from the rest of France. Then the conclusion would be false, even though the premises would both still be true. Thus, it is *logically possible* for the premises of this argument to be true and the conclusion false. The argument is not valid. If an argument is not valid, it is called INVALID. As we shall see, this term is a little misleading, because less than perfect arguments can be very useful. But before we do that, we need to look more at the concept of validity.

In general, then, the *actual* truth or falsity of the premises, if known, do not tell you whether or not an inference is valid. There is one exception: when the premises are true and the conclusion is false, the inference cannot be valid, because valid reasoning can only yield a true conclusion when beginning from true premises.

Figure 2.4 has another invalid argument:

In this case, we can see that the argument is invalid by looking at the truth of the premises and conclusion. We know the premises are true. We know that the conclusion is false. This is the one circumstance that a valid argument is supposed to make impossible.

Some invalid arguments are hard to detect because they resemble valid arguments. Consider the one in Figure 2.5

P_1: London is in England.
P_2: Beijing is in China.

C: Paris is in France.

Figure 2.3: An **invalid** argument.

P_1: All dogs are mammals
P_2: All dogs are animals

C: All animals are mammals.

Figure 2.4: An **invalid** argument.

This reasoning is not valid since the premises do not *definitively* support the conclusion. To see this, assume that the premises are true and then ask, ”Is it possible that the conclusion could be false in such a situation?”. There is no inconsistency in taking the premises to be true without taking the conclusion to be true. The first premise says that the stimulus package will allow the U.S. to avoid a depression, but it does not say that a stimulus package is the *only* way to avoid a depression. Thus, the mere fact that there is no stimulus package does not necessarily mean that a depression will occur.

Here is another, trickier, example. I will give it first in ordinary language.

> *A pundit is speaking on a cable news show* If the U.S. economy were in recession and inflation were running at more than 4%, then the value of the U.S. dollar would be falling against other major currencies. But this is not happening — the dollar continues to be strong. So, the U.S. is not in recession.

The conclusion is ”The U.S. economy is not in recession.” If we put the argument in canonical form, it looks like figure 2.6

The conclusion does not follow necessarily from the premises. It does follow necessarily from the premises that (i) the U.S. economy is not in recession or (ii) inflation is running at more than 4%, but they do not guarantee (i) in particular, which is the conclusion. For all the premises say, it is possible that the U.S. economy is in recession but inflation is less than 4%. So, the inference does not *necessarily* establish that the U.S. is not in recession. A parallel inference would be ”Jack needs eggs and milk to make an omelet. He can’t make an omelet. So, he doesn’t have eggs.”.

If an argument is not only valid, but also has true premises, we call it SOUND. “Sound” is the highest compliment you can pay an argument. If logic is the study of virtue in argument, sound arguments are the most virtuous. We said in Section 2.1 that there were two ways an argument could go wrong, either by having false premises or weak inferences. Sound arguments have true

P_1: An economic stimulus package will allow the U.S. to avoid a depression.
P_2: There is no economic stimulus package

C: The U.S. will go into a depression.

Figure 2.5: An **invalid** argument

P_1: If the U.S. were in a recession with more than 4% inflation, then the dollar would be falling
P_2: The dollar is not falling

C: The U.S. is not in a recession.

Figure 2.6: An **invalid** argument

premises and undeniable inferences. If someone gives a sound argument in a conversation, you have to believe the conclusion, or else you are irrational.

The argument on the left in Figure 2.7 is valid, but not sound. The argument on the right is both valid and sound.

Both arguments have the exact same form. They say that a thing belongs to a general category and everything in that category has a certain property, so the thing has that property. Because the form is the same, it is the same valid inference each time. The difference in the arguments is not the validity of the inference, but the truth of the second premise. People are not carrots, therefore the argument on the left is not sound. People are mortal, so the argument on the right is sound.

Often it is easy to tell the difference between validity and soundness if you are using completely silly examples. Things become more complicated with false premises that you might be tempted to believe, as in the argument in Figure 2.8.

You might have a general sense that the argument in Figure 2.8 is bad—you shouldn't assume that someone drinks Guinness just because they are Irish. But the argument is completely valid (at least when it is expressed this way.) The inference here is the same as it was in the previous two arguments. The problem is the first premise. Not all Irishmen drink Guinness, but if they did, and Smith was an Irishman, he would drink Guinness.

The important thing to remember is that validity is not about the actual truth or falsity of the statements in the argument. Instead, it is about the way the premises and conclusion are put together. It is really about the *form* of the argument. A valid argument has perfect logical form. The premises and conclusion have been put together so that the truth of the premises is incompatible with the falsity of the conclusion.

A general trick for determining whether an argument is valid is to try to come up with just one way in which the premises could be true but the conclusion false. If you can think of one (just one! anything at all! but no violating the laws of physics!), the reasoning is *invalid.*

P_1: Socrates is a person.
P_2: All people are carrots.

C: Therefore, Socrates is a carrot.

Valid, but not sound

P_1: Socrates is a person.
P_2: All people are mortal.

C: Therefore, Socrates is mortal.

Valid and sound

Figure 2.7: These two arguments are valid, but only the one on the right is sound

P_1: Every Irishman drinks Guinness
P_2: Smith is an Irishman

C: Smith drinks Guinness.

Figure 2.8: An argument that is **valid** but not *sound*

2.3 Strong, Cogent, Deductive, Inductive

We have just seen that sound arguments are the very best arguments. Unfortunately, sound arguments are really hard to come by, and when you do find them, they often only prove things that were already quite obvious, like that Socrates (a dead man) is mortal. Fortunately, arguments can still be worthwhile, even if they are not sound. Consider this one:

P_1: In January 1997, it rained in San Diego.
P_2: In January 1998, it rained in San Diego.
P_3: In January 1999, it rained in San Diego.

C: It rains every January in San Diego.

This argument is not valid, because the conclusion could be false even though the premises are true. It is possible, although unlikely, that it will fail to rain next January in San Diego. Moreover, we know that the weather can be fickle. No amount of evidence should convince us that it rains there *every* January. Who is to say that some year will not be a freakish year in which there is no rain in January in San Diego? Even a single counterexample is enough to make the conclusion of the argument false.

Still, this argument is pretty good. Certainly, the argument could be made stronger by adding additional premises: In January 2000, it rained in San Diego. In January 2001... and so on. Regardless of how many premises we add, however, the argument will still not be deductively valid. Instead of being valid, this argument is strong. An argument is STRONG if the premises would make the conclusion more likely, were they true. In a strong argument, the premises don't guarantee the truth of the conclusion, but they do make it a good bet. If an argument is strong, and it has true premises, we say that it is COGENT Cogency is the equivalent of soundness in strong arguments. If an inference is neither valid, nor strong, we say it is WEAK. In a weak argument, the premises would not even make the conclusion likely, even if they were true.

You may have noticed that the word "likely" is a little vague. How likely do the premises have to make the conclusion before we can count the argument as strong? The answer is a very unsatisfying "it depends." It depends on what is at stake in the decision to believe the conclusion. What happens if you are wrong? What happens if you are right? The phrase "make the conclusion a good bet" is really quite apt. Whether something is a good bet depends a lot on how much money is at stake and how much you are willing to lose. Sometimes people feel comfortable taking a bet that has a 50% chance of doubling their money, sometimes they don't.

The vagueness of the word "likely" brings out an interesting feature of strong arguments: some strong arguments are stronger than others. The argument about rain in San Diego, above, has three premises referring to three previous Januaries. The argument is pretty strong, but it can become stronger if we go back farther into the past, and find more years where it rains in January. The more evidence we have, the better a bet the conclusion is. Validity is not like this. Validity is a black-or-white matter. You either have it, and you're perfect, or you don't, and you're nothing. There is no point in adding premises to an argument that is already valid.

Arguments that are valid, or at least try to be, are called DEDUCTIVE , and people who attempt to argue using valid arguments are said to be arguing *deductively.* The notion of validity we are using here is, in fact, sometimes called *deductive validity.* Deductive argument is difficult, because, as we said, in the real world sound arguments are hard to come by, and people don't always recognize them as sound when they find them. Arguments that purport to merely be strong rather than valid are called INDUCTIVE. The most common kind of inductive argument includes arguments like the one above about rain in San Diego, which generalize from many cases to a conclusion about all cases.

Deduction is possible in only a few contexts. You need to have clear, fixed meanings for all of your terms and rules that are universal and have no exceptions. One can find situations like this if you are dealing with things like legal codes, mathematical systems or logical puzzles. One can also create, as it were, a context where deduction is possible by imagining a universal, exceptionless rule, even if you know that no such rule exists in reality. In the example above about rain in San Diego, we can change the argument from inductive to deductive by adding a universal, exceptionless premise like "It always rains in January in San Diego." This premise is unlikely to be true, but it can make the inference valid.

Here is an example in which the context is an artificial code — the tax code:

> *From a the legal code posted on a government website* A tax credit for energy-efficient home improvement is available at 30% of the cost, up to $1,500 total, in 2009 & 2010, ONLY for existing homes, NOT new construction, that are your "principal residence" for Windows and Doors (including sliding glass doors, garage doors, storm doors and storm windows), Insulation, Roofs (Metal and Asphalt), HVAC: Central Air Conditioners, Air Source Heat Pumps, Furnaces and Boilers, Water Heaters: Gas, Oil, & Propane Water Heaters, Electric Heat Pump Water Heaters, Biomass Stoves.

This rule describes the conditions under which a person can or cannot take a certain tax credit. Such a rule can be used to reach a valid conclusion that the tax credit can or cannot be taken.

As another example of an inference in an artificial situation with limited and clearly defined options, consider a Sudoku puzzle. The rules of Sudoku are that each cell contains a single number from 1 to 9, and each row, each column and each 9-cell square contain one occurrence of each number from 1 to 9. Consider the following partially completed board:

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

The following inference shows that, in the first column, a 9 must be entered below the 7:

> The 9 in the first column must go in one of the open cells in the column. It cannot go in the third cell in the column, because there is already a 9 in that 9-cell square. It cannot go in the eighth or ninth cell because each of these rows already contains a 9, and a row cannot contain two occurrences of the same number. Therefore, since there must be a 9 somewhere in this column, it must be entered in the seventh cell, below the 7.

The reasoning in this inference is valid: if the premises are true, then the conclusion must be true. Logic puzzles of all sorts operate by artificially restricting the available options in various ways. This then means that each conclusion arrived at (assuming the reasoning is correct) is necessarily true.

One can also create a context where deduction is possible by imagining a rule that holds without exception. This can be done with respect to any subject matter at all. Speakers often exaggerate the connecting premise in order to ensure that the justificatory or explanatory power of the inference is as strong as possible. Consider Smith's words in the following passage:

Smith: I'm going to have some excellent pizza this evening.

Jones: I'm glad to hear it. How do you know?

Smith: I'm going to Adriatico's. They always make a great pizza.

Here, Smith justifies his belief that the pizza will be excellent — it comes from Adriatico's, where the pizza, he claims, is *always* great: in the past, present and future.

As stated by Smith, the inference that the pizza will be great this evening is valid. However, making the inference valid in this way often means making the general premise false: it's not likely that the pizza is great *every single* time; Smith is overstating the case for emphasis. Note that Smith does not need to use a universal proposition in order to convince Jones that the pizza will *very likely* be good. The inference to the conclusion would be strong (though not valid) if he had said that the pizza is "almost always" great, or that the pizza has been great on all of the many occasions he has been at that restaurant in the past. The strength of the inference would fall to some extent—it would not be guaranteed to be great this evening—but a slightly weaker inference seems appropriate, given that sometimes things go contrary to expectation.

Sometimes the laws of nature make constructing contexts for valid arguments more reasonable. Now consider the following passage, which involves a scientific law:

> Jack is about to let go of Jim's leash. The operation of gravity makes all unsupported objects near the Earth's surface fall toward the center of the Earth. Nothing stands in the way. Therefore, Jim's leash will fall.

(Or, as Spock said in a Star Trek episode, "If I let go of a hammer on a planet that has a positive gravity, I need not see it fall to know that it has in fact fallen.") The inference above is represented in canonical form as follows:

P_1: Jack is about to let go of Jim's leash.
P_2: The operation of gravity makes all unsupported objects near the Earth's surface fall toward the center of the Earth.
P_3: Nothing stands in the way of the leash falling.

C: Jim's leash will fall toward the center of the Earth.

As stated, this argument is valid. That is, if you pretend that they are true or accept them "for the sake of argument", you would *necessarily* also accept the conclusion. Or, to put it another way, there is no way in which you could hold the premises to be true and the conclusion false.

Although this argument is valid, it involves idealizing assumptions similar to the ones we saw in the pizza example. P_2 states a physical law which is about as well confirmed as any statement about the world around us you care to name. However, physical laws make assumptions about the situations they apply to—they typically neglect things like wind resistance. In this case, the idealizing assumption is just that nothing stands in the way of the leash falling. This can be checked just by looking, but this check can go wrong. Perhaps there is an invisible pillar underneath Jack's hand? Perhaps a huge gust of wind will come? These events are much less likely than Adriatico's making a lousy pizza, but they are still possible.

Thus we see that using scientific laws to create a context where deductive validity is possible is a much safer bet than simply asserting whatever exceptionless rule pops into your head. However, it still involves improving the quality of the inference by introducing premises that are less likely to be true.

So deduction is possible in artificial contexts like logical puzzles and legal codes. It is also possible in cases where we make idealizing assumptions or imagine exceptionless rules. The rest of

P_1: 92% of Republicans from Texas voted for Bush in 2000.	P_1: Just over half of drivers are female.
P_2: Jack is a Republican from Texas.	P_2: There's a person driving the car that just cut me off.
C: Jack voted for Bush.	C: The person driving the car that just cut me off is female.
A **strong** argument	A **weak** argument

Figure 2.9: Neither argument is valid, but one is strong and one is weak

the time we are dealing with induction. When we do induction, we try for strong inferences, where the premises, assuming they are true, would make the truth of the conclusion very likely, though not necessary. Consider the two arguments in Figure 2.9

Note that the premises in neither inference *guarantee* the truth of the conclusion. For all the premises in the first one say, Jack could be one of the 8% of Republicans from Texas who did not vote for Bush; perhaps, for example, Jack soured on Bush, but not on Republicans in general, when Bush served as governor. Likewise for the second; the driver could be one of the 49%.

So, neither inference is valid. But there is a big difference between how much support the premises, if true, would give to the conclusion in the first and how much they would in the second. The premises in the first, assuming they are true, would provide very strong reasons to accept the conclusion. This, however, is not the case with the second: if the premises in it were true then they would give only weak reasons for believing the conclusion. thus, the first is strong while the second is weak.

As we said earlier, there there are only two options with respect to validity—valid or not valid. On the other hand, strength comes in degrees, and sometimes arguments will have percentages that will enable you to exactly quantify their strength, as in the two examples in Figure 2.9.

However, even where the degree of support is made explicit by a percentage there is no firm way to say at what degree of support an inference can be classified as strong and below which it is weak. In other words, it is difficult to say whether or not a conclusion is *very likely* to be true. For example, In the inference about whether Jack, a Texas Republican, voted for Bush. If 92% of Texas Republicans voted for Bush, the conclusion, if the premises are granted, would very probably be true. But what if the number were 85%? Or 75%? Or 65%? Would the conclusion very likely be true? Similarly, the second inference involves a percentage greater than 50%, but this does not seem sufficient. At what point, however, would it be sufficient?

In order to answer this question, go back to basics and ask yourself: "If I accept the truth of the premises, would I then have sufficient reason to believe the conclusion?". If you would not feel safe in adopting the conclusion as a belief as a result of the inference, then you think it is weak, that is, you do not think the premises give sufficient support to the conclusion.

Note that the same inference might be weak in one context but strong in another, because the degree of support needed changes. For example, if you merely have a deposit to make, you might accept that the bank is open on Saturday based on your memory of having gone to the bank on Saturday at some time in the past. If, on the other hand, you have a vital mortgage payment to make, you might not consider your memory sufficient justification. Instead, you will want to call up the bank and increase your level of confidence in the belief that it will be open on Saturday.

Most inferences (if successful) are strong rather than valid. This is because they deal with situations which are in some way open-ended or where our knowledge is not precise. In the example of Jack voting for Bush, we know only that 92% of Republicans voted for Bush, and so there is no definitive connection between being a Texas Republican and voting for Bush. Further, we have only statistical information to go on. This statistical information was based on polling or surveying a sample of Texas voters and so is itself subject to error (as is discussed in the chapter on induction in the complete version of this text. A more precise version of the premise might be "92% $\pm$ 3% of Texas Republicans voted for Bush.".

At the risk of redundancy, let's consider a variety of examples of valid, strong and weak inferences, presented in standard form.

P_1: David Duchovny weighs more than 200 pounds.

C: David Duchovny weighs more than 150 pounds.

The inference here is valid. It is valid because of the number system (here applied to weight): 200 is more than 150. It might be false, as a matter of fact, that David Duchovny weighs more than 200 pounds, and false, as a matter of fact, that David Duchovny weighs more than 150 pounds. But if you *suppose* or *grant* or *imagine* that David Duchovny weighs more than 200 pounds, it would then *have* to be true that David Duchovny weighs more than 150 pounds. Next:

P_1: Armistice Day is November 11th, each year.

P_2: Halloween is October 31st, each year.

C: Armistice Day is later than Halloween, each year.

This inference is valid. It is valid because of order of the months in the Gregorian calendar and the placement of the New Year in this system. Next:

P_1: All people are mortal.

P_2: Professor Pappas is a person.

C: Professor Pappas is mortal.

As written, this inference is valid. If you accept for the sake of argument that all men are mortal (as the first premise says) and likewise that Professor Pappas is a man (as the second premise says), then you would have to accept also that Professor Pappas is mortal (as the conclusion says). You could not consistently both (i) affirm that all men are mortal and that Professor Pappas is a man and (ii) deny that Professor Pappas is mortal. If a person accepted these premises but denied the conclusion, that person would be making a mistake in logic.

This inference's validity is due to the fact that the first premise uses the word "all". You might, however, wonder whether or not this premise is true, given that we believe it to be true only on our experience of men *in the past*. This might be a case of over-stating a premise, which we mentioned earlier. Next:

P_1: In 1933, it rained in Columbus, Ohio on 175 days.
P_2: In 1934, it rained in Columbus, Ohio on 177 days.
P_3: In 1935, it rained in Columbus, Ohio on 171 days.

C: In 1936, it rained in Columbus, Ohio on at least 150 days.

This inference is strong. The premises establish a record of days of rainfall that is well above 150. It is possible, however, that 1936 was exceptionally dry, and this possibility means that the inference does not achieve validity. Next:

P_1: The Bible says that homosexuality is an abomination.

C: Homosexuality is an abomination.

This inference is an appeal to a source. In brief, you should think about whether the source is reliable, is biased, and whether the claim is consistent with what other authorities on the subject say. You should apply all these criteria to this argument for yourself. You should ask what issues, if any, the Bible is reliable on. If you believe humans had any role in writing the Bible, you can ask about what biases and agendas they might have had. And you can think about what other sources—religious texts or moral experts—say on this issue. You can certainly find many who disagree. Given the controversial nature of this issue, we will not give our evaluation. We will only encourage you to think it through systematically.

P_1: Some professional philosophers published books in 2007.
P_2: Some books published in 2007 sold more than 100,000 copies.

C: Some professional philosophers published books in 2007 that sold more than 100,000 copies.

This reasoning is weak. Both premises use the word "some" which doesn't tell you a lot about many professional philosophers published books and how many books sold more than 100,000 copies in 2007. This means that you cannot be confident that even one professional philosopher sold more than 100,000 copies. Next:

P_1: Lots of Russians prefer vodka to bourbon.

C: George Bush was the President of the United States in 2006.

No one (in her right mind) would make an inference like this. It is presented here as an example only: it is clearly weak. It's hard to see how the premise justifies the conclusion to any extent at all.

To sum up this section, we have seen that there are two styles of reasoning, deductive and inductive. The former tries to use valid arguments, while the latter contents itself to give arguments that are merely strong. The section of this book on formal logic will deal entirely with deductive reasoning. Historically, most of formal logic has been devoted to the study of deductive arguments, although many great systems have been developed for the formal treatment of inductive logic. On the other hand, the sections of this book on informal logic and critical thinking will focus mostly on inductive logic, because these arguments are more readily available in the real world.

Key Terms

Cogent

Deductive

Fallacy

Fallacy of mistaking the conclusion for the argument

Inductive

Invalid

Sound

Strong

Valid

Weak

Chapter 3

What is Formal Logic?

3.1 Formal as in Concerned with the Form of Things

Formal logic is distinguished from other branches of logic by the way it achieves content neutrality. Back on page 3, we said that a distinctive feature of logic is that it is neutral about the content of the argument it evaluates. If a kind of argument is strong—say, a kind of statistical argument—it will be strong whether it is applied to sports, politics, science or whatever. Formal logic takes radical measures to ensure content neutrality: it removes the parts of a statement that tie it to particular objects in the world and replaces them with abstract symbols. (See page 3)

Consider the two arguments from Figure 2.7 again:

P_1: Socrates is a person.
P_2: All persons are mortal.
———————————
C: Socrates is mortal.

P_1: Socrates is a person.
P_2: All people are carrots.
———————————
C: Socrates is a carrot.

These arguments are both valid. In each case, if the premises were true, the conclusion would have to be true. (In the case of the first argument, the premises are actually true, so the argument is sound, but that is not what we are concerned with right now.) What makes these arguments valid is that they are put together the right way. Another way of thinking about this is to say that they have the same logical form. Both arguments can be written like this:

P_1: S is M.
P_2: All M are P.
———————————
C: S is P.

In both arguments S stands for Socrates and M stands for person. In the first argument, P stands for mortal; in the second, P stands for carrot. (The reason we chose these letters will become clear in Chapters 4 and **??**.) The letters 'S', 'M', and 'P' are variables. They are just like the variables you may have learned about in algebra class. In algebra, you had equations like $y = 2x + 3$, where x and y were variables that could stand for any number. Just as x could stand for any number in algebra, 'S' can stand for any name in logic. In fact, this is one of the original uses of variables. Long before variables were used to stand for numbers in algebra, they were used to stand for classes of things, like people or carrots, by Aristotle in his book the *Prior Analytics* (c. 350 BCE/1984b). At about the same time, over in India, the ancient grammarian and linguist Pāṇini was also using variables to represent possible sounds that could be used in different forms of a word (Pāṇini, 2015). Both thinkers introduce their variables fairly causally, as if their readers would be familiar with the idea, so it may be that people prior to them actually invented the variable.

Whoever invented it, the variable was one of the most important conceptual innovations in human history, right up there with the invention of the zero, or alphabetic writing. The importance of the variable for the history of mathematics is obvious. But it was also incredibly important in one of its original fields of application, logic. For one thing, it allows logicians to be more content neutral. We can set aside any associations we have with people, or carrots, or whatever, when we are analyzing an argument. More importantly, once we set aside content in this way, we discover that something incredibly powerful is left over, the logical structure of the sentence itself. This is what we investigate when we study formal logic. In the case of the two arguments above, identifying the logical structure of statements reveals not only that the two arguments have the same logical form, but they have an impeccable logical form. Both arguments are valid, and any other arguments that have this form will be valid.

When Aristotle introduced the variable to the study of logic he used it the way we did in the argument above. His variables stood for names and categories in simple two-premise arguments called syllogisms. The system of logic Aristotle outlined became the dominant logic in the Western world for more than two millennia. It was studied and elaborated on by philosophers and logicians from Baghdad to Paris. The thinkers that carried on Aristotelian tradition were divided by language and religion. They were pagans, Muslims, Jews, and Christians writing typically in Greek, Latin or Arabic. But they were all united by the sense that the tools Aristotle had given them allowed them to see something profound about the nature of reality. They were looking at abstract structures which somehow seemed to be at the foundation of things. As the philosopher and historian of logic Catarina Dutilh Novaes points out, the logic that the thinkers of all these religious traditions were pursuing was formal in that it concerned the *forms* of things Dutilh Novaes (2011). As formal logic evolved, however, the idea of being "formal" would take on an additional meaning.

3.2 Formal as in Strictly Following Rules

Despite its historical importance, Aristotelean logic has largely been superseded. Starting in the 19th century people learned to do more than simply replace categories with variables. They learned to replicate the whole structure of sentences with a formal system that brought out all sorts of features of the logical form of arguments. The result was the creation of entire artificial languages. An ARTIFICIAL LANGUAGE is a language that was consciously developed by identifiable individuals for some purpose. Esperanto, for instance, is an artificial language developed by Ludwig Lazarus Zamenhof in the 19th century with the hope of promoting world peace by creating a common language for all. J.R.R. Tolkien invented several languages to flesh out the fictional world of his fantasy novels, and even created timelines for their evolution. For Tolkien, the creation of languages was an art form in itself, “An art for which life is not long enough, indeed: the construction of imaginary languages in full or outline for amusement, for the pleasure of the constructor or even conceivably of any critic that might occur” (Tolkein 1931/2016). And it is an art that is really beginning to catch on, especially with Hollywood commissioning languages to be constructed for blockbuster films.

Artificial languages contrast with NATURAL LANGUAGES, which develop spontaneously and are learned by infants as their first language. Natural languages include all the well-known languages spoken around the world, like English or Japanese or Arabic. It also includes more recently developed languages and evolved spontaneously amongst groups of people. For instance, whenever you put deaf children together, for instance in a boarding school, they will spontaneously develop their own sign language. This phenomenon was important for the development of American Sign Language (ASL) and is part of why ASL counts as a *natural* language. For similar reasons Nicaraguan Sign Language counts as a natural language, even though it emerged very recently—in the late 1970s and 80s, when the new Sandinista government set up schools for the deaf for the first time. Natural languages can also develop by creolization, when languages merge and children grow up speaking the merged language as their first language. Haitian Creole is the most famous example of this.

The languages developed by logicians are artificial, not natural. Their goal is not to promote global harmony, like Zamenhof’s Esperanto. Nor are they creating art for art’s sake, as Tolkein was, although logical languages can have a great deal of beauty. When the languages first started being developed in the late 19th and early 20th centuries, the goal was, in fact, to have a logically pure language, free of the irrationalities the plague natural languages. More specifically, they had two distinct goals: first, remove all ambiguity and vagueness, and second, to make the logical structure of the language immediately apparent, so that the language wore its logical structure on its face, as it were. If such a language could be developed, it would help us solve all kinds of problems. The logician and philosopher Rudolf Carnap, for instance, felt that the right artificial language could simply make philosophical problems disappear (Carnap, 1928).

The languages developed by logicians in the late 19th and early 20th centuries got labeled formal languages, in part because the logicians in question were working in the tradition of formal logic that was already established. A shift began to happen here with the meaning of formal, however, a change which is well documented by Dutilh Novaes 2011. Logicians began to hope that the languages that were being developed were so logical that everything about them could be characterized by a machine. A machine could be used to create sentences in this language, and then again to identify all the valid arguments in this language. This brings out another sense of the word “formal.” As Dutilh Novaes puts it (2011) instead of being “formal” in the sense of concerning the forms of things, logic was formal in the sense that it followed rules perfectly precisely. You might compare this to the way a “formal hearing” in a court of law follows the rule of law to the letter.

For the purposes of this textbook, we will say that the core idea of a FORMAL LANGUAGE is that it is an artificial language designed to bring out the logical structure of ideas and remove all the ambiguity and vagueness that plague natural languages like English. We will further add that sometimes, formal languages are languages that can be implemented by a machine. Creating formal languages always involves all kinds of trade offs. On the one hand, we are trying to create a language that makes a logical structure clear and obvious. This will require simplifying things, removing excess baggage from the language. On the other hand, we want to make the language perfectly precise, free of vagueness and ambiguity. This will mean adding complexity to the language. The other thing was that it was very important for the people developing these languages that you be able to prove the all the truths of mathematics in them. This meant that the languages had to have a certain scope.

This was a trade off no logician was ever able to get perfectly correct, because, as it turns out, a logically pure language is impossible. No formal language can do everything that a natural language can do. Logicians became convinced of this, naturally enough, because of a pair of logical proofs. In 1931, the logician Kurt Gödel showed that you couldn't do all of mathematics in a consistent logical system, which was enough to persuade most of the logicians engaged in the project to drop it. There is a more general problem with the idea of a purely logical language, though, which is that that many of the features logicians were trying to remove from language were actually necessary to make it function. Arika Okrent puts the point quite well. For Okrent, the failure of artificial languages is precisely what illuminates the virtues of natural language.

> [By studying artificial languages we] gain a deeper appreciation of natural language and the messy qualities that give it so much flexibility and power and that a simple communication device. The ambiguity and lack of precision allow to serve as a instrument of thought *formation*, of experimentation and discovery. We don't know exactly what we mean before we speak; we can figure it out as we go along,. We can talk just to talk, to be social, to feel connected, to participate. At the same time natural language still works as an instrument of thought transmission, one that can be *made* extremely precise and reliable when we need it to be, or left loose and sloppy when we can't spare the time or effort (Okrent, 2009)

The languages developed in the late 19th and early 20th centuries had goals that were theoretical, rather than practical. They languages were meant to improve our understanding of the world for the sake of improving our understanding of the world. They failed at this theoretical goal, but they wound up having a practical spin-off of world-historical proportions, which is why formal logic is a thriving discipline to this day. Remember that in this period people started thinking of formal languages as languages that could be implemented mechanically. At first, the idea of a a mechanistic language was a metaphor. The rules that were being followed to the letter were to be followed by a human being actually writing down symbols. This human being was generally referred to as a "computer," because they were computing things. The world changed when a logician named Alan Turing started using literal machines to be computers.

In the 1930s, Turing developed the idea of a reasoning machine that could compute any function. At first, this was just an abstract idea: it involved an infinite stretch of tape. But during World War II, Turing went to work the British code breaking effort at Bletchley Park. The Nazis encoded messages using a device called the Enigma Machine. The Allies had captured one, but since they settings on the machine were reshuffled for each message, it didn't do them much good. Turing, together with people like the mathematicians Gordon Welchman and Joan Clarke, managed to build another machine that could test Enigma settings rapidly to identify the configuration being used. People had made computing machines before, but now the science of logic was so much more advanced that they real power of mechanical computing could be exploited. The human computers became the fully programmable machines we know today, and the formal languages logicians created for theoretical reasons came the computer languages the world of the 21st century depends on. (All of this information, plus lots of fascinating pictures and diagrams, is available at www.turing.org.uk.)

In this book we will be developing two formal languages, called SL and QL. Part **??** develops SL.In SL, the smallest units are individual statements. Simple statements are represented as letters and connected with logical connectives like *and* and *not* to make more complex statements. Part **??** develops QL. In QL, the basic units are objects, properties of objects, and relations between objects.

3.3 On Learning a Formal Logical System

You may be reading this book because you have a keen interest in logic and are excited to learn more about it. You may also be reading this book because it was assigned in a class that you need to fulfill a distribution requirement. As the chapters on formal logic roll on, and the pages begin to fill up with unfamiliar squiggles, you may even begin to question whether the study of logic is for you. Rest assured, if you have a human brain capable of reading this sentence, you are also capable of doing formal logic—and you can benefit from doing so, too. In this section, we are going to talk about why you can be confident in your ability to do logic, even if you are new to it. We are also going to offer some strategies for studying formal logic, so even if you are already quite confident in your abilities, it will be worth reading the rest of this section.

All of the basic mental skills used in a formal logical system are just that: basic mental skills. They are things you do whenever you use language. A basic part of formal logic is using abstract symbols to refer to a group of things that aren't specified. So earlier we used "P" in place of the words "mortal" and "carrot" and a whole bunch of other words that might occupy that spot in an argument. This is the same thing you do when you use a word like "dog" to refer to Spot and Fido and a whole bunch of other dogs that you don't know about. We are also going to spend time transforming things in one logical form into another. Again, this is something you already do when you speak. You know that "Jane gave the ball to Sally" can be changes to "Sally was given the ball by Jane" without changing its meaning. The kinds of things we are doing in this text are no different.

One phenomenon that can hold people back when studying logic even though they are perfectly capable of doing it is called "stereotype threat." Just reading this far in the text may have given you a false stereotype about logicians: they are white and male. You have probably also encountered corresponding false stereotypes about women and minorities—that they are irrational or not good at things like math. You hopefully don't believe those stereotypes, but you've probably encountered them. Stereotype threat is a way that a stereotype can hold people back, even when they don't believe in the stereotype. That's why the word "threat" is in there.

3.4 More Logical Notions for Formal Logic

Part I covered the basic concepts you need to study any kind of logic. When we study formal logic, we will be interested in some additional logical concepts, which we will explain here.

Truth values

A truth value is the status of a statement as true or false. Thus the truth value of the sentence "All dogs are mammals" is "True," while the truth value of "All dogs are reptiles" is false. More precisely, a TRUTH VALUE is the status of a statement with relationship to truth. We have to say this, because there are systems of logic that allow for truth values besides "true" and "false," like "maybe true," or "approximately true," or "kinda sorta true." For instance, some philosophers have claimed that the future is not yet determined. If they are right, then statements about *what will be the case* are not yet true or false. Some systems of logic accommodate this by having an additional truth value. Other formal languages, so-called paraconsistent logics, allow for statements that are both true *and* false. We won't be dealing with those in this textbook, however. For our purposes, there are two truth values, "true" and "false," and every statement has exactly one of these. Logical systems like ours are called BIVALENT.

Tautology, contingent statement, contradiction

In considering arguments formally, we care about what would be true *if* the premises were true. Generally, we are not concerned with the actual truth value of any particular statements—whether they are *actually* true or false. Yet there are some statements that must be true, just as a matter of logic.

Consider these statements:

(a) It is raining.

(b) Either it is raining, or it is not.

(c) It is both raining and not raining.

In order to know if statement (a) is true, you would need to look outside or check the weather channel. Logically speaking, it might be either true or false. Statements like this are called *contingent* statements.

Statement (b) is different. You do not need to look outside to know that it is true. Regardless of what the weather is like, it is either raining or not. If it is drizzling, you might describe it as partly raining or in a way raining and a way not raining. However, our assumption of bivalence means that we have to draw a line, and say at some point that it is raining. And if we have not crossed this line, it is not raining. Thus the statement "either it is raining or it is not" is always going to be true, no matter what is going on outside. A statement that has to be true, as a matter of logic is called a TAUTOLOGY or logical truth.

You do not need to check the weather to know about statement (c), either. It must be false, simply as a matter of logic. It might be raining here and not raining across town, it might be raining now but stop raining even as you read this, but it is impossible for it to be both raining and not raining here at this moment. The third statement is *logically false*; it is false regardless of what the world is like. A logically false statement is called a CONTRADICTION.

We have already said that a contingent statement is one that could be true, or could be false, as far as logic is concerned. To be more precise, we should define a CONTINGENT STATEMENT as a statement that is neither a tautology nor a contradiction. This allows us to avoid worrying about what it means for something to be logically possible. We can just piggyback on the idea of being logically necessary or logically impossible.

A statement might *always* be true and still be contingent. For instance, it may be the case that in no time in the history of the universe was there ever an elephant with tiger stripes. Elephants only ever evolved on Earth, and there was never any reason for them to evolve tiger stripes. The statement "Some elephants have tiger stripes," is therefore always false. It is, however, still a contingent statement. The fact that it is always false is not a matter of logic. There is no contradiction in considering a possible world in which elephants evolved tiger stripes, perhaps to hide in really tall grass. The important question is whether the statement *must* be true, just on account of logic.

When you combine the idea of tautologies and contradictions with the notion of deductive validity, as we have defined it, you get some curious results. For one thing, any argument with a tautology in the conclusion will be valid, even if the premises are not relevant to the conclusion. This argument, for instance, is valid.

P_1: There is coffee in the coffee pot.

P_2: There is a dragon playing bassoon on the armoire.

C: All bachelors are unmarried men.

The statement "All bachelors are unmarried men" is a tautology. No matter what happens in the world, all bachelors have to be unmarried men, because that is how the word "bachelor" is defined. But if the conclusion of the argument is a tautology, then there is no way that the premises could be true and the conclusion false. So the argument must be valid.

Even though it is valid, something seems really wrong with the argument above. The premises are not relevant to the conclusion. Each sentence is about something completely different. This notion of relevance, however, is something that we don't have the ability to capture in the kind of simple logical systems we will be studying. The logical notion of validity we are using here will not capture everything we like about arguments.

Another curious result of our definition of validity is that any argument with a contradiction in the premises will also be valid. In our kind of logic, once you assert a contradiction, you can say anything you want. This is weird, because you wouldn't ordinarily say someone who starts out with contradictory premises is arguing well. Nevertheless, an argument with contradictory premises is valid.

Logically Equivalent and Contradictory Pairs of Sentences

We can also ask about the logical relations *between* two statements. For example:

(a) John went to the store after he washed the dishes.

(b) John washed the dishes before he went to the store.

These two statements are both contingent, since John might not have gone to the store or washed dishes at all. Yet they must have the same truth value. If either of the statements is true, then they both are; if either of the statements is false, then they both are. When two statements necessarily have the same truth value, we say that they are LOGICALLY EQUIVALENT.

On the other hand, if two sentences must have opposite truth values, we say that they are CONTRADICTORIES. Consider these two sentences

(a) Susan is taller than Monica.

(b) Susan is shorter or the same height as Monica.

One of these sentences must be true, and if one of the sentences is true, the other one is false. It is important to remember the difference between a single sentence that is a *contradiction* and a pair of sentences that are *contradictory.* A single sentence that is a contradiction is in conflict with itself, so it is never true. When a pair of sentences is contradictory, one must always be true and the other false.

Consistency

Consider these two statements:

(a) My only brother is taller than I am.

(b) My only brother is shorter than I am.

Logic alone cannot tell us which, if either, of these statements is true. Yet we can say that *if* the first statement (a) is true, *then* the second statement (b) must be false. And if (b) is true, then (a) must be false. It cannot be the case that both of these statements are true. It is possible, however that both statements can be false. My only brother could be the same height as I am.

If a set of statements could not all be true at the same time, they are said to be INCONSISTENT. Otherwise, they are CONSISTENT.

We can ask about the consistency of any number of statements. For example, consider the following list of statements:

(a) There are at least four giraffes at the wild animal park.

(b) There are exactly seven gorillas at the wild animal park.

(c) There are not more than two Martians at the wild animal park.

(d) Every giraffe at the wild animal park is a Martian.

Statements (a) and (d) together imply that there are at least four Martian giraffes at the park. This conflicts with (c), which implies that there are no more than two Martian giraffes there. So the set of statements (a)–(d) is inconsistent. Notice that the inconsistency has nothing at all to do with (b). Statement (b) just happens to be part of an inconsistent set.

Sometimes, people will say that an inconsistent set of statements "contains a contradiction." By this, they mean that it would be logically impossible for all of the statements to be true at once. A set can be inconsistent even when all of the statements in it are either contingent or tautologous. When a single statement is a contradiction, then that statement alone cannot be true.

Key Terms

Artificial language

Consistent

Contingent statement

Contradiction

Contradictories

Formal language

Inconsistent

Logically equivalent

Natural language

Tautology

Truth value

Chapter 4

Categorical Logic

4.1 Quantified Categorical Statements

We saw that a statement was a unit of language that could be true or false. In this chapter and the next we are going to look at a particular kind of statement, called a quantified categorical statement, and begin to develop a formal theory of how to create arguments using these statements. This kind of logic is generally called "categorical" or "Aristotelian" logic, because it was originally invented by the great logician and philosopher Aristotle in the fourth century BCE. This kind of logic dominated the European and Islamic worlds for 20 centuries afterward, and was expanded in all kinds of fascinating ways, some of which we will look at here.

Consider the following propositions:

(a) All dogs are mammals.

(b) Most physicists are male.

(c) Few teachers are rock climbers.

(d) No dogs are cats.

(e) Some Americans are doctors.

(f) Some adults are not logicians.

(g) Thirty percent of Canadians speak French.

(h) One chair is missing.

These are all examples of quantified categorical statements. A QUANTIFIED CATEGORICAL STATEMENT is a statement that makes a claim about a certain quantity of the members of a class or group. (Sometimes we will just call these "categorical statements") Statement (a), for example, is about the class of dogs and the class of mammals. These statements make no mention of any particular members of the categories or classes or types they are about. The propositions are also *quantified* in that they state *how many* of the things in one class are also members of the other. For instance, statement (b) talks about *most* physicists, while statement (c) talks about *few* teachers.

Categorical statements can be broken down into four parts: the quantifier, the subject term, the predicate term, and the copula. The QUANTIFIER is the part of a categorical sentence that specifies a portion of a class. It is the "how many" term. The quantifiers in the sentences above are all, most, few, no, some, thirty percent, and one. Notice that the "no" in sentence (d) counts as a quantifier, the same way zero counts as a number. The subject and predicate terms are the two classes the statement talks about. The SUBJECT CLASS is the first class mentioned in a quantified categorical statement, and the PREDICATE CLASS is the second. In sentence (e), for instance, the subject class is the class of Americans and the predicate class is the class of doctors. The COPULA is simply the form of the verb "to be" that links subject and predicate. Notice that the quantifier is always referring to the subject. The statement "Thirty percent of Canadians speak French" is saying something about a portion of Canadians, not about a portion of French speakers.

Sentence (g) is a little different than the others. In sentence (g) the subject is the class of Canadians and the predicate is the class of people who speak French. That's not quite the way it is written, however. There is no explicit copula, and instead of giving a noun phrase for the predicate term, like "people who speak French," it has a verb phrase, "speak French." If you are asked to identify the copula and predicate for a sentence like this, you should say that the copula is implicit and transform the verb phrase into a noun phrase. You would do something similar for sentence (h): the subject term is "chair," and the predicate term is "things that are missing." We will go into more detail about these issues in Section 4.5.

In the previous chapter we noted that formal logic achieves content neutrality by replacing some or all of the ordinary words in a statement with symbols. For categorical logic, we are only going to be making one such substitution. Sometimes we will replace the classes referred to in a quantified categorical statement with capital letters that act as variables. Typically we will use the letter S when referring to the class in the subject term and P when referring to the predicate term, although sometimes more letters will be needed. Thus the sentence "Some Americans are doctors," above, will sometimes become "Some S are P." The sentence "No dogs are cats" will sometimes become"No S is P."

4.2 Quantity, Quality, Distribution, and Venn Diagrams

Ordinary English contains all kinds of quantifiers, including the counting numbers themselves. In this chapter and the next, however, we are only going to deal with two quantifiers: "all," and "some." We are restricting ourselves to the quantifiers "all" and "some" because they are the ones that can easily be combined to create valid arguments using the system of logic that was invented

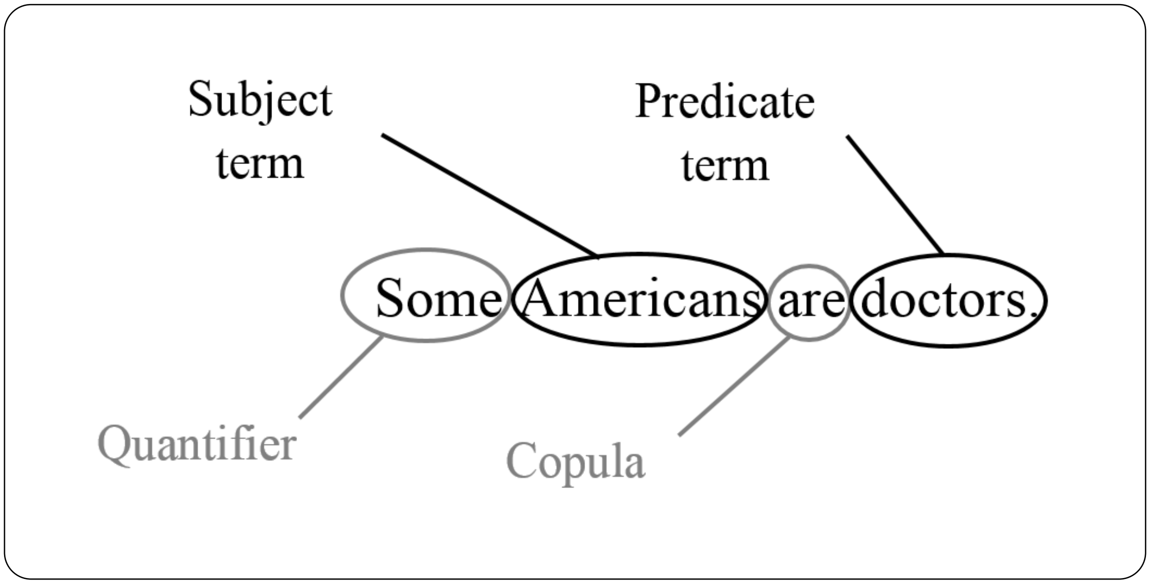


Figure 4.1: Parts of a quantified categorical statement.

by Aristotle.

The quantifier used in a statement is said to give the QUANTITY of the statement. Statements with the quantifier "All" are said to be "UNIVERSAL" and those with the quantifier "some" are said to be "PARTICULAR."

Here "some" will just mean "at least one." So, "some people in the room are standing" will be true even if there is only one person standing. Also, because "some" means "at least one," it is compatible with "all" statements. If I say "some people in the room are standing" it might actually be that *all* people in the room are standing, because if all people are standing, then at least one person is standing. This can sound a little weird, because in ordinary circumstances, you wouldn't bother to point out that something applies to some members of a class when, in fact, it applies to all of them. It sounds odd to say "*some* dogs are mammals," when in fact they *all* are. Nevertheless, when "some" means "at least one" it is perfectly true that some dogs are mammals.

In addition to talking about the quantity of statements, we will talk about their QUALITY. The quality of a statement refers to whether the statement is negated. Statements that include the words "no" or "not" are NEGATIVE, and other statements are AFFIRMATIVE. Combining quantity and quality gives us four basic types of quantified categorical statements, which we call the STATEMENT MOODS or just "moods." The four moods are labeled with the letters A, E, I, and O. Statements that are universal and affirmative are MOOD-A STATEMENTS. Statements that are universal and negative are MOOD-E STATEMENTS. Particular and affirmative statements are MOOD-I STATEMENTS, and particular and negative statements are MOOD-O STATEMENTS. (See Table 4.1.)

Mood	Form	Example
A	All S are P	All dogs are mammals.
E	No S are P	No dogs are reptiles.
I	Some S are P	Some birds can fly.
O	Some S are not P	Some birds cannot fly.

Table 4.1: The four moods of a categorical statement

Aristotle didn't actually use those letters to name the kinds of categorical propositions. His later followers writing in Latin came up with the idea. They remembered the labels because the "A" and the "I" were in the Latin word "**a**ff**i**rmo," ("I affirm") and the "E" and the "O" were in the Latin word "n**e**g**o**" ("I deny").

The DISTRIBUTION of a categorical statement refers to how the statement describes its subject and predicate class. A term in a sentence is said to be distributed if a claim is being made about the whole class. In the sentence "All dogs are mammals," the subject class, dogs, is distributed, because the quantifier "All" refers to the subject. The sentence is asserting that every dog out there is a mammal. On the other hand, the predicate class, mammals, is not distributed, because the sentence isn't making a claim about all the mammals. We can infer that at least some of them are dogs, but we can't infer that all of them are dogs. So in mood-A statements, only the subject is distributed.

On the other hand, in an I sentence like "Some birds can fly" the subject is not distributed. The quantifier "some" refers to the subject, and indicates that we are not saying something about all of that subject. We also aren't saying anything about all flying things, either. So in mood-I statements, neither subject nor predicate is distributed.

Even though the quantifier always refers to the subject, the predicate class can be distributed as well. This happens when the statement is negative. The sentence "No dogs are reptiles" is making a claim about all dogs: they are all not reptiles. It is also making a claim about all reptiles: they are all not dogs. So mood-E statements distribute both subject and predicate. Finally, negative particular statements (mood-O) have only the predicate class distributed. The statement "some birds cannot fly" does not say anything about all birds. It does, however say something about all flying things: the class of all flying things excludes some birds.

The quantity, quality, and distribution of the four forms of a categorical statement are given in Table 4.2. The general rule to remember here is that universal statements distribute the subject, and negative statements distribute the predicate.

In 1880 English logician John Venn published two essays on the use of diagrams with circles to represent categorical propositions (Venn 1880a, 1880b). Venn noted that the best use of such diagrams so far had come from the brilliant Swiss mathematician Leonhard Euler, but they still had many problems, which Venn felt could be solved by bringing in some ideas about logic from his fellow English logician George Boole. Although Venn only claimed to be building on the long

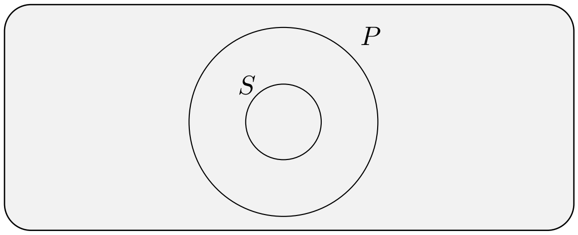


Figure 4.2: Euler Circles

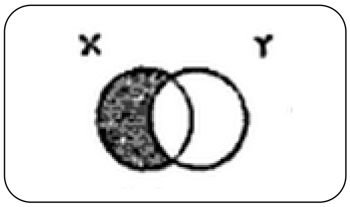


Figure 4.3: Venn's original diagram for an mood-A statement (Venn 1880a).

logical tradition he traced, since his time these kinds of circle diagrams have been known as VENN DIAGRAMS.

In this section we are going to learn to use Venn diagrams to represent our four basic types of categorical statement. Later in this chapter, we will find them useful in evaluating arguments. Let us start with a statement in mood A: "All S are P." We are going to use one circle to represent S and another to represent P. There are a couple of different ways we could draw the circles if we wanted to represent "All S are P." One option would be to draw the circle for S entirely inside the circle for P, as in Figure 4.2

It is clear from Figure 4.2 that all S are in fact P. And outside of college logic classes, you may have seen people use a diagram like this to represent a situation where one group is a subclass of another. You may have even seen people call concentric circles like this a Venn diagram. But Venn did not think we should put one circle entirely inside the other if we just want to represent

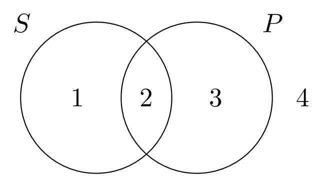
Mood	Form	Quantity	Quality	Terms Distributed
A	All S are P	Universal	Affirmative	S
E	No S are P	Universal	Negative	S and P
I	Some S are P	Particular	Affirmative	None
O	Some S are not P	Particular	Negative	P

Table 4.2: Quantity, quality, and distribution.

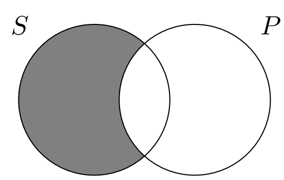
"All S is P." Technically speaking Figure 4.2 shows Euler circles.

Venn pointed out that the circles in Figure 4.2 don't just say that "All S are P." They also says that "All P are S" is false. But we don't necessarily know that if we have only asserted "All S are P." The statement "All S are P" leaves it open whether the S circle should be smaller than or the same size as the P circle.

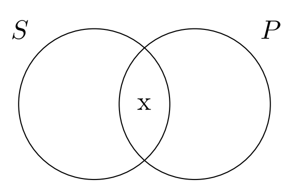
Venn suggested that to represent just the content of a single proposition, we should always begin by drawing partially overlapping circles. This means that we always have spaces available to represent the four possible ways the terms can combine:



Area 1 represents things that are S but not P; area 2, things that are S and P; area 3, things that are just P; and area 4 represents things that are neither S nor P. We can then mark up these areas to indicate whether something is there or could be there. We shade a region of the diagram to represent the claim that nothing can exist in that region. For instance, if we say "All S are P," we are asserting that nothing can exist that is in the S circle unless it is also in the P circle. So we shade out the part of the S circle that doesn't overlap with P.



If we want to say that something does exist in a region, we put an "x" in it. This is the diagram for "Some S are P":



If a region of a Venn diagram is blank, if it is neither shaded nor has an x in it, it could go either way. Maybe such things exist, maybe they do not.

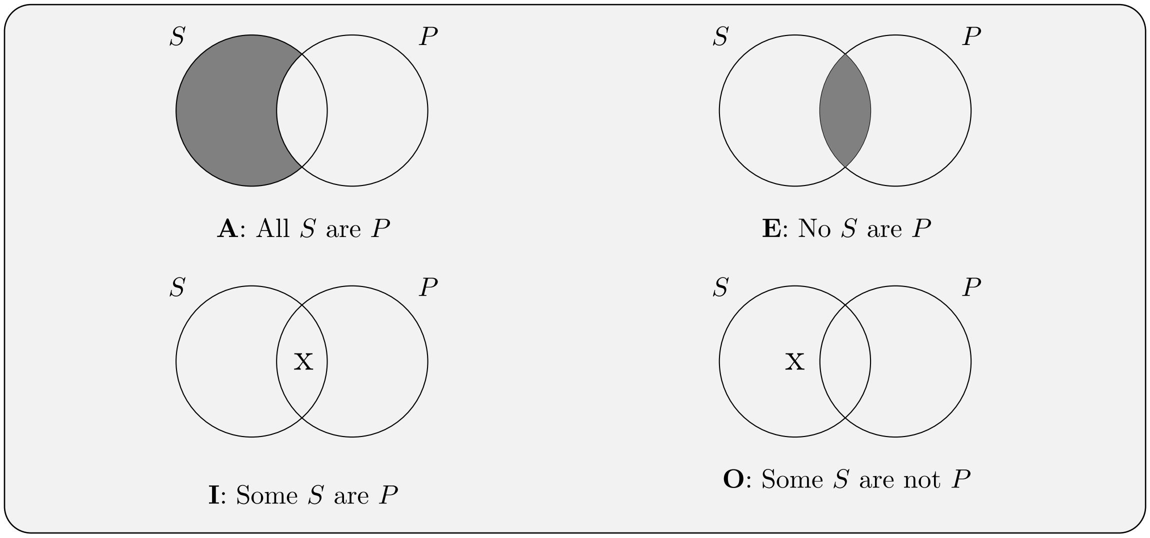
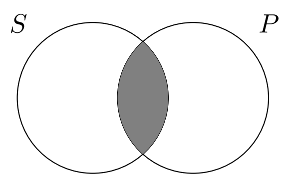


Figure 4.4: Venn Diagrams for the Four Basic Forms of a Categorical Statement

The Venn diagrams for all four basic forms of categorical statements are in Figure 4.4. Notice that when we draw diagrams for the two universal forms, A and E, we do not draw any x's. For these forms we are only ruling out possibilities, not asserting that things actually exist. This is part of what Venn learned from Boole, and we will see its importance in Section **??**.

Finally, notice that so far, we have only been talking about categorical statements involving the variables S and P. Sometimes, though, we will want to represent statements in regular English. To do this, we will include a dictionary saying what the variables S and P represent in this case. For instance, this is the diagram for "No dogs are reptiles."



S: Dogs
P: Reptiles

Evaluating Short Arguments

In the real world, the inferences we make are messy and hard to classify. Right now we are just going to deal with a limited subset of inferences. Let's start start with the uncontroversial premise

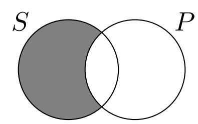
"All dogs are mammals." Can we infer from this that all non-mammals are non-dogs? In canonical form, the argument would look like this.

P_1: All dogs are mammals

C: All non-mammals are non-dogs.

Evaluating an immediate inference like this is a four step process. First, identify the subject and predicate classes. Second, draw the Venn diagram for the premise. Third, see if the Venn diagram shows that the conclusion must be true. If it must be, then the argument is valid. Finally, if the argument is valid, identify the process that makes it valid. (You can skip this step if the argument is invalid.)

For the argument above, the result of the first two steps would look like this:



S: Dogs
P: Mammals

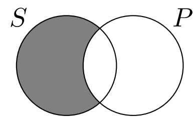
The Venn diagram for the premise shades out the possibility that there are dogs that aren't mammals. For step three, we ask, does this mean the conclusion must be true? In this case, it does. The same shading implies that everything that is not a mammal must also not be a dog. In fact, the Venn diagram for the premise and the Venn diagram for the conclusion are the same. So the argument is valid. This means that we must go on to step four and identify the process that makes it valid. In this case, the conclusion is created by reversing subject and predicate and taking their complements, which means that this is a valid argument by contraposition.

Now, remember what it means for an argument to be valid. As we said on page 22, an argument is valid if it is impossible for the premises to be true and the conclusion false. This means that we can have a valid argument with false premises, so long as it is the case that *if* the premises were true, the conclusion would have to be true. So if the argument above is valid, then so is this one:

P_1: All dogs are reptiles.

C: All non-reptiles are non-dogs.

The premise is now false: all dogs are not reptiles. However, *if* all dogs were reptiles, then it would also have to be true that all non-reptiles are non-dogs. The Venn diagram works the same way.



S: Dogs
P: Reptiles

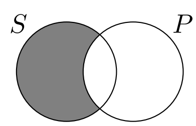
The Venn diagram for the premise still matches the Venn diagram for the conclusion. Only the labels have changed. The fact that this argument form remains true even with a false premise is just a variation on a theme we saw in Figure 2.7 when we saw a valid argument (with false premises) for the conclusion "Socrates is a carrot." So arguments by transposition, just like any argument, can be valid even if they have false premises. The same is true for arguments by conversion and obversion.

Arguments like these can also be invalid, even if they have true premises and a true conclusion. Remember that A statements are not logically equivalent to their converse. So this is an invalid argument with a true premise and a false conclusion:

P_1: All dogs are mammals.

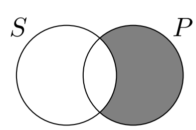
C: All mammals are dogs.

Our Venn diagram test shows that this is invalid. Steps one and two give us this for the premise:



S: Dogs
P: Mammals

But this is the Venn diagram for the conclusion:



S: Dogs
P: Mammals

This is an argument by conversion on an mood-A statement, which is invalid. The argument remains invalid, even if we substitute in a predicate where the conclusion happens to be true. For instance this argument is invalid.

P_1: All dogs are *Canis familiaris.*

C: All *Canis familiaris* are dogs.

The Venn diagrams for the premise and conclusion of this argument will be just like the ones for the previous argument, just with different labels. So even though the argument has a true premise and a true conclusion, it is still invalid, because it is possible for an argument of this form to have a true premise and a false conclusion. This is an unreliable argument form that just happened, in this instance, not to lead to a false conclusion. This again is just a variation on a theme we saw in *The Basics of Evaluating Argument*, when we saw an invalid argument for the conclusion that Paris was in France.

4.3 Aristotelian Syllogism

So far we have just been looking at very short arguments using categorical statements. The arguments just had one premise and a conclusion that was often logically equivalent to the premise. For most of the history of logic in the West, however, the focus has been on arguments that are a step more complicated called CATEGORICAL SYLLOGISMS. A categorical syllogism is a two-premise argument composed of categorical statements. Aristotle began the study of this kind of argument in his book the *Prior Analytics* (c.350 BCE/1984b). This work was refined over the centuries by many thinkers in the Pagan, Christian, Jewish, and Islamic traditions until it reached the form it is in today.

There are actually all kinds of two-premise arguments using categorical statements, but Aristotle only looked at arguments where each statement is in one of the moods A, E, I, or O. The arguments also had to have exactly three terms, arranged so that any two pairs of statements will share one term. Let's call a categorical syllogism that fits this more narrow description an ARISTOTELIAN SYLLOGISM Here is a typical Aristotelian syllogism using only mood-A sentences:

P_1: All mammals are vertebrates.
P_2: All dogs are mammals.

C: All dogs are vertebrates.

Notice how the statements in this argument overlap each other. Each statement shares a term with the other two. Premise 2 shares its subject term with the conclusion and its predicate with Premise 1. Thus there are only three terms spread across the three statements. Aristotle dubbed these the major, middle, and minor premises, but there was initially some confusion about how to define them. In the 6th century, the Christian philosopher John Philoponus, drawing on the work of his pagan teacher Ammonius, decided to arbitrarily designate the MAJOR TERM as the predicate of the conclusion, the MINOR TERM as the subject of the conclusion, and the MIDDLE TERM as the one term of the Aristotelian syllogism that does not appear in the conclusion. So in the argument above, the major term is "vertebrate," the middle term is "mammal," and the minor term is "dog." We can also define the MAJOR PREMISE as the one premise in an Aristotelian syllogism that names the major term, and the MINOR PREMISE as the one premise that names the minor term. So in the argument above, Premise 1 is the major premise and Premise 2 is the minor premise.

4.4 Validity and the Counterexample Method

Let's go back again to the definition of validity. (It is always good for beginning student to reinforce their understanding of validity). We did this in the last chapter in Section 4.2, and we are doing it again now. Validity is a fundamental concept in logic that can be confusing. A valid argument is not necessarily one with true premises or a true conclusion. An argument is valid if the premises *would* make the conclusion true *if* the premises were true.

This means that, as we have seen before, there can be valid arguments with false conclusions. Consider this argument:

> No reptiles are chihuahuas. But all dogs are reptiles. Therefore, no dogs are chihuahuas.

This seems silly, if only because the conclusion is false. We know that some dogs are chihuahuas. But the argument is still valid. In fact, it shares a form with an argument that makes perfect sense:

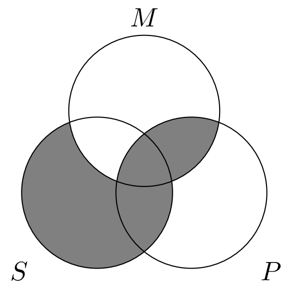
> No reptiles are dogs, but all chameleons are reptiles. Therefore, no dogs are chameleons.

Both of these arguments have the following form:

P_1: No M are P.
P_2: All S are M.

C: No S are P.

This form is valid, whether the subject and predicate term are dogs and chameleons, or dogs and chihuahuas, which you can see from this Venn diagram.



This means you can't assume an argument is invalid because it has a false conclusion. The reverse is also true. You can't assume an argument is valid just because it has a true conclusion. Consider this

> All cats are animals, and some animals are dogs. Therefore no dogs are cats.

Makes sense, right? Everything is true. But the argument isn't valid. The premises aren't making the conclusion true. Other arguments with the same form have true premises and a false conclusion. Like this one.

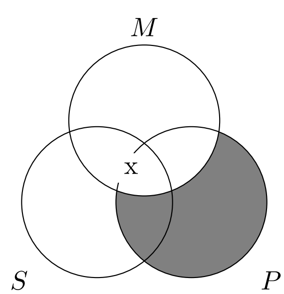
> All chihuahuas are animals, and some animals are dogs. Therefore, no dogs are chihuahuas.

Really, the arguments in both these passages have the same form:

P_1: All P are M.
P_2: Some M are S.

C: No S are P.

This in an invalid form, and it remains invalid whether the P stands for cats or chihuahuas. You can see this in the Venn diagram:



All these examples bring out an important fact about the kind of logic we are doing in this chapter and the last one: this is *formal* logic. As we discussed, formal logic is a way of making our investigation content neutral. By using variables for terms instead of noun phrases in English we can show that certain ways of arguing are good or bad regardless of the topic being argued about. Parts

The examples above also show us another way of proving that an argument given in English is invalid, called the counterexample method. As we have just seen, if you are given an argument in English with, say, false premises and a false conclusion, you cannot determine immediately whether the argument is valid. However, we can look at arguments that have the same form, and use them to see whether the argument is valid. If we can find an argument that has the exact same form as a given argument but has true premises and a false conclusion, then we know the argument is invalid. We just did that with the argument above. We were given an argument with true premises and a true conclusion involving cats, dogs, and animals. We were able to show this argument invalid by finding an argument with the same form that has true premises and a false conclusion, this time involving chihuahuas, dogs, and animals.

More precisely, we can define the COUNTEREXAMPLE METHOD as a method for determining if an argument with ordinary English words for terms is valid, where one consistently substitutes other English terms for the terms in the given argument to see if one can find an argument with the same form that has true premises and a false conclusion. Let's run through a couple more examples to see how this works.

First consider this argument in English:

> All tablet computers are computers. We know this because a computer is a kind of machine, and some machines are not tablet computers.

Every statement in this argument is true, but it doesn't seem right. The premises don't really relate to the conclusion. That means you can probably find an argument with the same form that has true premises and a false conclusion. Let's start by putting the argument in canonical form. Notice that the English passage had the conclusion first.

P_1: All computers are machines.
P_2: Some machines are not tablet computers.

C: All tablet computers are computers.

Let's find substitutes for "machines," "computers," and "tablet computers" that will give us true premises and a false conclusion. It is easiest to work with really common sense categories, like "dog" and "cat." It is also easiest to start with a false conclusion and then try to find a middle term that will give you true premises. "All dogs are cats" is a nice false conclusion to start with:

P_1: All cats are M.

P_2: Some M are not dogs.

C: All dogs are cats.

So what can we substitute for M (which used to be "machines") that will make P_1 and P_2 true? "Animals" works fine.

P_1: All cats are animals.
P_2: Some animals are not dogs.

C: All dogs are cats.

There you have it: a counterexample that shows the argument invalid. Let's try another one.

> Some diseases are viral, therefore some things caused by bacteria are not things that are caused by viruses, because all diseases are bacterial.

This will take a bit more unpacking. You can see from the indicator words that the conclusion is in the middle. We also have to fix "viral" and "things that are caused by viruses" so they match, and the same is true for "bacterial" and "things that are caused by bacteria." Once we have the sentences in canonical form, the argument will look like this:

P_1: Some diseases are things caused by viruses.
P_2: All diseases are things causes by bacteria.

C: Some things caused by bacteria are not things caused by viruses.

Once you manage to think through the complicated wording here, you can see that P_1 and the conclusion are true. Some diseases come from viruses, and not everything that comes from a bacteria comes from a virus. But P_2 is false. All diseases are not caused by bacteria. In fact, P_1 contradicts P_2. But none of this is enough to show the argument is invalid. To do that, we need to find an argument with the same form that has true premises and a false conclusion.

Let's go back to the simple categories: "dogs," "animals," etc. We need a false conclusion. Let's go with "Some dogs are not animals."

P_1: Some M are dogs.
P_2: All M are animals.

C: Some dogs are not animals.

We need a middle term that will make the premises true. It needs to be a class that is more general than "dogs" but more narrow than "animals." "Mammals" is a good standby here.

P_1: Some mammals are dogs.

P_2: All mammals are animals.

C: Some dogs are not animals.

And again we have it, a counterexample to the given syllogism.

4.5 Transforming English into Logically Structured English

Because the four basic forms are stated using variables, they have a great deal of generality. We can expand on that generality by showing how many different kinds of English sentences can be transformed into sentences in our four basic forms. We already touched on this a little in section 4.1, when we look at sentences like "Thirty percent of Canadians speak French." There we saw that the predicate was not explicitly a class. We needed to change "speak French" to "people who speak French." In this section, we are going to expand on that to show how ordinary English sentences can be transformed into something we will call "logically structured English." LOGICALLY STRUCTURED ENGLISH is English that has been put into a standardized form that allows us to see its logical structure more clearly and removes ambiguity. Doing this is a step towards the creation of formal languages, which we will start doing soon.

Transforming English sentences into logically structured English is fundamentally a matter of understanding the meaning of the English sentence and then finding the logically structured English statements with the same or similar meaning. Sometimes this will require judgment calls. English, like any natural language, is fraught with ambiguity. One of our goals with logically structured English is to reduce the amount of ambiguity. Clarifying ambiguous sentences will always require making judgments that can be questioned. Things will only get harder when we start using full blown formal languages, which are supposed to be completely free of ambiguity.

To transform a quantified categorical statement into logically structured English, we have to put all of its elements in a fixed order and be sure they are all of the right type. All statements must begin with the quantifies "All" or "Some" or the negated quantifier "No." Next comes the subject term, which must be a plural noun, a noun phrase, or a variable that stands for any plural noun or noun phrase. Then comes the copula "are" or the negated copula "are not." Last is the predicate term, which must also be a plural noun or noun phrase. We also specify that you can only say "are not" with the quantifier "some," that way the universal negative statement is always phrased "No S are P," not "All S are not P." Taken together, these criteria define the STANDARD FORM FOR A CATEGORICAL STATEMENT in logically structured English.

The subsections below identify different kinds of changes you might need to make to put a statement into logically structured English. Sometimes translating a sentence will require using multiple changes.

Nonstandard Verbs

In section 4.1 we saw that "Some Canadians speak French" has a verb phrase "speaks French" instead of a copula and a plural noun phrase. To transform these sentences into logically structured English, you need to add the copula and turn all the terms into plural nouns or plural noun phrases.

Below are some examples

English	Logically Structured English
No cats bark.	No cats are animals that bark.
All birds can fly.	All birds are animals that can fly.
Some thoughts should be left unsaid.	Some thoughts are things that should be left unsaid.

Adding a plural noun phrase means you have to come up with some category, like "people" or "animals." When in doubt, you can always use the most general category, "things."

Implicit Noun Phrases

Sometimes you just have an adjective for the predicate, and you need to turn it into a noun, as in the examples below.

English	Logically Structured English
Some roses are red.	Some roses are red flowers.
Football players are strong.	All football players are strong persons.
Some names are hurtful.	Some names are hurtful things.

Again, you will have to come up with a category for the predicate, and when it doubt, you can just use "things."

Unexpressed Quantifiers

Sometimes categorical generalizations come without an explicit quantifier, which you need to add.

English	Logically Structured English
Boots are footwear.	All boots are footwear.
Giraffes are tall.	All giraffes are tall things.
A dog is not a cat.	No dogs are cats.

English	Logically Structured English
A lion is a fierce creature.	All lions are fierce creatures.

Notice that in the second sentence we had to make two changes, adding both the words "All" and "things."

In the last two sentences, the indefinite article "a" is being used to create a kind of generic sentence. Not all sentences using the indefinite article work this way. If a story begins "A man is walking down the street," it is not talking about all men generically. It is introducing some specific man. For this kind of statement, see the subsection on singular propositions. You will have to use your good judgment and understanding of context to know how the indefinite article is being used.

Nonstandard Quantifiers

English has many alternate ways of saying "all" and "some." You need to change these when translating to logically structured English.

English	Logically Structured English
Every day is a blessing.	All days are blessings.
Whatever is a dog is not a cat.	No dogs are cats.
Not a single dog is a cat.	No dogs are cats.
There are Americans that are doctors.	Some Americans are doctors.
Someone in America is a doctor.	Some Americans are doctors.
At least a few Americans are doctors.	Some Americans are doctors.
Not everyone who is an adult is a logician.	Some adults are not logicians.
Most people with a PhD in psychology are female.	Some people with a PhD in psychology are female.
Among the things that Sylvia inherited was a large mirror	Some things that Sylvia inherited were large mirrors

Notice in the last case we are losing quite a bit of information when we transform the sentence into logically structured English. "Most" means more that fifty percent, while "some" could be any percentage less than a hundred. This is simply a price we have to pay in creating a standard logical form. As we will see when we move to constructing artificial languages in later chapters, no logical language has the expressive richness of a natural language.

Singular Propositions

Aristotle treated sentences about individual things, like specific people, differently than either general or particular categorical statements. A statement like "Socrates is mortal," for Aristotle, was neither A, E, I, nor O. We can expand the power of logically structured English by bringing these kind of singular propositions into our system of categorical propositions. Using phrases like "All things identical to…" we can turn singular terms into general ones.

English	Logically Structured English
Socrates is mortal.	All persons identical with Socrates are mortal.
The Empire State Building is tall.	All things identical to The Empire State Building are tall things.
Ludwig was not happy.	No persons identical with Ludwig are happy.
A man is walking down the street.	Some men are things that are walking down the street.

Adverbs and Pronouns

In English we use specific adverbs like "everywhere" and "always" to create quantified statements about place and time. We can transform these into logically structured English by talking about "all places" or "all times" and things like that. English also has specific pronouns for quantified statements about people or things, such as "everyone" or "nothing."

English	Logically Structured English
"Whenever you need me, I'll be there." – Michael Jackson	All times that you need me are times that I will be there.
"We are never, ever, ever getting back together." – Taylor Swift	No times are times when we will get back together.
"Whoever fights with monsters should be careful lest he thereby become a monster." –Friedrich Nietzsche	All persons who fight with monsters are persons who should be careful lest they become a monster.
"What does not destroy me, makes me stronger." –Friedrich Nietzsche	All things that do not destroy me are things that make me stronger.

Conditional Statements

A conditional is a statement of the form "If … then … ." They will become a big focus of our attention starting in Chapter *Sentential Logic* when we begin introducing modern formal languages. They are not given special treatment in the Aristotelian tradition, however. Instead, where we can, we just treat them as categorical generalizations:

English	Logically Structured English
If something is a cat, then it is a feline.	All cats are feline.

English	Logically Structured English
If something is a dog, then it's not a cat.	No dogs are cats.

Exclusive Propositions

Phrases like "only," "none but," or "none except" are used in English to create exclusive propositions. They are applied to the predicate term and exclude everything but the predicate term from the subject term.

English	Logically Structured English
Only people over 21 may drink.	All people who drink are over 21.
No one, except those with a ticket, may enter the theater.	All people who enter the theater have a ticket.
None but the strong survive.	All people who survive are strong people.

The important thing to see here is that words like "only" are actually modifying the predicate, and not the subject. So when you translate them into logically structured English, the order of the words often gets turned around. In "only people over 21 may drink" the predicate is actually "people over 21."

"The Only"

Sentences with "The only" are a little different than sentences that just have "only" in them. The sentence "Humans are the only animals that talk on cell phones" should be translated as "All animals who talk on cell phones are humans." In this sentence, "the only" introduces the subject, rather than the predicate.

English	Logically Structured English
Humans are the only animals who talk on cell phones.	All animals who talk on cell phones are human.
Shrews are the only venomous mammal in North America.	All venomous mammals in North America are shrews.

Key Terms

Affirmative

Complement

Contradictories

Contraposition

Contraries

Converse

Copula

Distribution

Existential import

Logically structured English

Mood-A statement

Mood-E statement

Mood-I statement

Mood-O statement

Negative

Obverse

Particular

Predicate class

Quality

Quantified categorical statement

Quantifier

Quantity

Square of opposition

Standard form categorical statement

Subalternation

Subcontraries

Subject class

Truth value

Universal

Vacuous truth

Venn diagram

Appendix A

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