Quantum correlations between two non-interacting atoms under the influence of a thermal environment*

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By considering a double Jaynes–Cummings model, we investigate the dynamics of quantum correlations, such as the quantum discord and the entanglement, for two atoms in their respective noisy environments, and study the effect of the purity and the cavity temperature on the quantum correlations. The results show that the entanglement suffers sudden death and revival, however the quantum discord can still reveal the quantum correlations between the two atoms in the region where the entanglement is zero. Moreover, when the temperature of each cavity is high the entanglement dies out in a short time, but the quantum discord still survives for quite a long time. It means that the quantum discord is more resistant to environmental disturbance than the entanglement at higher temperatures.

Keywords: quantum discord, entanglement, double Jaynes–Cummings model

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1. Introduction

Entanglement, quantum discord and quantum dissension of a given system are different ways of charecterizing the quantum correlations presented in that system. The quantum entanglement corresponds to the global states of a composite system, which cannot be written as a product of the states of the individual subsystems. This characteristic yields correlations between quantum systems that cannot be found in any classical system. Moreover, entanglement, as a new quantum resource which cannot be performed by means of classical resources, plays a key role in quantum information processing.[1−4] The quantum discord is an independent measure of the quantum correlations, which might include the entanglement and is defined as the difference between the total and the classical correlations for a given state. It has a significant application in the deterministic quantum computation with one qubit (DQC1).[5−7] Moreover, the quantum discord can also be used to measure the quantum correlation between relatively accelerated observers,[8] to get a better understanding of the quantum phase transition,[9] and to define the class of the initial system-bath states for which the quantum dynamics is equivalent to a completely positive map.[10] Very recently, the quantum discord has been investigated widely due to its potential value in quantum information theory.[11−17] Quantum dissension is a similar notion of quantum discord but excludes entanglement, and is used to quantify the quantum correlation in multipartite systems.[18,19] The present paper studies both the entanglement and the discord dynamics of two isolated two-level atoms (such as two qubits located anywhere in a large network[20,21]) each separately interacting with a cavity field that is initially in the single-mode thermal state. A thermal field, which frequently appears in problems of decoherence, is a highly chaotic field and provides us with minimal information about the field. It can help us to understand the decoherence mechanisms and to simulate the interaction of the environment (thermal noise) with the quantum bits (atoms interacting in a cavity, for example).

As we all know, a realistic quantum system unavoidably interacts with the environment, which can

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lead to noise related decoherence, and entanglement is very sensitive to the system–environment interaction and the initial state of the system. When we consider an initial entangled state exposed to local noisy environments the phenomenon of entanglement sudden death (ESD) often occurs, due to the decoherence.[20−23] Fortunately, it has been demonstrated that the mixed separable (unentangled) states can also have nonclassical correlation[24−27] and the use of such states can improve some computational tasks (compared to the classical case).[5−7] Such a performance improvement is attributed to the correlations not presented in the classical systems. These quantum correlations can be identified through quantum discord. Recently, much research has shown that quantum discord is more robust against decoherence than entanglement.[28,29] It is natural for us to study the quantum correlations under the influence of the mixed thermal environment. Despite the fact that a thermal field alone may induce an entanglement between two atoms,[30] the thermal noise generally has destructive effects. Therefore, it would be interesting to investigate the quantum correlation of two noninteracting atoms under the influence of the thermal environment. Is there any difference between quantum discord and entanglement in this case? To the author's knowledge, these problems have never been discussed before. In the present paper, we consider a double Jaynes–Cummings (JC) model system,[20,21] which consists of two physically independent ideal cavities, each containing a two-level atom. Thereafter, there is no physical interaction between the atoms, and with each interacting solely with the corresponding cavity field. The two atoms have been initially prepared in the Werner mixed state, and each cavity field is in the single-mode thermal state. We study both the entanglement and the discord dynamics of the two atoms under the same conditions, and compare their time dependence characteristics. We also investigate the dynamics of the quantum discord of two qubits, in which the ESD can occur, and analyze the effects of the purity and the temperature (mixedness) on the quantum correlation.

2. Measurements of quantum correlation

In this section, we briefly review the measurements of the quantum correlation considered in the present study.

2.1. Quantum entanglement

Several different measurements have been proposed to identify the entanglement between two qubits, and we choose the Wootters entanglement measurement.[31,32] The concurrence C is defined as

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \qquad (1)$$

where $\lambda_1, \dots, \lambda_4$ are the eigenvalues of matrix $\tilde{\rho} = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, in which ρ is the density matrix representing the quantum state, and the matrix elements are taken in the basis of $|ee\rangle$, $|eg\rangle$, $|ge\rangle$, $|gg\rangle$. The range of the concurrence is from 0 to 1. For the unentangled qubits, $C = 0$, whereas $C = 1$ for the maximally entangled qubits.

2.2. Quantum discord

The quantum discord, first introduced by Ollivier and Zurek,[24] is defined as the difference between the total correlation and the classical correlation

$$\mathcal{D}(\rho_{\rm AB}) = \mathcal{I}(\rho_{\rm AB}) - \mathcal{Q}(\rho_{\rm AB}). \qquad (2)$$

Here, the total correlation between two subsystems A and B of a bipartite quantum system $\rho_{\rm AB}$ is measured by the quantum mutual information

$$\mathcal{I}(\rho_{\rm AB}) = S(\rho_{\rm A}) + S(\rho_{\rm B}) - S(\rho_{\rm AB}), \qquad (3)$$

where $S(\rho_{\rm AB}) = -\operatorname{Tr}(\rho_{\rm AB}\log_2 \rho_{\rm AB})$ is the Von Neumann entropy,[33] $\rho_{\rm A} = \operatorname{Tr}_{\rm B}(\rho_{\rm AB})$ and $\rho_{\rm B} = \operatorname{Tr}_{\rm A}(\rho_{\rm AB})$ are the reduced density operators of subsystems A and B, respectively. The measurement of the classical correlation is introduced implicitly by Olliver and Zurek and interpreted explicitly by Henderson and Vedral.[34] The classical correlation between two subsystems A and B may be defined as

$$\mathcal{Q}(\rho_{\rm AB}) = \max_{\{\Pi_k\}} \Big[S(\rho_{\rm A}) - \sum_k p_k S(\rho_k)\Big], \qquad (4)$$

where $\{\Pi_k\}$ is a complete set of projectors to measure subsystem B, $\rho_k = \operatorname{Tr}_{\rm B}[(I_{\rm A} \otimes \Pi_k)\rho_{\rm AB}(I_{\rm A} \otimes \Pi_k)]/p_k$

is the state of subsystem A after the measurement resulting in outcome k with probability $p_k = \mathrm{Tr}_{\mathrm{AB}}[(I_{\mathrm{A}} \otimes \Pi_k)\rho_{\mathrm{AB}}(I_{\mathrm{A}} \otimes \Pi_k)]$, and I_{A} denotes the identity operator for subsystem A. Here, maximizing the quantity represents the most information gained about system A as a result of the perfect measurement $\{\Pi_k\}$.

It can be shown that the quantum discord is zero for states with only classical correlations and nonzero for states with quantum correlations.

3. Dynamics of quantum correlations

We consider a double JC model, which consists of two identical two-level atoms labeled A and B and two single-mode thermal cavity fields labeled a and b. Each atom–cavity system is isolated (atom A interacting only with field a, and similarly for atom B and field b). There is no coupling (direct or indirect interaction) between atoms A and B. The dynamics of this model is given by the double JC Hamiltonian

$$H = H_{\mathrm{A}} + H_{\mathrm{B}}. \tag{5}$$

The Hamiltonians (under the rotating wave approximation and assuming $\hbar = 1$) are[35]

$$\begin{aligned} H_j &= \omega_j \sigma_j^z + \nu_j a_j^+ a_j + g_j(a_j^+ \sigma_j^- + \sigma_j^+ a_j), \\ j &= \mathrm{A},\ \mathrm{B}, \end{aligned} \tag{6}$$

where ω_j is the transition frequency between the atomic excited state and the ground state, ν_j is the field frequency, and a_j^+ (a_j) is the photon creation (annihilation) operator of the cavity mode. The effective coupling constant between the cavity field and the atom is g_j, and σ_j^z, σ_j^+, σ_j^- are the atomic inversion, raising, and lowering operators, respectively. For simplicity, we assume that $g_{\mathrm{A}} = g_{\mathrm{B}} = g$, $\omega_{\mathrm{A}} = \omega_{\mathrm{B}} = \omega$, $\nu_{\mathrm{A}} = \nu_{\mathrm{B}} = \nu$, and $\omega = \nu$ (on-resonance interaction). Using the standard techniques, it can be shown that H_j gives rise to the following time evolution operator in the interaction picture:

$$\begin{aligned} U_{\mathrm{I}j} &= \cos(\sqrt{a_j^+ a_j + 1}gt)|e\rangle_j\langle e| \\ &\quad + \cos(\sqrt{a_j^+ a_j}gt)|g\rangle_j\langle g| \\ &\quad - \mathrm{i}\,\frac{\sin(\sqrt{a_j^+ a_j + 1}gt)}{\sqrt{a_j^+ a_j + 1}} a_j|e\rangle_j\langle g| \\ &\quad - \mathrm{i}\,\frac{\sin(\sqrt{a_j^+ a_j}gt)}{\sqrt{a_j^+ a_j}} a_j^+|g\rangle_j\langle e|. \end{aligned} \tag{7}$$

We assume that the two atoms are initially in the Werner mixed state[36]

$$\rho_{\mathrm{AB}}(0) = r|\Phi(0)\rangle_{\mathrm{ABAB}}\langle\Phi(0)| + \frac{1-r}{4} I_{\mathrm{AB}}, \tag{8}$$

where r indicates the purity of the initial state, I is a 4×4 identity matrix, and $|\Phi(0)\rangle_{\mathrm{AB}} = (|eg\rangle + |ge\rangle)/\sqrt{2}$ is a superposition of Bell states.

The two cavity fields a and b are initially in the single-mode thermal field states

$$\begin{aligned} \rho_{\mathrm{fa}}(0) &= \sum_n P_n|n\rangle\langle n|, \\ \rho_{\mathrm{fb}}(0) &= \sum_m P_m|m\rangle\langle m|. \end{aligned} \tag{9}$$

The weight functions P_n and P_m are

$$P_n = \frac{\bar{n}^n}{(1+\bar{n})^{n+1}}, \quad P_m = \frac{\bar{m}^m}{(1+\bar{m})^{m+1}}, \tag{10}$$

where

$$\bar{n} = [\exp(\hbar\omega_/\kappa_{\mathrm{B}}T_{\mathrm{a}}) - 1]^{-1}$$

and

$$\bar{m} = [\exp(\hbar\omega_/\kappa_{\mathrm{B}}T_{\mathrm{b}}) - 1]^{-1}$$

are the mean photon numbers of the two thermal cavity fields a and b, corresponding to equilibrium cavity temperatures T_{a} and T_{b}, respectively, and k_{B} is the Boltzmann constant.

The calculation of the density operator follows the traditional route in quantum optics. By using the evolution operator, we can generate the density operator evolution of the system at time t

$$\begin{aligned} &\rho(t) \\ &= U_{\mathrm{IA}} \otimes U_{\mathrm{IB}}[\rho_{\mathrm{AB}}(0) \otimes \rho_{\mathrm{fa}}(0) \otimes \rho_{\mathrm{fb}}(0)] U_{\mathrm{IB}}^+ \otimes U_{\mathrm{IA}}^+. \end{aligned} \tag{11}$$

We can obtain the reduced density matrix $\rho_{\mathrm{IAB}}(t)$ of the two atoms at time t by tracing over the field variables. In atomic bases $|ee\rangle$, $|eg\rangle$, $|ge\rangle$, $|gg\rangle$, the reduced density matrix $\rho_{\mathrm{IAB}}(t)$ can be written as

$$\rho_{\mathrm{IAB}}(t) = \mathrm{Tr}_{\mathrm{f}}[\rho(t)] = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{12}$$

where the matrix elements are expressed as

$$\rho_{11} = \Sigma_n \Sigma_m \Big(\frac{1-r}{4} P_n P_m C_{n+1}^2 C_{m+1}^2$$

$$+\frac{1+r}{4}P_nP_{m-1}C_{n+1}^2S_{m-1}^2$$
$$+\frac{1+r}{4}P_{n-1}P_mS_{n-1}^2C_{m+1}^2$$
$$+\frac{1-r}{4}P_{n-1}P_{m-1}S_{n-1}^2S_{m-1}^2\Big),$$

$$\rho_{22} = \Sigma_n\Sigma_m\Big(\frac{1-r}{4}P_nP_mC_{n+1}^2S_{m+1}^2 + \frac{1+r}{4}P_nP_mC_{n+1}^2C_m^2 + \frac{1+r}{4}P_{n-1}P_mS_{n-1}^2S_{m+1}^2 + \frac{1-r}{4}P_{n-1}P_mS_{n-1}^2C_m^2\Big),$$

$$\rho_{23} = \Sigma_n\Sigma_m\Big(\frac{r}{2}P_nP_mC_n^2C_m^2C_{n+1}^2C_{m+1}^2\Big),$$

$$\rho_{33} = \Sigma_n\Sigma_m\Big(\frac{1-r}{4}P_nP_mS_{n+1}^2C_{m+1}^2 + \frac{1+r}{4}P_nP_{m-1}S_{n+1}^2S_{m-1}^2 + \frac{1+r}{4}P_nP_mC_n^2C_{m+1}^2 + \frac{1-r}{4}P_nP_{m-1}C_n^2S_{m-1}^2\Big),$$

$$\rho_{32} = \rho_{23}^*, \quad \rho_{44} = 1-\rho_{11}-\rho_{22}-\rho_{33}. \tag{13}$$

Here $*$ stands for the complex conjugate, P_n and P_m are given by Eq. (10), and the time-dependent functions are given by

$$C_n = \cos[\sqrt{n}gt], \quad C_m = \cos[\sqrt{m}gt],$$
$$S_n = \sin[\sqrt{n}gt], \quad S_m = \sin[\sqrt{m}gt]. \tag{14}$$

The concurrence for the density matrix of Eq. (12) is given by

$$C(t) = 2\max\{0, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\}. \tag{15}$$

For the calculation of the amount of the discord, we propose a complete set of orthogonal projectors $\{\Pi_1 = |\theta_\parallel\rangle\langle\theta_\parallel|, \Pi_2 = |\theta_\perp\rangle\langle\theta_\perp|\}$ for a local measurement performed on subsystem B, where $|\theta_\parallel\rangle = \cos\theta|0\rangle + \mathrm{e}^{\mathrm{i}\phi}\sin\theta|1\rangle$ and $|\theta_\perp\rangle = -\cos\theta|1\rangle + \mathrm{e}^{-\mathrm{i}\phi}\sin\theta|0\rangle$. Then we can evaluate the maximum of $\mathcal{Q}(\rho_{\mathrm{AB}})$ numerically by varying angles θ and ϕ from 0 to 2π, respectively.

The dynamics of the quantum correlations of the two isolated two-level atoms versus gt are plotted in Fig. 1 for different values of purity r and given $\bar{n} = \bar{m} = 1$, and in Fig. 2 for different values of mean photon numbers and given $r = 1$. Both figures indicate that the entanglement suffers sudden death. In this case, the entanglement can terminate abruptly in finite time and will remain zero for a period of time before it recovers, and the time periods are related to the initial atomic state and the values of the mean photon numbers (corresponding to the cavity temperature). However, the quantum discord can still capture the quantum correlations between the two atoms in some states that are not entangled.

The influence of the purity on the quantum correlations is shown in Fig. 1 for $\bar{n} = \bar{m} = 1$. It is interesting to notice from Fig. 1(a) that the quantum discord is not zero for purity $r = 0.3$. These non zero values of the quantum discord are a strong signature for the presence of non classical correlations in the region of $r < 1/3$, where the entanglement is zero. This is one step toward evidence supporting the fact that the quantum discord and the entanglement are not synonymous. Figures 1(b) and 1(c) display the dependence of the quantum correlations on the purity when the two isolated atoms are initially entangled. From these figures, we can find that the larger the initial concurrence of the two atoms (corresponding to a larger value of the purity), the larger the possibility of the revival for the entanglement.

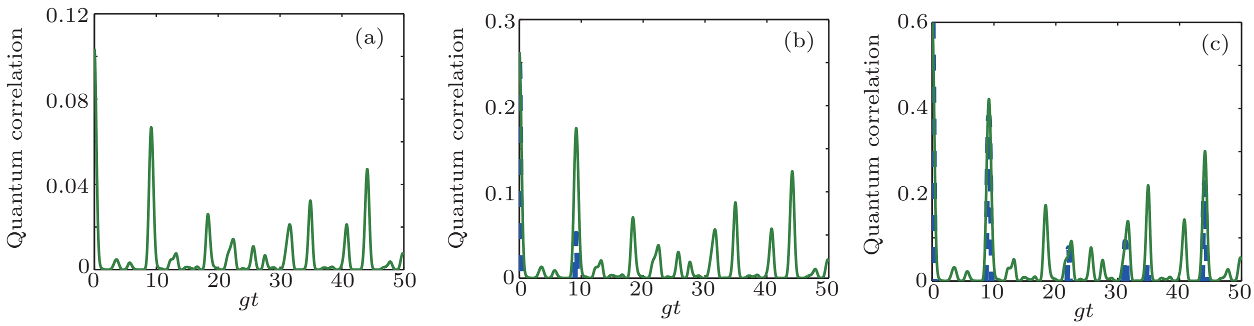

Fig. 1. (colour online) Dynamics of concurrence (dotted line) and quantum discord (solid line) of the two atoms versus scaled time gt with the mean photon numbers $\bar{n} = \bar{m} = 1$ and the purity r being (a) 0.3, (b) 0.5, and (c) 0.8.

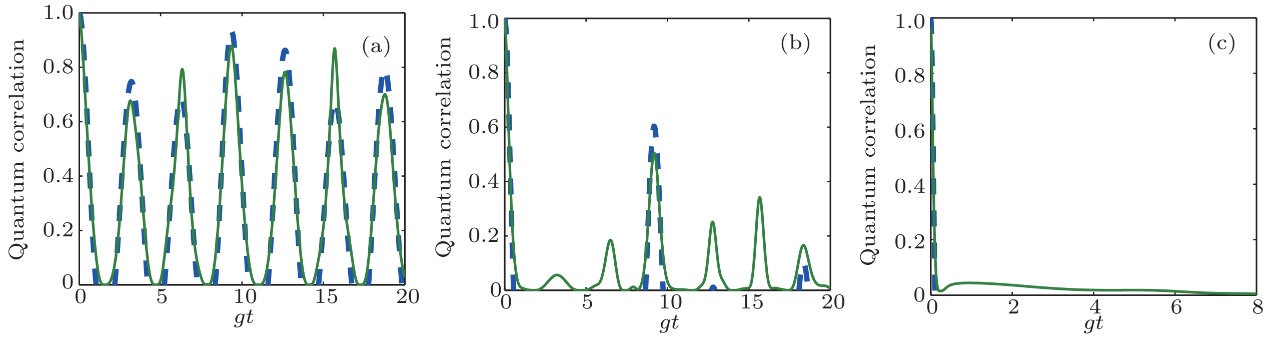

Fig. 2. (colour online) Dynamics of concurrence (dotted line) and quantum discord (solid line) of the two atoms versus scaled time gt with the purity $r = 1$ and the mean photon numbers ($\bar{n} = \bar{m}$) being (a) 0.1, (b) 1, and (c) 40.

Figure 2 displays how the mean photon numbers affect the evolution of the quantum correlation when $r = 1$, which corresponds to the initial atomic state $|\Phi(0)\rangle_{\rm AB} = (|eg\rangle + |ge\rangle)/\sqrt{2}$. The thermal state is a weighted mixture of the Fock states with weight P_n (or P_m) given by Eq. (10). The weight factor P_n (or P_m) is a decreasing function of n (or m). When the mean photon numbers are very small, corresponding to the vacuum dominance, the evolution of the entanglement is similar to that of the quantum discord, as shown in Fig. 2(a). As the mean photon numbers get moderately larger to 1, P_n (or P_m) becomes more pronounced. Despite the fact that the entanglement decays to zero, the quantum discord is sustained in this case, as can be seen from Fig. 2(b). With the increase of the mean photon number (see Fig. 2(c) for $\bar{n} = \bar{m} = 40$, which means that the temperatures of the cavities are high.) in each cavity, the entanglement suffers sudden death and does not recover. This is because when the mean photon number increases, the weight factor P_n (or P_m) of the single-mode thermal field becomes highly flat, which washes out the quantum correlation. It is interesting to note that although the entanglement type quantum correlations die out in a short time, not all types of quantum correlations are quickly lost because of the high temperature in the thermal noisy environment. That means that the quantum discord is more robust than the entanglement for higher temperatures.

It is worth mentioning that for the initial state

$$\rho_{\rm AB}(0) = r|\Psi(0)\rangle_{\rm ABAB}\langle\Psi(0)| + \frac{1-r}{4}I_{\rm AB},$$

where $|\Psi(0)\rangle = (|ee\rangle + |gg\rangle)/\sqrt{2}$, the behaviors of the quantum discord and the entanglement will be similar to those we have shown above.

4. Conclusion

We have calculated the dynamics of quantum correlations, such as the quantum discord and the entanglement for two non-interacting atoms in their corresponding noisy environments. Under the influence of the purity and the cavity temperature, the sudden death and the revival of the quantum correlations are compared. The results show that the entanglement suffers sudden death and revival. The larger the initial concurrence of the two atoms, the larger the possibility of the revival for the entanglement. Secondly, the discord is more robust than the entanglement in a noisy environment and is more comprehensive to describe the quantum correlation involved in the system. Therefore, the quantum discord could be considered, in this scenario, as a better indicator of the quantum correlations than the entanglement. Even where there is no entanglement, a large amount of quantum discord exists, which is a strong signature for the presence of non classical correlations. Finally, when the temperature of each cavity is high, the entanglement dies out in a short time, but the quantum discord still survives for quite a long time. It means that the quantum discord is more resistant to environmental disturbance than the entanglement at higher temperatures. These results indicate that in a thermal noise environment the quantum discord can be a more valuable resource for quantum information technologies than the entanglement.

References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)

[2] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 *Rev. Mod. Phys.* **81** 865

[3] Lettner M, Mücke M, Riedl S, Vo C, Hahn C, Baur S, Bochmann J, Ritter S, Dürr S and Rempe G 2011 *Phys. Rev. Lett.* **106** 210503

[4] Chen Q Y, Fang M F, Xiao X and Zhou X F 2011 *Chin. Phys.* B **20** 050302

[5] Datta A, Shaji A and Caves C M 2008 *Phys. Rev. Lett.* **100** 050502

[6] Lanyon B P, Barbieri M, Almeida M P and White A G 2008 *Phys. Rev. Lett.* **101** 200501

[7] Werlang T, Trippe C, Ribeiro G A P and Rigolin G 2010 *Phys. Rev. Lett.* **105** 095702

[8] Datta A 2009 *Phys. Rev.* A **80** 052304

[9] Sarandy M S 2009 *Phys. Rev.* A **80** 022108

[10] Shabani A and Lidar D A 2009 *Phys. Rev. Lett.* **102** 100402

[11] Luo S and Sun W 2010 *Phys. Rev.* A **82** 012338

[12] Madhok V and Datta A 2010 *arXiv:1008.4135*

[13] Lu X M, Ma J, Xi Z and Wang X 2011 *Phys. Rev.* A **83** 012327

[14] Galve F, Giorgi G L and Zambrini R 2011 *Phys. Rev.* A **83** 012102

[15] Giorda P and Paris M G A 2010 *Phys. Rev. Lett.* **105** 020503

[16] Streltsov A, Kampermann H and Bruß D 2011 *Phys. Rev. Lett.* **106** 160401

[17] Wang L C, Shen J and Yi X X 2011 *Chin. Phys.* B **20** 050306

[18] Modi K, Paterek T, Son W, Vedral V and Williamson M 2010 *Phys. Rev. Lett.* **104** 080501

[19] Chakrabarty I, Agrawal P and Pati A K 2010 *arXiv: 1006.5784*

[20] Yönac M and Eberly J H 2008 *Opt. Lett.* **33** 270

[21] Yönac M, Yu T and Eberly J H 2007 *J. Phys.* B **40** S45

[22] Yu T and Eberly J H 2004 *Phys. Rev. Lett.* **93** 140404

[23] Almeida M P, de Melo F, Hor-Meyll M, Salles A, Walborn S P, Souto Riberio P H and Davidovich L 2007 *Science* **316** 579

[24] Ollivier H and Zurek W H 2001 *Phys. Rev. Lett.* **88** 017901

[25] Dakic B, Vedral V and Brukner C 2010 *Phys. Rev. Lett.* **105** 190502

[26] Ferraro A, Aolita L, Cavalcanti D, Cucchietti F M and Acin A 2010 *Phys. Rev.* A **81** 052318

[27] Liu B Q, Shao B and Zou J 2010 *Phys. Rev.* A **82** 062119

[28] Werlang T, Souza S, Fanchini F F and Villas Boas C J 2009 *Phys. Rev.* A **80** 024103

[29] Mazzola L, Piilo J and Maniscalco S 2010 *Phys. Rev. Lett.* **104** 200401

[30] Kim M S, Lee J, Ahn D and Knight P L 2002 *Phys. Rev.* A **65** 040101

[31] Hill S 1997 *Phys. Rev. Lett.* **78** 5022

[32] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245

[33] Vedral V 2002 *Rev. Mod. Phys.* **74** 197

[34] Henderson L and Vedral V 2001 *J. Phys.* A **34** 6899

[35] Jaynes E T and Cummings F W 1963 *Proc. IEEE* **51** 89

[36] Werner R F 1989 *Phys. Rev.* A **40** 4277