**Dynamics of Quantum Systems Modeled by JaynesCummings Hamiltonian under the influence of MemoryChannels**



Timeline: August 1, 2023 → June 30, 2024

**Jotter**

Apart from current work done by me, here are some ideas that can help me solve the issues at hand.

May 6, 2024

I am not getting a trace preserved density matrix!

• Perhaps I have mistaken the calculation for correlated and uncorrelated parts?• Have I misunderstood the concept of memory channel/parameter?

• Are two qubits really getting entangled in our problem or is it just the n state and g or e state getting entangled?• Reference to ma’am’s article, only use correlated noise channels instead of both

I was stuck in a calculation regarding the evolution of state. I seem to have figured it out.

It is just applying procedure of generating matrix of an operator and calculating expectation values for each state.

May 15, 2024

Atanu sir taught me the proper way to work around these Hamiltonians. My mistake -

• I made mistake while finding eigenvectors of the the Hamiltonian.• My method did work out, but is long and messy.

What I learned -

• The method to find out eigenvectors of an operator

• Identifying the subspace of an infinite subspace. In our case it is a 6 dimensional subspace.• Hamiltonian without the interaction term $(H\_0)$ is a diagonal matrix. Interactions perturb the system and introduce off diagonal elements. The total Hamiltonian contains off diagonal elements. This is because $(H\_0)$ and interaction Hamiltonian have both different basis.

• When expressing the interaction Hamiltonian in $H\_0$’s basis, it becomes off diagonal. on diagonalizing it, $H\_0$ becomes off diagonal.

• To diagonalize, we need to find eigenvectors of a Hamiltonian and create a matrix S with it. Then, $SMS^{-1}$ will give $M\_D$, the diagonalized matrix.

• It is easier for calculation and after performing calculations, we can change it back to $M$, by doing , $S^{-1} M\_DS$.

May 18, 2024

I tried to write the interaction Hamiltonian in the $H\_0$’s basis. I was able to get the required state after time evolution.However, I am still not convinced I have done it right. I am confused on how to take the density matrix.

May 19, 2024

I have solved the problem that I was stuck with. It was a silly thing I forgot to notice. The evolved state was asuperposition of three states n+1, n, n-1. I overlooked the summation and started getting ahead of myself, overthinking it.So as of now, this problem has been solved. We will proceed to applying Kraus operators on our evolved matrix.

[Progress](file:///app/Dynamics%2520of%2520Quantum%2520Systems%2520Modeled%2520by%2520Jaynes%2520Cumm%2520b7bbf2c9b7c544d3910d563dddd51699/Progress%25200078213f2a81426ba82123de47910b9d.csv)

**Calculations for density operator**

Lets revisit the calculations:

Initially, our system is in the state:

$$ \psi(0) = \sum\_{n=0}^{\infty}P\_n |n,e\rangle\

$$

Hamiltonian can be described as:

$$ H\_{af}= \lambda (a^\dagger |g\rangle\langle e| + a |e\rangle\langle g|) = a^\dagger \sigma\_- + a \sigma\_+ =\begin{pmatrix} 0&a^\dagger\a & 0 \end{pmatrix} $$

It’s evolution can be given as:

$$ \psi(t) = U(t)\psi(0) \ U(t) = e^{-iHt} = \sum\_{n=0}^{\infty}\frac{(-iHt)^n}{n!} \hspace{1cm} ...(1) \ $$

On expansion of (1), Hamiltonian can be seen taking odd and even powers.

Odd powers of $H$ give change $|g\rangle$ to $|e\rangle$ and $|e\rangle$ to $|g\rangle$; Whereas, even powers of $H$only change the photon number, $n$, while identity acts on $|g\rangle$ and $|e\rangle$, hence they remain the same.

$$ H|e\rangle = (a^\dagger \sigma\_- + a \sigma\_+)|e\rangle \ = a^\dagger|g\rangle $$

and,

$$ H|g\rangle = (a^\dagger \sigma\_- + a \sigma\_+)|g\rangle \ = a|e\rangle $$

For even powers acting on $|g\rangle$,

$$ H^2|g\rangle = H(a^\dagger \sigma\_- + a \sigma\_+)|g\rangle \ = H(a|e\rangle) = a (H|e\rangle)\ = a^\dagger a |g\rangle\

$$

and,

$$ \ H^4|g\rangle = H^2.H^2|g\rangle \ = H^2(a^\dagger a |g\rangle) =(a^\dagger a )^2|g\rangle

$$

Therefore,

$$ H^{n} |g \rangle = (a^\dagger a)^{n/2} |g \rangle = (\sqrt{a^\dagger a})^n |g \rangle \ \textit{where n is even} $$

Similarly, for $|e \rangle$ ;

$$ H^{n} |e \rangle = (aa^\dagger)^{n/2} |e \rangle = (\sqrt{a^\dagger a + 1})^n |e \rangle \ \textit{where n is even} $$

For odd powers,

$$ H^3|g\rangle = H.H^2|g\rangle \ = H(a^\dagger a|g\rangle) = a^\dagger a (H|e\rangle)\ = aa^\dagger a |e\rangle $$

and,

$$ H^5|g\rangle = H.H^4|g\rangle \ = H(a^\dagger a)^2|g\rangle = a^\dagger a (H|e\rangle)\ = a(a^\dagger a)^2|e\rangle $$

therefore, it is evident:

$$ H^{n} |g \rangle = a(a^\dagger a)^{\frac{n-1}{2}} |g \rangle = a(\sqrt{a^\dagger a})^n-1 |e \rangle \ \textit{where n isodd} $$



and,

$$ H^{n} |e \rangle = a^\dagger(aa^\dagger)^{\frac{n-1}{2}} |e \rangle = a^\dagger(\sqrt{a^\dagger a + 1})^n |g \rangle\ \textit{where n is odd} $$

In matrix form, the Unitary Time Evolution operator can be written as:

$$ U(t) = \sum\_{i,j}c\_{ij} |i\rangle \langle j|\ where,c\_{ij} = \langle j|U(t)|i\rangle\ $$

for states $|e \rangle,|g \rangle$; it takes the following matrix form:

$$ \begin{pmatrix} \langle g|U(t)|g\rangle&\langle g|U(t)|e\rangle\\\langle e|U(t)|g\rangle&\langle e|U(t)|e\rangle\\end{pmatrix} $$

Solving for each, we get:

$$ \langle g|U(t)|g\rangle = \langle g| e^{-iHt}|g\rangle \ = \langle g|[1-it H - \frac{(it)^2H^2}{2!} + \frac{(it)^3H^3}{3!}+\frac{(it)^4H^4}{4!} + ...]|g \rangle \ = \langle g|[1-\frac{t^2H^2}{2!} ...] - i[tH -\frac{t^3H^3}{3!}....] |g\rangle \

$$

Since, odd powers will yield an $|e\rangle$ leading to a 0 on inner product with $\langle g|$; only even powers willcontribute:

$$ = \langle g|[1-\frac{t^2H^2}{2!} + \frac{t^4H^4}{4!} ...] |g\rangle \ = \langle g | [|g\rangle - \frac{t^2H^2|g \rangle }{2!}+\frac{t^4H^4|g\rangle }{4!} - \frac{t^6H^6|g\rangle }{6!}... \ =1-\frac{t^2(\sqrt{a^\dagger a})^2}{2!} +\frac{t^4(\sqrt{a^\dagger a})^4}{4!} - \frac{t^6(\sqrt{a^\dagger a})^6}{6!} ...\ = \sum \frac{(\sqrt{a^\dagger a})^nt^n}{n!} \ = cos(t\sqrt{a^\dagger a}) $$

similarly,

$$ \langle e|U(t)|e\rangle = \langle e| e^{-iHt}|e\rangle = cos(t(\sqrt{aa^\dagger}) \ = cos(t\sqrt{a^\dagger a +1}) $$

and,

$$ \langle g|U(t)|e\rangle = \langle g| e^{-iHt}|e\rangle \ = \langle g|[1-itH - \frac{(it)^2H^2}{2!} + \frac{(it)^3H^3}{3!}+\frac{(it)^4H^4}{4!} + ...]|e \rangle \ = \langle g|[1-\frac{t^2H^2}{2!} ...] -i[tH -\frac{t^3H^3}{3!}....] |e\rangle \

$$

Since, even powers will yield an $|e\rangle$ leading to a 0 on inner product with $\langle g|$; only odd powers willcontribute:

$$ = \langle g|-i[tH -\frac{t^3H^3}{3!}....] |e\rangle \ = \langle g|-i[ta^\dagger|g\rangle -\frac{t^3a^\dagger(aa^\dagger)|g\rangle }{3!}....]\ =-ia^\dagger[t -\frac{t^3(aa^\dagger)| }{3!}....]\ =-ia^\dagger\sum \frac{(\sqrt{a^\dagger a + 1}))^{n-1}t^n}{n!} \ =\frac{-ia^\dagger}{\sqrt{a^\dagger a + 1}}\sum \frac{(\sqrt{a^\dagger a + 1}))^{n}t^n}{n!} \

=\frac{-ia^\dagger}{\sqrt{a^\dagger a +1}}sin(t\sqrt{a^\dagger a+1}) $$

similarly,

$$ \langle e|U(t)|g\rangle = \langle e| e^{-iHt}|g\rangle \ =\frac{-ia}{\sqrt{a^\dagger a }}sin(t\sqrt{a^\dagger a})$$

**Therefore the time evolution operator takes the following matrix form :**

$$ \begin{pmatrix} cos(\tau\sqrt{a^\dagger a})&\frac{-ia^\dagger}{\sqrt{a^\dagger a +1}}sin(\tau\sqrt{a^\dagger a+1})\\\frac{-ia}{\sqrt{a^\dagger a }}sin(\tau\sqrt{a^\dagger a}) &cos(\tau\sqrt{a^\dagger a +1})\ \end{pmatrix}\ =cos(\tau\sqrt{a^\dagger a})|g\rangle\langle g| - \frac{ia^\dagger}{\sqrt{a^\dagger a +1}}sin(\tau\sqrt{a^\dagger a+1})|g\rangle\langle e| \- \frac{ia}{\sqrt{a^\dagger a }}sin(\tau\sqrt{a^\dagger a})|e\rangle\langle g| + cos(\tau\sqrt{a^\dagger a+1})|e\rangle\langle e| $$

Now, the evolution of our state takes the form:

$$ \ket{\psi (t)} = U(t)\ket{\psi(0)}\ = \sum\_{n=0}^{\infty}{P\_n}\bigg[cos(\tau\sqrt{a^\dagger a})|g\rangle\langle g|n,e\rangle - \frac{ia^\dagger}{\sqrt{a^\dagger a +1}}sin(\tau\sqrt{a^\dagger a+1})|g\rangle\langle e|n,e\rangle \- \frac{ia}{\sqrt{a^\dagger a }}sin(\tau\sqrt{a^\dagger a})|e\rangle\langle g|n,e\rangle + cos(\tau\sqrt{a^\dagger a +1})|e\rangle\langle e|n,e\rangle \bigg]\ =\sum\_{n=0}^{\infty}{P\_n}\bigg[0- \frac{ia^\dagger}{\sqrt{a^\dagger a +1}}sin(\tau\sqrt{a^\dagger a+1})|g\rangle\langle e|n,e\rangle - 0 + cos(\tau\sqrt{a^\dagger a +1})|e\rangle\langle e|n,e\rangle\bigg]\=\sum\_{n=0}^{\infty}{P\_n}\bigg[cos(\tau\sqrt{n +1})|n,e\rangle -i \ sin(\tau\sqrt{n+1})|n+1,g\rangle\bigg] $$

We can now obtain the density operator of the above state as:

$$ \ket{\psi(t)}=\sum\_{n=0}^{\infty}{P\_n}\bigg[cos(\tau\sqrt{n+1 })\ket{n,e}-i \ sin(\tau\sqrt{n+1})\ket{n+1,g}\bigg]\\bra{\psi(t)}=\sum\_{n=0}^{\infty}{P\_n}\bigg[cos(\tau\sqrt{n+1 })\bra{n,e}+i \ sin(\tau\sqrt{n+1})\bra{n+1,g}\bigg]\\ket{\psi(t)}\bra{\psi(t)} = \sum\_{n=0}^{\infty}{P\_n}\sum\_{n=0}^{\infty}{P\_n}\bigg[cos(\tau\sqrt{n+1 })\ket{n,e}-i \sin(\tau\sqrt{n+1})\ket{n+1,g}\bigg]\bigg[cos(\tau\sqrt{n+1 })\bra{n,e}+i \ sin(\tau\sqrt{n+1})\bra{n+1,g}\bigg]\

$$

Since the eigen states of photon operator are $\ket{n-1},\ket{n} ,\ket{n-1}$, $n$ takes the values $n-1,n,n+1$.(Shore,Knight,1993)

Therefore,

$$ \ket{\psi(t)}\bra{\psi(t)} =\sum\_{n=0}^{\infty}{P\_n}^2\bigg[K\_1\ket{n,e}\bra{n,e}+K\_2\ket{n,e}\bra{n+1,g}+K\_3\ket{n+1,g}\bra{n,e}+K\_4\ket{n+1,e}\bra{n+1,g}\bigg]\ =P\_{n-1}^2\bigg[cos^2(\tau\sqrt{n })\ket{n-1,e}\bra{n-1,e}+i\ cos(\tau\sqrt{n})\ sin(\tau\sqrt{n})\ket{n-1,e}\bra{n,g}-i\ cos(\tau\sqrt{n})\ sin(\tau\sqrt{n})\ket{n,g}\bra{n-1,e}+sin^2(\tau\sqrt{n })\ket{n,g}\bra{n,g}\bigg]\ + {P\_n}^2\bigg[cos^2(\tau\sqrt{n+1 })\ket{n,e}\bra{n,e}+i\ cos(\tau\sqrt{n+1 })\ sin(\tau\sqrt{n+1})\ket{n,e}\bra{n+1,g}-i\ cos(\tau\sqrt{n+1 })\ sin(\tau\sqrt{n+1})\ket{n+1,g}\bra{n,e}+sin^2(\tau\sqrt{n+1 })\ket{n+1,g}\bra{n+1,g}\bigg] + P\_{n+1}^2\bigg[cos^2(\tau\sqrt{n+2 })\ket{n+1,e}\bra{n+1,e}+i\ cos(\tau\sqrt{n+2 })\ sin(\tau\sqrt{n+2})\ket{n+1,e}\bra{n+2,g}-i\ cos(\tau\sqrt{n+2 })\ sin(\tau\sqrt{n+2})\ket{n+2,g}\bra{n+1,e}+sin^2(\tau\sqrt{n+2 })\ket{n+2,g}\bra{n+2,g}\bigg] $$

A $2 \times 2$ subspace in our $2 \times \infty$ atom-field space can be expressed as:

$$ \begin{bmatrix} g\e \end{bmatrix} \otimes \begin{bmatrix} n\n+1 \end{bmatrix} = \begin{bmatrix} n,g\n,e\n+1,g\n+1,e \end{bmatrix} $$

For this subspace, our density operator takes the form:

$$

\ket{\psi(t)}\bra{\psi(t)} = \begin{bmatrix}\kappa\_1 &0&0&0\0& \kappa\_2 &\kappa\_3&0\0&-\kappa\_3&\kappa\_4&0\0&0&0&\kappa\_5\end{bmatrix}\ = \kappa\_1\ket{n,g}\bra{n,g}+\kappa\_2\ket{n,e}\bra{n,e}+\kappa\_3\big[ \ket{n,e}\bra{n+1,g}-\ket{n+1,g}\bra{n,e}\big]+\kappa\_4\ket{n+1,g}\bra{n+1,g}+\kappa\_5\ket{n+1,e}\bra{n+1,e} $$

where,

$$ \kappa\_1= \frac{P\_{n-1}^2}{N}sin^2(\tau\sqrt{n })\ \\kappa\_2=\frac{P^2\_n}{N}cos^2(\tau\sqrt{n+1 }) \\kappa\_3=i\frac{P^2\_n}{N} cos(\tau\sqrt{n+1 })\ sin(\tau\sqrt{n+1}) \\kappa\_4=\frac{P^2\_n}{N}sin^2(\tau\sqrt{n+1 }) \\kappa\_5=\frac{P^2\_{n+1}}{N}cos^2(\tau\sqrt{n+2 })

$$

Where $N$ is the normalization coefficient:

$$ N = P\_{n-1}^2sin^2(\tau\sqrt{n }) + P\_n^2[sin^2(\tau\sqrt{n+1}) + cos^2(\tau\sqrt{n+1})] + P\_{n+1}^2cos^2(\tau\sqrt{n+2})\ =P\_{n-1}^2sin^2(\tau\sqrt{n }) + P\_n^2 + P\_{n+1}^2cos^2(\tau\sqrt{n+2})\

$$

**Kraus operators**

$$

\begin{pmatrix} \left( (-1 + \lambda)^2 \mu \kappa\_1 + (-1 + \lambda) \lambda (-1 + \mu) \kappa\_2 + (-1 + \lambda)\lambda (-1 + \mu) \kappa\_4 + \lambda^2 \mu \kappa\_5 \right) & 0 & 0 & 0 \ 0 & \left( (-1 + \lambda) \lambda (-1 +\mu) \kappa\_1 + (-1 + \lambda)^2 \mu \kappa\_2 + \lambda (-1 + 2\lambda) \mu \kappa\_3 \right) & -\left( (-1 + \lambda)\lambda \mu \kappa\_3 \right) & 0 \ 0 & \left( \lambda (-1 + 2\lambda) \mu \kappa\_3 \right) & \left( (-1 + \lambda)\lambda (-1 + \mu) \kappa\_1 + \lambda^2 \mu \kappa\_2 + (-1 + \lambda) \lambda (-1 + \mu) \kappa\_4 + \lambda^2 \mu\kappa\_5 \right) & 0 \ 0 & 0 & 0 & \left( \lambda^2 \mu \kappa\_1 + (-1 + \lambda) \lambda (-1 + \mu) \kappa\_2 + (-1 +\lambda) \lambda (-1 + \mu) \kappa\_4 + \lambda^2 \mu \kappa\_5 \right) \end{pmatrix}

$$