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Section 2.1

14. find the solution set for the following equation

$$4x_1 + 3x_2 + 2x_3 = 1$$
$$2x_3 = 1 - 4x_1 - 3x_2$$
$$x_3 = \frac{1 - 4x_1 - 3x_2}{2}$$
$$x_3 = \frac{1 - 4x_1 - 3(2k-1)}{2}$$
$$x_3 = -2x_1 - 3k - 1$$

$$\left[\begin{array}{l} x_1 = x_1 \\ x_2 = 2k - 1 \\ x_3 = -2x_1 - 3k - 1 \end{array}\right.$$

- infinite solutions due to having more free variables than equations in system $(2 > 1)$

30. find the augmented matrix for the given linear system.

$$\begin{aligned} a - 2b + d &= 2 \\ -a + b - c - 3d &= 1 \end{aligned} \Rightarrow \left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ -1 & 1 & -1 & -3 & 1 \end{array}\right]$$

32. find the linear system given the augmented matrix.

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & 3 & 1 & 2 \\ 1 & 1 & 2 & 1 & -1 & 4 \\ 0 & 1 & 0 & 2 & 3 & 0 \end{array}\right] \Rightarrow \begin{aligned} v - w + 3y + z &= 2 \\ v + w + 2x + y - z &= 4 \\ w + 2y + 3z &= 0 \end{aligned}$$

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Section 2.2

2. determine wether a give matrix is in REF, also state if it is in RREF

$$\begin{bmatrix} 7 & 0 & 1 & 0 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ No, there is no leading 1 on row 1.

4. (same as #2)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ Yes, this matrix is also in RREF as well.

12. (a) Use elementary row operation to transform the given matrix into REF

1

$$\begin{array}{c} R_1 \\ R_2 \end{array} \left[\begin{array}{ccc|c} 2 & -4 & -6 & 6 \\ 3 & 1 & 6 & 6 \end{array}\right] \quad R_1 = \frac{1}{2} R_1$$

$\downarrow$ 2

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & 3 \\ 3 & 1 & 6 & 6 \end{array}\right] \quad R_2 = R_2 - 3R_1$$

3 $\downarrow$

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & 3 \\ 0 & -5 & 15 & -3 \end{array}\right] \quad R_2 = -\frac{1}{5} R$$

4

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & 3 \\ 0 & 1 & -3 & 3/5 \end{array}\right] \quad \text{in REF}$$

14. (a) (same as #12)

1

$$\left[\begin{array}{cc|c} -2 & -4 & 7 \\ -3 & -6 & 10 \\ 1 & 2 & -3 \end{array}\right] \quad \begin{array}{l} R_2 = -R_2 + 3R_3 \\ R_1 \leftrightarrow R_3 \end{array} \rightarrow$$

2

$$\left[\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & 0 & 1 \\ -2 & -4 & 7 \end{array}\right] \quad \begin{array}{l} R_3 = R_3 + 2R_1 \\ R_3 = R_3 - R_2 \end{array}$$

3

$$\left[\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right] \quad \text{in REF}$$

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16. Which elementary row operation "undoes" each given operation

$R_i \longleftrightarrow R_j$ gets undone by $R_j \longleftrightarrow R_i$

kR_i gets undone by $\frac{1}{k}R_i$

$R_i + kR_j$ gets undone by $R_i - kR_j$

20. What is the net effect of the given operation:

$R_2 + R_1$, $R_1 - R_2$, $R_2 + R_1$, $-R_1$

$R_1 = -(R_1 - R_2)$

$R_2 = (\cancel{R_2} + R_1 + (R_1 - \cancel{R_2})) \rightsquigarrow 2R_1$

24. What are the possible RREF of a 3x3 matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

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26. Solve the system of equations by either gaussian or gauss-jordan elimination:

$$\begin{aligned} x - y + z &= 0 \\ -x + 3y + z &= 5 \\ 3x + y + 7z &= 2 \end{aligned} = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 3 & 1 & 5 \\ 3 & 1 & 7 & 2 \end{array}\right]$$

① $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 3 & 1 & 5 \\ 3 & 1 & 7 & 2 \end{array}\right]$ $R_2 = R_2 + R_1$, $R_3 = R_3 - 3R_1$ $\Rightarrow$ ② $\left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 2 & 2 & 5 \\ 0 & 4 & 10 & 2 \end{array}\right]$ $R_3 = R_3 - 2R_2$, $R_2 = \frac{1}{2}R_2$

③ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 5/2 \\ 0 & 0 & 6 & -8 \end{array}\right]$ $R_3 = \frac{1}{6}R_3$ $\Rightarrow$ ④ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 5/2 \\ 0 & 0 & 1 & -8/6 \end{array}\right]$

$x - y + z = 0$
$y + z = 5/2$
$z = -8/6$

$y + (-8/6) = 5/2$
$+8/6 \quad +8/6$

$y = \frac{23}{6}$

$x - \frac{23}{6} - \frac{8}{6} = 0 \qquad x = \frac{31}{6}$

$$\begin{bmatrix} x = \frac{31}{6} \\ y = \frac{23}{6} \\ z = -\frac{8}{6} \end{bmatrix}$$

30. (same as #26)

$$\begin{aligned} -x_1 + 3x_2 - 2x_3 + 4x_4 &= 0 \\ 2x_1 - 6x_2 + x_3 - 2x_4 &= -3 \\ x_1 - 3x_2 + 4x_3 - 8x_4 &= 2 \end{aligned} \qquad \left[\begin{array}{cccc|c} -1 & 3 & -2 & 4 & 0 \\ 2 & -6 & 1 & -2 & -3 \\ 1 & -3 & 4 & -8 & 2 \end{array}\right]$$

① $\left[\begin{array}{cccc|c} -1 & 3 & -2 & 4 & 0 \\ 2 & -6 & 1 & -2 & -3 \\ 1 & -3 & 4 & -8 & 2 \end{array}\right]$ $R_2 = R_2 - 2R_1$, $R_3 = R_3 + R_1$ $\Rightarrow$ ② $\left[\begin{array}{cccc|c} -1 & 3 & -2 & 4 & 0 \\ 0 & 0 & -7 & 14 & -7 \\ 0 & 0 & 2 & -4 & 2 \end{array}\right]$

$R_3 = R_3 + \frac{2}{7}R_2$, $R_1 = R_1 - \frac{2}{7}R_2$ $\Rightarrow$ ③ $\left[\begin{array}{cccc|c} -1 & 3 & 0 & 0 & 0 \\ 0 & 0 & -7 & 14 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ $R_1 = -R_1$, $R_2 = -\frac{1}{7}R_2$

$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 + 3x_2 = 0$
$x_3 - 2x_4 = 1$

$$\begin{bmatrix} x_1 = -3x_2 \\ x_3 = 2x_4 + 1 \end{bmatrix}$$

x_2 and x_4 are free variables, infinetly many solutions

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40. for what values of k will there be 1 solution, no solution, or infinitly many solutions

$$kx + 2y = 3$$
$$2x - 4y = -6$$

Let $k = -1$

$$\begin{aligned} -x + 2y &= 3 \\ 2x - 4y &= -6 \end{aligned} \quad \underset{\div -2}{\Rightarrow} \quad \begin{aligned} -x + 2y &= 3 \\ -x + 2y &= 3 \end{aligned}$$

- infinitly many solutions for $k = -1$
- unique solutions for $k \neq -1$
- no values of k lead to no solution

42. (same as #40)

$$x - 2y + 3z = 2$$
$$x + y + 2 = k$$
$$2x - y + 4z = k^2$$

- no solution $k = 3$ or $-\frac{4}{3}$
- infinite $k = 0, 2,$ or -2
- unique k = none of above

1

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 1 & 1 & 2 & k \\ 2 & -1 & 4 & k^2 \end{array}\right] \quad \begin{aligned} R_3 &= R_3 - 2R_2 \\ R_2 &= R_2 - R_1 \end{aligned}$$

2

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & -1 & -1 & k - 2 \\ 0 & -3 & 0 & k^2 - 2k \end{array}\right]$$

3

$$\begin{aligned} R_2 &= R_2 - \tfrac{1}{3}R_3 \\ R_3 &= \tfrac{1}{3}R_3 \end{aligned} \quad \left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 0 & -1 & \frac{k^2}{3} - \frac{k}{3} - 2 \\ 0 & 1 & 0 & \frac{k^2 - 2k}{3} \end{array}\right]$$

4

$$R_1 = R_1 + 2R_3$$
$$R_1 = R_1 + 3R_2$$
$$R_2 = -R_2$$
$$R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & k^2 - \frac{5}{3}k - 4 \\ 0 & 1 & 0 & \frac{k^2}{3} - \frac{2k}{3} \\ 0 & 0 & 1 & \frac{k^2}{3} - \frac{k}{3} - 2 \end{array}\right]$$