### EXERCISE 2 - GLOBAL OPTIMIZATION

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## 1 Reformulation in the canonical form of Linear Programming

$$\max_{\theta, S_I} \ card(S_I) \tag{1}$$

$$s.t \quad |x_i + T_x - x_i'| \le \delta. \quad \forall i \in S_I$$
 (2)

$$|y_i + T_y - y_i'| \le \delta \quad \forall i \in S_I \tag{3}$$

with the identification binary variable  $z_i=1$  if the i-th correspondence is an inlier. Otherwise,  $z_i=0$ ; And the auxiliary variables  $w_ix=z_iT_xw_iy=z_iT_y$ 

$$Tx.lb \ge Tx \ge Tx.ub$$
 (4)

$$Ty.lb \ge Ty \ge Ty.ub$$
 (5)

$$z_i \in (0,1) \tag{6}$$

converted in the canonical form with relaxation  $0 \le z_i \le 1$ :

$$\max_{\theta, \mathbf{Z}} \quad \sum_{i=1}^{n} z_i \tag{7}$$

$$(x_i - x_i' - \delta)z_i + w_x i \le 0 \tag{8}$$

$$(x_i - x_i' - \delta)z_i + w_{xi} \le 0 \tag{9}$$

$$-(x_i - x_i' + \delta)z_i - w_{xi} \le 0 \tag{10}$$

$$(y_i - y_i' + \delta)z_i + w_{yi} \le 0 \tag{11}$$

$$-(y_i - y_i' + \delta)z_i - w_{yi} \le 0 \tag{12}$$

$$z_i T_x . lb - w_{xi} \le 0 (13)$$

$$-z_i T_x . ub + w_{xi} \le 0 \tag{14}$$

$$T_x + z_i T_x . ub - w_{xi} \le T_x . ub \tag{15}$$

$$-T_x - z_i T_x . lb + w_{xi} \le -T_x . lb \tag{16}$$

$$T_y + z_i T_y . ub - w_{yi} \le T_y . ub \tag{17}$$

$$-T_y - z_i T_y . lb + w_{yi} \le -T_y . lb \tag{18}$$

$$1 \ge z_i \ge 0 \tag{19}$$

$$Tx.lb \ge Tx \ge Tx.ub$$
 (20)

$$Ty.lb \ge Ty \ge Ty.ub$$
 (21)

in matrix form:

$$\max_{x} < f, x > \tag{22}$$

with

$$x = [Tx.opt, Ty.opt, z, w_x, w_y]$$
(23)

$$f = [0, 0, 1_n, 0_{2n}] (24)$$

s.t:

$$Ax \le b \tag{25}$$

$$lb \le x \le ub \tag{26}$$

 $\quad \text{with} \quad$ 

$$b = [0_{8n}; Tx.ub_n; -Tx.lb_n; Ty.ub_n; -Ty.lb_n];$$
(27)

$$lb = [Tx.lb; Ty.lb; 0_n; -Inf_{2n}];$$
 (28)

$$ub = [Tx.ub; Ty.ub; 1_n; Inf_{2n}];$$
 (29)

# $2 \quad todoplot$