

Structure Learning



Dr. Cengiz Öztireli

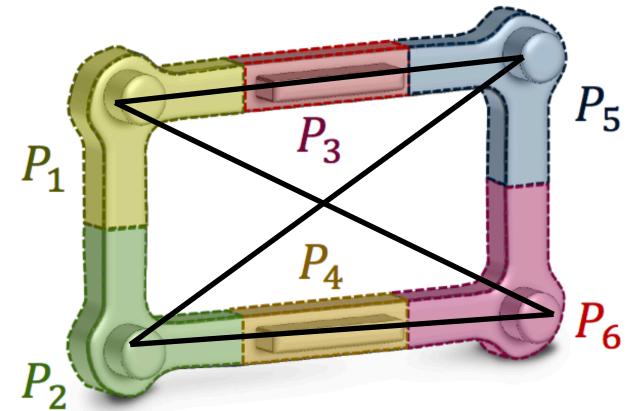
This Lecture

- Structure in general
- Symmetries
- Repeated structures
- Structure & process
- Structure & function



This Lecture

- Higher level & semantic structure
 - Parts of the geometry
 - Relations among the parts
 - They can encode context
 - Extract relations
 - Preserve relations

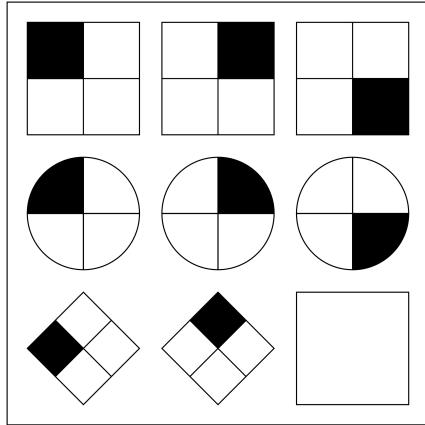


Structure

- Structure (Oxford dict.): The arrangement of and relations between the parts or elements of something complex.
- Challenges
 - Extracting structure
 - Preserving structural constraints
 - Designing algorithms & metaphores

Structure

- Structure is fundamental and beautiful



IQ Test



World is structured



Art is structured

Structure

Semantic



Functional



Economic



Structure

- Applications: modeling by example



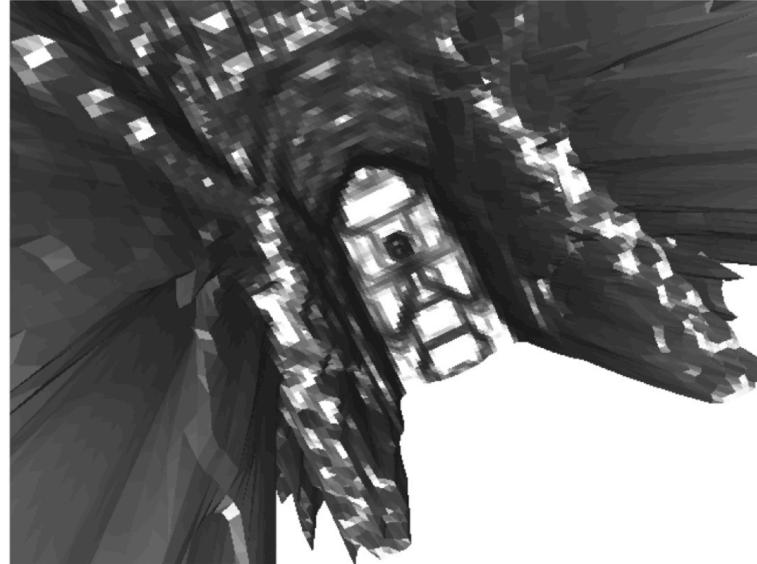
Structure

- Applications: symmetry detection & symmetrization



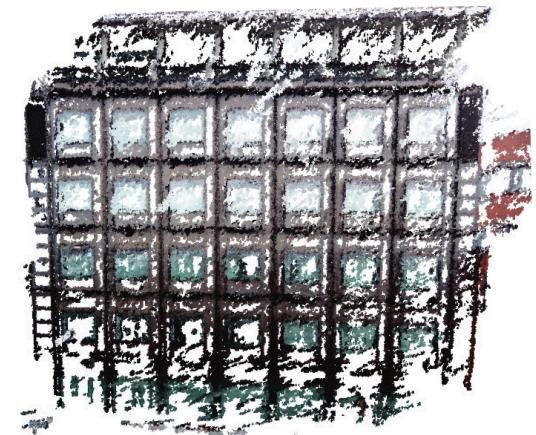
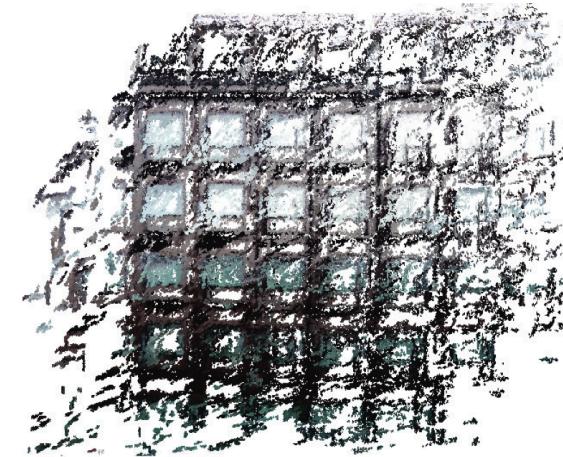
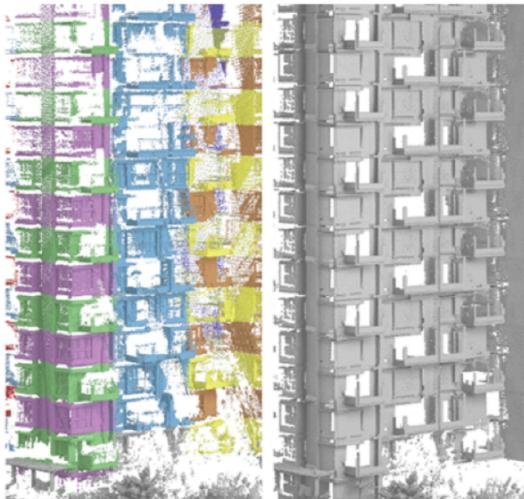
Structure

- Applications: 3D reconstruction



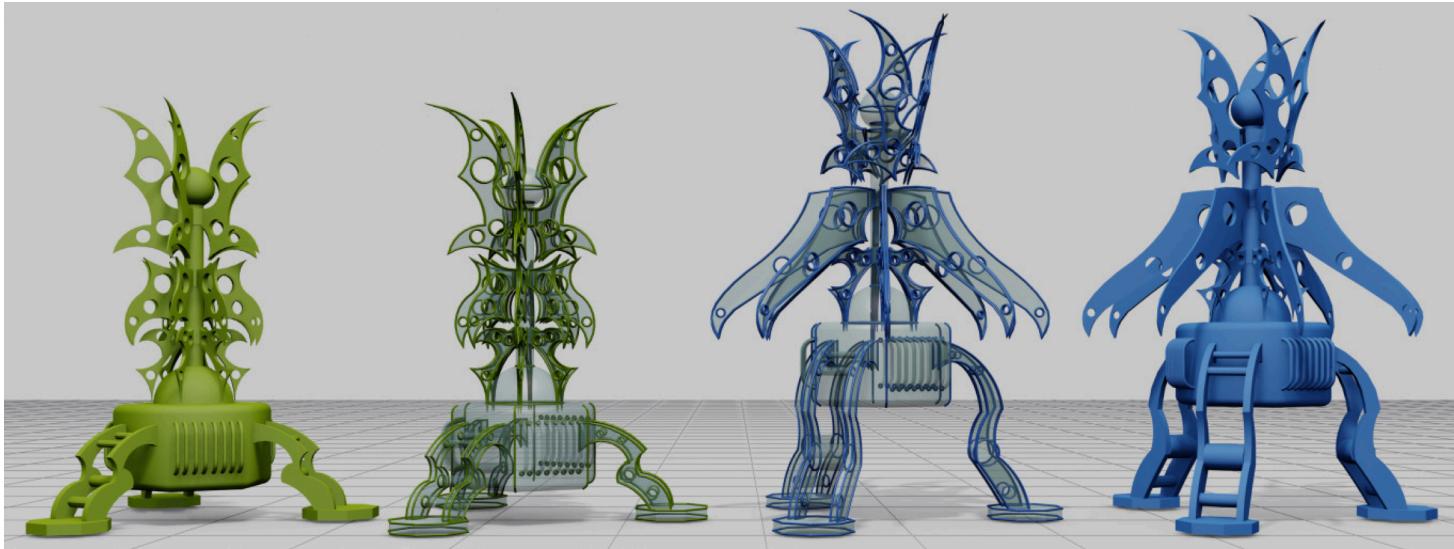
Structure

- Applications: scan enhancement



Structure

- Applications: structure-aware shape editing



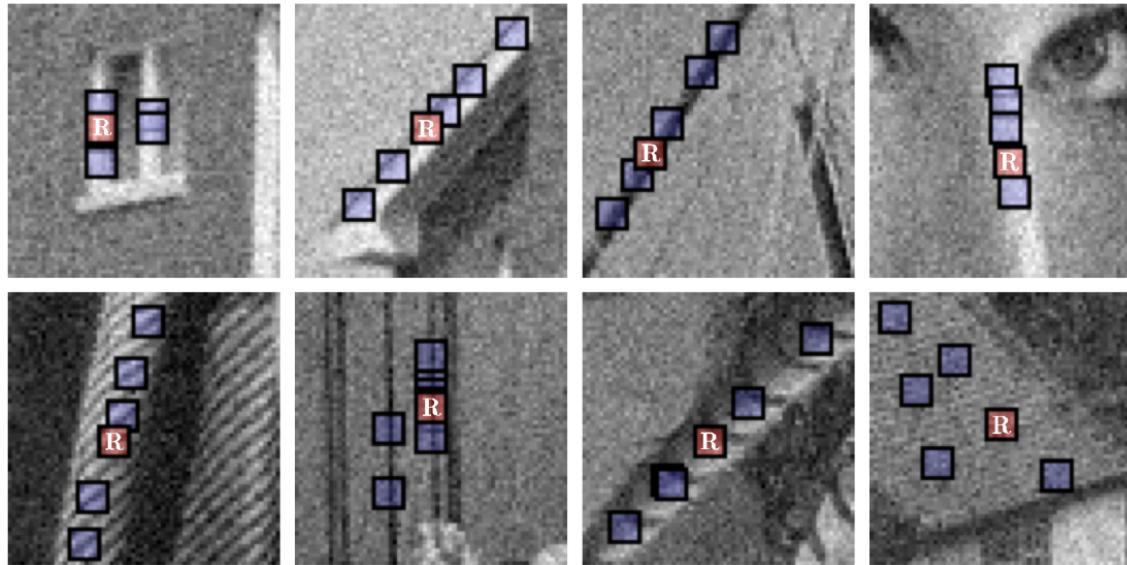
Structure

- Applications: scene understanding & synthesis



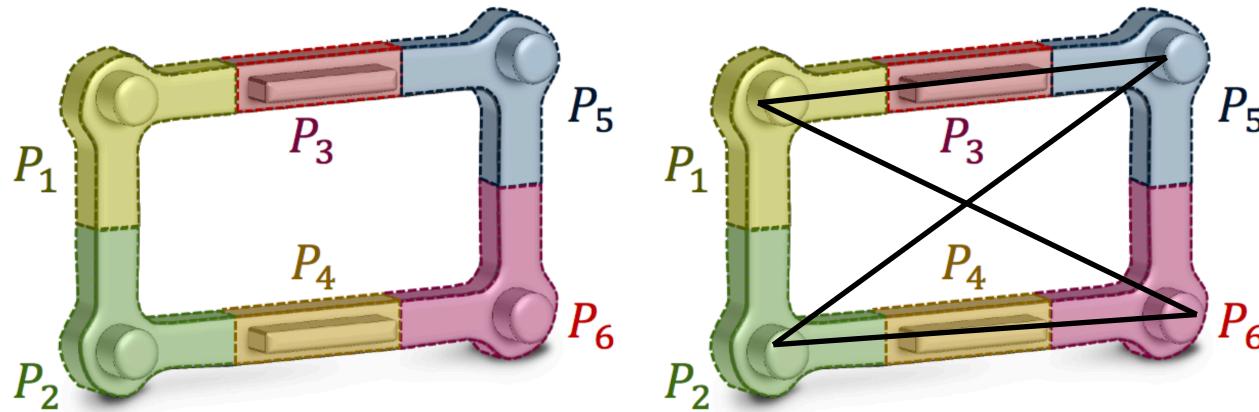
Structure

- Applications: image smoothing



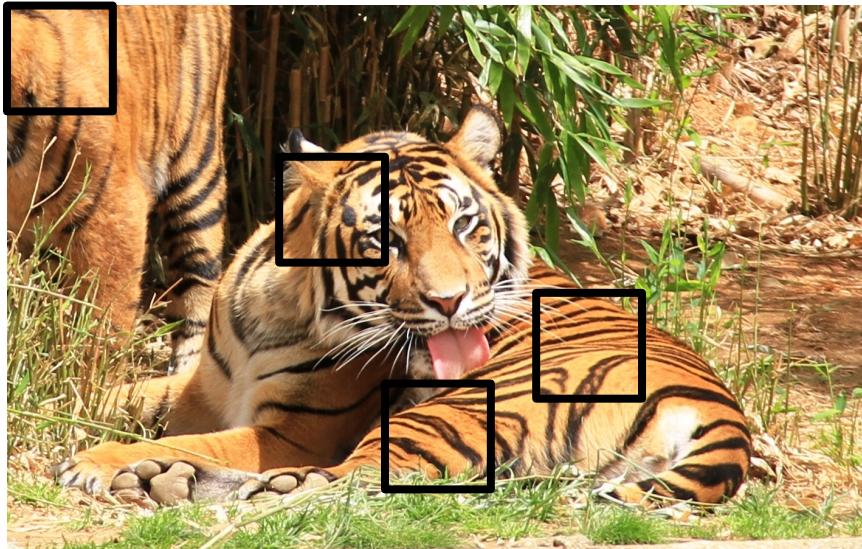
Structure Discovery

- Two main steps:
 - Get & parameterize parts
 - Form relations among the parts



Structure Discovery

- Example: non-local means filtering



Parts: patches

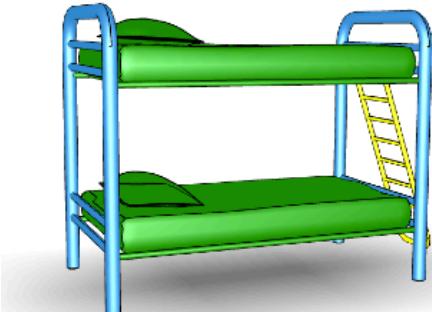
Relations:

$$r(x, y) = \|\mathbf{x} - \mathbf{y}\|^2$$

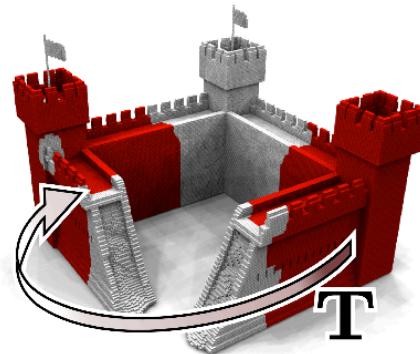
Structure Discovery

- Identify parts

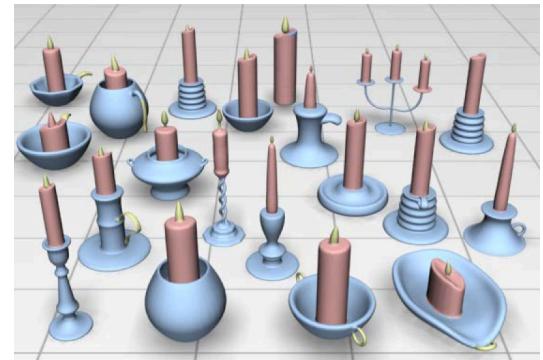
User-given



Model-based



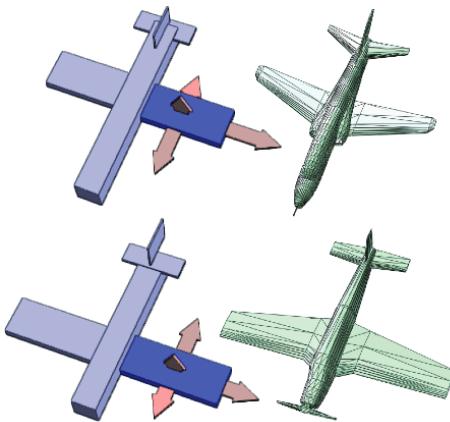
Data-based



Structure Discovery

- Parameterize the parts – model-based

Bounding-box

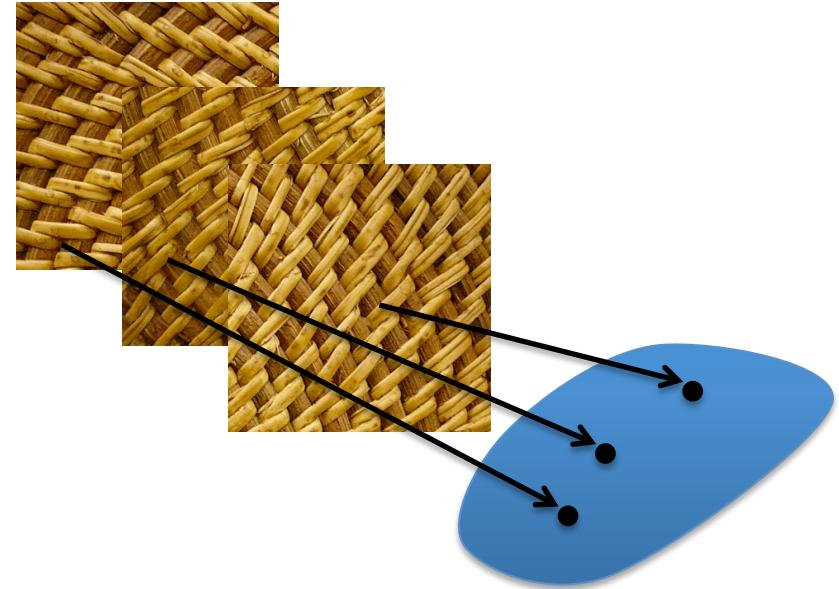
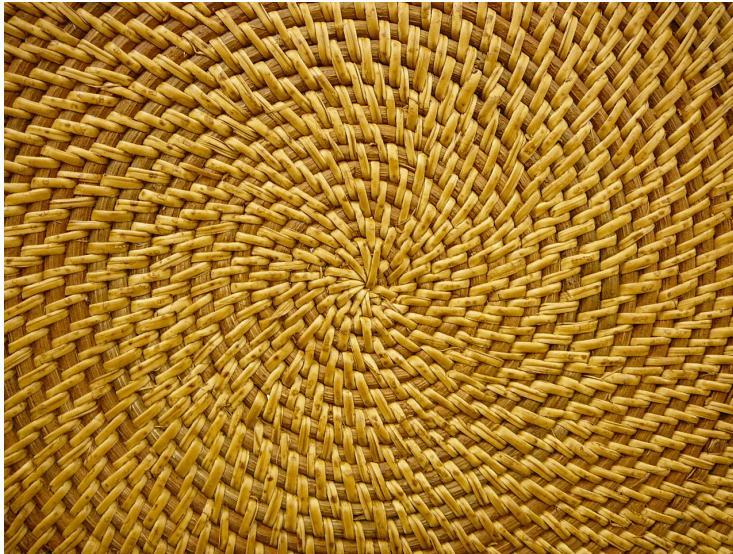


Rigid transformations



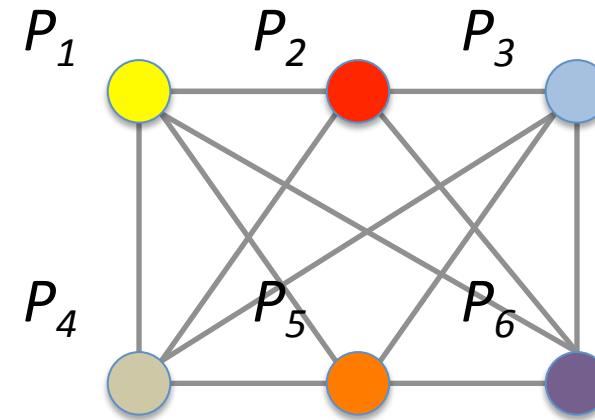
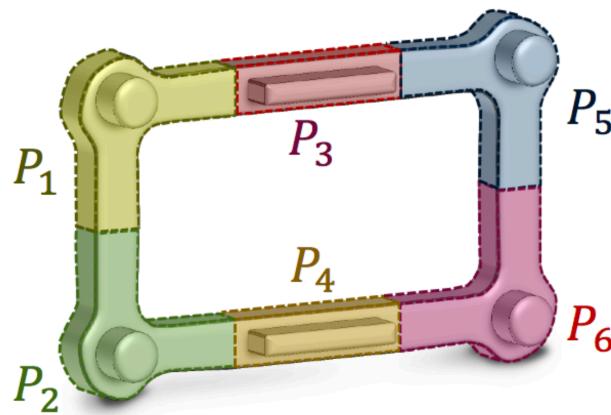
Structure Discovery

- Parameterize the parts – data-based



Structure Discovery

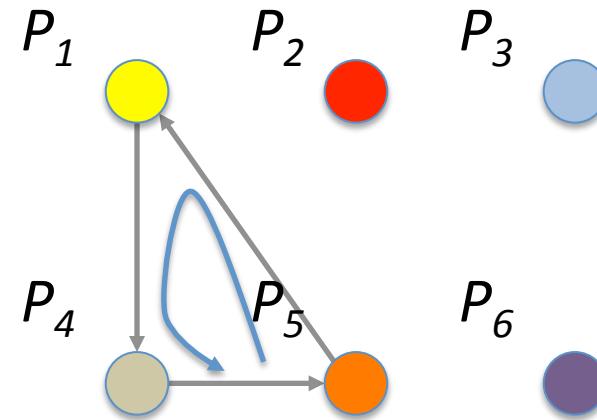
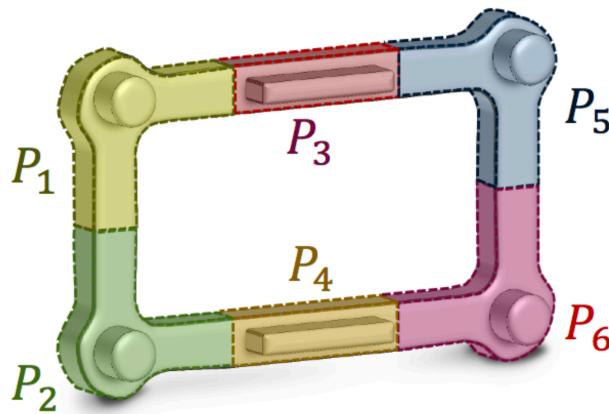
- Establish relations



Pair-wise relations

Structure Discovery

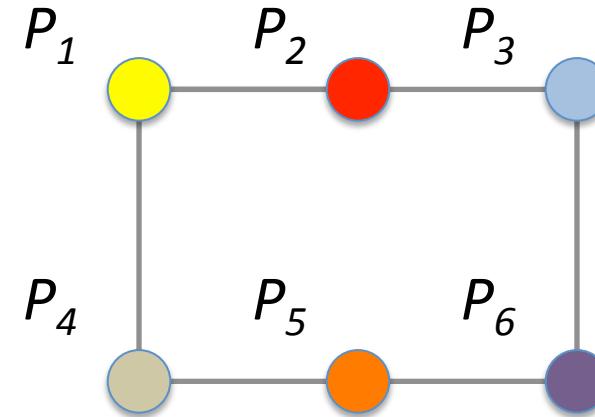
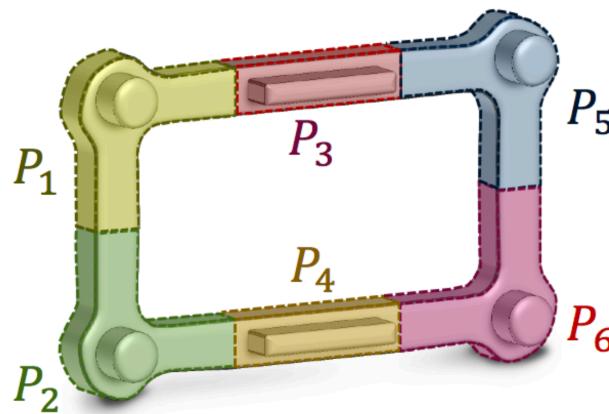
- Establish relations



Higher order relations

Structure Discovery

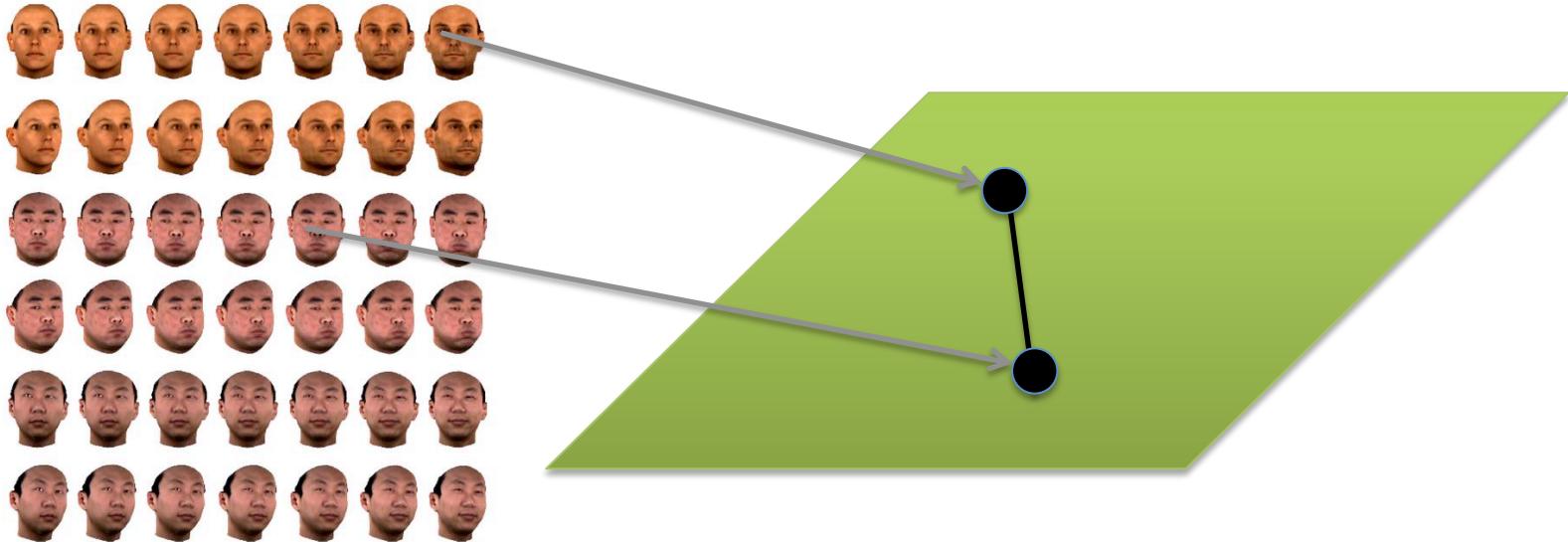
- Establish relations



Markov-random fields

Structure Discovery

- Establish relations



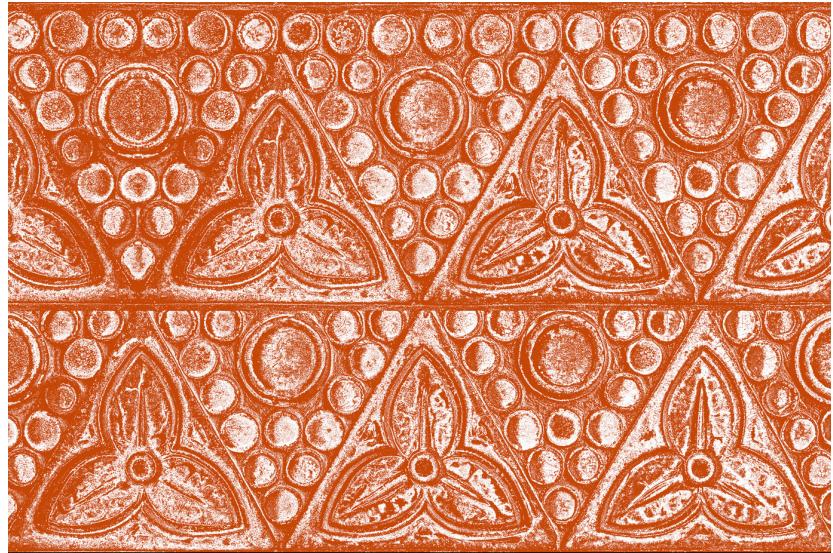
Symmetry

- A fundamental structural relation: symmetry



Symmetry

- A fundamental structural relation: symmetry



Symmetry

- What is symmetry?

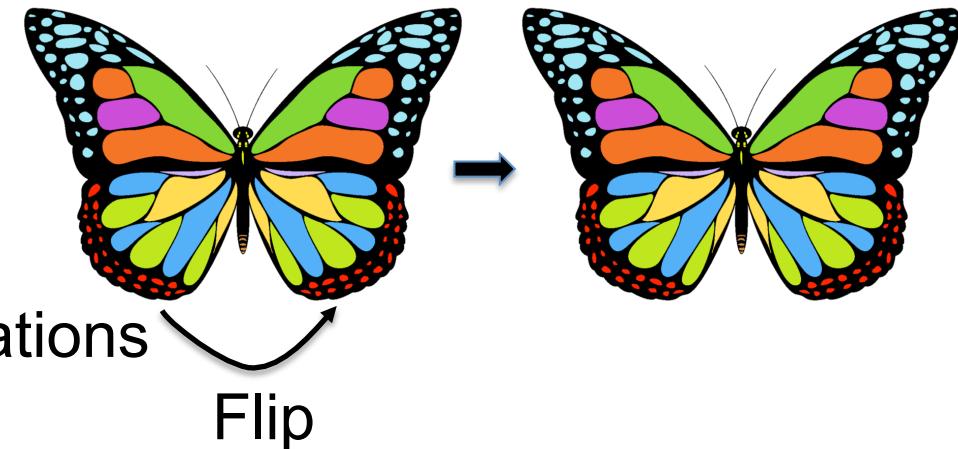
Given an object M

And some transformation T

$$M = T(M)$$

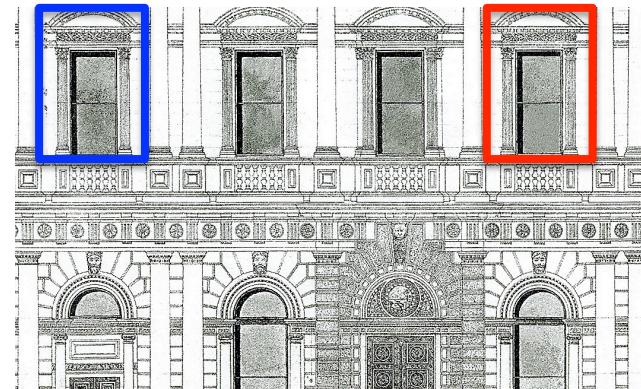
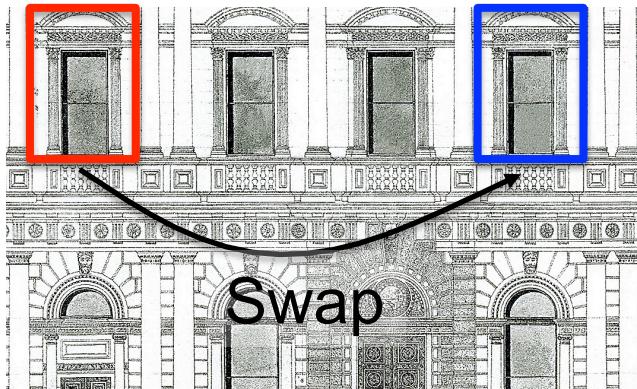
Space of symmetry transformations

$$T \in \mathcal{S}$$



Symmetry

$$M = T(M)$$



Symmetry

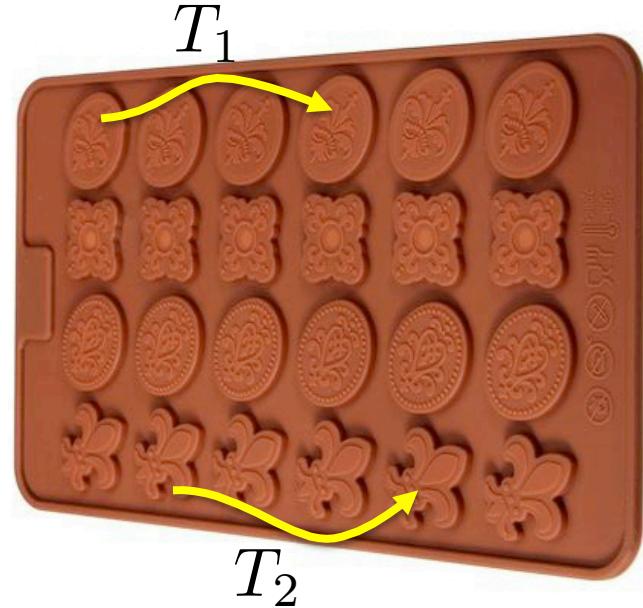
- Properties of symmetry transformations

Identity element

$$I(M) = M \quad I \in \mathcal{S}$$

Closure

$$T_1 \in \mathcal{S} \quad T_2 \in \mathcal{S} \rightarrow T_1 T_2 \in \mathcal{S}$$



Symmetry

- Properties of symmetry transformations

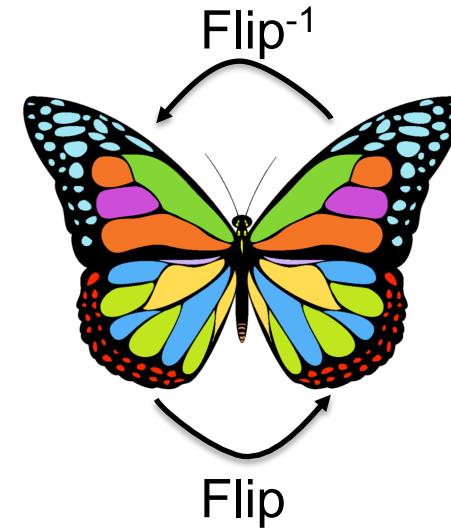
Inverse element

$$T \in \mathcal{S} \rightarrow T^{-1} \in \mathcal{S}$$

such that $TT^{-1} = T^{-1}T = I$

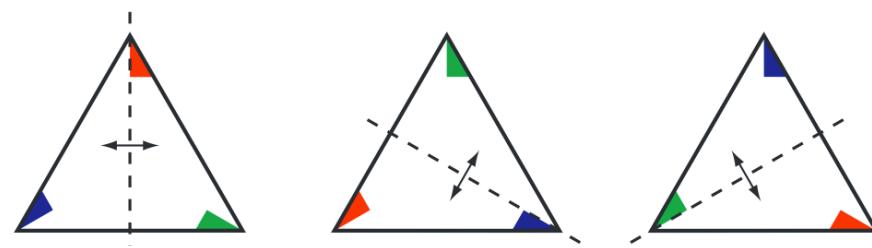
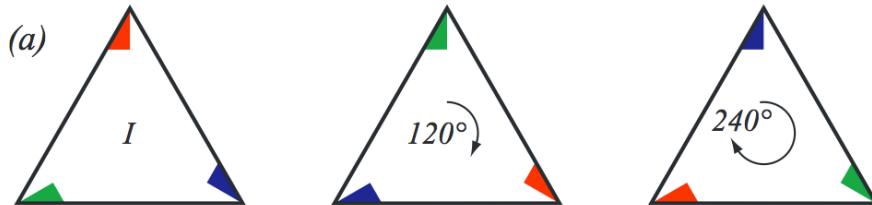
Associativity

$$(T_1 T_2) T_3 = T_1 (T_2 T_3)$$

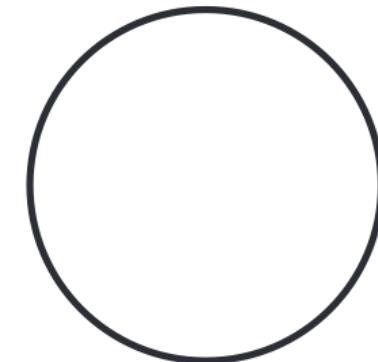


Symmetry

- Examples of symmetry groups



dihedral group D_3



infinite group $O(2)$

Symmetry

- Measuring symmetry

$$d(M, T(M))$$

ϵ – symmetric

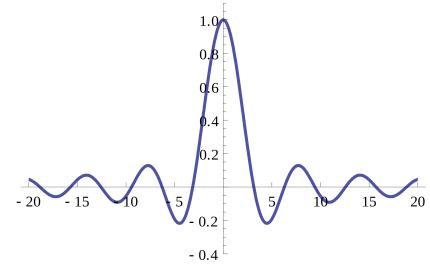
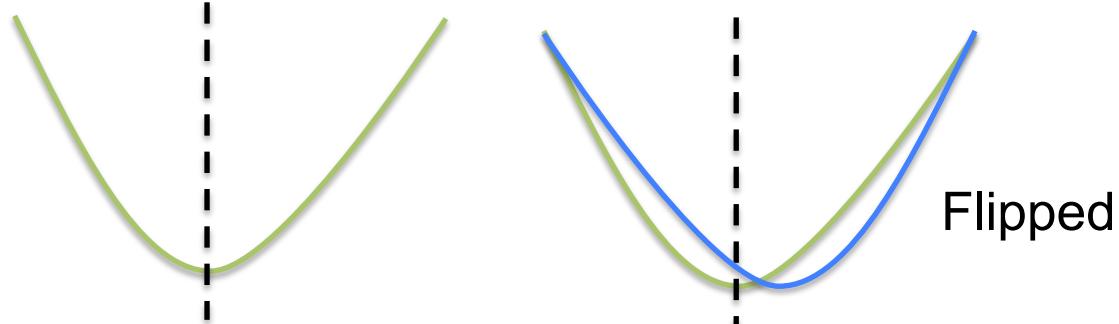
$$d(M, T(M)) < \epsilon$$



Symmetry

- Measuring symmetry

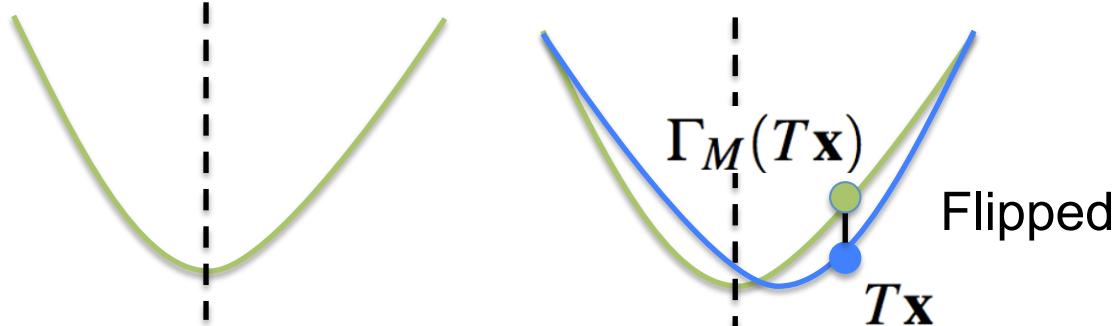
$$d(f, T) = \left\| f - \frac{f + T(f)}{2} \right\| = \left\| \frac{f - T(f)}{2} \right\|$$



Symmetry

- Measuring symmetry

$$d(M, T(M)) = \int_{\mathbf{x} \in M} \|T\mathbf{x} - \Gamma_M(T\mathbf{x})\|^2 d\mathbf{x}$$



Symmetry Detection

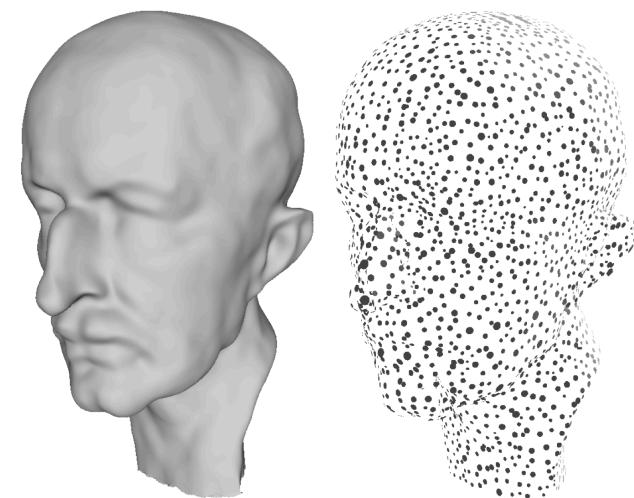
- First try: brute force algorithm

Sample n points

For each combination of 3 points

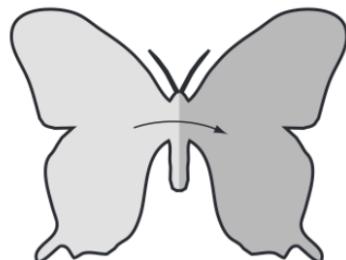
Check if $d(M, T(M)) < \epsilon$

Complexity: $O(n^4)$

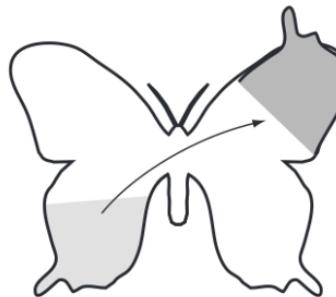


Symmetry Detection

- Step 0: decide on the symmetry group



reflection



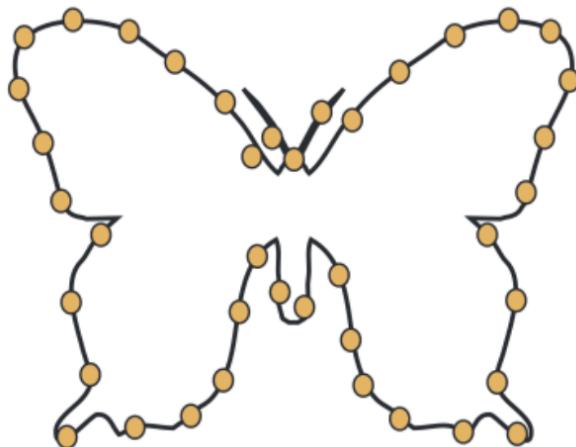
reflection + rotation + translation



scale + rotation + translation

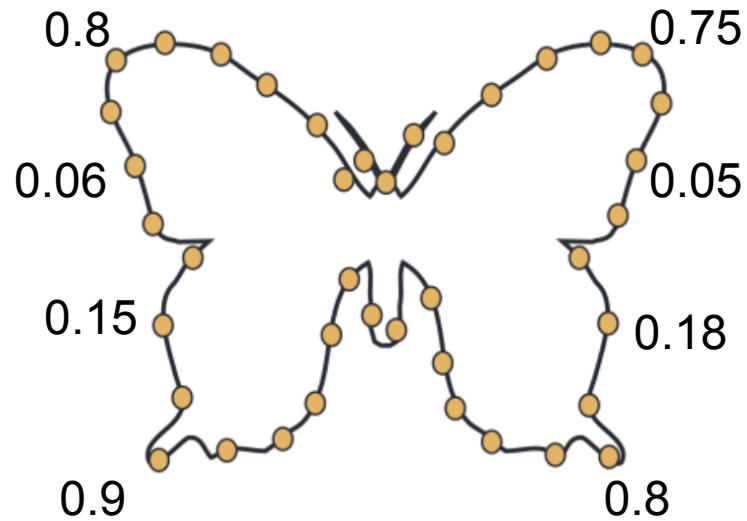
Symmetry Detection

- Step 1: sample points on the model



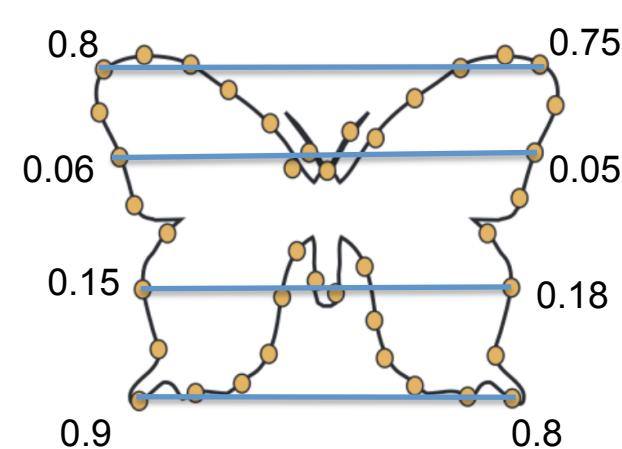
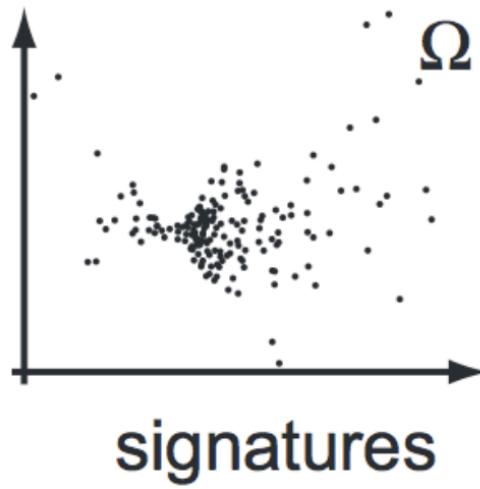
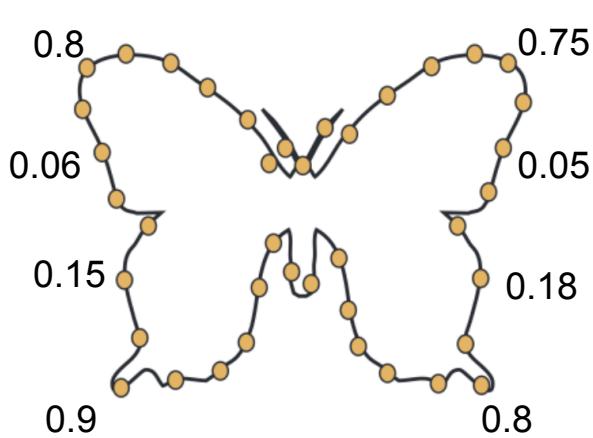
Symmetry Detection

- Step 2: compute invariant signatures



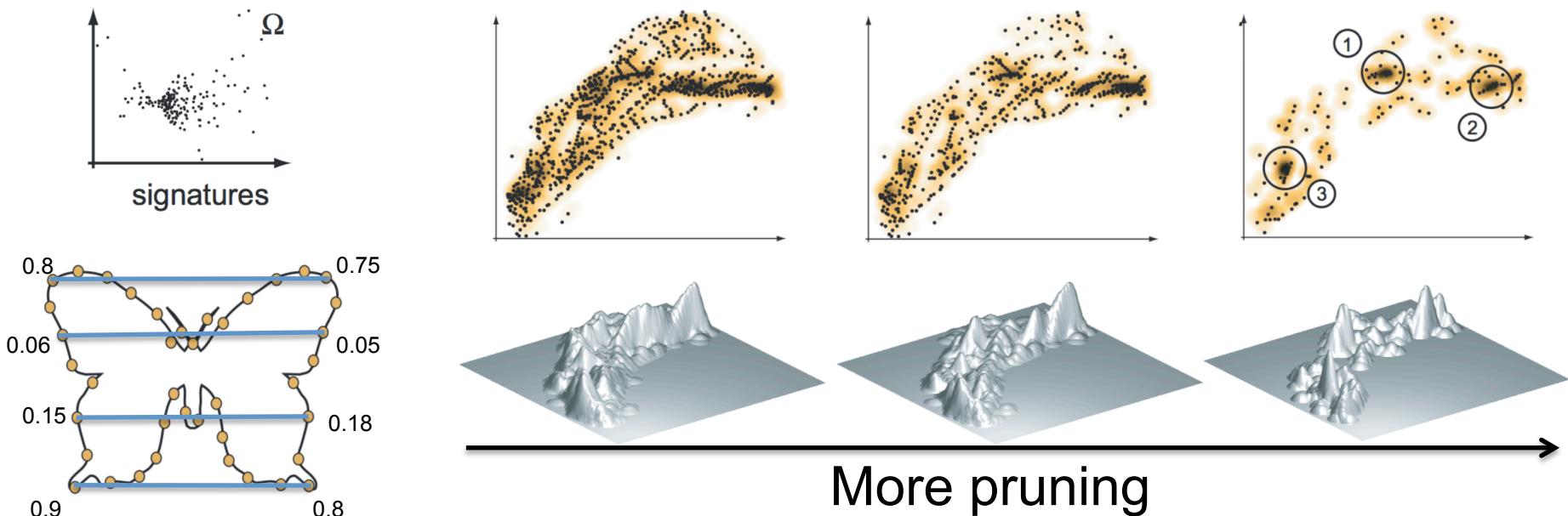
Symmetry Detection

- Step 3: map all pairs of points to the signature space and pair points



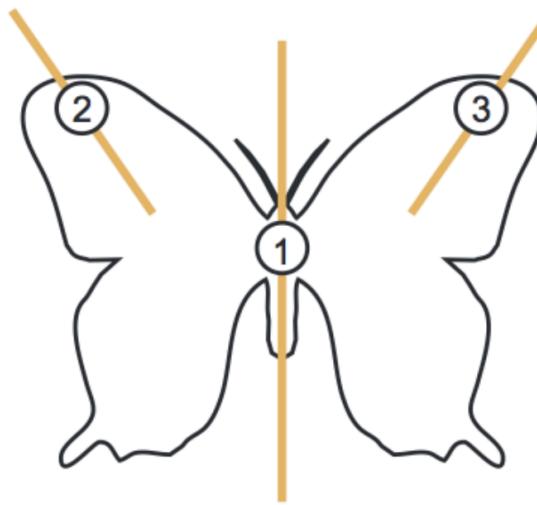
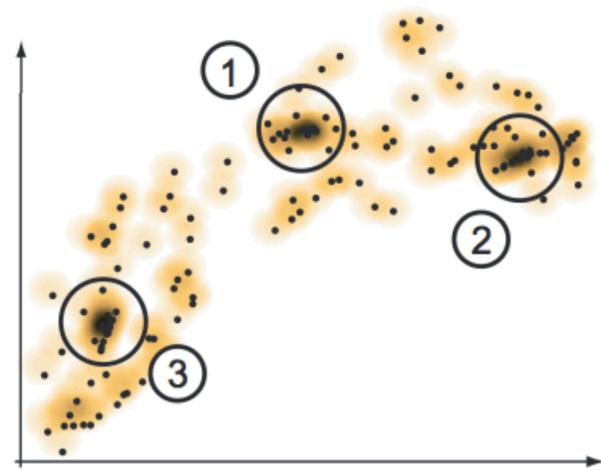
Symmetry Detection

- Step 4: compute transformations and prune



Symmetry Detection

- Step 5: cluster to detect symmetries



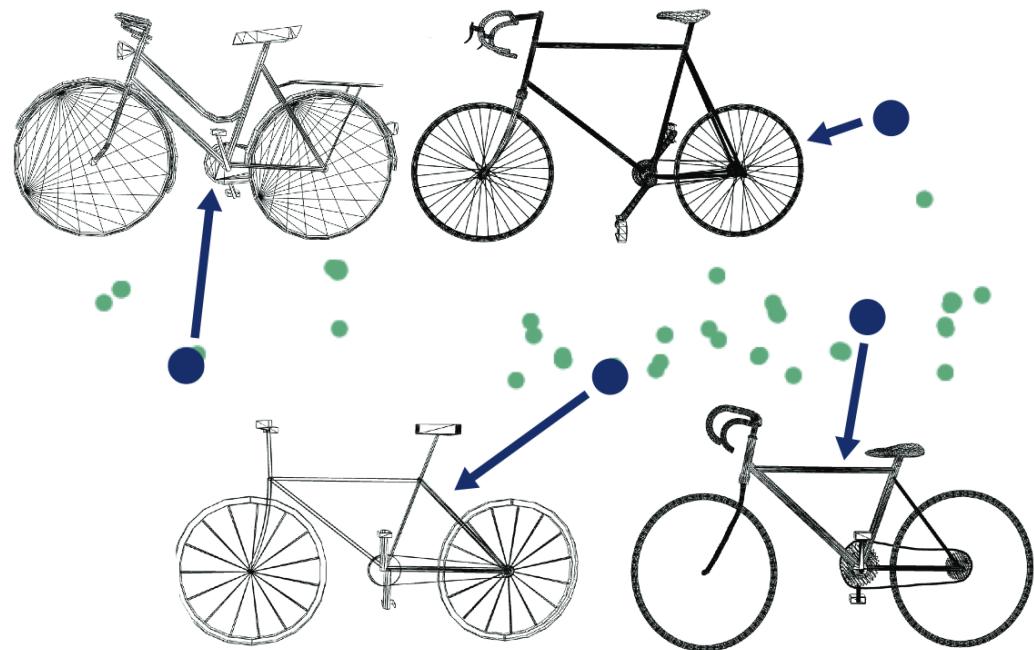
Data Based Structure

- Learn structure from provided models



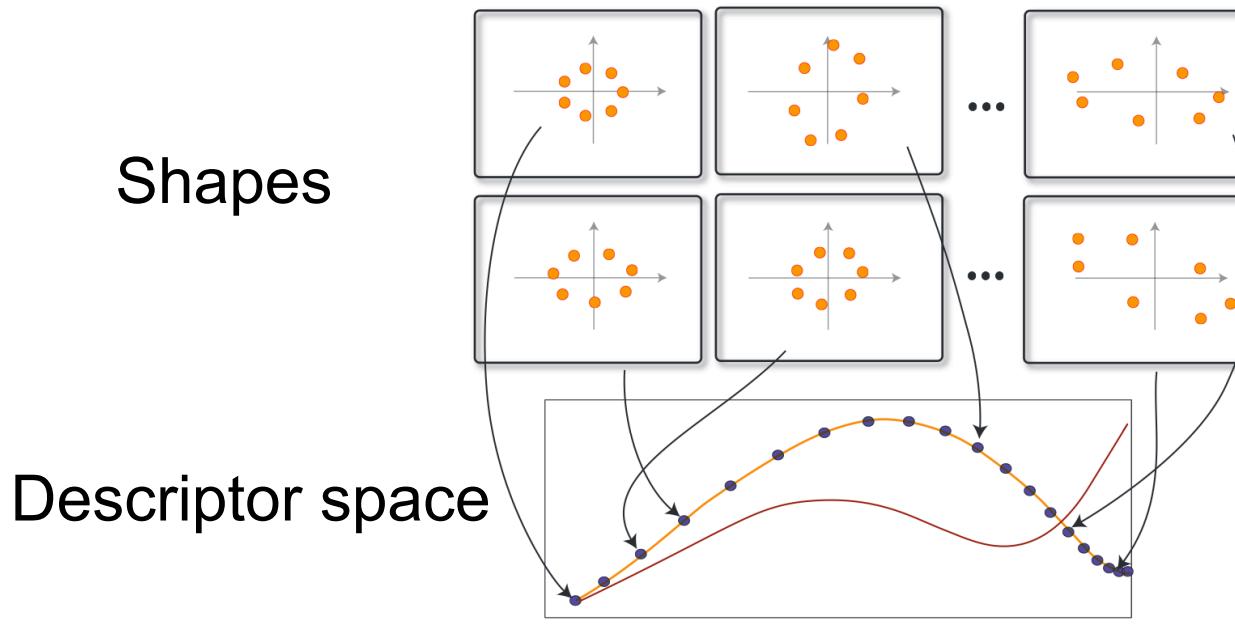
Data Based Structure

- Compute descriptors for each shape
- Project into a subspace using PCA



Data Based Structure

- Extract underlying degrees of freedom



Exam

- Everything from the lectures & exercises
- Closed-book
- Random questions from a box
- 20 minutes
- We will send out a schedule
- At least 30 minutes earlier than the scheduled time
- Questions are always welcome

Conclusions

- Mathematical foundations
 - Representation
 - Approximation
 - Optimization
 - Transformation
 - Integration & differentiation
 - Structure extraction

Conclusions

- Mathematical foundations
 - Deep connections to various fields
 - Strong emphasis on numerical methods
 - Very important: learn on the correct level
 - Very important: don't be afraid to go deep
 - Very important: don't be lazy to go broad

Some references

- Applications:
 - Dense 3D Reconstruction of Symmetric Scenes from a Single Image
 - Modeling by Example
 - iWIRES: An Analyze-and-Edit Approach to Shape Manipulation
 - Example-based Synthesis of 3D Object Arrangements
 - Partial and Approximate Symmetry Detection for 3D Geometry
 - Image Denoising by Aparse 3D Transform-domain Collaborative Filtering
- Surveys:
 - Structure-aware Shape Processing
 - Symmetry in 3D Geometry: Extraction and Applications
 - Computational Symmetry in Computer Vision and Computer Graphics
- Symmetry:
 - Partial and Approximate Symmetry Detection for 3D Geometry