Mathematical Foundations of Computer Graphics & Vision

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Today!

1) Handout, exercise 6;

Primal Dual Algorithm

$$\min_{x \in X} \max_{y \in Y} \langle Kx, y \rangle + G(x) - F^*(y)$$

Algorithm 1 Primal Dual algorithm

- **Initialization:** Choose $\sigma, \tau > 0$, $\theta \in [0,1]$, $(x^0, y^0) \in X \times Y$, and set $\bar{x}^0 = x^0$
- **Iterations:** $(n > 0)$, update x^n, y^n and $\bar{x}^n$ as follows

$$\begin{cases} y^{n+1} & = & \operatorname{prox}_{\sigma F^*}\left(y^n + \sigma K \bar{x}^n\right) \\ x^{n+1} & = & \operatorname{prox}_{\tau G}\left(x^n - \tau K^* y^{n+1}\right) \\ \bar{x}^{n+1} & = & x^{n+1} + \theta\left(x^{n+1} - x^n\right) \end{cases}$$

Goal of the homework

- Theory
  - Convexity
- Applications
  - Segmentation
  - Inpainting

Part 1

- Are the following function convex?

  - $x \mapsto \sin(x)$
  - $x \mapsto x^2$
  - $x \mapsto \sin(x) + x^2$

- Show convexity of the ROF functional

$$E_{ROF}\left(I_u\right) = \int_\Omega \left[\left| \nabla I_u(\mathbf{x}) \right| + \parallel I_u(\mathbf{x}) - I_0(\mathbf{x}) \parallel_2^2 \right] d\mathbf{x}$$

Part 2 – 3 – Total Variation

- A word on total variation

$$\min_{x \in X} \lambda G(x) + \| \nabla x \|_1$$

- Primal Dual formulation?

$$\| \nabla x \|_1 = \max_{y \in Y} \langle \nabla x, y \rangle \text{ where } Y = \{ y \in D_X | \, \| y \|_\infty \leq 1 \}$$

Part 2-3 – Total Variation

- Primal Dual TV

$$\min_{x \in X} \max_{y \in D_X} \langle \nabla x, y \rangle + \lambda G(x) - \delta_Y(y)$$

$$\delta_Y(y) = \begin{cases} 0 & \text{if } y \in Y \\ \infty & \text{if } y \notin Y \end{cases}$$

Part 2 - Segmentation

- Revisit the interactive segmentation
- Find a function that is close to 0 in background and close to 1 in foreground

$$G(x) = \langle x, f \rangle + \delta_{[0,1]}(x)$$

$$f_i = \log H_{bg}(I_i) - \log H_{fg}(I_i)$$

Part 2-Segmentation

Part 2-Segmentation

Part 3 - Inpainting

- Filling missing parts of images

$$G(x) = \frac{1}{2} \sum_{i,j \in \mathcal{D}_I \setminus \mathcal{I}} \frac{1}{2} (I_{i,j} - x_{i,j})$$

Where $\mathcal{I}$ is the inpainting region

$$\min_{x \in X} \max_{y \in D_X} \langle \nabla x, y \rangle + \frac{\lambda}{2} \sum_{i,j \in \mathcal{D}_I \setminus \mathcal{I}} \frac{1}{2} (I_{i,j} - x_{i,j}) - \delta_Y(y)$$

Part 3 – Task 1

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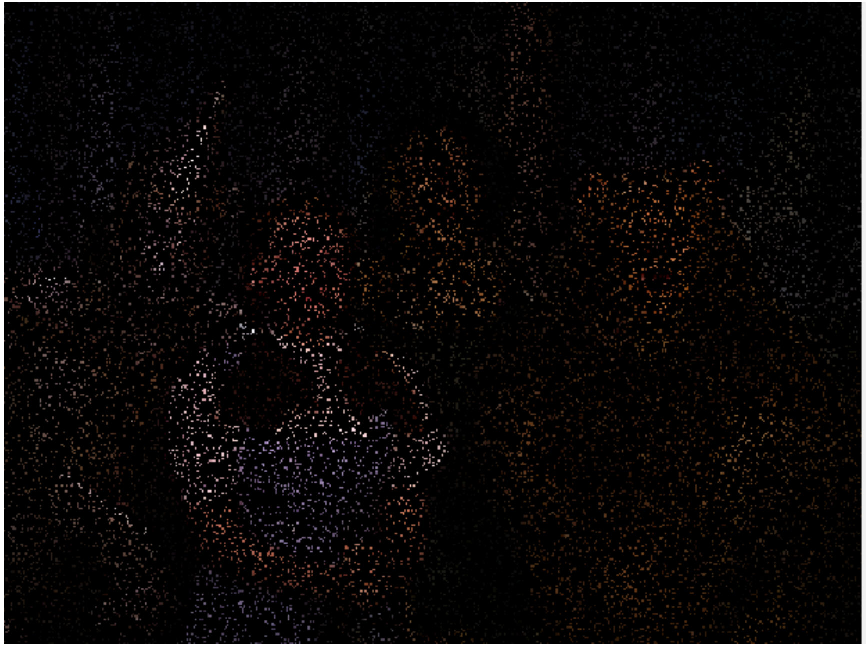


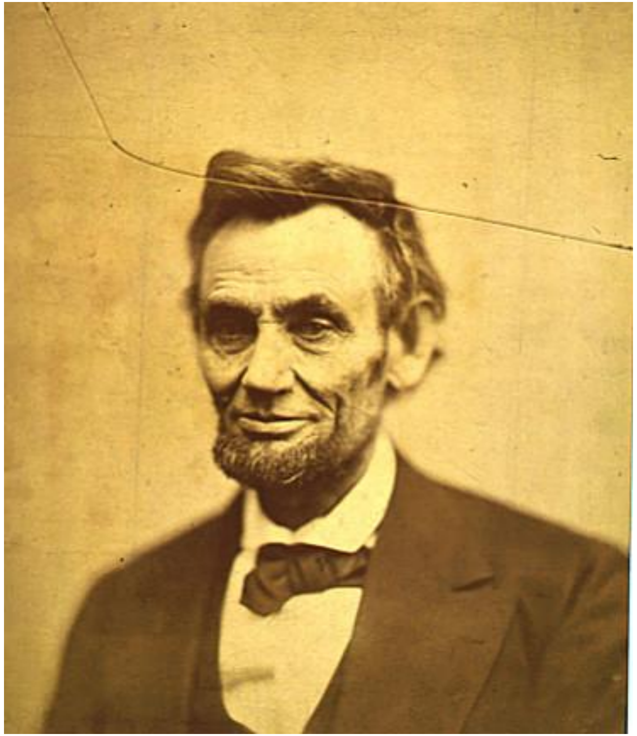
Part 3 – Task 1

Part 3 – Task 2

- Interactive inpainting: remove artifacts
- Weighted TV

$$\min_{x \in X} \max_{y \in D_X} \frac{1}{\| \nabla I_0 \|_1 + 1} \langle \nabla x, y \rangle + \lambda G(x) - \delta_Y(y)$$

Part 3 – Task 2



Part 3 – Task 2