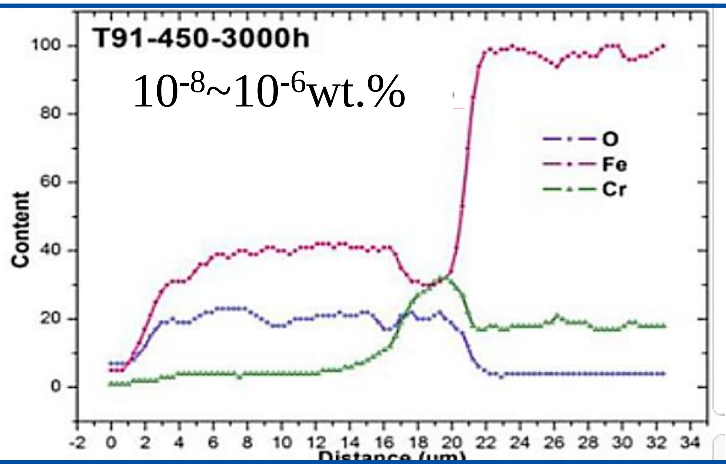
- **The** **Lead-Cooled Fast Reactors** **(LFR) has been acknowledged as the most accessible small modulus** **Gen-IV** **Reactor**

- **As the core component of the 1st hydraulic loop, the** **main coolant pump** **(MCP) suffers from the severe** **corrosion** **caused by the high-temperature, high-density, high-velocity coolant**

- **Principal of MOO design of MCP: Lower the velocity + Satisfy the practical requirement**



>500°C, 2m/s → Corrosion

High Density → Vibrate, Fatigue…

→ Considering the Inner Flow Field !

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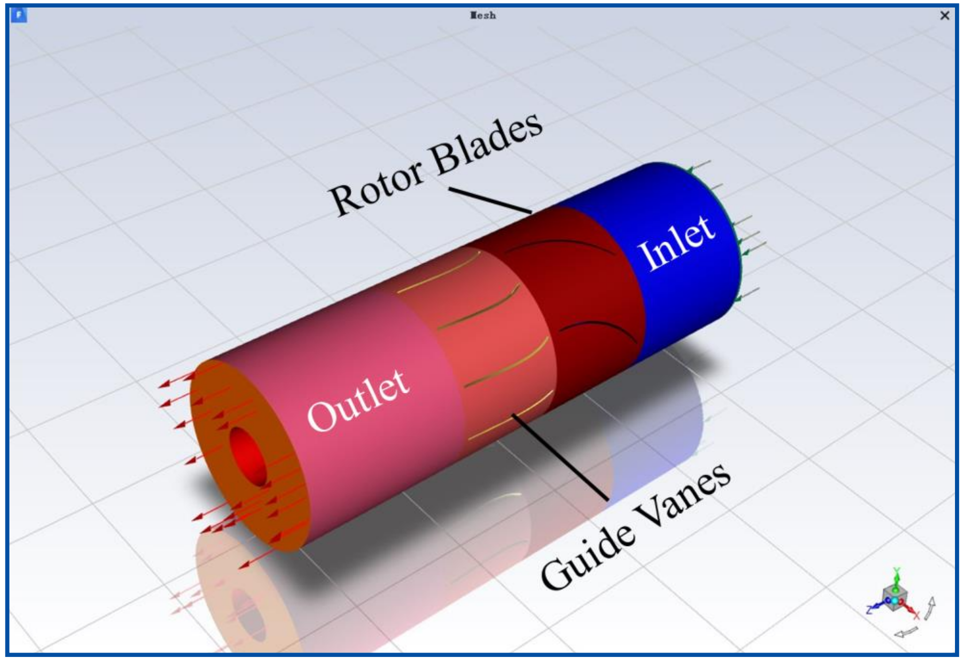
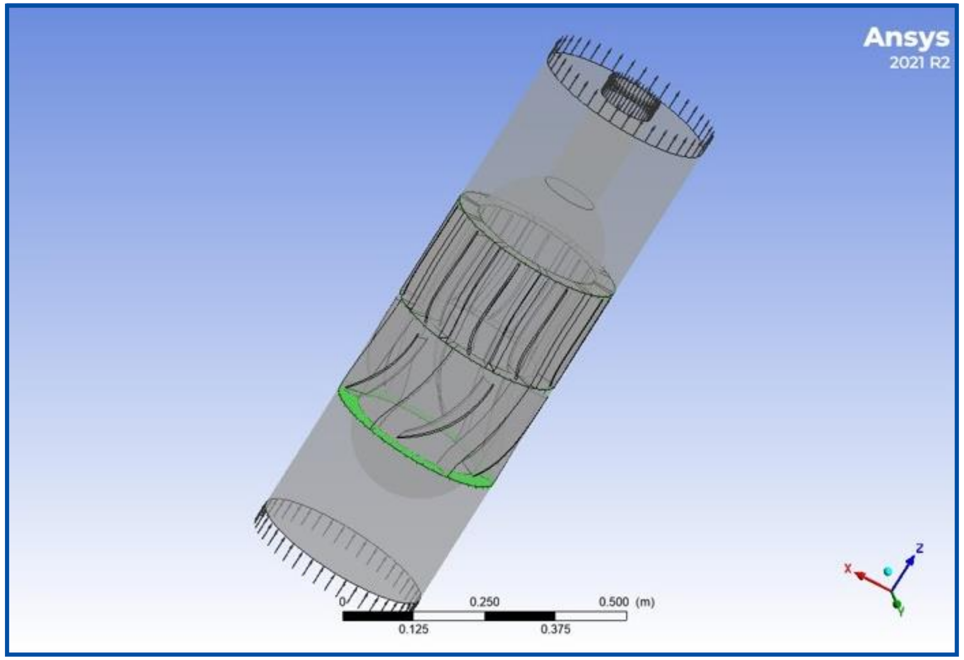
❖ Traditional Methods to Obtain Inner Flow Field within the Blade Region:





Ansys Fluent: FVM

Ansys CFX: FEM-FVM

- **High-quality mesh + Discretized Navier-Stokes Equations + Iterative Solving → High Computational Cost**
- **Pump design considering flow field: Change the operating/structural para → Excessive simulation tasks**

→ Improve Single Simulation Speed

→ Surrogate Modeling

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- **Surrogate Modeling: Creating an approximate model *f*(*operation*, *structure*)**

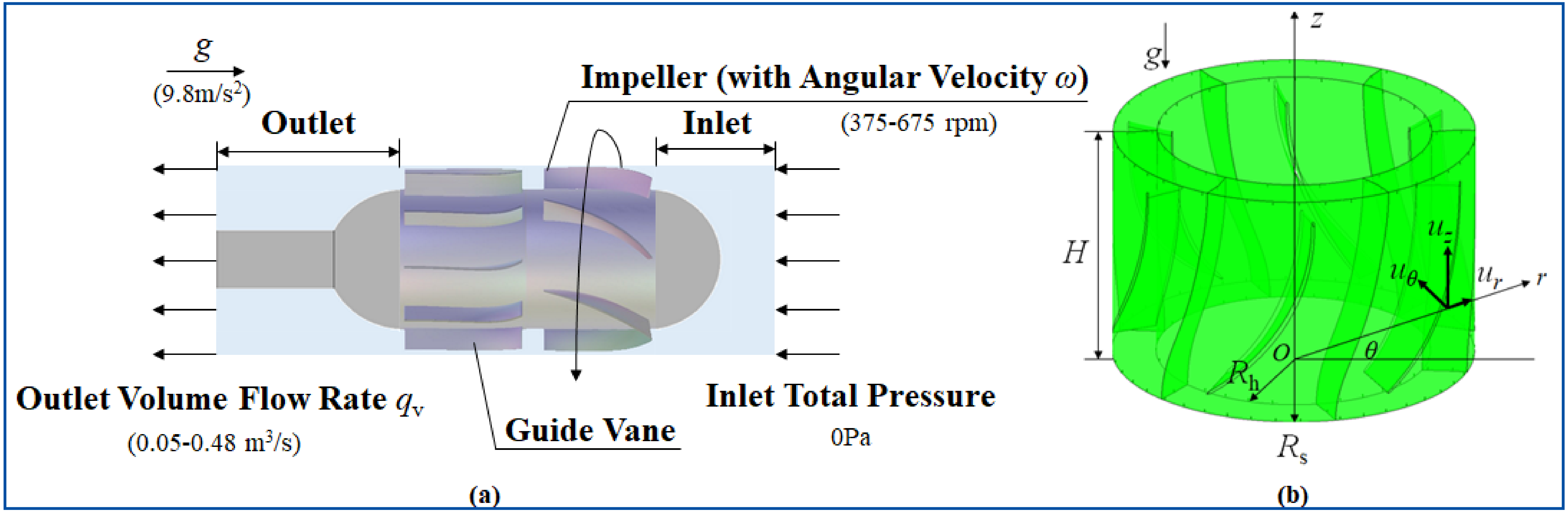
Surrogate Methods	With Data?	With Physics Constraints?	Modeling Strategy	Training Cost
ROM	Yes	No (implicit/explicit)	Linear Decomposition, Perform Truncation	Medium or Low
Pure NN	Large Dataset Required	No	Fitting Input-Output Mapping	Medium (Depends on the dataset)
PINN	Small Dataset is Acceptable	Yes	Fitting physical laws and data simultaneously	High (Differentiation)

- **Generalization Ability:** model's capacity to make accurate predictions under new conditions.
- PINN has the best generalization ability, however incorporating **operating/structural parameters** into PINNs significantly **enlarges the solution space**, making training more challenging and less efficient.

→ **ROM(Ensure the General Flow & Reduce the Solution Space)**
+ PINN(Regress ROM Coeff. with small Data) ?

2 Methodology

❖ **Example: an axial main coolant pump (MCP) in a lead-cooled fast reactor (LFR)**



- **Coolant: lead-bismuth eutectic (LBE) at 350°C, with the density 10278kg/m^3, dynamic viscosity 0.0016 Pa s**

❖ **Objective: Achieving real-time solution of the flow field in the impeller area at different rotating speeds and outlet flow rates**

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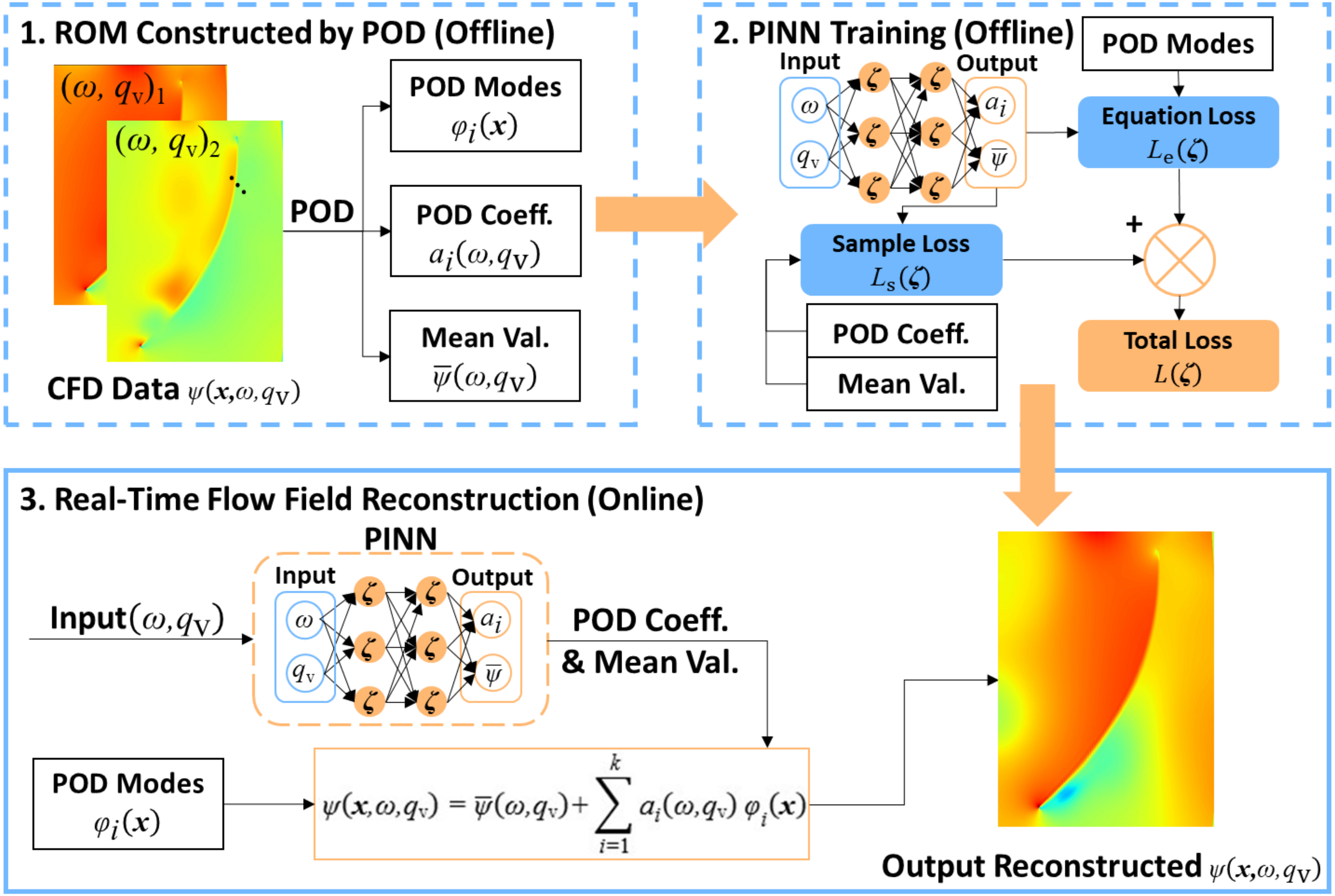
POD-PINN Approach

POD: To discover the **common features known as Modes/Modals** of the flow field in multiple operating conditions.

PINN: To re-construct the modals properly by **regressing accurate modal coefficients** according to the operating parameter input into the model.



POD Reducing Order Process

- **Objective: Getting the first k main modes $\varphi_i(\boldsymbol{x})$**
  - **Creating the simulation set $\{\psi(\boldsymbol{x},\omega,q_{\mathrm{V}})|(\omega,q_{\mathrm{V}})_{\boldsymbol{j}}\}$ and forming a "Snapshot Matrix" $\boldsymbol{S}$;**
  - **Performing SVD to $\boldsymbol{S}-\bar{\psi}(\omega,q_{\mathrm{V}})$ and extract the largest k singular values (indicating that they account for the largest energy proportion of the system) with the corresponding modal vector (the row of the right matrix) which can be later interpolated to modals $\varphi_i(\boldsymbol{x})$**
  - **Calculate the modal coefficients $a_i(\omega,q_{\mathrm{V}})$, forming a dataset $\{(a_i,\bar{\psi})(\omega,q_{\mathrm{V}})_j\}$ for PINN.**

PINN Training Process

- **Objective: Getting the best estimation of $a_i(\omega,q_{\mathrm{V}};\boldsymbol{\zeta})$ and $\bar{\psi}(\omega,q_{\mathrm{V}};\boldsymbol{\zeta})$ by adjusting network parameter $\boldsymbol{\zeta}$ (MOO Problem)**
  - **Sub-task1: Fitting the dataset $\{(a_i,\bar{\psi})(\omega,q_{\mathrm{V}})_j\}$**
  - **Sub-task2: Minimize the governing equations (coordinates on the pump axial):**

- The Continuum Equation:

$$F_C(u_r,u_\theta,u_z)=\frac{1}{r}\frac{\partial(ru_r)}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{\partial u_z}{\partial z}=0$$

- The r-Momentum Equation:

$$F_R(u_r,u_\theta,u_z,p)=u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial\theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}+\frac{1}{\rho}\frac{\partial p}{\partial r}-\frac{\mu}{\rho}\left(\nabla^2u_r-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial\theta}\right)-\omega^2r+2\omega u_\theta=0$$

- The θ-Momentum Equation:

$$F_\theta(u_r,u_\theta,u_z,p)=u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial\theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_\theta u_r}{r}+\frac{1}{\rho r}\frac{\partial p}{\partial\theta}-\frac{\mu}{\rho}\left(\nabla^2u_\theta-\frac{u_\theta}{r^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial\theta}\right)-2\omega u_r=0$$

- The z-Momentum Equation:

$$F_Z(u_r,u_\theta,u_z,p)=u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial\theta}+u_z\frac{\partial u_z}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}-\frac{\mu}{\rho}\nabla^2u_z+g=0$$

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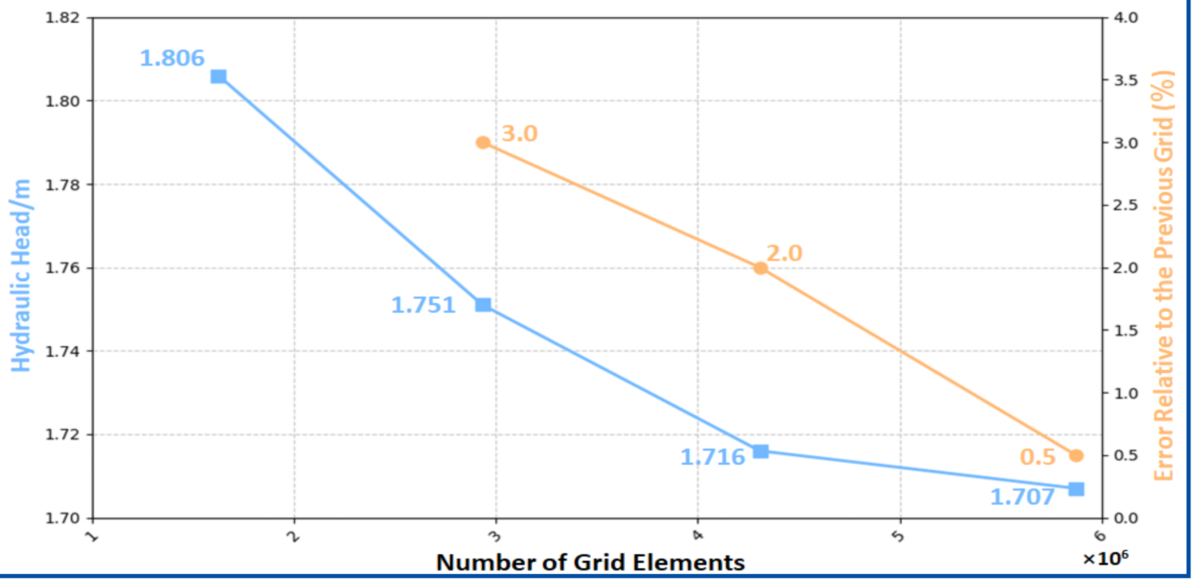
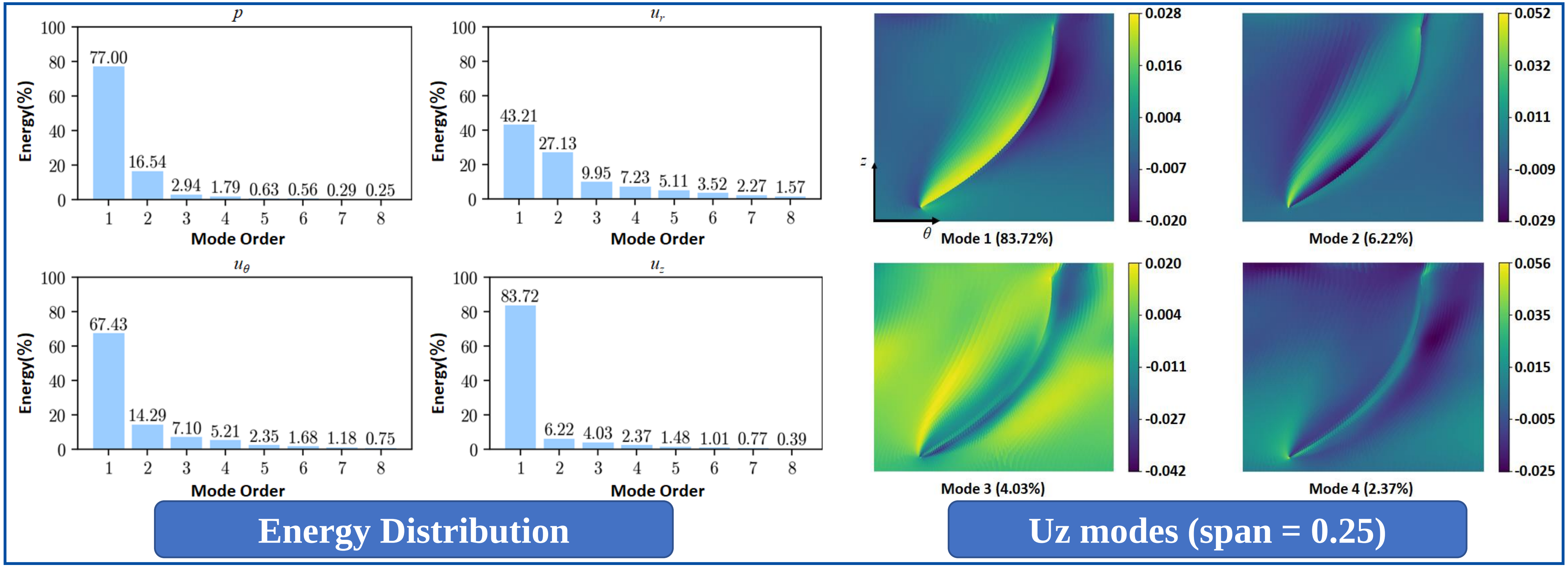
Dataset Construction

Num. i,j	1	2	3	4	5
Rotor Speed ω_i/RPM	375	450	525	600	675
Flow Rate $q_{v,j}/(m^3\ s^{-1})$	0.05	0.16	0.27	0.37	0.48

- 5*5 = 25 Simulations with Ansys CFX were carried out
- Set all j=1 as the testing set (Did not participate in the sampling loss function)
- The grid independence was validated at 420rpm, 0.099m^3/s



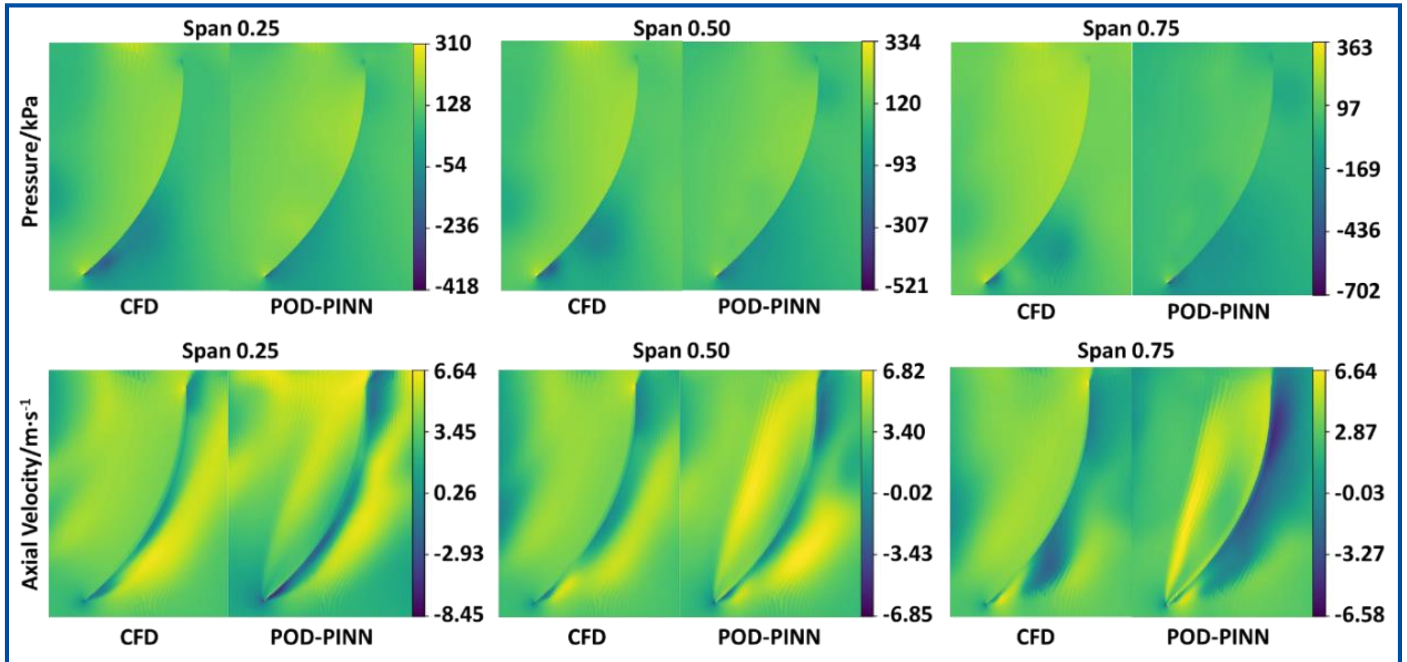
POD Result



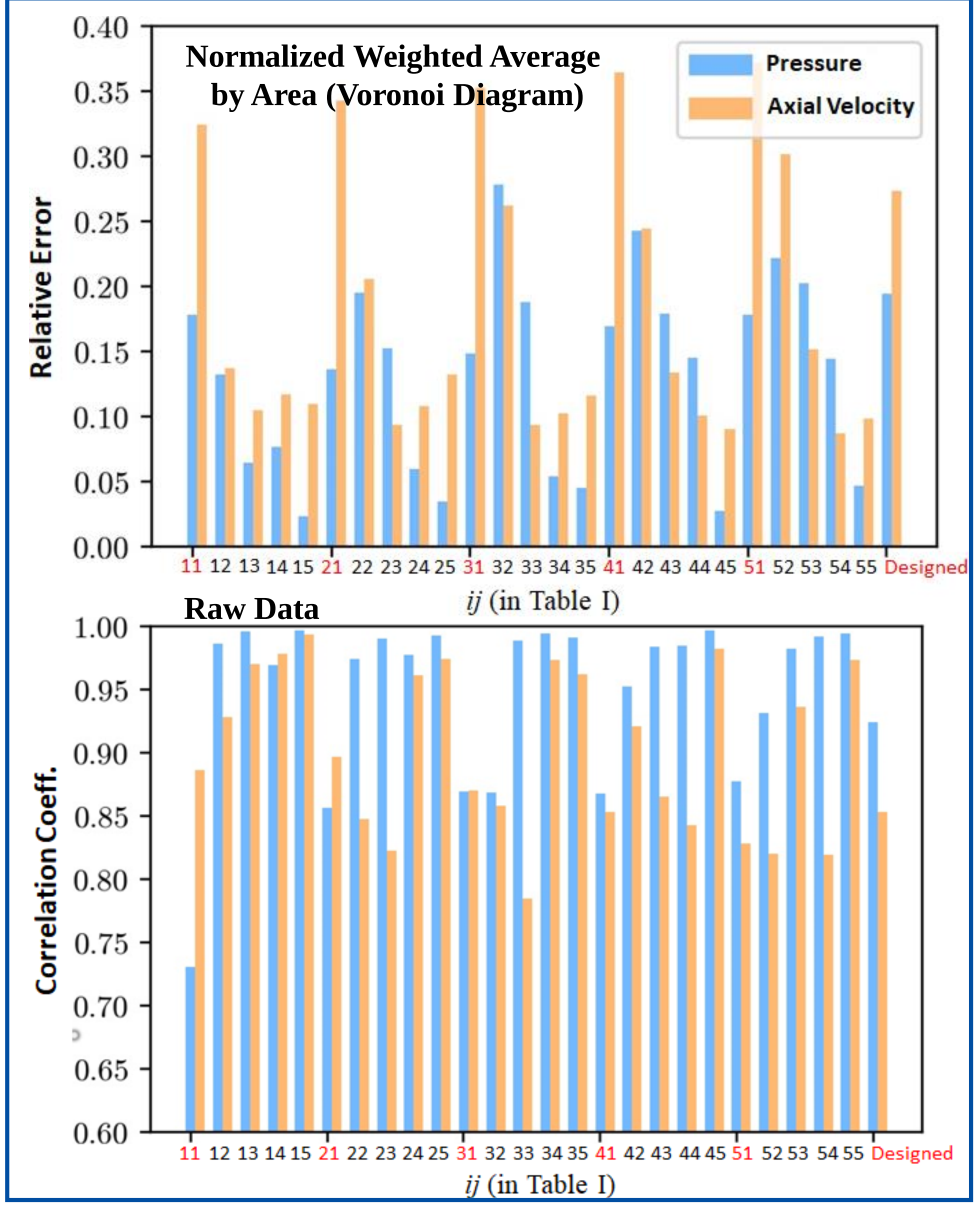
- **Axial pump has little radial velocity. (u_r is not strictly cared, cylindrical-layer independence assumption)**
- **The modal distinction of other physical quantities is relatively high. First 4 modes are selected for later reconstruction (more than 95% energy proportion)**

PINN Result

Span = 0.25, 0.50, 0.75 in Average:



420rpm, 0.099m³/s Reconstruction (Low q_v)



- **Relative Error < 35%; Pearson correlation coefficient > 70%;**
- **Errors mostly occurs in the operating condition where the flow rate is relatively low.**

3 Numerical Results

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- **The details for the following discussion are available at the Project Repository:**
  - **In order to verify the impact of different training strategy, interpolation method and dataset partitioning methods on model performance, the simulation data of another axial flow LBE pump with span=0.65 is used.**
  - **Simulate the same conditions as the previous model, but only use *ij*=13, 24, 32, 43, 52 as testing sets to avoid extrapolation.**

Num. i,j	1	2	3	4	5
Rotor Speed ω_i/RPM	375	450	525	600	675
Flow Rate $q_{v,j}/(m^3\ s^{-1})$	0.05	0.16	0.27	0.37	0.48

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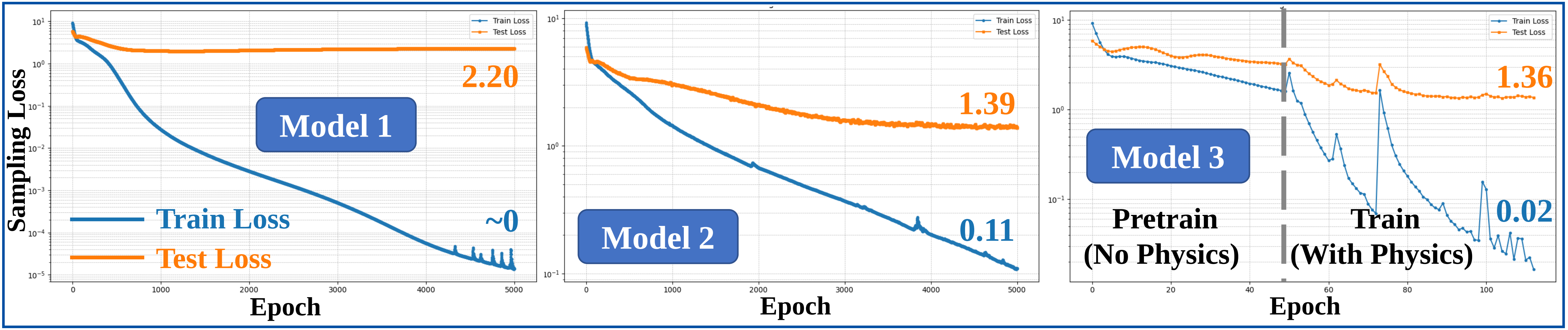
❖ Influence of the Training Strategy: Focus on the Loss Curves



- **Model 1 (without physics loss): Overfitted, the Testing Loss Stopped Going Down at the ~500th Epoch;**

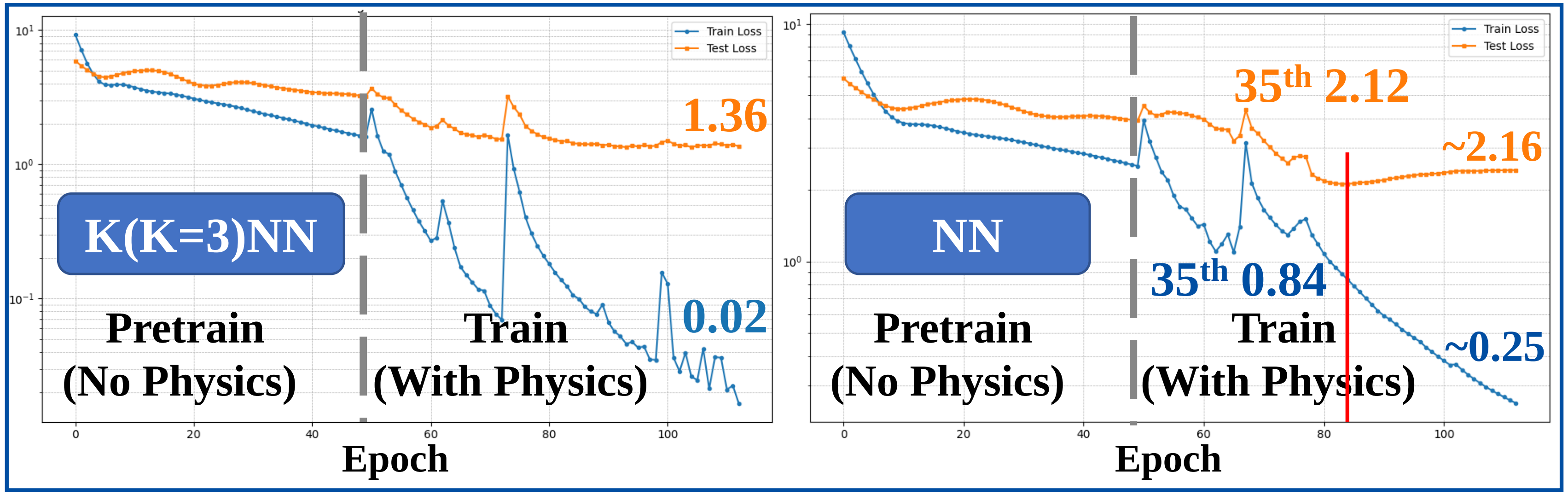
- **Model 2 (with physics loss, randomly applied on several spatial coordinates at each epoch): Underfitted;**

✓ **To improve Model 2 (Train loss was high), pretraining and larger learning rate were applied to Model 3. Unless otherwise specified, all other hyperparameters are the same (including random seed).**

- **Model 3 (Pretrain + with physics loss, at each spatial coordinates): Has the best generalizability.**

Influence of the Interpolation Methods:

- **The POD method obtains a mapping between a coordinate list and the corresponding modal values.**
- **Interpolation is required to ensure the prediction capability for any coordinates and to ensure that the derivative of the equation loss function is calculable.**



KNN	NN
nearest points to estimate	Neural network to regress
C1(Coord. set) C2 (Other)	C∞
Slower when training	~2 times faster than KNN
More accurate Well done	Less accurate Underfitted

- **The KNN and NN interpolator obtained similar loss curves.**
- **NN interpolator sacrificed its accuracy to achieve better landscape and speed.**

3 Numerical Results

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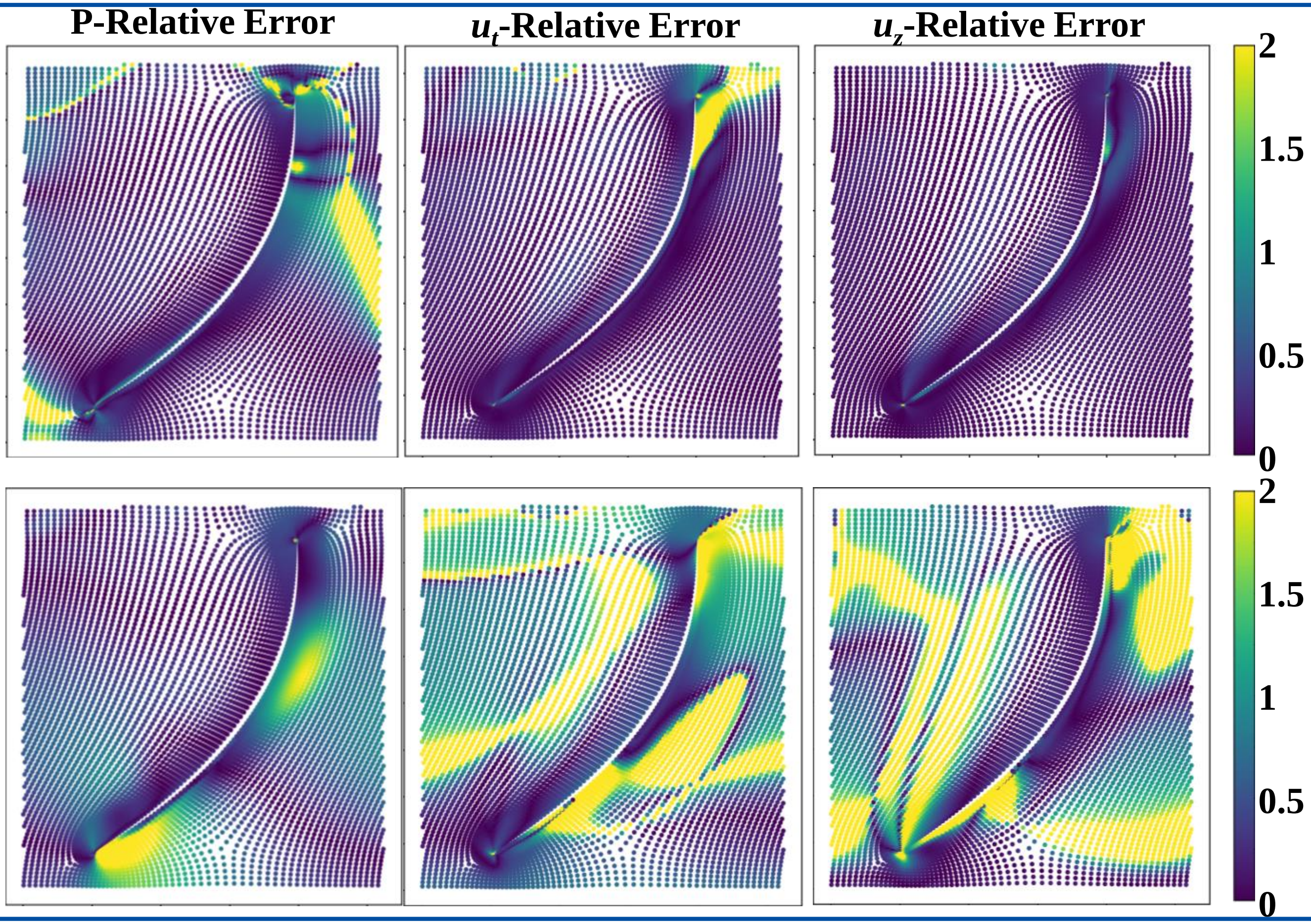
❖ Influence of the Dataset Partitioning Methods: Not Significant

600rpm
$0.27m^3/s$
Test

600rpm
$0.05m^3/s$
Train



Item	P	u_t	u_z
Pearson Correlation Coefficient	0.953	0.950	0.762
Normalized Area Weighted Relative Error	2.27%	5.01%	5.96%

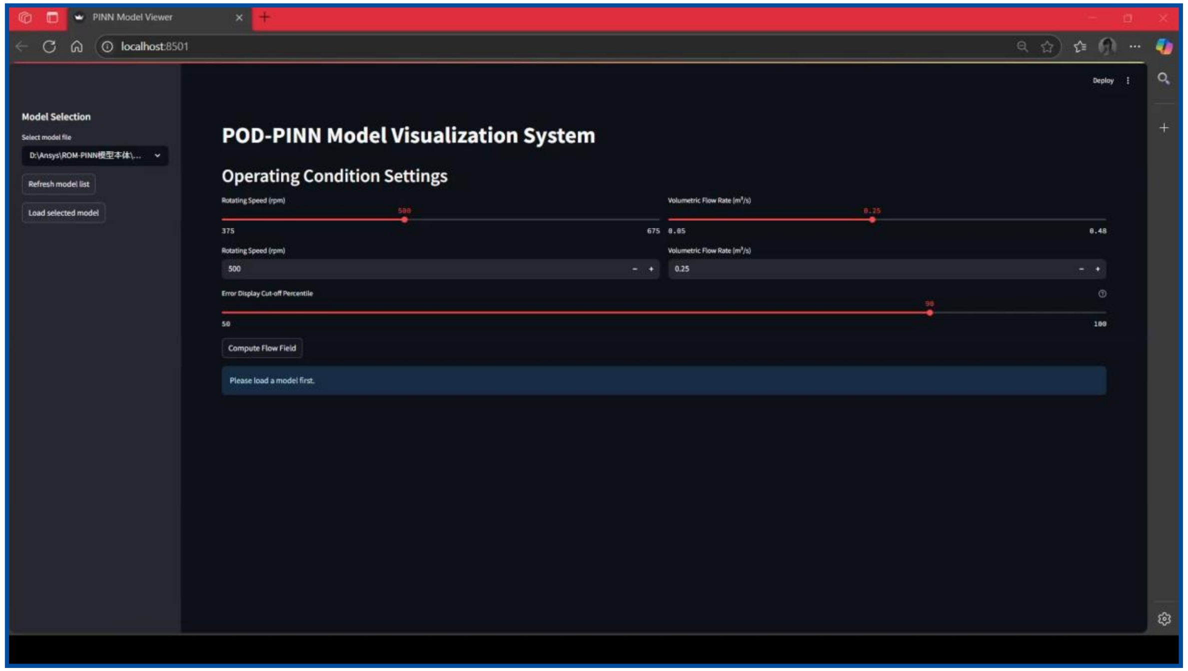
Item	P	u_t	u_z
Pearson Correlation Coefficient	0.745	0.739	0.881
Normalized Area Weighted Relative Error	13.73%	20.84%	22.69%

- The source of error has little to do with the coefficients, but more to do with the modes. Low flow rate flow patterns are difficult to accurately grasp using the first four modes (95% energy proportion)
- Low flow rate → Flow dead zone → velocity ~0, local error ↑ & Pattern Difference with larger q_v (inaccurate mode)

4 Conclusion

- Proposed a hybrid reduced-order model combining **POD and PINN** for fast flow field reconstruction near the MCP impeller in LFRs.

- Achieved relative reconstruction errors **~15%** at the impeller, with test data errors not exceeding **35%~40%**, and Pearson correlation coefficients generally above **0.8**.



- Flow field reconstructed in **~8ms**, significantly faster than traditional CFD (plotting time not included).

- Using **global physics constraints + pretraining** can enhance the generalizability and fitting performance.

- While KNN interpolation is more accurate, NN interpolation is faster (More potential to develop).

- Low q_v causes dead zones (**new flow regimes**, **insufficient data**), leading to the inaccurate capture of modes.