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1 C++

1.1 C++ template

```
#include <bits/stdc++.h>
using namespace std;

//IMPRESINDIBLES PARA ICPC
#define form(i, s, e) for(int i = s; i < e; i++)
#define icin(x) \
    int x; \
    cin >> x;
#define llcin(x) \
    long long x; \
    cin >> x;
#define scin(x) \
    string x; \
    cin >> x;
#define endl '\n'
#define S second
#define F first
```

```

#define pb push_back
#define sz(x) x.size()
#define all(x) x.begin(),x.end()

typedef long long ll;
typedef vector<int> vi;
typedef vector<vi> vvi;
typedef pair<int,int> pii;

const ll INF = 1e9+7; //tambien es primo
const double PI = acos(-1);
//UTILES
#define DBG(x) cerr << #x << ' ' << (x) << endl
#define coutDouble cout << fixed << setprecision(17)
#define numtobin(n) bitset<32>(n).to_string()
#define bintoint(bin_str) stoi(bin_str, nullptr, 2) //
bin_str should be a STRING
#define LSONe(S) ((S) & ~(S))

typedef double db;
typedef vector<string> vs;
typedef vector<ll> vll;
typedef vector<vll> vvll;
typedef pair<int,bool> pib;
typedef pair<ll,ll> pll;
typedef vector<pii> vpii;
typedef vector<pib> vpib;
typedef vector<pll> vpll;

int main() {
    ios::sync_with_stdio(0);
    cin.tie(0);
    cout.tie(0);

    icin(nn0)
    while (nn0--) {
    }

    return 0;
}

```

1.2 Librerías sino da bits/stdc++

```

// En caso de que no sirva #include <bits/stdc++.h>
#include <algorithm>
#include <iostream>
#include <iterator>
#include <sstream>
#include <fstream>
#include <cassert>
#include <climits>
#include <cstdlib>

```

```

#include <cstring>
#include <string>
#include <cstdio>
#include <vector>
#include <cmath>
#include <queue>
#include <deque>
#include <stack>
#include <list>
#include <map>
#include <set>
#include <bitset>
#include <iomanip>
#include <unordered_map>

////
#include <tuple>
#include <random>
#include <chrono>

```

1.3 Tasks.json LINUX

```

{
  "tasks": [
    {
      "type": "cppbuild",
      "label": "C/C++: g++ compilar archivo activo",
      "command": "/usr/bin/g++",
      "args": [
        "-fdiagnostics-color=always",
        "-g",
        "${file}",
        "-o",
        "${fileDirname}/${fileBasenameNoExtension}"
      ],
      "options": {
        "cwd": "${fileDirname}"
      },
      "problemMatcher": [
        "$gcc"
      ],
      "group": "build",
      "detail": "Tarea generada por el depurador."
    },
    {
      "label": "Run test cases (linux)",
      "type": "shell",
      "command": "g++ \"${file}\" -O2 -std=c++17 -Wall -o\n\"${fileBasenameNoExtension}\" && for f in ${\nfileBasenameNoExtension}*.in; do echo ===== \"${f\n\n\" =====; \"./${fileBasenameNoExtension}\" < \"\n${f}\" > \"${f%.in}.tmp\"; cat \"${f%.in}.tmp\";\necho; done",
      "problemMatcher": [],
    }
  ]
}

```

```

    "group": {
      "kind": "test",
      "isDefault": true
    },
    "version": "2.0.0"
  },
  "version": "2.0.0"
}

```

1.4 Comando para comparar salidas

1.4.1 Linux

```

./programa < in.txt > myout.txt
diff -u out.txt myout.txt

```

1.4.2 Windows

```

algo2.exe < in.txt > myout.txt
fc myout.txt out.txt

```

1.5 C++

1.5.1 Ejemplo problematico if-else

A primera vista podrías pensar que el else se asocia al primer if (if (a > 0)), pero en realidad se asocia al segundo (if (b > 0)).

```

if (a > 0)
  if (b > 0)
    cout << "Ambos positivos";
else
  cout << "a no es positivo";

```

1.5.2 vector

```

// RESIZE
vector<int> v = {1, 2, 3};
v.resize(5, 9); // ahora: {1, 2, 3, 9, 9}
v.resize(2);    // ahora: {1, 2}

// ASSIGN
vector<int> v = {1, 2, 3};
v.assign(5, 9); // ahora: {9, 9, 9, 9, 9}

```

2 Grafos

Simplest Trick
to find
PreOrder
InOrder
PostOrder

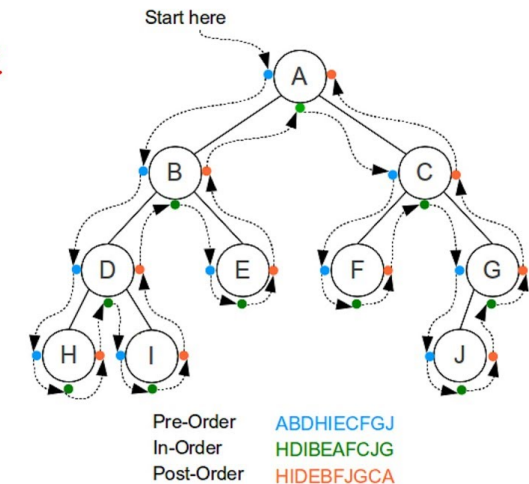


Figure 1: Pre-In-Post Orders DFS

2.1 DFS cpbook

```

enum { UNVISITED = -1, VISITED = -2 };
// basic flags

// these variables have to be global to be easily
// accessible by our recursion (other ways exist)
vector<vii> AL;
vi dfs_num;

void dfs(int u) {
  normal usage
  printf(" %d", u); // this
  vertex is visited
  dfs_num[u] = VISITED; // mark
  u as visited
  for (auto &[v, w] : AL[u]) // C++17
    style, w ignored
    if (dfs_num[v] == UNVISITED) // to
      avoid cycle
      dfs(v); //
      recursively visits v
}

int main() {

```

```

/*
// Undirected Graph in Figure 4.1
9
1 1 0
3 0 0 2 0 3 0
2 1 0 3 0
3 1 0 2 0 4 0
1 3 0
0
2 7 0 8 0
1 6 0
1 6 0
*/

freopen("dfs_cc_in.txt", "r", stdin);

int V; scanf("%d", &V);
AL.assign(V, vii());
for (int u = 0; u < V; ++u) {
    int k; scanf("%d", &k);
    while (k--) {
        int v, w; scanf("%d %d", &v, &w);
        AL[u].emplace_back(v, w);
    }
}

printf("Standard DFS Demo (the input graph must be
UNDIRECTED)\n");
dfs_num.assign(V, UNVISITED);
int numCC = 0;
for (int u = 0; u < V; ++u) // for
    each u in [0..V-1]
    if (dfs_num[u] == UNVISITED) // if
        that u is unvisited
        printf("CC %d:", ++numCC), dfs(u), printf("\n"); //
        3 lines here!
printf("There are %d connected components\n", numCC);

return 0;
}

```

2.2 DFS iterativo - Lucas

```

vector<bool> vis;
void dfs(int start, vector<vector<int>> & adj, int v){
    // v = Vertices
    stack<int> s;
    s.push(start);
    vis[start] = true;
    int cont = 1;
    while (!s.empty()){
        int prox = s.top();
        if (!adj[prox].empty()){
            if (vis[adj[prox].back()] == false){
                vis[adj[prox].back()] = true;

```

```

                s.push(adj[prox].back());
            }
            else{
                adj[prox].pop_back();
            }
        }
        else{
            s.pop();
        }
    }
}

```

2.3 BFS - camino mas corto - O(V+E)

```

// inside int main()---no recursion
vi dist(V, INF); dist[s] = 0; // initial distances
queue<int> q; q.push(s); // start from source
while (!q.empty()) { // queue: layer by layer!
    int u = q.front(); q.pop(); // C++17 style, w ignored
    for (auto &[v, w] : AL[u]) {
        if (dist[v] != INF) continue; // ALREADY VISITED, skip
        dist[v] = dist[u]+1; // now set dist[v] != INF
        q.push(v); // for the next iteratio
    }
}

```

2.4 BFS - bipartito check - Lucas - O(V+E)

```

// Realiza una BFS desde el nodo 'src' en un grafo
// dirigido o no dirigido
// representado como lista de adyacencia.
// Parametros:
//   n : numero de nodos (0 .. n-1)
//   adj : vector de vectores, donde adj[u] contiene
//         todos los v tales que u -> v
//   src : nodo de partida
// Devuelve:
//   true si es bipartito y false si no lo es
bool bfs(int n, vector<pair<vector<int>, char>> &adj, int
src)
{
    queue<int> q;
    q.push(src);
    char decision = 'a';
    bool bipartito = true;

    while (!q.empty())
    {
        int u = q.front();
        q.pop();
        if (adj[u].second == 'c')
        {

```

```

        adj[u].second = decision;
    }
    if (adj[u].second == 'a')
        decision = 'b';
    else
        decision = 'a';
    for (int v : adj[u].first)
    {
        if (adj[v].second == 'c')
        {
            q.push(v);
            adj[v].second = decision;
        }
        if (adj[u].second == adj[v].second)
        {
            bipartito = false;
            break;
        }
    }
}

return bipartito;
}

int main()
{
    ios::sync_with_stdio(false);
    cin.tie(nullptr);
    int n, m;
    // Leer numero de nodos y aristas
    cin >> n >> m;
    // Construir lista de adyacencia
    vector<pair<vector<int>, char>> adj(n);
    // a= 1er conjunto
    // b = 2do
    // c = sin conjunto
    for (int i = 0; i < m; ++i)
    {
        int u, v;
        cin >> u >> v;
        adj[u].first.push_back(v);
        adj[v].first.push_back(u);
    }
    // inicializacion en c para saber si no esta
    // explorado
    for (int i = 0; i < n; i++)
        adj[i].second = 'c';

    bool es_bipartito = true;
    // Iterar por todos los nodos para manejar grafos no
    // conexos
    for (int i = 0; i < n; ++i)
    {
        // Si el nodo 'i' no ha sido coloreado, iniciar
        // un BFS desde el
    }
}

```

```

    if (adj[i].second == 'c')
    {
        // Si cualquier componente no es bipartita,
        // el grafo entero no lo es
        if (!bfs(n, adj, i))
        {
            es_bipartito = false;
            break; // Podemos detenernos en cuanto
            // encontramos un fallo
        }
    }
}
cout << "res: " << es_bipartito << endl;
return 0;
}

```

2.5 BFS - cpbook - camino mas corto - Bipartito check O(V+E)

```

const int INF = 1e9; // INF = 1B, not 2^31-1 to avoid
// overflow
vi p; // addition:parent vector

void printPath(int u) { // extract info from vi p
    if (p[u] == -1) { printf("%d", u); return; }
    printPath(p[u]); // output format: s -> ... -> t
    printf(" %d", u);
}

int main() {
    /*
    // Graph in Figure 4.3, format: list of unweighted
    // edges
    // This example shows another form of reading graph
    // input
    13 16
    0 1 1 2 2 3 0 4 1 5 2 6 3 7 5 6
    4 8 8 9 5 10 6 11 7 12 9 10 10 11 11 12
    */
    freopen("bfs_in.txt", "r", stdin);

    int V, E; scanf("%d %d", &V, &E);
    vector<vii> AL(V, vii());
    for (int i = 0; i < E; ++i) {
        int a, b; scanf("%d %d", &a, &b);
        AL[a].emplace_back(b, 0);
        AL[b].emplace_back(a, 0);
    }

    // as an example, we start from this source, see Figure
    // 4.3
}

```

```

int s = 5;
// BFS routine inside int main() -- we do not use
// recursion
vi dist(V, INF); dist[s] = 0;           // INF =
// 1e9 here
queue<int> q; q.push(s);
p.assign(V, -1);                       // p is
// global

int layer = -1;                        // for
// output printing
bool isBipartite = true;               //
// additional feature

while (!q.empty()) {
    int u = q.front(); q.pop();
    if (dist[u] != layer) printf("\nLayer %d: ", dist[u])
    ;
    layer = dist[u];
    printf("visit %d, ", u);
    for (auto &[v, w] : AL[u]) {        // C++17
        // style, w ignored
        if (dist[v] == INF) {
            dist[v] = dist[u]+1;       // dist[
            // v] != INF now
            p[v] = u;                  //
            // parent of v is u
            q.push(v);                 // for
            // next iteration
        }
        else if ((dist[v]%2) == (dist[u]%2)) // same
            // parity
            isBipartite = false;
    }
}

printf("\nShortest path: ");
printPath(7), printf("\n");
printf("isBipartite? %d\n", isBipartite);

return 0;
}

```

2.6 DFS - detect cycle - $O(V+E)$

```

vector<vector<int>> adj(5);
int n;
vector<char> state(5);
/*
a = no visitado
b = visitando
c = visitado
*/

```

```

bool dfs_detect_cycle(int node)
{
    if(state[node] == 'b')
        return true;
    state[node] = 'b';
    for(auto i: adj[node])
    {
        if(dfs_detect_cycle(i))
        {
            return true;
        }
    }
    state[node] = 'c';
    return false;
}

int main()
{
    ios::sync_with_stdio(0); cin.tie(0); cout.tie(0);
    n = 5;
    adj[1].push_back(2);
    // Componente 2 (con ciclo)
    adj[3].push_back(4);
    adj[4].push_back(0);
    // adj[0].push_back(3); // CON ESTO SI HAY CICLO

    form(i,0,5) state[i] = 'a';
    int i;
    for( i=0; i < 5; i++)
    {
        if(state[i] == 'a')
            if(dfs_detect_cycle(i))
            {
                cout << "Hay ciclo" << endl;
                return 0;
            }
    }
    if(i == 5)
        cout << "NO hay ciclo" << endl;

    return 0;
}

```

2.7 Dijkstra - $O((E + V)\log V)$

- Para pesos ≥ 0 porque generan ciclos infinitos, pero si hay negativos pero no pueden generar ciclos entonces si se puede usar Dijkstra. - $O((E + V)\log V)$ implementación con Heap Binario (priorityQueue C++). - $O(E + V(\log V))$ implementación con Heap Fibonacci (Teórico) que soporta decreaseKey.

```

vector<long long> dist;
struct cmp {
    bool operator()(const pair<int, long long>& a, const
        pair<int, long long>& b) const {

```

```

        return a.second > b.second;
    }
};
priority_queue<pair<int, long long>, vector<pair<int,
    long long>>, cmp> q;
void dijkstra(int n, vector<vector<pair<int, long long>>>
    &adj, int src)
{
    dist.resize(n+1, -1);
    dist[src] = 0;
    q.push({src, 0});
    while (!q.empty())
    {
        auto u = q.top();
        q.pop();
        if (u.second > dist[u.first])
        {
            continue; // Ya encontramos un camino mas
                       // corto a 'u', ignoramos este.
        }
        for (auto v : adj[u.first])
        {
            if (dist[v.first] > dist[u.first] + v.second
                or dist[v.first] == -1)
            {
                dist[v.first] = dist[u.first] + v.second;
                q.push({v.first, dist[v.first]});
            }
        }
    }
    true;
}

int main()
{
    ios::sync_with_stdio(false);
    cin.tie(nullptr);
    int n, m;
    cin >> n >> m;
    int u, v;
    long long p;
    vector<vector<pair<int, long long>>> adj(n+1); //nodo
    destino, peso
    for (int i = 0; i < m; ++i)
    {
        cin >> u >> v >> p;
        adj[u].push_back({v, p});
    }
    dijkstra(n, adj, 1); // desde nodo origen a todos los
    demas
    for (int i = 1; i <= n; ++i)
    {
        cout << dist[i] << " ";
    }
}

```

```

        return 0;
    }
}

```

2.8 Topological sort dfs $O(V+E)$

- Esta version no puede detectar ciclos entonces primero EJECUTAR CYCLE CHECK. - Si existe una arista $u \rightarrow v$ entonces u aparece antes que v en el orden topológico. - Ya que es recursivo no soporta grafos muy grandes mayores o iguales 10^5 nodos. - El orden topológico solo existe en un DAG (Grafo Dirigido Acíclico). Si el grafo tiene un ciclo, no puedes ordenarlo.

```

// Para que exista un orden topologico
// el grafo tiene que ser DAG(grafo aciclico dirigido)
vi g[nax], ts;
bool seen[nax];
void dfs(int u) {
    seen[u] = true;
    for(int v: g[u])
        if (!seen[v])
            dfs(v);
    ts.pb(u);
}
void topo(int n) {
    for(i,n) if (!seen[i]) dfs(i);
    reverse(all(ts));
}

```

2.9 Topological sort bfs (Kahn's algorithm con queue $O(V+E)$ / con priority_queue $O((V+E)\log V)$)

- Si existe una arista $u \rightarrow v$ entonces u aparece antes que v en el orden topológico. - Gracias a que se puede usar una priority_queue en vez de una queue, obtenemos el orden lexicográficamente menor, mayor o cualquier otro orden dependiendo de la comparación que definamos en la priority_queue. - El orden topológico solo existe en un DAG (Grafo Dirigido Acíclico). Si el grafo tiene un ciclo, no puedes ordenarlo. - Al final del algoritmo, si el número de nodos en tu lista de orden topológico no es igual al número total de nodos en el grafo (N), entonces existe un ciclo. Los nodos que se quedaron con indegree $\neq 0$ son parte del ciclo o son alcanzables desde uno.

```

// enqueue vertices with zero incoming degree into a (
// priority) queue pq
priority_queue<int, vi, greater<int>> pq; // min
priority_queue
for (int u = 0; u < N; ++u)
    if (in_degree[u] == 0) // next to
        be processed

```



```

    pq.push(u); // smaller
    index first
while (!pq.empty()) { // Kahn's
    algorithm
    int u = pq.top(); pq.pop();
    printf(" %s", u); // process u here
    for (auto &v : AL[u]) {
        --in_degree[v]; // virtually
        remove u->v
        if (in_degree[v] > 0) continue; // not a
        candidate, skip
        pq.push(v); // enqueue v
        in pq
    }
}

```

2.10 Tarjan cpbook - Strongly Connected Components $O(V+E)$

```

enum { UNVISITED = -1 };
int dfsNumberCounter, numSCC;
vector<vii> AL, AL_T;
vi dfs_num, dfs_low, S, visited; //
    global variables
stack<int> St;
void tarjanSCC(int u) {
    dfs_low[u] = dfs_num[u] = dfsNumberCounter; //
    dfs_low[u] <= dfs_num[u]
    dfsNumberCounter++; //
    increase counter
    St.push(u); //
    remember the order
    visited[u] = 1;
    for (auto &[v, w] : AL[u]) {
        if (dfs_num[v] == UNVISITED)
            tarjanSCC(v);
        if (visited[v]) //
            condition for update
            dfs_low[u] = min(dfs_low[u], dfs_low[v]);
    }
    if (dfs_low[u] == dfs_num[u]) { // a
        root/start of an SCC
        ++numSCC; // when
        recursion unwinds
        while (1) {
            int v = St.top(); St.pop(); visited[v] = 0;
            if (u == v) break;
        }
    }
}

```

```

}
}
void Kosaraju(int u, int pass) { // pass = 1 (original),
    2 (transpose)
    dfs_num[u] = 1;
    vii &neighbor = (pass == 1) ? AL[u] : AL_T[u]; // by
    ref to avoid copying
    for (auto &[v, w] : neighbor) // C++17
        style, w ignored
        if (dfs_num[v] == UNVISITED)
            Kosaraju(v, pass);
    S.push_back(u); // as in finding topological order in
    Section 4.2.5
}
int main() {
    int N, M;
    while (scanf("%d %d", &N, &M), (N || M)) {
        AL.assign(N, vii());
        AL_T.assign(N, vii()); // the transposed graph
        while (M--) {
            int V, W, P; scanf("%d %d %d", &V, &W, &P); --V; --
            W; // to 0-based indexing
            AL[V].emplace_back(W, 1); // always
            AL_T[W].emplace_back(V, 1);
            if (P == 2) { // if this is two way, add the
                reverse direction
                AL[W].emplace_back(V, 1);
                AL_T[V].emplace_back(W, 1);
            }
        }
        // run Tarjan's SCC code here
        dfs_num.assign(N, UNVISITED); dfs_low.assign(N, 0);
        visited.assign(N, 0);
        while (!St.empty()) St.pop();
        dfsNumberCounter = numSCC = 0;
        for (int u = 0; u < N; ++u)
            if (dfs_num[u] == UNVISITED)
                tarjanSCC(u);
        // // run Kosaraju's SCC code here
        // S.clear(); // first pass: record the post-order of
        // original graph
        // dfs_num.assign(N, UNVISITED);
        // for (int u = 0; u < N; ++u)
        //     if (dfs_num[u] == UNVISITED)
        //         Kosaraju(u, 1);
        // int numSCC = 0; // second pass: explore SCCs using
        // first pass order
        // dfs_num.assign(N, UNVISITED);
        // for (int i = N-1; i >= 0; --i)
        //     if (dfs_num[S[i]] == UNVISITED)
        //         numSCC++, Kosaraju(S[i], 2); // on
        // transposed graph
    }
}

```



```

    // if SCC is only 1, print 1, otherwise, print 0
    printf("%d\n", numSCC );
}
return 0;
}

```

3 Data Structures

3.1 unordered_map<clave,valor>

Almacena pares clave valor.

```

// hacer siempre RESERVE
unordered_map<int,int> a;
a.reserve(n*1.33); // IMPORTANTEEEEEEE
n = 1e6 aprox 42.6 MB

```

n = 3e6 aprox 128 MB

n = 5e6 aprox 213 MB (aún puede entrar, pero ojo con pila, I/O buffers, otros contenedores).

3.1.1 Ejemplo basico Contar frecuencias

```

int n;
cin >> n;
vector<int> arr(n);
for (int &x : arr)
    cin >> x;

unordered_map<int,int> freq; // <clave, valor>
freq.reserve(n*1.33); // evita rehash

for (int x : arr)
    freq[x]++;

for (auto &p : freq)
    cout << p.first << " aparece " << p.second << " veces\n";

```

3.1.2 Buscar existencia de una llave

```

unordered_map<string,int> id;
id.reserve(1e5);

id["uva"] = 10;
id["manzana"] = 20;

// Con count
if (id.count("uva")) cout << "uva existe\n";

```

3.1.3 Transformar índices dispersos a continuos

```

vector<int> vals = {1000, 5000, 1000, 42};
unordered_map<int,int> comp;
comp.reserve(vals.size()*1.33);

int id = 0;
for (int v : vals)
    if (!comp.count(v))
        comp[v] = id++;

/*
    Antes -> Ahora
    1000 = 1
    5000 = 2
    42 = 3
*/

for (int v : vals)
    cout << v << " -> " << comp[v] << "\n";

```

3.1.4 Hashing pair

```

struct pair_hash {
    size_t operator()(const pair<int,int>& p) const {
        return ((long long)p.first << 32) ^ p.second;
    }
};

int main()
{
    unordered_map<pair<int,int>, int, pair_hash> edge_cost;
    edge_cost.reserve(1e6);
    // Muy usado para representar grafos dispersos.
    edge_cost[{1,2}] = 5;
    edge_cost[{2,3}] = 10;

    cout << edge_cost[{1,2}] << "\n"; // 5
}

```

3.2 unordered_set<clave>

// hacer siempre RESERVE

No existe acceso aleatorio con [] (índices), pero se puede iterar con for auto.

```

int n = 3e5;
vi a = {1,2,3,42,42,42};
unordered_set<int> s; // <T>
s.reserve(n * 1.3); // evita rehash

// insert(T)
for (int x : a)
    s.insert(x);

```

```
//VERIFICAR EXISTENCIA
if (s.find(42) != s.end())
    cout << "42 existe" << endl;

//Iterar para ver claves existentes
for(auto x : s)
    cout << x << " ";

return 0;
```

3.3 unordered_multimap<clave, valores>

// hacer siempre RESERVE
Una misma clave puede tener varios valores asociados

3.3.1 Ejemplo básico

```
multimap<int, string> mm;

// insertar pares (clave, valor)
mm.insert({1, "uva"});
mm.insert({2, "manzana"});
mm.insert({2, "pera"});
mm.insert({3, "melon"});

// Iterar (se imprime ordenado por clave)
for (auto &p : mm)
    cout << p.first << " -> " << p.second << "\n";

/*
1 -> uva
2 -> manzana
2 -> pera
3 -> melon
*/
```

3.3.2 Buscar por clave

```
multimap<int, string> mm;
// insertar pares (clave, valor)
mm.insert({1, "uva"});
mm.insert({2, "manzana"});
mm.insert({2, "pera"});
mm.insert({3, "melon"});

// Buscar la primera aparicion de clave 2
auto it = mm.find(2);
if (it != mm.end())
    cout << "Encontrado: " << it->second << "\n";

// Contar cuantos con clave=2
cout << "Claves con 2: " << mm.count(2) << "\n";

// Obtener todos los con clave=2
```

```
auto [ini, fin] = mm.equal_range(2);
for (auto it = ini; it != fin; ++it)
    cout << it->second << " ";

/*
SALIDA
Encontrado: manzana
Claves con 2: 2
manzana pera
*/
```

3.3.3 Delete

```
mm.erase(2); // borra *todas* las entradas con clave=2

// Si quieres borrar solo uno:
auto it = mm.find(2);
if (it != mm.end())
    mm.erase(it);
```

3.4 Union find - cpbook

Cada que unimos dos Sets del mismo RANK(rank=r=altura - size=s=vertices) nuestro rank aumenta en +1. Entonces para formar un RANK r se necesitan por lo menos 2^r vertices.

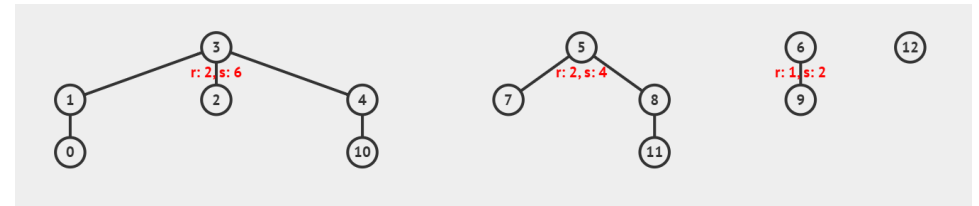


Figure 2: Inicializacion de Union-Find. Cada nodo es su propio padre.

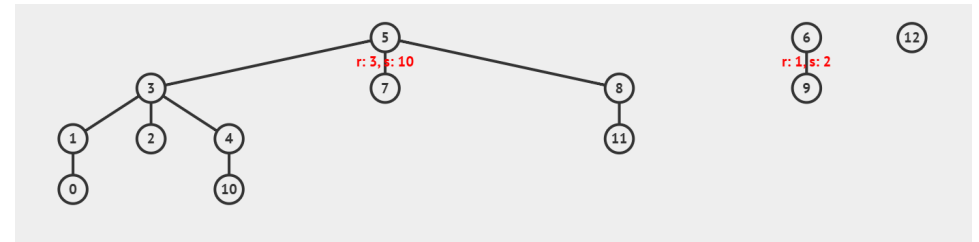


Figure 3: Union-Find despues de unir 3 y 5

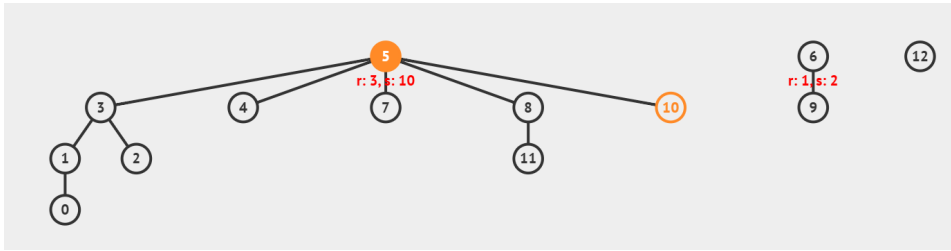


Figure 4: Union-Find después findSet(10) con Path Compression

```
// Union-Find Disjoint Sets Library written in OOP manner
, using both path compression and union by rank
heuristics

#include <bits/stdc++.h>
using namespace std;

typedef vector<int> vi;

class UnionFind {                                // OOP
    style
private:
    vi p, rank, setSize;                          // vi p
    int numSets;                                  // is the key part
public:
    UnionFind(int N) {
        p.assign(N, 0); for (int i = 0; i < N; ++i) p[i] = i;
        rank.assign(N, 0);                        // optional speedup
        setSize.assign(N, 1);                      // optional feature
        numSets = N;                              // optional feature
    }

    int findSet(int i) {
        return (p[i] == i) ? i : (p[i] = findSet(p[i])); // Path Compression
    }

    bool isSameSet(int i, int j) { return findSet(i) == findSet(j); }

    int numDisjointSets() { return numSets; }        // optional
    int sizeOfSet(int i) { return setSize[findSet(i)]; } // optional

    void unionSet(int i, int j) {
        if (isSameSet(i, j)) return;                // i and j are in same set
    }
}
```

```
int x = findSet(i), y = findSet(j);                // find both rep items
if (rank[x] > rank[y]) swap(x, y);                 // keep x 'shorter' than y
p[x] = y;                                           // set x under y
if (rank[x] == rank[y]) ++rank[y];                 // optional speedup
setSize[y] += setSize[x];                          // combine set sizes at y
--numSets;                                         // a union reduces numSets
}

int main() {
    printf("Assume that there are 5 disjoint sets initially\n");
    UnionFind UF(17); // create 5 disjoint sets
    UF.unionSet(1,2);
    UF.unionSet(3,4);
    UF.unionSet(1,3);
    UF.unionSet(5,6);
    UF.unionSet(7,8);
    UF.unionSet(5,7);
    UF.unionSet(1,5);

    UF.unionSet(9,10);
    UF.unionSet(11,12);
    UF.unionSet(9,11);
    UF.unionSet(13,14);
    UF.unionSet(15,16);
    UF.unionSet(13,16);
    UF.unionSet(9,13);

    UF.unionSet(9,1);
    UF.findSet(10);
    UF.findSet(11);

    int a = 1 + 2;
    printf("isSameSet(0, 3) = %d\n", UF.isSameSet(0, 3));
    // will return 0 (false)
    printf("isSameSet(4, 3) = %d\n", UF.isSameSet(4, 3));
    // will return 1 (true)
    for (int i = 0; i < 5; i++) // findSet will return 1 for {0, 1} and 3 for {2, 3, 4}
        printf("findSet(%d) = %d, sizeOfSet(%d) = %d\n", i, UF.findSet(i), i, UF.sizeOfSet(i));
    UF.unionSet(0, 3);
    printf("%d\n", UF.numDisjointSets()); // 1
    for (int i = 0; i < 5; i++) // findSet will return 3 for {0, 1, 2, 3, 4}
        printf("findSet(%d) = %d, sizeOfSet(%d) = %d\n", i, UF.findSet(i), i, UF.sizeOfSet(i));
    return 0;
}
```



```

class SegmentTree {                                // OOP
    style
private:
    int n;                                         // n = (
        int)A.size()
    vi A, st, lazy;                               // the
        arrays

    int l(int p) { return p<<1; }                 // go to
        left child
    int r(int p) { return (p<<1)+1; }             // go to
        right child

    int conquer(int a, int b) {
        if (a == -1) return b;                    //
        corner case
        if (b == -1) return a;
        return min(a, b);                        // RMQ
    }

    void build(int p, int L, int R) {              // O(n)
        if (L == R)
            st[p] = A[L];                         // base
        case
        else {
            int m = (L+R)/2;
            build(l(p), L, m);
            build(r(p), m+1, R);
            st[p] = conquer(st[l(p)], st[r(p)]);
        }

    void propagate(int p, int L, int R) {
        if (lazy[p] != -1) {                      // has a
            lazy flag
            st[p] = lazy[p];                      // [L..R
            ] has same value
            if (L != R)                           // not a
                leaf
                lazy[l(p)] = lazy[r(p)] = lazy[p]; //
                propagate downwards
            else                                   // L ==
                R, a single index
                A[L] = lazy[p];                   // time
                to update this
                lazy[p] = -1;                      // erase
                lazy flag
        }

    int RMQ(int p, int L, int R, int i, int j) {   // O(log
        n)
        propagate(p, L, R);                      // lazy
        propagation
        if (i > j)
            return -1;                            // infeasible
        if ((L >= i) && (R <= j))

```

```

        return st[p];                            // found the segment
    int m = (L+R)/2;
    int left = RMQ(l(p), L, m, i, min(m, j));
    int right = RMQ(r(p), m+1, R, max(i, m+1), j);
    return conquer(left, right);
}

void update(int p, int L, int R, int i, int j, int val)
{ // O(log n)
    propagate(p, L, R);                          // lazy
    propagation
    if (i > j) return;
    if ((L >= i) && (R <= j)) {                   // found
        the segment
        lazy[p] = val;                           //
        update this
        propagate(p, L, R);                      // lazy
        propagation
    }
    else {
        int m = (L+R)/2;
        update(l(p), L, m, i, min(m, j), val);
        update(r(p), m+1, R, max(i, m+1), j, val);
        int lsubtree = (lazy[l(p)] != -1) ? lazy[l(p)] : st
            [l(p)];
        int rsubtree = (lazy[r(p)] != -1) ? lazy[r(p)] : st
            [r(p)];
        st[p] = conquer(lsubtree, rsubtree);
    }
}

public:
    SegmentTree(int sz) : n(sz), A(n), st(4*n), lazy(4*n,
        -1) {}

    SegmentTree(const vi &initialA) : SegmentTree((int)
        initialA.size()) {
        A = initialA;
        build(1, 0, n-1);
        true;
    }

    void update(int i, int j, int val) { update(1, 0, n-1,
        i, j, val); }

    int RMQ(int i, int j) { return RMQ(1, 0, n-1, i, j); }
};

int main() {
    vi A = {18, 17, 13, 19, 15, 11, 20, 99};    // make
        n a power of 2
    SegmentTree st(A);
    st.update(4, 7, 2);

    st.RMQ(1, 2);
    printf("          idx      0, 1, 2, 3, 4, 5, 6, 7\n");
    ;

```

```

printf("                A is {18,17,13,19,15,11,20,oo}\n"
);
printf("RMQ(1, 3) = %d\n", st.RMQ(1, 3));           // 13
printf("RMQ(4, 7) = %d\n", st.RMQ(4, 7));           // 11
printf("RMQ(3, 4) = %d\n", st.RMQ(3, 4));           // 15
st.update(5, 5, 77);                                //
    update A[5] to 77
printf("                idx    0, 1, 2, 3, 4, 5, 6, 7\n")
;
printf("Now, modify A into {18,17,13,19,15,77,20,oo}\n"
);
printf("RMQ(1, 3) = %d\n", st.RMQ(1, 3));           //
    remains 13
printf("RMQ(4, 7) = %d\n", st.RMQ(4, 7));           // now
    15
printf("RMQ(3, 4) = %d\n", st.RMQ(3, 4));           //
    remains 15

st.update(0, 3, 30);                                //
    update A[0..3] to 30
printf("                idx    0, 1, 2, 3, 4, 5, 6, 7\n")
;
printf("Now, modify A into {30,30,30,30,15,77,20,oo}\n"
);
printf("RMQ(1, 3) = %d\n", st.RMQ(1, 3));           // now
    30
printf("RMQ(4, 7) = %d\n", st.RMQ(4, 7));           //
    remains 15
printf("RMQ(3, 4) = %d\n", st.RMQ(3, 4));           //
    remains 15

st.update(3, 3, 7);                                  //
    update A[3] to 7
printf("                idx    0, 1, 2, 3, 4, 5, 6, 7\n")
;
printf("Now, modify A into {30,30,30, 7,15,77,20,oo}\n"
);
printf("RMQ(1, 3) = %d\n", st.RMQ(1, 3));           // now 7
printf("RMQ(4, 7) = %d\n", st.RMQ(4, 7));           //
    remains 15
printf("RMQ(3, 4) = %d\n", st.RMQ(3, 4));           // now 7

return 0;
}

```

3.7 Order Statistics Tree

3.7.1 Quick Selct - cpbook

Ranking(v) = posicion del elemento v si el arreglo estuviese ordenado.

```

int Partition(int A[], int l, int r) {
    int p = A[l];                                // p is
        the pivot
    int m = l;                                    // S1
        and S2 are empty
    for (int k = l+1; k <= r; ++k) {              //
        explore unknown region
        if (A[k] < p) {                            // case
            2
            ++m;
            swap(A[k], A[m]);
        } // notice that we do nothing in case 1: a[k] >= p
    }
    swap(A[l], A[m]);                            // swap
        pivot with a[m]
    return m;                                    //
        return pivot index
}

int RandPartition(int A[], int l, int r) {
    int p = l + rand() % (r-l+1);                //
        select a random pivot
    swap(A[l], A[p]);                            // swap
        A[p] with A[l]
    return Partition(A, l, r);
}

int QuickSelect(int A[], int l, int r, int k) {    //
    expected O(n)
    if (l == r) return A[l];
    int q = RandPartition(A, l, r);              // O(n)
    if (q+1 == k)
        return A[q];
    else if (q+1 > k)
        return QuickSelect(A, l, q-1, k);
    else
        return QuickSelect(A, q+1, r, k);
}

int main() {
    int A[] = { 2, 8, 7, 1, 5, 4, 6, 3 };
    nth_element(A,A+4, A+8);
    printf("%d\n", A[4]);
    //output: 5
    for(auto i:A)
        cout << i << ", ";
    //output: [3,2,1,4,5,7,6,8]
    return 0;
}

```

3.8 Ordered Statistics Tree - bits/extc++.h


```

#include <bits/stdc++.h>
using namespace std;

#include <bits/extc++.h> // pbds
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
            tree_order_statistics_node_update> ost;

int main() {
    int n = 9;
    int A[] = { 2, 4, 7, 10, 15, 23, 50, 65, 71}; // as in
                                                Chapter 2
    ost tree;
    for (int i = 0; i < n; ++i) // O(n
        log n)
        tree.insert(A[i]);
    // O(log n) select
    cout << *tree.find_by_order(0) << "\n"; // 1-
        smallest = 2
    cout << *tree.find_by_order(n-1) << "\n"; // 9-
        smallest/largest = 71
    cout << *tree.find_by_order(4) << "\n"; // 5-
        smallest = 15
    // O(log n) rank
    cout << tree.order_of_key(2) << "\n"; // index
        0 (rank 1)
    cout << tree.order_of_key(71) << "\n"; // index
        8 (rank 9)
    cout << tree.order_of_key(15) << "\n"; // index
        4 (rank 5)

    return 0;
}

```

3.9 Priority Queue

```

struct Node {
    int id;
};
// si la funcion devuelve true, a tiene menor prioridad
// que b
struct cmp {
    bool operator()(const Node& a, const Node& b) const {
        return a.id < b.id;
    }
};

priority_queue<Node, vector<Node>, cmp> pq; // max-heap
// por id

int main() {
    priority_queue<int, vector<int>> pq_min; // por defecto
// es max-heap
    priority_queue<int, vector<int>, greater<int>> pq_min;
// min-heap
}

```

```

// Create O(n)
vector<int> d =
    {12, 3, 4, 3, 3, 5, 34, 343, 5325, 235, 23452, 3532};
priority_queue<int> a (d.begin(), d.end());
// Create O(n * log n)
vector<int> d =
    {12, 3, 4, 3, 3, 5, 34, 343, 5325, 235, 23452, 3532};
priority_queue<int> a;
for(auto i: d)
    a.push(i);
return 0;
}

```

3.10 Trie-Recursivo-Lucas

```

struct Node
{
    Node* sig_cero = NULL; Node* sig_uno = NULL;
    int cont_cero = 0, cont_uno = 0;
};

class Trie
{
private:
    Node* r;
    void Add(Node*& node, int i)
    {
        if(i > 63) return;
        if(node == NULL) node = new Node();
        if(last_num[i] == '0')
        {
            node->cont_cero++;
            Add(node->sig_cero, i+1);
        }
        else
        {
            node->cont_uno++;
            Add(node->sig_uno, i+1);
        }
    }

    void Delete(Node*& node, int i)
    {
        if(i > 63) return;
        if(node == NULL) node = new Node();
        if(last_num[i] == '0')
        {
            node->cont_cero--;
            Delete(node->sig_cero, i+1);
        }
        else
        {
            node->cont_uno--;
            Delete(node->sig_uno, i+1);
        }
    }
}

```

```

}
void Max_xor(Node*& node, int i)
{
    if(i > 63)
        return;
    if(node == NULL)
        return;
    if(last_num[i] == '0')
    {
        if(node->cont_uno > 0)
        {
            ans.pb('1');
            Max_xor(node->sig_uno, i+1);
        }
        else
        {
            ans.pb('0');
            Max_xor(node->sig_cero, i+1);
        }
    }
    else
    {
        if(node->cont_cero > 0)
        {
            ans.pb('0');
            Max_xor(node->sig_cero, i+1);
        }
        else
        {
            ans.pb('1');
            Max_xor(node->sig_uno, i+1);
        }
    }
}
public:
string ans;
string last_num;
Trie()
{
    r = new Node();
}
void add(int x)
{
    this->last_num = numtobin(x); //128 bits
    Add(r, 0);
}
void deletee(int x)
{
    this->last_num = numtobin(x); //128 bits
    Delete(r, 0);
}
void max_xor(int x)
{
    this->last_num = numtobin(x); //128 bits

```

```

        ans.clear();
        Max_xor(r, 0);
    }
};

```

3.11 Trie-Iterativo-Mati

```

struct nodo {
    nodo* hijos[2];
    int cont;

    nodo() {
        hijos[0] = hijos[1] = nullptr;
        cont = 0;
    }
};

class ArbolBin {
private:
    nodo* raiz;

    void borrarNodo(nodo* n) {
        if(!n) return;

        borrarNodo(n->hijos[0]);
        borrarNodo(n->hijos[1]);
        delete n;
    }

public:
    ArbolBin() {
        raiz = new nodo();
    }

    ~ArbolBin() {
        borrarNodo(raiz);
    }

    void insertar(int x) {
        nodo* nodoActual = raiz;
        int bitActual;
        for(int i = MB - 1; i >= 0; i--) {
            bitActual = (x >> i) & 1;

            if(nodoActual->hijos[bitActual] == nullptr) {
                nodoActual->hijos[bitActual] = new nodo();
            }

            nodoActual = nodoActual->hijos[bitActual];
            nodoActual->cont++;
        }
    }
}

```

```

void borrar(int x) {
    nodo* nodoActual = raiz;
    int bitActual;
    for(int i = MB - 1; i >= 0; i--) {
        bitActual = (x>>i) & 1;

        nodoActual = nodoActual->hijos[bitActual];
        nodoActual->cont--;
    }
}

int consulta(int x) {
    nodo* nodoActual = raiz;
    int bitDeseado, res = 0;
    for(int i = MB - 1; i >= 0; i--) {
        bitDeseado = ((x>>i) & 1)^1;

        if(nodoActual->hijos[bitDeseado] != nullptr and
           nodoActual->hijos[bitDeseado]->cont > 0) {
            res = res|(1<<i);
        }
        else {
            bitDeseado = bitDeseado^1;
        }

        nodoActual = nodoActual->hijos[bitDeseado];
    }
    return res;
}
};

```

3.12 Suffix Tree

```

const int N=1000000, // maximum possible number of
nodes in suffix tree
INF=10000000000; // infinity constant
string a; // input string for which the suffix tree
is being built
int t[N][26], // array of transitions (state, letter)
l[N], // left...
r[N], // ...and right boundaries of the substring
of a which correspond to incoming edge
p[N], // parent of the node
s[N], // suffix link
tv, // the node of the current suffix (if we're
mid-edge, the lower node of the edge)
tp, // position in the string which corresponds
to the position on the edge (between l[tp] and r[
tp], inclusive)
ts, // the number of nodes
la; // the current character in the string

void ukkadd(int c) { // add character s to the tree

```

```

    suff;; // we'll return here after each
    transition to the suffix (and will add character
    again)
    if (r[tp]<tp) { // check whether we're still within
    the boundaries of the current edge
        // if we're not, find the next edge. If it doesn'
        t exist, create a leaf and add it to the tree
        if (t[tp][c]==-1) {t[tp][c]=ts;l[ts]=la;p[ts++]=
        tv;tv=s[tp];tp=r[tp]+1;goto suff;}
        tv=t[tp][c];tp=l[tp];
    } // otherwise just proceed to the next edge
    if (tp==-1 || c==a[tp]-'a')
        tp++; // if the letter on the edge equal c, go
        down that edge
    else {
        // otherwise split the edge in two with middle in
        node ts
        l[ts]=l[tp];r[ts]=tp-1;p[ts]=p[tp];t[ts][a[tp]-'a
        ']=tv;
        // add leaf ts+1. It corresponds to transition
        through c.
        t[ts][c]=ts+1;l[ts+1]=la;p[ts+1]=ts;
        // update info for the current node - remember to
        mark ts as parent of tv
        l[tp]=tp;p[tp]=ts;t[p[tp]][a[l[tp]]-'a']=ts;ts
        +=2;
        // prepare for descent
        // tp will mark where are we in the current
        suffix
        tv=s[p[ts-2]];tp=l[ts-2];
        // while the current suffix is not over, descend
        while (tp<=r[ts-2]) {tv=t[tp][a[tp]-'a'];tp+=r[tp
        ]-l[tp]+1;}
        // if we're in a node, add a suffix link to it,
        otherwise add the link to ts
        // (we'll create ts on next iteration).
        if (tp==r[ts-2]+1) s[ts-2]=tv; else s[ts-2]=ts;
        // add tp to the new edge and return to add
        letter to suffix
        tp=r[tp]-(tp-r[ts-2])+2;goto suff;
    }
}

void build() {
    ts=2;
    tv=0;
    tp=0;
    fill(r,r+N,(int)a.size()-1);
    // initialize data for the root of the tree
    s[0]=1;
    l[0]=-1;
    r[0]=-1;
    l[1]=-1;
    r[1]=-1;
    memset (t, -1, sizeof t);

```

```

fill(t[1],t[1]+26,0);
// add the text to the tree, letter by letter
for (la=0; la<(int)a.size(); ++la)
    ukkadd (a[la]-'a');
}

```

3.13 Custom Hash

```

struct custom_hash {
    static ll splitmix64(ll x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(ll x) const {
        static const ll FIXED_RANDOM = chrono::
            steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
unordered_map<ll,int, custom_hash> mapa;

```

4 Math

4.1 Prime Numbers 1-2000

```

2 3 5 7 11 13 17 19 23 29
31 37 41 43 47 53 59 61 67 71
73 79 83 89 97 101 103 107 109 113
127 131 137 139 149 151 157 163 167 173
179 181 191 193 197 199 211 223 227 229
233 239 241 251 257 263 269 271 277 281
283 293 307 311 313 317 331 337 347 349
353 359 367 373 379 383 389 397 401 409
419 421 431 433 439 443 449 457 461 463
467 479 487 491 499 503 509 521 523 541
547 557 563 569 571 577 587 593 599 601
607 613 617 619 631 641 643 647 653 659
661 673 677 683 691 701 709 719 727 733
739 743 751 757 761 769 773 787 797 809
811 821 823 827 829 839 853 857 859 863
877 881 883 887 907 911 919 929 937 941
947 953 967 971 977 983 991 997 1009 1013

```

```

1019 1021 1031 1033 1039 1049 1051 1061 1063 1069
1087 1091 1093 1097 1103 1109 1117 1123 1129 1151
1153 1163 1167 1181 1187 1193 1201 1213 1217 1223
1229 1231 1237 1249 1259 1277 1279 1283 1289 1291
1297 1301 1303 1307 1319 1321 1327 1361 1367 1373
1381 1399 1409 1423 1427 1429 1433 1439 1447 1451
1453 1459 1471 1481 1483 1487 1489 1493 1499 1511
1523 1531 1543 1549 1553 1559 1567 1571 1579 1583
1597 1601 1607 1609 1613 1619 1621 1627 1637 1657
1663 1667 1669 1693 1699 1709 1721 1723 1733
1741 1747 1753 1759 1777 1783 1787 1789 1801 1811
1823 1831 1847 1861 1867 1871 1873 1877 1879 1889
1901 1907 1913 1931 1933 1949 1951 1973 1979 1987

```

```

970'997 971'483 921'281'269 999'279'733
1'000'000'009 1'000'000'021 1'000'000'409
1'005'012'527

```

4.2 Serie de Fibonacci 0-20

```

Def: F(0)=0 , F(1)=1 , F(n)=F(n-1)+F(n-2)
F(0) = 0
F(1) = 1
F(2) = 1
F(3) = 2
F(4) = 3
F(5) = 5
F(6) = 8
F(7) = 13
F(8) = 21
F(9) = 34
F(10) = 55
F(11) = 89
F(12) = 144
F(13) = 233
F(14) = 377
F(15) = 610
F(16) = 987
F(17) = 1597
F(18) = 2584
F(19) = 4181

```

F (20) = 6765

4.3 Factorial 0-20

Def: $n!=n(n-1)!$
0! = 1
1! = 1
2! = 2
3! = 6
4! = 24
5! = 120
6! = 720
7! = 5040
8! = 40320
9! = 362880
10! = 3628800
11! = 39916800
12! = 479001600
13! = 6227020800
14! = 87178291200
15! = 1307674368000
16! = 20922789888000
17! = 355687428096000
18! = 6402373705728000
19! = 121645100408832000
20! = 2432902008176640000

4.4 Numeros Triangulares 1-20

Def: $T(n)=n(n+1)/2$

T (1) = 1
T (2) = 3
T (3) = 6
T (4) = 10
T (5) = 15
T (6) = 21
T (7) = 28
T (8) = 36

T (9) = 45
T (10) = 55
T (11) = 66
T (12) = 78
T (13) = 91
T (14) = 105
T (15) = 120
T (16) = 136
T (17) = 153
T (18) = 171
T (19) = 190
T (20) = 210

4.5 Numeros Cuadrados 1-20

Def: $Q(n)=n^2$

Q (1) = 1
Q (2) = 4
Q (3) = 9
Q (4) = 16
Q (5) = 25
Q (6) = 36
Q (7) = 49
Q (8) = 64
Q (9) = 81
Q (10) = 100
Q (11) = 121
Q (12) = 144
Q (13) = 169
Q (14) = 196
Q (15) = 225
Q (16) = 256
Q (17) = 289
Q (18) = 324
Q (19) = 361
Q (20) = 400

4.6 Simple Sieve of Eratosthenes $O(n \cdot \log(\log(n)))$ - con $n=1e7$ 1.25 MB

```
#define tam 1e7
vector< bool > criba(tam , true);

void criba_function()
{
    criba[0]=false;
    criba[1]=false;
    /*( i*i < tam) equivalente a (i <= sqrt(tam))*/
    for(int i = 2; i*i <= tam ; i++)
    {
        if(!criba[i]) continue;
        for(int j = 2; i*j <= tam ; j++)
            criba[i * j] = false;
    }
}
```

4.7 Smallest Prime Factor AND Sieve of Eratosthenes $O(n)$ - con $n=1e7$ 45 MB

```
/* O(n)
// pr contains prime numbers
// lp[i] == i if i is prime
// else lp[i] is minimum prime factor of i
const int nax = 1e7;
int lp[nax+1]; //because lp is an array nax have to be
less than 1e7 or change to a vector(nax+1,0)
vector<int> pr; // It can be sped up if change for an
array

void sieve(){
    form(i,2,nax){
        if (lp[i] == 0) {
            lp[i] = i; pr.pb(i);
        }
        for (int j=0, mult= i*pr[j]; j<sz(pr) && pr[j]<=lp[i]
            && mult<nax; ++j, mult= i*pr[j])
            lp[mult] = pr[j];
        }
    }
```

4.8 Smallest Prime Factor Piton++

```
/* O(n)
// pr contains prime numbers
// lp[i] == i if i is prime
// else lp[i] is minimum prime factor of i
const int nax = 1e7;
int lp[nax+1]; //because lp is an array nax have to be
less than 1e7 or change to a vector(nax+1,0)
vector<int> pr; // It can be sped up if change for an
array

void sieve(){
    form(i,2,nax){
        if (lp[i] == 0) {
            lp[i] = i; pr.pb(i);
        }
        for (int j=0, mult= i*pr[j]; j<sz(pr) && pr[j]<=lp[i]
            && mult<nax; ++j, mult= i*pr[j])
            lp[mult] = pr[j];
        }
    }
```

```
vector<int> pr; /* It can be sped up if change for an
array */

void sieve(){
    form(i,2,nax){
        if (lp[i] == 0) {
            lp[i] = i; pr.pb(i);
        }
        for (int j=0, mult= i*pr[j]; j<sz(pr) && pr[j]<=lp[i]
            && mult<nax; ++j, mult= i*pr[j])
            lp[mult] = pr[j];
        }
    }
```

4.9 Combinatorics

4.9.1 Next permutation

```
int main() {
    vector<int> perm = {1, 2, 3};
    sort(perm.begin(), perm.end());
    do {
        for (int x : perm)
            cout << x << ' ';
        cout << '\n';
    } while (next_permutation(perm.begin(), perm.end()));

    int arr[] = {2,3,4,5,1,0};
    sort(arr,arr+6); /* arr+CANTIDAD DE ELEMENTOS */
    do
    {
        for (int x : perm)
            cout << x << ' ';
        cout << '\n';
    } while (next_permutation(arr,arr+6));
    return 0;
}
```

5 Dynamic Programming

6 Otros

6.1 Binary Search

```
vi acu(int(1e5));
int c(int l, int r)
{
    if(l > 0) return acu[r] - acu[l-1];
    return acu[r];
}
```

```
int t ;
int bs(int l, int r)
{
    int i = l, j = r;
    int n , mitad;
    while (i != j)
    {
        n = j - i;
        mitad = i + n/2;
        if(c(l,mitad) == t) return mitad - 1 + 1;
        if(c(l,mitad) > t) j = mitad;
        else i = mitad + 1;
    }
    return c(l,i) <= t ? i - 1 + 1 : (c(l,i - 1) ? i - 1 :
        - 1) ;
}

int main() {
    ios::sync_with_stdio(0);
    cin.tie(0);
    cout.tie(0);

    icin(n)
    cin >> t;
    vi nums(n);
    form(i,0,n) cin >> nums[i];

    acu[0] = nums[0];
    form(i,1,n) acu[i] = acu[i-1] + nums[i];
    int maxi = 0;
    form(i,0,n)
        maxi = max(maxi , bs(i, n-1));
    cout << maxi << endl;
    return 0;
}
```

Decimal - Binary - Octal - Hex – ASCII
Conversion Chart

| Decimal | Binary | Octal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII |
|---------|----------|-------|-----|-------|---------|----------|-------|-----|-------|---------|----------|-------|-----|-------|---------|----------|-------|-----|-------|
| 0 | 00000000 | 000 | 00 | NUL | 32 | 00100000 | 040 | 20 | SP | 64 | 01000000 | 100 | 40 | @ | 96 | 01100000 | 140 | 60 | ` |
| 1 | 00000001 | 001 | 01 | SOH | 33 | 00100001 | 041 | 21 | ! | 65 | 01000001 | 101 | 41 | A | 97 | 01100001 | 141 | 61 | a |
| 2 | 00000010 | 002 | 02 | STX | 34 | 00100010 | 042 | 22 | " | 66 | 01000010 | 102 | 42 | B | 98 | 01100010 | 142 | 62 | b |
| 3 | 00000011 | 003 | 03 | ETX | 35 | 00100011 | 043 | 23 | # | 67 | 01000011 | 103 | 43 | C | 99 | 01100011 | 143 | 63 | c |
| 4 | 00000100 | 004 | 04 | EOT | 36 | 00100100 | 044 | 24 | \$ | 68 | 01000100 | 104 | 44 | D | 100 | 01100100 | 144 | 64 | d |
| 5 | 00000101 | 005 | 05 | ENQ | 37 | 00100101 | 045 | 25 | % | 69 | 01000101 | 105 | 45 | E | 101 | 01100101 | 145 | 65 | e |
| 6 | 00000110 | 006 | 06 | ACK | 38 | 00100110 | 046 | 26 | & | 70 | 01000110 | 106 | 46 | F | 102 | 01100110 | 146 | 66 | f |
| 7 | 00000111 | 007 | 07 | BEL | 39 | 00100111 | 047 | 27 | ' | 71 | 01000111 | 107 | 47 | G | 103 | 01100111 | 147 | 67 | g |
| 8 | 00001000 | 010 | 08 | BS | 40 | 00101000 | 050 | 28 | (| 72 | 01001000 | 110 | 48 | H | 104 | 01101000 | 150 | 68 | h |
| 9 | 00001001 | 011 | 09 | HT | 41 | 00101001 | 051 | 29 |) | 73 | 01001001 | 111 | 49 | I | 105 | 01101001 | 151 | 69 | i |
| 10 | 00001010 | 012 | 0A | LF | 42 | 00101010 | 052 | 2A | * | 74 | 01001010 | 112 | 4A | J | 106 | 01101010 | 152 | 6A | j |
| 11 | 00001011 | 013 | 0B | VT | 43 | 00101011 | 053 | 2B | + | 75 | 01001011 | 113 | 4B | K | 107 | 01101011 | 153 | 6B | k |
| 12 | 00001100 | 014 | 0C | FF | 44 | 00101100 | 054 | 2C | , | 76 | 01001100 | 114 | 4C | L | 108 | 01101100 | 154 | 6C | l |
| 13 | 00001101 | 015 | 0D | CR | 45 | 00101101 | 055 | 2D | - | 77 | 01001101 | 115 | 4D | M | 109 | 01101101 | 155 | 6D | m |
| 14 | 00001110 | 016 | 0E | SO | 46 | 00101110 | 056 | 2E | . | 78 | 01001110 | 116 | 4E | N | 110 | 01101110 | 156 | 6E | n |
| 15 | 00001111 | 017 | 0F | SI | 47 | 00101111 | 057 | 2F | / | 79 | 01001111 | 117 | 4F | O | 111 | 01101111 | 157 | 6F | o |
| 16 | 00010000 | 020 | 10 | DLE | 48 | 00110000 | 060 | 30 | 0 | 80 | 01010000 | 120 | 50 | P | 112 | 01110000 | 160 | 70 | p |
| 17 | 00010001 | 021 | 11 | DC1 | 49 | 00110001 | 061 | 31 | 1 | 81 | 01010001 | 121 | 51 | Q | 113 | 01110001 | 161 | 71 | q |
| 18 | 00010010 | 022 | 12 | DC2 | 50 | 00110010 | 062 | 32 | 2 | 82 | 01010010 | 122 | 52 | R | 114 | 01110010 | 162 | 72 | r |
| 19 | 00010011 | 023 | 13 | DC3 | 51 | 00110011 | 063 | 33 | 3 | 83 | 01010011 | 123 | 53 | S | 115 | 01110011 | 163 | 73 | s |
| 20 | 00010100 | 024 | 14 | DC4 | 52 | 00110100 | 064 | 34 | 4 | 84 | 01010100 | 124 | 54 | T | 116 | 01110100 | 164 | 74 | t |
| 21 | 00010101 | 025 | 15 | NAK | 53 | 00110101 | 065 | 35 | 5 | 85 | 01010101 | 125 | 55 | U | 117 | 01110101 | 165 | 75 | u |
| 22 | 00010110 | 026 | 16 | SYN | 54 | 00110110 | 066 | 36 | 6 | 86 | 01010110 | 126 | 56 | V | 118 | 01110110 | 166 | 76 | v |
| 23 | 00010111 | 027 | 17 | ETB | 55 | 00110111 | 067 | 37 | 7 | 87 | 01010111 | 127 | 57 | W | 119 | 01110111 | 167 | 77 | w |
| 24 | 00011000 | 030 | 18 | CAN | 56 | 00111000 | 070 | 38 | 8 | 88 | 01011000 | 130 | 58 | X | 120 | 01111000 | 170 | 78 | x |
| 25 | 00011001 | 031 | 19 | EM | 57 | 00111001 | 071 | 39 | 9 | 89 | 01011001 | 131 | 59 | Y | 121 | 01111001 | 171 | 79 | y |
| 26 | 00011010 | 032 | 1A | SUB | 58 | 00111010 | 072 | 3A | : | 90 | 01011010 | 132 | 5A | Z | 122 | 01111010 | 172 | 7A | z |
| 27 | 00011011 | 033 | 1B | ESC | 59 | 00111011 | 073 | 3B | ; | 91 | 01011011 | 133 | 5B | [| 123 | 01111011 | 173 | 7B | { |
| 28 | 00011100 | 034 | 1C | FS | 60 | 00111100 | 074 | 3C | < | 92 | 01011100 | 134 | 5C | \ | 124 | 01111100 | 174 | 7C | |
| 29 | 00011101 | 035 | 1D | GS | 61 | 00111101 | 075 | 3D | = | 93 | 01011101 | 135 | 5D |] | 125 | 01111101 | 175 | 7D | } |
| 30 | 00011110 | 036 | 1E | RS | 62 | 00111110 | 076 | 3E | > | 94 | 01011110 | 136 | 5E | ^ | 126 | 01111110 | 176 | 7E | ~ |
| 31 | 00011111 | 037 | 1F | US | 63 | 00111111 | 077 | 3F | ? | 95 | 01011111 | 137 | 5F | _ | 127 | 01111111 | 177 | 7F | DEL |

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Figure 7: Ascii code

| Tipo | Tam. Bits | Dígitos de precisión | Rango | |
|------------------------|-----------|----------------------|----------------------------|----------------------------|
| | | | Min | Max |
| Bool | 8 | 0 | | 1 |
| Char | 8 | 2 | -128 | 127 |
| Signed char | 8 | 2 | -128 | 127 |
| unsigned char | 8 | 2 | 0 | 255 |
| short int | 16 | 4 | -32,768 | 32,767 |
| unsigned short int | 16 | 4 | 0 | 65,535 |
| Int | 32 | 9 | -2,147,483,648 | 2,147,483,647 |
| unsigned int | 32 | 9 | 0 | 4,294,967,295 |
| long int | 32 | 9 | -2,147,483,648 | 2,147,483,647 |
| unsigned long int | 32 | 9 | 0 | 4,294,967,295 |
| long long int | 64 | 18 | -9,223,372,036,854,775,808 | 9,223,372,036,854,775,807 |
| unsigned long long int | 64 | 18 | 0 | 18,446,744,073,709,551,615 |
| Float | 32 | 6 | 1.17549e-38 | 3.40282e+38 |
| Double | 64 | 15 | 2.22507e-308 | 1.79769e+308 |

Figure 8: Data types limits

Simplest **Trick**
to find
PreOrder
InOrder
PostOrder

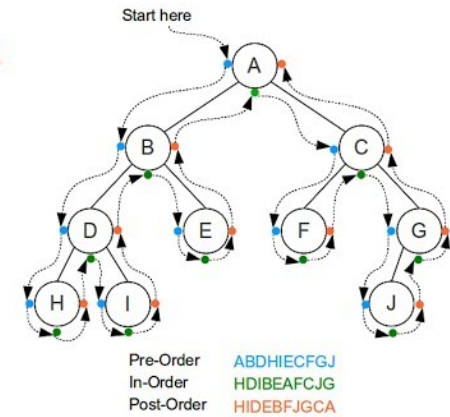


Figure 10: Pre-order, In-order y Post-order

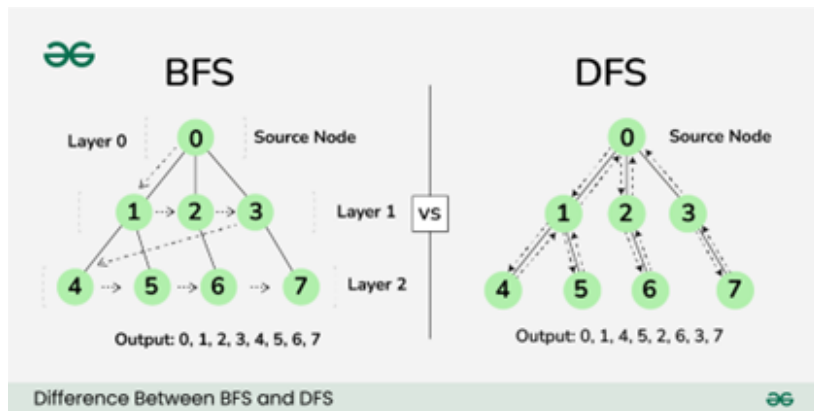


Figure 9: DFS y BFS

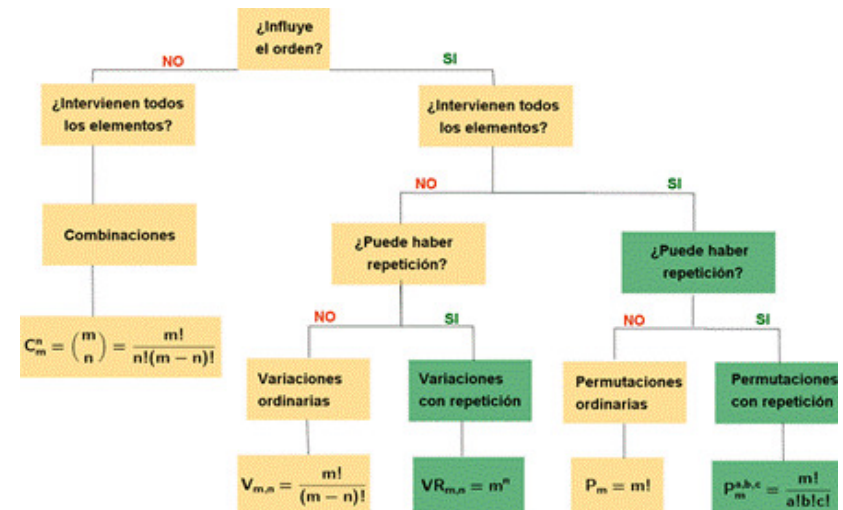


Figure 11: Combinatorics

The following are true involving modular arithmetic:

1. $(a + b) \% m = ((a \% m) + (b \% m)) \% m$
 Example: $(15 + 29) \% 8$
 $= ((15 \% 8) + (29 \% 8)) \% 8 = (7 + 5) \% 8 = 4$
2. $(a - b) \% m = ((a \% m) - (b \% m)) \% m$
 Example: $(37 - 15) \% 6$
 $= ((37 \% 6) - (15 \% 6)) \% 6 = (1 - 3) \% 6 = -2 \text{ or } 4$
3. $(a \times b) \% m = ((a \% m) \times (b \% m)) \% m$
 Example: $(23 \times 12) \% 5$
 $= ((23 \% 5) \times (12 \% 5)) \% 5 = (3 \times 2) \% 5 = 1$

Figure 12: Modulo properties

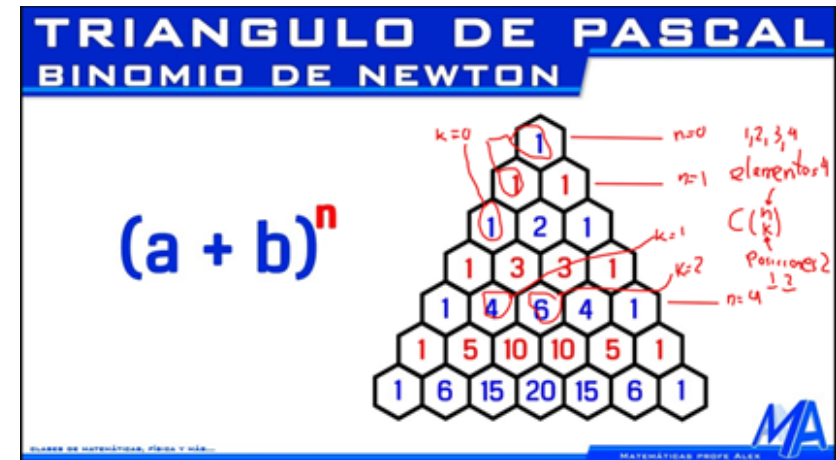


Figure 13: Pascal's triangle