Work Integrated Learning Programmes Division
M. Tech. in Data Science and Engineering

Assignment 2

DSECL ZC416 - Mathematical Foundations for Data Science

Instructions

1. Use any programming language (other than Excel) of your choice. Attach only the relevant data in your submission and no need to submit the entire code unless asked for specifically.

2. By random entries, I mean a system generated random number whose format is nn.dddd unless specified otherwise in the question. No marks would be awarded for deterministic entries.

3. Use 5 places of decimal in your computations and report them in your answers.

4. Answers should be in line with the deliverables asked in the question. Do not write the answers for question x in question y (where x $\neq$ y).

5. This is not a group activity. Each student should do the problems and submit individually.

6. Assignments have to be handwritten and uploaded as a single pdf file with name BITSID.pdf

7. Submissions beyond 25th of February, 2024 19.00 hrs would not be graded.

8. Assignments sent via email / other electronic forms would not be accepted.

9. Copying is strictly prohibited. Adoption of unfair means would lead to disciplinary action.

Answer all the questions

Q1) Power Method and Associated problems (2 marks)

i) Generate using code a random integer matrix C of size 4×3 and a matrix A_1 defined as $A_1 = C^T C$ and workout its characteristic equation. Using any software package, determine the eigenvalues and eigenvectors.

   Deliverables: The matrices C and A_1, the computation of the characteristic equation, the eigenvalues and eigenvectors as obtained from the package. (0.5 marks)

ii) Write a code in your chosen programming language to implement the Power method and use it to derive the largest eigenvalue λ_1 and corresponding eigenvector x_1 of A_1. Find $\hat{x}_1 = \dfrac{x_1}{\|x_1\|_2}$. Compare the values obtained in i) with these values.

Deliverables: The handwritten code that implements the Power method, the first 10 iterates of eigenvalue generated by the algorithm and the final λ_1 and $\hat{x}_1$ and a comment on the comparison. (0.5 marks)

iii) Write a code to construct matrix $A_2 = (A_1 - \hat{x}_1\hat{x}_1^T A_1)$. Use the Power method code written in ii) and use it to derive the largest eigenvalue λ_2 and corresponding eigenvector x_2 of A_2. Find $\hat{x}_2 = \dfrac{x_2}{\|x_2\|_2}$. Compare the values obtained in i) with these values.

Deliverables: The first 10 iterates of λ_2 and $\hat{x}_2$ and a comment on the comparison. (0.5 marks)

iv) Write a code to construct matrix $A_3 = (A_1 - \hat{x}_1\hat{x}_1^T A_1 - \hat{x}_2\hat{x}_2^T A_1)$. Use the Power method code written in ii) and use it to derive the largest eigenvalue λ_3 and corresponding eigenvector x_3 of A_3. Find $\hat{x}_3 = \dfrac{x_3}{\|x_3\|_2}$. Compare the values obtained in i) with these values.

Deliverables: The first 10 iterates of λ_3 and $\hat{x}_3$ and a comment on the comparison. (0.5 marks)

Q2) Gradient Descent using Armijo's Rule (3 marks)

Consider the function $f(x, y) = 10x^4 - 20x^2y + x^2 + 10y^2 - 2x + 1$. Write a code in your chosen programming language to find a stationary point of this function using gradient descent where the step size α is chosen using Armijos Rule described in Section 4.4.3.3 in T2.

Deliverables: The handwritten code used to find the minimum by Armijo's Rule enabled gradient descent. The first 10 iterates of the gradient descent algorithm where the individual iterates of $x, y, \alpha, f(x, y)$ should be d=written (the output of the code) separately to show the progress of gradient descent scheme. Also mention the optimal points x^*, y^* and $f(x^*, y^*)$ obtained by the converged algorithm and all the parameters chosen in the Armijo's Rule. (3 marks)

Q3) Gradient Descent. Please use the file "assign2_q3.py" provided along with this assignment. (3 marks)

The python source file implements the gradient descent algorithm. You are expected to fill in the missing lines of code to get the algorithm to work as expected. The script takes no additional parameters.
There are four places where you need to add your lines of code. Your lines of code should go in-between the lines with the comment:

"##### To be Updated #####"
and
"########################"

NOTE: You should NOT modify any other lines of the script.

Deliverables: **A)** A screenshot of the 3D plot obtained when you execute the python script after adding your lines of code. **B)** A screenshot of the console output obtained obtained when you execute the python script after adding your lines of code. **C)** The equations of the Loss function and gradient of loss function on which the gradient descent was performed. **D)** A snapshot of the lines of code that was added by you in the designated places. Do not write the entire code. (3 marks)

For your benefit, we have given below the sample output plot and console output for a fictitious student id "023xx12345".

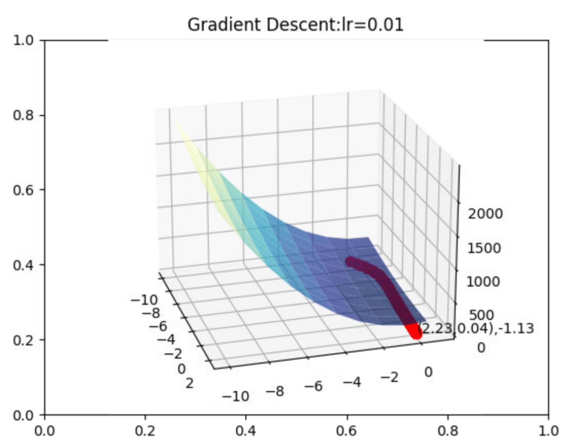

Figure 1: sample output plot of $student\_id$ = "$023xx12345$"

```
root@b7657211e2a9:/home/main/dropbox/gradDesc# python3 assign_2_sol1.
py
GD Start Point: [[-7.2685]
 [-0.8265]] [[170.16106]]
Num steps: 184
Converged Point: [[2.22622306]
 [0.04009413]] [[-1.12859173]]
```

Figure 2: sample console output of $student\_id =$ "023xx12345"

Q4) Gradient Descent. Please refer to the file "assign2_q3.py" provided along with this assignment. (2 marks)

Repeat the exercise in (Q3) for the loss function given below:

$L = 3 * x^2 + 2 * y^2 + 20 * \cos(x) * \cos(y)$

Modify the file "assign2_q3.py" only in-between the lines marked with comment as described in Q3. Do not modify any other line of the script.

Deliverables: **A)** A screenshot of the 3D plot obtained when you execute the python script after adding your lines of code. **B)** A screenshot of the console output obtained obtained when you execute the python script after adding your lines of code. **C)** The equations of the Loss function and gradient of loss function on which the gradient descent was performed. **D)** A snapshot of the lines of code that was added by you in the designated places. **E)** State whether the algorithm converged to the global minima, a local minima or if it failed to converge in your case. (2 marks)

```
Given below are the recommended versions of python
and python packages for Q3 and Q4
Python 3.8.10 (default, Nov 22 2023, 10:22:35)
[GCC 9.4.0] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> import numpy
>>> numpy.__version__
'1.24.4'
>>> import matplotlib
>>> matplotlib.__version__
'3.7.4'
```