a) $T(z) = 1 + z^{-1} + z^{-2} + z^{-3} = z^3 + z^2 + z + 1$

$T(z) = e^{j\omega}$

$H(\omega) = \dfrac{1}{e^0} + e^{j\omega} + e^{j\omega 2} + e^{3j\omega}$

$= e^{j1.5\omega}\left(e^{j1.5\omega} + e^{j \cdot 0.5\omega} + e^{-j0.5\omega} + e^{-1.5j\omega}\right)$

$\angle H(\omega) = 1.5\omega$

$|H(\omega)| = \left|e^{j1.5\omega}\right|\left|e^{j\omega \cdot 1.5} + e^{j \cdot 0.5\omega} + e^{-j0.5\omega} + e^{-1.5j\omega}\right|$

$\underbrace{}_{1}$

$\left(e^{j \cdot 1.5\omega} + e^{-1.5\omega j}\right) + \left(e^{j0.5\omega} + e^{-j0.5\omega}\right)$

$= 2\cos(1.5\omega) + 2\cos(0.5\omega)$

$\Rightarrow |H(\omega)| = 2 \cdot |\cos(1.5\omega) + \cos(0.5\omega)|$

b) $z^4 + z^3 + z^2 + z + 1$

$= e^{4j\omega} + e^{3j\omega} + e^{2j\omega} + e^{j\omega} + e^0 = e^{2j\omega}\left(e^{2j\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-2j\omega}\right)$

$\Rightarrow \angle H(\omega) = 2\omega$

$\Rightarrow |H(\omega)| = \left|e^{2j\omega}\right| \underbrace{\left|e^{2j\omega} + e^{-2j\omega} + e^{j\omega} + e^{-j\omega} + 1\right|}$

$\qquad\qquad\quad \underbrace{1} \qquad \left|2\cos(2j\omega) + 2\cos(j\omega) + 1\right|$

c) $T(z) = z - 1 \longrightarrow$ normalizada

$T(w)$ es $T(z)$ con, $z = e^{jw}$

$|T(w)| = |e^{jw} - 1| = \underbrace{|e^{0.5jw}|}_{=1} \cdot |e^{0.5jw} - e^{-0.5jw}|$

$= |\cos(0.5w) + j \cdot \text{sen}(0.5w) - [\cos(0.5w) - j \cdot \text{sen}(0.5w)]|$

$= |2j \, \text{sen}(0.5w)|$

$\Rightarrow |T(w)| = 2 \cdot |j \, \text{sen}(0.5w)|$

$\Rightarrow \sphericalangle T(z) = 0.5w$

d) $T(z) = z^2 - 1$

$H(\omega) = e^{2j\omega} - e^0 = e^{j\omega}\left(e^{j\omega} + e^{-j\omega}\right)$

$\Rightarrow \angle H(\omega) = \omega \qquad \Rightarrow |H(\omega)| = \underbrace{|e^{j\omega}|}_{1}\ \underbrace{|e^{j\omega} + e^{-j\omega}|}_{|2\cos(j\omega)|}$