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## 1、文章信息

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题目：Distributed Nash Equilibrium Seeking by a Consensus Based Approach

## 2、背景、目的及结论

### 背景

In this paper, Nash equilibrium seeking among a network of players is considered. Different from many existing works on Nash equilibrium seeking in noncooperative games, the players considered in this paper **cannot directly observe** the actions of the players who are not their neighbors. Instead, the players are supposed to be capable of communicating with each other via an undirected and connected communication graph.

### 目的

By a synthesis of a **leader-following consensus protocol** and **the gradient play**, a distributed Nash equilibrium seeking strategy is proposed for the noncooperative games. Analytical analysis on the convergence of the players' actions to the Nash equilibrium is conducted via Lyapunov stability analysis.

### 结论

For games with **nonquadratic payoffs**, where multiple isolated Nash equilibria may coexist in the game, a **local convergence** result is derived under certain conditions. Then, **a stronger condition is provided to derive a nonlocal convergence** result for the nonquadratic games. For **quadratic games**, it is shown that the proposed seeking strategy enables the players' actions to converge to the **Nash equilibrium globally** under the given conditions.

## 3、其他文献的方法与局限性

| 文献  | 方法   | 局限性 |
|---|--|-----|
| [2]Finding multiple Nash equilibria in pool-based markets: A stochastic EPEC approach | The authors formulated pure strategy Nash equilibrium seeking as a mixed-integer linear programming problem for pool-based electricity market. |     |

| 文献   | 方法   | 局限性 |
|--|--|-----|
| [3]On the characterization of local Nash equilibria in continuous games  | Gradient play was leveraged for finding differential Nash equilibrium in continuous games.   |     |
| [4]Dynamic fictitious play, dynamic gradient play, and distributed convergence to Nash equilibria                  | Dynamic fictitious play and gradient paly were exploited for a continuous-time form of repeated matrix games.  |     |
| [5]Multi-agent differential graphical games: Online adaptive learning solution for synchronization with optimality | Policy evaluation and policy improvement were utilized for the computation of the Nash equilibrium in differential graphical games.  |     |
| [6]Flexible Nash seeking using stochastic difference inclusion   | The discrete-time stochastic algorithm developed, it allows the players to take actions in both simultaneous and asynchronous fashions.  |     |
| [7]Designing Games for Distributed Optimization  | A distributed process was proposed to obtain the equilibrium.  |     |
| [10]Designing Games for Distributed Optimization   | Utilizing the saddle point dynamics, convergence to the Nash equilibrium of a two-network zero-sum game.   |     |
| [11]Distributed convergence to Nash equilibria in two-network zero-sum games                                       | Switching communications were considered for the two-network zero-sum game.  |     |
| [12]-[13]  | Nash equilibrium seeking in generalized convex games. Solving the generalized convex game by a discrete-time distributed algorithm.  |     |
| [14]On the solution of affine generalized Nash equilibrium problems with shared constraints by Lemke's method      | Nash equilibrium seeking in generalized convex games. Lemke's method was adapted for the computation of generalized Nash equilibrium in convex games with quadratic payoffs subject to linear constraints. |     |

| 文献  | 方法   | 局限性   |
|---|--|---|
| [1]Nash<br>equilibrium<br>seeking in non-<br>cooperative<br>games   | integrator-type extremum seeking   |   |
| [16]Stochastic<br>Nash Equilibrium<br>Seeking for<br>Games with<br>General<br>Nonlinear<br>Payoffs                          | discrete-time extremum seeking   |   |
| [17]Shahshahani<br>gradient-like<br>extremum<br>seeking   | Shahshahani gradient-like extremum seeking   |   |
| [18]Lie bracket<br>approximation of<br>extremum<br>seeking systems  | Lie bracket approximation-based extremum seeking   | A common<br>characteristic of<br>these methods is<br>that no explicit<br>model information is<br>required for the<br>implementation of<br>the methods.<br>Requiring the<br>players'<br>observations over<br>their opponents'<br>actions to search for<br>the Nash<br>equilibrium. |
| [21]Existence<br>and Uniqueness<br>of Equilibrium<br>Points for<br>Concave N-<br>Person Games                               | The players' payoff functions are determined by the players' own<br>actions, together with the other players' actions.   |   |
| [22]Nash<br>equilibrium<br>seeking by a<br>gossip-based<br>algorithm  | Solving games without using full information from all the players<br>was generalized, where the players' payoff functions depend on<br>the players' actions in a more general manner. The Nash<br>equilibrium was characterized by a variational inequality and a<br>gossip-based algorithm was proposed. The players were<br>equipped with a waking clock and they updated their actions<br>asynchronously to reach the Nash equilibrium. |   |
| [23]-[30], [42],<br>[43]  | Utilizing a consensus protocol to broadcast local information.   | [21], [22] Full<br>communication is<br>impractical in many<br>engineering<br>systems.   |
| [42]Distributed<br>Control of<br>Networked<br>Dynamical<br>Systems: Static<br>Feedback,<br>Integral Action<br>and Consensus | A class of consensus controllers were proposed for networked<br>dynamical systems.   |   |

| 文献  | 方法  | 局限性 |
|---|---|-----|
| [43]Consensus + Innovations Approach for Distributed Multiagent Coordination in a Microgrid       | A consensus-based approach was studied for distributed coordination of the generation, load, and storage devices in a microgrid.  |     |
| [26]Robust consensus tracking of a class of second-order multi-agent dynamic systems              | Leader-following consensus concerns with the synchronization of the agents' states to a common value, which is equal to the reference signal provided by the leader.  |     |
| [31]Distributed learning in large-scale multi-agent games: A modified fictitious play approach    | Distributed learning for games on the communication graph was investigated.   |     |
| [32]Empirical Centroid Fictitious Play: An Approach for Distributed Learning in Multi-Agent Games | For large-scale multiagent games by an adaptation of the fictitious play.   |     |
| [33]A gossip algorithm for aggregative games on graphs  | The authors considered Nash equilibrium seeking for aggregative games on the graph, where each player's payoff function depends on their own actions and an aggregate of all the players' actions. A discrete-time gossip algorithm was proposed to solve it. |     |
| [34]Game Design and Analysis for Price-Based Demand Response: An Aggregate Game Approach          | For the energy consumption game in smart grids, the continuous-time method was based on a dynamic average consensus protocol and the primal-dual dynamics.  |     |

## 4、文章好在哪里

优点:

1. Nash equilibrium seeking for noncooperative games, where the players have no direct access to the actions of the players who are not their neighbors, is investigated in this paper. Based on a leader-following consensus protocol and the gradient play, a Nash equilibrium seeking strategy is designed. In the proposed algorithm, the players only need to communicate with their neighbors on their estimates of the players' actions. Avoiding full communication among the players broadens the applicability of game theory to engineering systems where only local communication is attainable.

2. The convergence of the players' actions to the Nash equilibrium by utilizing the proposed seeking strategy is analytically explored. Based on the Lyapunov stability analysis, it is shown that the proposed method enables the players' actions to converge to the Nash equilibrium under certain conditions.

**不足:**

1. The players in this paper are supposed to be equipped with an undirected and connected communication graph. Can I extent it to directed graph? In other words, can I conclude a same conclusion in directed graph?
2. The designed strategy in this paper is perfect in quadratic games, but not in nonquadratic games. Can I relax the conditions of the strategy or design a good strategy for nonquadratic games?

## **5、自我想法**

1. 文中没有假设收益函数的具体形式，只需要与邻居通信，去估计其他玩家的行动，就可以达到纳什均衡了。这说明在满足本文给出的假设的前提下，函数的形式可以是任意的。
- 2.