Distributed Nash Equilibrium Seeking For Second-Order Systems With Boundary Unknown Dynamics

author

Abstract—This paper investigates the distributed Nash equilibrium seeking problem for second-order systems with unknown dynamics. A chattering free continuous function method is proposed to achieve distributed Nash equilibrium seeking for second-order players by using a time-varying boundary layer technique. The proposed method is able to prevent chattering in practical engineering applications. Moreover, by Lyapunov stability analysis, which shows that the proposed method can drive the actions of all players to a small neighborhood of a Nash equilibrium point by adjusting the control gains properly. Finally, a simulation example is provided to verify the result.

Index Terms—distributed Nash equilibrium seeking; second-order systems; unknown dynamics; boundary layer technique.

I. INTRODUCTION

OVER the past two decades, game theory has spread across diverse research fields, including biology, economics, and computer science. As game theory evolves, the quest for Nash equilibrium in noncooperative games gains significance in theory and practice, as evidenced by studies [1], [2], [3], [4], [5], [6]. Substantial progress has been made in distributed control and optimization of networked systems based on Nash equilibrium seeking and its applications [7], [8], [9], [10], [11]. For example, building upon the framework introduced in [12], numerous consensus-based distributed Nash equilibrium seeking strategies have been devised, such as distributed Nash equilibrium seeking in multiagent game under switching communication topologies digraph [13] and fully distributed Nash equilibrium seeking [14]. Nevertheless, the majority of existing findings don't account for the influence of system disturbances, which is unrealistic considering that many practical engineering systems frequently encounter disturbances.

System dynamics are often influenced by external disturbances, leading to significant interest in disturbance attenuation algorithms in engineering applications [15], [16], [17]. Common disturbance estimation and attenuation methods include disturbance observer based control, active disturbance rejection control, disturbance accommodation control and composite hierarchical antidisturbance control just a few [15]. For instance, disturbance accommodation control has been widely used in trajectory-driven adaptive control of autonomous unmanned aerial vehicles [18], floating offshore wind turbines [19] and integration of a fuel cell into the power system [20]. Furthermore, disturbance attenuation algorithms have been extensively studied in the context of multi-agent systems. For instance, .

Inspired by the significance of disturbance attenuation algorithms in practical engineering applications, distributed Nash equilibrium seeking strategies were proposed based on reduced-order disturbance observers and signum function [21], extended state observer [22], unknown dynamics estimator [23]. This paper aims to address the distributed Nash equilibrium seeking problem for second-order systems with unknown boundary disturbances by a chattering free continuous function method. Compared to most of existing literature, this paper's contributions are summarized as follows:

1) This paper proposes a chattering free continuous function method by using a time-varying boundary layer technique that achieve disturbance rejection distributed Nash equilibrium seeking for second-order players. The proposed method is able to prevent chattering in practical engineering applications, which is in contrast to the signum function method in [21].Moreover, the method in [24] requires distributed estimation of all players' actions and some specific objective functions determined by the players, while the proposed method in this paper does not. Therefore, compared to the methods in [21], [22], [23], [24], the proposed method in this paper requires less computation.
2) Due to high frequency switching of discontinuous terms, the controller may induce chattering in practical implementations which is detrimental to actuators. Compared [21], this paper proposes a continuous function method, chattering is avoided.
3) By Lyapunov stability analysis, which shows that the proposed method can drive the actions of all players to a small neighborhood of a Nash equilibrium point by adjusting the control gains properly.

The rest of this paper is organized as follows. Section II gives notations and preliminaries. Problem statement is introduced in Section III and main results are given in Section IV where we utilize boundary layer technique to design a continuous function method for distributed Nash equilibrium seeking for second-order players. Simulation studies are presented in Section V and conclusions are

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drawn in Section VI.

II. NOTATIONS AND PRELIMINARIES

Notations: The real number set is denote as $\mathbb{R}$. $\|z\|$ denotes the ℓ_2-norm of z. $[z_i]_{vec}$ where $i \in \{1, 2, ..., N\}$ is defined as a column vector whose dimension is $N \times 1$ and the ith element is z_i. diag$\{k_i\}$ for $i \in \{1, 2, ..., N\}$ is a diagonal matrix whose dimension is $N \times N$ and the ith diagonal element is k_i. diag$\{a_{ij}\}$ where $i, j \in \{1, 2, ..., N\}$ gives a diagonal matrix whose dimension is $N^2 \times N^2$ and diagonal elements are $a_{11}, a_{12}, ..., a_{1N}, a_{21}, ..., a_{NN}$, successively. $\mathcal{A} = [a_{ij}]$ is a matrix whose (i, j)th entry is a_{ij}. Given that matrix Q is symmetric and real, $\lambda_{min}(Q)(\lambda_{max}(Q))$ stands for the smallest(largest) eigenvalue of Q. $max_{i\in\{1,2,...,N\}}\{l_i\}$ denotes the largest value of l_i for $i \in \{1, 2, ..., N\}$. $\mathbf{I}_{N\times N}$ is an identity matrix with its dimension being $N \times N$ and $\mathbf{1}(\mathbf{0})$ is a column vector with its entries being 1(0). Moreover, $\otimes$ is the Kronecker product.

Algebraic Graph Theory: A graph $\mathcal{G}$ is given by $\mathcal{G} = (\mathcal{V}, \mathcal{E}_g)$, in which $\mathcal{V} = \{1, 2, ..., N\}$, $\mathcal{E}_g \subseteq \mathcal{V} \times \mathcal{V}$ respectively are the node set and edge set. The edge $(i, j) \in \mathcal{E}_g$ indicates are the node j can receive information from node i, but not necessarily vice versa. The in-neighbor set of node i is given as $\mathcal{N}_i^{in} = \{j|(j, i) \in \mathcal{E}_g\}$. A directed path is a sequence of edges of the form $(i_1, i_2), (i_2, i_3), ...$ A directed graph is strongly connected if for every pair of two distinct nodes, there is a path. Let $\mathcal{A} = [a_{ij}]$ be the adjacency matrix in which $a_{ij} > 0$ if $(i, j) \in \mathcal{E}_g$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \text{diag}\{d_i\}$ and $d_i = \sum_{j=1}^N a_{ij}$.

III. PROBLEM STATEMENT

In the concerned game, N players with labels from 1 to N are engaged and each player i has a local objective function $f_i(\mathbf{x}) : \mathbb{R}^N \to \mathbb{R}$, in which $\mathbf{x} = [x_1, x_2, ..., x_N]^T$ and $x_i \in \mathbb{R}$ is the action of player i[1]. Moreover, for second-order players, player i's action is governed by

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i + d_i(t) \end{cases} \tag{1}$$

where $v_i(t)$ is the velocity-like state of player i, u_i is the control input and $d_i(t)$ is the disturbance for $i \in \mathcal{V}$. Each player i aims to minimize its own objective function $f_i(\mathbf{x})$ by adjusting its action $x_i(t)$, that is

$$\begin{aligned} &\min_{x_i} && f_i(\mathbf{x}) \\ &\text{s.t.} && (1) \quad \text{for second-order players} \end{aligned} \tag{2}$$

Definition 1: An action profile $\mathbf{x}^* = (x_i^*, \mathbf{x}_{-i}^*)$ is a Nash equilibrium if for all $i \in \mathcal{V}$, we have

$$f_i(x_i^*, \mathbf{x}_{-i}^*) \le f_i(x_i, \mathbf{x}_{-i}^*), \forall x_i \in \mathbb{R} \tag{3}$$

where $\mathbf{x}_{-i} = [x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_N]^T$.

[1]For the sake of simplicity in presentation, let's assume that x_i is one-dimensional. However, it is important to note that the methods and results presented herein are directly applicable to accommodate games with multi-dimensional actions.

To facilitate later analysis, we made the following assumptions.

Assumptions 1: For $i \in \mathcal{V}$, $f_i(\mathbf{x})$ is $\mathcal{C}^2$ and $\nabla f_i(\mathbf{x})$ is globally Lipschitz with l_i.

Assumptions 2: The digraph $\mathcal{G}$ is strongly connected.

Lemma 1: Let D be a nonnegative diagonal matrix and $\mathcal{H} = \mathcal{L} + D$. Under Assumptions 2, there are symmetric positive definite matrices P and Q such that

$$\mathcal{H}^T P + P\mathcal{H} = Q \tag{4}$$

Assumptions 3: For $\mathbf{x}, \mathbf{z} \in \mathbb{R}^N$,

$$(\mathbf{x} - \mathbf{z})^T ([\nabla_i f_i(\mathbf{x})]_{vec} - [\nabla_i f_i(\mathbf{z})]_{vec}) \ge m\|\mathbf{x} - \mathbf{z}\|^2 \tag{5}$$

where $\nabla_i f_i(\mathbf{x}) = \partial f_i(\mathbf{x})/\partial x_i$ and m is a positive constant.

IV. MAIN RESULTS

Assumptions 4: For $i \in \mathcal{V}$, $\nabla_{ij}^2 f_i(\mathbf{x}) = \partial^2 f_i(\mathbf{x})/\partial x_i \partial x_j$ is bounded.

Assumptions 5: The disturbance $d(t)$ is bounded, i.e., $\forall i \in \mathcal{V}, |d_i(t)| \le \bar{d}_i(t)$ for a positive constant $\bar{d}_i$.

To realize asymptotic Nash equilibrium seeking for games with second-order integrator-type players distributively, the control input is designed as

$$\begin{cases} u_i = -\tau k_i s_i - \beta_i \xi_i(t) \\ s_i = v_i + \nabla_i f_i(\mathbf{y}_i) \\ \dot{y}_{ij} = -\theta\bar{\theta}_{ij} \left(\sum_{k=1}^N a_{ik}(y_{ij} - y_{kj}) + a_{ij}(y_{ij} - x_j) \right) \\ \dot{\varrho}_i(t) = -\alpha_i \varrho_i(t) \text{ with } \varrho_i(t) > 0 \end{cases} \tag{6}$$

where $i, j \in \mathcal{V}, \theta, \tau$ are adjustable positive parameters, $\bar{\theta}_{ij}, k_i, \alpha_i$ are fixed positive parameters, $\beta_i \ge \bar{d}_i$ is a positive constant, s_i and $\varrho_i(t)$ represents an auxiliary variable for player i and $\xi_i(t) = s_i(t)/(|s_i(t)| + \varrho_i(t))$.

By (1) and (6), the closed-loop system is

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{v} \\ \dot{\mathbf{v}} = -\tau\mathbf{k}\mathbf{s} - [\beta_i\xi_i(t)]_{vec} + \mathbf{d}(t) \\ \mathbf{s} = \mathbf{v} + [\nabla_i f_i(\mathbf{y}_i)]_{vec} \\ \dot{\mathbf{y}} = -\theta\bar{\theta}(\mathcal{L} \otimes \mathbf{I}_{N\times N} + \mathcal{A}_q)\tilde{\mathbf{y}} \end{cases} \tag{7}$$

where $\mathbf{k} = \text{diag}\{k_i\}, \bar{\theta} = \text{diag}\{\bar{\theta}_{ij}\}, \mathcal{A}_q = \text{diag}\{a_{ij}\}, \mathbf{y} = [\mathbf{y}_1^T, \mathbf{y}_2^T, ..., \mathbf{y}_N^T]^T$ and $\tilde{\mathbf{y}} = \mathbf{y} - \mathbf{1}_N \otimes \mathbf{x}$.

Let $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}^*$.

Theorem 1: Under Assumptions 1-5, there exist a $\theta^* > 0$ such that for each $\theta > \theta^*$, there exists a $\tau^* > 0$ such that for each $\tau > \tau^*$, players' actions globally exponentially converge to the Nash equilibrium by (7).

Proof: Define $V = V_1 + V_2 + V_3$, where

$$\begin{aligned} V_1(t) &= \frac{1}{2}\tilde{\mathbf{x}}^T\tilde{\mathbf{x}} \\ V_2(t) &= \frac{1}{2}\tilde{\mathbf{y}}^T P\tilde{\mathbf{y}} \\ V_3(t) &= \frac{1}{2}\mathbf{s}^T\mathbf{s} + \sum_{i=1}^N \frac{\beta_i}{\alpha_i}\varrho_i(t) \end{aligned} \tag{8}$$

Then, taking the time derivative of V_1 yields

$$\begin{aligned}\dot{V}_1 &= \tilde{\mathbf{x}}^T(\mathbf{s} - [\nabla_i f_i(\mathbf{y}_i)]_{vec}) \\ &\leq -m\|\tilde{\mathbf{x}}\|^2 + \max_{i\in\mathcal{V}}\{l_i\}\|\tilde{\mathbf{x}}\|\|\tilde{\mathbf{y}}\| + \|\tilde{\mathbf{x}}\|\|\mathbf{s}\|\end{aligned} \tag{9}$$

where in the inequality, we have used Assumptions 3 and $\|[\nabla_i f_i(\mathbf{y}_i)]_{vec} - [\nabla_i f_i(\mathbf{x})]_{vec}\| \leq \max_{i\in\mathcal{V}}\{l_i\}\|\tilde{\mathbf{y}}\|$.

Then, taking the time derivative of V_2 yields

$$\begin{aligned}\dot{V}_2 =& \dot{\tilde{\mathbf{y}}}^T P\tilde{\mathbf{y}} + \tilde{\mathbf{y}}^T P\dot{\tilde{\mathbf{y}}} \\ =& -\theta\tilde{\mathbf{y}}^T Q\tilde{\mathbf{y}} - 2\tilde{\mathbf{y}}^T P\mathbf{1}_N \otimes \mathbf{s} \\ &+ 2\tilde{\mathbf{y}}^T P\mathbf{1}_N \otimes [\nabla_i f_i(\mathbf{y}_i)]_{vec} \\ \leq& -(\theta\lambda_{\min}(Q) - 2\sqrt{N}\|P\|\max_{i\in\mathcal{V}}\{l_i\})\|\tilde{\mathbf{y}}\|^2 \\ &+ 2\sqrt{N}\|P\|\|\tilde{\mathbf{y}}\|\|\mathbf{s}\| + 2N\|P\|\max_{i\in\mathcal{V}}\{l_i\}\|\tilde{\mathbf{y}}\|\|\tilde{\mathbf{x}}\|.\end{aligned} \tag{10}$$

where in the inequality, by Assumption 2 and Lemma 1, we let $P, Q \in \mathbb{R}^{N^2\times N^2}$ and $Q = P\bar{\theta}(\mathcal{L}\otimes\mathbf{I}_{N\times N} + \mathcal{A}_q) + (\mathcal{L}\otimes\mathbf{I}_{N\times N} + \mathcal{A}_q)^T\bar{\theta}P = Q$.

Moreover, taking the time derivative of V_3 yields

$$\begin{aligned}\dot{V}_3 &= \mathbf{s}^T(\dot{\mathbf{v}} + \bar{H}(\mathbf{y})\dot{\mathbf{y}}) - \sum_{i=1}^N \beta_i\varrho_i(t) \\ &= \mathbf{s}^T(-\tau\mathbf{k}\mathbf{s} - [\beta_i\xi_i(t)]_{vec} + \mathbf{d}(t) + \bar{H}(\mathbf{y})\dot{\mathbf{y}}) - \sum_{i=1}^N \beta_i\varrho_i(t)\end{aligned} \tag{11}$$

where $\bar{H}(\mathbf{y}) = [h_{ij}]$ in which for $i \neq j$, $h_{ij} = \mathbf{0}_N^T$ and for $i = j$, $h_{ii} = [\nabla^2_{i1}f_i(\mathbf{y}_i), \nabla^2_{i2}f_i(\mathbf{y}_i), \ldots, \nabla^2_{iN}f_i(\mathbf{y}_i)]$ $\nabla^2_{ij}f_i(\mathbf{y}_i) = \frac{\partial^2 f_i(\mathbf{x})}{\partial x_i \partial x_i}|_{\mathbf{x}=\mathbf{y}_i}$. By Assumption 4, $\|\bar{H}\mathbf{y}\|$ is bounded. Hence, There are some positive constant L_1 meet $\|\bar{H}(\mathbf{y})\|\|\bar{\boldsymbol{\theta}}(\mathcal{L}\otimes\mathbf{I}_{N\times N} + \mathcal{A}_q)\| \leq L_1$. Therefore,

$$\begin{aligned}\dot{V}_3 \leq& -\tau\lambda_{\min}(\mathbf{k})\|\mathbf{s}\|^2 + \theta L_1\|\mathbf{s}\|\|\tilde{\mathbf{y}}\| \\ &- \sum_{i=1}^N \frac{(\beta_i - |d_i(t)|)|s_i(t)|(|s_i(t)| + \varrho_i(t)) + \beta_i\varrho_i^2(t)}{|s_i(t)| + \varrho_i(t)}\end{aligned} \tag{12}$$

Hence,

$$\begin{aligned}\dot{V} \leq& -m\|\tilde{\mathbf{x}}\|^2 - \tau\lambda_{\min}(\mathbf{k})\|\mathbf{s}\|^2 \\ &- (\theta\lambda_{\min}(Q) - 2\sqrt{N}\|P\|\max_{i\in\mathcal{V}}\{l_i\})\|\tilde{\mathbf{y}}\|^2 \\ &+ (\max_{i\in\mathcal{V}}\{l_i\} + 2N\|P\|\max_{i\in\mathcal{V}}\{l_i\})\|\tilde{\mathbf{x}}\|\|\tilde{\mathbf{y}}\| \\ &+ (2\sqrt{N}\|P\| + \theta L_1)\|\tilde{\mathbf{y}}\|\|\mathbf{s}\| + \|\tilde{\mathbf{x}}\|\|\mathbf{s}\| \\ &- \sum_{i=1}^N \frac{(\beta_i - |d_i(t)|)|s_i(t)|(|s_i(t)| + \varrho_i(t)) + \beta_i\varrho_i^2(t)}{|s_i(t)| + \varrho_i(t)} \\ \leq& -\lambda_{\min}(F)\|\mathbf{E}_1\|^2 - \tau\lambda_{\min}(\mathbf{k})\|\mathbf{s}\|^2 \\ &+ (1 + 2\sqrt{N}\|P\| + \theta L_1)\|\mathbf{E}_1\|\|\mathbf{s}\|\end{aligned} \tag{13}$$

where $F = \begin{bmatrix} m & a \\ a & \theta\lambda_{\min}(Q) - 2\sqrt{N}\|P\|\max_{i\in\mathcal{V}}\{l_i\}\end{bmatrix}$ $\mathbf{E}_1 = [\tilde{\mathbf{x}}^T, \tilde{\mathbf{y}}^T]^T$ $a = -\frac{\max_{i\in\mathcal{V}}\{l_i\} + 2N\|P\|\max_{i\in\mathcal{V}}\{l_i\}}{2}$, choose $\theta > \frac{(\max_{i\in\mathcal{V}}\{l_i\} + 2N\|P\|\max_{i\in\mathcal{V}}(l_i))^2}{4m\lambda_{\min}(Q)} + \frac{2\sqrt{N}\|P\|\max_{i\in\mathcal{V}}\{l_i\}}{\lambda_{\min}(Q)}$. Then, $\lambda_{min}(F)$ is a real positive number and $\lambda_{min}(F) > 0$. Thus,

$$\dot{V}(t) \leq -\lambda_{\min}(G)\|\mathbf{E}\|^2 \tag{14}$$

where $G = \begin{bmatrix} \lambda_{\min}(F) & -\frac{1+2\sqrt{N}\|P\|+\theta L_1}{2} \\ -\frac{1+2\sqrt{N}\|P\|+\theta L_1}{2} & \tau\lambda_{\min}(\mathbf{k})\end{bmatrix}$, $\mathbf{E} = [\tilde{\mathbf{x}}^T, \tilde{\mathbf{y}}^T, \tilde{\mathbf{s}}^T]^T$. By choosing $\tau > \frac{(1+2\sqrt{N}\|P\|+\theta L_1)^2}{4\lambda_{\min}(F)\lambda_{\min}(\mathbf{k})}$, $\lambda_{min}(G)$ is a real positive number and $\lambda_{min}(G) > 0$.

The conclusion can be drawn.

V. SIMULATION STUDIES

VI. CONCLUSIONS

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