Distribution for products in asymmetric e^+e^- collider: an example in B and L violating τ decay

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1 Abstract

The aim of this thesis is to provide a computational analysis of the distributions for the momenta of particles, produced up to n-th order two body decays, and flight distances requirement in asymmetric collider. In particular it is analyzed the case of $e^+ + e^-$ in $\tau^+ + \tau^-$ with one τ decaying in $\Lambda^0 + \pi$ as in Ref.[1] in order to reproduce the Monte Carlo simulations of Fig.(1) in the reference.

The study is helpful in understanding the detector configuration. Known the lifetime of each particle it is possible to compute the distribution of flight length of each particle in the k-th step of the process, giving information not only at which distance from the collision center it is most probably to find that particular vertex but also what kind of particles are required to be analyzed for a particular collision given the dimension of the detector. Then simulating a large number of events it is indeed possible to determine the energy distribution and the 3-momenta module distribution in the laboratory frame of any particle involved so it is possible, once chosen the size of the detector, to choose which kind of identification method is better to be implemented in.

2 General Method and Considerations

The electron has an energy $E_e >> m_e$ ans so it has the positron, in this condition the energy of the positron E_p is:

$$E_p \sim \frac{s}{4E_e}$$

Where s is the Mandelstam variable, this is enough to calculate the speed and the relative γ_{cm} of the center of mass frame:

$$\beta_{cm} \sim \frac{E_e - E_p}{E_e + E_p} \qquad \gamma_{cm} = \frac{1}{\sqrt{1-\beta_{cm}^2}} \tag{1}$$

Provided this data it is possible to start constructing our method of analysis. In the further analysis it is assumed that the differential cross section in the C.M. (center of mass) frame for the process can be factorized as:

$$d\sigma(e^+ + e^- \longrightarrow f + \bar{f}) = f(\cos(\theta))g(\phi)\ d\phi d\cos(\theta) \tag{2}$$

Where θ is the polar angle and ϕ is the azimuthal angle while $f(\cos(\theta))$ and $g(\phi)$ are two limited function in the interested interval. This is the only condition to apply the Acceptance-rejection method as in Ref.[2] to generate a $\cos(\theta)$ and a ϕ according to the required distribution. Chosen a constant normalized distribution uniform distribution $h(x)$, and a constant C equal to the supremum of $f(\cos(\theta))$ (or $g(\phi)$) per the length of the interval (in case of the f function the interval is [-1,1] because a $\cos(\theta)$ is generated not θ, instead for the g-function is [0,2π]), a random number r between 0 and 1 is generated along with a random number x generated in the interval of definition for the considered function. Then x is accepted if it saturate the condition:

$$rCh(x) < f(x) \longrightarrow \sup_{x\in interval}\{f(x)\}r < f(x) \tag{3}$$

It is now fairly easy to solve the scattering in the C.M. frame and then perform a boost to take back the 4-momenta in the LAB frame. It is important to recall the expression for a generic Lorentz's Boost in the $\vec{v}$ direction.

$$B(\vec{v}) = \begin{bmatrix} \gamma & -\gamma v_x/c & -\gamma v_y/c & -\gamma v_z/c \\ -\gamma v_x/c & 1+(\gamma-1)\frac{v_x^2}{v^2} & (\gamma-1)\frac{v_x v_y}{v^2} & (\gamma-1)\frac{v_x v_z}{v^2} \\ -\gamma v_y/c & (\gamma-1)\frac{v_y v_x}{v^2} & 1+(\gamma-1)\frac{v_y^2}{v^2} & (\gamma-1)\frac{v_y v_z}{v^2} \\ -\gamma v_z/c & (\gamma-1)\frac{v_z v_x}{v^2} & (\gamma-1)\frac{v_z v_y}{v^2} & 1+(\gamma-1)\frac{v_z}{v^2} \end{bmatrix} \tag{4}$$

Now suppose the f particle produced in the above analyzed collision decays itself as follow: $f \rightarrow X + b$, where X and b are two other particles. The decay is completely solved in the frame at rest with respect to the f, supposing that the differential decay length for the considered decay can be factorized as the one in formula (2), the same method can be applied to generate a random $\cos\theta$ and a random ϕ according to the proper distribution. In this way it is possible to compute the distribution of the components in the COM or in the LAB frame by performing a boost with the proper β (here c=1) in the $-\vec{\beta}$ direction. And so on one can proceed supposing for example the X to decay in two particles with the same restriction on the decay length etc. until the n-th decay.

Then it is possible to calculate the flight length distribution provided the mean lifetime, recalling that defined the mean flight length as:

$$<l>= \gamma\tau c\beta$$

Where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ in which β is the speed of the particle in the LAB frame, τ is the particle mean lifetime, a particle decays in a length distributed as:

$$f(L) = \frac{1}{<l>} e^{-\frac{L}{<l>}} \tag{5}$$

It has to be noticed that the dimension is $[L^{-1}]$ because the probability is sibling to f(L)dL that is dimensionless. So for each value of the 3-momenta $\vec{p}$ of the selected particle we can compute the mean fight length and then generate a casual Fight length according to the distribution in equation (5) with the Inverse Transform method of Ref.[2] .

3 Example $e^+ + e^- \rightarrow \tau^+ + \tau^-$ and $\tau \rightarrow \Lambda^0 + \pi$

With reference to the BaBar's collaboration experiment at PEP-II (Ref. [1]) it had been studied the decay $\tau \rightarrow \Lambda^0 + \pi$ with an asymmetric collider e^+e^- which e^-'s beam energy was 9.0 GeV and the center of mass energy was the $\Upsilon(4S)$ level: 10.56 GeV. Other useful data are:

- the π's mass: 0.139 GeV;
- the τ's mass: 1.77 GeV;
- the Λ^0's mass: 1.11 GeV.

Here the values in equation (1) for the case studied:

$$E_{pos} = 3.09\ GeV \quad \beta_{cm} = 0.487 \quad \gamma_{cm} = 1.14$$

The differential cross section for the process $e^+ + e^- \rightarrow \tau^+ + \tau^-$ in the C.M. frame is:

$$\frac{d\sigma}{d\Omega} = N_c \frac{\alpha^2}{4s} \beta[1 + \cos^2(\theta) + (1-\beta^2)\sin^2(\theta)\ Q_f^2 \tag{6}$$

Where s is the Mandelstam variable, β is v/c for the produced fermions in the c.m., θ is the c.m scattering angle, Q_f is the charge of the lepton and N_c is 1 for charged leptons and 3 for quarks. We procede with analytic calculations and Monte Carlo simulations as well.

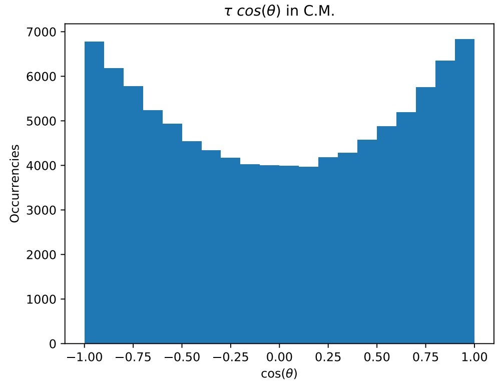

Figure 1: 100000 events simulation

In Figure 1 it is shown the result of the extraction of $\cos(\theta)$ under the distribution in Eq.(6).Then assume a uniform distribution for $\tau's$ decay. With easy calculation it is possible to show that Λ^0's 3-momenta kinematics lower limit is 1.8 GeV/c in the c.m. frame and to further visualize this limit it has been plotted the distribution for the Λ^0 3-momenta in the c.m. frame in Figure 2.

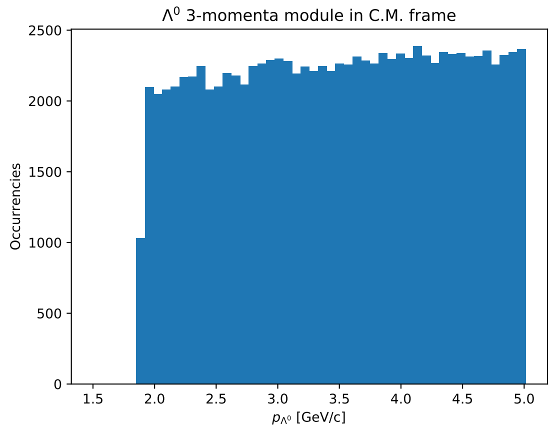

Figure 2: 100000 events simulation

In fact no events with lower momenta than the kinematics limit were obtained. Then there have been reported the distributions for the mean flight length and the flight length for both the τ's and the Λ^0 in the laboratory frame, where the distances are measured. The flight length of the Λ^0 is required to be $L_{\Lambda^0} > 1$ cm so the efficiency is about 97%.

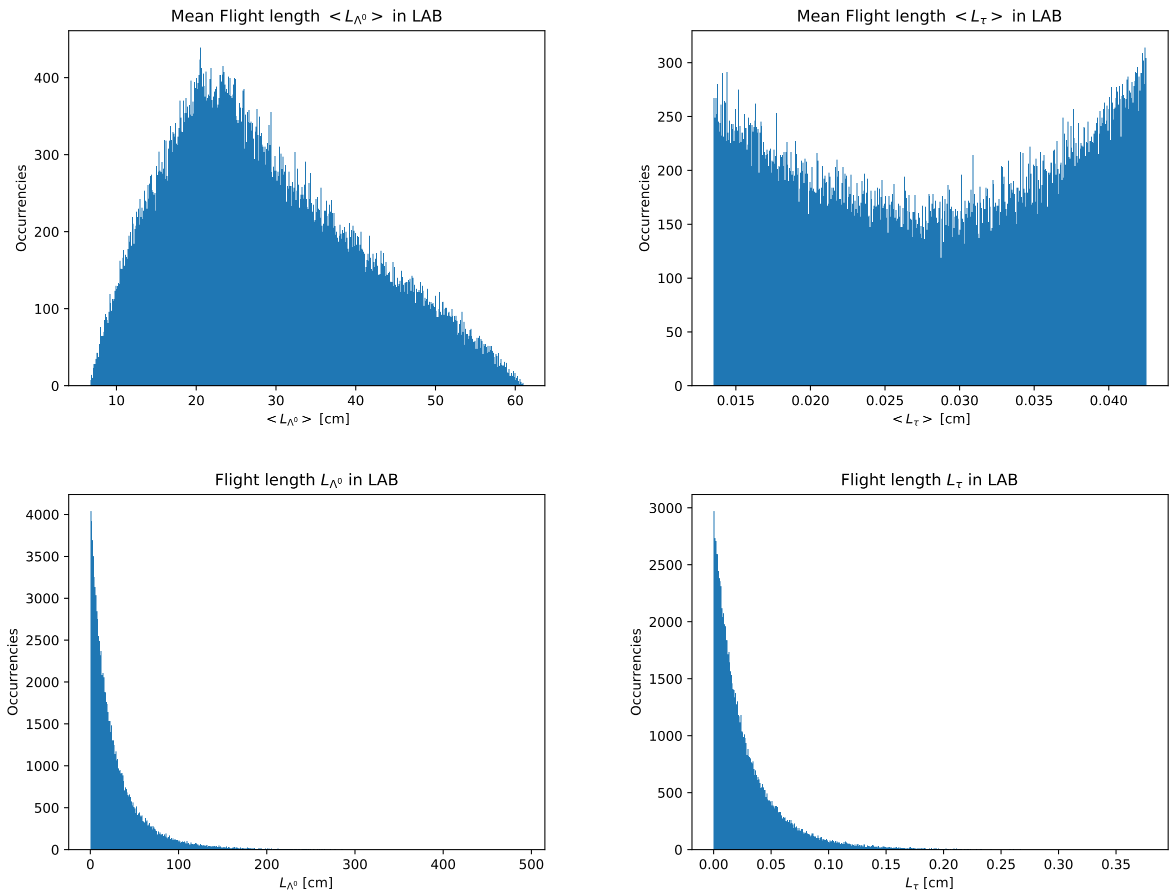

Figure 3: 100000 events simulation

The full output of the program is in section 5. As it can be seen by the graphs, within a sphere of radius 0.2 cm from the collisions center there are all the τ's decay. Because of the fact that the proton doesn't decay and the π has a very long lifetime it is not needed to go at higher order in the calculation to gauge which particle are expected to be detected (there should always be a proton) and the π is always detected within the dimension of the detector.

4 References

[1] Babar Collaboration, `https://arxiv.org/abs/hep-ex/0607040v1`

[2] `https://pdg.lbl.gov/2020/reviews/rpp2020-rev-monte-carlo-techniques.pdf`

5 Program and other graphs

The code and the other plots have been made available at the following repository on GitHub: `https://github.com/Lollo0900/BachelorThesis` .