

Problem 1: Calculate the guided mode of slab SOI waveguides

Problem 2: Study the propagation of optical pulses in dispersive fibers

Problem 3: Optical bio-sensor based on silicon micro-rings

Problem 1: Slab waveguides

Optical modes of silicon waveguides

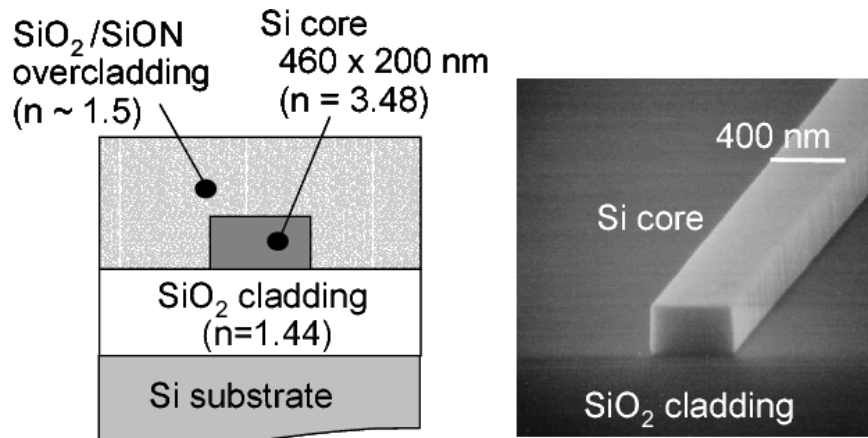


Fig. 1. Schematic of silicon photonic wire waveguide.

<https://ieeexplore.ieee.org/document/1708225>

Example of application:

Fig.1 shows a typical example of a 2D silicon waveguide used for guiding light in photonic integrated circuits made in CMOS compatible photonic platform.

To get familiar with waveguides we will however study a simpler example of silicon waveguide: the slab waveguide as shown in Fig.2. Our analysis neglects the Si substrate because it is enough separated by the slab core.

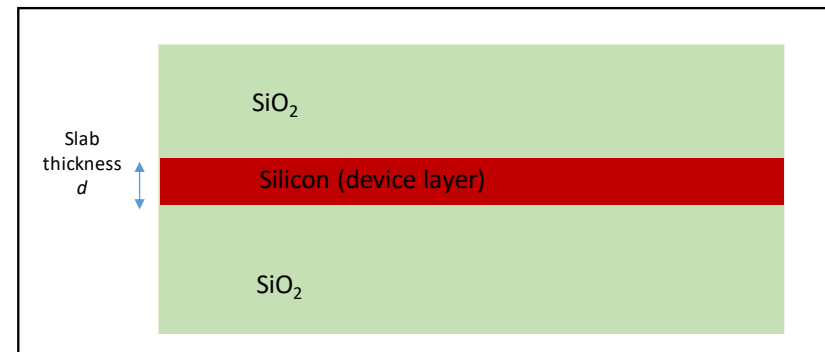


Fig. 2: SOI slab waveguide

Optical modes of silicon waveguides

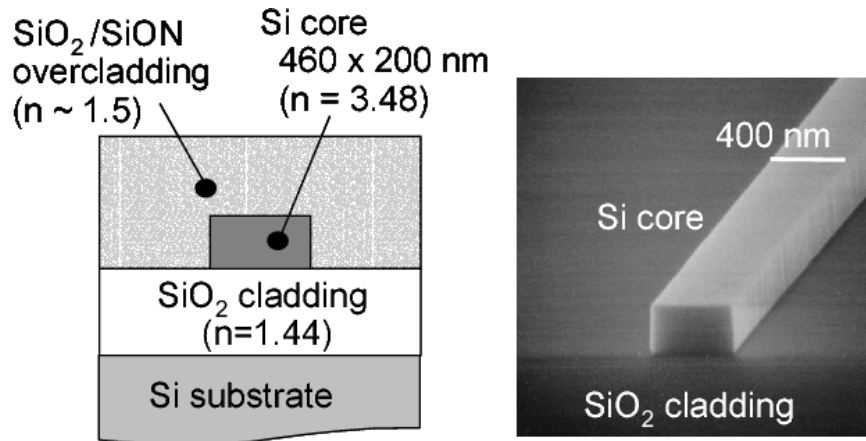


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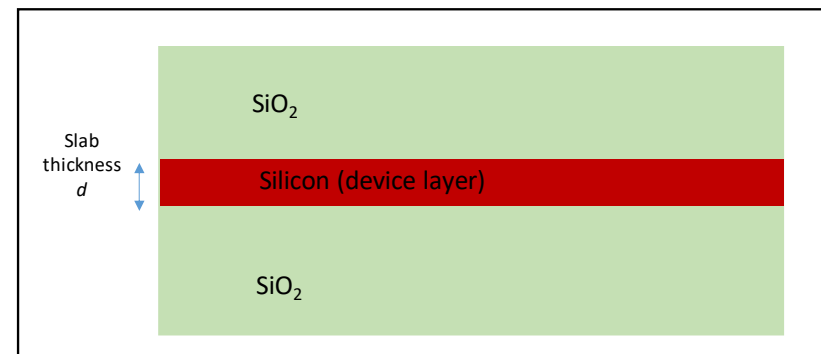


Fig. 2: SOI slab waveguide

Applications of silicon waveguides

Photonic integrated circuit for beam steering in LiDAR applications

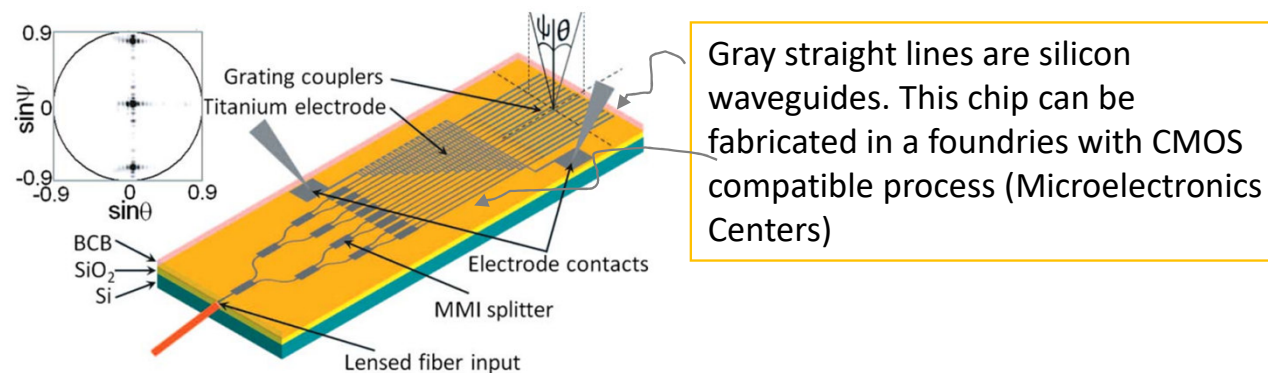
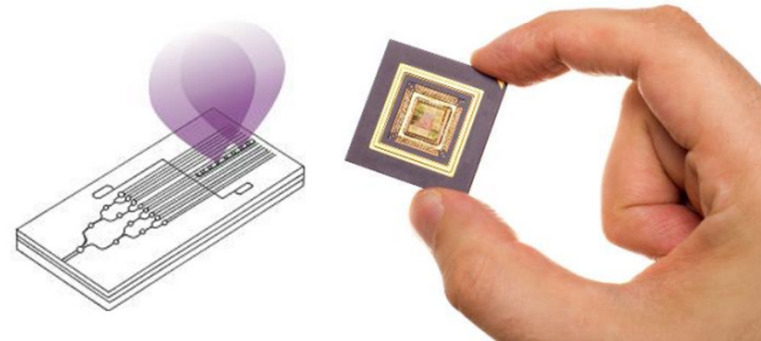
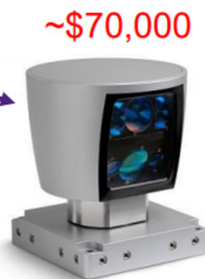


Fig. 1. (Color online) Schematic of the beam steering component. The inset shows the far-field image.

Karel Van Acoleyen, Wim Bogaerts, Jana Jágorská, Nicolas Le Thomas, Romuald Houdré, and Roel Baets, "Off-chip beam steering with a one-dimensional optical phased array on silicon-on-insulator," Opt. Lett. 34, 1477-1479 (2009)

Advantages of photonic integration

- LIDAR (Light Detection And Ranging) is a critical device for self-driving cars
 - Bulky and clumsy, with 64 lasers
 - Contains moving parts
 - Very expensive, costs more than the car itself
 - Difficult for commercialization
- LIDAR on-chip is an alternative solution to commercialize the technology
 - Compact, integrated on a chip
 - Solid and durable, no moving parts
 - Can be produced cheaply on a large-scale
 - Low prototype efficiency, ≈ 2 meters!



Problem 1: *Requirements*

1. Calculate and report a graph of **the effective refractive index** of the fundamental TE modes at 1.55 μm of the slab of Fig. 2-slide 4 **as function of the slab thickness d**
2. Select a slab thickness to have only two guided modes (TE_0 and TE_1) and plot **the intensity of the field components**: $E_y(x)$, $H_x(x)$ and $H_z(x)$ of TE_0 and TE_1 .
3. For mode TE_0 , calculate the analytical expression of the optical confinement factor (Γ_{TE0}) in the silicon core and plot the optical confinement factor as function of the slab thickness.
4. Assume that the loss for Si and SiO2 at 1550 nm are: $\alpha_{\text{Si}} = 0.3\text{cm}^{-1}$ and $\alpha_{\text{SiO2}} = 0.15\text{cm}^{-1}$; calculate the modal loss of TE_0 and plot the modal loss in cm^{-1} and in dB/cm as function of the slab thickness.
Modal loss are defined as: $\alpha_{\text{modal}} = \Gamma_{\text{core}}\alpha_{\text{Si}} + \Gamma_{\text{cladding}}\alpha_{\text{SiO2}}$
5. From point 4, the insertion loss as function of the slab thickness for a waveguide of length $L=5\text{mm}$.
6. Select a slab thickness to have a single mode waveguide with Γ_{TE0} in the range 70-80% at 1550 nm.
7. Plot for the waveguide of point 6, **the effective refractive index of TE_0 and TE_1 as function of the pulsation** (ie: $n_{\text{eff}}(\omega)$) and **the effective refractive index as function of wavelength** (ie: $n_{\text{eff}}(\lambda)$). Limit both graphs in the wavelength range [850 nm-1620 nm].

Problem 1.1 : *Tips to find n_{eff}*

TE mode Eigenvalue equation

$$\tan(k_x d) = \frac{k_x(\gamma + \delta)}{k_x^2 - \gamma\delta}$$



$$k_x d = \text{atan}\left[\frac{k_x(\gamma + \delta)}{k_x^2 - \gamma\delta}\right] + m\pi = \text{atan}\left[\frac{\frac{\gamma}{k_x} + \frac{\delta}{k_x}}{1 - \left(\frac{\gamma}{k_x}\right)\left(\frac{\delta}{k_x}\right)}\right] + m\pi = \text{atan}\left(\frac{\gamma}{k_x}\right) + \text{atan}\left(\frac{\delta}{k_x}\right) + m\pi$$

$$= \Phi_A + \Phi_B + m\pi, \quad m = 0, 1, 2, \dots$$

$$\text{atan}\left(\frac{x_1 + x_2}{1 - x_1 x_2}\right) = \text{atan}(x_1) + \text{atan}(x_2)$$

$$\Phi_A = \text{atan}\left(\frac{\gamma}{k_x}\right)$$

$$\Phi_B = \text{atan}\left(\frac{\delta}{k_x}\right)$$

$$\delta = k_0 \sqrt{n_{\text{eff}}^2 - n_3^2}$$

$$k_x = k_0 \sqrt{n_1^2 - n_{\text{eff}}^2}$$

$$\gamma = k_0 \sqrt{n_{\text{eff}}^2 - n_2^2}$$

Cont.

- Define in Matlab an array of values for n_{eff} between n_{cladding} and n_{core} :

For example: `neff=linspace (ncladding,ncore,1e4)`

- Write the function that calculates d as function of n_{eff} implementing the expression evidenced in green in the previous slide

$d=...$

- Plot n_{eff} on y-axis and d on x-axis →

Matlab command: `plot(d,neff)`

You get n_{eff} as function of the slab thickness d

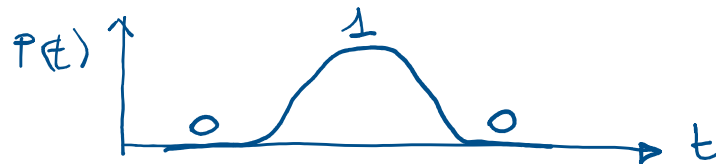
Theoretical questions related to problem 1

- *What is the difference between single mode and multi-mode waveguides?*
- *If the input wavelength increases, the waveguide becomes closer to a multi-mode or a single mode waveguide?*
- *Can you explain with your results what is GVD?*
- *Can you explain with your results what is intermodal dispersion?*
- *Can you explain why it is not a good idea to use this waveguides to guide light at 850nm?*
- *Can you list the differences between a silicon photonic wire and an optical fiber?*

Problem 2:
Study the propagation of
optical pulses in dispersive
waveguides

Applications of problem 2

- The aim of this problem is quantifying the impact of chromatic dispersion of SMF on the propagation of an optical pulse. An optical pulse can be considered a simple model for the data sequence 010. This data sequence is distorted if the propagation length is long and/or the bit rate is high.



- Learn reading optical fiber datasheet. We consider a SMF -28, the datasheet can be download from <https://www.corning.com/media/worldwide/coc/documents/Fiber/SMF-28%20ULL.pdf>

Problem 2: Requirements

1. Plot with Matlab **the input Gaussian pulse for different values of τ_0** ($\tau_0=1$ ns, 0.5 ns, 0.1 ns, 10 ps) and **the corresponding optical spectrum** (ie: absolute value of Fourier transform of the optical pulse versus wavelength). The pulse can be normalized to maximum peak amplitude equal to 1.
2. Write and plot **the propagation constant $\beta(\omega)$** around **1625 nm of the SMF-28** (use the data sheet to find the fiber parameters). Note: the dispersion in the data sheet is defined as: $D = -\frac{2\pi c}{\lambda^2} \cdot \frac{\partial^2 \beta}{\partial \omega^2}$
3. Using Matlab FFT and IFFT, calculate and plot **the temporal intensity of the output pulses for and the optical spectrum for different lengths L** (L=1m, 10 m, 1km, 80km). Consider two cases: $\tau_0=1$ ns and $\tau_0=10$ ns . Include in the calculation the fiber loss. Explain the results obtained. *Tips: plot the pulses as function of $\tilde{t} = t - \tau_g$ where τ_g is the group delay of the different lengths. In this way you can overlap and compare pulses at different propagation distances.*
4. Make a plot of **the ratio between the output pulse FWHM and τ_0 versus the fiber length**. Explain the result obtained. Consider in the plot all the four cases $\tau_0=1$ ns, 0.5 ns, 0.1 ns, 10 ps.
5. Optional: plot the frequency chirp of the output pulse

Problem 2: *TIPS on the use of FFT with Matlab*

1. Define the temporal variable $t=0:dt:Tend$, where dt is the time sampling step and $Tend$ is the temporal window you are considering. The total number of time samples is therefore: $N=length(t)$.

2. **Warning:** chose dt to sample pulses correctly!! The narrower is the pulse, the shorter is dt !

3. Write the Gaussian pulse $p(t)=...$

4. Set the frequency range in the frequency domain as:

$$f_range=linespace(-0.5*1/dt,0.5*1/dt, N)$$

5. Fourier transform of $p(t)$ using the Matlab function FFT as:

$$P_f=fftshift(fft(p(t)))/N;$$

(To reconstruct the pulse in the temporal domain (for example after propagation) use using the Matlab function IFFTSHIFT and IFFT)

Problem 3:
silicon photonics ring
resonator as optical bio-sensor

Applications of problem 3

Ring Resonator Sensor

Fig.1 shows an example of micro-ring resonator; the waveguides are silicon waveguides on SiO_2 BOX. This ring can be employed as a bio-sensor according to the principle illustrated in Fig.2. By measuring the wavelength shift of the resonance wavelength, we get information about the density (or type) of the bio-analyte (ie: the target) deposited and bound to the receptor. The target, bound to the receptor, causes a variation of the refractive index of the top cladding layer; this variation of refractive index gives the shift of the ring resonance wavelength.

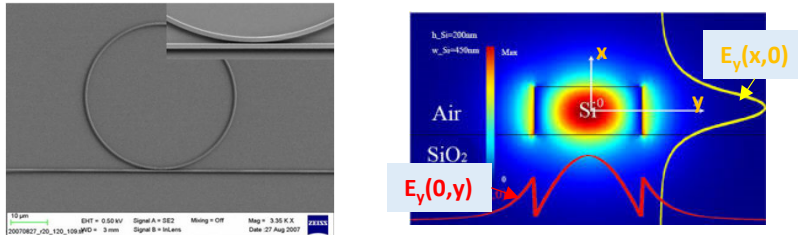


FIG.1 a) SEM photographs of the silicon microring resonator with a radius of 20 μm . Inset is a zoom-in view of the coupling region. Image from [10.1109/JLT.2008.2005510](https://doi.org/10.1109/JLT.2008.2005510). **b)** example of the distribution of the electric field of the fundamental TE mode of the silicon waveguide

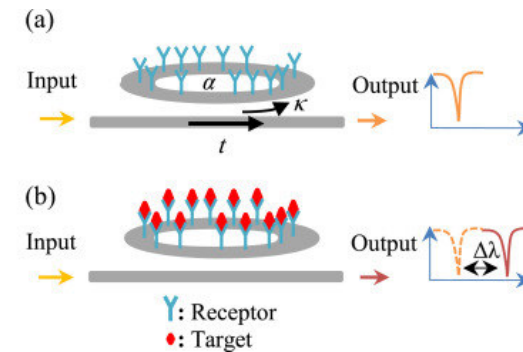


FIG. 2 a) Ring response with deposited receptor layer and **(b)** ring response after the target has deposited on the receptor layer. Image from <https://doi.org/10.1016/j.optcom.2015.12.007>

Cont.

- The refractive index of the silicon ring waveguide in absence of the target is n_{eff0} . The refractive index is changed by the bound target as $n_{eff} = n_{eff,0} + \Gamma_{clad}\Delta n_{clad}$, where Δn_{clad} is the variation of the refractive index of the cladding induced by the bound target. The **wavelength shift** is:

$$\Delta\lambda = \Gamma_{clad}\Delta n_{clad} \frac{\lambda_0}{n_{eff,0}} \quad (1)$$

where Γ_{clad} is the optical confinement factor in the cladding layer

- The **sensitivity of the sensor** is defined as :

$$S = \frac{\Delta\lambda}{\Delta n_{clad}}$$

- The parameter accounting for the measurement system resolution and the capacity of the ring resonator to filter out the noise is defined **Detection Limit (DL)** . DL depends on the ring parameters and on the sensitivity:

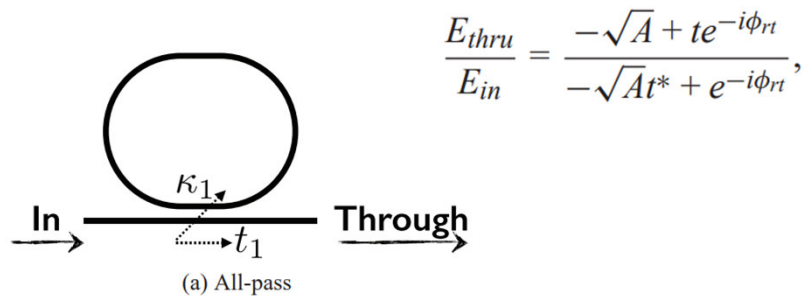
$$DL = \frac{\lambda_0}{Q \cdot S}$$

- The DL improves increasing the Q factor of the ring, the smaller is DL the better it is. High Q rings must therefore be designed for high performance sensors.
- The target however can increase the ring waveguide loss, because the target may absorb the light in the cladding layer. Therefore, an increase of optical loss degrades DL because reduce the Q of the ring.

Problem 3: Requirements

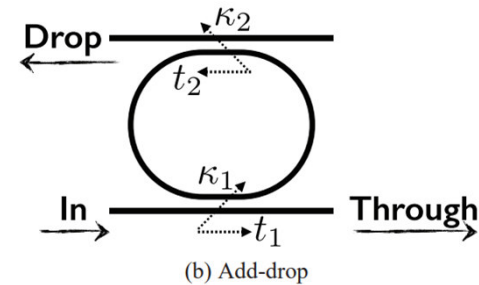
- Consider a silicon micro-ring in the add-drop configuration (see slide 19) with the following parameters:
 - Bus waveguide to ring waveguide coupling coefficient $k^2=0.01$
 - $n_{eff,0}=2.6$ $n_g=3.2$ *waveguide loss= 0.5dB/cm*
 - Operating wavelength 1.55 μm
 - Ring radius $R=8 \mu\text{m}$
1. Plot **the power transmission spectrum at the through and drop ports as function of wavelength** (consider an appropriate wavelength span) and compare with the case of the same ring with $k^2=0.1$. Comment the results obtained. Which case is the best for using the ring as sensor?
 2. Assuming the analyte causes additional material loss (α_{analyte}) in the range between 0-30 cm^{-1} , plot **the ring Q versus the material loss for three different values of $\Gamma_{\text{clad}}=1\%$, 5% and 10%.**
 3. Plot for the case in (2) the **sensor DL** and **sensitivity** and comment on the results obtained.

Rings in All-pass or Add-Drop configurations



$$\frac{E_{thru}}{E_{in}} = t - \frac{k^2 a e^{-j\phi}}{1 - a t e^{-j\phi}}$$

$$FWHM = \frac{(1 - ta)\lambda_m^2}{\pi n_g L_{rt} \sqrt{ta}}$$



$$\frac{E_{thru}}{E_{in}} = \frac{t_1 - t_2^* \sqrt{A} e^{i\phi_{rt}}}{1 - \sqrt{A} t_1^* t_2^* e^{i\phi_{rt}}}$$

$$\frac{E_{drop}}{E_{in}} = \frac{-\kappa_1^* \kappa_2 A^{\frac{1}{4}} e^{i\phi_{rt}/2}}{1 - \sqrt{A} t_1^* t_2^* e^{i\phi_{rt}}},$$

$$FWHM = \frac{(1 - t_1 t_2 a)\lambda_m^2}{\pi n_g L_{rt} \sqrt{t_1 t_2 a}}$$

Assume $k_1=k_2$ $t_1 = t_2 = t_1^* = t_2^*$

$$\phi_{rt} = \beta L_{rt}$$

$$A = e^{-\alpha L_{rt}}.$$

$$L_{rt} = 2\pi R$$

$$a = \sqrt{A}$$