Rozwiązywanie równań i układów równań nieliniowych

19 maja 2022

1 Problem 1

1.1 Opis problemu

Główną ideą zadanie jest wyznaczenie pierwiastków równania $f(x) = 0$ w zadanym przedziale metodą Newtona oraz metodą siecznych. Dla metody Newtona punkty startowe wybierane będą rozpoczynając od wartości końców przedziału, zmniejszając je o 0.1 w kolejnych eksperymentach numerycznych. Odpowiednio dla metody siecznej jeden z końców przedziału stanowić powinna wartość punktu startowego dla metody Newtona, a drugi - początek, a następnie koniec przedziału [a, b].

Badana funkcja:

$$f(x) = x^n + x^m$$

$$f'(x) = nx^{(n-1)} + mx^{(m-1)}$$

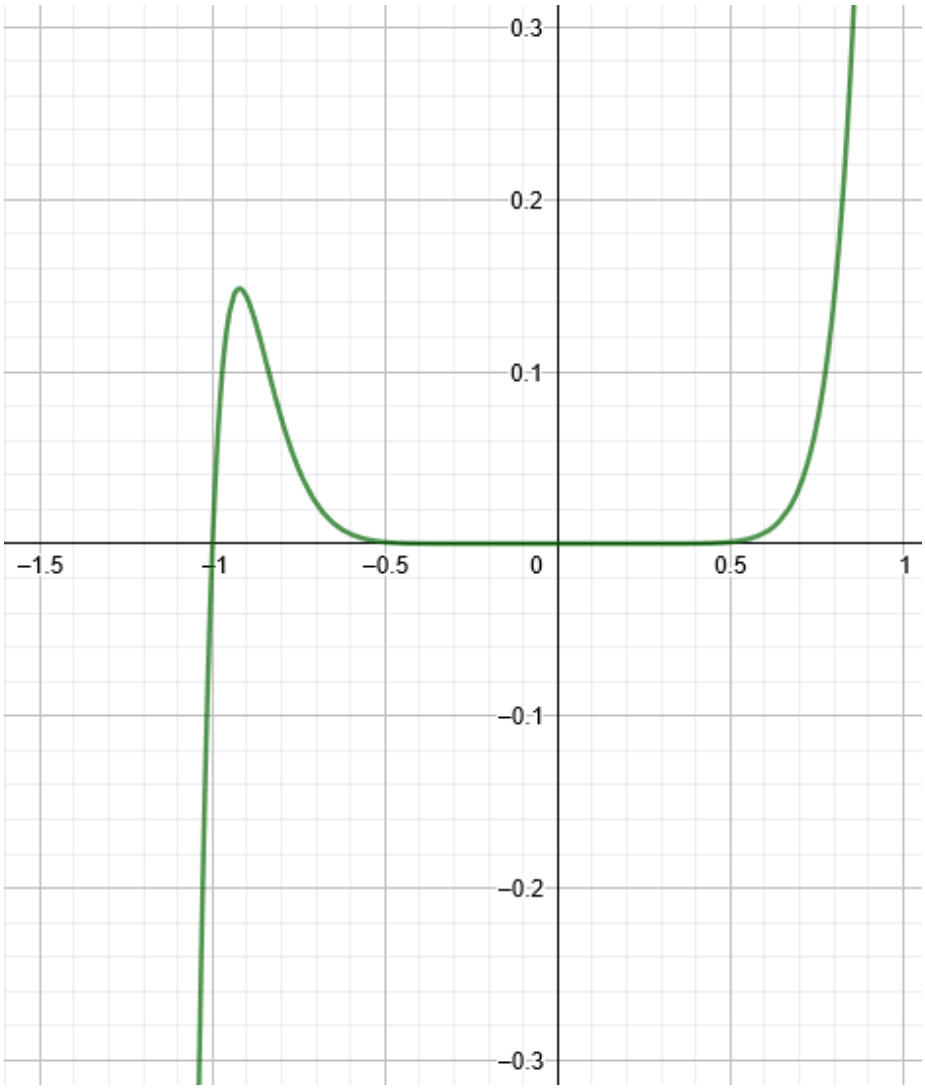
Gdzie $n = 15$, $m = 10$ oraz $x \in [-1.5, 0.3]$.

Liczba iteracji dla obu tych metod (dla różnych dokładności ρ) zostanie porównana, stosując kryteria stopu:

1. $\left|x_{(i+1)} - x_{(i)}\right| < \rho$
2. $|f(x_i)| < \rho$

1.2 Opracowanie

Wykres badanej funkcji wygląda następująco:



Rysunek 1: Funkcja f

1.2.1 Metoda Newtona

Żeby zastosować metodę Newtona muszą być spełnione warunki (jednak na potrzebny zadania je ignorujemy):

1. funkcja jest ciągła,
2. w przedziale znajduje się dokładnie jeden pierwiastek,
3. funkcja ma różne znaki na krańcach przedziału,

4. pierwsza i druga pochodna funkcji mają stały znak w tym przedziale.

Poniżej znajdują się tabele z wynikami oraz liczbami iteracji dla różnych punktów startowych, dokładności i kryteriów stopu (kolumny - dokładność warunku stopu, wiersze - punkt startowy, pierwsza wartość to liczba iteracji, druga to wyliczona wartość):

	0.01	0.0001
-1.5	9 , -1.0002874196526226	11 , -1.000000000011706
-1.4	8 , -1.0002324713110928	10 , -1.0000000000050169
-1.3	7 , -1.0001155692216488	9 , -1.0000000000003073
-1.2	6 , -1.0000226461422514	7 , -1.000000006152385
-1.1	4 , -1.0001611737724945	6 , -1.0000000000011613
-1.0	1 , -1.0	1 , -1.0
-0.9	19 , -0.08588432405196786	63 , -0.0008328830846807948
-0.8	21 , -0.08232443336397852	64 , -0.0008870670204558646
-0.7	20 , -0.08290827817358438	63 , -0.0008933580825082613
-0.6	18 , -0.08905531556392565	62 , -0.0008636343990309718
-0.5	17 , -0.08302512528112482	60 , -0.000894617136871563
-0.4	15 , -0.08224132694859262	58 , -0.0008861715296672138
-0.3	12 , -0.08470093552096244	56 , -0.0008214069357594659
-0.2	8 , -0.08608975894723488	52 , -0.0008348753307038678
-0.1	2 , -0.08099992842712822	45 , -0.000872795172735189
0.0	- , -	- , -
0.1	1 , 0.09000004999925001	45 , 0.000872797540867176
0.2	8 , 0.08609712349169579	52 , 0.000834947821600728
0.3	12 , 0.08475669916682832	56 , 0.0008219486904814841

Tabela 1: Wyniki dla kryterium 1

	0.000001	0.00000001
[-1.5, 0.3]	1 , 0.2999999719783287	202 , 1.2347184284733514e-07
[-1.5, 0.2]	1 , 0.19999999954202397	1 , 0.19999999954202397
[-1.5, 0.1]	1 , 0.0999999999995792	1 , 0.0999999999995792
[-1.5, 0.0]	1 , 0.0	1 , 0.0
[-1.5, -0.1]	1 , -0.1000000000003682	1 , -0.1000000000003682
[-1.5, -0.2]	1 , -0.2000000003499929	1 , -0.2000000003499929
[-1.5, -0.3]	1 , -0.30000001859054404	202 , -1.2339064682943556e-07
[-1.5, -0.4]	1 , -0.4000003002460556	206 , -1.2271185187385738e-07
[-1.5, -0.5]	145 , -1.3161745867565867e-05	209 , -1.228490181943292e-07
[-1.5, -0.6]	148 , -1.2600322423135904e-05	211 , -1.2651956438673627e-07
[-1.5, -0.7]	150 , -1.2515478045913677e-05	213 , -1.2566764383372642e-07
[-1.5, -0.8]	151 , -1.2844719872314041e-05	214 , -1.2897355387755059e-07
[-1.5, -0.9]	149 , -1.2925666442189665e-05	212 , -1.2978633663146324e-07
[-1.5, -1.0]	1 , -1.0	1 , -1.0
[-1.5, -1.1]	9 , -1.0000000000300757	10 , -1.0
[-1.5, -1.2]	11 , -1.000000000345665	12 , -1.0000000000000022
[-1.5, -1.3]	13 , -1.0000000000288647	14 , -1.0
[-1.5, -1.4]	15 , -1.0000000000000167	15 , -1.0000000000000167

Tabela 2: Wyniki dla kryterium 1

	0.01	0.0001
0.3	12 , 0.08475669916682832	56 , 0.0008219486904814841
0.2	8 , 0.08609712349169579	52 , 0.000834947821600728
0.1	1 , 0.09000004999925001	45 , 0.000872797540867176
0.0	- , -	- , -
-0.1	2 , -0.08099992842712822	45 , -0.000872795172735189
-0.2	8 , -0.08608975894723488	52 , -0.0008348753307038678
-0.3	12 , -0.08470093552096244	56 , -0.0008214069357594659
-0.4	15 , -0.08224132694859262	58 , -0.0008861715296672138
-0.5	17 , -0.08302512528112482	60 , -0.000894617136871563
-0.6	18 , -0.08905531556392565	62 , -0.0008636343990309718
-0.7	20 , -0.08290827817358438	63 , -0.0008933580825082613
-0.8	21 , -0.08232443336397852	64 , -0.0008870670204558646
-0.9	19 , -0.08588432405196786	63 , -0.0008328830846807948
-1.0	1 , -1.0	1 , -1.0
-1.1	4 , -1.0001611737724945	6 , -1.0000000000011613
-1.2	6 , -1.0000226461422514	7 , -1.000000006152385
-1.3	7 , -1.0001155692216488	9 , -1.0000000000003073
-1.4	8 , -1.0002324713110928	10 , -1.0000000000050169
-1.5	9 , -1.0002874196526226	11 , -1.000000000011706

Tabela 3: Wyniki dla kryterium 1

	0.000001	0.00000001
0.3	99 , 8.856713858468886e-06	143 , 8.589008432188226e-08
0.2	95 , 8.99678292368562e-06	139 , 8.724843731991304e-08
0.1	89 , 8.464161265561398e-06	133 , 8.208321240026561e-08
0.0	- , -	- , -
-0.1	89 , -8.464138300039502e-06	133 , -8.208298968666736e-08
-0.2	95 , -8.99600181515869e-06	139 , -8.724086233461781e-08
-0.3	99 , -8.850876308497756e-06	143 , -8.5833473295799e-08
-0.4	102 , -8.59385869556889e-06	146 , -8.334098400468788e-08
-0.5	104 , -8.675761975557713e-06	148 , -8.41352604978538e-08
-0.6	106 , -8.375299523210722e-06	150 , -8.122145456711725e-08
-0.7	107 , -8.663551997098671e-06	151 , -8.401685133434527e-08
-0.8	108 , -8.60254292998989e-06	152 , -8.342520142873587e-08
-0.9	106 , -8.974534838975916e-06	150 , -8.70326812390175e-08
-1.0	1 , -1.0	1 , -1.0
-1.1	6 , -1.0000000000011613	7 , -1.0
-1.2	8 , -1.0000000000000004	8 , -1.0000000000000004
-1.3	9 , -1.0000000000003073	10 , -1.0
-1.4	10 , -1.0000000000050169	11 , -1.0
-1.5	11 , -1.000000000011706	12 , -1.0

Tabela 4: Wyniki dla kryterium 1

Obliczenia dla warunku 2.

	0.01	0.0001
-1.5	9 , -1.0002874196526226	10 , -1.0000009876775848
-1.4	8 , -1.0002324713110928	9 , -1.000000646585915
-1.3	7 , -1.0001155692216488	8 , -1.000000160037582
-1.2	5 , -1.0013859088559562	7 , -1.000000006152385
-1.1	4 , -1.0001611737724945	5 , -1.0000003110803821
-1.0	1 , -1.0	1 , -1.0
-0.9	1 , -0.5774524132499013	5 , -0.37580494501631845
-0.8	2 , -0.6174177115731144	7 , -0.36015709148461034
-0.7	1 , -0.6221346579399515	6 , -0.36272218473057516
-0.6	1 , -0.5373591740626698	4 , -0.38976043090696477
-0.5	1 , -0.4491803278688525	3 , -0.36323560303624336
-0.4	1 , -0.35979200519987004	1 , -0.35979200519987004
-0.3	1 , -0.2699634166537027	1 , -0.2699634166537027
-0.2	1 , -0.1799967984632624	1 , -0.1799967984632624
-0.1	1 , -0.08999994999925	1 , -0.08999994999925
0.0	- , -	- , -
0.1	1 , 0.09000004999925001	1 , 0.09000004999925001
0.2	1 , 0.18000319846473695	1 , 0.18000319846473695
0.3	1 , 0.2700363176222668	1 , 0.2700363176222668

Tabela 5: Wyniki dla kryterium 2

	0.000001	0.00000001
-1.5	11 , -1.000000000011706	11 , -1.000000000011706
-1.4	10 , -1.0000000000050169	10 , -1.0000000000050169
-1.3	8 , -1.000000160037582	9 , -1.0000000000003073
-1.2	7 , -1.000000006152385	8 , -1.0000000000000004
-1.1	6 , -1.0000000000011613	6 , -1.0000000000011613
-1.0	1 , -1.0	1 , -1.0
-0.9	9 , -0.24634401634621444	14 , -0.1454470491483192
-0.8	11 , -0.2361275739023406	15 , -0.15490974066747534
-0.7	10 , -0.2378030391537506	14 , -0.15600842545278332
-0.6	9 , -0.22988719320527118	13 , -0.15081744087385382
-0.5	7 , -0.23813836080231085	11 , -0.15622830979730745
-0.4	5 , -0.2358890854993795	9 , -0.1547533504367877
-0.3	2 , -0.24294767798588834	7 , -0.14344287504086967
-0.2	1 , -0.1799967984632624	3 , -0.1457949721622596
-0.1	1 , -0.08999994999925	1 , -0.08999994999925
0.0	- , -	- , -
0.1	1 , 0.09000004999925001	1 , 0.09000004999925001
0.2	1 , 0.18000319846473695	3 , 0.14580502481244703
0.3	2 , 0.24305203085834612	7 , 0.14353511314444486

Tabela 6: Wyniki dla kryterium 2

	0.01	0.0001
0.3	1 , 0.2700363176222668	1 , 0.2700363176222668
0.2	1 , 0.18000319846473695	1 , 0.18000319846473695
0.1	1 , 0.09000004999925001	1 , 0.09000004999925001
0.0	- , -	- , -
-0.1	1 , -0.08999994999925	1 , -0.08999994999925
-0.2	1 , -0.1799967984632624	1 , -0.1799967984632624
-0.3	1 , -0.2699634166537027	1 , -0.2699634166537027
-0.4	1 , -0.35979200519987004	1 , -0.35979200519987004
-0.5	1 , -0.4491803278688525	3 , -0.36323560303624336
-0.6	1 , -0.5373591740626698	4 , -0.38976043090696477
-0.7	1 , -0.6221346579399515	6 , -0.36272218473057516
-0.8	2 , -0.6174177115731144	7 , -0.36015709148461034
-0.9	1 , -0.5774524132499013	5 , -0.37580494501631845
-1.0	1 , -1.0	1 , -1.0
-1.1	4 , -1.0001611737724945	5 , -1.0000003110803821
-1.2	5 , -1.0013859088559562	7 , -1.000000006152385
-1.3	7 , -1.0001155692216488	8 , -1.000000160037582
-1.4	8 , -1.0002324713110928	9 , -1.000000646585915
-1.5	9 , -1.0002874196526226	10 , -1.0000009876775848

Tabela 7: Wyniki dla kryterium 2

	0.000001	0.00000001
0.3	2 , 0.24305203085834612	7 , 0.14353511314444486
0.2	1 , 0.18000319846473695	3 , 0.14580502481244703
0.1	1 , 0.09000004999925001	1 , 0.09000004999925001
0.0	- , -	- , -
-0.1	1 , -0.08999994999925	1 , -0.08999994999925
-0.2	1 , -0.1799967984632624	3 , -0.1457949721622596
-0.3	2 , -0.24294767798588834	7 , -0.14344287504086967
-0.4	5 , -0.2358890854993795	9 , -0.1547533504367877
-0.5	7 , -0.23813836080231085	11 , -0.15622830979730745
-0.6	9 , -0.22988719320527118	13 , -0.15081744087385382
-0.7	10 , -0.2378030391537506	14 , -0.15600842545278332
-0.8	11 , -0.2361275739023406	15 , -0.15490974066747534
-0.9	9 , -0.24634401634621444	14 , -0.1454470491483192
-1.0	1 , -1.0	1 , -1.0
-1.1	6 , -1.0000000000011613	6 , -1.0000000000011613
-1.2	7 , -1.000000006152385	8 , -1.0000000000000004
-1.3	8 , -1.000000160037582	9 , -1.0000000000003073
-1.4	10 , -1.0000000000050169	10 , -1.0000000000050169
-1.5	11 , -1.000000000011706	11 , -1.000000000011706

Tabela 8: Wyniki dla kryterium 2

1.2.2 Metoda Siecznych

Żeby zastosować metodę siecznych muszą być spełnione warunki (jednak na potrzebny zadania je ignorujemy):

1. funkcja jest ciągła,
2. funkcja ma różne znaki na krańcach przedziału,

Poniżej znajdują się tabele z wynikami oraz liczbami iteracji dla różnych punktów startowych, dokładności i kryteriów stopu (kolumny - dokładność warunku stopu, wiersze - przedział startowy, pierwsza wartość to liczba iteracji, druga to wyliczona wartość):

	0.01	0.0001
[-1.5, 0.3]	1 , 0.2999999719783287	1 , 0.2999999719783287
[-1.5, 0.2]	1 , 0.19999999954202397	1 , 0.19999999954202397
[-1.5, 0.1]	1 , 0.0999999999995792	1 , 0.0999999999995792
[-1.5, 0.0]	1 , 0.0	1 , 0.0
[-1.5, -0.1]	1 , -0.1000000000003682	1 , -0.1000000000003682
[-1.5, -0.2]	1 , -0.2000000003499929	1 , -0.2000000003499929
[-1.5, -0.3]	1 , -0.30000001859054404	1 , -0.30000001859054404
[-1.5, -0.4]	1 , -0.4000003002460556	1 , -0.4000003002460556
[-1.5, -0.5]	1 , -0.5000024880872789	1 , -0.5000024880872789
[-1.5, -0.6]	1 , -0.6000131991976202	1 , -0.6000131991976202
[-1.5, -0.7]	1 , -0.7000494407743927	1 , -0.7000494407743927
[-1.5, -0.8]	1 , -0.8001328759633891	88 , -0.0012792299168158175
[-1.5, -0.9]	1 , -0.9002252333668005	86 , -0.0012872915386244574
[-1.5, -1.0]	1 , -1.0	1 , -1.0
[-1.5, -1.1]	1 , -1.0983271885127561	8 , -1.0000001212313034
[-1.5, -1.2]	1 , -1.192548559411928	10 , -1.000000549797303
[-1.5, -1.3]	9 , -1.0035519725131916	12 , -1.000000118555283
[-1.5, -1.4]	11 , -1.001177702285624	13 , -1.0000011733648262

Tabela 9: Wyniki dla kryterium 1

	0.000001	0.00000001
[-1.5, 0.3]	1 , 0.2999999719783287	202 , 1.2347184284733514e-07
[-1.5, 0.2]	1 , 0.19999999954202397	1 , 0.19999999954202397
[-1.5, 0.1]	1 , 0.0999999999995792	1 , 0.0999999999995792
[-1.5, 0.0]	1 , 0.0	1 , 0.0
[-1.5, -0.1]	1 , -0.1000000000003682	1 , -0.1000000000003682
[-1.5, -0.2]	1 , -0.2000000003499929	1 , -0.2000000003499929
[-1.5, -0.3]	1 , -0.30000001859054404	202 , -1.2339064682943556e-07
[-1.5, -0.4]	1 , -0.4000003002460556	206 , -1.2271185187385738e-07
[-1.5, -0.5]	145 , -1.3161745867565867e-05	209 , -1.228490181943292e-07
[-1.5, -0.6]	148 , -1.2600322423135904e-05	211 , -1.2651956438673627e-07
[-1.5, -0.7]	150 , -1.2515478045913677e-05	213 , -1.2566764383372642e-07
[-1.5, -0.8]	151 , -1.2844719872314041e-05	214 , -1.2897355387755059e-07
[-1.5, -0.9]	149 , -1.2925666442189665e-05	212 , -1.2978633663146324e-07
[-1.5, -1.0]	1 , -1.0	1 , -1.0
[-1.5, -1.1]	9 , -1.0000000000300757	10 , -1.0
[-1.5, -1.2]	11 , -1.000000000345665	12 , -1.0000000000000022
[-1.5, -1.3]	13 , -1.0000000000288647	14 , -1.0
[-1.5, -1.4]	15 , -1.0000000000000167	15 , -1.0000000000000167

Tabela 10: Wyniki dla kryterium 1

	0.01	0.0001
[-1.5, 0.3]	1 , 0.2999999719783287	1 , 0.2999999719783287
[-1.4, 0.3]	1 , 0.2999999205423745	1 , 0.2999999205423745
[-1.3, 0.3]	1 , 0.29999974677043706	1 , 0.29999974677043706
[-1.2, 0.3]	1 , 0.2999990365064661	1 , 0.2999990365064661
[-1.1, 0.3]	1 , 0.2999947667271144	1 , 0.2999947667271144
[-1.0, 0.3]	2 , -1.0	2 , -1.0
[-0.9, 0.3]	1 , 0.3000497480711615	1 , 0.3000497480711615
[-0.8, 0.3]	1 , 0.3000902025881659	1 , 0.3000902025881659
[-0.7, 0.3]	1 , 0.30025194678580824	75 , 0.0013181345683313255
[-0.6, 0.3]	1 , 0.30095634345809513	76 , 0.0012270729075232
[-0.5, 0.3]	1 , 0.3050369852262945	76 , 0.001237122001267884
[-0.4, 0.3]	13 , 0.130589916926152	76 , 0.0013112562952161217
[-0.3, 0.3]	3 , 0.29999999999998295	3 , 0.29999999999998295
[-0.2, 0.3]	4 , -0.18396575136485996	72 , -0.0012735023351856303
[-0.1, 0.3]	2 , -0.10001352000369917	2 , -0.10001352000369917
[0.0, 0.3]	2 , 0.0	2 , 0.0
[0.1, 0.3]	2 , 0.09999324328861631	2 , 0.09999324328861631
[0.2, 0.3]	2 , 0.19660117605336058	71 , 0.0012474157991641647

Tabela 11: Wyniki dla kryterium 1

	0.000001	0.00000001
[-1.5, 0.3]	1 , 0.2999999719783287	202 , 1.2347184284733514e-07
[-1.4, 0.3]	1 , 0.2999999205423745	202 , 1.2347182971148565e-07
[-1.3, 0.3]	1 , 0.29999974677043706	202 , 1.234717853290779e-07
[-1.2, 0.3]	1 , 0.2999990365064661	202 , 1.2347160392290603e-07
[-1.1, 0.3]	139 , 1.229666167208727e-05	202 , 1.234705133661465e-07
[-1.0, 0.3]	2 , -1.0	2 , -1.0
[-0.9, 0.3]	139 , 1.229805990576891e-05	202 , 1.2348455300024245e-07
[-0.8, 0.3]	139 , 1.2299088250296855e-05	202 , 1.234948785853621e-07
[-0.7, 0.3]	139 , 1.2303195882740216e-05	202 , 1.2353612323370862e-07
[-0.6, 0.3]	139 , 1.2321012472311298e-05	202 , 1.2371501922348515e-07
[-0.5, 0.3]	139 , 1.2421915204825858e-05	202 , 1.2472818137398164e-07
[-0.4, 0.3]	139 , 1.3166296043781797e-05	203 , 1.22891488599575e-07
[-0.3, 0.3]	3 , 0.29999999999998295	3 , 0.29999999999998295
[-0.2, 0.3]	135 , -1.2787209349288837e-05	198 , -1.2839609196217587e-07
[-0.1, 0.3]	124 , -1.3184402473145165e-05	188 , -1.230604902725048e-07
[0.0, 0.3]	2 , 0.0	2 , 0.0
[0.1, 0.3]	124 , 1.3182273716086579e-05	188 , 1.2304062089368122e-07
[0.2, 0.3]	134 , 1.252527500642357e-05	197 , 1.2576601490189444e-07

Tabela 12: Wyniki dla kryterium 1

Obliczenia dla warunku 2.

	0.01	0.0001
[-1.5, 0.3]	0 , 0.3	0 , 0.3
[-1.5, 0.2]	0 , 0.2	0 , 0.2
[-1.5, 0.1]	0 , 0.1	0 , 0.1
[-1.5, 0.0]	0 , 0.0	0 , 0.0
[-1.5, -0.1]	0 , -0.1	0 , -0.1
[-1.5, -0.2]	0 , -0.2	0 , -0.2
[-1.5, -0.3]	0 , -0.3	0 , -0.3
[-1.5, -0.4]	0 , -0.4	2 , -0.35979213983546243
[-1.5, -0.5]	0 , -0.5	4 , -0.39040130801355694
[-1.5, -0.6]	0 , -0.6	7 , -0.37416450906711657
[-1.5, -0.7]	2 , -0.6221547686232306	9 , -0.3716091577181563
[-1.5, -0.8]	4 , -0.5960998703447792	10 , -0.3814304816183517
[-1.5, -0.9]	2 , -0.5762442355591834	8 , -0.3837915650717631
[-1.5, -1.0]	0 , -1.0	0 , -1.0
[-1.5, -1.1]	6 , -1.000490206554177	8 , -1.0000001212313034
[-1.5, -1.2]	8 , -1.000879416840034	10 , -1.000000549797303
[-1.5, -1.3]	10 , -1.000488466044324	12 , -1.000000118555283
[-1.5, -1.4]	11 , -1.001177702285624	13 , -1.0000011733648262

Tabela 13: Wyniki dla kryterium 2

	0.000001	0.00000001
[-1.5, 0.3]	4 , 0.23501985880182222	10 , 0.1518402028750265
[-1.5, 0.2]	0 , 0.2	4 , 0.1566484373406491
[-1.5, 0.1]	0 , 0.1	0 , 0.1
[-1.5, 0.0]	0 , 0.0	0 , 0.0
[-1.5, -0.1]	0 , -0.1	0 , -0.1
[-1.5, -0.2]	0 , -0.2	4 , -0.15663886236268393
[-1.5, -0.3]	4 , -0.23491080994718538	10 , -0.15174366116087418
[-1.5, -0.4]	8 , -0.23390946353709313	14 , -0.15090947305716007
[-1.5, -0.5]	11 , -0.23417810827818566	17 , -0.15107818251905067
[-1.5, -0.6]	13 , -0.2411783165995013	19 , -0.15559243636836842
[-1.5, -0.7]	15 , -0.23955341019879936	21 , -0.15454468991624457
[-1.5, -0.8]	16 , -0.24585891973154145	23 , -0.1474389924878853
[-1.5, -0.9]	14 , -0.247409148038453	21 , -0.14836818794539236
[-1.5, -1.0]	0 , -1.0	0 , -1.0
[-1.5, -1.1]	8 , -1.0000001212313034	9 , -1.0000000000300757
[-1.5, -1.2]	11 , -1.000000000345665	11 , -1.000000000345665
[-1.5, -1.3]	12 , -1.000000118555283	13 , -1.0000000000288647
[-1.5, -1.4]	14 , -1.0000000011778807	14 , -1.0000000011778807

Tabela 14: Wyniki dla kryterium 2

	0.01	0.0001
[-1.5, 0.3]	0 , 0.3	0 , 0.3
[-1.4, 0.3]	0 , 0.3	0 , 0.3
[-1.3, 0.3]	0 , 0.3	0 , 0.3
[-1.2, 0.3]	0 , 0.3	0 , 0.3
[-1.1, 0.3]	0 , 0.3	0 , 0.3
[-1.0, 0.3]	0 , 0.3	0 , 0.3
[-0.9, 0.3]	0 , 0.3	0 , 0.3
[-0.8, 0.3]	0 , 0.3	0 , 0.3
[-0.7, 0.3]	0 , 0.3	0 , 0.3
[-0.6, 0.3]	0 , 0.3	0 , 0.3
[-0.5, 0.3]	0 , 0.3	0 , 0.3
[-0.4, 0.3]	0 , 0.3	0 , 0.3
[-0.3, 0.3]	0 , 0.3	0 , 0.3
[-0.2, 0.3]	0 , 0.3	0 , 0.3
[-0.1, 0.3]	0 , 0.3	0 , 0.3
[0.0, 0.3]	0 , 0.3	0 , 0.3
[0.1, 0.3]	0 , 0.3	0 , 0.3
[0.2, 0.3]	0 , 0.3	0 , 0.3

Tabela 15: Wyniki dla kryterium 2

	0.000001	0.00000001
[-1.5, 0.3]	4 , 0.23501985880182222	10 , 0.1518402028750265
[-1.4, 0.3]	4 , 0.23501983456326772	10 , 0.15184018672364935
[-1.3, 0.3]	4 , 0.23501975266727598	10 , 0.1518401321526053
[-1.2, 0.3]	4 , 0.23501941793015638	10 , 0.15183990910197134
[-1.1, 0.3]	4 , 0.23501740559353826	10 , 0.1518385681918165
[-1.0, 0.3]	1 , -1.0	1 , -1.0
[-0.9, 0.3]	4 , 0.23504331175527898	10 , 0.15185583083011273
[-0.8, 0.3]	4 , 0.23506236427442123	10 , 0.15186852680361118
[-0.7, 0.3]	4 , 0.23513846435458302	10 , 0.15191923975013016
[-0.6, 0.3]	4 , 0.23546847770243468	10 , 0.15213920366035943
[-0.5, 0.3]	4 , 0.2373354293059705	10 , 0.15338494394360608
[-0.4, 0.3]	4 , 0.2509878244141485	11 , 0.15112793875644817
[-0.3, 0.3]	- , -	- , -
[-0.2, 0.3]	1 , -0.20879915027668156	6 , -0.15805545446787816
[-0.1, 0.3]	1 , -0.10000675766070799	1 , -0.10000675766070799
[0.0, 0.3]	1 , 0.0	1 , 0.0
[0.1, 0.3]	1 , 0.09999662110206764	1 , 0.09999662110206764
[0.2, 0.3]	1 , 0.1982390234528264	5 , 0.15448866784507018

Tabela 16: Wyniki dla kryterium 2

2 Problem 2

2.1 Opis problemu

Główną ideą problemu jest rozwiązanie układu równań metodą Newtona.

$$\begin{cases} x_1^2 + x_2^2 + x_3 = 1 \\ 2x_1^2 + x_2^2 + x_3^3 = 2 \\ 3x_1 - 2x_2^3 - 2x_3^2 = 3 \end{cases}$$

Wektory spełniające układ równań:

$$\left[1, -1, -1\right], \left[1, 0, 0\right], \left[0.953156, -0.428689, -0.0922802\right]$$

Eksperymenty zostaną przeprowadzone dla różnych wektorów początkowych.

1. $\left\|X_{(i+1)} - X_{(i)}\right\| < \rho$
2. $\|F(X_i)\| < \rho$

2.2 Opracowanie

Niech

$$F(X) = \begin{bmatrix} f_1(X) \\ f_2(X) \\ f_3(X) \end{bmatrix} = \begin{bmatrix} x_1^2 + x_2^2 + x_3 = 1 \\ 2x_1^2 + x_2^2 + x_3^3 = 2 \\ 3x_1 - 2x_2^3 - 2x_3^2 = 3 \end{bmatrix}$$

Metoda Newtona dla układów równań jest analogiczna jak dla równania nieliniowego z tą różnicą, że zamiast pochodnej używany jest jakobian macierzy, w tym przypadku:

$$J(X) = \begin{bmatrix} 2x_1 & 2x_2 & 1 \\ 4x_1 & 2x_2 & 3x_3^2 \\ 3 & -6x_2^2 & -4x_3 \end{bmatrix}$$

Wówczas

$$X_{k+1} = X_k - \frac{F(X_k)}{J(X_k)}$$

Czyli

$$X_{k+1} = X_k - J(X_k)^{-1} F(X_k)$$

Gdzie $J(X_k)^{-1}F(X_k)$ jest rozwiązaniem układu równań $J(X_k)S = F(X_k)$, więc

$$X_{k+1} = X_k - S$$

Wektor początkowy	L. iteracji, Wynik
[-1.0, -1.0, -1.0]	106 , [1. -1. -1.]
[-1.0, -1.0, -0.3]	254 , [1. -1. -1.]
[-1.0, -1.0, 0.3]	215 , [1. -1. -1.]
[-1.0, -1.0, 1.0]	24 , [1. -1. -1.]
[-1.0, -0.3, -1.0]	8 , [1. -1. -1.]
[-1.0, -0.3, -0.3]	342 , [1. -1. -1.]
[-1.0, -0.3, 0.3]	9 , [1. -1. -1.]
[-1.0, -0.3, 1.0]	353 , [1. -1. -1.]
[-1.0, 0.3, -1.0]	31 , [1. -1. -1.]
[-1.0, 0.3, -0.3]	129 , [1. -1. -1.]
[-1.0, 0.3, 0.3]	80 , [1. -1. -1.]
[-1.0, 0.3, 1.0]	176 , [1. -1. -1.]
[-1.0, 1.0, -1.0]	15 , [1. -1. -1.]
[-1.0, 1.0, -0.3]	29 , [1.00000000e+00 8.73838008e-07 2.38725030e-16]
[-1.0, 1.0, 0.3]	58 , [1. -1. -1.]
[-1.0, 1.0, 1.0]	126 , [1. -1. -1.]
[-0.3, -1.0, -1.0]	71 , [1. -1. -1.]
[-0.3, -1.0, -0.3]	358 , [1. -1. -1.]
[-0.3, -1.0, 0.3]	118 , [1. -1. -1.]
[-0.3, -1.0, 1.0]	38 , [1.00000000e+00 8.48959093e-07 -1.02803848e-17]
[-0.3, -0.3, -1.0]	93 , [1. -1. -1.]
[-0.3, -0.3, -0.3]	32 , [1. -1. -1.]
[-0.3, -0.3, 0.3]	9 , [1. -1. -1.]
[-0.3, -0.3, 1.0]	37 , [1. -1. -1.]
[-0.3, 0.3, -1.0]	148 , [1. -1. -1.]
[-0.3, 0.3, -0.3]	20 , [1. -1. -1.]
[-0.3, 0.3, 0.3]	69 , [1. -1. -1.]
[-0.3, 0.3, 1.0]	81 , [1. -1. -1.]
[-0.3, 1.0, -1.0]	24 , [1.00000000e+00 8.52286434e-07 2.22404201e-16]
[-0.3, 1.0, -0.3]	165 , [1.00000000e+00 5.36569338e-07 -1.99809146e-16]
[-0.3, 1.0, 0.3]	101 , [1. -1. -1.]
[-0.3, 1.0, 1.0]	165 , [1. -1. -1.]

Wektor początkowy	L. iteracji, Wynik
[0.3, -1.0, -1.0]	21 , [1. -1. -1.]
[0.3, -1.0, -0.3]	147 , [1. -1. -1.]
[0.3, -1.0, 0.3]	63 , [1. -1. -1.]
[0.3, -1.0, 1.0]	396 , [1.00000000e+00 6.40652918e-07 2.04896485e-16]
[0.3, -0.3, -1.0]	220 , [1.00000000e+00 6.03236699e-07 1.06891993e-17]
[0.3, -0.3, -0.3]	11 , [1. -1. -1.]
[0.3, -0.3, 0.3]	34 , [1. -1. -1.]
[0.3, -0.3, 1.0]	14 , [1. -1. -1.]
[0.3, 0.3, -1.0]	130 , [1. -1. -1.]
[0.3, 0.3, -0.3]	24 , [1.00000000e+00 5.42508179e-07 2.41224404e-16]
[0.3, 0.3, 0.3]	123 , [1. -1. -1.]
[0.3, 0.3, 1.0]	177 , [1. -1. -1.]
[0.3, 1.0, -1.0]	134 , [1. -1. -1.]
[0.3, 1.0, -0.3]	23 , [1.00000000e+00 5.74285178e-07 1.94318265e-16]
[0.3, 1.0, 0.3]	23 , [1.00000000e+00 7.46004508e-07 -1.00047675e-16]
[0.3, 1.0, 1.0]	736 , [1.00000000e+00 7.73411964e-07 -4.55281571e-18]
[1.0, -1.0, -1.0]	1 , [1. -1. -1.]
[1.0, -1.0, -0.3]	7 , [0.9531556 -0.42868942 -0.09228022]
[1.0, -1.0, 0.3]	7 , [0.9531556 -0.42868942 -0.09228022]
[1.0, -1.0, 1.0]	24 , [1.00000000e+00 5.85365451e-07 -3.00940094e-16]
[1.0, -0.3, -1.0]	186 , [1. -1. -1.]
[1.0, -0.3, -0.3]	7 , [0.9531556 -0.42868942 -0.09228022]
[1.0, -0.3, 0.3]	21 , [1.00000000e+00 6.07994622e-07 9.44864089e-17]
[1.0, -0.3, 1.0]	113 , [1. -1. -1.]
[1.0, 0.3, -1.0]	34 , [1. -1. -1.]
[1.0, 0.3, -0.3]	20 , [1.00000000e+00 9.66979817e-07 -1.27336830e-16]
[1.0, 0.3, 0.3]	22 , [1.00000000e+00 8.95178294e-07 -2.26539514e-19]
[1.0, 0.3, 1.0]	49 , [1. -1. -1.]
[1.0, 1.0, -1.0]	243 , [1. -1. -1.]
[1.0, 1.0, -0.3]	22 , [1.00000000e+00 7.15529230e-07 -2.92212534e-19]
[1.0, 1.0, 0.3]	22 , [1.00000000e+00 8.92027679e-07 2.76139201e-16]
[1.0, 1.0, 1.0]	41 , [1. -1. -1.]

Tabela 17: Wyniki dla kryterium 1

Wektor początkowy	L. iteracji, Wynik
[-1.0, -1.0, -1.0]	105 , [1. -1. -1.]
[-1.0, -1.0, -0.3]	253 , [1. -1. -1.]
[-1.0, -1.0, 0.3]	214 , [1. -1. -1.]
[-1.0, -1.0, 1.0]	23 , [1. -1. -1.]
[-1.0, -0.3, -1.0]	7 , [1. -1. -1.]
[-1.0, -0.3, -0.3]	341 , [1. -1. -1.]
[-1.0, -0.3, 0.3]	8 , [1. -1. -1.]
[-1.0, -0.3, 1.0]	352 , [1. -1. -1.]
[-1.0, 0.3, -1.0]	30 , [1. -1. -1.]
[-1.0, 0.3, -0.3]	128 , [1. -1. -1.]
[-1.0, 0.3, 0.3]	79 , [1. -1. -1.]
[-1.0, 0.3, 1.0]	175 , [1. -1. -1.]
[-1.0, 1.0, -1.0]	14 , [1. -1. -1.]
[-1.0, 1.0, -0.3]	20 , [1.00000000e+00 8.73838008e-07 2.38725030e-16]
[-1.0, 1.0, 0.3]	57 , [1. -1. -1.]
[-1.0, 1.0, 1.0]	125 , [1. -1. -1.]
[-0.3, -1.0, -1.0]	70 , [1. -1. -1.]
[-0.3, -1.0, -0.3]	357 , [1. -1. -1.]
[-0.3, -1.0, 0.3]	117 , [1. -1. -1.]
[-0.3, -1.0, 1.0]	29 , [1.00000000e+00 8.48959093e-07 -1.02803848e-17]
[-0.3, -0.3, -1.0]	92 , [1. -1. -1.]
[-0.3, -0.3, -0.3]	31 , [1. -1. -1.]
[-0.3, -0.3, 0.3]	8 , [1. -1. -1.]
[-0.3, -0.3, 1.0]	36 , [1. -1. -1.]
[-0.3, 0.3, -1.0]	147 , [1. -1. -1.]
[-0.3, 0.3, -0.3]	19 , [1. -1. -1.]
[-0.3, 0.3, 0.3]	69 , [1. -1. -1.]
[-0.3, 0.3, 1.0]	80 , [1. -1. -1.]
[-0.3, 1.0, -1.0]	15 , [1.00000000e+00 8.52286434e-07 2.22404201e-16]
[-0.3, 1.0, -0.3]	155 , [1.00000000e+00 5.36569338e-07 -1.99809146e-16]
[-0.3, 1.0, 0.3]	100 , [1. -1. -1.]
[-0.3, 1.0, 1.0]	164 , [1. -1. -1.]

[0.3, -1.0, -1.0]	20 , [1. -1. -1.]
[0.3, -1.0, -0.3]	146 , [1. -1. -1.]
[0.3, -1.0, 0.3]	62 , [1. -1. -1.]
[0.3, -1.0, 1.0]	386 , [1.00000000e+00 6.40652918e-07 2.04896485e-16]
[0.3, -0.3, -1.0]	210 , [1.00000000e+00 6.03236699e-07 1.06891993e-17]
[0.3, -0.3, -0.3]	11 , [1. -1. -1.]
[0.3, -0.3, 0.3]	33 , [1. -1. -1.]
[0.3, -0.3, 1.0]	13 , [1. -1. -1.]
[0.3, 0.3, -1.0]	129 , [1. -1. -1.]
[0.3, 0.3, -0.3]	14 , [1.00000000e+00 5.42508179e-07 2.41224404e-16]
[0.3, 0.3, 0.3]	123 , [1. -1. -1.]
[0.3, 0.3, 1.0]	176 , [1. -1. -1.]
[0.3, 1.0, -1.0]	133 , [1. -1. -1.]
[0.3, 1.0, -0.3]	13 , [1.00000000e+00 5.74285178e-07 1.94318265e-16]
[0.3, 1.0, 0.3]	13 , [1.00000000e+00 7.46004508e-07 -1.00047675e-16]
[0.3, 1.0, 1.0]	726 , [1.00000000e+00 7.73411964e-07 -4.55281571e-18]
[1.0, -1.0, -1.0]	1 , [1. -1. -1.]
[1.0, -1.0, -0.3]	6 , [0.9531556 -0.42868942 -0.09228022]
[1.0, -1.0, 0.3]	6 , [0.9531556 -0.42868942 -0.09228022]
[1.0, -1.0, 1.0]	14 , [1.00000000e+00 5.85365451e-07 -3.00940094e-16]
[1.0, -0.3, -1.0]	185 , [1. -1. -1.]
[1.0, -0.3, -0.3]	6 , [0.9531556 -0.42868942 -0.09228022]
[1.0, -0.3, 0.3]	11 , [1.00000000e+00 6.07994622e-07 9.44864089e-17]
[1.0, -0.3, 1.0]	112 , [1. -1. -1.]
[1.0, 0.3, -1.0]	33 , [1. -1. -1.]
[1.0, 0.3, -0.3]	11 , [1.00000000e+00 9.66979817e-07 -1.27336830e-16]
[1.0, 0.3, 0.3]	13 , [1.00000000e+00 8.95178294e-07 -2.26539514e-19]
[1.0, 0.3, 1.0]	48 , [1. -1. -1.]
[1.0, 1.0, -1.0]	242 , [1. -1. -1.]
[1.0, 1.0, -0.3]	12 , [1.00000000e+00 7.15529230e-07 -2.92212534e-19]
[1.0, 1.0, 0.3]	13 , [1.00000000e+00 8.92027679e-07 2.76139201e-16]
[1.0, 1.0, 1.0]	40 , [1. -1. -1.]

Tabela 18: Wyniki dla kryterium 2

2.3 Wnioski

Zostały znalezione wszystkie rozwiązania układu równań. Niedokładność $= 0.000001$ jest w pełni wystarczający dla w.w wektorów początkowych niezależnie od kryterium stopu.