

New Distance Formulas

April 13, 2025

1 Document Analysis

Total documents processed: 2

- *ratio_defibosuite_defibo* Total unique terms: 378
 - Term 0: 0
 - Term 1: 000095604
 - Term 2: 000101584
 - Term 3: 000197188
 - Term 4: 0002987
 - Term 5: 000489623
 - Term 6: 0004957
 - Term 7: 0007944
 - Term 8: 00129
 - Term 9: 001373265
 - ... (and 368 more terms)

2 Available Metrics

- **Euclidean** (Vector Space, Implemented): Straight-line distance (L2 norm)
- **Manhattan** (Vector Space, Implemented): Sum of absolute differences (L1 norm)
- **Chebyshev** (Vector Space, Implemented): Maximum of absolute differences (L norm)
- **Minkowski** (Vector Space, Implemented): Generalized distance with $p=3$

- **Mahalanobis** (Vector Space, Implemented): Distance accounting for covariance
- **Cosine** (Vector Space, Implemented): Cosine of the angle between vectors
- **Hamming** (String Space, Implemented): Number of differing positions
- **Levenshtein** (String Space, Implemented): Minimum edit distance
- **KL Divergence** (Probability Space, Implemented): Divergence between distributions
- **Jensen-Shannon** (Probability Space, Implemented): Symmetrized KL divergence
- **Bhattacharyya** (Probability Space, Implemented): Similarity between distributions
- **Hellinger** (Probability Space, Implemented): Bounded similarity between distributions
- **Wasserstein** (Probability Space, Implemented): Optimal transport distance
- **Alcubierre** (Vector Space, Implemented): Warped distance inspired by Alcubierre metric
- **Persistent Homology** (Topological Space, Implemented): Topological distance using persistent homology
- **Neural Network** (Vector Space, Implemented): Learned distance using a neural network
- **Fibonacci** (Sequence Space, Implemented): Distance based on Fibonacci progression

3 Selected Metrics

- Fibonacci

4 Analysis Steps

4.1 New Distance Formulas for Feature j=0, Config 1

4.2 Final Equation for Feature j=0, Config 1

Final threshold equation: $x_0 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$ Evaluated
threshold: $x_0 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$, Result: $[TrueFalse]$

4.3 Properties of the Main Formula for Feature j=0, Config 1

4.3.1 Document $\text{ratio}_{de\text{fib}\text{o}}$

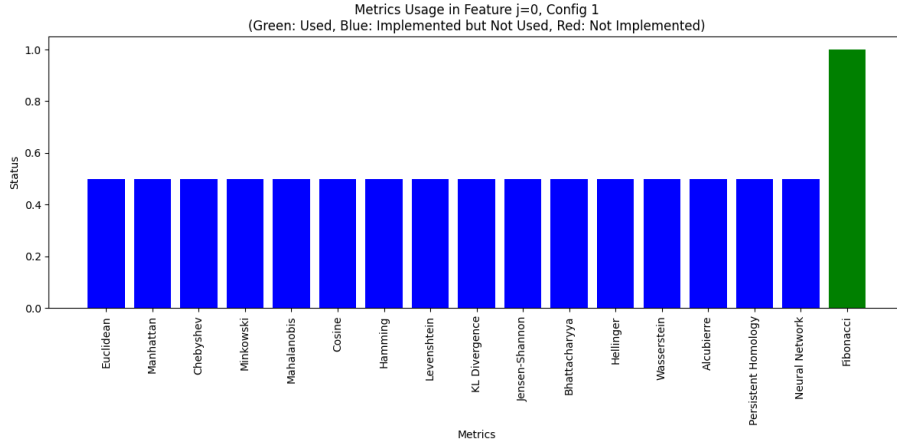
- **Norms for x_0 :**
 - L_1 Norm: 19.3907
 - L_2 Norm: 19.3907
 - L_∞ Norm: 19.3907
- **Norms for $x_0 - \theta$:**
 - L_1 Norm: 19.3907
 - L_2 Norm: 19.3907
 - L_∞ Norm: 19.3907
- **Inner Product:** $\langle x_0, \theta \rangle = 0.0000$
- **Induced Norm of x_0 :** $\sqrt{\langle x_0, x_0 \rangle} = 19.3907$
- **Induced Metric:** $\sqrt{\langle x_0 - \theta, x_0 - \theta \rangle} = 19.3907$
- **Transformations and Embeddings:** PCA with 2 dimensions
- **Scale and Units:** Features standardized using StandardScaler (mean=0, std=1)
- **Underlying Geometry/Topology:**
 - Persistent Homology: Number of H0 components: 2, H1 components: 0
 - Geometry Note: Data embedded in a vector space with Euclidean geometry, transformed using PCA

4.3.2 Document $\text{suite}_{de\text{fib}\text{o}}$

- **Norms for x_0 :**
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 - L_2 Norm: 19.3907
 - L_∞ Norm: 19.3907
- **Norms for $x_0 - \theta$:**
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- **Inner Product:** $\langle x_0, \theta \rangle = -0.0000$
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 - Geometry Note: Data embedded in a vector space with Euclidean geometry, transformed using PCA

4.4 Metrics Usage for Feature j=0, Config 1



4.5 New Distance Formulas for Feature j=0, Config 2

4.6 Final Equation for Feature j=0, Config 2

Final threshold equation: $x_0 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$ Evaluated threshold: $x_0 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$, Result: $[TrueFalse]$

4.7 Properties of the Main Formula for Feature j=0, Config 2

4.7.1 Document $\text{ratio}_{de\text{fibonacci}}$

- Norms for x_0 :

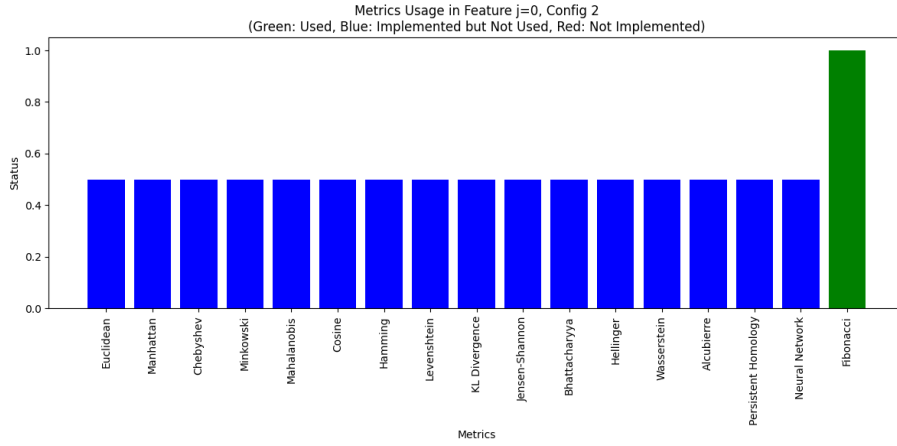
- L_1 Norm: 19.3907
- L_2 Norm: 19.3907
- L_∞ Norm: 19.3907
- **Norms for $x_0 - \theta$:**
 - L_1 Norm: 19.3907
 - L_2 Norm: 19.3907
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- **Inner Product:** $\langle x_0, \theta \rangle = 0.0000$
- **Induced Norm of x_0 :** $\sqrt{\langle x_0, x_0 \rangle} = 19.3907$
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4.7.2 Document suite_{defibo}

- **Norms for x_0 :**
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- **Scale and Units:** Features standardized using StandardScaler (mean=0, std=1)
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 - Geometry Note: Data embedded in a vector space with Euclidean geometry, transformed using PCA

4.8 Metrics Usage for Feature j=0, Config 2



4.9 New Distance Formulas for Feature j=0, Config 3

4.10 Final Equation for Feature j=0, Config 3

Final threshold equation: $x_0 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$ Evaluated threshold: $x_0 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$, Result: $[TrueFalse]$

4.11 Properties of the Main Formula for Feature j=0, Config 3

4.11.1 Document ratio_{de $fibo$}

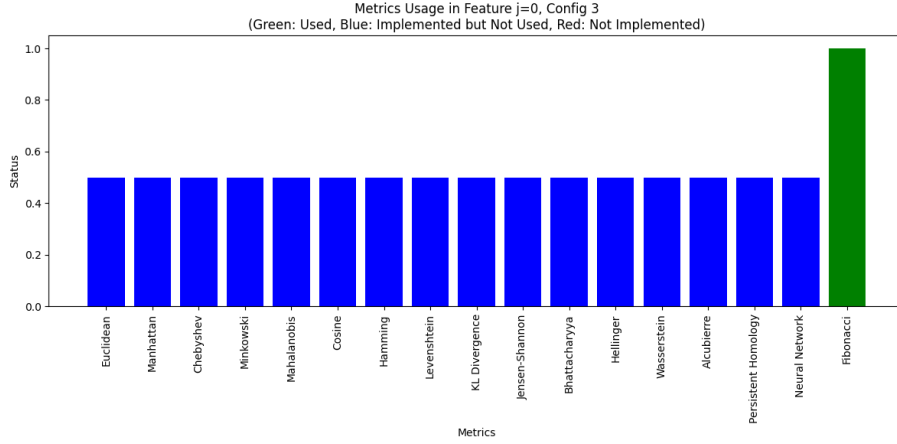
- **Norms for x_0 :**
 - L_1 Norm: 19.3907
 - L_2 Norm: 19.3907
 - L_∞ Norm: 19.3907
- **Norms for $x_0 - \theta$:**

- L_1 Norm: 19.3907
- L_2 Norm: 19.3907
- L_∞ Norm: 19.3907
- **Inner Product:** $\langle x_0, \theta \rangle = 0.0000$
- **Induced Norm of x_0 :** $\sqrt{\langle x_0, x_0 \rangle} = 19.3907$
- **Induced Metric:** $\sqrt{\langle x_0 - \theta, x_0 - \theta \rangle} = 19.3907$
- **Transformations and Embeddings:** PCA with 2 dimensions
- **Scale and Units:** Features standardized using StandardScaler (mean=0, std=1)
- **Underlying Geometry/Topology:**
 - Persistent Homology: Number of H0 components: 2, H1 components: 0
 - Geometry Note: Data embedded in a vector space with Euclidean geometry, transformed using PCA

4.11.2 Document suite_{defibo}

- **Norms for x_0 :**
 - L_1 Norm: 19.3907
 - L_2 Norm: 19.3907
 - L_∞ Norm: 19.3907
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 - L_1 Norm: 19.3907
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- **Transformations and Embeddings:** PCA with 2 dimensions
- **Scale and Units:** Features standardized using StandardScaler (mean=0, std=1)
- **Underlying Geometry/Topology:**
 - Persistent Homology: Number of H0 components: 2, H1 components: 0
 - Geometry Note: Data embedded in a vector space with Euclidean geometry, transformed using PCA

4.12 Metrics Usage for Feature j=0, Config 3



4.13 New Distance Formulas for Feature j=1, Config 1

4.14 Final Equation for Feature j=1, Config 1

Final threshold equation: $x_1 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$ Evaluated
 threshold: $x_1 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$, Result: $[FalseFalse]$

4.15 Properties of the Main Formula for Feature j=1, Config 1

4.15.1 Document ratio_{defibo}

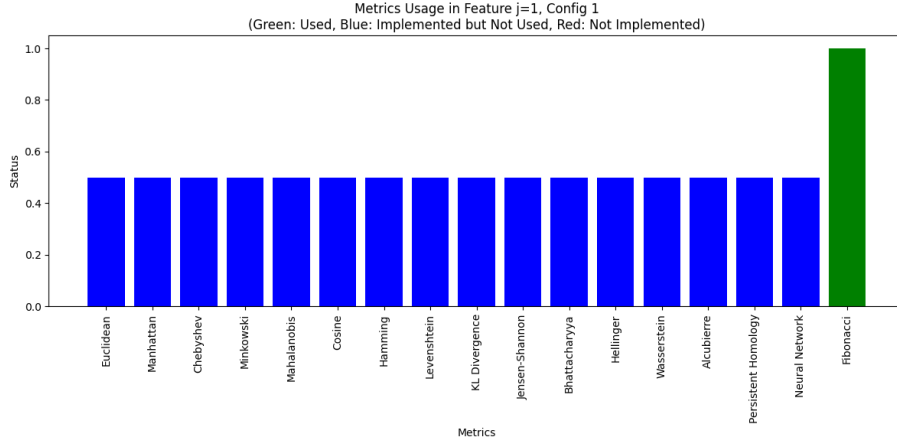
- Norms for x_1 :
 - L_1 Norm: 0.0000
 - L_2 Norm: 0.0000
 - L_∞ Norm: 0.0000
- Norms for $x_1 - \theta$:
 - L_1 Norm: 0.0000
 - L_2 Norm: 0.0000
 - L_∞ Norm: 0.0000
- Inner Product: $\langle x_1, \theta \rangle = 0.0000$
- Induced Norm of x_1 : $\sqrt{\langle x_1, x_1 \rangle} = 0.0000$
- Induced Metric: $\sqrt{\langle x_1 - \theta, x_1 - \theta \rangle} = 0.0000$

- **Transformations and Embeddings:** PCA with 2 dimensions
- **Scale and Units:** Features standardized using StandardScaler (mean=0, std=1)
- **Underlying Geometry/Topology:**
 - Persistent Homology: Number of H0 components: 2, H1 components: 0
 - Geometry Note: Data embedded in a vector space with Euclidean geometry, transformed using PCA

4.15.2 Document suite_{deibo}

- **Norms for x_1 :**
 - L_1 Norm: 0.0000
 - L_2 Norm: 0.0000
 - L_∞ Norm: 0.0000
- **Norms for $x_1 - \theta$:**
 - L_1 Norm: 0.0000
 - L_2 Norm: 0.0000
 - L_∞ Norm: 0.0000
- **Inner Product:** $\langle x_1, \theta \rangle = 0.0000$
- **Induced Norm of x_1 :** $\sqrt{\langle x_1, x_1 \rangle} = 0.0000$
- **Induced Metric:** $\sqrt{\langle x_1 - \theta, x_1 - \theta \rangle} = 0.0000$
- **Transformations and Embeddings:** PCA with 2 dimensions
- **Scale and Units:** Features standardized using StandardScaler (mean=0, std=1)
- **Underlying Geometry/Topology:**
 - Persistent Homology: Number of H0 components: 2, H1 components: 0
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4.16 Metrics Usage for Feature j=1, Config 1



4.17 New Distance Formulas for Feature j=1, Config 2

4.18 Final Equation for Feature j=1, Config 2

Final threshold equation: $x_1 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$ Evaluated
threshold: $x_1 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$, Result: $[FalseFalse]$

4.19 Properties of the Main Formula for Feature j=1, Config 2

4.19.1 Document ratio_{defibo}

- Norms for x_1 :

- L_1 Norm: 0.0000
- L_2 Norm: 0.0000
- L_∞ Norm: 0.0000

- Norms for $x_1 - \theta$:

- L_1 Norm: 0.0000
- L_2 Norm: 0.0000
- L_∞ Norm: 0.0000

- Inner Product: $\langle x_1, \theta \rangle = 0.0000$

- Induced Norm of x_1 : $\sqrt{\langle x_1, x_1 \rangle} = 0.0000$

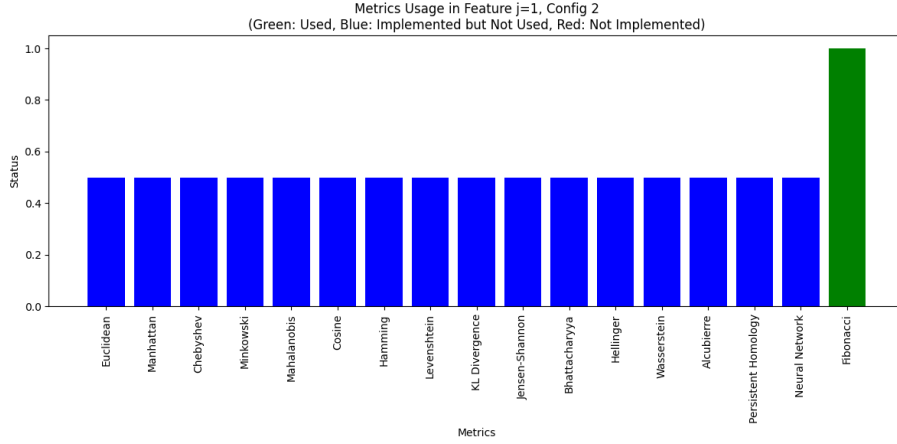
- Induced Metric: $\sqrt{\langle x_1 - \theta, x_1 - \theta \rangle} = 0.0000$

- **Transformations and Embeddings:** PCA with 2 dimensions
- **Scale and Units:** Features standardized using StandardScaler (mean=0, std=1)
- **Underlying Geometry/Topology:**
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4.19.2 Document suite_{deibo}

- **Norms for x_1 :**
 - L_1 Norm: 0.0000
 - L_2 Norm: 0.0000
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- **Norms for $x_1 - \theta$:**
 - L_1 Norm: 0.0000
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 - L_∞ Norm: 0.0000
- **Inner Product:** $\langle x_1, \theta \rangle = 0.0000$
- **Induced Norm of x_1 :** $\sqrt{\langle x_1, x_1 \rangle} = 0.0000$
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- **Scale and Units:** Features standardized using StandardScaler (mean=0, std=1)
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4.20 Metrics Usage for Feature j=1, Config 2



4.21 New Distance Formulas for Feature j=1, Config 3

4.22 Final Equation for Feature j=1, Config 3

Final threshold equation: $x_1 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$ Evaluated
threshold: $x_1 \geq \mathbb{R} \times (1 - \frac{1}{N} \sum_{k=1}^N d_{\text{Fibonacci}}(x_i, x_k))$, Result: $[FalseFalse]$

4.23 Properties of the Main Formula for Feature j=1, Config 3

4.23.1 Document ratio_{defibo}

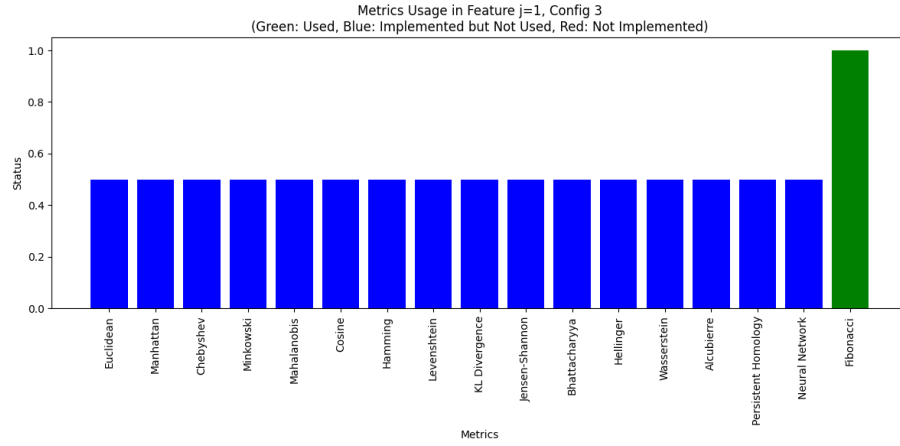
- Norms for x_1 :
 - L_1 Norm: 0.0000
 - L_2 Norm: 0.0000
 - L_∞ Norm: 0.0000
- Norms for $x_1 - \theta$:
 - L_1 Norm: 0.0000
 - L_2 Norm: 0.0000
 - L_∞ Norm: 0.0000
- Inner Product: $\langle x_1, \theta \rangle = 0.0000$
- Induced Norm of x_1 : $\sqrt{\langle x_1, x_1 \rangle} = 0.0000$
- Induced Metric: $\sqrt{\langle x_1 - \theta, x_1 - \theta \rangle} = 0.0000$

- **Transformations and Embeddings:** PCA with 2 dimensions
- **Scale and Units:** Features standardized using StandardScaler (mean=0, std=1)
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4.23.2 Document suite_{deibo}

- **Norms for x_1 :**
 - L_1 Norm: 0.0000
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 - L_∞ Norm: 0.0000
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4.24 Metrics Usage for Feature j=1, Config 3



5 Metric Validity Summary

- **Fibonacci:** ✓ (Valid metric)