Assignment # 1: Static Aeroelasticity
Class: Structural Dynamics and Aeroelasticity,
Prof. Giuseppe Quaranta
A.Y. 2022/23

April 12, 2023

Take the last four figures of your person code ABCD, and assemble these two numbers: DA and CB[1]

$$\begin{cases} 00 \leq DA < 50 & x_1 = 1.2 \\ 50 \leq DA < 99 & x_1 = 0.7 \\ 00 \leq CB < 50 & x_2 = 1.1 \\ 50 \leq CB < 99 & x_2 = 0.6 \end{cases} \tag{1}$$

Consider an aircraft with a *forward* swept wing and a *rigid* canard surface sketched in Figure 1, with the following geometric properties: wing semi-span $b = 6.10$ m, wing chord $c = 3.05$ m, sweep angle $\Lambda = -30$, fuselage length $L_f = 9.15$ m, canard chord $c_c = 3.05$ m canard semi-span $b_c = 1.525$ m. The wing is attached to the fuselage on the point with coordinate $(x_w, y_w) = (0, 0)$, while the canard is attached at $(x_c, y_c) = (-4.525, 0)$. On the wing tips, there are two ailerons with a chord equal to $c_a = \frac{1}{4}c$ and semi-span equal to $b_a = \frac{1}{2}b$. The elastic axis of the wing is positioned at 50% of the chord.

The wing is characterized by the following structural properties (with $\bar{y}$ the axis aligned with the wing beam structure)

- Bending stiffness $EI_w(\bar{y}) = x_1 \times 4.5\, 10^6$ N m^2
- Torsional stiffness $GJ_w(\bar{y}) = x_2 \times 7.0\, 10^6$ N m^2

The fuselage is rigid in bending and has the following torsional stiffness

- Fuselage torsional stiffness $GJ_f(x) = 12.0\, 10^6$ N m^2

[1] Example: $10134997 \rightarrow A = 4$, $B = 9$, $C = 9$, $D = 7$, so the two numbers will be $DA = 74$, $CB = 99$.

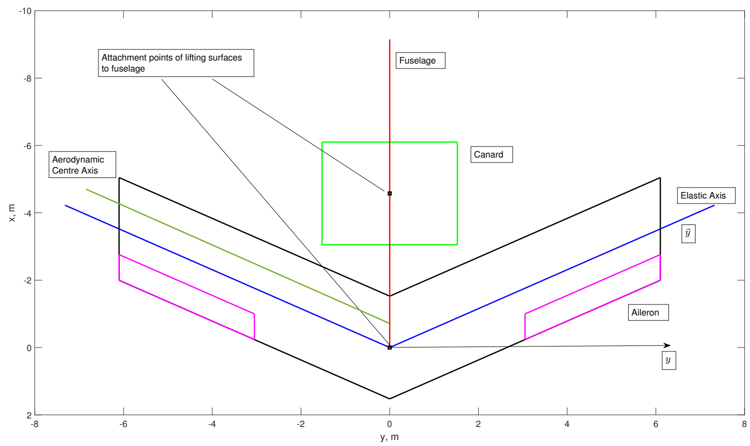

Figure 1: Aircraft planform.

Consider the aircraft subject to an incompressible airflow (i.e. $M = 0$) and use the strip theory with $C^0_{L\alpha} = 2\pi$ to model the aerodynamics for both the wing and the canard. The aerodynamic centre must be considered always placed at 1/4 of the chord. To compute the aerodynamic coefficients for the aileron, the expression provided in class (set of slides #2) must be used.

1. Compute the lowest divergence dynamic pressure of the aircraft using the Ritz-Galerkin method. Use as shape functions sinusoidal functions appropriate for the boundary conditions of the problem under analysis (remember that the aircraft is flying and not constrained in a wind tunnel). Select the number of shape functions to reach a sufficient precision.

2. Consider to rotate the ailerons by a $\beta = 1°$ (right downward, left upward). Compute the angular roll speed reached at regime by the aircraft when flying at 85% of the divergence dynamic pressure and compare it with the regime angular roll speed for the rigid aircraft. (WARNING: the canard's effect, and the effect of the flexible fuselage, must be considered. SUGGESTION: Use as shape functions to model the fuselage torsion appropriate trigonometric functions, i.e. $\sin \omega x$ and/or $\cos \omega x$ with appropriate spatial frequencies ω).

3. Plot in this condition (i.e. regime after application of $\beta = 1°$) the diagram of the torsional moment as a function of x applied to the fuselage.

4. Compute the bending moment generated by the aileron rotation applied at the wing root.

5. Make the hypothesis that it is possible to design the wing structure using composite materials so that coupling between bending and torsion exists.

Consider that the material used in the wing structure leads to the following constitutive law

$$\begin{Bmatrix} M_b \\ T \end{Bmatrix} = \begin{bmatrix} EI_w & C_w \\ C_w & GJ_w \end{bmatrix} \begin{Bmatrix} w'' \\ \theta' \end{Bmatrix} \tag{2}$$

Try to estimate a value of C_w, if exists, sufficient to have a divergences speed of the forward swept wing above that of the equivalent straight wing.

The methods necessary to solve all steps must be written by hand on paper and a copy must be submitted together with the numeric solutions. All integrals could be solved using the symbolic integrator of MATLAB (or any other symbolic solver). The eigenvalues can be computed using the MATLAB `eig` routine (or any other numerical tool). All routines used to compute the solution must be provided too so that they can be run to verify how they work.

Clarifications on the text may be asked until April 21, 2023. To submit the responses and the photos of the written solution you will have to connect to a MS Form page. The answers will have to be submitted after 10 days starting from April 22, 2023 (so by 1:00 pm of May 02, 2023). A to the MS Form page to submit the answers will be provided on April 22, 2023.