Binary Arithmetic

Binary Addition

Rules:
- 0 + 0 = 0
- 1 + 0 = 1
- 0 + 1 = 1
- 1 + 1 = 10 (0 with carry 1)
- 1 + 1 + 1 = 11 (1 with carry 1)

Carrying just like in decimal:

Ignore if overflows

$$
\begin{aligned}
&0000\ 0011 = 3_{10} \\
&+ \\
&0000\ 0101 = 5_{10} \\
&= \\
&0000\ 1000 = 8_{10}
\end{aligned}
$$

Could also convert from binary to decimal then add:

$$
\begin{aligned}
&0111\ 1000 = 64 + 32 + 16 + 8 = 120_{10} \\
&+ \\
&0101\ 0101 = 64 + 16 + 4 + 1 = 85_{10} \\
&= \\
&1100\ 1101 = 128 + 64 + 8 + 4 + 1 = 205_{10}
\end{aligned}
$$

Unsigned vs Signed

Unsigned numbers always positive or zero

Leftmost bit in signed number is sign; 0 = positive, 1 = negative
1000 0000 = −128
0111 1111 = +127

Binary Subtraction (Two's Complement Notation)

Steps:
1. Take the 2nd number, convert with 2's complement
2. Add 1st number to 2's complement of 2nd

$$
\begin{aligned}
&= 01 - 10 \\
&= 10_{10} - 13_{10} \\
&= 10_{10} + (-13_{10})
\end{aligned}
$$

How to take 2's Complement

Steps:
1. Flip every bit to its opposite
2. Add 1 to the binary

0011 0011

1. $\rightarrow$ 1100 1100

2. $\rightarrow$ 1100 1101

Convert 2's Complement to Decimal

Leftmost bit is −128 (1) or +127 (0)

Add the same way:

$\rightarrow$ 1100 1101

$\rightarrow -128 + 64 + 8 + 4 + 1$

$= -51_{10}$

Subtraction

Example 1:

0000 1011 − 0000 0010

1. Second to complement: 0000 0010 $\rightarrow$ 1111 1101
2. Add to first: 0000 1011 + 1111 1101

$=$ 0000 1001

Example 2:

0000 1100 − 0000 1001

1. Second to complement: 0000 1001 $\rightarrow$ 1111 0111
2. Add to first: 0000 1100 + 1111 0111

$=$ 0000 0011