Prove:

$n! > 2^n$ for $n \geq 4$

Base Case:

$n = 4$

$4! > 2^4$

$24 > 16$

Hypothesis:

$k! > 2^k$ is TRUE for $k \geq 4$

Proof:

$(k+1)! > 2^{k+1}$

$(k+1)k! > 2(2^k)$

$(k+1)2^k > 2(2^k)$

$k \geq 4 \therefore k+1 > 2$

$(k+1)! > k!(k+1) > 2^k(k+1) > 2(2^k) > 2^k$

Prove:

$2^{2n} - 1$ is divisible by 3

Base Case:

$n = 1$

$2^{2(1)} - 1$

$4 - 1 = \frac{3}{3} = 1$

Hypothesis:

$2^{2k} - 1$ is divisible by 3 is TRUE for $k \geq 1$

Proof:

$2^{2(k+1)} - 1$

$2^{2k+2} - 1$

$2^2(2^k) - 1$

$4(2^k) - 1$

$(3+1)2^k - 1$

$3(2^k) + 2^k - 1$

Prove

```
fact(n)
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Base case

$n = 0$

```
fact(0)
    if 0 == 0:
        return 1
```

Hypothesis:

`fact(k)` is TRUE for $k \geq 0$

Proof:

```
fact(k + 1):
    if k + 1 == 0 false
    else:
        return k * fact(k + 1 - 1)
```