Capacitance

C = capacitance; a measure of an object(s) ability to store charge

$$C = \frac{q}{\Delta V} = \frac{q}{V}$$

$$C > 0$$

Unts: 1 C/V = 1 farad = 1 F

Two parallel large charged plates

$$|\vec{E}| = \frac{\sigma}{\varepsilon_0}$$

$$\Delta V = Ed$$

$$C = \varepsilon_0 \frac{A}{d}$$

Two closely spaced charged cylinders

$$C = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{r_0}{r_i}\right)}$$

Stored Energy

$$U = \frac{1}{2}CV^2$$

$$U = \frac{q^2}{2C}$$

$$U = \frac{1}{2}qV$$

Combinations of Capacitors

$$\Delta V = \frac{q}{C_{\text{eq}}}$$

1. Series connection (single path)

$$\frac{1}{C_{\text{eq}}} = \sum_{i=1}^{N} \frac{1}{C_i}$$

2. Parallel connection (multiple paths)

$$C_{\text{eq}} = \sum_{i=1}^{N} C_i$$

Dielectrics

- insert an insulator between plates
- increases capacitance
- allows for more charge to be stored at a given charging voltage

κ = dielectric constant, a measure of how well an insulator reduces V

$$\kappa = \frac{V_0}{V}$$

$$\kappa \geq 1$$

Different materials have different κ values

$$\vec{E}_d = \text{displacement field}$$

$$\vec{E}_{\text{total}} = \vec{E}_0 - \vec{E}_d$$

$$\sigma_d = \sigma\left(\frac{k-1}{k}\right)$$

$$\varepsilon = \text{permittivity of the dielectric}$$

$$\varepsilon = \kappa\varepsilon_0$$

$$C = \kappa C_0$$

$$U = \frac{U_0}{\kappa}$$

$$C = \kappa\varepsilon_0\frac{A}{d}$$

$$V_{\max} = E_{\max}d$$

Dielectric Strength

- maximum $\vec{E}$-field that a dielectric can tolerate before breaking down
- Dry air: $E_{\max}$ = 3.0×10^6 V/m

Energy Density

u = energy density = $\frac{U}{\text{volume between plates}}$

$$u = \frac{1}{2}\varepsilon_0 E^2$$