Circular Motion

(x, y) becomes (R, θ)

$\vec{r} = R\hat{r}$

$R = \sqrt{x^2 + y^2}$

$\hat{r} =$ direction of radius

We assume R is constant and ω is constant

$\vec{r} = R\cos(\theta)\hat{i} + R\sin(\theta)\hat{j}$

$\vec{v} = \dfrac{d}{dt}\vec{r} = R\left(-\sin\theta\dfrac{d\theta}{dt}\hat{i} + \cos\theta\dfrac{d\theta}{dt}\hat{j}\right)$

$\dfrac{d\theta}{dt} = \omega = \underline{\text{angular velocity}}$

$\vec{v} = R\omega\left(-\sin\theta\hat{i} + \cos\theta\hat{j}\right)$

$|\vec{v}| = \sqrt{(R\omega)^2\left[(-\sin\theta)^2 + \cos^2\theta\right]}$

$|\vec{v}| = R\omega$

$\omega = \dfrac{|v|}{R}$

Acceleration direction is always towards the center

$\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}\left[R\omega(-\sin\theta\hat{i} + \cos\theta\hat{j})\right]$

$\vec{a} = R\omega(-\cos\theta\omega\hat{i} - \sin\theta\omega\hat{j})$