# Introduction to Periodic Motion

$$\boxed{\omega = \frac{2\pi}{T}}$$

Assume no "extra forces"

$F_f = 0$

$F_A = 0$

$$\sum \vec{F} = \vec{F}_S = m\vec{a}$$

$$\boxed{-k\Delta\vec{x} = m\vec{a}}$$

$$\boxed{\vec{a} = -\frac{k}{m}\Delta\vec{x}}$$

Cannot use kinematic eqns, $\vec{a} \neq$ constant

$$\vec{a} = \frac{d^2\vec{x}}{dt^2}$$

$$-k\Delta\vec{x} = m\frac{d^2\vec{x}}{dt^2}$$

Gives back original fxn with (-) after 2 derivatives:

$x(t) = A\sin(\omega t)$

$x(t) = A\cos(\omega t)$

$$\frac{dx}{dt} = A\omega\cos(\omega t)$$

$$\frac{d^2x}{dt^2} = A\omega - \sin(\omega t)\,\omega$$

$$\frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t)$$

$$-k\Delta x = m\frac{d^2x}{dt^2}$$

$-k\left[A\sin\left(\omega t\right)\right] = m\left[-A\omega^2\sin\left(wt\right)\right]$

$\boxed{k = m\omega^2}$

$\boxed{\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}}$

Spring system, max $\Delta x$, $t = 0$, $v = 0$

$x(t) = A\sin\left(\omega t\right)$

$x(0) = x_{\max} = A\sin\left(\omega \cdot 0\right)$

$x_{\max} = 0$, doesn't make sense for sin!

$x(t) = A\cos\left(\omega t\right)$

$x(0) = x_{\max} = A\cos\left(\omega \cdot 0\right)$

$x_{\max} = A$, does make sense for cos!

Spring system, equilibrium, $v \neq 0$, $t = 0$, $x = 0$

$x(t) = A\sin\left(\omega t\right)$

$x(0) = A\sin\left(0\right)$

$0 = 0$

Spring system, some amplitude but not full, $\frac{A}{2}$

$\boxed{x(t) = A\sin\left(\omega t + \phi\right)}$, $\phi$ is a phase shift

$x(0) = \frac{A}{2} = A\sin\left(\omega \cdot 0 + \phi\right)$

$\frac{1}{2} = \sin\left(\phi\right)$

$\phi = \frac{\pi}{6}$ rad