Rotational Motion

Still assuming R is constant.

The angle θ makes an arc: Δs

$\Delta s = R \cdot \theta$

$$v = \frac{d\Delta s}{dt} = R\frac{d\theta}{dt}$$

Remember: $\frac{d\theta}{dt} = \omega$

If radian $= \Delta s = R$ then $\theta = 1$ radian, and $\theta = \frac{\Delta s}{R}$

v (tangential velocity) $= R\omega$

$$a = \frac{dv}{dt} = R\frac{d\omega}{dt}$$

We're going to call $\dfrac{d\omega}{dt}$: α (angular acceleration)
Assume α is constant

Therefore, $\underline{\vec{a}_t \text{ (tangential acceleration)} = R\alpha}$

If we take:

$$\omega = \frac{d\theta}{dt}$$

$$d\theta = w\,dt$$

$$\int d\theta = \int \omega\,dt$$

Using:

$$\alpha = \frac{d\omega}{dt}$$

$$d\omega = \alpha\,dt$$

$$\int d\omega = \int \alpha\,dt$$

$$\Delta\omega = \alpha t$$

$$\omega = \omega_0 + \alpha t$$

$$\int d\theta = \int \omega \, dt$$

$$\omega = \omega_0 + \alpha t$$

$$\Delta\theta = \int (\omega_0 + \alpha t)\ dt = \omega_0 t + \frac{1}{2}\alpha t^2$$

Angular Kinematics

1. $\vec{\theta} = \vec{\theta}_0 + \vec{\omega}_0 t + \frac{1}{2}\vec{\alpha} t^2$
2. $\vec{\omega} = \vec{\omega}_0 + \vec{\alpha} t$
3. $\omega^2 = {\omega_0}^2 + 2\alpha\Delta\theta$