Gravitational Interactions

$\vec{F}_G = \frac{-Gm_1M-2}{r^2}\hat{r}$

$\vec{F}_{G1} = \frac{Gm_1m_2}{r^2}\hat{i}$

Positive because m_1 is being pulled to the right by m_2

$\vec{F}_{G2} = -\vec{F}_{G1}$, Newton's 3rd Law

When three masses:

$\sum \vec{F}_1 = +\vec{F}_{G1,2} + \vec{F}_{G1,3}$

$= \frac{Gm_1m_2}{{r_1}^2} + \frac{Gm_1m_3}{(r_1+r_2)^2}$

$\sum \vec{F}_2 = -\vec{F}_{G2,1} + \vec{F}_{G2,3}$

$= \frac{-Gm_2m_1}{r_1^2} + \frac{Gm_2m_3}{{r_2}^2}$

When infinite masses:

$dF_G = \frac{Gm_1dm}{r^2}$

$F_G = \int_a^{a+l} \frac{Gm_1dm}{r^2}$

a = first point mass, l = final point mass

Earth around Sun:

$\sum_E \vec{F} = \frac{Gm_sm_E}{{R_E}^2} = m_Ea_c$

$a_c = R_E\omega^2$

Important:

$\omega^2 = \frac{Gm_s}{{R_E}^3}$

$$\sum_{\text{object}} \vec{F} = \vec{F}_{Go,s} + \vec{F}_{Go,E}$$

$$\sum_{o} \vec{F} = \frac{-Gm_s m_o}{(R_E - d)^2} + \frac{Gm_E m_o}{d^2} = -m_o a_c$$

Negative acceleration because it's towards the center of the circle

$$\frac{-Gm_s}{(R_E - d)^2} + \frac{Gm_E}{d^2} = -(R_E - d)\omega^2$$

$$\frac{-Gm_s}{(R_E - d)^2} + \frac{Gm_E}{d^2} = -(R_E - d)\frac{Gm_s}{{R_E}^3}$$

$$\frac{-m_s}{(R_E - d)^2} + \frac{m_E}{d^2} = \frac{-(R_E - d)m_s}{{R_E}^3}$$

$$\frac{1}{(R_E - d)^2} = \frac{1}{{R_E}^2} \cdot \frac{1}{(1 - \frac{d}{R_E})^2}$$

Assumption:

$$\frac{d}{R_E} << 1$$

$$\frac{1}{(1+x)^n} \approx 1 - nx + O(x^2)$$

$$\frac{1}{(R_E - d)^2} \approx \frac{1}{{R_E}^2}(1 - 2(\frac{-d}{R_E}))$$

$$= \frac{1}{{R_E}^2}(1 + \frac{2d}{R_E})$$

$$\frac{-m_s}{{R_E}^2}\left[1 + \frac{2d}{R_E}\right] + \frac{m_E}{d^2} = \frac{-(R_E - d)m_s}{{R_E}^3}$$

First $\frac{m_s}{{R_E}^2}$ cancels with $\frac{-(R_E)m_s}{{R_E}^3}$ after distribution

$$\frac{-2m_s d}{{R_E}^3} + \frac{m_E}{d^2} = \frac{m_s d}{{R_E}^3}$$

$$\frac{m_E}{d^2} = \frac{3m_s d}{{R_E}^3}$$

$$d = \left(\frac{1}{3}\frac{m_E}{m_s}\right)^{\frac{1}{3}} R_E$$