Lab Report

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Introduction

In this lab, we will conduct three experiments to verify that the total energy of a closed system is constant; in other words:

$$U_A + KE_A = U_B + KE_B$$

First, we will drop a puck from rest on an incline, and calculate the total energy at our starting point and the total energy at an arbitrary Point B.

Next, we will launch the puck with some initial velocity. Again, we will measure the total energy at the starting point and measure the total energy at an arbitrary Point B.

Finally, we will use a pulley system to attach two masses together. One will remain on the table, while we drop the other from rest. Again, we will measure the total energy at the starting point and measure the total energy at an arbitrary Point B.

Raw Data

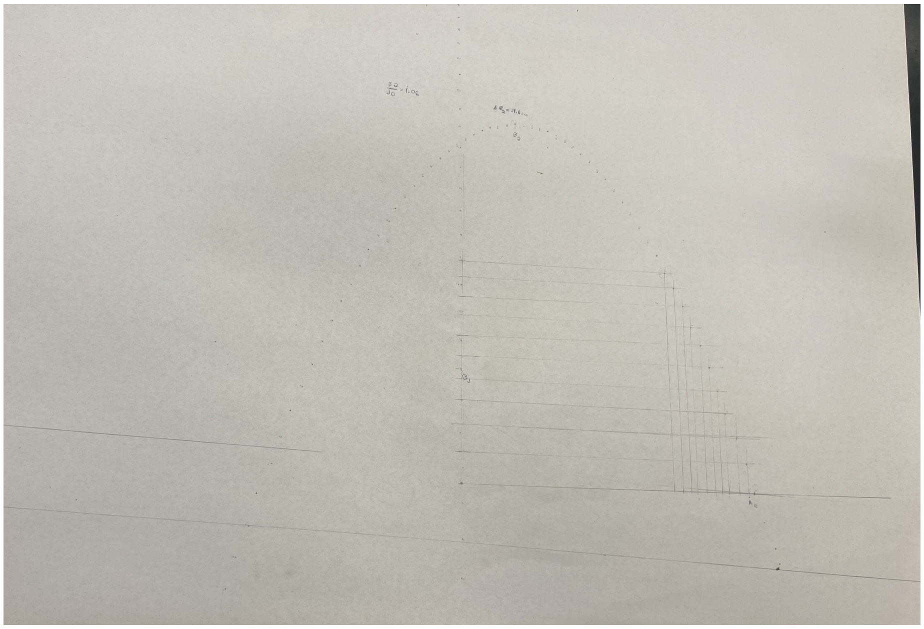


Figure 1: System 1 and System 2 Paper

For both System 1 and System 2, our group placed the puck on top of this paper. A spark timer created dots on the paper at the puck's position in 30Hz intervals. The vertical series of dots represents System 1, in which we dropped the puck on the incline from rest. The upside down parabola-like traces represent System 2, in which we launched the puck.

For both System 1 and System 2, the mass of the puck was 5.4×10^{-1} kg

For both System 1 and System 2, $\Delta R = 29.2$ cm

For System 3, the mass of the cart was 2.467×10^{-1} kg

For System 3, the mass of the hanging weight was 2.0×10^{-2} kg

Data Analysis

System 1

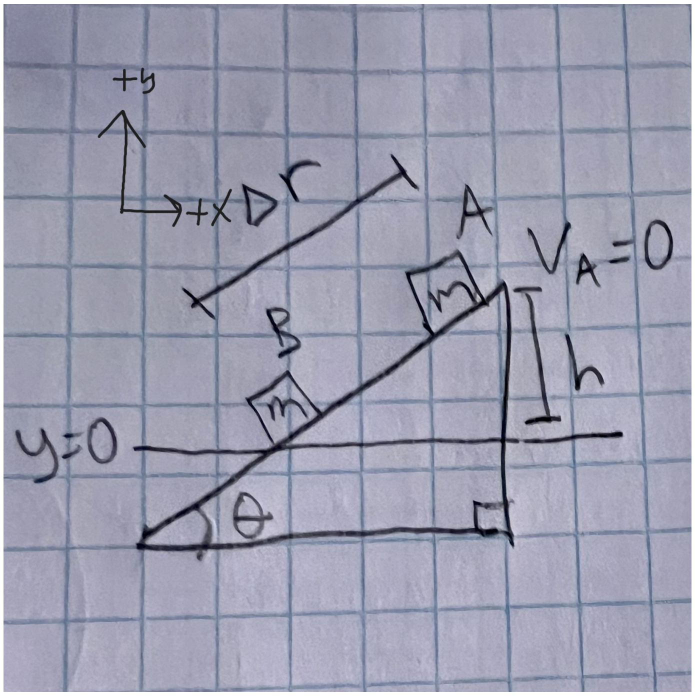


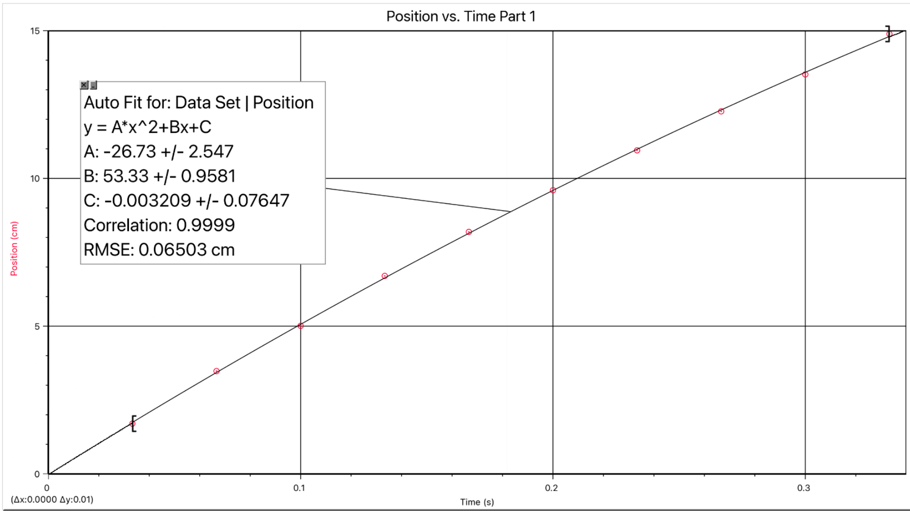
Figure 2: Visual of System 1

Since we dropped the puck from rest, Point A has no kinetic energy. However, it does have gravitational potential energy:

$$E_A = mgh = mg\Delta r \sin\theta$$

Because Point B is at y-position 0, it has no potential energy. However, it does have kinetic energy:

$$E_B = \frac{1}{2}m{v_B}^2$$



Graph 1: System 1 Position vs. Time

Since acceleration is constant in this system, we could use the kinematic equations for calculations on our data:

$$y = y_o + v_o t + \frac{1}{2}at^2$$

Because of this, we used a curve fit on the data in the form of:

$$C + Bt + At^2$$

According to the kinematic equation above, the A value in our curve fit represents $\frac{1}{2}a$, meaning the acceleration in our system is twice our A value from Graph 1:

$$A = -26.73 \text{ cm/s}^2$$

$$2A = -53.46 \text{ cm/s}^2$$

$$\vec{a}_{\text{system}} = -5.346 \times 10^{-1} \text{ m/s}^2$$

$$|a| = 5.346 \times 10^{-1} \text{ m/s}^2$$

Since there is an incline, the magnitude of our acceleration is $a = gsin\theta$.

Since we know the value of a and the value of g, we could solve for θ in our system:

$$a = g\sin\theta$$

$$\theta = \arcsin\left(\frac{a}{g}\right) = \arcsin\left(\frac{5.346 \times 10^{-1}\ \mathrm{m/s}^2}{9.8\ \mathrm{m/s}^2}\right) = 3.13°$$

Since $y = 0$ at Point B, $v_B = v_o$

According to the kinematic equation:

$$y = y_o + v_o t + \frac{1}{2}at^2$$

The B value in our curve fit would represent the initial velocity. According to Graph 1, our B value is 53.55 cm/s. Therefore, our velocity at Point B is:

$$v_B = 5.355 \times 10^{-1}\ \mathrm{m/s}$$

Now, we have all of the unknown values from the energy equations earlier:

$$E_A = mgh = mg\Delta r \sin\theta$$

$$E_B = \frac{1}{2}m{v_B}^2$$

Plugging in these values we get:

$$E_A = mg\Delta r \sin\theta$$

$$E_A = 5.4 \times 10^{-1}\ \mathrm{kg} \times 9.8\ \mathrm{m/s}^2 \times 29.2\ \mathrm{cm} \times \frac{\mathrm{m}}{100\ \mathrm{cm}} \times \sin(3.13°)$$

$$E_A = 8.44 \times 10^{-2}\ \mathrm{J}$$

$$E_B = \frac{1}{2}m{v_B}^2$$

$$E_B = \frac{1}{2} \times 5.4 \times 10^{-1}\ \mathrm{kg} \times \left(5.355 \times 10^{-1}\ \mathrm{m/s}\right)^2$$

$$E_B = 7.74 \times 10^{-2}\ \mathrm{J}$$

Then, we found the percent difference between these two energies. Since both energies were found theoretically, we weighted them the same.

$$\%\ \mathrm{Error} = \frac{(E_A - E_B)}{(E_A + E_B)/2} \times 100\% = 8.65\%$$

System 2

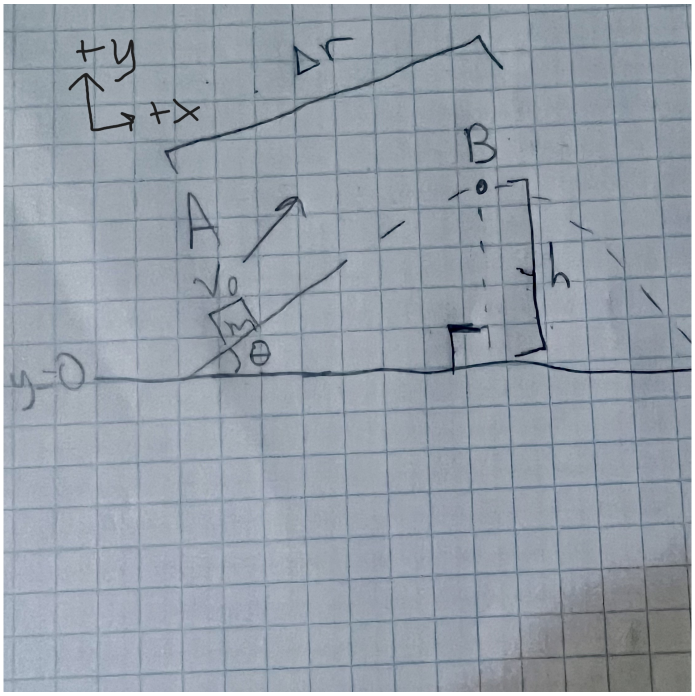


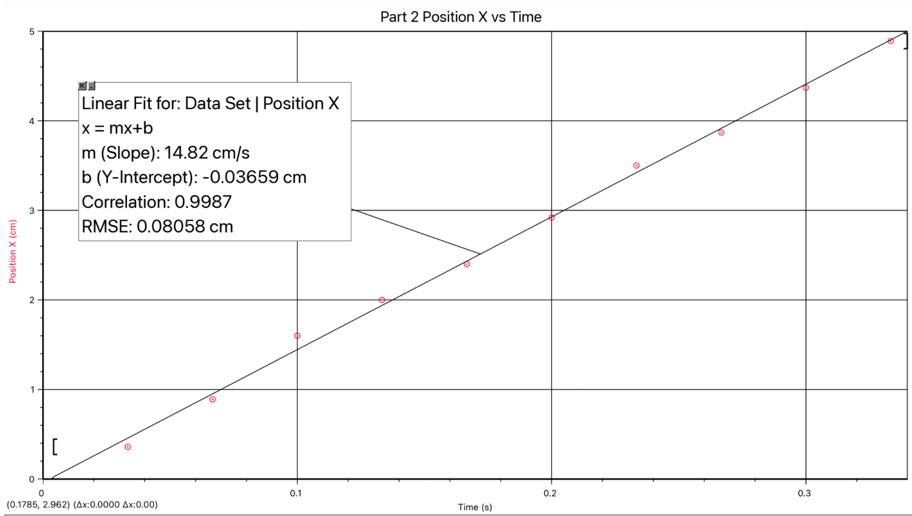
Figure 3: Overview of System 2

We designated that Point A, the starting position was at $y = 0$, Point A only had kinetic energy and no potential energy. Therefore,

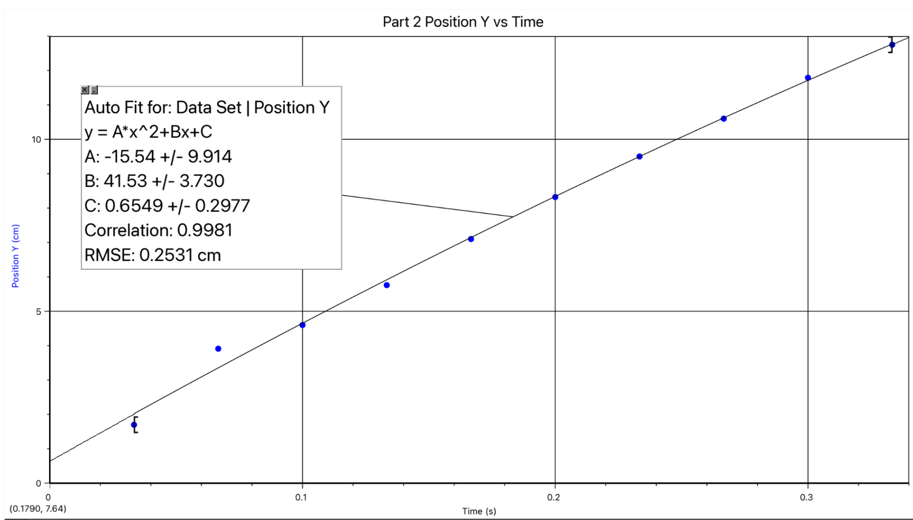
$$E_A = \frac{1}{2}m{v_o}^2 = \frac{1}{2}m\left({v_{ox}}^2 + {v_{oy}}^2\right)$$

However, the puck still had some velocity in the x-direction at Point B while also having some potential energy. We assumed that the acceleration in the x-direction was 0. Therefore the total energy at Point B was:

$$E_B = mgh + \frac{1}{2}m{v_{ox}}^2$$



Graph 2: System 2 X-Position vs. Time



Graph 3: System 2 Y-Position vs. Time

Since we assumed velocity was constant, the slope of Graph 2 would represent the velocity in the x-direction at all points. We used a linear fit on Graph 2 and found that our slope or $v_x = 14.82$ cm/s $= 1.482 \times 10^{-1}$ m/s

Assuming acceleration is constant, we could use the kinematic equations:

$$y = y_o + v_o t + \frac{1}{2}at^2$$

Using the curve fit, we found the B value from the curve fit, 41.53 cm/s = 4.153×10^{-1} m/s to be our v_{oy}

According to Figure 3: $h = \Delta r \sin\theta$

$$h = \Delta r \sin\theta$$

We setup our System 2 such that we had the same Δr from Part 1, 29.2 cm. Then, we used the same process from Part 1 to find θ:

$$|\text{A Value}| = \frac{1}{2}|a_{\text{system}}|$$

$$|a| = 2 \times |-15.54 \text{ cm/s}| \times \frac{\text{m}}{100 \text{ cm}} = 3.11 \times 10^{-1} \text{ m/s}^2$$

$$a = g\sin\theta$$

$$\theta = \arcsin\left(\frac{a}{g}\right) = \arcsin\left(\frac{3.11 \times 10^{-1}\text{m/s}^2}{9.8\text{m/s}^2}\right) = 1.82°$$

Now, we had all the values we needed to determine the total energy at Points A and B:

$$E_A = \frac{1}{2}m\left(v_{ox}^2 + v_{oy}^2\right)$$

$$E_A = \frac{1}{2} \times 5.4 \times 10^{-1} \text{ kg}\left((1.482 \times 10^{-1}\text{m/s})^2 + (4.153 \times 10^{-1} \text{ m/s})^2\right)$$

$$E_A = 5.25 \times 10^{-2} \text{ J}$$

$$E_B = mgh + \frac{1}{2}m{v_{ox}}^2$$

$$E_B = m\left(g\Delta r\sin\theta + \frac{1}{2}{v_{ox}}^2\right)$$

$$E_B = 5.4{\times}10^{-1} \text{ kg}\left(9.8 \text{ m/s}^2 \times 29.2 \text{ cm} \times \sin(1.82°) \times \frac{\text{m}}{100 \text{ cm}} + \frac{1}{2}(1.482 \times 10^{-1}\text{m/s})^2\right)$$

$$E_B = 5.50 \times 10^{-2} \text{ J}$$

Then, we found the percent difference between these two energies. Since both energies were found theoretically, we weighted them the same.

$$\% \text{ Error} = \frac{(E_A - E_B)}{(E_A + E_B)/2} \times 100\% = -4.65\%$$

System 3

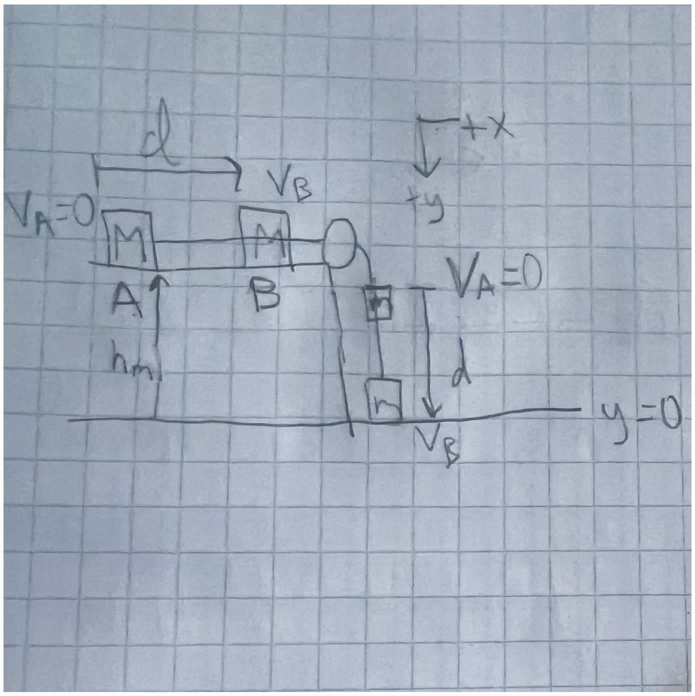


Figure 4: Overview of System 3

At Point A in Figure 4, there was no kinetic energy. However, there was potential energy from both the large mass and the hanging mass. We set our $y = 0$ point at the height of the hanging mass at Point B. Using this coordinate system:

$$E_A = M_g h_m + mgd$$

At Point B, there is some kinetic energy, but there is no potential energy in the hanging mass since it's at $y = 0$:

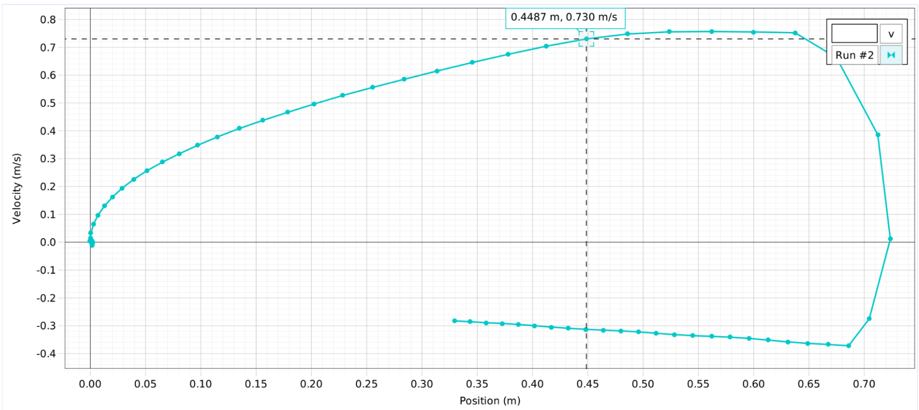
$$E_B = M_g h_m + \frac{1}{2}(M + m)\, {v_B}^2$$

The conservation of energy equation would be valid if these energy equations were equal to each other:

$$M_g h_m + mgd = M_g h_m + \frac{1}{2}(M + m)\, {v_B}^2$$

$$mgd = \frac{1}{2}(M + m)\, {v_B}^2$$

The PASCO Smart Cart tracked its own velocity and position and created Graph 4:



Graph 4: System 3 Cart Velocity vs Cart Position

We picked some arbitrary Point B before the hanging mass hit the ground, where it's displacement $d = 4.487 \times 10^{-1}$ m and $v_B = 7.30 \times 10^{-1}$ m/s

$$E_A = \left(2.0 \times 10^{-2}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)\left(4.487 \times 10^{-1}\ \text{m}\right)$$

$$E_A = 8.79 \times 10^{-2}\ \text{J}$$

$$E_B = \frac{1}{2}\left(2.673 \times 10^{-1}\ \text{kg} + 2.0 \times 10^{-2}\ \text{kg}\right)\left(7.30 \times 10^{-1}\ \text{m/s}\right)^2$$

$$E_B = 7.66 \times 10^{-2}\ \text{J}$$

Then, we found the percent difference between these two energies. Since both energies were found theoretically, we weighted them the same.

$$\%\ \text{Error} = \frac{(E_A - E_B)}{(E_A + E_B)/2} \times 100\% = 13.7\%$$

Conclusion

In System 1, we calculated the theoretical total energy at the moment an object is dropped on an incline and the total energy at an arbitrary moment after. We found that the percent error between these two totals was 8.65%

In System 2, we calculated the theoretical total energy at the moment we launched at object with some initial velocity and the total energy when that object reached its peak height. The calculated percent error between these two totals was -4.65%

In System 3, we calculated the theoretical total energy at the moment we released a PASCO Smart Cart attached to a hanging mass and the total energy at an arbitrary time before the hanging object hit the ground. The calculated percent error between these two totals was 13.7%

Based on these relatively low percent errors, we could conclude that in general, the total energy in a closed system is constant, and the equation for the conservation of energy is true:

$$U_A + KE_A = U_B + KE_B$$

Although we concluded that the conservation of energy equation is true, our experiments and calculations could have had some error. For example, we did not take into account non-conservative forces, assuming that they were negligible.

We used air tables and the PASCO Smart Cart to try to minimize the work from friction and air resistance, but these non-conservative forces, may have contributed to the percent errors between our total energies. Additionally, in our procedure for System 1 and System 2, we manually measured the displacement of the object, introducing some error due from the lack of precision of the ruler.

The larger percent error found in System 3 may have come from the technologies involved. The timing from the computer recording starting and the cart actually moving could have mismeasured initial position and velocity. If we chose a different arbitrary Point B to perform calculations on, we may have had more or less accurate results.