SOURCES OF MAGNETIC FIELD

28

VP28.2.1. **IDENTIFY:** We want to calculate the magnetic field due to a moving charged particle.

SET UP: $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$, $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$. Our target variable is the magnetic field at various points.

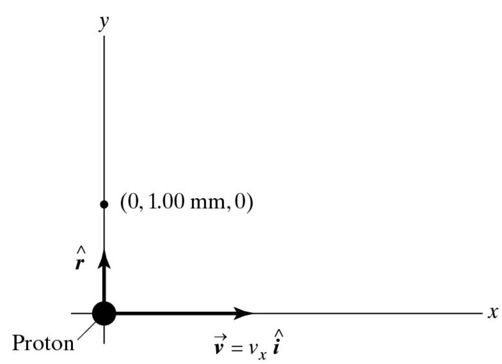


Figure VP28.2.1a

EXECUTE: **(a)** At (0, 1.00 mm, 0). See Fig. 28.2.1a. Use $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$ with $q = e$, $\phi = 90°$, and the given values for v and r. By the right-hand rule, the direction is along the $+z$ axis. The result is $\vec{B} = 3.20\times10^{-15}\ \text{T}\ \hat{i}$.

(b) At (0, 0, 2.00 mm). The approach is the same as in (a) except r = 2.00 mm. The cross product is in the $-y$-direction, so we get $\vec{B} = -8.00\times10^{-16}\ \text{T}\ \hat{j}$.

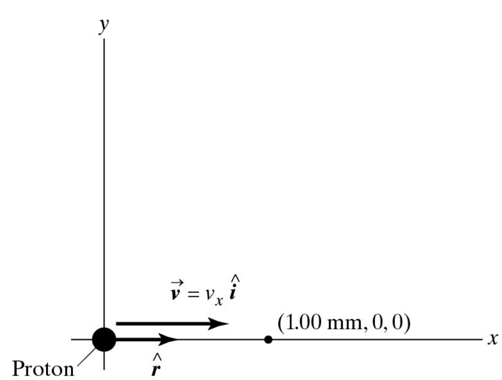


Figure VP28.2.1b

(c) At (1.00 mm, 0, 0). As Fig. VP28.2.1b shows, $\phi = 0$, so $B = 0$.

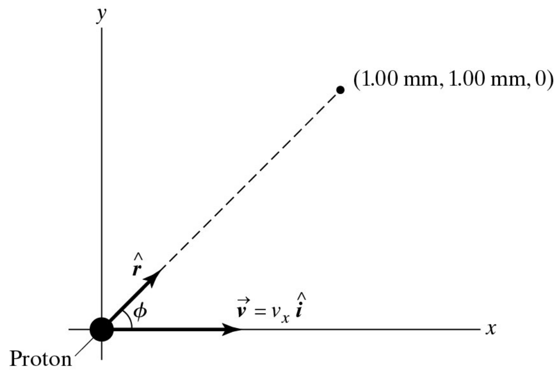


Figure VP28.2.1c

(d) At (1.00 mm, 1.00 mm, 0). See Fig. VP28.2.1c. Use $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$ with $\phi = 45.0°$ and $r = \sqrt{2}$ mm. The direction is along the $+z$-axis, so $\vec{B} = 1.13\times10^{-15}\ \text{T}\ \hat{k}$.

EVALUATE: As we see, the magnetic fields produced by individual charges are very small.

VP28.2.2. IDENTIFY: We want to find the magnetic force between two moving charged particles.

SET UP: Our target variable is the force that the proton exerts on the electron and that the electron exerts on the proton. $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$, $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$, $\vec{F} = q\vec{v}\times\vec{B}$, $F = |q|vB\sin\phi$.

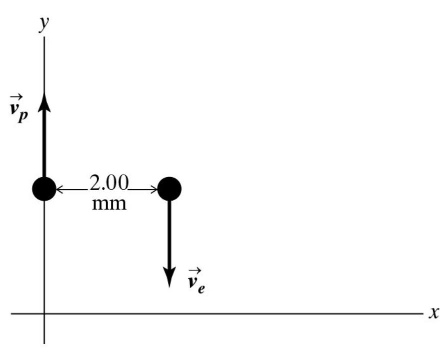


Figure VP28.2.2

EXECUTE: **(a)** We want the force that the proton exerts on the electron. See Fig. VP28.2.2. $\vec{F}_{\text{on e}} = q_e\vec{v}_e\times\vec{B}_p$, where $\vec{B}_p = \dfrac{\mu_0}{4\pi}\dfrac{q_p\vec{v}_p\times\hat{r}}{r^2}$. Putting these quantities together gives $F_{\text{on e}} = (ev)\left(\dfrac{\mu_0 ve}{4\pi r^2}\right) = \dfrac{\mu_0}{4\pi}\dfrac{v^2e^2}{r^2}$. The field due to the proton points in the $-z$-direction by the right-hand rule. The velocity of the electron is in the $-y$-direction, so the force on the electron (which is *negatively charged*) is in the $-x$-direction. Using $r = 2.00$ mm, $\phi = 90°$, and $v_e = 420$ km/s gives $F_{\text{on e}} = 1.13\times10^{-28}$ N in the $-x$-direction, or $-\hat{i}$ direction.

(b) We want the force that the electron exerts on the proton. Follow exactly the same approach as in part (a). The force on the proton has the same magnitude as the force on the electron but it points in the opposite direction. So $F_{\text{on p}} = 1.13\times10^{-28}$ N in the $+\hat{i}$ direction.

EVALUATE: The two forces are equal but opposite, which agrees with Newton's third law (action-reaction). Also note that the two charges attract each other, but this is *not* the same as the Coulomb force. That force would be $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{r^2} = \;= 2.30\times10^{-22}\text{ N},$ which is roughly 2 million times stronger than the magnetic attraction.

VP28.2.3. IDENTIFY: We are looking at the magnetic field due to a very small wire segment.

SET UP: $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\hat{l}\times\hat{r}}{r^2}$.

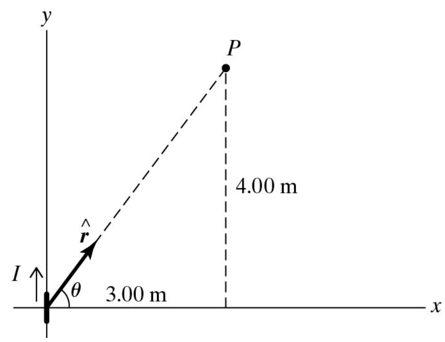


Figure VP28.2.3

EXECUTE: **(a)** We want $\hat{r}$. See Fig. VP28.2.3. We see that $\theta = \arctan(4.00/3.00) = 53.13°$. Also $|\hat{r}| = 1$. So $\hat{r} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j} = \;= \cos 53.13°\,\hat{i} + \sin 53.13°\,\hat{j} = 0.600\,\hat{i} + 0.800\,\hat{j}$.

(b) We want $\vec{B}$. Use $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\hat{l}\times\hat{r}}{r^2}$. $d\vec{l} = dl\hat{j} = (0.00200\text{ m})\hat{j}$. Using $\hat{r}$ and $d\vec{l}$ and taking the cross product with the given numbers, we get $d\vec{B} = -2.88\times10^{-11}\text{ T }\hat{k}$.

EVALUATE: This is a very small field, but the wire segment is also very small.

VP28.2.4. IDENTIFY: We want to find the magnetic force on a moving charged particle due to a tiny current-carrying wire segment.

SET UP: $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\hat{l}\times\hat{r}}{r^2}$, $\vec{F} = q\vec{v}\times\vec{B}$, $F = |q|vB\sin\phi$. Sketch the situation as in Fig. VP28.2.4.

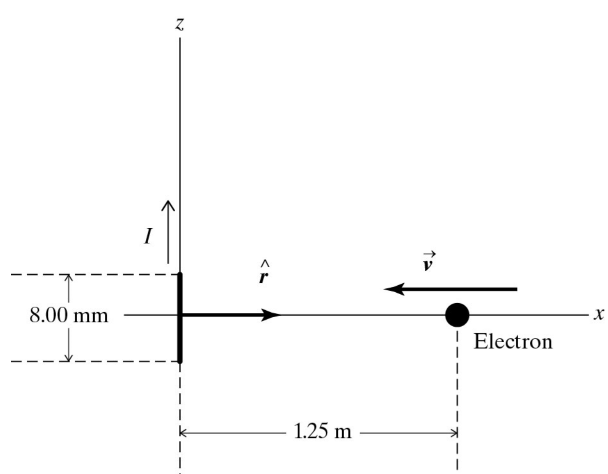


Figure VP28.2.4

EXECUTE: $\vec{F}_{\text{one}} = q_e \vec{v}_e \times \vec{B}_{\text{wire}}$. At the location of the electron, the field due to the wire points into the paper, which is in the $-y$-direction. Therefore the magnetic force on the electron is in the $+z$-direction by the right-hand rule (remember that the electron is *negative*). Combining $F = |q| vB \sin\phi$ and $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$ gives $\vec{F}_{\text{one}} = \frac{\mu_0}{4\pi}\frac{evdl}{r^2}\hat{k}$. Using the given numbers, we have $\vec{F} = 9.83\times 10^{-23}\ \text{N}\ \hat{k}$.

EVALUATE: Magnetic forces are typically very small compared to electrostatic forces, but not always.

VP28.5.1. **IDENTIFY** and **SET UP:** We want to find the magnetic field due to a long current-carrying wire. $B = \frac{\mu_0}{2\pi}\frac{I}{r}$.

EXECUTE: Solve for I and use the given numbers. $I = \frac{2\pi r B}{\mu_0} = 2.24\ \text{A}$.

EVALUATE: This result is typical of many household currents, so it is clear that they produce small magnetic fields. This one is about 30 μT.

VP28.5.2. **IDENTIFY:** We want the net magnetic field $\vec{B}$ produced at various points by two long current-carrying wires.

SET UP: $B = \frac{\mu_0}{2\pi}\frac{I}{r}$. The net field is the vector sum of the two individual fields $\vec{B}_1$ and $\vec{B}_4$. Let the paper be the xy-plane with the currents coming out of the paper, as shown in Fig. VP28.5.2.

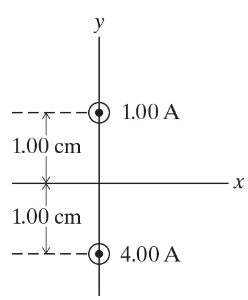


Figure VP28.5.2

EXECUTE: **(a)** At (0, 0, 0). This point is midway between the wires. $\vec{B}_1$ points in the $+x$-direction and $\vec{B}_4$ points in the $-x$-direction. $B = B_1 - B_4$. Using $B = \frac{\mu_0}{2\pi}\frac{I}{r}$ we get $B = \frac{\mu_0}{2\pi r}(I_1 - I_4)$. Using the given numbers gives $\vec{B} = -6.00\times 10^{-5}\ \text{T}\ \hat{i}$.

(b) At (0, 2.00 cm, 0). This point is on the y-axis 1.00 cm above the 1.00 A wire. Both fields point in the $-x$-direction. $B = B_1 + B_4$. Using $B = \frac{\mu_0}{2\pi}\frac{I}{r}$ for each wire gives $B = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} + \frac{I_4}{r_4}\right)$ which gives $\vec{B} = -4.67\times 10^{-5}\ \text{T}\ \hat{i}$.

(c) At (0, –2.00 cm, 0). This point is 1.00 cm below the 4.00 A wire. Both fields point in the $+x$-direction, so $B = B_1 + B_4$ and $B = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} + \frac{I_4}{r_4}\right)$. Using the numbers gives $\vec{B} = 8.67\times 10^{-5}\ \text{T}\ \hat{i}$.

EVALUATE: The net field in (c) is the greatest of the three since this point is closer to the larger current, so this is a reasonable result.

VP28.5.3. **IDENTIFY:** We use the magnetic field due to a very long current-carrying wire.

SET UP: $B = \frac{\mu_0}{2\pi}\frac{I}{r}$. The net field $\vec{B}$ is the vector sum of the two individual fields $\vec{B}_1$ and $\vec{B}_2$. We know $I_1 = 2.00$ A and the net field, and we want to find I_2. See Fig. VP28.5.3.

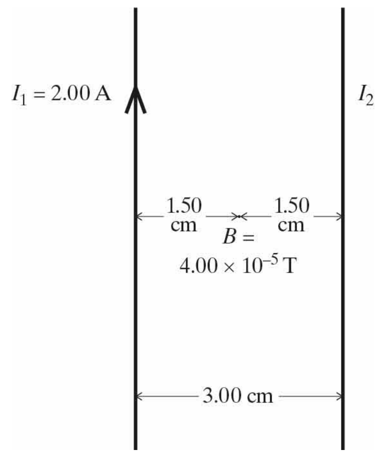


Figure VP28.5.3

EXECUTE: **(a)** Both currents are in the same direction. Their magnetic fields point in opposite directions in the region between the wires, so $B = B_2 - B_1$. Using $B = \frac{\mu_0}{2\pi}\frac{I}{r}$ for each field and solving for I_2 gives $I_2 = \frac{Br}{\mu_0/2\pi} + I_1$. Using $r = 1.50$ cm and the other given numbers, we get $I_2 = 5.00$ A.

(b) The currents are in opposite directions. Now both fields point into the paper, so $B = B_2 + B_1$. Using $B = \frac{\mu_0}{2\pi}\frac{I}{r}$ for each field and solving for I_2 gives $I_2 = \frac{Br}{\mu_0/2\pi} - I_1$. Using $r = 1.50$ cm and the other given numbers, gives $I_2 = 1.00$ A.

EVALUATE: A clear sketch is important to help decided which way the fields point and if their magnitudes should be added or subtracted.

VP28.5.4. **IDENTIFY:** This problem involves the magnetic force between two long current-carrying wires.

SET UP: $F/L = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$. The target variable is I_2.

EXECUTE: The wires attract each other so their currents must be in the *same* direction, which is *upward*. Solving $F/L = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$ for I_2 gives $I_2 = \frac{r(F/L)}{I_1(\mu_0/2\pi)}$. Using the given quantities with $r = 0.900$ m gives $I_2 = 3.29 \times 10^4$ A.

EVALUATE: Like currents (i.e. in the same direction) attract while unlike currents (opposite direction) repel.

VP28.10.1. **IDENTIFY:** This problem deals with the magnetic field inside a current-carrying conductor.

SET UP: The target variable is the magnetic field at various distances from the axis. Inside $B = \frac{\mu_0 I}{2\pi}\frac{r}{R^2}$ and outside $B = \frac{\mu_0}{2\pi}\frac{I}{r}$.

EXECUTE: (a) Inside: Use $B = \frac{\mu_0 I}{2\pi}\frac{r}{R^2}$ with the given quantities, giving $I = 4.94\ \mu\text{T}$.

(b) At the surface: Use either formula with $r = R$. $B = \frac{\mu_0}{2\pi}\frac{I}{R} = 8.89\ \mu\text{T}$.

(c) Outside: Use $B = \frac{\mu_0}{2\pi}\frac{I}{r} = 6.67\ \mu\text{T}$.

EVALUATE: Using $B = \frac{\mu_0 I}{2\pi}\frac{r}{R^2}$ at the surface gives $B = \frac{\mu_0 I}{2\pi}\frac{R}{R^2} = \frac{\mu_0 I}{2\pi R}$, which is the equation for outside the cylinder evaluated when $r = R$. Note that B is largest at the *surface* of the cylinder.

VP28.10.2. IDENTIFY: We are dealing with the magnetic field produced by a hollow cylindrical conductor. We will need to use Ampere's law to find the field.

SET UP: Ampere's law is $\oint \vec{B} \cdot d\hat{l} = \mu_0 I_{\text{encl}}$, $J = I/A$. We want to find B at several locations.

EXECUTE: (a) For $r < R_1$: Use $\oint \vec{B} \cdot d\hat{l} = \mu_0 I_{\text{encl}}$ and as an integration path use a circle of radius $r < R_1$. From Ampere's law this gives $B(2\pi r) = \mu_0 I_{\text{encl}} = 0$, which means that $B = 0$

For $R_1 < r < R_2$: Use the same path as above except $R_1 < r < R_2$. $I_{\text{encl}} = JA_{\text{encl}} = J\left(\pi r^2 - \pi R_1^2\right) = J\pi\left(r^2 - R_1^2\right)$. Ampere's law becomes $B(2\pi r) = \mu_0 J\pi\left(r^2 - R_1^2\right)$. This gives $B = \frac{\mu_0 J}{2r}\left(r^2 - R_1^2\right)$.

For $r > R_2$: Use the same path as before except $r > R_2$. $I_{\text{encl}} = J\pi\left(R_2^2 - R_1^2\right)$, so $B(2\pi r) = \mu_0 J\pi\left(R_2^2 - R_1^2\right)$. This gives $B = \frac{\mu_0 J\left(R_2^2 - R_1^2\right)}{2r}$.

(b) Where is B a maximum? For $r < R_1$, $B = 0$ so it can't be in that region. For $r > R_2$, B decreases as r increases so it cannot be outside. Investigate $R_1 < r < R_2$. For a maximum, $dB/dr = 0$. Using $B = \frac{\mu_0 J}{2r}\left(r^2 - R_1^2\right)$, $\frac{dB}{dr} = \frac{\mu_0 J}{2}\left(1 + \frac{R_1^2}{r^2}\right) = 0$. There are no real solutions to this equation, so there is no *relative* maximum in this region. But at $r = 0$, $B = 0$ and at $r = R_2$, $B = \frac{\mu_0 J}{2}\left(R_2 - \frac{R_1^2}{R_2}\right)$ which is greater than zero. So in this region B keeps increasing as r increases and reaches its maximum value at $r = R_2$. For $r > R_2$, B decreases with r, so B_{max} occurs at $r = R_2$.

EVALUATE: Our result is physically reasonable. As r increases from R_1 to R_2, more and more current contributes to the field, so B increases.

VP28.10.3. IDENTIFY and SET UP: We have the magnetic field inside a solenoid. $B = \mu_0 nI$. The current is the target variable.

EXECUTE: Solve for I and use the given numbers. $I = \frac{B}{\mu_0 n} = 3.18\ \text{A}$.

EVALUATE: This is a reasonable amount of current.

VP28.10.4. IDENTIFY and SET UP: We have a toroidal solenoid. $B = \frac{\mu_0 NI}{2\pi r}$.

EXECUTE: (a) We want N. Solve for N and use the given values. $N = \frac{Br}{I(\mu_0/2\pi)} = 833$ turns.

(b) We want to find the maximum and minimum field inside the toroidal solenoid. B_{max} occurs at r = 6.00 cm. Evaluate $B = \dfrac{\mu_0 NI}{2\pi r}$ when r = 6.00 cm, giving B_{max} = 2.33 mT. B_{min} occurs when r = 8.00 cm. Using this value gives B_{min} = 1.75 mT.

EVALUATE: For an ideal toroidal solenoid, B = 0 outisde of it (r > 8.00 cm) and inside the opening (r < 6.00 cm). For a *real* toroidal solenoid this is not quite true.

28.1. IDENTIFY and **SET UP:** Use $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$ to calculate $\vec{B}$ at each point.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}, \text{ since } \hat{r} = \frac{\vec{r}}{r}.$$

$\vec{v} = (8.00\times10^6 \text{ m/s})\hat{j}$ and $\vec{r}$ is the vector from the charge to the point where the field is calculated.

EXECUTE: **(a)** $\vec{r} = (0.500 \text{ m})\hat{i}, r = 0.500 \text{ m}.$

$\vec{v}\times\vec{r} = vr\hat{j}\times\hat{i} = -vr\hat{k}.$

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k} = -(1\times10^{-7} \text{ T}\cdot\text{m/A})\frac{(6.00\times10^{-6} \text{ C})(8.00\times10^6 \text{ m/s})}{(0.500 \text{ m})^2}\hat{k}.$$

$\vec{B} = -(1.92\times10^{-5} \text{ T})\hat{k}.$

(b) $\vec{r} = -(0.500 \text{ m})\hat{j}, r = 0.500 \text{ m}.$

$\vec{v}\times\vec{r} = -vr\hat{j}\times\hat{j} = 0$ and $\vec{B} = 0.$

(c) $\vec{r} = (0.500 \text{ m})\hat{k}, r = 0.500 \text{ m}.$

$\vec{v}\times\vec{r} = vr\hat{j}\times\hat{k} = vr\hat{i}.$

$$\vec{B} = (1\times10^{-7} \text{ T}\cdot\text{m/A})\frac{(6.00\times10^{-6} \text{ C})(8.00\times10^6 \text{ m/s})}{(0.500 \text{ m})^2}\hat{i} = +(1.92\times10^{-5} \text{ T})\hat{i}.$$

(d) $\vec{r} = -(0.500 \text{ m})\hat{j} + (0.500 \text{ m})\hat{k}, r = \sqrt{(0.500 \text{ m})^2 + (0.500 \text{ m})^2} = 0.7071 \text{ m}.$

$\vec{v}\times\vec{r} = v(0.500 \text{ m})(-\hat{j}\times\hat{j} + \hat{j}\times\hat{k}) = (4.00\times10^6 \text{ m}^2/\text{s})\hat{i}.$

$$\vec{B} = (1\times10^{-7} \text{ T}\cdot\text{m/A})\frac{(6.00\times10^{-6} \text{ C})(4.00\times10^6 \text{ m}^2/\text{s})}{(0.7071 \text{ m})^3}\hat{i} = +(6.79\times10^{-6} \text{ T})\hat{i}.$$

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{v}$ and $\vec{r}$. $B = 0$ along the direction of $\vec{v}$.

28.2. IDENTIFY: A moving charge creates a magnetic field as well as an electric field.

SET UP: The magnetic field caused by a moving charge is $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$, and its electric field is

$E = \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{r^2}$ since $q = e$.

EXECUTE: Substitute the appropriate numbers into the above equations.

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2} = \frac{4\pi\times10^{-7} \text{ T}\cdot\text{m/A}}{4\pi}\frac{(1.60\times10^{-19}\text{C})(2.2\times10^6\text{m/s})\sin 90°}{(5.3\times10^{-11}\text{m})^2} = 13 \text{ T, out of the page.}$$

$$E = \frac{1}{4\pi_0}\frac{e}{r^2} = \frac{(9.00\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{C})}{(5.3\times10^{-11}\text{m})^2} = 5.1\times10^{11}\text{N/C, toward the electron.}$$

EVALUATE: There are enormous fields within the atom!

28.3. IDENTIFY: A moving charge creates a magnetic field.

SET UP: The magnetic field due to a moving charge is $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$.

EXECUTE: Substituting numbers into the above equation gives

(a) $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2} = \dfrac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\dfrac{(1.6\times10^{-19}\text{C})(3.0\times10^{7}\,\text{m/s})\sin 30^\circ}{(2.00\times10^{-6}\text{m})^2}.$

$B = 6.00\times10^{-8}$ T, out of the paper, and it is the same at point B.

(b) $B = (1.00\times10^{-7}\ \text{T}\cdot\text{m/A})(1.60\times10^{-19}\ \text{C})(3.00\times10^{7}\ \text{m/s})/(2.00\times10^{-6}\ \text{m})^2.$

$B = 1.20\times10^{-7}$ T, out of the page.

(c) $B = 0$ T since $\sin(180^\circ) = 0$.

EVALUATE: Even at high speeds, these charges produce magnetic fields much less than the earth's magnetic field.

28.4. **IDENTIFY:** Both moving charges produce magnetic fields, and the net field is the vector sum of the two fields.

SET UP: Both fields point out of the paper, so their magnitudes add, giving

$$B = B_{\text{alpha}} + B_{\text{el}} = \frac{\mu_0 v}{4\pi r^2}(e\sin 40^\circ + 2e\sin 140^\circ).$$

EXECUTE: Factoring out an e and putting in the numbers gives

$$B = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.60\times10^{-19}\ \text{C})(2.50\times10^{5}\ \text{m/s})}{(8.65\times10^{-9}\ \text{m})^2}(\sin 40^\circ + 2\sin 140^\circ).$$

$B = 1.03\times10^{-4}$ T $= 0.103$ mT, out of the page.

EVALUATE: At distances very close to the charges, the magnetic field is strong enough to be important.

28.5. **IDENTIFY:** Apply $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\vec{r}}{r^3}.$

SET UP: Since the charge is at the origin, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

EXECUTE: **(a)** $\vec{v} = v\vec{i}$, $\vec{r} = r\hat{i}$; $\vec{v}\times\vec{r} = 0$, $B = 0$.

(b) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{j}$; $\vec{v}\times\vec{r} = vr\hat{k}$, $r = 0.500$ m.

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^{5}\ \text{m/s})}{(0.500\ \text{m})^2} = 1.31\times10^{-6}\ \text{T}.$$

q is negative, so $\vec{B} = -(1.31\times10^{-6}\ \text{T})\hat{k}$.

(c) $\vec{v} = v\hat{i}$, $\vec{r} = (0.500\ \text{m})(\hat{i} + \hat{j})$; $\vec{v}\times\vec{r} = (0.500\ \text{m})v\hat{k}$, $r = 0.7071$ m.

$$B = \left(\frac{\mu_0}{4\pi}\right)\left(|q||\vec{v}\times\vec{r}|/r^3\right) = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(0.500\ \text{m})(6.80\times10^{5}\ \text{m/s})}{(0.7071\ \text{m})^3}.$$

$B = 4.62\times10^{-7}$ T. $\vec{B} = -(4.62\times10^{-7}\ \text{T})\hat{k}$.

(d) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{k}$; $\vec{v}\times\vec{r} = -vr\hat{j}$, $r = 0.500$ m.

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^{5}\ \text{m/s})}{(0.500\ \text{m})^2} = 1.31\times10^{-6}\ \text{T}.$$

$\vec{B} = (1.31\times10^{-6}\ \text{T})\hat{j}$.

EVALUATE: In each case, $\vec{B}$ is perpendicular to both $\vec{r}$ and $\vec{v}$.

28.6. IDENTIFY: Apply $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$. For the magnetic force, apply the results of Example 28.1, except here the two charges and velocities are different.

SET UP: In part (a), $r = d$ and $\vec{r}$ is perpendicular to $\vec{v}$ in each case, so $\frac{|\vec{v}\times\vec{r}|}{r^3} = \frac{v}{d^2}$. For calculating the force between the charges, $r = 2d$.

EXECUTE: **(a)** $B_{\text{total}} = B + B' = \frac{\mu_0}{4\pi}\left(\frac{qv}{d^2} + \frac{q'v'}{d^2}\right)$.

$$B = \frac{\mu_0}{4\pi}\left(\frac{(8.0\times10^{-6}\text{ C})(4.5\times10^{6}\text{ m/s})}{(0.120\text{ m})^2} + \frac{(3.0\times10^{-6}\text{ C})(9.0\times10^{6}\text{ m/s})}{(0.120\text{ m})^2}\right) = 4.38\times10^{-4}\text{ T}.$$

The direction of $\vec{B}$ is into the page.

(b) Following Example 28.1 we can find the magnetic force between the charges:

$$F_B = \frac{\mu_0}{4\pi}\frac{qq'vv'}{r^2} = (10^{-7}\text{ T}\cdot\text{m/A})\frac{(8.00\times10^{-6}\text{ C})(3.00\times10^{-6}\text{ C})(4.50\times10^{6}\text{ m/s})(9.00\times10^{6}\text{ m/s})}{(0.240\text{ m})^2}.$$

$F_B = 1.69\times10^{-3}$ N. The force on the upper charge points up and the force on the lower charge points down. The Coulomb force between the charges is

$F_C = k\frac{q_1q_2}{r^2} = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(8.0\times10^{-6}\text{ C})(3.0\times10^{-6}\text{ C})}{(0.240\text{ m})^2} = 3.75\text{ N}$. The force on the upper charge points up and the force on the lower charge points down. The ratio of the Coulomb force to the magnetic force is $\frac{F_C}{F_B} = \frac{c^2}{v_1v_2} = \frac{3.75\text{ N}}{1.69\times10^{-3}\text{ N}} = 2.22\times10^{3}$; the Coulomb force is much larger.

(c) The magnetic forces are reversed in direction when the direction of only one velocity is reversed but the magnitude of the force is unchanged.

EVALUATE: When two charges have the same sign and move in opposite directions, the force between them is repulsive. When two charges of the same sign move in the same direction, the force between them is attractive.

28.7. IDENTIFY: We want the magnetic field due to a moving charge.

SET UP: $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$, $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$.

EXECUTE: **(a)** The unit vector points from the proton to point *P*, which is to the left.

(b) We want *B*. $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$ and ϕ is either 0 or 180°. In either case $\sin\phi = 0$, so $B = 0$.

(c) We want $\vec{B}$. $\phi = 90°$ and $q = e$. Using the given quantities $B = \frac{\mu_0}{4\pi}\frac{ev}{r^2} = 1.21$ pT. By the right-hand rule, the cross product in $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$ is out of the page, so $\vec{B}$ is out of the page.

(d) For an electron, only the charge and mass are different, so *B* would be the same but the direction would be reversed. So $B = 1.21$ pT, into the page.

EVALUATE: The magnetic field due to a negative charge is opposite to the direction of $\vec{v}\times\hat{r}$.

28.8. IDENTIFY: Both moving charges create magnetic fields, and the net field is the vector sum of the two. The magnetic force on a moving charge is $F_{\text{mag}} = qvB\sin\phi$ and the electrical force obeys Coulomb's law.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$.

EXECUTE: **(a)** Both fields are into the page, so their magnitudes add,

giving $B = B_e + B_p = \dfrac{\mu_0}{4\pi}\left(\dfrac{ev}{r_e^2}+\dfrac{ev}{r_p^2}\right)\sin 90°.$

$$B = \frac{\mu_0}{4\pi}(1.60\times10^{-19}\,\text{C})(735{,}000\text{ m/s})\left[\frac{1}{(5.00\times10^{-9}\,\text{m})^2}+\frac{1}{(4.00\times10^{-9}\,\text{m})^2}\right].$$

$$B = 1.21\times10^{-3}\text{ T} = 1.21\text{ mT, into the page.}$$

(b) Using $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$, where $r = \sqrt{41}$ nm and $\phi = 180° - \arctan(5/4) = 128.7°$, we get

$$B = \frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(1.6\times10^{-19}\,\text{C})(735{,}000\text{ m/s})\sin 128.7°}{(\sqrt{41}\times10^{-9}\,\text{m})^2} = 2.24\times10^{-4}\text{ T, into the page.}$$

(c) $F_{mag} = qvB\sin 90° = (1.60\times10^{-19}\text{ C})(735{,}000\text{ m/s})(2.24\times10^{-4}\text{ T}) = 2.63\times10^{-17}\text{ N}$, in the $+x$-direction.

$$F_{elec} = (1/4\pi\epsilon_0)e^2/r^2 = \frac{(9.00\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{C})^2}{(\sqrt{41}\times10^{-9}\,\text{m})^2} = 5.62\times10^{-12}\text{ N, at } 129°$$

counterclockwise from the $+x$-axis.

EVALUATE: The electric force is over 200,000 times as strong as the magnetic force.

28.9. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\phi}{r^2}$.

EXECUTE: Applying the law of Biot and Savart gives

(a) $dB = \dfrac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\dfrac{(10.0\text{ A})(0.00110\text{ m})\sin 90°}{(0.0500\text{ m})^2} = 4.40\times10^{-7}\text{ T}$, out of the paper.

(b) The same as above, except $r = \sqrt{(5.00\text{ cm})^2 + (14.0\text{ cm})^2}$ and $\phi = \arctan(5/14) = 19.65°$, giving $dB = 1.67\times10^{-8}\text{ T}$, out of the page.

(c) $dB = 0$ since $\phi = 0°$.

EVALUATE: This is a very small field, but it comes from a very small segment of current.

28.10. **IDENTIFY:** Apply the Biot-Savart law.

SET UP: Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{qd\vec{l}\times\vec{r}}{r^3}$. $r = \sqrt{(-0.730\text{ m})^2 + (0.390\text{ m})^2} = 0.8276\text{ m}.$

EXECUTE:

$d\vec{l}\times\vec{r} = [0.500\times10^{-3}\text{ m}]\hat{j}\times[(-0.730\text{ m})\hat{i} + (0.390\text{ m})\hat{k}] = (+3.65\times10^{-4}\text{ m}^2)\hat{k} + (+1.95\times10^{-4}\text{ m}^2)\hat{i}.$

$d\vec{B} = (1.00\times10^{-7}\text{ T}\cdot\text{m/A})\dfrac{5.40\text{ A}}{(0.8276\text{ m})^3}[(3.65\times10^{-4}\text{ m}^2)\hat{k} + (1.95\times10^{-4}\text{ m}^2)\hat{i}].$

$d\vec{B} = (1.86\times10^{-10}\text{ T})\hat{i} + (3.48\times10^{-10}\text{ T})\hat{k}.$

EVALUATE: The magnetic field lies in the xz-plane.

28.11. **IDENTIFY** and **SET UP:** The magnetic field produced by an infinitesimal current element is given by $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\vec{l}\times\hat{r}}{r^2}$.

As in Example 28.2, use $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\vec{l}\times\hat{r}}{r^2}$ for the finite 0.500-mm segment of wire since the $\Delta l = 0.500\text{-mm}$ length is much smaller than the distances to the field points.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l} \times \vec{r}}{r^3}$$

I is in the $+z$-direction, so $\Delta\vec{l} = (0.500\times10^{-3}\ \text{m})\hat{k}$.

EXECUTE: **(a)** The field point is at $x = 2.00$ m, $y = 0$, $z = 0$ so the vector $\vec{r}$ from the source point (at the origin) to the field point is $\vec{r} = (2.00\ \text{m})\hat{i}$.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\ \text{m})(2.00\ \text{m})\hat{k} \times \hat{i} = +(1.00\times10^{-3}\ \text{m}^2)\hat{j}$.

$$\vec{B} = \frac{(1\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\ \text{A})(1.00\times10^{-3}\ \text{m}^2)}{(2.00\ \text{m})^3}\hat{j} = (5.00\times10^{-11}\ \text{T})\hat{j}.$$

(b) $\vec{r} = (2.00\ \text{m})\hat{j}$, $r = 2.00$ m.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\ \text{m})(2.00\ \text{m})\hat{k} \times \hat{j} = -(1.00\times10^{-3}\ \text{m}^2)\hat{i}$.

$$\vec{B} = \frac{(1\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\ \text{A})(-1.00\times10^{-3}\ \text{m}^2)}{(2.00\ \text{m})^3}\hat{i} = -(5.00\times10^{-11}\ \text{T})\hat{i}.$$

(c) $\vec{r} = (2.00\ \text{m})(\hat{i} + \hat{j})$, $r = \sqrt{2}(2.00\ \text{m})$.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\ \text{m})(2.00\ \text{m})\hat{k} \times (\hat{i} + \hat{j}) = (1.00\times10^{-3}\ \text{m}^2)(\hat{j} - \hat{i})$.

$$\vec{B} = \frac{(1\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\ \text{A})(1.00\times10^{-3}\ \text{m}^2)}{\left[\sqrt{2}(2.00\ \text{m})\right]^3}(\hat{j} - \hat{i}) = (-1.77\times10^{-11}\ \text{T})(\hat{i} - \hat{j}).$$

(d) $\vec{r} = (2.00\ \text{m})\hat{k}$, $r = 2.00$ m.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\ \text{m})(2.00\ \text{m})\hat{k} \times \hat{k} = 0$; $\vec{B} = 0$.

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{r}$ and $\Delta\vec{l}$. $B = 0$ along the length of the wire.

28.12. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\phi}{r^2}$.

Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the law of Biot and Savart for the 12.0-A current

gives $dB = \dfrac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\dfrac{(12.0\ \text{A})(0.00150\ \text{m})\left(\dfrac{2.50\ \text{cm}}{8.00\ \text{cm}}\right)}{(0.0800\ \text{m})^2} = 8.79\times10^{-8}\ \text{T}$.

The field from the 24.0-A segment is twice this value, so the total field is 2.64×10^{-7} T, into the page.

EVALUATE: The rest of each wire also produces field at P. We have calculated just the field from the two segments that are indicated in the problem.

28.13. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\phi}{r^2}$. Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the Biot and Savart law, where $r = \frac{1}{2}\sqrt{(3.00\ \text{cm})^2 + (3.00\ \text{cm})^2} = 2.121\ \text{cm}$, we have $dB = 2\dfrac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\dfrac{(28.0\ \text{A})(0.00200\ \text{m})\sin 45.0°}{(0.02121\ \text{m})^2} = 1.76\times10^{-5}\ \text{T}$, into the paper.

EVALUATE: Even though the two wire segments are at right angles, the magnetic fields they create are in the same direction.

28.14. **IDENTIFY:** This problem involves the magnetic field due to a long wire and the force on a charge moving in that field.

SET UP: $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$, $\vec{F} = q\vec{v}\times\vec{B}$, $\sum F_x = ma_x$, $\sum F_y = ma_y$. We want the velocity components of the particle. The given acceleration is so large that we can neglect the effects of gravity.

EXECUTE: The magnetic field at the location of the particle (call this point P) points in the $+z$-direction. At that point we use the given quantities and $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ to find $B = 0.150$ mT.

Applying $\sum F_x = ma_x$ gives $qv_yB = ma_x$, so $v_y = \dfrac{ma_x}{qB}$. Using the given values we get $v_y = -12.5$ km/s. Now apply $\sum F_y = ma_y$. $qv_xB = ma_y$, which gives $v_x = \dfrac{ma_y}{qB}$. Using the given values gives $v_x = -22.5$ km/s.

EVALUATE: Notice that the x component of the velocity affects the y component of the acceleration and likewise v_y affects a_x.

28.15. **IDENTIFY:** We can model the lightning bolt and the household current as very long current-carrying wires.

SET UP: The magnetic field produced by a long wire is $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: Substituting the numerical values gives

(a) $B = \dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(20{,}000\ \text{A})}{2\pi(5.0\ \text{m})} = 8\times10^{-4}\ \text{T}.$

(b) $B = \dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(10\ \text{A})}{2\pi(0.050\ \text{m})} = 4.0\times10^{-5}\ \text{T}.$

EVALUATE: The field from the lightning bolt is about 20 times as strong as the field from the household current.

28.16. **IDENTIFY:** The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: First find the current: $I = (8.20\times10^{18}\ \text{el/s})(1.60\times10^{-19}\ \text{C/el}) = 1.312\ \text{A}.$

Now find the magnetic field: $\dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1.312\ \text{A})}{2\pi(0.0400\ \text{m})} = 6.56\times10^{-6}\ \text{T} = 6.56\ \mu\text{T}.$

Since electrons are negative, the conventional current runs from east to west, so the magnetic field above the wire points toward the north.

EVALUATE: This magnetic field is much less than that of the earth, so any experiments involving such a current would have to be shielded from the earth's magnetic field, or at least would have to take it into consideration.

28.17. **IDENTIFY:** We can model the current in the heart as that of a long straight wire. It produces a magnetic field around it.

SET UP: For a long straight wire, $B = \dfrac{\mu_0 I}{2\pi r}$. $\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$. 1 gauss $= 10^{-4}$ T.

EXECUTE: Solving for the current gives

$$I = \frac{2\pi rB}{\mu_0} = \frac{2\pi(0.050\ \text{m})(1.0\times10^{-9}\ \text{T})}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}} = 25\times10^{-5}\ \text{A} = 250\ \mu\text{A}.$$

EVALUATE: By household standards, this is a very small current. But the magnetic field around the heart $(\approx 10\ \mu\text{G})$ is also very small.

28.18. **IDENTIFY:** For each wire $B = \dfrac{\mu_0 I}{2\pi r}$, and the direction of $\vec{B}$ is given by the right-hand rule (Figure 28.6 in the textbook). Add the field vectors for each wire to calculate the total field.

(a) SET UP: The two fields at this point have the directions shown in Figure 28.18a.

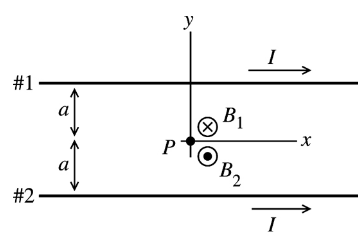


Figure 28.18a

EXECUTE: At point P midway between the two wires the fields $\vec{B}_1$ and $\vec{B}_2$ due to the two currents are in opposite directions, so $B = B_2 - B_1$.

But $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi a}$, so $B = 0$.

(b) SET UP: The two fields at this point have the directions shown in Figure 28.18b.

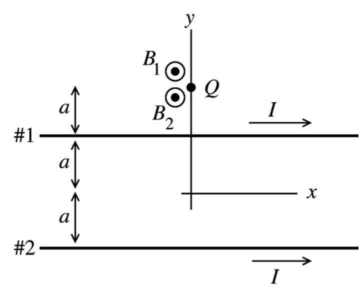


Figure 28.18b

EXECUTE: At point Q above the upper wire $\vec{B}_1$ and $\vec{B}_2$ are both directed out of the page ($+z$-direction), so $B = B_1 + B_2$.

$$B_1 = \frac{\mu_0 I}{2\pi a},\ B_2 = \frac{\mu_0 I}{2\pi(3a)}.$$

$$B = \frac{\mu_0 I}{2\pi a}\left(1 + \tfrac{1}{3}\right) = \frac{2\mu_0 I}{3\pi a};\ \vec{B} = \frac{2\mu_0 I}{3\pi a}\hat{\boldsymbol{k}}.$$

(c) SET UP: The two fields at this point have the directions shown in Figure 28.18c.

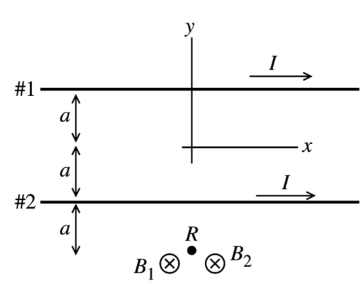


Figure 28.18c

EXECUTE: At point R below the lower wire $\vec{B}_1$ and $\vec{B}_2$ are both directed into the page ($-z$-direction), so $B = B_1 + B_2$.

$$B_1 = \frac{\mu_0 I}{2\pi(3a)},\ B_2 = \frac{\mu_0 I}{2\pi a}.$$

$B_1 = \dfrac{\mu_0 I}{2\pi a}\left(1+\tfrac{1}{3}\right) = \dfrac{2\mu_0 I}{3\pi a}$; $\vec{B} = -\dfrac{2\mu_0 I}{3\pi a}\hat{\boldsymbol{k}}$.

EVALUATE: In the figures we have drawn, $\vec{B}$ due to each wire is out of the page at points above the wire and into the page at points below the wire. If the two field vectors are in opposite directions the magnitudes subtract.

28.19. IDENTIFY: The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field.

SET UP: For the wire, $B_{\text{wire}} = \dfrac{\mu_0 I}{2\pi r}$ and the direction of B_{wire} is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook. $\vec{B}_0 = (1.50\times10^{-6}\ \text{T})\hat{\boldsymbol{i}}$.

EXECUTE: **(a)** At (0, 0, 1 m), $\vec{B} = \vec{B}_0 - \dfrac{\mu_0 I}{2\pi r}\hat{\boldsymbol{i}} = (1.50\times10^{-6}\ \text{T})\hat{\boldsymbol{i}} - \dfrac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{\boldsymbol{i}} = -(1.0\times10^{-7}\ \text{T})\hat{\boldsymbol{i}}$.

(b) At (1 m, 0, 0), $\vec{B} = \vec{B}_0 + \dfrac{\mu_0 I}{2\pi r}\hat{\boldsymbol{k}} = (1.50\times10^{-6}\ \text{T})\hat{\boldsymbol{i}} + \dfrac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{\boldsymbol{k}}$.

$\vec{B} = (1.50\times10^{-6}\ \text{T})\hat{\boldsymbol{i}} + (1.6\times10^{-6}\ \text{T})\hat{\boldsymbol{k}} = 2.19\times10^{-6}$ T, at $\theta = 46.8°$ from x to z.

(c) At $(0, 0, -0.25\ \text{m})$, $\vec{B} = \vec{B}_0 + \dfrac{\mu_0 I}{2\pi r}\hat{\boldsymbol{i}} = (1.50\times10^{-6}\ \text{T})\hat{\boldsymbol{i}} + \dfrac{\mu_0(8.00\ \text{A})}{2\pi(0.25\ \text{m})}\hat{\boldsymbol{i}} = (7.9\times10^{-6}\ \text{T})\hat{\boldsymbol{i}}$.

EVALUATE: At point c the two fields are in the same direction and their magnitudes add. At point a they are in opposite directions and their magnitudes subtract. At point b the two fields are perpendicular.

28.20. IDENTIFY: The magnetic field is that of a long current-carrying wire.

SET UP: $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{(2.0\times10^{-7}\ \text{T}\cdot\text{m/A})(150\ \text{A})}{8.0\ \text{m}} = 3.8\times10^{-6}\ \text{T}$. This is 7.5% of the earth's field.

EVALUATE: Since this field is much smaller than the earth's magnetic field, it would be expected to have less effect than the earth's field.

28.21. IDENTIFY: $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

SET UP: Call the wires a and b, as indicated in Figure 28.21. The magnetic fields of each wire at points P_1 and P_2 are shown in Figure 28.21a. The fields at point 3 are shown in Figure 28.21b.

EXECUTE: **(a)** At P_1, $B_a = B_b$ and the two fields are in opposite directions, so the net field is zero.

(b) $B_a = \dfrac{\mu_0 I}{2\pi r_a}$. $B_b = \dfrac{\mu_0 I}{2\pi r_b}$. $\vec{B}_a$ and $\vec{B}_b$ are in the same direction so

$$B = B_a + B_b = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_a}+\frac{1}{r_b}\right) = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\ \text{A})}{2\pi}\left[\frac{1}{0.300\ \text{m}}+\frac{1}{0.200\ \text{m}}\right] = 6.67\times10^{-6}\ \text{T}.$$

$\vec{B}$ has magnitude $6.67\ \mu\text{T}$ and is directed toward the top of the page.

(c) In Figure 28.21b, $\vec{B}_a$ is perpendicular to $\vec{r}_a$ and $\vec{B}_b$ is perpendicular to $\vec{r}_b$. $\tan\theta = \dfrac{5\ \text{cm}}{20\ \text{cm}}$ and $\theta = 14.04°$. $r_a = r_b = \sqrt{(0.200\ \text{m})^2 + (0.050\ \text{m})^2} = 0.206\ \text{m}$ and $B_a = B_b$.

$$B = B_a\cos\theta + B_b\cos\theta = 2B_a\cos\theta = 2\left(\frac{\mu_0 I}{2\pi r_a}\right)\cos\theta = \frac{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.0\ \text{A})\cos 14.04°}{2\pi(0.206\ \text{m})} = 7.54\ \mu\text{T}$$

B has magnitude $7.53\ \mu\text{T}$ and is directed to the left.

EVALUATE: At points directly to the left of both wires the net field is directed toward the bottom of the page.

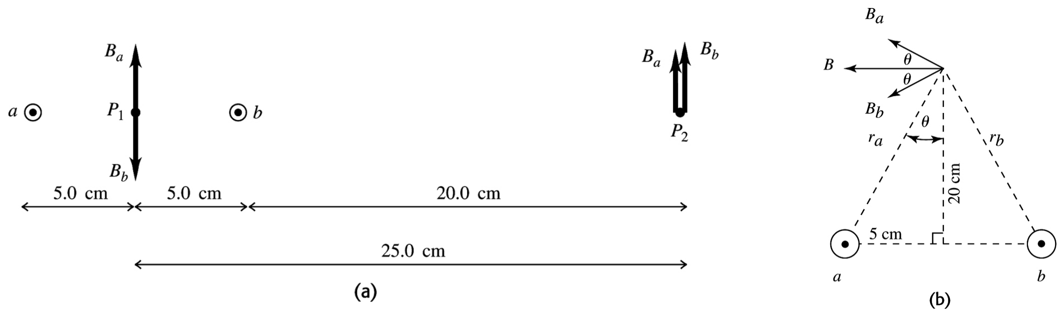


Figure 28.21

28.22. **IDENTIFY:** Each segment of the rectangular loop creates a magnetic field at the center of the loop, and all these fields are in the same direction.

SET UP: The field due to each segment is $B = \dfrac{\mu_0 I}{4\pi}\dfrac{2a}{x\sqrt{x^2+a^2}}$. $\vec{B}$ is into paper so I is clockwise around the loop.

EXECUTE: Long sides: $a = 4.75$ cm. $x = 2.10$ cm. For the two long sides,

$$B = 2(1.00\times10^{-7}\ \text{T}\cdot\text{m/A})I\frac{2(4.75\times10^{-2}\ \text{m})}{(2.10\times10^{-2}\ \text{m})\sqrt{(0.0210\ \text{m})^2+(0.0475\ \text{m})^2}} = (1.742\times10^{-5}\ \text{T/A})I.$$

Short sides: $a = 2.10$ cm. $x = 4.75$ cm. For the two short sides,

$$B = 2(1.00\times10^{-7}\ \text{T}\cdot\text{m/A})I\frac{2(2.10\times10^{-2}\ \text{m})}{(4.75\times10^{-2}\ \text{m})\sqrt{(0.0475\ \text{m})^2+(0.0210\ \text{m})^2}} = (3.405\times10^{-6}\ \text{T/A})I.$$

Using the known field, we have $B = (2.082\times10^{-5}\ \text{T/A})I = 5.50\times10^{-5}$ T, which gives $I = 2.64$ A.

EVALUATE: This is a typical household current, yet it produces a magnetic field which is about the same as the earth's magnetic field.

28.23. **IDENTIFY:** The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

SET UP: For each wire, $B = \dfrac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook.

EXECUTE: **(a)** and **(b)** $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.

(c) The fields due to each wire are sketched in Figure 28.23.

$$B = B_a\cos45^\circ + B_b\cos45^\circ + B_c\cos45^\circ + B_d\cos45^\circ = 4B_a\cos45^\circ = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos45^\circ.$$

$r = \sqrt{(10\ \text{cm})^2+(10\ \text{cm})^2} = 10\sqrt{2}\ \text{cm} = 0.10\sqrt{2}\ \text{m}$, so

$$B = 4\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(100\ \text{A})}{2\pi(0.10\sqrt{2}\ \text{m})}\cos45^\circ = 4.0\times10^{-4}\ \text{T, to the left.}$$

EVALUATE: In part (c), if all four currents are reversed in direction, the net field at the center of the square would be to the right.

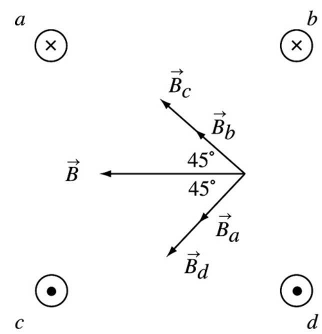
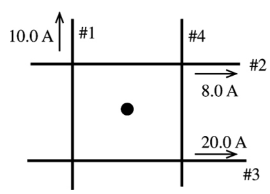


Figure 28.23

28.24. **IDENTIFY:** Use $B = \dfrac{\mu_0 I}{2\pi r}$ and the right-hand rule to determine the field due to each wire. Set the sum of the four fields equal to zero and use that equation to solve for the field and the current of the fourth wire.

SET UP: The three known currents are shown in Figure 28.24.



$\vec{B}_1 \otimes, \vec{B}_2 \otimes, \vec{B}_3 \odot$

$B = \dfrac{\mu_0 I}{2\pi r}$; $r = 0.200$ m for each wire.

Figure 28.24

EXECUTE: Let $\odot$ be the positive z-direction. $I_1 = 10.0\text{ A}, I_2 = 8.0\text{ A}, I_3 = 20.0\text{ A}$. Then

$B_1 = 1.00\times 10^{-5}\text{ T},\ B_2 = 0.80\times 10^{-5}\text{ T},\ \text{and}\ B_3 = 2.00\times 10^{-5}\text{ T}.$

$B_{1z} = -1.00\times 10^{-5}\text{ T}, B_{2z} = -0.80\times 10^{-5}\text{ T}, B_{3z} = +2.00\times 10^{-5}\text{ T}.$

$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0.$

$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0\times 10^{-6}\text{ T}.$

To give $\vec{B}_4$ in the $\otimes$ direction the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r} \text{ so } I_4 = \frac{rB_4}{(\mu_0/2\pi)} = \frac{(0.200\text{ m})(2.0\times 10^{-6}\text{ T})}{(2\times 10^{-7}\text{ T}\cdot\text{m/A})} = 2.0\text{ A}.$$

EVALUATE: The fields of wires #2 and #3 are in opposite directions and their net field is the same as due to a current $20.0\text{ A} - 8.0\text{ A} = 12.0\text{ A}$ in one wire. The field of wire #4 must be in the same direction as that of wire #1, and $10.0\text{ A} + I_4 = 12.0\text{ A}$.

28.25. **IDENTIFY:** The net magnetic field at any point is the vector sum of the magnetic fields of the two wires.

SET UP: For each wire $B = \dfrac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is determined by the right-hand rule described in the text. Let the wire with 12.0 A be wire 1 and the wire with 10.0 A be wire 2.

EXECUTE: **(a)** Point Q: $B_1 = \dfrac{\mu_0 I_1}{2\pi r_1} = \dfrac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(12.0\text{ A})}{2\pi(0.15\text{ m})} = 1.6\times 10^{-5}\text{ T}.$

The direction of $\vec{B}_1$ is out of the page. $B_2 = \dfrac{\mu_0 I_2}{2\pi r_2} = \dfrac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(10.0\text{ A})}{2\pi(0.080\text{ m})} = 2.5\times 10^{-5}\text{ T}.$

The direction of $\vec{B}_2$ is out of the page. Since $\vec{B}_1$ and $\vec{B}_2$ are in the same direction,

$B = B_1 + B_2 = 4.1\times 10^{-5}\text{ T}$ and $\vec{B}$ is directed out of the page.

Point P: $B_1 = 1.6\times10^{-5}$ T, directed into the page. $B_2 = 2.5\times10^{-5}$ T, directed into the page.

$B = B_1 + B_2 = 4.1\times10^{-5}$ T and $\vec{B}$ is directed into the page.

(b) $\vec{B}_1$ is the same as in part (a), out of the page at Q and into the page at P. The direction of $\vec{B}_2$ is reversed from what it was in (a) so is into the page at Q and out of the page at P.

Point Q: $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions so $B = B_2 - B_1 = 2.5\times10^{-5}\ \text{T} - 1.6\times10^{-5}\ \text{T} = 9.0\times10^{-6}$ T and $\vec{B}$ is directed into the page.

Point P: $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions so $B = B_2 - B_1 = 9.0\times10^{-6}$ T and $\vec{B}$ is directed out of the page.

EVALUATE: Points P and Q are the same distances from the two wires. The only difference is that the fields point in either the same direction or in opposite directions.

28.26. **IDENTIFY:** This problem involves the magnetic field of a short current-carrying wire.

SET UP: $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ (infinite wire), $B = \dfrac{\mu_0 I}{2\pi}\dfrac{2a}{x\sqrt{x^2+a^2}}$ (finite wire). We want the magnetic field this wire produces at point P(0, 5.00 cm).

EXECUTE: **(a)** Finite length: Use $B = \dfrac{\mu_0 I}{2\pi}\dfrac{2a}{x\sqrt{x^2+a^2}}$ with x = 5.00 cm and a = 10.0 cm. This gives B = 28.6 μT.

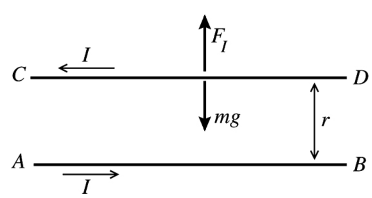
(b) Infinite length: Use $B = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$ with r = 5.00 cm, giving B = 32.0 μT.

$$\frac{\Delta B}{B} = \frac{32.0\ \mu\text{T} - 28.6\ \mu\text{T}}{28.6\ \mu\text{T}} = 0.119 = 11.9\%.$$

EVALUATE: Using the less accurate infinite-length approximation gives an easier calculation but a less accurate answer.

28.27. **IDENTIFY:** The wire CD rises until the upward force F_I due to the currents balances the downward force of gravity.

SET UP: The forces on wire CD are shown in Figure 28.27.



Currents in opposite directions so the force is repulsive and F_I is upward, as shown.

Figure 28.27

$\dfrac{F}{L} = \dfrac{\mu_0 I'I}{2\pi r}$ says $F_I = \dfrac{\mu_0 I^2 L}{2\pi h}$ where L is the length of wire CD and h is the distance between the wires.

EXECUTE: $mg = \lambda L g$.

Thus $F_I - mg = 0$ says $\dfrac{\mu_0 I^2 L}{2\pi h} = \lambda L g$ and $h = \dfrac{\mu_0 I^2}{2\pi g\lambda}$.

EVALUATE: The larger I is or the smaller λ is, the larger h will be.

28.28. **IDENTIFY:** Apply $\dfrac{F}{L} = \dfrac{\mu_0 I'I}{2\pi r}$ for the force from each wire.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: On the top wire $\frac{F}{L}=\frac{\mu_0 I^2}{2\pi}\left(\frac{1}{d}-\frac{1}{2d}\right)=\frac{\mu_0 I^2}{4\pi d}$, upward. On the middle wire, the magnetic forces cancel so the net force is zero. On the bottom wire $\frac{F}{L}=\frac{\mu_0 I^2}{2\pi}\left(\frac{1}{d}-\frac{1}{2d}\right)=\frac{\mu_0 I^2}{4\pi d}$, downward.

EVALUATE: The net force on the middle wire is zero because at the location of the middle wire the net magnetic field due to the other two wires is zero.

28.29. IDENTIFY: Apply $\frac{F}{L}=\frac{\mu_0 I'I}{2\pi r}$.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: **(a)** $F=\frac{\mu_0 I_1 I_2 L}{2\pi r}=\frac{\mu_0(5.00\text{ A})(2.00\text{ A})(1.20\text{ m})}{2\pi(0.400\text{ m})}=6.00\times10^{-6}\text{ N}$, and the force is repulsive since the currents are in opposite directions.

(b) Doubling the currents makes the force increase by a factor of four to w

EVALUATE: Doubling the current in a wire doubles the magnetic field of that wire. For fixed magnetic field, doubling the current in a wire doubles the force that the magnetic field exerts on the wire.

28.30. IDENTIFY: Apply $\frac{F}{L}=\frac{\mu_0 I'I}{2\pi r}$.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: **(a)** $\frac{F}{L}=\frac{\mu_0 I_1 I_2}{2\pi r}$ gives $I_2=\frac{F}{L}\frac{2\pi r}{\mu_0 I_1}=(4.0\times10^{-5}\text{ N/m})\frac{2\pi(0.0250\text{ m})}{\mu_0(0.60\text{ A})}=8.33\text{ A}$.

(b) The two wires repel so the currents are in opposite directions.

EVALUATE: The force between the two wires is proportional to the product of the currents in the wires.

28.31. IDENTIFY: We can model the current in the brain as a ring. Since we know the magnetic field at the center of the ring, we can calculate the current.

SET UP: At the center of a ring, $B=\frac{\mu_0 I}{2R}$. In this case, t.

EXECUTE: Solving for I gives $I=\frac{2RB}{\mu_0}=\frac{2(8\times10^{-2}\text{ m})(3.0\times10^{-12}\text{ T})}{4\times10^{-7}\text{ T}\cdot\text{m/A}}=3.8\times10^{-7}\text{ A}$.

EVALUATE: This current is about a third of a microamp, which is a very small current by household standards. However, the magnetic field in the brain is a very weak field, about a hundreth of the earth's magnetic field.

28.32. IDENTIFY: The magnetic field at the center of a circular loop is $B=\frac{\mu_0 I}{2R}$. By symmetry each segment of the loop that has length Δl contributes equally to the field, so the field at the center of a semicircle is $\frac{1}{2}$ that of a full loop.

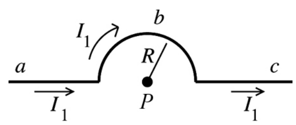
SET UP: Since the straight sections produce no field at P, the field at P is $B=\frac{\mu_0 I}{4R}$.

EXECUTE: $B=\frac{\mu_0 I}{4R}$. The direction of $\vec{B}$ is given by the right-hand rule: $\vec{B}$ is directed into the page.

EVALUATE: For a quarter-circle section of wire the magnetic field at its center of curvature is $B=\frac{\mu_0 I}{8R}$.

28.33. **IDENTIFY:** Calculate the magnetic field vector produced by each wire and add these fields to get the total field.

SET UP: First consider the field at P produced by the current I_1 in the upper semicircle of wire. See Figure 28.33a.

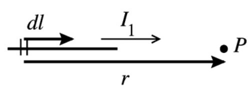


Consider the three parts of this wire:
a: long straight section
b: semicircle
c: long, straight section

Figure 28.33a

Apply the Biot-Savart law $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l}\times\hat{r}}{r^2} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l}\times\vec{r}}{r^3}$ to each piece.

EXECUTE: Part a: See Figure 28.33b.

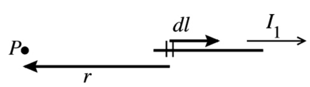


$d\vec{l}\times\vec{r} = 0,$
so $dB = 0.$

Figure 28.33b

The same is true for all the infinitesimal segments that make up this piece of the wire, so $B = 0$ for this piece.

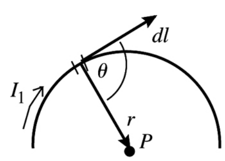
Part c: See Figure 28.33c.



$d\vec{l}\times\vec{r} = 0,$
so $dB = 0$ and $B = 0$ for this piece.

Figure 28.33c

Part b: See Figure 28.33d.



$d\vec{l}\times\vec{r}$ is directed into the paper for all infinitesimal segments that make up this semicircular piece, so $\vec{B}$ is directed into the paper and $B = \int dB$ (the vector sum of the $d\vec{B}$ is obtained by adding their magnitudes since they are in the same direction).

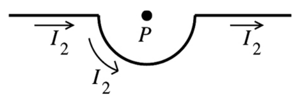
Figure 28.33d

$\left|d\vec{l}\times\vec{r}\right| = rdl\sin\theta.$ The angle θ between $d\vec{l}$ and $\vec{r}$ is 90° and $r = R,$ the radius of the semicircle. Thus $\left|d\vec{l}\times\vec{r}\right| = R\,dl.$

$$dB = \frac{\mu_0}{4\pi}\frac{I\left|d\vec{l}\times\vec{r}\right|}{r^3} = \frac{\mu_0 I_1}{4\pi}\frac{R}{R^3}dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)dl.$$

$$B = \int dB = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)\int dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)(\pi R) = \frac{\mu_0 I_1}{4R}.$$

(We used that $\int dl$ is equal to $\pi R,$ the length of wire in the semicircle.) We have shown that the two straight sections make zero contribution to $\vec{B},$ so $B_1 = \mu_0 I_1/4R$ and is directed into the page.



For current in the direction shown in Figure 28.33e, a similar analysis gives $B_2 = \mu_0 I_2/4R$, out of the paper.

Figure 28.33e

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions, so the magnitude of the net field at P is $B = |B_1 - B_2| = \dfrac{\mu_0 |I_1 - I_2|}{4R}$.

EVALUATE: When $I_1 = I_2, B = 0$.

28.34. IDENTIFY: Apply $B_x = \dfrac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}$.

SET UP: At the center of the coil, $x = 0$. a is the radius of the coil, 0.0240 m.

EXECUTE: **(a)** $B_x = \mu_0 NI/2a$, so $I = \dfrac{2aB_x}{\mu_0 N} = \dfrac{2(0.024\text{ m})(0.0770\text{ T})}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(800)} = 3.68\text{ A}$.

(b) At the center, $B_c = \mu_0 NI/2a$. At a distance x from the center,

$B_x = \dfrac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}} = \left(\dfrac{\mu_0 NI}{2a}\right)\left(\dfrac{a^3}{(x^2+a^2)^{3/2}}\right) = B_c\left(\dfrac{a^3}{(x^2+a^2)^{3/2}}\right)$. $B_x = \frac{1}{2}B_c$ says $\dfrac{a^3}{(x^2+a^2)^{3/2}} = \frac{1}{2}$, and

$(x^2 + a^2)^3 = 4a^6$. Since $a = 0.024$ m, $x = 0.0184\text{ m} = 1.84$ cm.

EVALUATE: As shown in Figure 28.14 in the textbook, the field has its largest magnitude at the center of the coil and decreases with distance along the axis from the center.

28.35. IDENTIFY: The field at the center of the loops is the vector sum of the field due to each loop. They must be in opposite directions in order to add to zero.

SET UP: Let wire 1 be the inner wire with diameter 20.0 cm and let wire 2 be the outer wire with diameter 30.0 cm. To produce zero net field, the fields $\vec{B}_1$ and $\vec{B}_2$ of the two wires must have equal magnitudes and opposite directions. At the center of a wire loop $B = \dfrac{\mu_0 I}{2R}$. The direction of $\vec{B}$ is given by the right-hand rule applied to the current direction.

EXECUTE: $B_1 = \dfrac{\mu_0\, I}{2R_1}$, $B_2 = \dfrac{\mu_0\, I}{2R_2}$. $B_1 = B_2$ gives $\dfrac{\mu_0\, I_1}{2R_1} = \dfrac{\mu_0\, I_2}{2R_2}$. Solving for I_2 gives

$I_2 = \left(\dfrac{R_2}{R_1}\right) I_1 = \left(\dfrac{15.0\text{ cm}}{10.0\text{ cm}}\right)(12.0\text{ A}) = 18.0\text{ A}$. The directions of I_1 and of its field are shown in Figure 28.35. Since $\vec{B}_1$ is directed into the page, $\vec{B}_2$ must be directed out of the page and I_2 is counterclockwise.

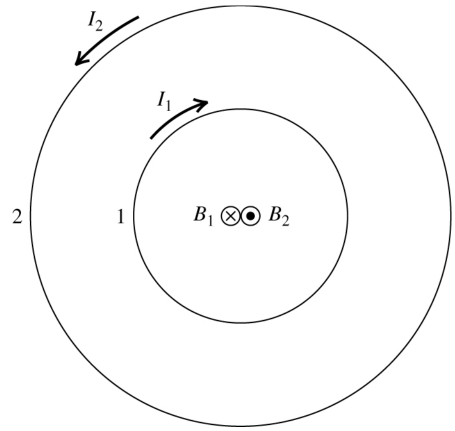


Figure 28.35

EVALUATE: The outer current, I_2, must be larger than the inner current, I_1, because the outer ring is larger than the inner ring, which makes the outer current farther from the center than the inner current is.

28.36. **IDENTIFY** and **SET UP:** The magnetic field at a point on the axis of N circular loops is given by $B_x = \dfrac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$. Solve for N and set $x = 0.0600$ m.

EXECUTE: $N = \dfrac{2B_x(x^2+a^2)^{3/2}}{\mu_0 I a^2} = \dfrac{2(6.39\times10^{-4}\ \text{T})\left[(0.0600\ \text{m})^2+(0.0600\ \text{m})^2\right]^{3/2}}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.50\ \text{A})(0.0600\ \text{m})^2} = 69.$

EVALUATE: At the center of the coil the field is $B_x = \dfrac{\mu_0 N I}{2a} = 1.8\times10^{-3}\ \text{T}$. The field 6.00 cm from the center is a factor of $1/2^{3/2}$ times smaller.

28.37. **IDENTIFY:** Apply Ampere's law.

SET UP: $\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$.

EXECUTE: **(a)** $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} = 3.83\times10^{-4}\ \text{T}\cdot\text{m}$ and $I_{\text{encl}} = 305\ \text{A}$.

(b) $-3.83\times10^{-4}\ \text{T}\cdot\text{m}$ since at each point on the curve the direction of $d\vec{l}$ is reversed.

EVALUATE: The line integral $\oint \vec{B}\cdot d\vec{l}$ around a closed path is proportional to the net current that is enclosed by the path.

28.38. **IDENTIFY:** Apply Ampere's law.

SET UP: From the right-hand rule, when going around the path in a counterclockwise direction currents out of the page are positive and currents into the page are negative.

EXECUTE: Path a: $I_{\text{encl}} = 0 \Rightarrow \oint \vec{B}\cdot d\vec{l} = 0$.

Path b: $I_{\text{encl}} = -I_1 = -4.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = -\mu_0(4.0\ \text{A}) = -5.03\times10^{-6}\ \text{T}\cdot\text{m}$.

Path c: $I_{\text{encl}} = -I_1 + I_2 = -4.0\ \text{A} + 6.0\ \text{A} = 2.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0(2.0\ \text{A}) = 2.51\times10^{-6}\ \text{T}\cdot\text{m}$

Path d: $I_{\text{encl}} = -I_1 + I_2 + I_3 = 4.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = +\mu_0(4.0\ \text{A}) = 5.03\times10^{-6}\ \text{T}\cdot\text{m}$.

EVALUATE: If we instead went around each path in the clockwise direction, the sign of the line integral would be reversed.

28.39. **IDENTIFY:** Apply Ampere's law.

SET UP: To calculate the magnetic field at a distance r from the center of the cable, apply Ampere's law to a circular path of radius r. By symmetry, $\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$ for such a path.

EXECUTE: **(a)** For $a < r < b, I_{\text{encl}} = I \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B2\pi r = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}$.

(b) For $r > c,$ the enclosed current is zero, so the magnetic field is also zero.

EVALUATE: A useful property of coaxial cables for many applications is that the current carried by the cable doesn't produce a magnetic field outside the cable.

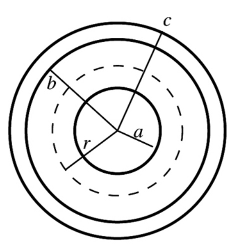
28.40. **IDENTIFY** and **SET UP:** At the center of a long solenoid $B = \mu_0 nI = \mu_0 \dfrac{N}{L} I$.

EXECUTE: $I = \dfrac{BL}{\mu_0 N} = \dfrac{(0.150 \text{ T})(0.550 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4000)} = 16.4 \text{ A}$.

EVALUATE: The magnetic field inside the solenoid is independent of the radius of the solenoid, if the radius is much less than the length, as is the case here.

28.41. **IDENTIFY:** Apply Ampere's law to calculate $\vec{B}$.

(a) SET UP: For $a < r < b$ the end view is shown in Figure 28.41a.



Apply Ampere's law to a circle of radius r, where $a < r < b$. Take currents I_1 and I_2 to be directed into the page. Take this direction to be positive, so go around the integration path in the clockwise direction.

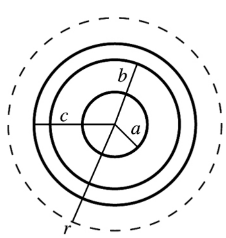
Figure 28.41a

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r), I_{\text{encl}} = I_1$.

Thus $B(2\pi r) = \mu_0 I_1$ and $B = \dfrac{\mu_0 I_1}{2\pi r}$.

(b) SET UP: <u>$r > c$</u>: See Figure 28.41b.



Apply Ampere's law to a circle of radius r, where $r > c$. Both currents are in the positive direction.

Figure 28.41b

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r), I_{\text{encl}} = I_1 + I_2$.

Thus $B(2\pi r) = \mu_0 (I_1 + I_2)$ and $B = \dfrac{\mu_0 (I_1 + I_2)}{2\pi r}$.

EVALUATE: For $a < r < b$ the field is due only to the current in the central conductor. For $r > c$ both currents contribute to the total field.

28.42. **IDENTIFY:** $B = \mu_0 nI = \dfrac{\mu_0 NI}{L}$.

SET UP: $L = 0.150$ m.

EXECUTE: $B = \dfrac{\mu_0 (600)(8.00\text{ A})}{(0.150\text{ m})} = 0.0402\text{ T}$.

EVALUATE: The field near the center of the solenoid is independent of the radius of the solenoid, as long as the radius is much less than the length, as it is here.

28.43. **IDENTIFY** and **SET UP:** The magnetic field near the center of a long solenoid is given by $B = \mu_0 nI$.

EXECUTE: **(a)** Turns per unit length $n = \dfrac{B}{\mu_0 I} = \dfrac{0.0270\text{ T}}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(12.0\text{ A})} = 1790\text{ turns/m}$.

(b) $N = nL = (1790\text{ turns/m})(0.400\text{ m}) = 716\text{ turns}$.

Each turn of radius *R* has a length $2\pi R$ of wire. The total length of wire required is

$N(2\pi R) = (716)(2\pi)(1.40\times10^{-2}\text{ m}) = 63.0\text{ m}$.

EVALUATE: A large length of wire is required. Due to the length of wire the solenoid will have appreciable resistance.

28.44. **IDENTIFY:** Outside an ideal toroidal solenoid there is no magnetic field and inside it the magnetic field is given by $B = \dfrac{\mu_0 NI}{2\pi r}$.

SET UP: The torus extends from $r_1 = 15.0$ cm to $r_2 = 18.0$ cm.

EXECUTE: **(a)** $r = 0.12$ m, which is outside the torus, so $B = 0$.

(b) $r = 0.16$ m, so $B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\mu_0 (250)(8.50\text{ A})}{2\pi(0.160\text{ m})} = 2.66\times10^{-3}\text{ T}$.

(c) $r = 0.20$ m, which is outside the torus, so $B = 0$.

EVALUATE: The magnetic field inside the torus is proportional to $1/r$, so it varies somewhat over the cross-section of the torus.

28.45. **IDENTIFY** and **SET UP:** Use the appropriate expression for the magnetic field produced by each current configuration.

EXECUTE: **(a)** $B = \dfrac{\mu_0 I}{2\pi r}$ so $I = \dfrac{2\pi r B}{\mu_0} = \dfrac{2\pi(2.00\times10^{-2}\text{ m})(37.2\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 3.72\times10^6\text{ A} = 3.72\text{ MA}$.

(b) $B = \dfrac{N\mu_0 I}{2R}$ so $I = \dfrac{2RB}{N\mu_0} = \dfrac{2(0.420\text{ m})(37.2\text{ T})}{(100)(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})} = 2.49\times10^5\text{ A} = 249\text{ kA}$.

(c) $B = \mu_0 \dfrac{N}{L} I$ so $I = \dfrac{BL}{\mu_0 N} = \dfrac{(37.2\text{ T})(0.320\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(40{,}000)} = 237\text{ A}$.

EVALUATE: Much less current is needed for the solenoid, because of its large number of turns per unit length.

28.46. **IDENTIFY:** Use $B = \dfrac{\mu_0 NI}{2\pi r}$, with μ_0 replaced by $\mu = K_\text{m}\mu_0$, with $K_\text{m} = 80$.

SET UP: The contribution from atomic currents is the difference between *B* calculated with μ and *B* calculated with μ_0.

EXECUTE: **(a)** $B = \dfrac{\mu NI}{2\pi r} = \dfrac{K_\text{m}\mu_0 NI}{2\pi r} = \dfrac{\mu_0 (80)(400)(0.25\text{ A})}{2\pi(0.060\text{ m})} = 0.0267\text{ T}$.

(b) The amount due to atomic currents is $B' = \dfrac{79}{80}B = \dfrac{79}{80}(0.0267\ \text{T}) = 0.0263\ \text{T}.$

EVALUATE: The presence of the core greatly enhances the magnetic field produced by the solenoid.

28.47. **IDENTIFY:** The magnetic field from the solenoid alone is $B_0 = \mu_0 nI$. The total magnetic field is $B = K_m B_0$. M is given by $\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$.

SET UP: $n = 6000$ turns/m.

EXECUTE: **(a)** (i) $B_0 = \mu_0 nI = \mu_0 (6000\ \text{m}^{-1})(0.15\ \text{A}) = 1.13\times10^{-3}\ \text{T}.$

(ii) $M = \dfrac{K_m - 1}{\mu_0}B_0 = \dfrac{5199}{\mu_0}(1.13\times10^{-3}\ \text{T}) = 4.68\times10^{6}\ \text{A/m}.$

(iii) $B = K_m B_0 = (5200)(1.13\times10^{-3}\ \text{T}) = 5.88\ \text{T}.$

(b) The directions of $\vec{B}$, $\vec{B}_0$ and $\vec{M}$ are shown in Figure 28.47. Silicon steel is paramagnetic and $\vec{B}_0$ and $\vec{M}$ are in the same direction.

EVALUATE: The total magnetic field is much larger than the field due to the solenoid current alone.

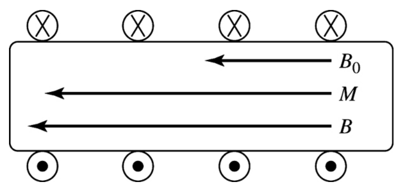


Figure 28.47

28.48. **IDENTIFY:** Apply $B = \dfrac{K_m \mu_0 NI}{2\pi r}$.

SET UP: K_m is the relative permeability and $\chi_m = K_m - 1$ is the magnetic susceptibility.

EXECUTE: **(a)** $K_m = \dfrac{2\pi rB}{\mu_0 NI} = \dfrac{2\pi(0.2500\ \text{m})(1.940\ \text{T})}{\mu_0(500)(2.400\ \text{A})} = 2021.$

(b) $\chi_m = K_m - 1 = 2020.$

EVALUATE: Without the magnetic material the magnetic field inside the windings would be $B/2021 = 9.6\times10^{-4}\ \text{T}$. The presence of the magnetic material greatly enhances the magnetic field inside the windings.

28.49. **IDENTIFY:** Moving charges create magnetic fields. The net field is the vector sum of the two fields. A charge moving in an external magnetic field feels a force.

(a) SET UP: The magnitude of the magnetic field due to a moving charge is $B = \dfrac{\mu_0}{4\pi}\dfrac{|q|v\sin\phi}{r^2}$. Both fields are into the paper, so their magnitudes add, giving $B_{\text{net}} = B + B' = \dfrac{\mu_0}{4\pi}\left(\dfrac{|q|v\sin\phi}{r^2} + \dfrac{|q'|v'\sin\phi'}{r'^2}\right)$.

EXECUTE: Substituting numbers gives

$$B_{\text{net}} = \frac{\mu_0}{4\pi}\left[\frac{(8.00\ \mu\text{C})(9.00\times10^4\ \text{m/s})\sin 90^\circ}{(0.300\ \text{m})^2} + \frac{(5.00\ \mu\text{C})(6.50\times10^4\ \text{m/s})\sin 90^\circ}{(0.400\ \text{m})^2}\right].$$

$B_{\text{net}} = 1.00\times10^{-6}\ \text{T} = 1.00\ \mu\text{T}$, into the paper.

(b) SET UP: The magnetic force on a moving charge is $\vec{F} = q\vec{v}\times\vec{B}$, and the magnetic field of charge q' at the location of charge q is into the page. The force on q is

$$\vec{F} = q\vec{v}\times\vec{B}' = (qv)\hat{i}\times\frac{\mu_0}{4\pi}\frac{q\vec{v}'\times\hat{r}}{r^2} = (qv)\hat{i}\times\left(\frac{\mu_0}{4\pi}\frac{qv'\sin\phi}{r^2}\right)(-\hat{k}) = \left(\frac{\mu_0}{4\pi}\frac{qq'vv'\sin\phi}{r^2}\right)\hat{j}$$

where ϕ is the angle between $\vec{v}'$ and $\hat{r}'$.

EXECUTE: Substituting numbers gives

$$\vec{F} = \frac{\mu_0}{4\pi}\left[\frac{(8.00\times10^{-6}\text{ C})(5.00\times10^{-6}\text{ C})(9.00\times10^{4}\text{ m/s})(6.50\times10^{4}\text{ m/s})}{(0.500\text{ m})^2}\left(\frac{0.400}{0.500}\right)\right]\hat{j}.$$

$\vec{F} = (7.49\times10^{-8}\text{N})\hat{j}.$

EVALUATE: These are small fields and small forces, but if the charge has small mass, the force can affect its motion.

28.50. **IDENTIFY:** Charge q_1 creates a magnetic field due to its motion. This field exerts a magnetic force on q_2, which is moving in that field.

SET UP: Find $\vec{B}_1$, the field produced by q_1 at the location of q_2. $\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{q_1\vec{v}_1\times\vec{r}_{1\to2}}{r^3_{1\to2}}$, since $\hat{r} = \vec{r}/r$.

EXECUTE: $\vec{r}_{1\to2} = (0.150\text{ m})\hat{i} + (-0.250\text{ m})\hat{j}$, so $r_{1\to2} = 0.2915$ m.

$\vec{v}_1\times\vec{r}_{1\to2} = [(9.20\times10^5\text{ m/s})\hat{i}]\times[(0.150\text{ m})\hat{i} + (-0.250\text{ m})\hat{j}] = (9.20\times10^5\text{ m/s})(-0.250\text{ m})\hat{k}.$

$$\vec{B}_1 = (1.00\times10^{-7}\text{ T}\cdot\text{m/A})\frac{(4.80\times10^{-6}\text{ C})(9.20\times10^5\text{ m/s})(-0.250\text{ m})}{(0.2915\text{ m})^3}\hat{k} = -(4.457\times10^{-6}\text{ T})\hat{k}.$$

The force that $\vec{B}_1$ exerts on q_2 is

$F_2 = q_2\vec{v}_2\times\vec{B}_1 = (-2.90\times10^{-6}\text{ C})(-5.30\times10^5\text{ m/s})(-4.457\times10^{-6}\text{ T})\hat{j}\times\hat{k} = -(6.85\times10^{-6}\text{ N})\hat{i}.$

EVALUATE: If we think of the moving charge q_1 as a current, we can use the right-hand rule for the direction of the magnetic field due to a current to find the direction of the magnetic field it creates in the vicinity of q_2. Then we can use the cross product right-hand rule to find the direction of the force this field exerts on q_2, which is in the $-x$-direction, in agreement with our result.

28.51. **IDENTIFY:** This problem involves the magnetic field due to two long wires. We want to know where the fields cancel completely.

SET UP: $B = \frac{\mu_0}{2\pi}\frac{I}{r}$. The fields completely cancel only if they are in opposite directions and of the same magnitude. Both conditions can be met only in the region between the wires.

EXECUTE: Call y the coordinate of the point where $B = 0$. At that point $B_1 = B_2$. This gives $\frac{\mu_0}{2\pi}\frac{I_1}{r_1} = \frac{\mu_0}{2\pi}\frac{I_2}{r_2}$, so $\frac{2}{r_1} = \frac{6}{r_2}$. $r_1 = y$ and $r_2 = 0.800\text{ m} - y$, so $\frac{1}{y} = \frac{3}{0.800\text{ m} - y}$. Solving for y gives us $y = 0.200$ m.

EVALUATE: The point where $B = 0$ must be closer to the smaller current than to the larger one. $y_1 = 0.200$ m and $y_2 = 0.600$ m, which agrees with this condition.

28.52. **IDENTIFY:** This problem involves the magnetic field due to two long wires. We want to know where the fields cancel completely.

SET UP: $B = \frac{\mu_0}{2\pi}\frac{I}{r}$. The fields completely cancel only if they are in opposite directions and of the same magnitude. Both conditions can be met only in the region below the 2.00 A current wire.

EXECUTE: Call y the coordinate of the point where $B = 0$. At that point $B_1 = B_2$. This gives $\frac{\mu_0}{2\pi}\frac{I_1}{r_1} = \frac{\mu_0}{2\pi}\frac{I_2}{r_2}$, so $\frac{2}{r_1} = \frac{6}{r_2}$. $r_1 = y$ and $r_2 = 0.800\text{ m} + y$, so $\frac{1}{y} = \frac{3}{0.800\text{ m} + y}$. Solving for y gives us $y = -0.400$ m.

EVALUATE: The point where $B = 0$ must be closer to the smaller current than to the larger one. $y_1 = 0.400$ m and $y_2 = 1.20$ m, which agrees with this condition.

28.53. **IDENTIFY:** In this problem, we are dealing with the magnetic force on a current-carrying wire.

SET UP: $F = IlB\sin\phi$, $B = \frac{\mu_0}{2\pi}\frac{I}{r}$ (long wire), $B = \frac{\mu_0 I}{2a}$ (center of circular loop).

EXECUTE: **(a)** We want the force. Combine $F = IlB\sin\phi$ and $B = \frac{\mu_0}{2\pi}\frac{I}{r}$ with ϕ = 90°. This gives

$F = IlB = I(2\pi R)\left(\frac{\mu_0 I}{2\pi d}\right) = \mu_0 I^2 R/d.$

(b) We want the force. At the center of the loop we use $B = \frac{\mu_0 I}{2a}$. In this case, $a = R$. Solve for I:

$I = \frac{2RB}{\mu_0}$. Express the area A in terms of R: $A = \pi R^2 \rightarrow R = \sqrt{A/\pi}$. Use the result from (a):

$F = \frac{\mu_0 I^2 R}{d} = \frac{\mu_0 (2RB/\mu_0)^2 \sqrt{A/\pi}}{d}$, which reduces to $F = \frac{4B^2}{\mu_0 d}\left(\frac{A}{\pi}\right)^{3/2}$.

(c) Solve for B: $B = \frac{1}{2}\sqrt{\mu_0 F d}\left(\frac{\pi}{A}\right)^{3/4}$.

(d) Estimate: F = 5 N.

(e) Estimate: A = 2 cm by 4 cm = 8 cm^2.

(f) $B = \frac{1}{2}\sqrt{\mu_0 F d}\left(\frac{\pi}{A}\right)^{3/4}$. Using the estimates and d = 25 μm, we get 3 mT.

EVALUATE: This field is about 3 mT and the earth's magnetic field is about 0.05 mT, so the magnet's field is roughly 60 times that of the earth.

28.54. **IDENTIFY:** The wire creates a magnetic field near it, and the moving electron feels a force due to this field.

SET UP: The magnetic field due to the wire is $B = \frac{\mu_0 I}{2\pi r}$, and the force on a moving charge is $F = |q|vB\sin\phi$.

EXECUTE: $F = |q|vB\sin\phi = (ev\mu_0 I\sin\phi)/2\pi r$. Substituting numbers gives

$F = (1.60\times10^{-19}\ \text{C})(6.00\times10^4\ \text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(8.60\ \text{A})(\sin 90°)/[2\pi(0.0450\ \text{m})].$

$F = 3.67\times10^{-19}$ N. From the right-hand rule for the cross product, the direction of $\vec{v}\times\vec{B}$ is opposite to the current, but since the electron is negative, the force is in the same direction as the current.

EVALUATE: This force is small at an everyday level, but it would give the electron an acceleration of over 10^{11} m/s^2.

28.55. **IDENTIFY:** Find the force that the magnetic field of the wire exerts on the electron.

SET UP: The force on a moving charge has magnitude $F = |q|vB\sin\phi$ and direction given by the right-hand rule. For a long straight wire, $B = \frac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** $a = \frac{F}{m} = \frac{|q|vB\sin\phi}{m} = \frac{ev}{m}\left(\frac{\mu_0 I}{2\pi r}\right)$. Substituting numbers gives

$$a = \frac{(1.6\times10^{-19}\text{C})(2.50\times10^5\ \text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(13.0\ \text{A})}{(9.11\times10^{-31}\ \text{kg})(2\pi)(0.0200\ \text{m})} = 5.7\times10^{12}\ \text{m/s}^2,$$ away from the wire.

(b) The electric force must balance the magnetic force. $eE = evB$, and

$E = vB = v\frac{\mu_0 I}{2\pi r} = \frac{(250{,}000\ \text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(13.0\ \text{A})}{2\pi(0.0200\ \text{m})} = 32.5$ N/C. The magnetic force is directed away from the wire so the force from the electric field must be toward the wire. Since the charge of the electron is negative, the electric field must be directed away from the wire to produce a force in the desired direction.

EVALUATE: (c) $mg = (9.11\times10^{-31}\text{ kg})(9.8\text{ m/s}^2) \approx 10^{-29}\text{ N}.$

$F_{\text{el}} = eE = (1.6\times10^{-19}\text{ C})(32.5\text{ N/C}) \approx 5\times10^{-18}\text{ N}.$ $F_{\text{el}} \approx 5\times10^{11}\,F_{\text{grav}},$ so we can neglect gravity.

28.56. IDENTIFY: The current in the wire creates a magnetic field, and that field exerts a force on the moving electron.

SET UP: The magnetic field due to the current in the wire is $B = \dfrac{\mu_0 I}{2\pi r}$. The force the field exerts on the electron is $\vec{F} = q\vec{v}\times\vec{B},$ where $q = -e$. The magnitude of a vector is $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$. The electron is on the $+y$-axis. The current is in the $-x$-direction so, by the right-hand rule, the magnetic field it produces at the location of the electron is in the $-z$-direction, so $\vec{B} = -\dfrac{\mu_0 I}{2\pi r}\,\hat{k}$.

EXECUTE: The magnitude of the magnetic field is $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{\mu_0(9.00\text{ A})}{2\pi(0.200\text{ m})} = 9.00\times10^{-6}\text{ T},$ so

$\vec{B} = -9.00\times10^{-6}\text{ T}\,\hat{k}$. The force on the electron is $\vec{F} = q\vec{v}\times\vec{B},$ so

$\vec{F} = q\vec{v}\times\vec{B} = -e(5.00\times10^4\text{ m/s}\,\hat{i} - 3.00\times10^4\text{ m/s}\,\hat{j})\times(-9.00\times10^{-6}\text{ T}\,\hat{k}).$

Taking out common factors gives $\vec{F} = (9\times10^{-2}e\text{ T}\cdot\text{m/s})(5\hat{i} - 3\hat{j})\times\hat{k}.$ Using the fact that $\boldsymbol{i}\times\boldsymbol{k} = -\boldsymbol{j}$ and $\boldsymbol{j}\times\boldsymbol{k} = \boldsymbol{i},$ we get $\vec{F} = (9\times10^{-2}e\text{ T}\cdot\text{m/s})(-5\hat{j} - 3\hat{i}).$ Using $e = 1.60\times10^{-19}\text{ C}$ gives

$\vec{F} = -4.32\times10^{-20}\text{ N}\,\hat{i} - 7.20\times10^{-20}\text{ N}\,\hat{j}.$

The magnitude of this force is

$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(-4.32\times10^{-20}\text{ N})^2 + (-7.20\times10^{-20}\text{ N})^2} = 8.40\times10^{-20}\text{ N}.$

EVALUATE: This is a small force on an everyday scale, but it would give the electron an acceleration of $a = F/m = (8.40\times10^{-20}\text{ N})/(9.11\times10^{-31}\text{ kg}) \approx 9\times10^{10}\text{ m/s}^2.$

28.57. IDENTIFY: This problem deals with the magnetic field in a cell phone.

SET UP and **EXECUTE: (a)** $P = 1.5$ W.

(b) $I = P/V = (1.5\text{ W})/(1.5\text{ V}) = 1.0$ A.

(c) Width = 20 cm.

(d) Estimate: Diameter = 3.0 cm. Treat the phone as a circular current loop. The field is given by $B = \dfrac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$, where a is the radius of the loop and x is the distance from its center along its axis. The estimated diameter is 3.0 cm, so $a = 1.5$ cm, and x is the distance from the phone to the middle of your head, which is 10 cm from our estimate in (c). Using these values,

$$B = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} = 0.14\ \mu\text{T}.$$

(e) $\dfrac{B_{\text{phone}}}{B_{\text{earth}}} = \dfrac{0.14\ \mu\text{T}}{50\ \mu\text{T}} \approx 3\times10^{-3} \approx 0.3\%.$ The magnetic field of the cell phone at the location of your brain is about 0.3% of the earth's magnetic field.

EVALUATE: This magnetic field is extremely weak compared to the earth's field which we experience all day every day.

28.58. IDENTIFY: Find the vector sum of the magnetic fields due to each wire.

SET UP: For a long straight wire $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule and is perpendicular to the line from the wire to the point where the field is calculated.

EXECUTE: **(a)** The magnetic field vectors are shown in Figure 28.58a.

(b) At a position on the x-axis $B_{\text{net}} = 2\dfrac{\mu_0 I}{2\pi r}\sin\theta = \dfrac{\mu_0 I}{\pi\sqrt{x^2+a^2}}\dfrac{a}{\sqrt{x^2+a^2}} = \dfrac{\mu_0 Ia}{\pi(x^2+a^2)}$, in the positive x-direction.

(c) The graph of B versus x/a is given in Figure 28.58b.

EVALUATE: **(d)** The magnetic field is a maximum at the origin, $x = 0$.

(e) When $x \gg a$, $B \approx \dfrac{\mu_0 Ia}{\pi x^2}$.

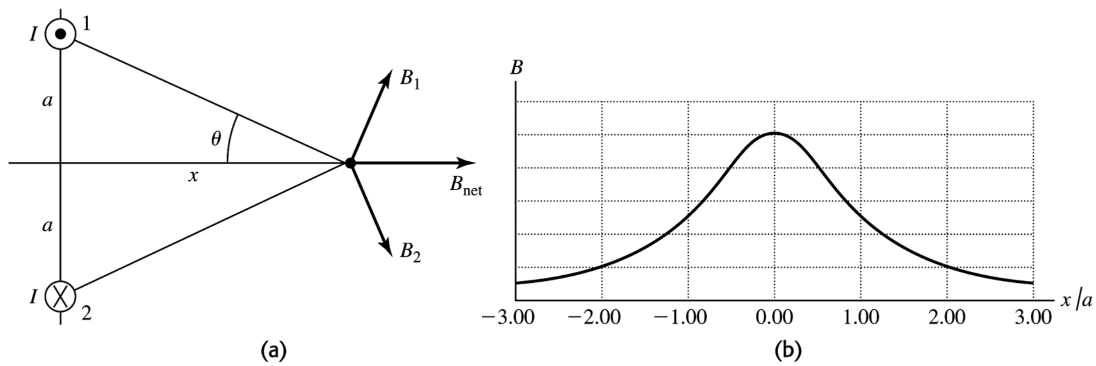
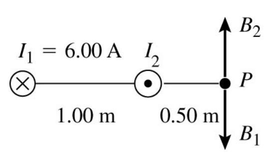


Figure 28.58

28.59. **IDENTIFY:** Use $B = \dfrac{\mu_0 I}{2\pi r}$ and the right-hand rule to calculate the magnitude and direction of the magnetic field at P produced by each wire. Add these two field vectors to find the net field.

(a) SET UP: The directions of the fields at point P due to the two wires are sketched in Figure 28.59a.



EXECUTE: $\vec{B}_1$ and $\vec{B}_2$ must be equal and opposite for the resultant field at P to be zero. $\vec{B}_2$ is to the upward so I_2 is out of the page.

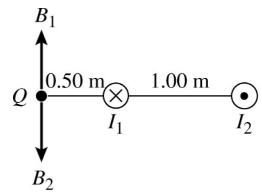
Figure 28.59a

$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0}{2\pi}\left(\frac{6.00\ \text{A}}{1.50\ \text{m}}\right) \qquad B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi}\left(\frac{I_2}{0.50\ \text{m}}\right).$$

$B_1 = B_2$ says $\dfrac{\mu_0}{2\pi}\left(\dfrac{6.00\ \text{A}}{1.50\ \text{m}}\right) = \dfrac{\mu_0}{2\pi}\left(\dfrac{I_2}{0.50\ \text{m}}\right)$.

$$I_2 = \left(\frac{0.50\ \text{m}}{1.50\ \text{m}}\right)(6.00\ \text{A}) = 2.00\ \text{A}.$$

(b) SET UP: The directions of the fields at point Q are sketched in Figure 28.59b.



EXECUTE: $B_1 = \dfrac{\mu_0 I_1}{2\pi r_1}$.

$$B_1 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{6.00\ \text{A}}{0.50\ \text{m}}\right) = 2.40\times10^{-6}\ \text{T}.$$

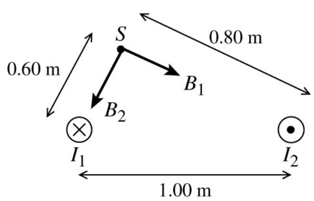
$$B_2 = \frac{\mu_0 I_2}{2\pi r_2}.$$

$$B_2 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{2.00\ \text{A}}{1.50\ \text{m}}\right) = 2.67\times10^{-7}\ \text{T}.$$

Figure 28.59b

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions and $B_1 > B_2$ so
$B = B_1 - B_2 = 2.40\times10^{-6}\ \text{T} - 2.67\times10^{-7}\ \text{T} = 2.13\times10^{-6}\ \text{T}$, and $\vec{B}$ is upward.

(c) SET UP: The directions of the fields at point S are sketched in Figure 28.59c.



EXECUTE: $B_1 = \dfrac{\mu_0 I_1}{2\pi r_1}$.

$$B_1 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{6.00\ \text{A}}{0.60\ \text{m}}\right) = 2.00\times10^{-6}\ \text{T}.$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2}.$$

$$B_2 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{2.00\ \text{A}}{0.80\ \text{m}}\right) = 5.00\times10^{-7}\ \text{T}.$$

Figure 28.59c

$\vec{B}_1$ and $\vec{B}_2$ are right angles to each other, so the magnitude of their resultant is given by
$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(2.00\times10^{-6}\ \text{T})^2 + (5.00\times10^{-7}\ \text{T})^2} = 2.06\times10^{-6}\ \text{T}$.

EVALUATE: The magnetic field lines for a long, straight wire are concentric circles with the wire at the center. The magnetic field at each point is tangent to the field line, so $\vec{B}$ is perpendicular to the line from the wire to the point where the field is calculated.

28.60. IDENTIFY: Consider the forces on each side of the loop.

SET UP: The forces on the left and right sides cancel. The forces on the top and bottom segments of the loop are in opposite directions, so the magnitudes subtract.

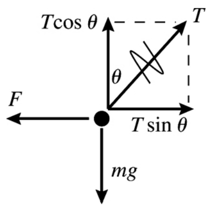
EXECUTE: $F = F_t - F_b = \left(\dfrac{\mu_0 I_{\text{wire}}}{2\pi}\right)\left(\dfrac{Il}{r_t} - \dfrac{Il}{r_b}\right) = \dfrac{\mu_0 I l I_{\text{wire}}}{2\pi}\left(\dfrac{1}{r_t} - \dfrac{1}{r_b}\right)$.

$F = \dfrac{\mu_0 (5.00\ \text{A})(0.200\ \text{m})(14.0\ \text{A})}{2\pi}\left(-\dfrac{1}{0.100\ \text{m}} + \dfrac{1}{0.026\ \text{m}}\right) = 7.97\times10^{-5}\ \text{N}$. The force on the top segment is toward the wire, so the net force is toward the wire.

EVALUATE: The net force on a current loop in a uniform magnetic field is zero, but the magnetic field of the wire is not uniform; it is stronger closer to the wire.

28.61. **IDENTIFY:** Apply $\sum \vec{F} = 0$ to one of the wires. The force one wire exerts on the other depends on I so $\sum \vec{F} = 0$ gives two equations for the two unknowns T and I.

SET UP: The force diagram for one of the wires is given in Figure 28.61.



The force one wire exerts on the other is $F = \left(\dfrac{\mu_0 I^2}{2\pi r}\right)L,$ where $r = 2(0.040\text{ m})\sin\theta = 8.362\times10^{-3}\text{ m}$ is the distance between the two wires.

Figure 28.61

EXECUTE: $\sum F_y = 0$ gives $T\cos\theta = mg$ and $T = mg/\cos\theta$.

$\sum F_x = 0$ gives $F = T\sin\theta = (mg/\cos\theta)\sin\theta = mg\tan\theta$.

And $m = \lambda L$, so $F = \lambda Lg\tan\theta$.

$$\left(\frac{\mu_0 I^2}{2\pi r}\right)L = \lambda Lg\tan\theta.$$

$$I = \sqrt{\frac{\lambda gr\tan\theta}{(\mu_0/2\pi)}}.$$

$$I = \sqrt{\frac{(0.0125\text{ kg/m})(9.80\text{ m/s}^2)(\tan 6.00°)(8.362\times10^{-3}\text{ m})}{2\times10^{-7}\,\text{T}\cdot\text{m/A}}} = 23.2\text{ A}.$$

EVALUATE: Since the currents are in opposite directions the wires repel. When I is increased, the angle θ from the vertical increases; a large current is required even for the small displacement specified in this problem.

28.62. **IDENTIFY:** Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l}\times\hat{r}}{r^2}$.

SET UP: The two straight segments produce zero field at P. The field at the center of a circular loop of radius R is $B = \dfrac{\mu_0 I}{2R}$, so the field at the center of curvature of a semicircular loop is $B = \dfrac{\mu_0 I}{4R}$.

EXECUTE: The semicircular loop of radius a produces field out of the page at P and the semicircular loop of radius b produces field into the page. Therefore, $B = B_a - B_b = \dfrac{1}{2}\left(\dfrac{\mu_0 I}{2}\right)\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \dfrac{\mu_0 I}{4a}\left(1 - \dfrac{a}{b}\right)$, out of page.

EVALUATE: If $a = b$, $B = 0$.

28.63. **IDENTIFY:** Apply Ampere's law to a circle of radius r.

SET UP: The current within a radius r is $I = \int \vec{J}\cdot d\vec{A}$, where the integration is over a disk of radius r.

EXECUTE: **(a)** $I_0 = \int \vec{J}\cdot d\vec{A} = \int\left(\dfrac{b}{r}e^{(r-a)/\delta}\right)rdrd\theta = 2\pi b\int_0^a e^{(r-a)/\delta}dr = 2\pi b\delta\, e^{(r-a)/\delta}\Big|_0^a = 2\pi b\delta(1 - e^{-a/\delta})$.

$I_0 = 2\pi(600\text{ A/m})(0.025\text{ m})(1 - e^{(0.050/0.025)}) = 81.5\text{ A}$.

(b) For $r \ge a$, $\text{r}\vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0$ and $B = \dfrac{\mu_0 I_0}{2\pi r}$.

(c) For $r \le a$, $I(r) = \int \vec{J}\cdot d\vec{A} = \int\left(\dfrac{b}{r'}e^{(r'-a)/\delta}\right)r'dr'd\theta = 2\pi b\int_0^r e^{(r-a)/\delta}dr = 2\pi b\delta e^{(r'-a)/\delta}\Big|_0^r$.

$I(r) = 2\pi b\delta(e^{(r-a)/\delta} - e^{-a/\delta}) = 2\pi b\delta e^{-a/\delta}(e^{r/\delta} - 1)$ and $I(r) = I_0 \dfrac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)}$.

(d) For $r \le a$, $\oint \vec{B} \cdot d\vec{l} = B(r)2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \dfrac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)}$ and $B = \dfrac{\mu_0 I_0 (e^{r/\delta} - 1)}{2\pi r(e^{a/\delta} - 1)}$.

(e) At $r = \delta = 0.025$ m, $B = \dfrac{\mu_0 I_0 (e-1)}{2\pi\delta(e^{a/\delta} - 1)} = \dfrac{\mu_0 (81.5\text{ A})}{2\pi(0.025\text{ m})} \dfrac{(e-1)}{(e^{0.050/0.025} - 1)} = 1.75\times10^{-4}$ T.

At $r = a = 0.050$ m, $B = \dfrac{\mu_0 I_0}{2\pi a} \dfrac{(e^{a/\delta} - 1)}{(e^{a/\delta} - 1)} = \dfrac{\mu_0 (81.5\text{ A})}{2\pi(0.050\text{ m})} = 3.26\times10^{-4}$ T.

At $r = 2a = 0.100$ m, $B = \dfrac{\mu_0 I_0}{2\pi r} = \dfrac{\mu_0 (81.5\text{ A})}{2\pi(0.100\text{ m})} = 1.63\times10^{-4}$ T.

EVALUATE: At points outside the cylinder, the magnetic field is the same as that due to a long wire running along the axis of the cylinder.

28.64. **IDENTIFY:** Both arcs produce magnetic fields at point *P* perpendicular to the plane of the page. The field due to arc *DA* points into the page, and the field due to arc *BC* points out of the page. The field due to *DA* has a greater magnitude than the field due to arc *BC*. The net field is the sum of these two fields.

SET UP: The magnitude field at the center of a circular loop of radius *a* is $B = \dfrac{\mu_0 I}{2\pi a}$. Each arc is 120°/360° = 1/3 of a complete loop, so the field due to each of them is $B = \dfrac{1}{3}\dfrac{\mu_0 I}{2\pi a} = \dfrac{\mu_0 I}{6\pi a}$.

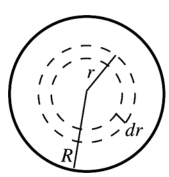
EXECUTE: The net field is

$B_{\text{net}} = B_{20} - B_{30} = \dfrac{\mu_0 (12.0\text{ A})}{6\pi}\left(\dfrac{1}{0.200\text{ m}} - \dfrac{1}{0.300\text{ m}}\right) = 4.19\times10^{-6}\text{ T} = 4.19\ \mu\text{T}$. Since $B_{20} > B_{30}$, the net field points into the page at *P*.

EVALUATE: The current in segments *CD* and *AB* produces no magnetic field at *P* because its direction is directly toward (or away from) point *P*.

28.65. **(a) IDENTIFY:** Consider current density *J* for a small concentric ring and integrate to find the total current in terms of α and *R*.

SET UP: We can't say $I = JA = J\pi R^2$, since *J* varies across the cross section.



To integrate *J* over the cross section of the wire, divide the wire cross section up into thin concentric rings of radius *r* and width *dr*, as shown in Figure 28.65.

Figure 28.65

EXECUTE: The area of such a ring is *dA*, and the current through it is $dI = J\,dA$; $dA = 2\pi r dr$ and

$dI = J\,dA = \alpha r(2\pi r\,dr) = 2\pi\alpha r^2 dr$.

$I = \int dI = 2\pi\alpha \int_0^R r^2 dr = 2\pi\alpha(R^3/3)$ so $\alpha = \dfrac{3I}{2\pi R^3}$.

(b) IDENTIFY and **SET UP:** (i) $r \le R$.

Apply Ampere's law to a circle of radius $r < R$. Use the method of part (a) to find the current enclosed by Ampere's law path.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \oint B\,dl = B\oint dl = B(2\pi r)$, by the symmetry and direction of $\vec{B}$. The current passing through the path is $I_{\text{encl}} = \int dl$, where the integration is from 0 to *r*.

$I_{\text{encl}} = 2\pi\alpha\int_0^r r^2 dr = \dfrac{2\pi\alpha r^3}{3} = \dfrac{2\pi}{3}\left(\dfrac{3I}{2\pi R^3}\right)r^3 = \dfrac{Ir^3}{R^3}$. Thus $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives

$B(2\pi r) = \mu_0\left(\dfrac{Ir^3}{R^3}\right)$ and $B = \dfrac{\mu_0 I r^2}{2\pi R^3}$.

(ii) **IDENTIFY** and **SET UP:** $r \ge R$.

Apply Ampere's law to a circle of radius $r > R$.

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = B\oint dl = B(2\pi r)$.

$I_{\text{encl}} = I$; all the current in the wire passes through this path. Thus $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives

$B(2\pi r) = \mu_0 I$ and $B = \dfrac{\mu_0 I}{2\pi r}$.

EVALUATE: Note that at $r = R$ the expression in (i) (for $r \le R$) gives $B = \dfrac{\mu_0 I}{2\pi R}$. At $r = R$ the expression in (ii) (for $r \ge R$) gives $B = \dfrac{\mu_0 I}{2\pi R}$, which is the same.

28.66. **IDENTIFY:** Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l}\times\hat{r}}{r^2}$.

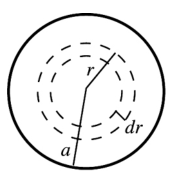
SET UP: The horizontal wire yields zero magnetic field since $d\vec{l}\times\vec{r} = 0$. The vertical current provides the magnetic field of half of an infinite wire. (The contributions from all infinitesimal pieces of the wire point in the same direction, so there is no vector addition or components to worry about.)

EXECUTE: $B = \frac{1}{2}\left(\dfrac{\mu_0 I}{2\pi a}\right) = \dfrac{\mu_0 I}{4\pi a}$ and is directed out of the page.

EVALUATE: In the equation preceding Eq. (28.8) the limits on the integration are 0 to a rather than $-a$ to a and this introduces a factor of $\frac{1}{2}$ into the expression for B.

28.67. **IDENTIFY:** Use the current density J to find dI through a concentric ring and integrate over the appropriate cross section to find the current through that cross section. Then use Ampere's law to find $\vec{B}$ at the specified distance from the center of the wire.

(a) SET UP:

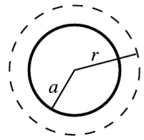


Divide the cross section of the cylinder into thin concentric rings of radius r and width dr, as shown in Figure 28.67a. The current through each ring is $dI = J\,dA = J2\pi r\,dr$.

Figure 28.67a

EXECUTE: $dI = \dfrac{2I_0}{\pi a^2}\left[1-(r/a)^2\right]2\pi r\,dr = \dfrac{4I_0}{a^2}\left[1-(r/a)^2\right]r\,dr$. The total current I is obtained by integrating dI over the cross section $I = \int_0^a dI = \left(\dfrac{4I_0}{a^2}\right)\int_0^a (1 - r^2/a^2)r\,dr = \left(\dfrac{4I_0}{a^2}\right)\left[\dfrac{1}{2}r^2 - \dfrac{1}{4}r^4/a^2\right]_0^a = I_0$, as was to be shown.

(b) SET UP: Apply Ampere's law to a path that is a circle of radius $r > a$, as shown in Figure 28.67b.



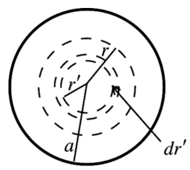
$\oint \vec{B} \cdot d\vec{l} = B(2\pi r).$

$I_{\text{encl}} = I_0$ (the path encloses the entire cylinder).

Figure 28.67b

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 I_0$ and $B = \dfrac{\mu_0 I_0}{2\pi r}.$

(c) SET UP:



Divide the cross section of the cylinder into concentric rings of radius r' and width dr', as was done in part (a). See Figure 28.67c. The current dI through each ring is

$$dI = \frac{4I_0}{a^2}\left[1-\left(\frac{r'}{a}\right)^2\right]r'\,dr'.$$

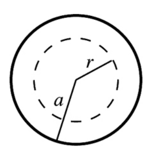
Figure 28.67c

EXECUTE: The current I is obtained by integrating dI from $r' = 0$ to $r' = r$:

$$I = \int dI = \frac{4I_0}{a^2}\int_0^r\left[1-\left(\frac{r'}{a}\right)^2\right]r'\,dr' = \frac{4I_0}{a^2}\left[\tfrac{1}{2}(r')^2 - \tfrac{1}{4}(r')^4/a^2\right]_0^r.$$

$$I = \frac{4I_0}{a^2}(r^2/2 - r^4/4a^2) = \frac{I_0 r^2}{a^2}\left(2-\frac{r^2}{a^2}\right).$$

(d) SET UP: Apply Ampere's law to a path that is a circle of radius $r < a$, as shown in Figure 28.67d.



$\oint \vec{B} \cdot d\vec{l} = B(2\pi r).$

$I_{\text{encl}} = \dfrac{I_0 r^2}{a^2}\left(2-\dfrac{r^2}{a^2}\right)$ (from part (c)).

Figure 28.67d

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 \dfrac{I_0 r^2}{a^2}(2 - r^2/a^2)$ and $B = \dfrac{\mu_0 I_0}{2\pi}\dfrac{r}{a^2}(2 - r^2/a^2).$

EVALUATE: Result in part (b) evaluated at $r = a$: $B = \dfrac{\mu_0 I_0}{2\pi a}.$ Result in part (d) evaluated at $r = a$: $B = \dfrac{\mu_0 I_0}{2\pi}\dfrac{a}{a^2}(2 - a^2/a^2) = \dfrac{\mu_0 I_0}{2\pi a}.$ The two results, one for $r > a$ and the other for $r < a$, agree at $r = a$.

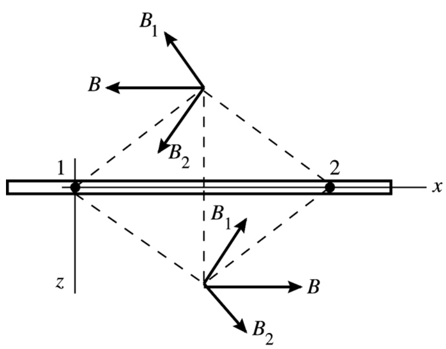
28.68. **IDENTIFY:** The net field is the vector sum of the fields due to the circular loop and to the long straight wire.

SET UP: For the long wire, $B = \dfrac{\mu_0 I_1}{2\pi D}$, and for the loop, $B = \dfrac{\mu_0 I_2}{2R}.$

EXECUTE: At the center of the circular loop the current I_2 generates a magnetic field that is into the page, so the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude: $\dfrac{\mu_0 I_1}{2\pi D} = \dfrac{\mu_0 I_2}{2R}.$ Thus, $I_1 = \dfrac{\pi D}{R} I_2.$

EVALUATE: If I_1 is to the left the two fields add.

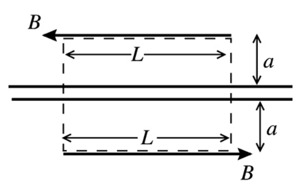
28.69. **IDENTIFY:** Use what we know about the magnetic field of a long, straight conductor to deduce the symmetry of the magnetic field. Then apply Ampere's law to calculate the magnetic field at a distance a above and below the current sheet.
SET UP: Do parts (a) and (b) together.



Consider the individual currents in pairs, where the currents in each pair are equidistant on either side of the point where $\vec{B}$ is being calculated. Figure 28.69a shows that for each pair the z-components cancel, and that above the sheet the field is in the $-x$-direction and that below the sheet it is in the $+x$-direction.

Figure 28.69a

Also, by symmetry the magnitude of $\vec{B}$ a distance a above the sheet must equal the magnitude of $\vec{B}$ a distance a below the sheet. Now that we have deduced the symmetry of $\vec{B}$, apply Ampere's law. Use a path that is a rectangle, as shown in Figure 28.69b.



$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}.$

Figure 28.69b

I is directed out of the page, so for I to be positive the integral around the path is taken in the counterclockwise direction.
EXECUTE: Since $\vec{B}$ is parallel to the sheet, on the sides of the rectangle that have length $2a$, $\oint \vec{B} \cdot d\vec{l} = 0.$ On the long sides of length L, $\vec{B}$ is parallel to the side, in the direction we are integrating around the path, and has the same magnitude, B, on each side. Thus $\oint \vec{B} \cdot d\vec{l} = 2BL.$ n conductors per unit length and current I out of the page in each conductor gives $I_{\text{encl}} = InL.$ Ampere's law then gives $2BL = \mu_0 InL$ and $B = \frac{1}{2}\mu_0 In.$
EVALUATE: Note that B is independent of the distance a from the sheet. Compare this result to the electric field due to an infinite sheet of charge in Chapter 22.

28.70. **IDENTIFY:** Find the vector sum of the fields due to each sheet.
SET UP: Problem 28.69 shows that for an infinite sheet $B = \frac{1}{2}\mu_0 In.$ If I is out of the page, $\vec{B}$ is to the left above the sheet and to the right below the sheet. If I is into the page, $\vec{B}$ is to the right above the sheet and to the left below the sheet. B is independent of the distance from the sheet. The directions of the two fields at points P, R and S are shown in Figure 28.70.
EXECUTE: **(a)** Above the two sheets, the fields cancel (since there is no dependence upon the distance from the sheets).
(b) In between the sheets the two fields add up to yield $B = \mu_0 nI,$ to the right.

(c) Below the two sheets, their fields again cancel (since there is no dependence upon the distance from the sheets).

EVALUATE: The two sheets with currents in opposite directions produce a uniform field between the sheets and zero field outside the two sheets. This is analogous to the electric field produced by large parallel sheets of charge of opposite sign.

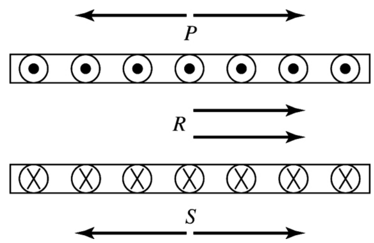


Figure 28.70

28.71. IDENTIFY: A charged cylindrical shell is rotating. This motion produces a magnetic field which exerts a torque on a very small disk at its midpoint.

SET UP and EXECUTE: **(a)** We want the current. $I = \dfrac{\Delta Q}{\Delta t}$. In one full rotation, charge Q_1 passes through in time T_1 which is the period of rotation of the cylinder. Thus $\Delta Q = Q_1$ and $\Delta T = T_1 = 2\pi/\omega_1$. So $I = \dfrac{Q_1}{(2\pi/\omega_1)} = \dfrac{Q_1\omega_1}{2\pi}$.

(b) We want B. Apply Ampere's law: $\oint \vec{B} \cdot d\hat{l} = \mu_0 I_{\text{encl}}$. The rectangular path of integration is similar to the one used in Example 28.9. The inner segment ab is on the axis of the cylinder, is equidistant from its ends, and has length $l \ll H$. $B = 0$ outside the cylinder, and the field is perpendicular to bc and da. Using the current from part (a), $\oint \vec{B} \cdot d\hat{l} = \mu_0 I_{\text{encl}}$ gives us $Bl = \mu_0 I \dfrac{l}{H} = \mu_0 \dfrac{Q_1\omega_1}{2\pi}\dfrac{l}{H}$. Solving for B and realizing that the field points along the $+z$-axis, we have $\vec{B} = \dfrac{\mu_0\omega_1 Q_1}{2\pi H}\hat{k}$.

(c) We want the torque on the disk. $\tau = \mu B \sin\phi$. Since the disk is very small, we can treat the magnetic field as uniform over its surface and equal to the field at the center of the cylinder. Using the given magnetic moment, our result from (b), and $\phi = \theta$, we get

$$\tau = \left(\frac{1}{4}Q_2\omega_2 R_2^2\right)\left(\frac{\mu_0 Q_1\omega_1}{2\pi H}\right)\sin\theta = \frac{\mu_0 Q_1 Q_2 \omega_1\omega_2 R_2^2}{8\pi H}\sin\theta.$$

(d) We want the angular momentum of the disk. $L = I\omega = \dfrac{1}{2}MR_2^2\omega_2$.

(e) We want the precession rate. From Section 10.7 we have

$$\Omega = \frac{\tau_z}{L_z} = \frac{\dfrac{\mu_0}{8\pi H}Q_1Q_2\omega_1\omega_2 R_2^2\sin\theta}{\dfrac{1}{2}MR_2^2\omega_2} = \frac{\mu_0}{4\pi H}\frac{Q_1Q_2\omega_1\sin\theta}{M}.$$

EVALUATE: The spinning charged cylinder behaves like a solenoid.

28.72. IDENTIFY and SET UP: We assume that both solenoids are ideal, in which case the field due to each one is given by $B = \mu_0 nI = \mu_0 \dfrac{N}{L}I$. The net field inside is the sum of both the fields.

EXECUTE: **(a)** The net field is $B=\mu_0\dfrac{N_1}{L}I_1+\mu_0\dfrac{N_2}{L}I_2=\dfrac{\mu_0}{L}[N_1I_1+N_2I_2].$ For the numbers in this problem, we have $BL/\mu_0=(0.00200\text{ A})N_1+N_2I_2.$ Therefore a graph of BL/μ_0 versus I_2 should be a straight line with slope equal to N_2 and y-intercept equal to (0.00200 A)N_1.

(b) Using the graph given with the problem, we calculate the slope using the points (5.00 mA, 16.00 A) and (2.00 mA, 8.00 A), which gives slope = (16.00 A – 8.00 A)/(5.00 mA – 2.00 mA) = 2667. Therefore N_2 = 2667 turns, which rounds to 2670 turns. To find the y-intercept, we use the point (5.00 mA, 16.00 A) and the slope to deduce the equation of the line. This gives $\dfrac{y-16.00\text{ A}}{x-0.00500\text{ A}}=2667,$ which simplifies to y = 2667x + 2.67. When x = 0, y = 2.67 A. As we saw, the y-intercept is equal to (0.00200 A)N_1, so N_1 = (2.67 A)/(0.00200 A) = 1335 turns, which rounds to 1340 turns.

(c) Now the fields are in opposite directions, so $B=\mu_0\dfrac{N_1}{L}I_1-\mu_0\dfrac{N_2}{L}I_2=\dfrac{\mu_0}{L}[N_1I_1-N_2I_2].$

B = [(μ_0)/(0.400 m)][(0.00200 A)(1335) – (0.00500 A)(2667)] $=-3.35\times10^{-5}$ T. The minus sign just tells us that the field due to I_2 is stronger than the field due to I_1. So the magnitude of the net field is $B=3.35\times0^{-5}\text{ T}=33.5\ \mu\text{T}.$

EVALUATE: As a check for N_1 in part (b), we could use a ruler to extrapolate the graph in the textbook back to its intersection with the y-axis to find the y-intercept. This method is not particularly accurate, but it should give reasonable agreement with the result for N_1 from part (b).

28.73. **IDENTIFY** and **SET UP:** The magnitude of the magnetic a distance r from the center of a very long current-carrying wire is $B=\dfrac{\mu_0 I}{2\pi r}.$ In this case, the measured quantity x is the distance from the *surface* of the cable, not from the center.

EXECUTE: **(a)** Multiplying the quantities given in the table in the problem, we get the following values for Bx in units of $\text{T}\cdot\text{cm},$ starting with the first pair: 0.812, 1.00, 1.09, 1.13, 1.16. As we can see, these values are not constant. However the last three values are nearly constant. Therefore Bx is not truly constant. The reason for this is that x is the distance from the *surface* of the cable, not from the center. In the formula $B=\dfrac{\mu_0 I}{2\pi r},$ r is the distance from the center of the cable. In that case, we would expect Br to be constant. For the last three points, it does appear that Bx is nearly constant. The reason for this is that the proper formula for the magnetic field for this cable is $B=\dfrac{\mu_0 I}{2\pi(R+x)},$ where R is the radius of the cable. As x gets large compared to R, $r\approx x$ and the magnitude approaches $\dfrac{\mu_0 I}{2\pi r}.$

(b) Using the equation appropriate for the cable and solving for x gives $x=(\mu_0 I/2\pi)\dfrac{1}{B}-R.$ A graph of x versus $1/B$ should have a slope equal to $\mu_0 I/2\pi$ and a y-intercept equal to $-R$. Figure 28.73 shows the graph of x versus $1/B$.

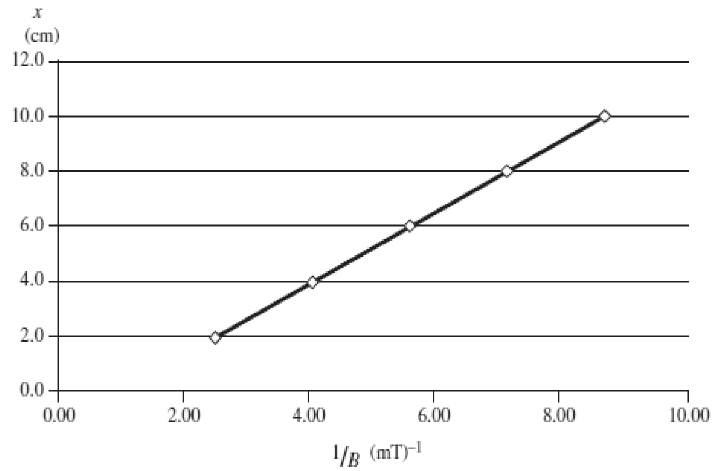


Figure 28.73

(c) The best-fit equation for this graph is $x = (1.2981\ \text{mT}\cdot\text{cm})\dfrac{1}{B} - 1.1914$ cm. The slope is $1.2981\ \text{mT}\cdot\text{cm} = 1.2981\times10^{-5}\ \text{T}\cdot\text{m}$. Since the slope is equal to $\mu_0 I/2\pi$, we have $\mu_0 I/2\pi =$ slope, which gives $I = 2\pi(\text{slope})/\mu_0 = 2\pi(1.2981\times10^{-5}\ \text{T}\cdot\text{m})/\mu_0 = 64.9\ \text{A}$, which rounds to 65 A. The y-intercept is $-R$, so $R = -(-1.1914\ \text{cm}) = 1.2$ cm.

EVALUATE: As we can see, the field within 2 cm or so of the surface of the cable would vary considerably from the value given by $B = \dfrac{\mu_0 I}{2\pi r}$.

28.74. **IDENTIFY** and **SET UP:** The wires repel each other since they carry currents in opposite directions, so the wires will move away from each other until the magnetic force is just balanced by the force due to the spring. The force per unit length between two parallel current-carrying wires of equal length and separation r is $\dfrac{F}{L} = \dfrac{\mu_0}{2\pi}\dfrac{I'I}{r}$. In this case, the currents are the same and the distance between the wires is $l_0 + x$, where x is the distance the spring stretches. Therefore the force is $F = \dfrac{\mu_0 I^2 L}{2\pi(l_0 + x)}$. The magnitude of the force that each spring exerts is $F = kx$, by Hooke's law. On each wire, $F_{\text{spr}} = F_{\text{mag}}$, and there are *two* spring forces on each wire. Therefore $\dfrac{\mu_0 I^2 L}{2\pi(l_0 + x)} = 2kx$.

EXECUTE: **(a)** We are given two cases with values for I and x, and each one leads to an equation involving l_0 and k. If we take the ratio of these two equations, common factors such as L will cancel. This gives us

$\dfrac{(13.1\ \text{A})^2(l_0 + 0.40\ \text{m})}{(8.05\ \text{A})^2(l_0 + 0.80\ \text{m})} = \dfrac{0.80\ \text{cm}}{0.40\ \text{cm}} = 2.0.$ Solving for l_0 gives $l_0 = 0.834$ cm, which rounds to 0.83 cm.

Now we can solve for k using this value for l_0 using $\dfrac{\mu_0 I^2 L}{2\pi(l_0 + x)} = 2kx$.

$$\frac{\mu_0(13.1\text{ A})^2(0.50\text{ m})}{2\pi(0.0080\text{ m}+0.00834\text{ m})}=2k(0.0080\text{ m}).$$ $k = 0.0656$ N/m, which rounds to 0.066 N/m.

(b) For a 12.0-A current, we have $\frac{\mu_0(12.0\text{ A})^2(0.50\text{ m})}{2\pi(x+0.00834\text{ m})}=2(0.0656\text{ N/m})x.$ Carrying out the multiplication and division and simplifying we get the quadratic equation
$x^2 + (0.00834\text{ m})x - 1.097\times10^{-4}\text{ m}^2 = 0.$

Using the quadratic formula and taking the positive solution gives $x = 0.0071$ m = 0.71 cm.
(c) To stretch the spring by 1.00 cm, the current must satisfy the equation

$$\frac{\mu_0 I^2(0.50\text{ m})}{2\pi(0.0100\text{ m}+0.00834\text{ m})}=2(0.0656\text{ N/m})(0.0100\text{ m}).$$ This gives $I = 15.5$ A, which rounds to 16 A.

EVALUATE: The spring force in part (c) is $kx = (0.0656\text{ N/m})(0.0100\text{ m}) = 6.56\times10^{-4}\text{ N}.$ This is a very small force resulting from a rather large 16-A current. This tells us that magnetic forces between parallel wires, such as extension cords, are not very significant for typical household currents.

28.75. **IDENTIFY:** The moving charges in the plasma cause a magnetic field.
SET UP and **EXECUTE:** **(a)** We want the charge density. There are n ions per unit volume, each with charge q, so $\rho = nq.$

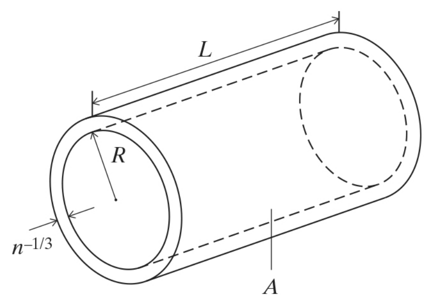


Figure 28.75a

(b) We want the surface charge density. See Fig. 28.75a. The "surface" has thickness equal to $n^{-1/3}$, so its volume is $V = 2\pi R n^{-1/3} L.$ $\sigma A = \rho V,$ so $\sigma = \frac{\rho V}{A} = \frac{\rho 2\pi R n^{-1/3} L}{2\pi R L} = \rho n^{-1/3} = (nq)n^{-1/3} = n^{2/3}q.$

(c) We want B at the surface. Use Ampere's law, taking an integration path around the surface. The current density is $J = \rho v.$ This gives $B 2\pi R = \mu_0 I = \mu_0(JA) = \mu_0 \rho v A = \mu_0 \rho v \pi R^2.$ Solve for B and using $\rho = nq.$ $B = \frac{1}{2}\mu_0 nqvR.$ By the right-hand rule, its direction is tangent to the circle. So we can express the field as $\vec{B} = \frac{1}{2}\mu_0 nqvR\hat{\phi}.$

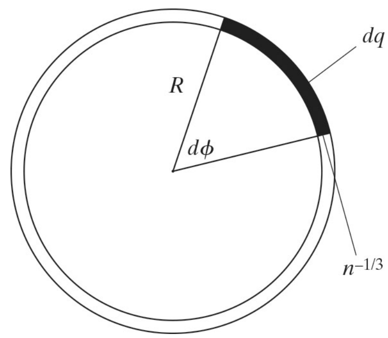


Figure 28.75b

(d) We want dI. Fig. 28.75b shows the geometric configuration. We know $\rho = nq$, $J = \rho v$, and $I = JA$. $dA = (Rd\phi)\left(n^{-1/3}\right)$. $dI = JdA = \rho v dA = \rho v\left(n^{-1/3}Rd\phi\right) = (qn)vn^{-1/3}Rd\phi = n^{2/3}qvRd\phi$.

(e) We want dF. $dF = dI(BL) = BL\left(n^{2/3}qvRd\phi\right) = \left(\frac{\mu_0}{2}nqvR\right)(L)\left(n^{2/3}qvRd\phi\right)$. This simplifies to $dF = \frac{\mu_0}{2}n^{5/3}q^2v^2R^2Ld\phi$.

(f) We want the force F and the pressure p. $F = \int_0^{2\pi} dF = \pi\mu_0 n^{5/3}q^2v^2R^2L$. The direction is inward toward the axis of the cylinder. $p = F/A$. Using F from above and $A = 2\pi RL$ gives $p = \frac{\mu_0}{2}n^{5/3}q^2v^2R$.

(g) We want the pressure. Using the numbers given in the problem, we get $p = 1.9\times10^{-5}$ Pa.

EVALUATE: This is a small pressure but it acts on very small objects (atoms).

28.76. **IDENTIFY:** We want to find the magnetic field inside a spinning charged cylinder.

SET UP and EXECUTE: **(a)** We want the surface charge density. $\sigma = \frac{Q}{A} = \frac{Q}{2\pi RW}$.

(b) We want dI. In one period T, the charge dq that passes along dx is $dq = \sigma dA = \sigma 2\pi R dx$. The time for this charge to pass by is $T = 2\pi/\omega$, so $dI = \frac{dq}{T} = \frac{\sigma 2\pi R dx}{2\pi/\omega} = \sigma\omega R dx$.

(c) We want dB_x at the origin. Eq. (28.15): $B_x = \frac{\mu_0 I a^2}{2\left(x^2+a^2\right)^{3/2}}$. Let a circular strip be at a distance x from the origin. In this case, $I \to dI, a \to R, x \to x$. Using this information and our results from parts (a) and (b), we get $dB_x = \frac{\mu_0 dI\, R^2}{2\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0\sigma\omega R dx R^2}{2\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0}{2}\left(\frac{Q}{2\pi RW}\right)\frac{\omega R^3 dx}{\left(x^2+R^2\right)^{3/2}}$, which reduces to $dB_x = \frac{\mu_0 Q\omega R^2 dx}{4\pi W\left(x^2+R^2\right)^{3/2}}$. By the right-hand rule, the direction is along the $+x$ axis.

(d) Integrate to find B_x. $B_x = 2\int_0^{W/2} dB_x = 2\int_0^{W/2} \frac{\mu_0 Q\omega R^2 dx}{4\pi W\left(x^2+R^2\right)^{3/2}}$. Using the integral tables in Appendix B we find $B_x = \frac{\mu_0}{2\pi}\frac{Q\omega}{\sqrt{W^2+4R^2}}$. The full field is $\vec{B} = \frac{\mu_0}{2\pi}\frac{Q\omega}{\sqrt{W^2+4R^2}}\hat{i}$.

EVALUATE: The rotating cylinder is quite similar to a solenoid.

28.77. **IDENTIFY:** A spinning spherical shell produces a magnetic moment.

SET UP and **EXECUTE:** $\vec{\mu} = \gamma\vec{L}$ where γ is the gyromagnetic ratio. $\gamma = g\dfrac{Q}{2M}$ where g is the g-factor.

(a) We want dI. Refer to Fig. 28.77 with the problem in the textbook. $dI = dq/T$, where $T =$ $T = 2\pi/\omega$. dq is the charge in one complete turn in time T. $q = \sigma dA = \left(\dfrac{Q}{4\pi R^2}\right)(Rd\theta)2\pi r$.

$r = R\sin\theta$. So $dI = \dfrac{\left(\dfrac{Q}{4\pi R^2}\right)(Rd\theta)(2\pi)(R\sin\theta)}{2\pi/\omega} = \dfrac{Q\omega}{4\pi}\sin\theta d\theta.$

(b) We want $d\mu$. $d\mu = AdI = \pi r^2 dI = \pi(R\sin\theta)^2\left(\dfrac{Q\omega}{4\pi}\sin\theta d\theta\right) = \dfrac{1}{4}Q\omega R^2\sin^3\theta\, d\theta.$ The direction is $+z$.

(c) We want $\vec{\mu}$. $\mu = \int d\mu = \int_0^\pi \dfrac{Q\omega R^2}{4}\sin^3\theta\, d\theta = \dfrac{Q\omega R^2}{4}\int_0^\pi \sin\theta(1-\cos^2\theta)d\theta = \dfrac{Q\omega R^2}{3}.$ The direction is $+z$.

(d) We want the angular momentum. $L = I\omega = \dfrac{2}{3}MR^2\omega.$

(e) We want γ. $\gamma = \dfrac{\mu}{L} = \dfrac{Q\omega R^2/3}{2MR^2\omega/3} = \dfrac{Q}{2M}.$

(f) We want g. $\gamma = g\dfrac{Q}{2M}$, $L = \dfrac{\mu}{\gamma}$, and $L = \dfrac{2}{3}MR^2\omega.$ Equate the two expressions for L and solve for g, which gives $g = 1$.

EVALUATE: If $g = 1$, $\gamma = \dfrac{Q}{2M}$ so $\mu = QL/2M$.

28.78. **IDENTIFY:** We are dealing with the magnetic field inside a rotating charged cylinder.

SET UP: $\vec{B} = \dfrac{\mu_0}{4\pi}\int\dfrac{\sigma\vec{v}\times\hat{r}}{r^2}dA.$ Refer to the Fig. 28.78 with the problem in the textbook.

EXECUTE: **(a)** Find σ. $\sigma = Q/A = \dfrac{Q}{2\pi RW}$.

(b) We want $\vec{r}$. ϕ is the angle with the $+y$-axis in the yz-plane. $\vec{r}$ is the vector from point (x, y, z) to the origin (0, 0, 0). So $r_x = -x$, $r_{y-} = -R\cos\phi$, and $r_z = -R\sin\phi$. Therefore

$\vec{r} = -(x\hat{i} + R\cos\phi\hat{j} + R\sin\phi\hat{k}).$

(c) We want $\vec{v}$. The velocity is in the yz-plane. v_y is negative when $0 < \phi \le \pi$. So $v_x = 0$, $v_y = -R\omega\sin\phi$, $v_z = R\omega\cos\phi$. So $\vec{v} = -R\omega\sin\phi\hat{j} + R\omega\cos\phi\hat{k}.$

(d) We want $\vec{v}\times\hat{r}$. $\hat{r} = \dfrac{\vec{r}}{r} = \dfrac{\vec{r}}{\sqrt{x^2+R^2}}$. Use this fact, along with the results from (b) and (c), to take the cross product $\vec{v}\times\hat{r}$. Combining and simplifying gives

$$\vec{v}\times\hat{r} = \frac{R\omega}{\sqrt{x^2+R^2}}\left[R\hat{i} - x\left(\cos\phi\hat{j} + \sin\phi\hat{k}\right)\right].$$

(e) Integrate to find $\vec{B}$. $\vec{B} = \dfrac{\mu_0}{4\pi}\int\dfrac{\sigma\vec{v}\times\hat{r}}{r^2}dA$ where $dA = R\,dx\,d\phi$. Use

$\vec{v}\times\hat{r} = \dfrac{R\omega}{\sqrt{x^2+R^2}}\left[R\hat{i} - x\left(\cos\phi\hat{j} + \sin\phi\hat{k}\right)\right]$ from part (d). The integral is

$\vec{B} = \dfrac{\mu_0 \sigma}{4\pi} \int_{-W/2}^{W/2} \int_{\phi=0}^{2\pi} \dfrac{R\omega}{\left(x^2+R^2\right)\sqrt{x^2+R^2}} \left[R\hat{i} - x\left(\cos\phi\hat{j} + \sin\phi\hat{k}\right)\right] R\,dx\,d\phi.$ Carrying out the integration

(which takes some time) yields $\vec{B} = \dfrac{\mu_0}{2\pi} \dfrac{Q\omega}{\sqrt{W^2+4R^2}}\hat{i}.$

EVALUATE: **(f)** Our answer agrees with the result of problem 28.76.

28.79. **IDENTIFY:** The current-carrying wires repel each other magnetically, causing them to accelerate horizontally. Since gravity is vertical, it plays no initial role.

SET UP: The magnetic force per unit length is $\dfrac{F}{L} = \dfrac{\mu_0}{2\pi}\dfrac{I^2}{d}$, and the acceleration obeys the equation $F/L = m/L\ a$. The rms current over a short discharge time is $I_0/\sqrt{2}$.

EXECUTE: **(a)** First get the force per unit length: $\dfrac{F}{L} = \dfrac{\mu_0}{2\pi}\dfrac{I^2}{d} = \dfrac{\mu_0}{2\pi d}\left(\dfrac{I_0}{\sqrt{2}}\right)^2 = \dfrac{\mu_0}{4\pi d}\left(\dfrac{V}{R}\right)^2 = \dfrac{\mu_0}{4\pi d}\left(\dfrac{Q_0}{RC}\right)^2.$

Now apply Newton's second law using the result above: $\dfrac{F}{L} = \dfrac{m}{L}a = \lambda a = \dfrac{\mu_0}{4\pi d}\left(\dfrac{Q_0}{RC}\right)^2$. Solving for a gives $a = \dfrac{\mu_0 Q_0^2}{4\pi\lambda R^2 C^2 d}$. From the kinematics equation $v_x = v_{0x} + a_x t$, we have $v_0 = at = aRC = \dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}.$

(b) Conservation of energy gives $\frac{1}{2}mv_0^2 = mgh$ and $h = \dfrac{v_0^2}{2g} = \dfrac{\left(\dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2}{2g} = \dfrac{1}{2g}\left(\dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2.$

EVALUATE: Once the wires have swung apart, we would have to consider gravity in applying Newton's second law.

28.80. **IDENTIFY:** Approximate the moving belt as an infinite current sheet.

SET UP: Problem 28.69 shows that $B = \frac{1}{2}\mu_0 In$ for an infinite current sheet. Let L be the width of the sheet, so $n = 1/L$.

EXECUTE: The amount of charge on a length Δx of the belt is $\Delta Q = L\Delta x\sigma$, so $I = \dfrac{\Delta Q}{\Delta t} = L\dfrac{\Delta x}{\Delta t}\sigma = Lv\sigma$. Approximating the belt as an infinite sheet $B = \dfrac{\mu_0 I}{2L} = \dfrac{\mu_0 v\sigma}{2}$. $\vec{B}$ is directed out of the page, as shown in Figure 28.80.

EVALUATE: The field is uniform above the sheet, for points close enough to the sheet for it to be considered infinite.

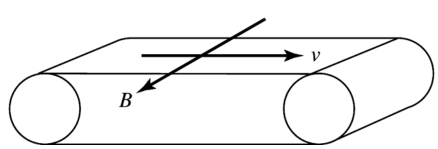


Figure 28.80

28.81. **IDENTIFY** and **SET UP:** This solenoid is not ideal since its width is fairly large compared to its length. But we can get a rough estimate using the ideal formula, $B = \mu_0 nI$.

EXECUTE: $B = \mu_0 nI = \mu_0 (1000\ \text{m}^{-1})I = 150\times10^{-6}\ \text{T}$, which gives $I = 0.12$ A, choice (b).

EVALUATE: This is a reasonable laboratory current of 120 mA.

28.82. IDENTIFY and SET UP: The magnetic field of an ideal solenoid is $B = \mu_0 nI$.

EXECUTE: Both solenoids have the same current, the same length, and the same number of turns, so the magnetic field inside both of them should be the same, which is choice (c).

EVALUATE: This answer is somewhat of an approximation. Even though both solenoids have the same current and same length and number of turns, the second (larger) solenoid is even farther from the ideal case than the first one. Therefore there would be some difference in the magnetic fields inside.

28.83. IDENTIFY and SET UP: The enclosure is no longer present to shield the solenoid from the earth's magnetic field of $50\ \mu\text{T}$, so net field inside is a sum of the solenoid field and the earth's field. Whether the earth's field adds or subtracts from the solenoid's field depends on the orientation of the solenoid. The magnetic field due to the solenoid is $150\ \mu\text{T}$.

EXECUTE: When the solenoid field is parallel to the earth's field, the net field is $150\ \mu\text{T} + 50\ \mu\text{T} = 200\ \mu\text{T}$. When the field's are antiparallel (opposite), the net field is $150\ \mu\text{T} - 50\ \mu\text{T} = 100\ \mu\text{T}$. So the field that the bacteria experience is between $100\ \mu\text{T}$ and $200\ \mu\text{T}$, which is choice (c).

EVALUATE: Since the earth's field is quite appreciable compared to the solenoid's field, it is important to shield the solenoid from external fields, such as that of the earth. The earth's field can make a difference of up to a factor of 2 in the field experienced by the bacteria.