Spring Potential Energy

Spring potential energy is a <u>restorative force</u> (it returns to equilibrium)

Hookes Law: $\vec{F}_s = -k\Delta\vec{x}$

$\Delta U = -W$

$\Delta U_s = -W_s$

$$W = \int_A^B \vec{F}_s \; d\vec{r}$$

$\vec{F}_s$ in the direction $\leftarrow$

$\Delta\vec{r}$ in the direction $\rightarrow$

$\angle$ between the vectors is 180°

$$W_s = \int_A^B |\vec{F}_s||d\vec{r}| \; \cos(180^\circ)$$

$$W_s = -\int_A^B k\Delta x \; dx$$

$$W_s = \left[-\frac{1}{2}k\Delta x^2\right]_A^B$$

Remember: $\Delta U_s = -W_s$

$$U_{sA} = \frac{1}{2}k(\Delta x_A)^2$$

$$U_s = \frac{1}{2}k(\Delta x)^2$$

$U_g = mgh$

Above two equations found using: $\Delta U = -W$

Only true for conservative forces

Also: $W = \Delta KE$

When we have only conservative forces:

$W_{tot} = \Delta KE$

$W_{tot} = -\Delta U$

Meaning that: $\Delta KE = -\Delta U$

$KE_f - KE_o = -(U_f - U_o)$

$KE_f + U_f = KE_o + U_o$

This is the conservation of energy

Let's say at Point A, all energy is kinetic

At Point B, all energy is potential

At Point C, kinetic energy and potential energy are equal

What's the velocity at Point C?

$E_{totc} = KE_c + U_c$

$KE_c = \frac{1}{2}mv_c^2$

Then, $KE_c = \frac{1}{2}mv_c^2 = \frac{1}{2}E_{tot} = \frac{1}{2}\left[\frac{1}{2}mv_a^2\right]$

Therefore, $v_c = \frac{v_a}{\sqrt{2}}$

Consider a spring:

- $\mu \neq 0$

- At point o, spring is fully extended
- At point A, spring is at equilibrium

We know that at Point o, $KE_o = 0$ (Fully extended)

We know that at point A, $U_{sA} = 0$ (Equilibrium)

When $\mu = 0$, $KE_o + U_{so} = KE_A + U_{sA}$

With friction, KE_A will be less because W_f is taking energy out of the system

$W_f + U_{so} = KE_A$

Place work from non-conservative forces with the initial energy of the system (because the work from friction is going to be negative)

$KE_o + U_o + W_{\mathrm{nc}} = KE_f + U_f$