Rocket Science

Conservation of Momentum comes directly from Newton's Third Law

If $\sum \vec{F} = 0$, $\vec{p}_o = \vec{p}_f$

$0 = -\Delta m u + (m - \Delta m)v$

u: speed of rock, v = speed of you

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}u + m\frac{d\vec{v}}{dt} = 0$$

Multiply everything by dt

$dmu + mdv = 0$

Divide everything by m

$$\frac{dm}{m}u + dv = 0$$

Assume u is constant, launching things out the back at a constant rate

u is kind of like average speed

Integrate both sides

$$\int_{m_o}^{m_f} \frac{dm}{m}u + \int_{v_o}^{v_f} dv = 0$$

$$u\,[\ln m]_{m_o}^{m_f} + \Delta v = 0$$

$$-u \ln\left(\frac{m_f}{m_o}\right) = \Delta v$$

<u>Important Equation</u>

$$\Delta v = u \ln\left(\frac{m_o}{m_f}\right)$$

If we use 80% of m_o as fuel. What is our max speed in terms of u?

$m_f = 0.2m_o$

$$\ln\left(\frac{m_o}{0.2m_o}\right) = \ln\left(\frac{1}{0.2}\right) = \ln 5$$

$\Delta v = \ln(5)\, u$

$\Delta v = 1.61u$

On Earth with <u>gravity</u>

$\sum \vec{F} = -mg$

$\frac{dm}{dt}u + m\frac{dv}{dt} = -mg$

Multiply by dt and divide by mass

$\frac{dm}{m}u + dv = -g\,dt$

Integrate all terms

$u \ln\left(\frac{m_f}{m_o}\right) + \Delta v = -gt$

Time matters because of an acceleration

$\Delta v = u \ln\left(\frac{m_o}{m_f}\right) - gt$