Perfetly Elastic Collisions

If $m_a = m_b$:

$v_{af} = 0$

$v_{bf} = 5\text{m/s}$

If $v_{af} < 0$:

$m_a > m_b$

$\vec{p}_o = \vec{p}_f$

$KE_o = KE_f$

Only really works for one-dimensional problems

Conservation of Momentum:

$m_a + v_{ao} + m_b v_{bo} = m_a v_{af} + m_b v_{bf}$

Conservation of Energy:

$\frac{1}{2} m v_{ao}^2 + \frac{1}{2} m_b v_{bo}^2 = \frac{1}{2} m_a v_{af}^2 + \frac{1}{2} m_b v_{bf}^2$

$m v_{ao}^2 + m_b v_{bo}^2 = m_a v_{af}^2 + m_b v_{bf}^2$

$m_a(v_{ao} - v_{af}) = m_b(v_{bf} - v_{bo})$

$m_a(v_{ao}^2 - v_{af}^2) = m_b(m_{bf}^2 - v_{bo}^2)$

$m_a((v_{ao} - v_{af})(v_{ao} + v_{af})) = m_b((v_{bf} v_{bo})(v_{bf} + v_{bo}))$

Use to cancel:

$m_a(v_{ao}^2 - v_{af}^2) = m_b(m_{bf}^2 - v_{bo}^2)$

Always true when Momentum and Energy are conserved:

Important Equation:

$v_{ao} + v_{af} = v_{bf} + v_{bo}$

$v_{bf} = v_{ao} + v_{af} - v_{bo}$

Plug into conservation of momentum:

$m_a v_{ao} + m_b v_{bo} = m_a v_{af} + m_b(v_{ao} + v_{af} - v_{bo})$

$(m_a - m_b)v_{ao} + 2m_b v_{bo} = (m_a + m_b)v_{af}$

Only in one dimension:

Usually helpful to change reference frame to make $v_{bo} = 0$

Equations to determine a_f and b_f:

$$v_{af} = \frac{m_a - m_b}{m_a + m_b} v_{ao} + \frac{2m_b}{m_a + m_b} v_{bo}$$

$$v_{bf} = \frac{2m_a}{m_a + m_b} v_{ao} + \frac{m_b - m_a}{m_a + m_b} v_{bo}$$

If $m_a = m_b$ and $v_{bo} = 0$:

$v_{af} = 0 + 0$

$v_{bf} = v_{ao} + 0$

For $v_{af} < 0$, m_a must be $< m_b$