Rotational Kinetic Energy

$KE = ?$

$KE = \frac{1}{2}mv^2$

$v = R\omega$

$KE = \frac{1}{2}m\left(R\omega\right)^2$

$KE = \frac{1}{2}\left(mR^2\right)\omega^2$

$KE = \frac{1}{2}I\omega^2$, I: moment of intertia, $I = mR^2$

$I_{pp} = mR^2$, pp = point particle, moment of intertia for a point particle

Earth is a point particle going around the sun. The Earth isn't one when we're standing on it. Point particle depends on the scale that we're looking at it.

"Rigid body" means $\omega_1 = \omega_2$. Planets are not a rigid body because each planet has a different orbital period.

$KE_{tot} = \frac{1}{2}\left(m_1{R_1}^2\right)\omega^2 + \frac{1}{2}\left(m_2{R_2}^2\right)\omega^2$

$\frac{1}{2}\left[m_1{R_1}^2 + m_2{R_2}^2\right]\omega^2$

Brackets = Moment of intertia for both objects

$KE_{tot} = \frac{1}{2}I_{tot}\omega^2$

$I_{tot} = \sum_{i=1}^{N} I_i$

$I = \sum_{i=1}^{10} m_i{r_i}^2$

As $n \to \infty$, $m \to dm$

$\int_0^M dmr^2$

Don't know how mass relates to distance, so can't integrate

linear density: $\lambda = \frac{dm}{dr}$, λ = lambda

How does a small change in mass relate to a small chance in distance?

$dm = \lambda dr$

Assume: λ is constant

$$I = \int_0^L \lambda dr\, r^2$$

$$I = \lambda \int_0^L r^2 dr$$

$$I = \frac{\lambda L^3}{3}$$

$$\lambda = \text{const.} = \frac{M}{L}$$

$$I = \frac{1}{3} M L^2$$

Rotating about center:

$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} \lambda r^2 \; dr$$

$$I = 2 \int_0^{\frac{L}{2}} \lambda r^2 \; dr$$

$$I = \left[\frac{\lambda r^3}{3}\right]_{-L/2}^{L/2} = \frac{\lambda}{3}\left[\frac{L^3}{8} - \frac{-L^3}{8}\right]$$

$$I = \frac{\lambda}{3} \frac{L^3}{4}$$

$$I = \frac{1}{12} M L^2$$