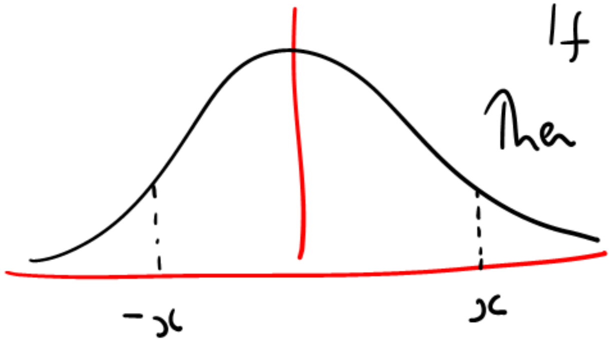
$$\rightarrow S_i = S_{i-1}\left(1 + r\,\delta t + \sigma\phi\sqrt{\delta t}\right)$$

$$S_t = S_{t-\delta t}\, e^{(r-\frac{1}{2}\sigma^2)\delta t + \sigma\phi\sqrt{\delta t}}$$

δt is Δ



If $x \sim N(0,1)$

Then so is $-x$

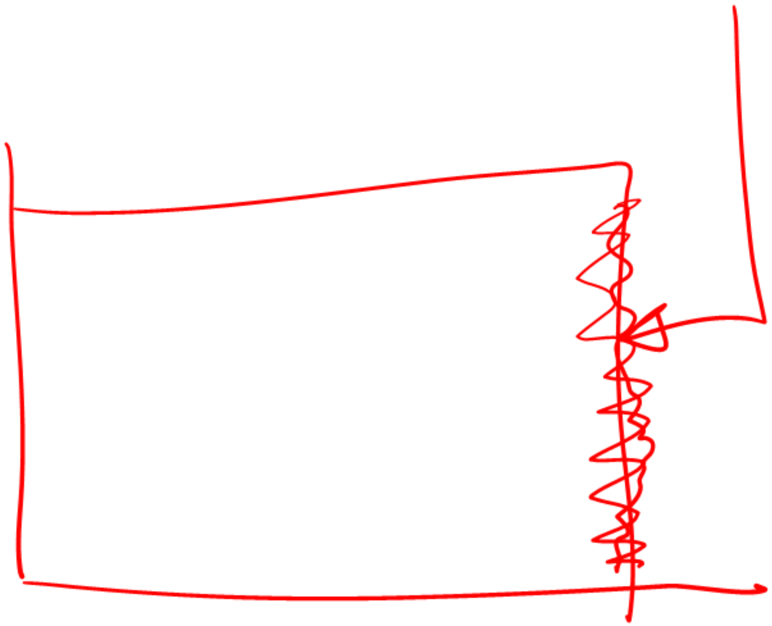
M.C:

1. Simulate many times the path
2. discount the payoff
3. Take average $\rightarrow V_1$
4. Redo 1 with $-\phi$ & repeat 2, 3 $\rightarrow V_2$
5. Option value $= \dfrac{V_1 + V_2}{2}$

$$\varepsilon \sim O\left(\frac{1}{\sqrt{N}}\right)$$

$$\varepsilon = \left| V_{MC} - V_{BS} \right|$$

$$\underbrace{\max(S - E_0)}$$



$$V(S,t) := V(n\delta S,\, T - m\delta t) := V_n^m \qquad 0 \le n \le N \quad 0 \le m \le M$$

$$\therefore \frac{\partial V}{\partial t} \sim \frac{V_n^m - V_n^{m+1}}{\delta t}\ ; \qquad \frac{\partial V}{\partial S} \sim \frac{V_{n+1}^m - V_{n-1}^m}{2\,\delta S}$$

$$\frac{\partial^2 V}{\partial S^2} \sim \frac{V_{n-1}^m - 2V_n^m + V_{n+1}^m}{\delta S^2}$$

Subst in BSE

$$\frac{V_n^m - V_n^{m+1}}{\delta t} + \frac{1}{2}\sigma^2 n^2 \cancel{\delta S^2}\, \frac{\left(V_{n-1}^m - 2V_n^m + V_{n+1}^m\right)}{\cancel{\delta S^2}} + (r-D)\, n \cancel{\delta S}\, \frac{\left(V_{n+1}^m - V_{n-1}^m\right)}{2\,\cancel{\delta S}} - rV_n^m = 0$$

$$V_n^{m+1} =$$

$$V_n^m + \frac{1}{2}\sigma^2 n^2 \delta t \left(V_{n-1}^m - 2V_n^m + V_{n+1}^m \right) + \frac{1}{2}(r-D)\, n\, \delta t \left(V_{n+1}^m - V_{n-1}^m \right) - r\,\delta t\, V_n^m = 0$$

$$V_{n-1}^m : \quad \frac{1}{2}\left((\sigma_n^m)^2 n^2 - (r^m - D_n^m)\, n \right) \delta t \qquad a_n$$

$$V_n^m : \quad 1 - \sigma^2 n^2 \delta t - r\,\delta t = 1 - \left((\sigma_n^m)^2 n^2 + r^m \right) \delta t \qquad b_n$$

$$V_{n+1}^m : \quad \frac{1}{2}\left((\sigma_n^m)^2 n^2 + (r^m - D_n^m)\, n \right) \delta t \qquad c_n$$

$$V_n^{m+1} = a_n V_{n-1}^m + b_n V_n^m + c_n V_{n+1}^m$$

$$V_n^{m+1} = F\left(V_{n-1}^m, V_n^m, V_{n+1}^m\right)$$

hence explicit scheme

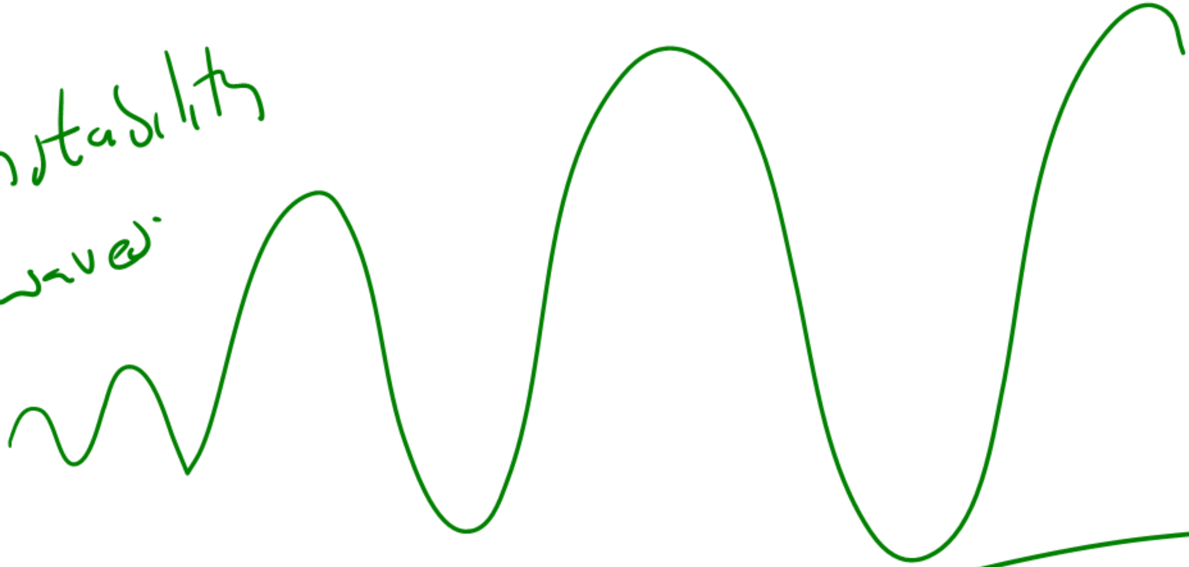
Suppose our B.S.E was

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2(S,t)\frac{\partial^2 V}{\partial S^2} + \left(r(t) - D(S,t)\right)\frac{\partial V}{\partial S} - r(t)V = 0$$

$$\boxed{\delta t < \frac{1}{\sigma^2 N^2}}$$

Fourier Stability Analysis

Instability waves.



- M.C
- hard core stat. analysis

at the upper boundary $S = S_\infty$ is $n = N$

$$V^{m+1}_{n=N} = a_n V^m_{\underbrace{n-1}_{N-1}} + b_n V^m_{\underbrace{n}_{N}} + c_n V^m_{\underbrace{n+1}_{N+1}}$$

Not defined for $n = N+1$

Using $\Gamma \to 0$ as $S \to \infty$

$$V^m_{N+1} = 2V^m_{N-1} - V^m_N$$

PLEASE CHECK