Python Labs

Credit Risk Analytics - II

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Credit Curve

In part two of this credit lab, we'll extend the understanding from the earlier lab to model hazard rates and fix missing values. Table below shows the term structure of credit spreads for two reference entities: Wells Fargo (WFC) - a highly rated institution and Clear Channel Communication (CCMO) - a highly leveraged entity.

Maturity	WFC	CCMO	$Z(0;T)$
1Y	50	751	0.97
2Y	77	1164	0.94
3Y	94	1874	0.92
5Y	125	4156	0.86
7Y	133	6083	0.81

The objective here is to bootstrap implied survival probabilities along with the term structure of hazard rate for

- WFC bank with recovery rate of 50%
- CCMO corporation with recovery rate of 10%

Similar to the previous lab module, we'll then study the effect of an increase in recovery rate on implied survival probabilities for different values of RR.

In [1]:
```
# Import libraries
import pandas as pd
import numpy as np
from numpy import *

# Plotting library
import matplotlib
import matplotlib.pyplot as plt

# Plot settings
matplotlib.rcParams['figure.figsize'] = [14.0, 8.0]
matplotlib.rcParams['font.size'] = 18
matplotlib.rcParams['lines.linewidth'] = 2.0

# Interactive plotting
import cufflinks as cf
cf.set_config_file(offline=True)
#cf.set_config_file(theme = 'pearl')
# cf.getThemes() ['ggplot', 'pearl', 'solar', 'space', 'white', 'polar', 'henani
```

In [2]:
```
# subsume list of inputs into a dataframe
cds = pd.DataFrame({'Maturity': [1, 2, 3, 5, 7],
                    'WFC': [50, 77, 94, 125, 133],
                    'CCMO': [751, 1164, 1874, 4156, 6083],
                    'Df': [0.97, 0.94, 0.92, 0.86, 0.81]})
# output
cds
```

Out[2]:

	Maturity	WFC	CCMO	Df
0	1	50	751	0.97
1	2	77	1164	0.94
2	3	94	1874	0.92
3	5	125	4156	0.86
4	7	133	6083	0.81

Interpolation

Given the missing data points in the CDS table, we'll have to perform interpolation to derive the missing tenor and spreads before we proceed with our analysis.

Discount Factors

For discounting factors, we'll do a log-linear interpolation for $\tau_i < \tau < \tau_{i+1}$,

$$lnDF(0,\tau) = \frac{\tau - \tau_i}{\tau_{i+1} - \tau_i}\, lnDF(0,\tau_{i+1}) + \frac{\tau_{i+1} - \tau}{\tau_{i+1} - \tau_i}\, lnDF(0,\tau_i)$$

We'll now create a User Defined Function (UDF) in Python to derive the discount factors.

In [3]:
```
def get_discount_factor(maturity, discountfactor, tenor):

    max_time_index = len(maturity) - 1

    if tenor == 0: Df = 1.
    if tenor > 0 and tenor < maturity[0]: Df = discountfactor[0]
    if tenor >= maturity[max_time_index]: Df = discountfactor[max_time_index]

    for i in range(0, max_time_index):
        if tenor >= maturity[i] and tenor < maturity[i+1]:
            term1 = ((tenor-maturity[i])/(maturity[i+1] - maturity[i]))*log(disc
            term2 = ((maturity[i+1]-tenor)/(maturity[i+1] - maturity[i]))*log(di
            lnDf = term1 + term2
            Df = exp(lnDf)

    return Df
```

In [4]:
```
t = np.arange(8)
Df = np.zeros(len(t))
```

```
for i in range(0, len(t)):
    Df[i] = get_discount_factor(cds.Maturity,cds.Df,t[i])
```

In [5]:
```
print(f'Discount Factors: {Df}')
```

```
Discount Factors: [1.         0.97       0.94       0.92       0.88949424 0.86
 0.83462566 0.81      ]
```

Credit Spreads

For credit spreads (or equally hazard rates), a linear interpolation is done either using the below formulation or by applying the method of your choice.

$$CDS(\tau) = \frac{\tau - \tau_i}{\tau_{i+1} - \tau_i} CDS_{i+1} + \frac{\tau_{i+1} - \tau}{\tau_{i+1} - \tau_i} CDS_i$$

In this case, we'll leverage the power of NumPy by directly applying the `interp` methods for interpolation of spread values.

In [6]:
```
# interpolation WFC spreads
wfc = np.interp(t,cds.Maturity,cds.WFC)

# set spreads to zero at t=0
wfc[0] = 0

# interpolation CCMO spreads
ccmo = np.interp(t,cds.Maturity,cds.CCMO)

# set spreads to zero at t=0
ccmo[0] = 0
```

In [7]:
```
# output the results
print(f'WFC Spreads: \t {wfc}')
print(f'CCMO Spreads: \t {ccmo}')
```

```
WFC Spreads:     [  0.    50.    77.    94.   109.5 125.   129.   133. ]
CCMO Spreads:    [   0.    751.   1164.   1874.   3015.   4156.   5119.5 6083. ]
```

In [8]:
```
# subsume list of inputs into a dataframe
df = pd.DataFrame({'Maturity': t,
                   'WFC': wfc,
                   'CCMO': ccmo,
                   'Df': Df})
# output
df
```

Out[8]:

	Maturity	WFC	CCMO	Df
0	0	0.0	0.0	1.000000
1	1	50.0	751.0	0.970000
2	2	77.0	1164.0	0.940000
3	3	94.0	1874.0	0.920000
4	4	109.5	3015.0	0.889494
5	5	125.0	4156.0	0.860000

	Maturity	WFC	CCMO	Df
6	6	129.0	5119.5	0.834626
7	7	133.0	6083.0	0.810000

Piecewise Constant Hazard Rate

We'll now modify the survial probability function that we built in the last lab to accomdate the calculation of intensity or hazard rates. The piecewise constant hazard rate is given by

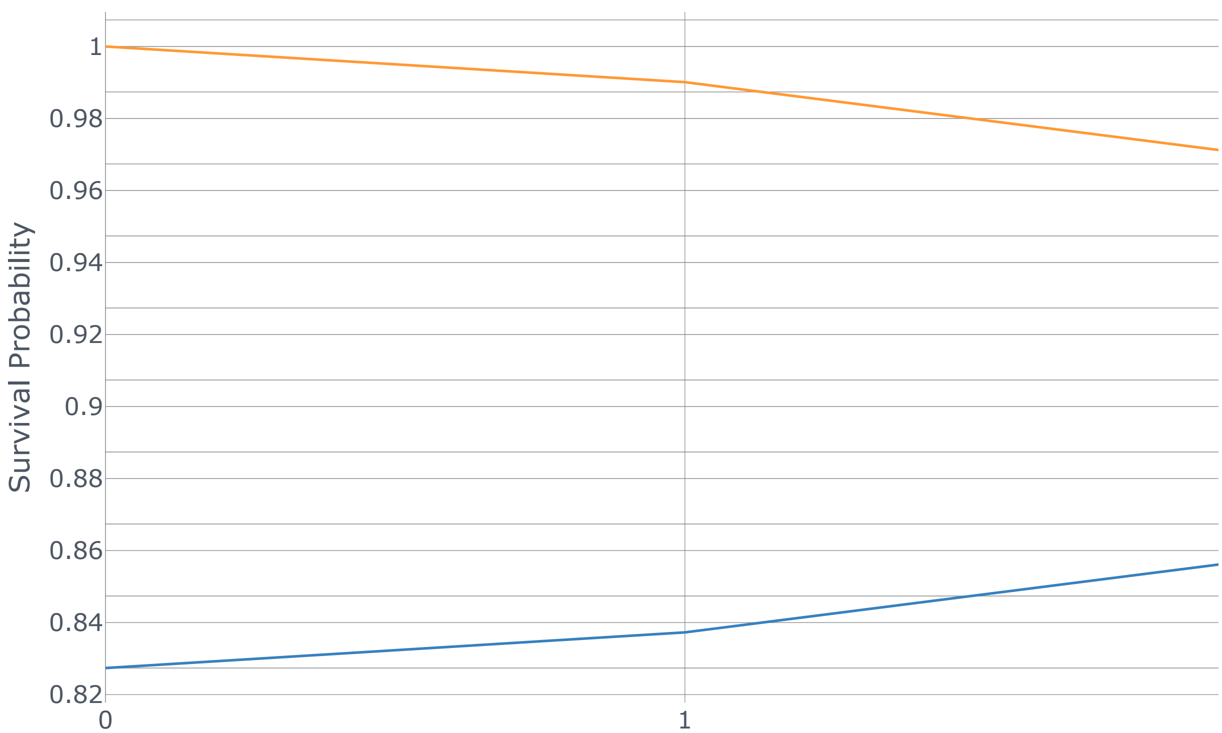
$$\lambda_m = \frac{-1}{\Delta t} \, log \frac{P(0, t_m)}{P(0, t_{m-1})}$$

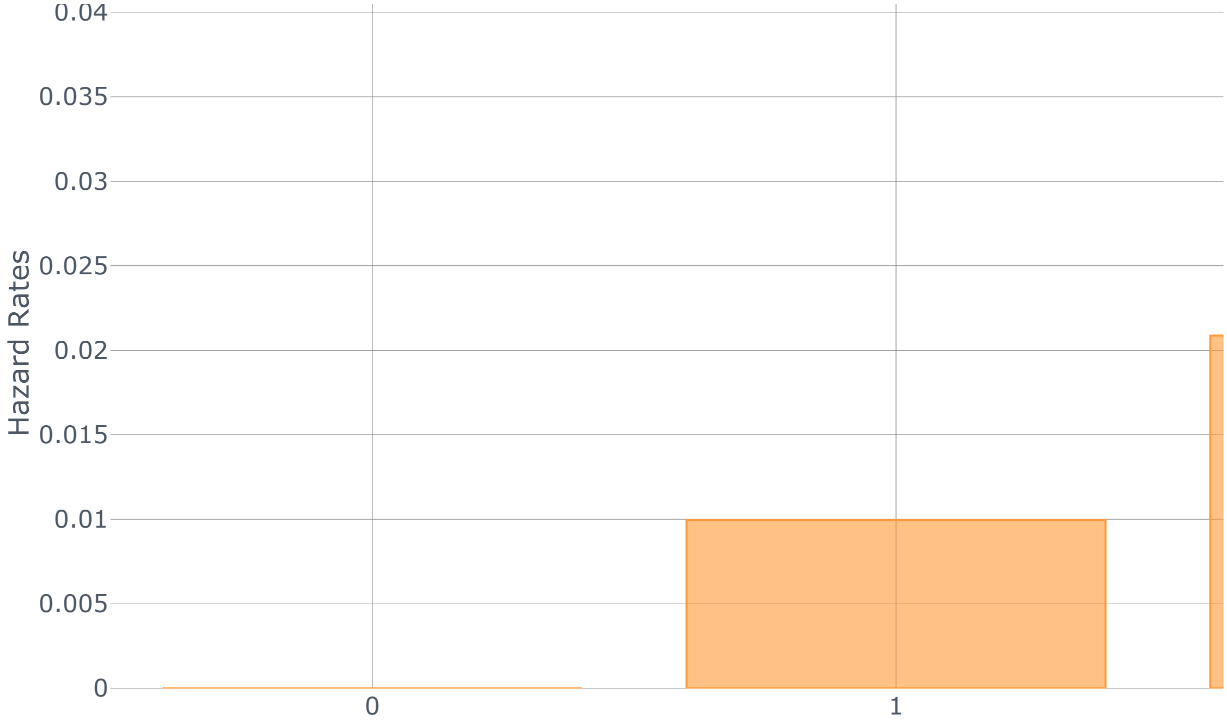
In [9]:
```
def get_survival_probability(maturity, discountfactor, spread, recovery, plot_pr

    # subsume list of inputs into a dataframe
    df = pd.DataFrame({'Maturity': maturity, 'Df': discountfactor, 'Spread': spr

    # convert bps to decimal
    df['Spread'] = df['Spread']/10000

    # specify delta_t
    df['Dt'] = df['Maturity'].diff().fillna(0)

    # loss rate
    L = 1.0 - recovery

    # initialize the variables
    term = term1 = term2 = divider = 0

    for i in range(0, len(df.index)):
        if i == 0: df.loc[i,'Survival'] = 1; df.loc[i, 'Hazard'] = 0
        if i == 1: df.loc[i,'Survival'] = L / (L+df.loc[i,'Dt']*df.loc[i,'Spread
            df.loc[i, 'Hazard'] = -log(df.loc[i,'Survival']/df.loc[i-1,'Survival
        if i > 1:
            terms = 0
            for j in range(1, i):
                term = df.loc[j,'Df']*(L*df.loc[j-1,'Survival'] - \
                                        (L + df.loc[j,'Dt']*df.loc[i,'Spre
                                        df.loc[j,'Survival'])
                terms = terms + term

            divider = df.loc[i,'Df']*(L+df.loc[i,'Dt']*df.loc[i,'Spread'])
            term1 = terms/divider

            term2 = (L*df.loc[i-1,'Survival']) / (L + (df.loc[i,'Dt'] * df.loc[i

            df.loc[i,'Survival'] = term1 + term2

            if (df.loc[i,'Survival'] >= 0 and df.loc[i-1,'Survival'] >= 0):
                df.loc[i, 'Hazard'] = -log(df.loc[i,'Survival']/df.loc[i-1,'Surv

    # derive probability of default
    df['Default'] = 1. - df['Survival']

    # derive marginal probability of default
```

```
    df['Marginal'] = df['Survival'].diff().fillna(0)

    if plot_prob:
        # plot survival probability
        df[['Survival', 'Default']].iplot(title='Survival vs Default Probability
                                          xTitle='CDS Maturity',
                                          yTitle='Survival Probability',
                                          secondary_y = 'Default',
                                          secondary_y_title='Default Probability

    if plot_hazard:
        # plot survival probability
        df['Hazard'].iplot(kind='bar', title='Term Structure of Hazard Rates',
                                          xTitle='CDS Maturity',
                                          yTitle='Hazard Rates')

    return df
```

In [10]:

```
WFC = get_survival_probability(df.Maturity,df.Df,df.WFC,0.50,plot_prob=True,plot
WFC
```





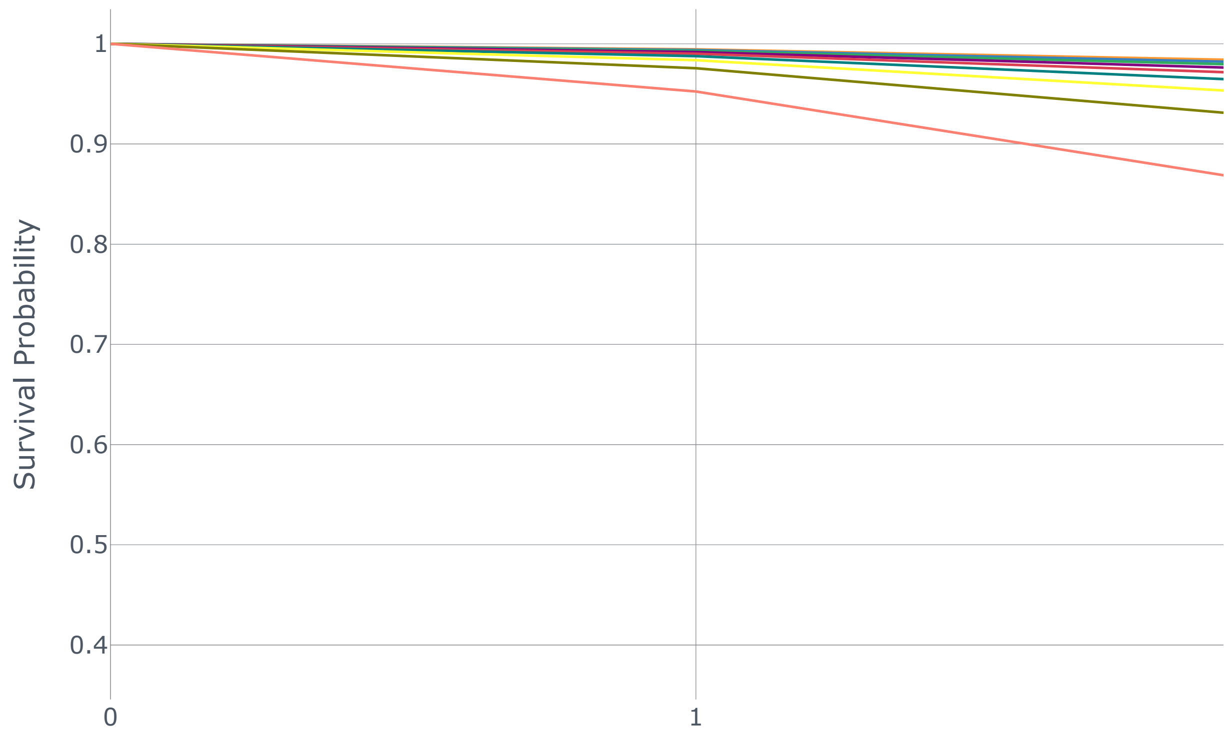
Out[10]:

	Maturity	Df	Spread	Dt	Survival	Hazard	Default	Marginal
0	0	1.000000	0.00000	0.0	1.000000	0.000000	0.000000	0.000000
1	1	0.970000	0.00500	1.0	0.990099	0.009950	0.009901	-0.009901
2	2	0.940000	0.00770	1.0	0.969649	0.020871	0.030351	-0.020450
3	3	0.920000	0.00940	1.0	0.944966	0.025785	0.055034	-0.024683
4	4	0.889494	0.01095	1.0	0.915366	0.031825	0.084634	-0.029600
5	5	0.860000	0.01250	1.0	0.880536	0.038793	0.119464	-0.034830
6	6	0.834626	0.01290	1.0	0.854360	0.030178	0.145640	-0.026176
7	7	0.810000	0.01330	1.0	0.827388	0.032079	0.172612	-0.026972

Survival Probability & Recovery Rate

In [11]:

```
# plot probabilities for various recovery rate
sp = pd.DataFrame()

for i in arange(0.1, 1, 0.1):
    sp[i] = get_survival_probability(df.Maturity,df.Df,df.WFC,i)['Survival']

sp.columns = around(sp.columns*100)
sp.iplot(title='Survival Probability for various Recovery Rate',
         xTitle='CDS Maturity',
         yTitle='Survival Probability',
         labels = sp.columns)
```



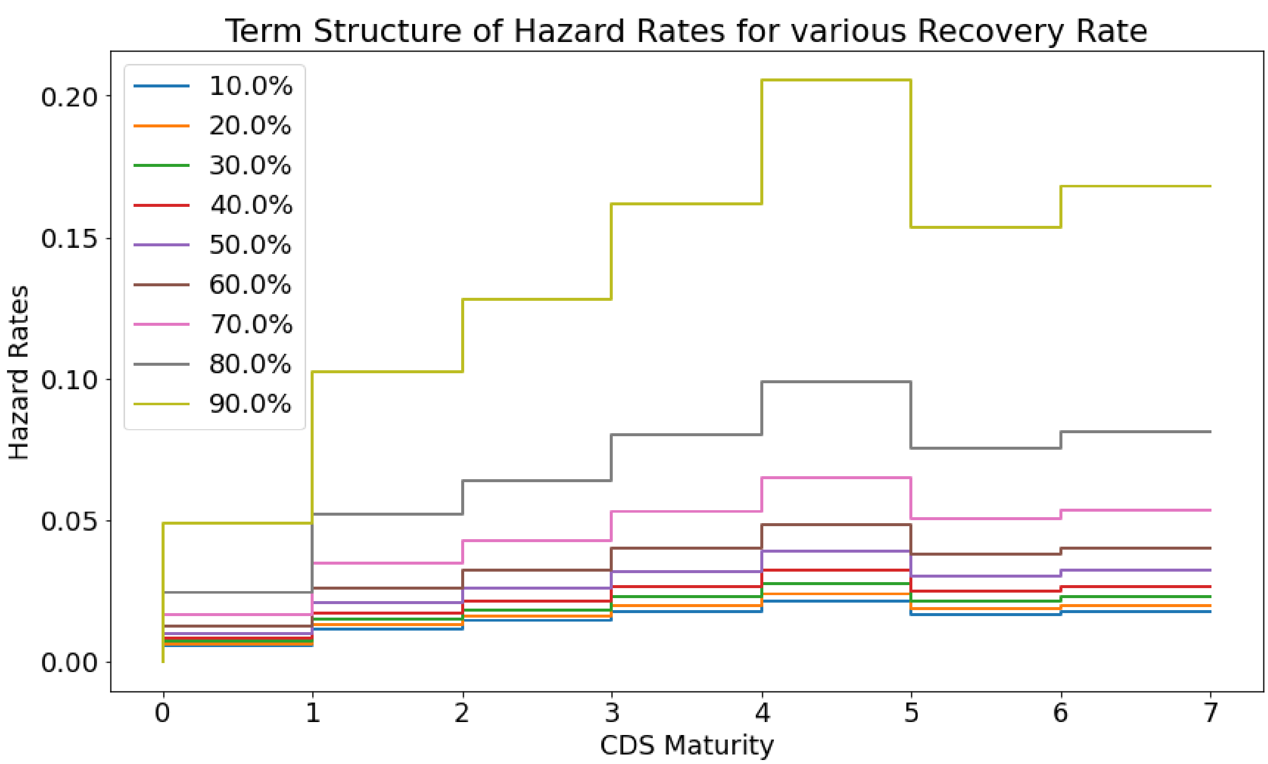
The low levels of recovery rates have almost no effect on the shape of the survival probability curve, while the high levels induce a skew. The relationship between credit spreads, hazard rates, and recovery rates is also approximated using the Credit Triangle CDS = λ * (1 − R) that applies over small timescale.

In [12]:
```
# surface plot
sp.iplot(kind='surface', title='Relationship of Survival Probability, Maturity &
```

Hazard Rate Visualization

Let's now plot the hazard rates for various recovery rates for WFC.

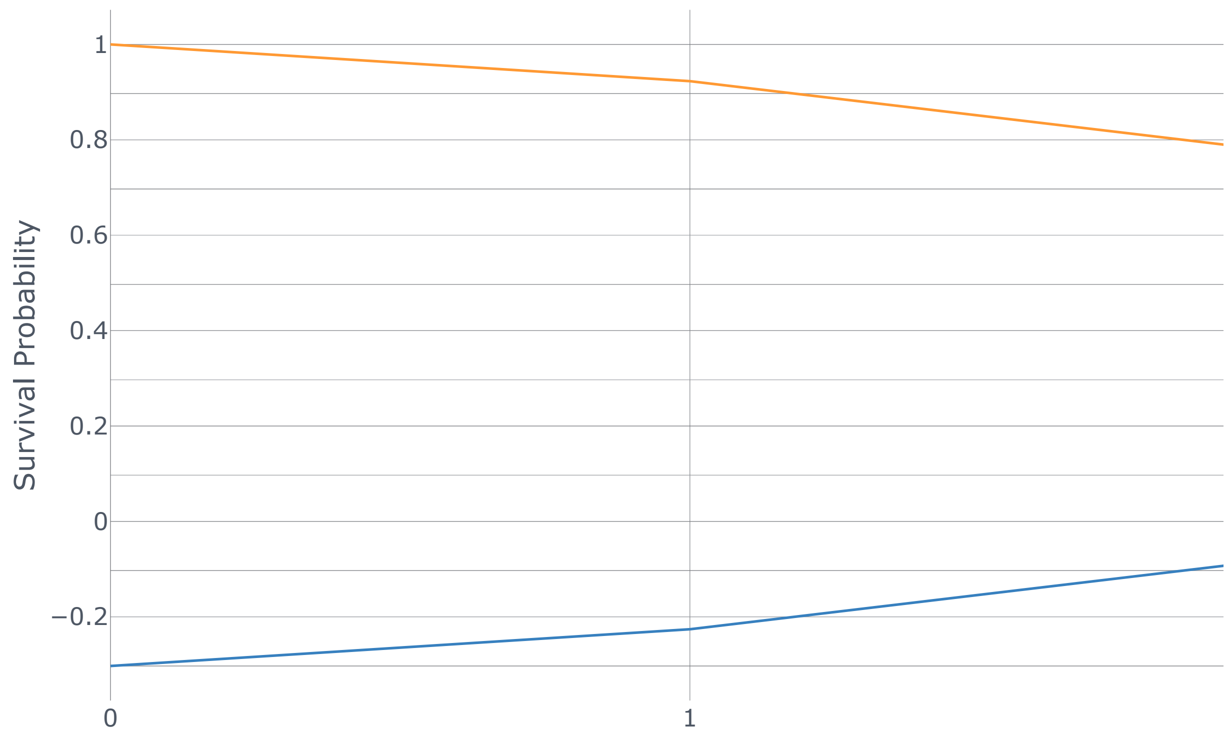
In [13]:
```
hz = pd.DataFrame()

for i in arange(0.1, 1, 0.1):
    hz[i] = get_survival_probability(df.Maturity,df.Df,df.WFC,i)['Hazard']
    plt.step(hz.index, hz[i], label =f'{i*100:0.4}%')

plt.title('Term Structure of Hazard Rates for various Recovery Rate')
plt.xlabel('CDS Maturity')
plt.ylabel('Hazard Rates')
plt.legend();
```



CCMO Observation

Let's now plot the survial and default probability of CCMO and study its readings.

In [14]:
```
CCMO = get_survival_probability(df.Maturity,df.Df,df.CCMO,0.10,plot_prob=True)
```



Cash Flows : Let's derive the expected present value of both the premium and default leg for 5Y. A negative implied survial probability highlight the occurrence of credit (default) event even before the end of five year. This is further substantiated from the expected PV plot below.

QuantLib Implementation

Given the maturity and market quoted spreads, we'll now derive the hazard rates and survival probability using QuantLib-Python and compare the result with the one derived above. For this exercise, we'll use the Trans-European Automated Real-time Gross settlement Express Transfer (TARGET) calendar. Further, we'll reprice the quoted CDS spreads employed for calibration and verify the outcome.

In [15]:
```
from QuantLib import *
```

In [16]:
```
# inputs
maturity = np.arange(6)
discountfactor = [0, 0.97, 0.94, 0.92, 0.89, 0.86]
cds_spread = [0, 50, 77, 94, 109.5, 125]
recovery = 0.40
```

In [17]:
```
sp = get_survival_probability(maturity,discountfactor,cds_spread,recovery)
sp
```

Out[17]:

	Maturity	Df	Spread	Dt	Survival	Hazard	Default	Marginal
0	0	0.00	0.00000	0.0	1.000000	0.000000	0.000000	0.000000
1	1	0.97	0.00500	1.0	0.991736	0.008299	0.008264	-0.008264
2	2	0.94	0.00770	1.0	0.974623	0.017406	0.025377	-0.017113
3	3	0.92	0.00940	1.0	0.953894	0.021498	0.046106	-0.020728
4	4	0.89	0.01095	1.0	0.928942	0.026506	0.071058	-0.024952
5	5	0.86	0.01250	1.0	0.899443	0.032271	0.100557	-0.029500

In [18]:
```
# set up
calendar = TARGET()

todaysDate = Date(14, December, 2020)
Settings.instance().evaluationDate = todaysDate

risk_free_rate = YieldTermStructureHandle(FlatForward(todaysDate, 0.0, Actual365
```

Hazard Rates

In [19]:
```
# CDS parameters
recovery_rate = 0.4
quoted_spreads = [0.005, 0.0077, 0.0094, 0.01095, 0.0125]
tenors = [Period(1, Years),
          Period(2, Years),
          Period(3, Years),
          Period(4, Years),
          Period(5, Years)]
maturities = [calendar.adjust(todaysDate + x, Following) for x in tenors]

instruments = [
    SpreadCdsHelper(
        QuoteHandle(SimpleQuote(s)),
        tenor,
        0,
        calendar,
        Annual,
        Following,
        DateGeneration.TwentiethIMM,
        Actual365Fixed(),
        recovery_rate,
        risk_free_rate,
    )
    for s, tenor in zip(quoted_spreads, tenors)
]

# calculate hazard rate
hazard_curve = PiecewiseFlatHazardRate(todaysDate, instruments, Actual365Fixed()
print("---"*10)
print("Calibrated hazard rate values: ")
print("---"*10)
for x in hazard_curve.nodes():
    print("hazard rate on %s is %.7f" % x)
```

```python
survival_probability = list(sp['Survival'])[1:]
temp = list(zip(maturities, survival_probability))

# calculate survival probability
print("---"*10)
print("Survival probability values: ")
print("---"*10)
counter = 1
for each in temp:
    print(f"{counter}Y survival probability:{hazard_curve.survivalProbability(ea
    counter = counter + 1
```

```
------------------------------
Calibrated hazard rate values:
------------------------------
hazard rate on December 14th, 2020 is 0.0083334
hazard rate on December 20th, 2021 is 0.0083334
hazard rate on December 20th, 2022 is 0.0174670
hazard rate on December 20th, 2023 is 0.0215308
hazard rate on December 20th, 2024 is 0.0263512
hazard rate on December 22nd, 2025 is 0.0318601
------------------------------
Survival probability values:
------------------------------
1Y survival probability:0.991701,
                expected 0.991736
2Y survival probability:0.974676,
                expected 0.974623
3Y survival probability:0.953978,
                expected 0.953894
4Y survival probability:0.929041,
                expected 0.928942
5Y survival probability:0.900041,
                expected 0.899443
```

Reprice Instruments

In [20]:

```python
# reprice instruments
nominal = 1000000.0
probability = DefaultProbabilityTermStructureHandle(hazard_curve)

# create a cds for every maturity:
all_cds = []
for maturity, s in zip(maturities, quoted_spreads):
    schedule = Schedule(
        todaysDate,
        maturity,
        Period(Annual),
        calendar,
        Following,
        Unadjusted,
        DateGeneration.TwentiethIMM,
        False,
    )
    cds = CreditDefaultSwap(Protection.Seller, nominal, s, schedule, Following,
    engine = MidPointCdsEngine(probability, recovery_rate, risk_free_rate)
    cds.setPricingEngine(engine)
    all_cds.append(cds)

print("---"*20)
print("Repricing of quoted CDS spreads employed for calibration: ")
print("---"*20)
```

```
for cds, tenor in zip(all_cds, tenors):
    print(f"{tenor} Fair spread : {cds.fairSpread():.6}")
    print(f"   NPV         : {cds.NPV():.6}")
    print(f"   default leg : {cds.defaultLegNPV():.6}")
    print(f"   coupon leg  : {cds.couponLegNPV():.6}")
    print("")
```

```
------------------------------------------------------------
Repricing of quoted CDS spreads employed for calibration:
------------------------------------------------------------
1Y Fair spread : 0.005
   NPV         : 1.39416e-08
   default leg : -5060.76
   coupon leg  : 5060.76

2Y Fair spread : 0.0077
   NPV         : 5.45697e-12
   default leg : -15362.3
   coupon leg  : 15362.3

3Y Fair spread : 0.0094
   NPV         : -4.36557e-11
   default leg : -27815.5
   coupon leg  : 27815.5

4Y Fair spread : 0.01095
   NPV         : 2.91038e-11
   default leg : -42736.6
   coupon leg  : 42736.6

5Y Fair spread : 0.0125
   NPV         : -4.29281e-10
   default leg : -60211.2
   coupon leg  : 60211.2
```

References

- CDS Market Quotes https://www.cnbc.com/sovereign-credit-default-swaps/
- QuantLib-SWIG https://github.com/lballabio
- Cufflinks documentation https://github.com/santosjorge/cufflinks and https://plotly.com/python/cufflinks/

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