SEQUENCES AND SERIES

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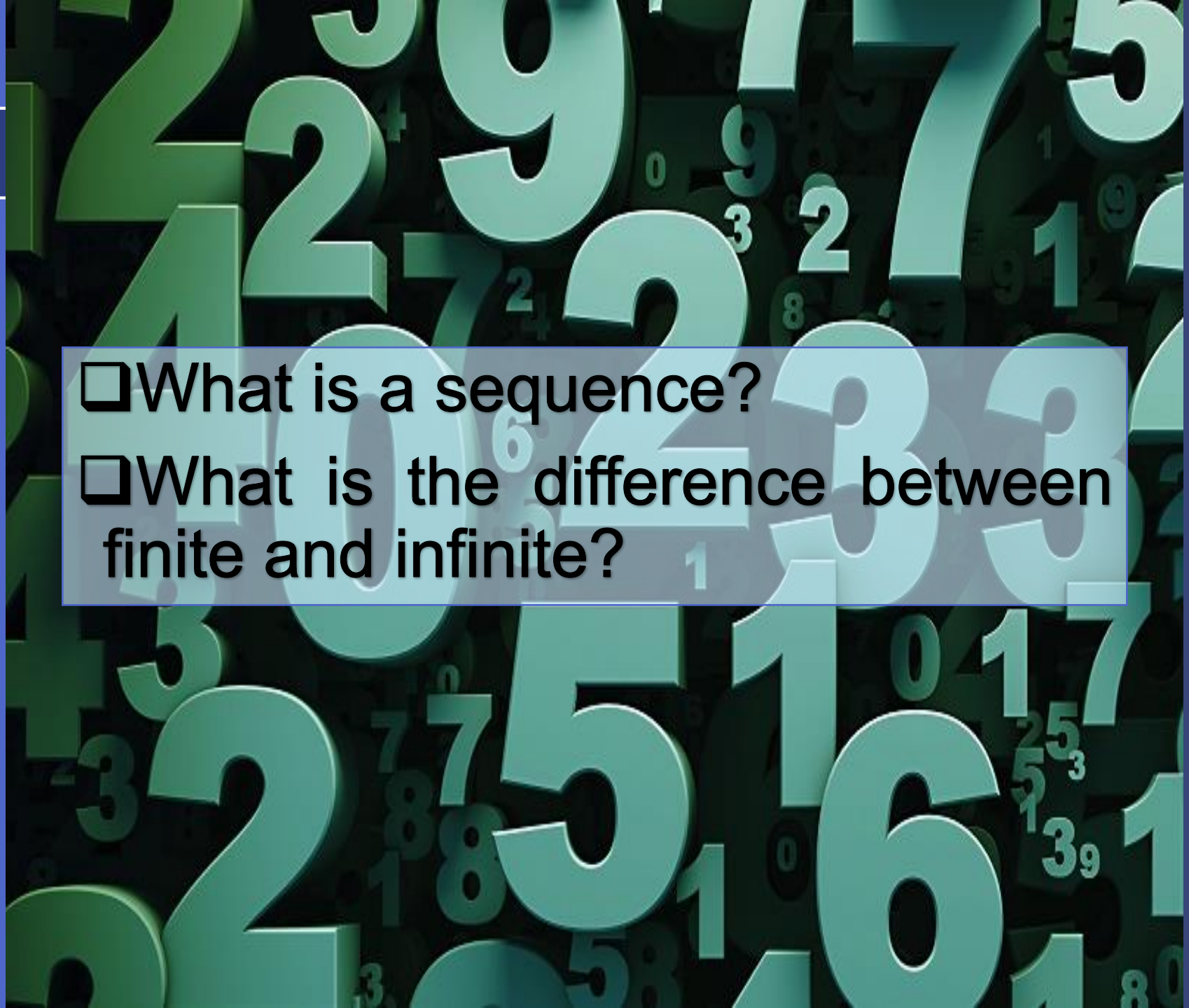

- What is a sequence?
- What is the difference between finite and infinite?

Sequence:

- A function whose domain is a set of consecutive integers (list of ordered numbers separated by commas).
- Each number in the list is called a ***term***.
- For Example:

Sequence 1

2,4,6,8,10

Term 1, 2, 3, 4, 5

Sequence 2

2,4,6,8,10,…

Term 1, 2, 3, 4, 5

Sequence:

- Unlike a set, order matters, and exactly the same elements can appear multiple times at different positions in the sequence.

- Most precisely, a sequence can be defined as a function whose domain is a countable totally ordered set such as the natural numbers.

Sequence:

Domain – relative position of each term (1,2,3,4,5) Usually begins with position 1 unless otherwise stated.

Range – the actual "terms" of the sequence (2,4,6,8,10)

Infinite Sequence:

- An *infinite sequence* is a function with domain the set of natural numbers $N = \{1,2,3, \ldots \ldots \ldots\}$.
- For example, consider the function "a" defined by

$$a(n) = n^2 \qquad (n = 1,2,3, \ldots \ldots)$$

Instead of the usual functional notation $a(n)$, for sequences we usually write

$$a_n = n^2$$

Infinite Sequence:

- That is, a letter with a subscript, such as a_n, is used to represent numbers in the range of a sequence. For the sequence defined by $a_n = n^2$,

$$a_1 = 1^2 = 1$$
$$a_2 = 2^2 = 4$$
$$a_3 = 3^2 = 9$$
$$a_4 = 4^2 = 16$$

GENERAL or nth Term

- A sequence is frequently defined by giving its range. The sequence on the given example can be written as

$$1, 4, 9, 16, \ldots\ldots\ldots, n^2, \ldots\ldots$$

Each number in the range of a sequence is a **term** of the sequence, with a_n the **nth term** or **general term** of the sequence.

The formula for the nth term generates the terms of a sequence by repeated substitution of counting numbers for n.

Finite Sequence:

- A *finite sequence* with m terms is a function with domain the set of natural numbers $\{1, 2, 3, \ldots \ldots, m\}$
- For example, $2, 4, 6, 8, 10$ is a finite sequence with 5 terms where

$$a_n = 2n, \text{ for } n = 1,2,3,4,5.$$

- In contrast, $2,4,6,8,10, \ldots \ldots$ is an infinite sequence where

$$a_n = 2n, \text{ for } n = 1,2,3,4,5 \ \ldots \ldots$$

Write the first six terms of $f(n) = (-3)^{n-1}$.

$f(1) = (-3)^{1-1} = 1$ — 1st term

$f(2) = (-3)^{2-1} = -3$ — 2nd term

$f(3) = (-3)^{3-1} = 9$ — 3rd term

$f(4) = (-3)^{4-1} = -27$ — 4th term

$f(5) = (-3)^{5-1} = 81$ — 5th term

$f(6) = (-3)^{6-1} = -243$ — 6th term

You are just substituting numbers into the equation to get your term.

Sample Problem

1.) Write the first five terms of the infinite sequence with general term $a_n = 2n - 1$.

Answer:

$$a_1 = 2(1) - 1 = 1$$
$$a_2 = 2(2) - 1 = 3$$
$$a_3 = 2(3) - 1 = 5$$
$$a_4 = 2(4) - 1 = 7$$
$$a_5 = 2(5) - 1 = 9$$

Thus, the first five terms are $1, 3, 5, 7, 9$ and the sequence is

$$1, 3, 5, 7, 9, \ldots \ldots 2n - 1, \ldots \ldots$$

Sample Problem

2.) A finite sequence has four terms, and the formula for the nth term is $x_n = (-1)^n \left(\frac{1}{2^{n-1}}\right)$. What is the sequence?

Answer:

$$x_1 = (-1)^1 \left(\frac{1}{2^{1-1}}\right) = -1$$

$$x_2 = (-1)^2 \left(\frac{1}{2^{2-1}}\right) = \frac{1}{2}$$

$$x_3 = (-1)^3 \left(\frac{1}{2^{3-1}}\right) = -\frac{1}{4}$$

$$x_4 = (-1)^4 \left(\frac{1}{2^{4-1}}\right) = \frac{1}{8}$$

Thus, the sequence is

$$-1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}$$

Exercise

Write the first five terms of the sequence.

1.) $a_n = 8 + 13n$

2.) $a_n = \frac{1}{n+1}$

3.) $a_n = 2^n + n$

Assignment

Research about the three types of sequence. Then, give 2 examples each.

- Arithmetic Sequence
- Geometric Sequence
- Harmonic Sequence

Use intermediate pad or yellow pad.

ARITHMETIC SEQUENCE

ARITHMETIC Sequence

- A sequence in which every term is created by adding or subtracting a definite number to the preceding number is an arithmetic sequence.

$$4, 9, 14, 19, 24, \ldots$$

arithmetic sequence

$$9 - 4 = 5$$
$$14 - 9 = 5$$
$$19 - 14 = 5$$
$$24 - 19 = 5$$

The **common difference**, d, is 5.

ARITHMETIC Sequence

› Formula:

$$a_n = a_1 + (n-1)d$$

› Where:

$a_1 =$ for the first term

$a_n = for\ the\ nth\ term$

$d = common\ difference$

$n = for\ the\ number\ of\ term\ from\ a_1 to\ a_n$

Sample Problem

1.) Find the first five terms of the sequence and determine if it is arithmetic.

$$a_n = 1 + (n - 1)4$$

$$a_1 = 1 + (1 - 1)4 = 1 + 0 = 1$$
$$a_2 = 1 + (2 - 1)4 = 1 + 4 = 5$$
$$a_3 = 1 + (3 - 1)4 = 1 + 8 = 9$$
$$a_4 = 1 + (4 - 1)4 = 1 + 12 = 13$$
$$a_5 = 1 + (5 - 1)4 = 1 + 16 = 17$$

$$\boldsymbol{d = 4}$$

This is an arithmetic sequence.

Sample Problem

2.) Find the 5th and 11th terms of the arithmetic sequence with the first term 3 and the common difference 4.

Answer:

$$a_1 = 3, \qquad d = 4$$
$$a_n = a_1 + (n-1)d$$

$$a_5 = 3 + (5-1)4$$
$$a_5 = 3 + 16$$
$$a_5 = 19$$

$$a_{11} = 3 + (11-1)4$$
$$a_{11} = 3 + 40$$
$$a_{11} = 43$$

Therefore, 19 and 43 are the 5th and the 11th terms of the sequence, respectively.

Exercise

Give the common difference and find the indicated term in each arithmetic sequence.

1.) $1, 5, 9, 13, \ldots (a_{10})$

2.) $13, 9, 5, 1, \ldots (a_{10})$

3.) $-8, -5, -2, 1, 4, \ldots (a_{12})$

4.) $5, 9, 13, 17, \ldots (a_{15})$

5.) $2, 6, 10, \ldots (a_{6})$

What is the nth term for each sequence below?

1.) $1, 5, 9, 13, \ldots$

2.) $13, 9, 5, 1, \ldots$

3.) $-7, -4, -1, 2, \ldots$

4.) $5, 3, 1, -1, -3$

5.) $2, 6, 10, \ldots$

ARITHMETIC Sequence

The **nth term** of an arithmetic sequence has the form

$$a_n = dn + c$$

where d is the common difference and

$$c = a_1 - d$$

$$2,8,14,20,26, \ldots$$

The nth term is $6n - 4$.

What is the nth term for each sequence below?

1.) $1, 5, 9, 13, \ldots$

2.) $13, 9, 5, 1, \ldots$

3.) $-7, -4, -1, 2, \ldots$

4.) $5, 3, 1, -1, -3$

5.) $2, 6, 10, \ldots$

Sample Problem

1.) Find the formula for the nth term of an arithmetic sequence whose common difference is 4 and whose first term is 15. Find the first five terms of the sequence.

$$a_n = dn + c$$

The first five terms are

15,19,23,27,31.

Exercise

1.) Find the formula for the nth term of an arithmetic sequence whose common difference is 3 and whose first term is 5. Find the first five terms of the sequence..

2.) The first term of an arithmetic sequence is equal to 6 and the common difference is equal to 3. Find a formula for the nth term of an arithmetic sequence.

3.) Find the formula for the nth term of an arithmetic sequence whose common difference is -18 and whose first term is 7. Find the first five terms of the sequence.